`thornado`-Hydro (xCFC)

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thornado

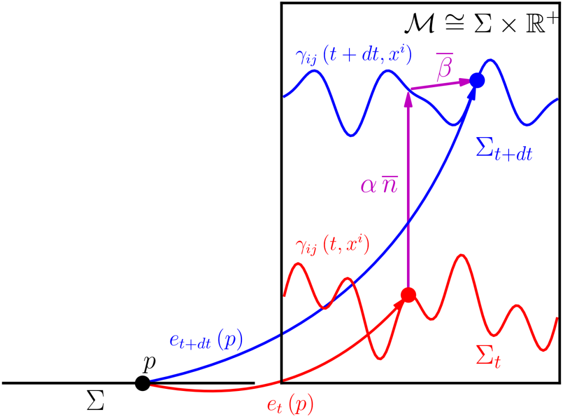
toolkit for high-order neutrino-radiation hydrodynamics

- DG
- SSPRK/IMEX
- GR (xCFC)
- Hydro[a] (Valencia)
- Neutrino transport[b] (M1)
- Interfaces to tabulated EoS/Opacities (weaklib: `https://github.com/starkiller-astro/weaklib`)
- Fluid self-gravity via Poseidon: `https://github.com/jrober50/Poseidon`
- GPUs via OpenACC or OpenMP pragmas
- MPI parallelism and AMR via AMReX: `https://github.com/AMReX-Codes/amrex`

[a] Endeve et al. (2019); Dunham et al. (2020); Pochik et al. (2021)

[b] Laiu et al. (2021)

3+1 Decomposition



$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu = -\alpha^2\,dt^2 + \gamma_{ij}\left(dx^i + \beta^i\,dt\right)\left(dx^j + \beta^j\,dt\right)$$

Conformally-Flat Condition

Developed by Wilson et al. (1996), extended by Cordero-Carrión et al. (2009)

$$\gamma_{ij}\left(x\right) = \psi^4\left(x\right)\,\overline{\gamma}_{ij}\left(x^i\right)$$
$$K = 0,\ \partial_t K = 0$$
$$\text{(Always and everywhere)}$$

- Exact in spherical symmetry!
- Hyperbolic $\rightarrow$ Elliptic equations
- Good for long-time simulations

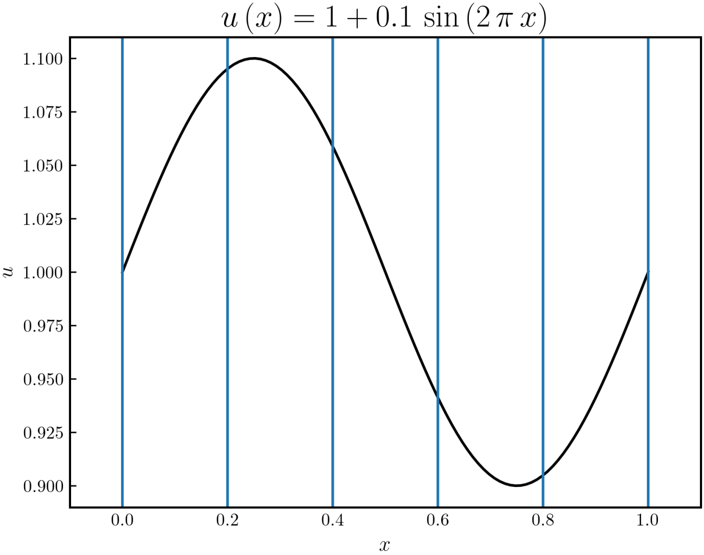
Special case: Schwarzchild spacetime in isotropic coordinates ($G = c = 1$)

$$\alpha = \left(1 + \frac{1}{2}\,\Phi\right)\left(1 - \frac{1}{2}\,\Phi\right)^{-1}$$
$$\psi = 1 - \frac{1}{2}\,\Phi$$
$$\beta^i = 0,$$

