

thornado-Hydro (xCFC)

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October 21, 2024



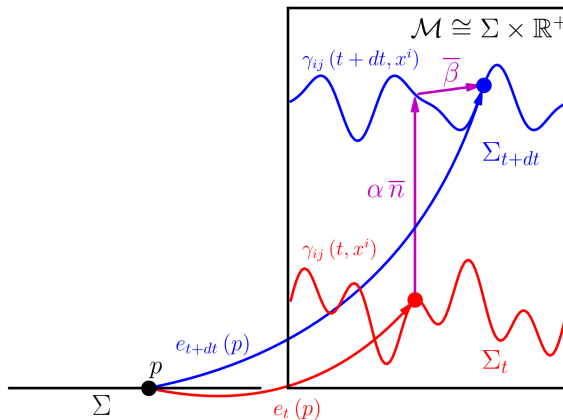
toolkit for high-order neutrino-radiation hydrodynamics

- DG
- SSPRK/IMEX
- GR (xCFC)
- Hydro^a (Valencia)
- Neutrino transport^b (M1)
- Interfaces to tabulated EoS/Opacities (weaklib: <https://github.com/starkiller-astro/weaklib>)
- Fluid self-gravity via Poseidon: <https://github.com/jrober50/Poseidon>
- GPUs via OpenACC or OpenMP pragmas
- MPI parallelism and AMR via AMReX: <https://github.com/AMReX-Codes/amrex>

^aEndeve et al. (2019); Dunham et al. (2020); Pochik et al. (2021)

^bLaiu et al. (2021)

3+1 Decomposition



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt) (dx^j + \beta^j dt)$$

Conformally-Flat Condition

Developed by Wilson et al. (1996),
extended by Cordero-Carrión et al.
(2009)

Special case: Schwarzschild spacetime
in isotropic coordinates ($G = c = 1$)

$$\gamma_{ij}(x) = \psi^4(x) \bar{\gamma}_{ij}(x^i)$$

$$K = 0, \quad \partial_t K = 0$$

(Always and everywhere)

$$\alpha = \left(1 + \frac{1}{2}\Phi\right) \left(1 - \frac{1}{2}\Phi\right)^{-1}$$

$$\psi = 1 - \frac{1}{2}\Phi$$

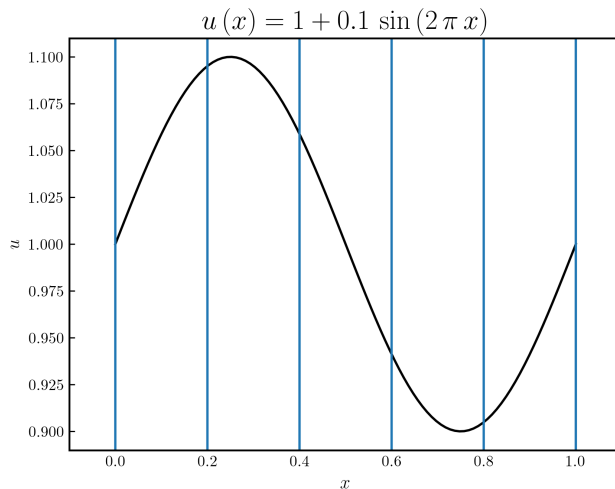
$$\beta^i = 0,$$

- Exact in spherical symmetry!
- Hyperbolic \rightarrow Elliptic equations
- Good for long-time simulations

