thornado-Hydro (xCFC)

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thornado

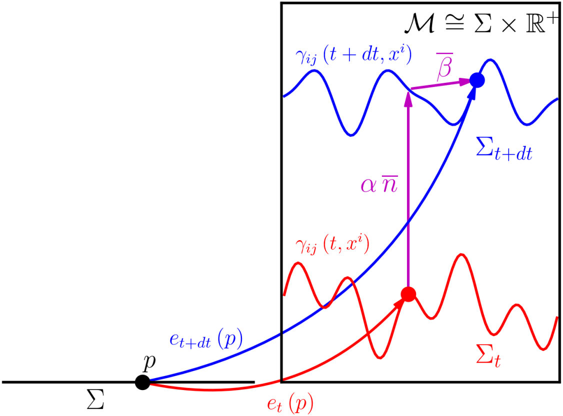
toolkit for high-order neutrino-radiation hydrodynamics

- DG
- SSPRK/IMEX
- GR (xCFC)
- Hydro[a] (Valencia)
- Neutrino transport[b] (M1)
- Interfaces to tabulated EoS/Opacities (weaklib: `https://github.com/starkiller-astro/weaklib`)
- Fluid self-gravity via Poseidon: `https://github.com/jrober50/Poseidon`
- GPUs via OpenACC or OpenMP pragmas
- MPI parallelism and AMR via AMReX: `https://github.com/AMReX-Codes/amrex`

[a] Endeve et al. (2019); Dunham et al. (2020); Pochik et al. (2021)

[b] Laiu et al. (2021)

3+1 Decomposition



$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu = -\alpha^2\,dt^2 + \gamma_{ij}\left(dx^i + \beta^i\,dt\right)\left(dx^j + \beta^j\,dt\right)$$

Conformally-Flat Condition

Developed by Wilson et al. (1996), extended by Cordero-Carrión et al. (2009)

$$\gamma_{ij}\left(x\right) = \psi^{4}\left(x\right)\,\overline{\gamma}_{ij}\left(x^{i}\right)$$
$$K = 0,\ \partial_{t} K = 0$$
$$\text{(Always and everywhere)}$$

- Exact in spherical symmetry!
- Hyperbolic $\rightarrow$ Elliptic equations
- Good for long-time simulations

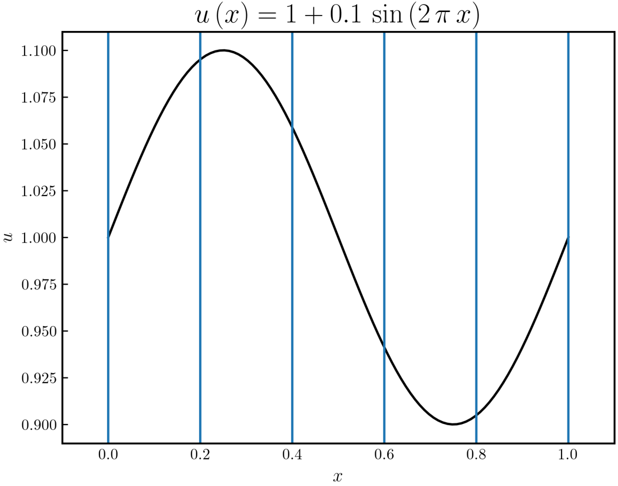
Special case: Schwarzchild spacetime in isotropic coordinates ($G = c = 1$)

$$\alpha = \left(1 + \frac{1}{2}\,\Phi\right)\left(1 - \frac{1}{2}\,\Phi\right)^{-1}$$
$$\psi = 1 - \frac{1}{2}\,\Phi$$
$$\beta^{i} = 0,$$