with

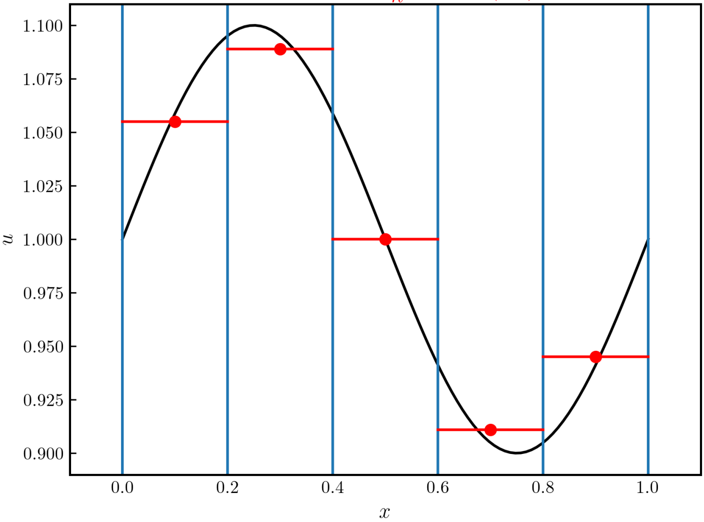
$$\Phi\left(r\right) := -\frac{M}{r}$$

Discontinuous Galerkin (DG) Method

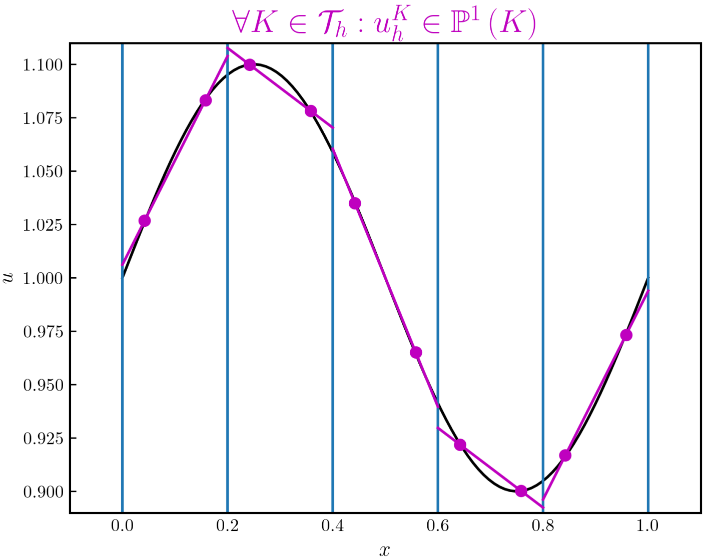


Discontinuous Galerkin (DG) Method

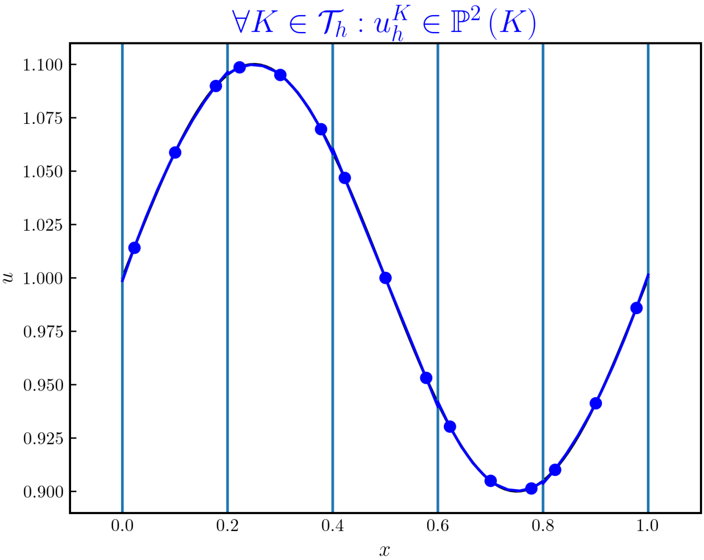
$$\forall K \in \mathcal{T}_h : u_h^K \in \mathbb{P}^0\left(K\right)$$