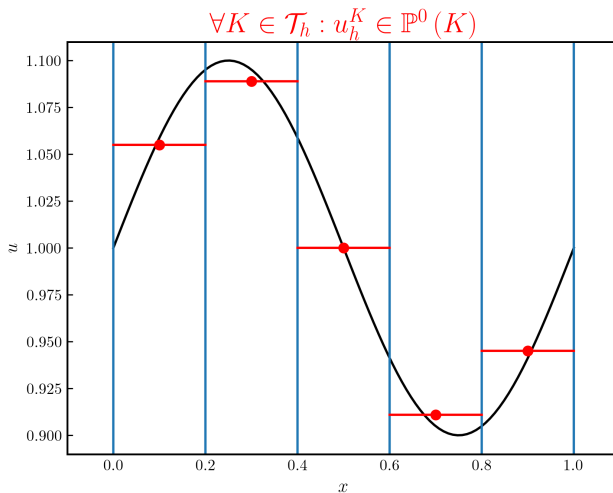
with

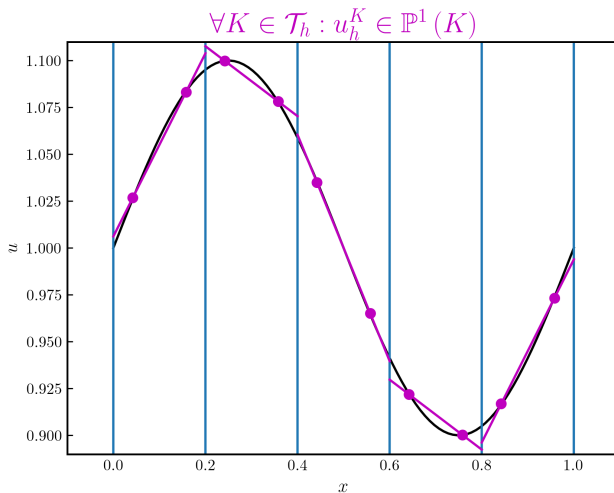
$$\Phi(r) := -\frac{M}{r}$$

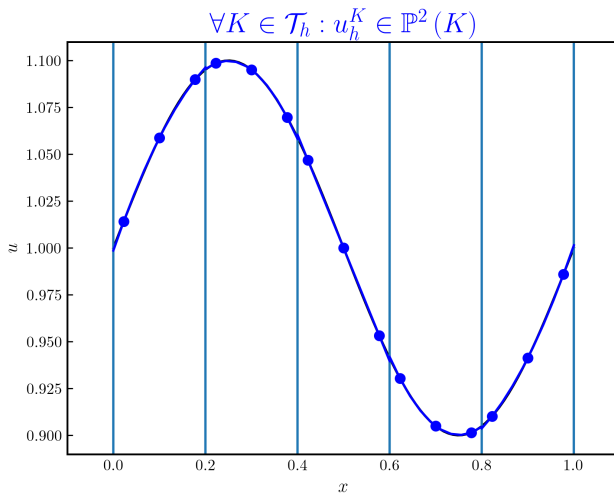
Discontinuous Galerkin (DG) Method

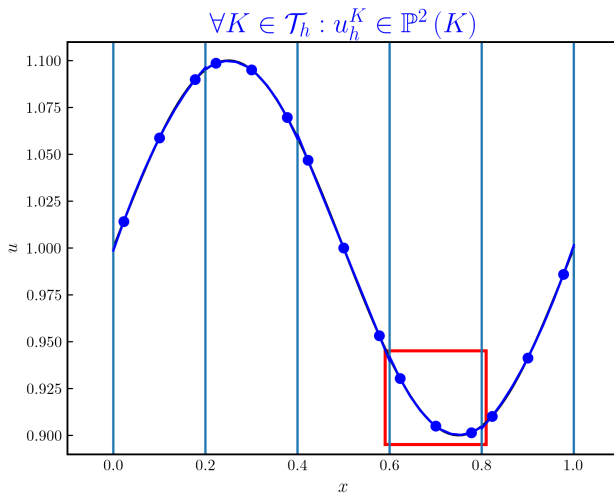


Finite-volume/DG(0)



