thornado-Hydro: A Discontinuous Galerkin Method for General Relativistic Hydrodynamics with an Eye towards Simulating Core-Collapse Supernovae

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Background/Motivation

- Stars with mass $\gtrsim 10\,\mathrm{M}_\odot$ explode as core-collapse supernovae, which:
  - help drive galactic chemical evolution
  - produce neutron stars
  - generate gravitational waves
  - are unique laboratories for fundamental neutrino physics
- Exactly how they explode is still an open question
- We want to understand the explosion mechanism and the observables core-collapse supernovae produce
  - To that end, we are developing solvers for gravity, hydrodynamics, and neutrino transport in the **`thornado`** framework

Relevant Equations

Evolution of spacetime

$$G^{\mu\nu} = 8\pi\left(T^{\mu\nu}_{\mathrm{Fluid}} + T^{\mu\nu}_{\mathrm{Neutrinos}}\right)$$

Conservation of rest-mass

$$\nabla_\nu J^\nu_{\mathrm{B}} = 0$$

Evolution of electron number

$$\nabla_\nu J^\nu_e = -m_{\mathrm{B}}\, L\left(\boldsymbol{U}, \boldsymbol{M}\right)$$

Conservation of energy/momentum

$$\nabla_\nu T^{\mu\nu}_{\mathrm{Fluid}} = -S^\mu\left(\boldsymbol{U}, \boldsymbol{M}\right)$$

$$\nabla_\nu T^{\mu\nu}_{\mathrm{Neutrinos}} = +S^\mu\left(\boldsymbol{U}, \boldsymbol{M}\right)$$

Fluid

Neutrino angular moments

Summary/Conclusions

- DG methods are well-suited for core-collapse supernova simulations
- **`thornado`** can evolve the fluid with dynamical spacetimes and tabulated nuclear equations of state
- **`thornado`** can evolve neutrinos in special relativity

Future Work

- Develop a more sophisticated bound-preserving limiter
- Evolve neutrino radiation field with GR (CFA)
- Couple neutrino transport to hydrodynamics and gravity
- Incorporate magnetic fields
- Core-collapse supernova simulations!

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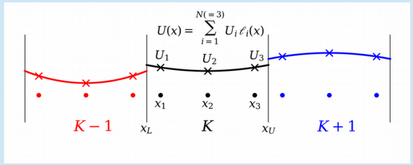
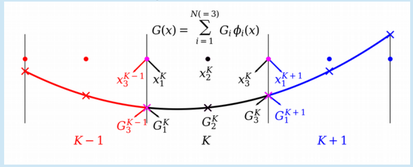
`thornado`

- toolkit for high-order neutrino-radiation hydrodynamics (github.com/endeve/thornado)
- **Conformally-flat approximation** (gravity, fluid, neutrinos)
- Discontinuous Galerkin (hyperbolic conservation laws)
  - Fluid and neutrinos
    - Discretize spatial domain
    - Approximate local solution with polynomial
      - Linear combination of Gauss-Legendre nodal values with Lagrange polynomials
      - **Solutions discontinuous at element interfaces**
    - Resolve interface discontinuities with approximate Riemann solver



- Continuous Galerkin/Spectral (elliptic equations)
  - Gravity
    - Discretize spatial domain
      - Finite element in radius
      - Spherical harmonic decomposition in angle
    - Approximate local solution with polynomial
      - Linear combination of Legendre-Gauss-Lobatto nodal values with modified Lagrange polynomials
      - **Solutions continuous at element interfaces (gravity assumed smooth)**



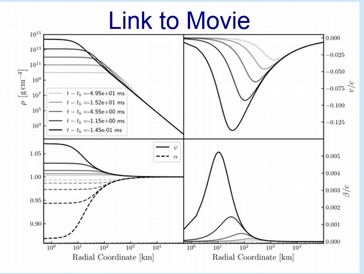
References

[1] Yahil, A.,1983, ApJ, **265** 1047
[2] Endeve et al., 2019, J. Phys. Conf. Ser., **1225** 012014
[3] Woosley & Heger, 2007, Phys. Reports, **442** 269
[4] Steiner et al., 2013, ApJ, **774** 17
[5] Pochik et al., 2021, ApJS, **253** 21
[6] Just et al, 2015, MNRAS, **453** 3386

Results (Hydro + Gravity)

- Self-similar collapse of polytrope [1], with GR-CFA
  - Dynamical spacetime
  - **Evolution of fluid remains self-similar**
  - Results are similar to those in [2]

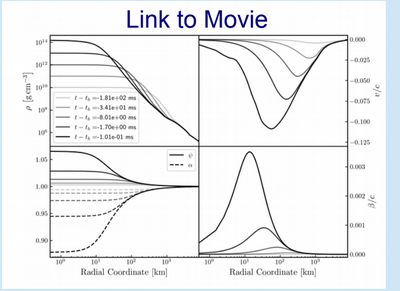
Link to Movie



Results (Hydro + Gravity + EOS)

- Adiabatic collapse of inner 8000 km of realistic $15\,M_\odot$ progenitor [3]
  - **Uses tabulated nuclear equation of state** [4] as implemented in weaklib (github.com/starkiller-astro/weaklib)
  - Entropy and electron fraction simply advect inwards, expected for adiabatic collapse
  - Compare to Newtonian results in [5] (also see David Pochik's poster: SP01.00028)

Link to Movie



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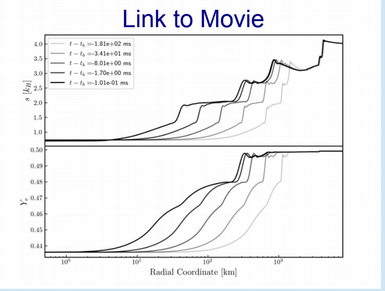

Results (Neutrino Transport)

- Streaming doppler shift [6]
  - Neutrinos traversing background of spatially-varying velocity
  - Plotted is neutrino number density as function of energy measured by comoving observer at maximum velocity
  - Numerical results agree well with analytic results
  - **Special relativistic**