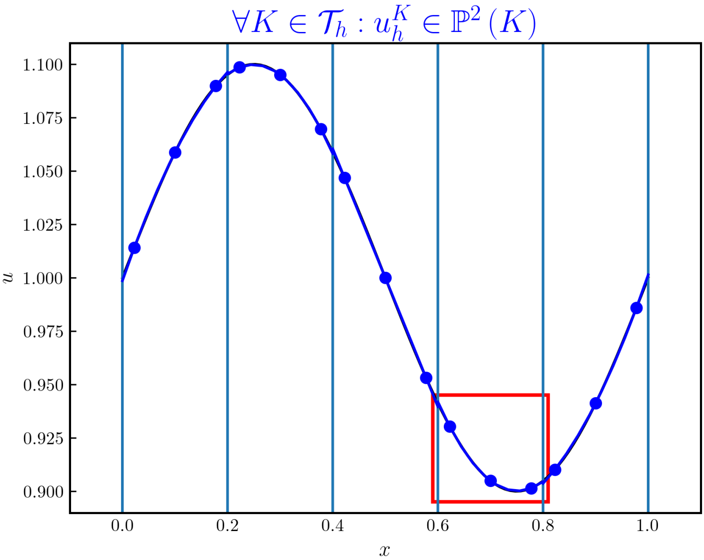
Discontinuous Galerkin (DG) Method



Discontinuous Galerkin (DG) Method

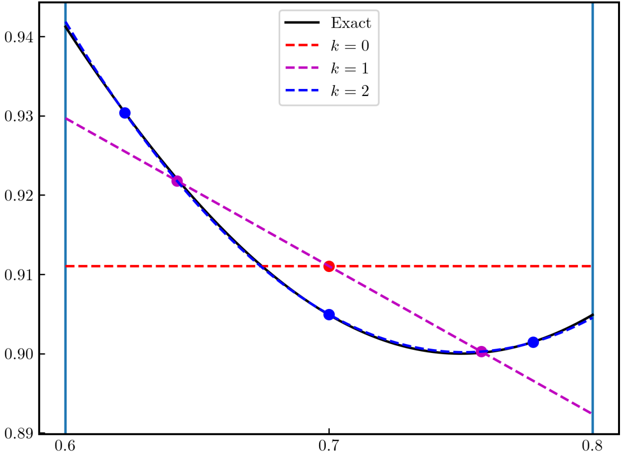


Discontinuous Galerkin (DG) Method



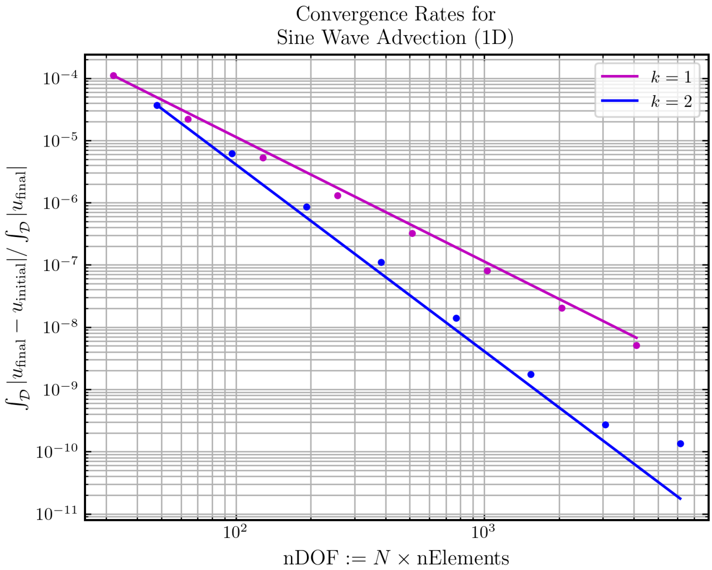
Discontinuous Galerkin (DG) Method

$$u_h(x,t) := \sum_{i=1}^{k+1} u_i(t)\ \ell_i(x)$$

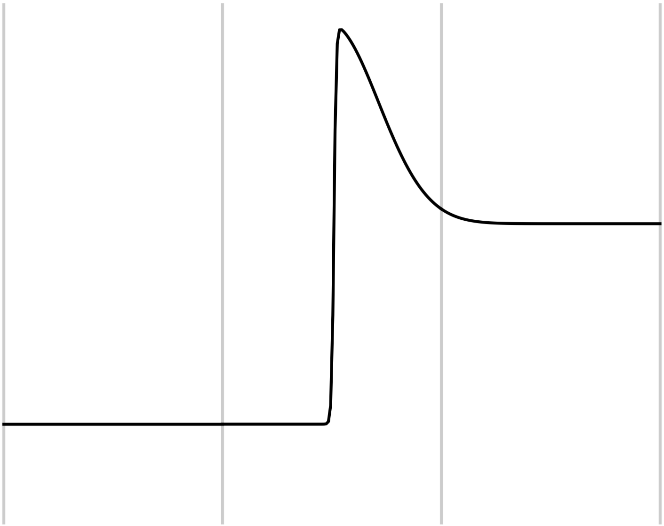


Time Stepping

- For hydro-only, strong-stabililty-preserving Runge–Kutta (SSPRK) methods (e.g., see Gottlieb et al. (2001))