with

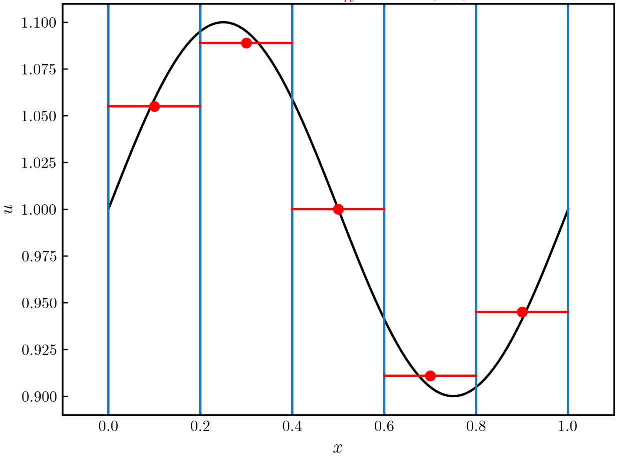
$$\Phi\left(r\right) := -\frac{M}{r}$$

Discontinuous Galerkin (DG) Method

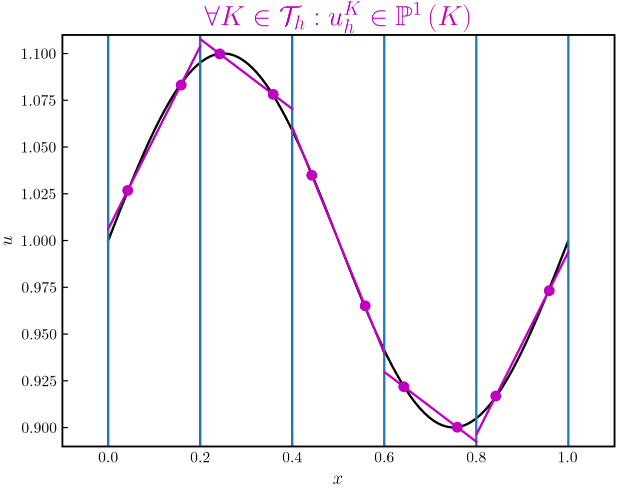


Finite-volume/DG(0)

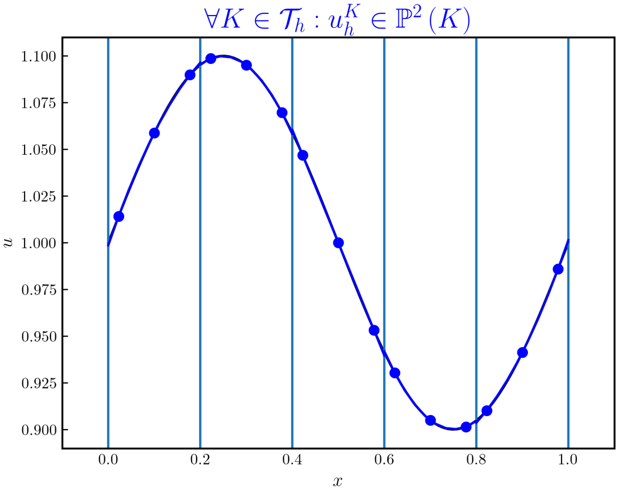
$$\forall K \in \mathcal{T}_h : u_h^K \in \mathbb{P}^0\left(K\right)$$



DG(1)

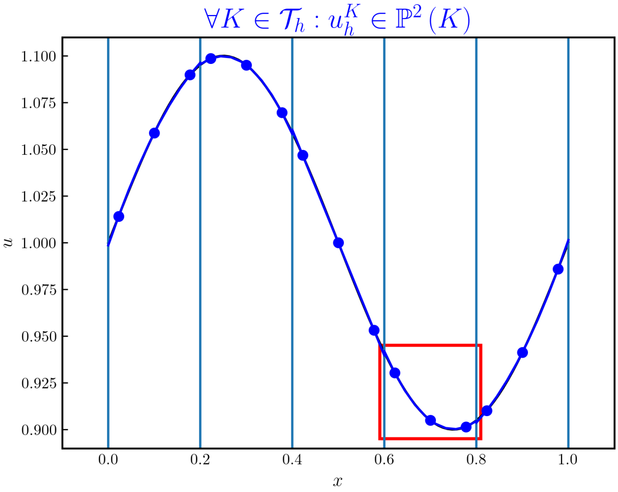

$\forall K \in \mathcal{T}_h : u_h^K \in \mathbb{P}^1(K)$
1.100
1.075
1.050
1.025
1.000
0.975
0.950
0.925
0.900
u
0.0
0.2
0.4
0.6
0.8
1.0
x