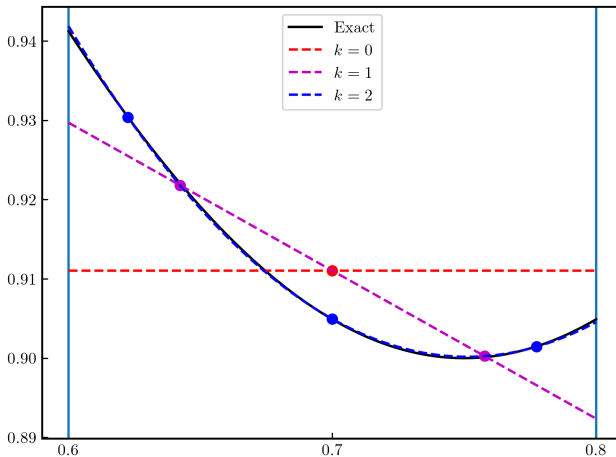




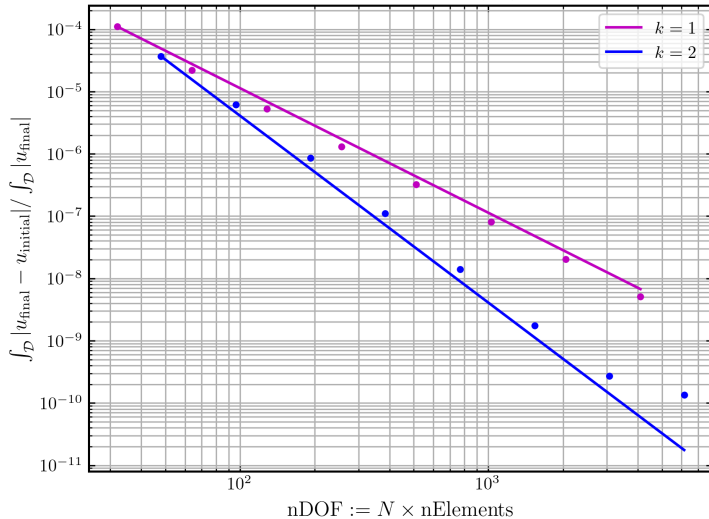


Discontinuous Galerkin (DG) Method

$$u_h(x, t) := \sum_{i=1}^{k+1} u_i(t) \ell_i(x)$$

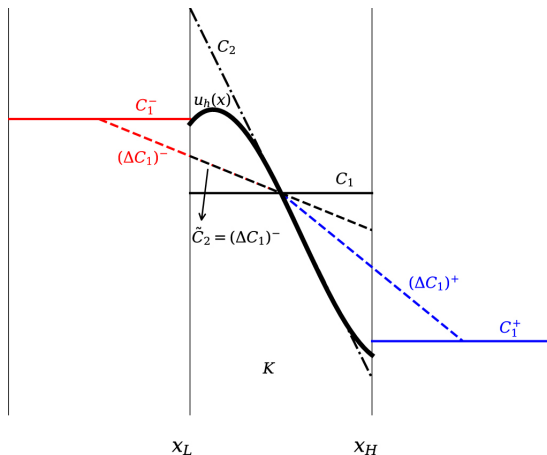


Convergence Rates for Sine Wave Advection (1D)



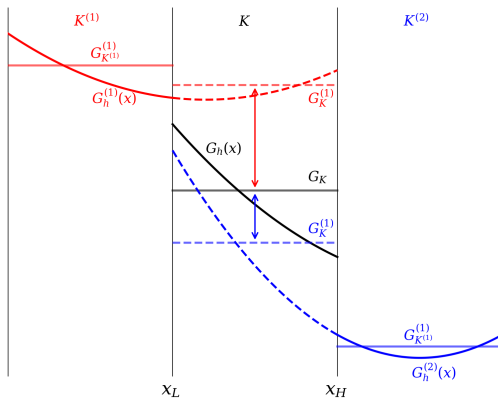
Slope Limiter (MinMod)

$$u_h(x, t) = \sum_{n=1}^N C_n(t) P_n(x) \implies \tilde{u}_h(x, t) = C_1(t) P_1(x) + \tilde{C}_2(t) P_2(x)$$



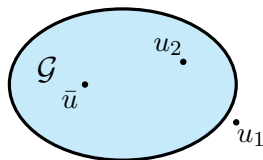
Troubled Cell Indicator

Proposed in Fu and Shu (2017)