    - Convex combinations of forward-Euler timesteps
- For hydro+neutrinos, ImEx methods (e.g., see Pareschi and Russo (2005))

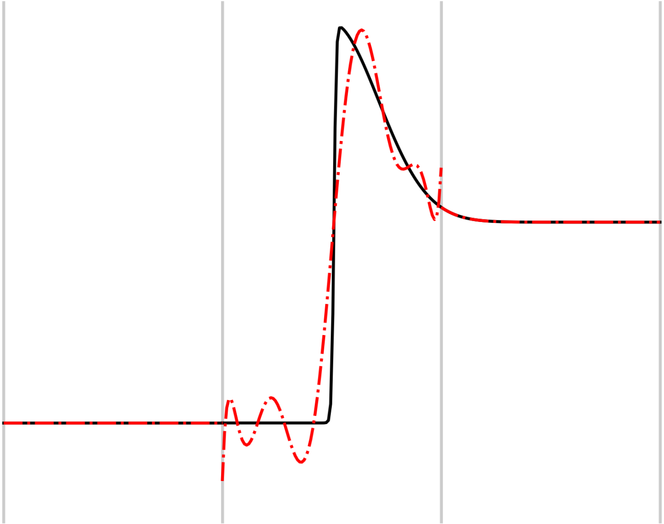

Convergence Rates for
Sine Wave Advection (1D)
$k = 1$
$k = 2$
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{2}
10^{3}
$\int_{\mathcal{D}} |u_{\text{final}} - u_{\text{initial}}| / \int_{\mathcal{D}} |u_{\text{final}}|$
nDOF $:= N \times$ nElements

Slope Limiter (MinMod)