DG(2)



$\forall K \in \mathcal{T}_h : u_h^K \in \mathbb{P}^2(K)$
1.100
1.075
1.050
1.025
1.000
0.975
0.950
0.925
0.900
u
0.0
0.2
0.4
0.6
0.8
1.0
x

DG(2)


$\forall K \in \mathcal{T}_h : u_h^K \in \mathbb{P}^2(K)$
1.100
1.075
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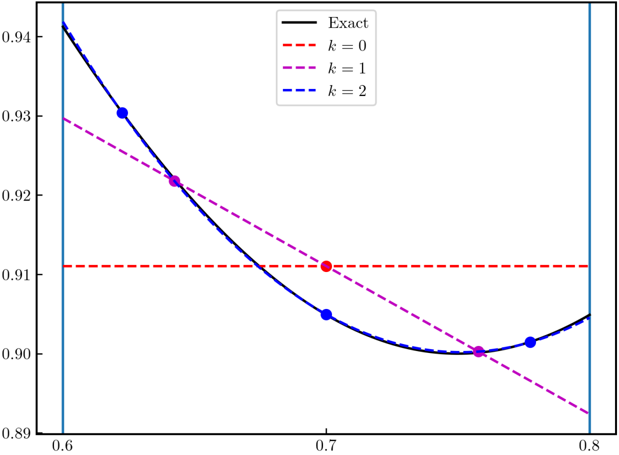
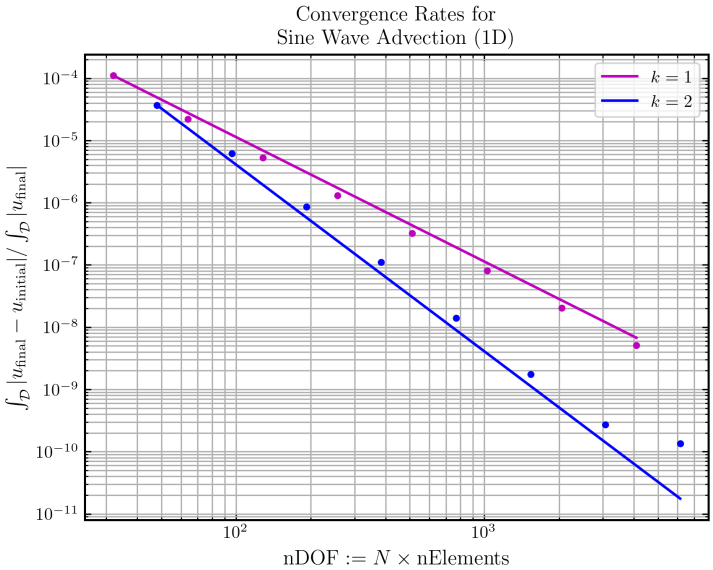
1.025
1.000
0.975
0.950
0.925
0.900
u
0.0
0.2
0.4
0.6
0.8
1.0
x

Discontinuous Galerkin (DG) Method

$$u_h\left(x,t\right) := \sum_{i=1}^{k+1} u_i\left(t\right)\ \ell_i\left(x\right)$$

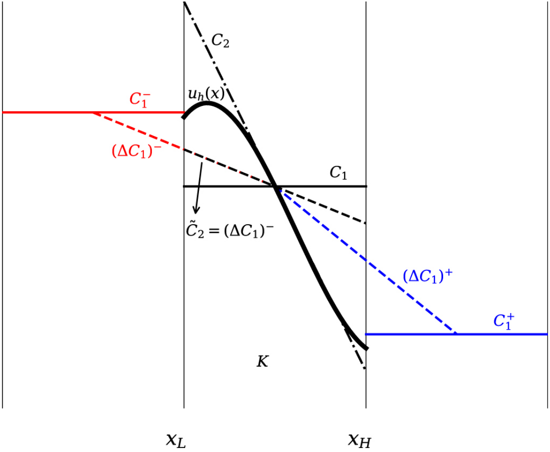




Convergence Rates for
Sine Wave Advection (1D)
$k = 1$
$k = 2$
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{2}
10^{3}
$\int_{\mathcal{D}} |u_{\mathrm{final}} - u_{\mathrm{initial}}| / \int_{\mathcal{D}} |u_{\mathrm{final}}|$
$\mathrm{nDOF} := N \times \mathrm{nElements}$