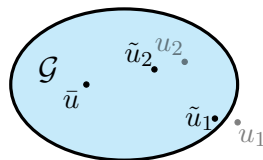
Bound-Enforcing Limiter (Λ_{BEL})

- Proposed in Qin et al. (2016)
- Define \mathcal{G} as the (convex) set of physically-admissible states when using an ideal EoS (positivity density and energy density, subluminal fluid velocity)



$$u_h = u_1 \ell_1 + u_2 \ell_2$$

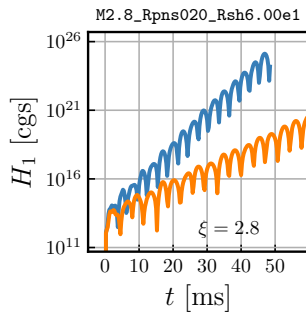
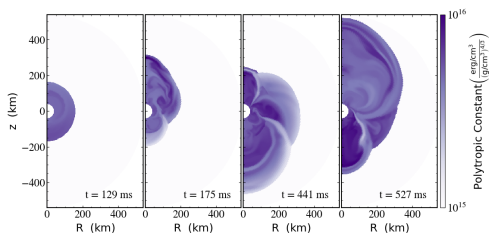
$$\xrightarrow{\Lambda_{\text{BEL}}}$$



$$\tilde{u}_h := \Lambda_{\text{BEL}}(u_h) = \tilde{u}_1 \ell_1 + \tilde{u}_2 \ell_2$$

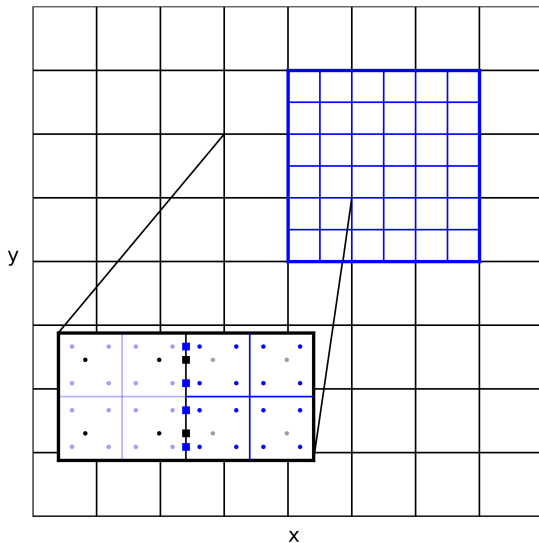
Standing Accretion Shock Instability

Used thornado to investigate the role of GR on the SASI¹

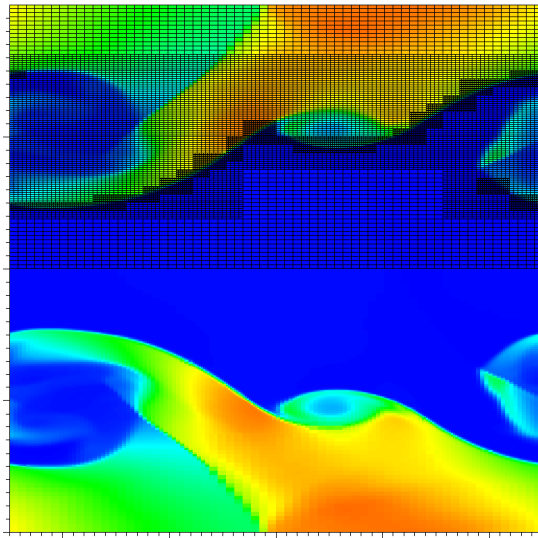


¹Dunham et al. (2020, 2024)