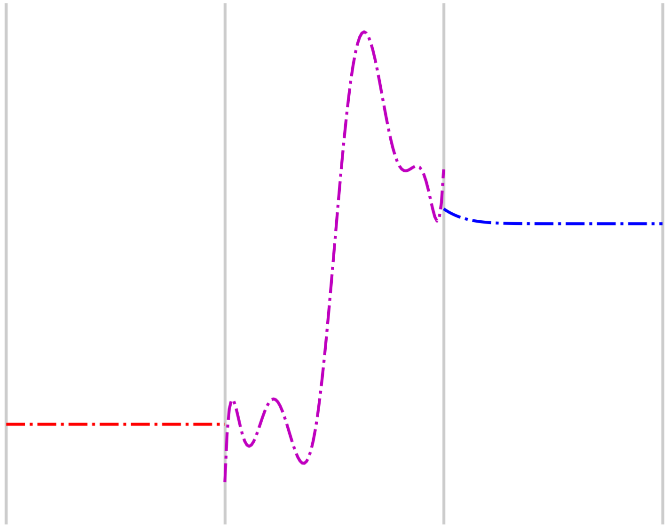


Slope Limiter (MinMod)

$k = 9$

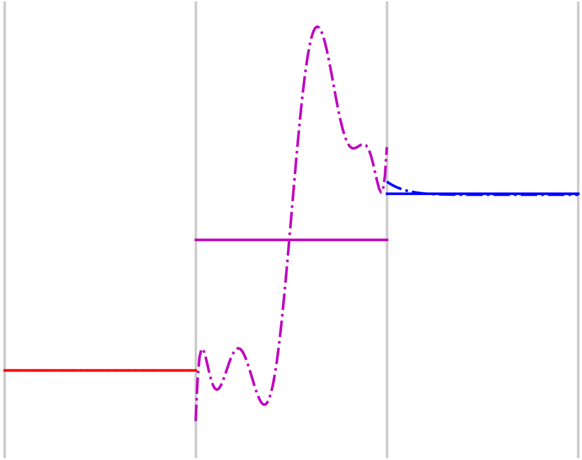


Slope Limiter (MinMod)



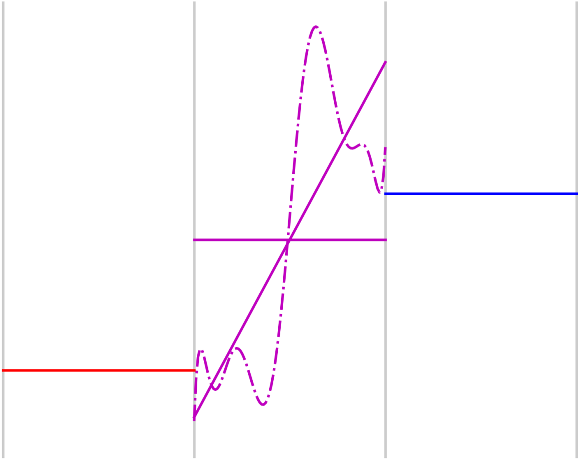
Slope Limiter (MinMod)

$$\bar{u}(t) := \frac{1}{V(t)} \int_K u_h(x,t)\, \sqrt{\gamma}(x,t)\, dx, \;\; V(t) := \int_K \sqrt{\gamma}(x,t)\, dx$$



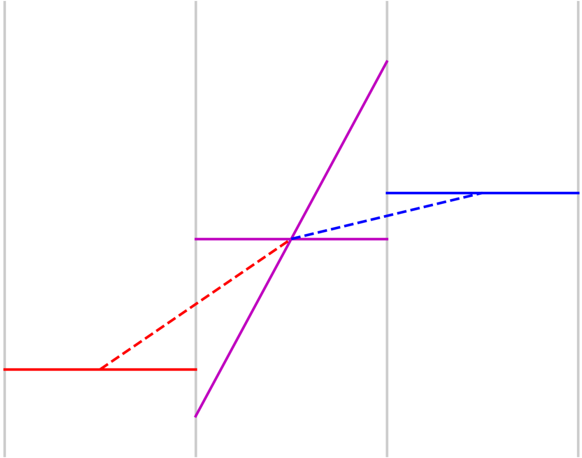
Slope Limiter (MinMod)

$$u_h(x,t) = \sum_{i=1}^{k+1} u_i(t)\ \ell_i(x) = \sum_{n=0}^{k} C_n(t)\ P_n(x)$$