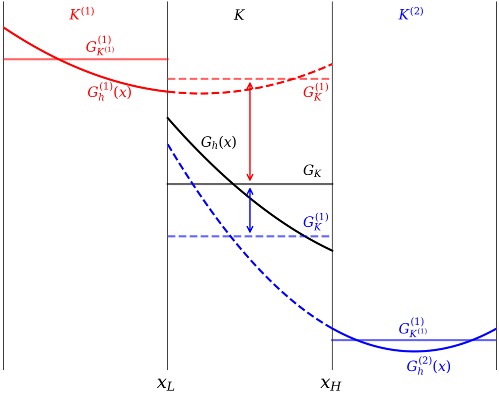
Slope Limiter (MinMod)

$$u_h\,(x,t) = \sum_{n=1}^{N} C_n\,(t)\;P_n\,(x) \implies \tilde{u}_h\,(x,t) = C_1\,(t)\;P_1\,(x) + \tilde{C}_2\,(t)\;P_2\,(x)$$



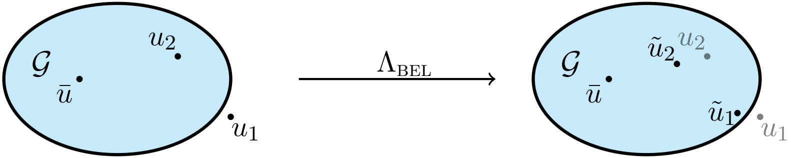
Troubled Cell Indicator

Proposed in Fu and Shu (2017)



Bound-Enforcing Limiter (Λ_{BEL})

- Proposed in Qin et al. (2016)
- Define $\mathcal{G}$ as the (convex) set of physically-admissible states when using an ideal EoS (positivity density and energy density, subluminal fluid velocity)

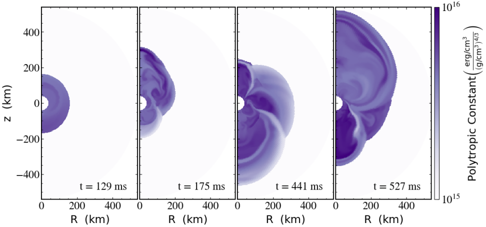


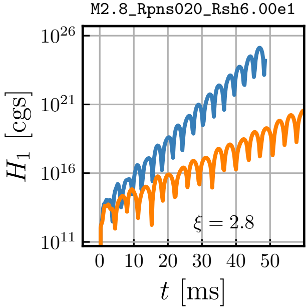
$u_h = u_1\,\ell_1 + u_2\,\ell_2$

$\tilde{u}_h := \Lambda_{\text{BEL}}\left(u_h\right) = \tilde{u}_1\,\ell_1 + \tilde{u}_2\,\ell_2$

Standing Accretion Shock Instability

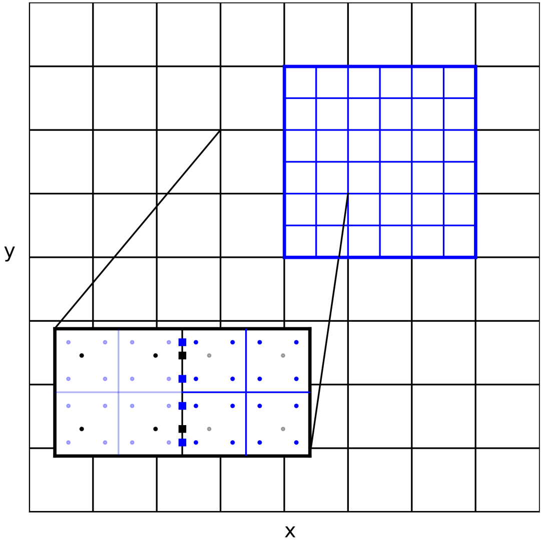
Used `thornado` to investigate the role of GR on the SASI[1]