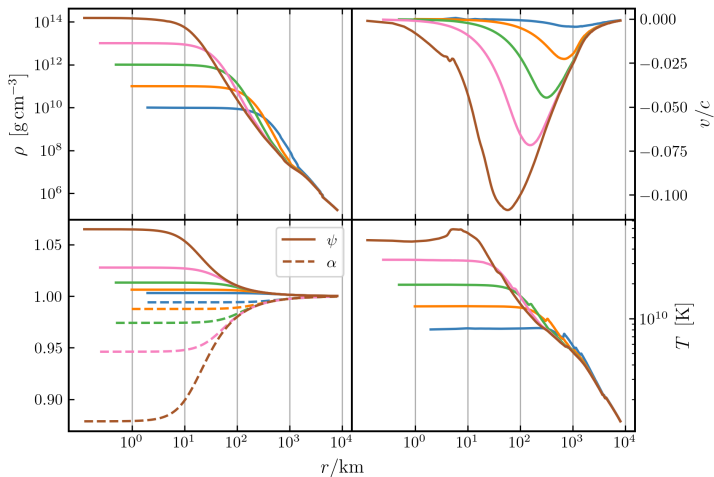
Mesh Refinement with AMReX



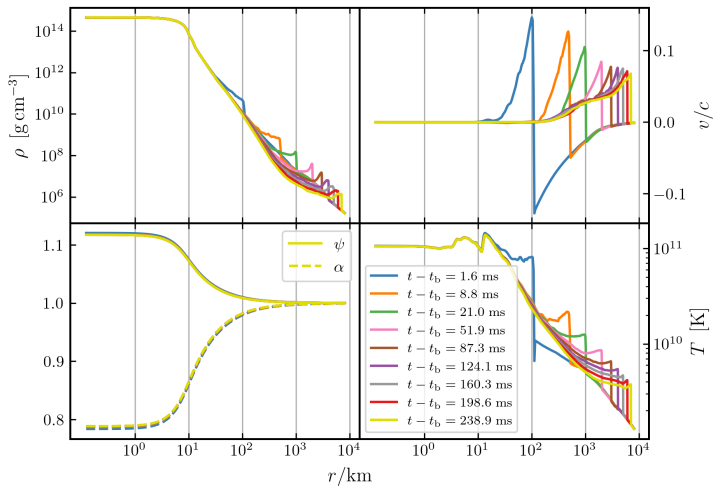
Kelvin–Helmholtz Instability



Adiabatic Collapse (Collapse Phase)



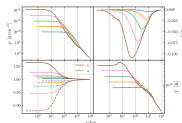
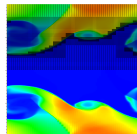
Adiabatic Collapse (Post-Bounce Phase)



- Eirik Endeve, Jesse Buffaloe, Samuel J. Dunham, Nick Roberts, Kristopher Andrew, Brandon Barker, David Pochik, Juliana Pulsinelli, and Anthony Mezzacappa. thornado-hydro: towards discontinuous Galerkin methods for supernova hydrodynamics. In *Journal of Physics Conference Series*, volume 1225 of *Journal of Physics Conference Series*, page 012014, May 2019. doi: 10.1088/1742-6596/1225/1/012014.
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- Isabel Cordero-Carrión, Pablo Cerdá-Durán, Harald Dimmelmeier, José Luis Jaramillo, Jérôme Novak, and Ericourgoulhon. Improved constrained scheme for the Einstein equations: An approach to the uniqueness issue. *Phys. Rev. D*, 79(2): 024017, January 2009. doi: 10.1103/PhysRevD.79.024017.
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- Tong Qin, Chi-Wang Shu, and Yang Yang. Bound-preserving discontinuous Galerkin methods for relativistic hydrodynamics. *Journal of Computational Physics*, 315:323–347, June 2016. doi: 10.1016/j.jcp.2016.02.079.
- Samuel J. Dunham, Eirik Endeve, Anthony Mezzacappa, John M. Blondin, Jesse Buffaloe, and Kelly Holley-Bockelmann. A Parametric Study of the SASI Comparing General Relativistic and Nonrelativistic Treatments. *ApJ*, 964(1):38, March 2024. doi: 10.3847/1538-4357/ad206c.

Summary

Can run multi-D pure hydro problems in GR with AMR



Can run 1D hydro+self-gravity problems in GR with AMR

Working on coupling GR transport to existing hydro+gravity modules