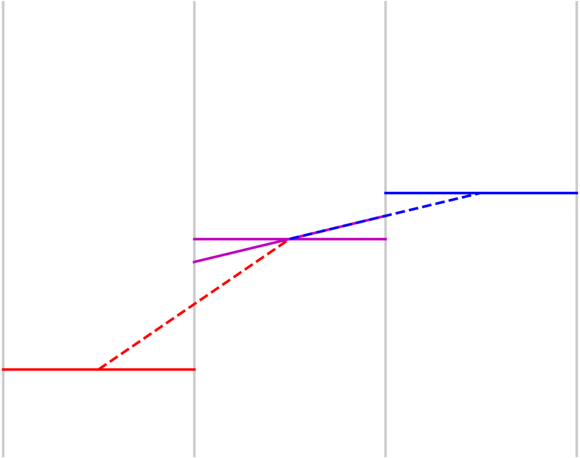
Slope Limiter (MinMod)

$-- : \bar{u}_K - \bar{u}_{K-1}$ $\quad$ $-- : \bar{u}_{K+1} - \bar{u}_K$



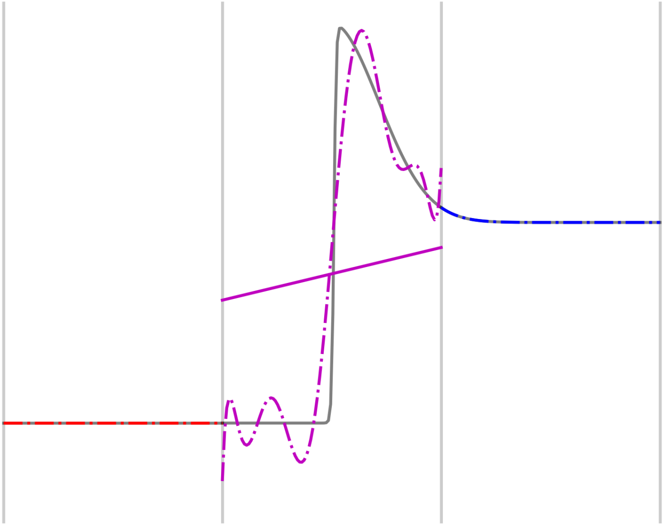
Slope Limiter (MinMod)

$$\sum_{n=1}^{k} C_n \, P_n = u_h \to \tilde{u}_h := \Lambda_{\text{SL}}\left(u_h\right) := C_0 \, P_0 + \tilde{C}_1 \, P_1$$



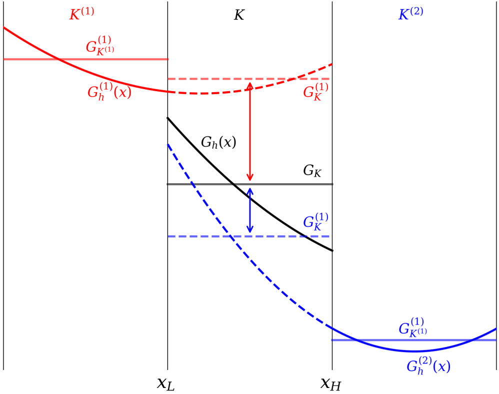
Slope Limiter (MinMod)

$k = 9$
$\tilde{k} = 1$



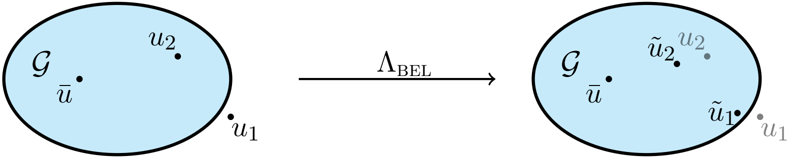
Troubled Cell Indicator

Proposed in Fu and Shu (2017)



Bound-Enforcing Limiter (Λ_{BEL})