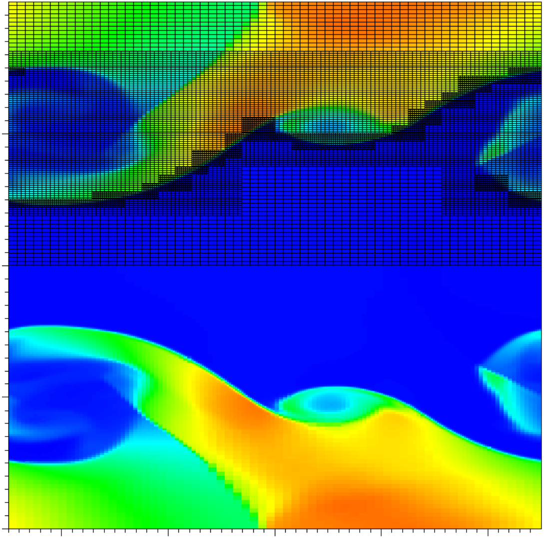


[1] Dunham et al. (2020, 2024)

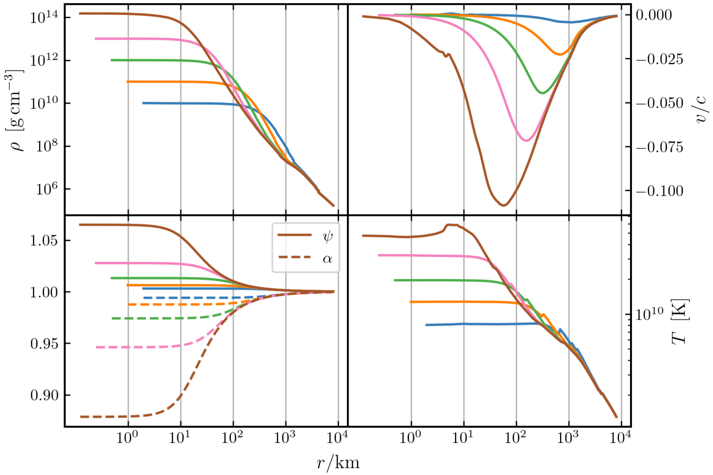
Mesh Refinement with AMReX



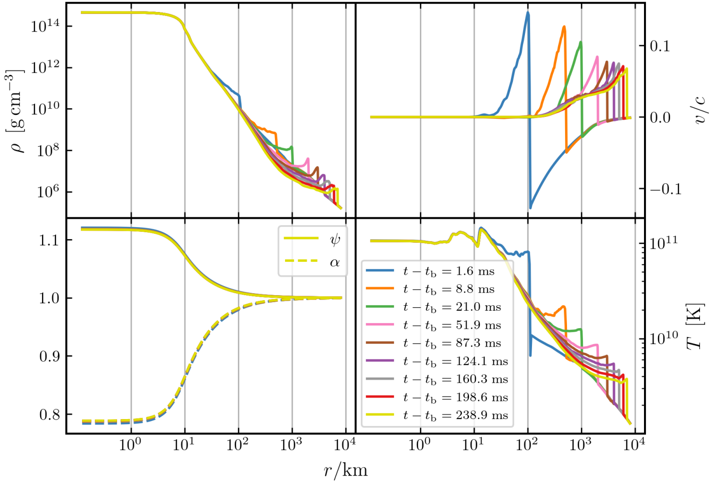
Kelvin–Helmholtz Instability



Adiabatic Collapse (Collapse Phase)



Adiabatic Collapse (Post-Bounce Phase)



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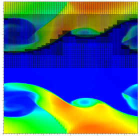
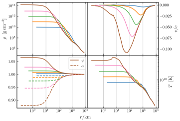
Summary

Can run multi-D pure hydro problems in GR with AMR





Can run 1D hydro+self-gravity problems in GR with AMR

Working on coupling GR transport to existing hydro+gravity modules