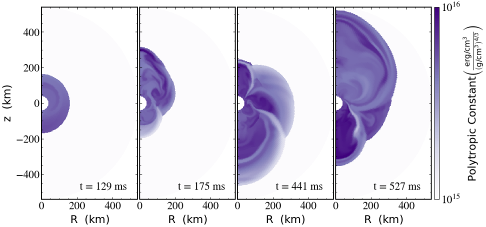
- Proposed in Qin et al. (2016)
- Define $\mathcal{G}$ as the (convex) set of physically-admissible states; i.e., positivity density and energy density, subluminal fluid velocity (ideal EoS only)

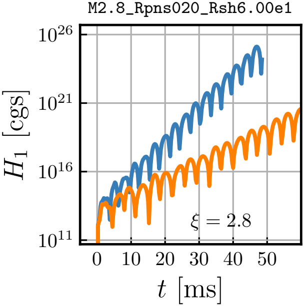


$$u_h = u_1\,\ell_1 + u_2\,\ell_2 \qquad \tilde{u}_h := \Lambda_{\mathrm{BEL}}\left(u_h\right) = \tilde{u}_1\,\ell_1 + \tilde{u}_2\,\ell_2$$

Standing Accretion Shock Instability

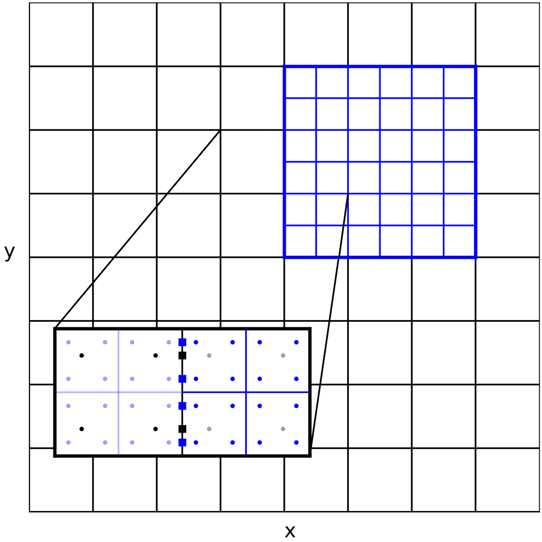
Used `thornado` to investigate the role of GR on the SASI[1]





[1]Dunham et al. (2020, 2024)

Mesh Refinement with AMReX



Conservative Interpolation (1D)

- Schaal et al. (2015)

Given a solution in the nodal points of a coarse element K, $\{U_j\}_{j=1}^{N}$, define the solution in the nodal points of the jth ($j \in \{1,2\}$) fine element $k^{(j)}$, $\left\{u_i^{(j)}\right\}_{i=1}^{N}$ via the minimization of an L^2-projection:

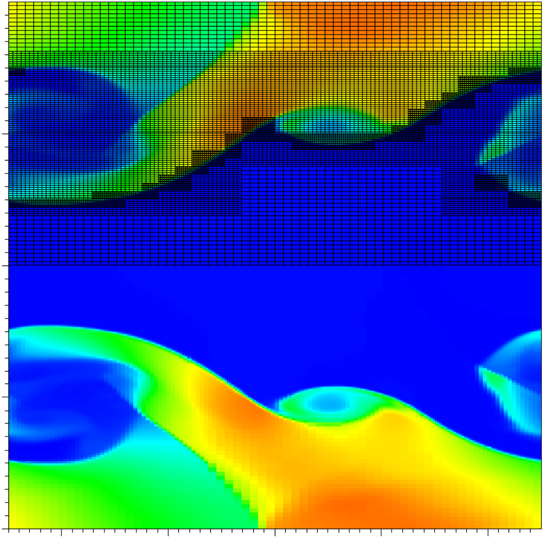
$$\forall i \in \{1, \cdots, N\} : \min_{\tilde{u}_i^{(j)}} \int_{k^{(j)}} \left(\tilde{U}_h - \tilde{u}_h^{(j)}\right)^2 dx,$$

where $\tilde{U} := \sqrt{\gamma}\, U$. For the geometry fields, we use the same procedure but do not pre-multiply with $\sqrt{\gamma}$.

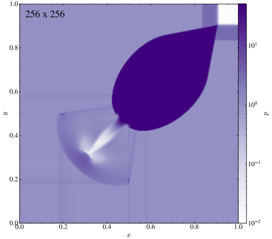
Flux Corrections

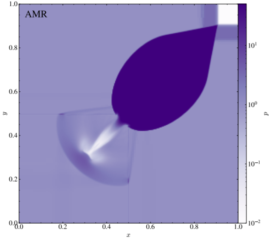
Show what we store in the flux register

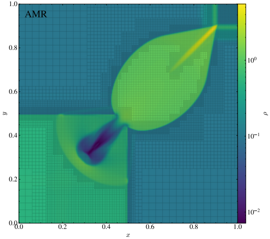
Kelvin–Helmholtz Instability



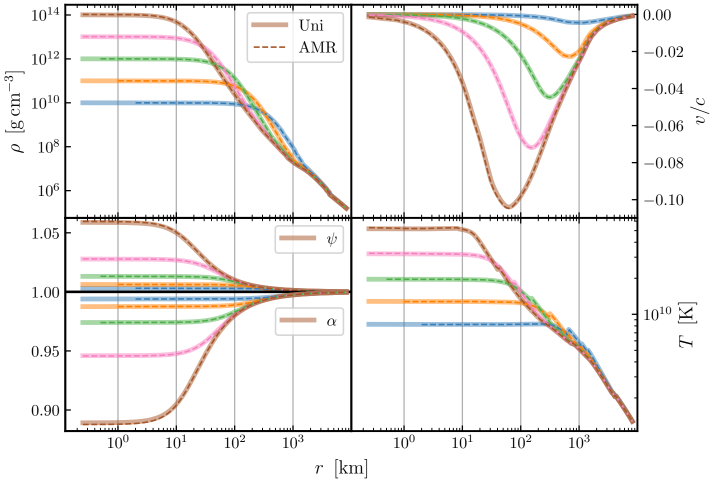
2D Blast Wave (Del Zanna and Bucciantini, 2002)



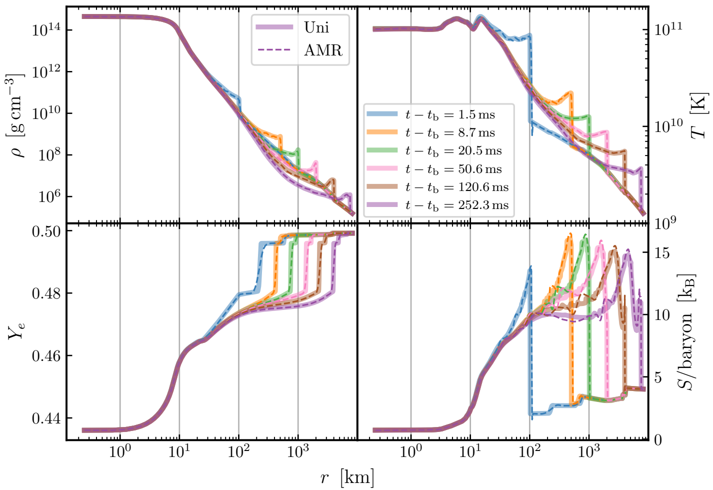




Adiabatic Collapse (Collapse Phase)



Adiabatic Collapse (Post-Bounce Phase)



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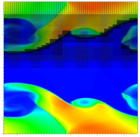
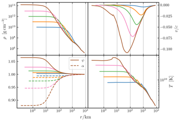
Summary

Can run multi-D pure hydro problems in GR with AMR





Can run 1D hydro+self-gravity problems in GR with AMR

Working on coupling GR transport to existing hydro+gravity modules