# thornado-Hydro (xCFC)

Samuel J. Dunham

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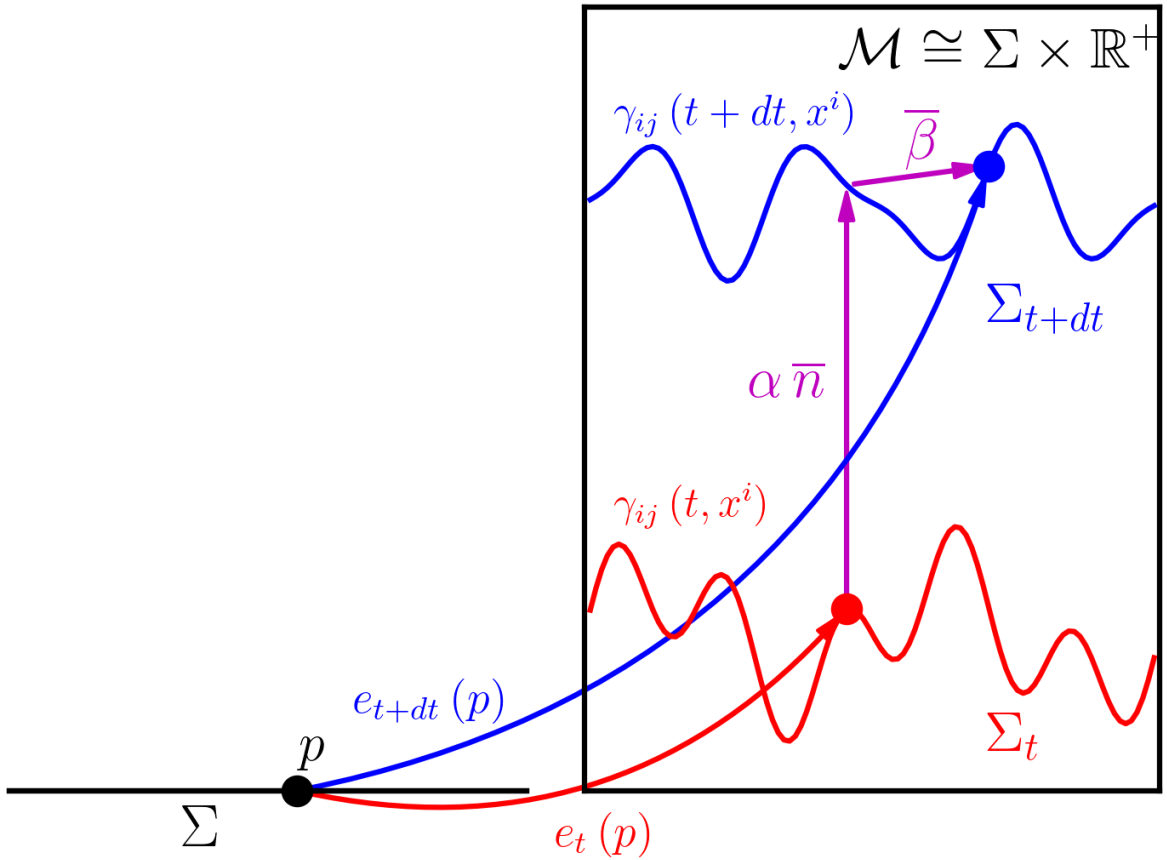
toolkit for high-order neutrino-radiation hydrodynamics

- DG
- SSPRK/IMEX
- GR (xCFC)
- Hydro[a] (Valencia)
- Neutrino transport[b] (M1)
- Interfaces to tabulated EoS/Opacities (weaklib: `https://github.com/starkiller-astro/weaklib`)
- Fluid self-gravity via Poseidon: `https://github.com/jrober50/Poseidon`
- GPUs via OpenACC or OpenMP pragmas
- MPI parallelism and AMR via AMReX: `https://github.com/AMReX-Codes/amrex`

[a] Endeve et al. (2019); Dunham et al. (2020); Pochik et al. (2021)

[b] Laiu et al. (2021)

# 3+1 Decomposition




$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu = -\alpha^2\,dt^2 + \gamma_{ij}\left(dx^i + \beta^i\,dt\right)\left(dx^j + \beta^j\,dt\right)$$

# Conformally-Flat Condition

Developed by Wilson et al. (1996), extended by Cordero-Carrión et al. (2009)

$$\gamma_{ij}(x) = \psi^4(x)\,\overline{\gamma}_{ij}\left(x^i\right)$$
$$K = 0,\ \partial_t K = 0$$
(Always and everywhere)

- Exact in spherical symmetry!
- Hyperbolic $\rightarrow$ Elliptic equations
- Good for long-time simulations

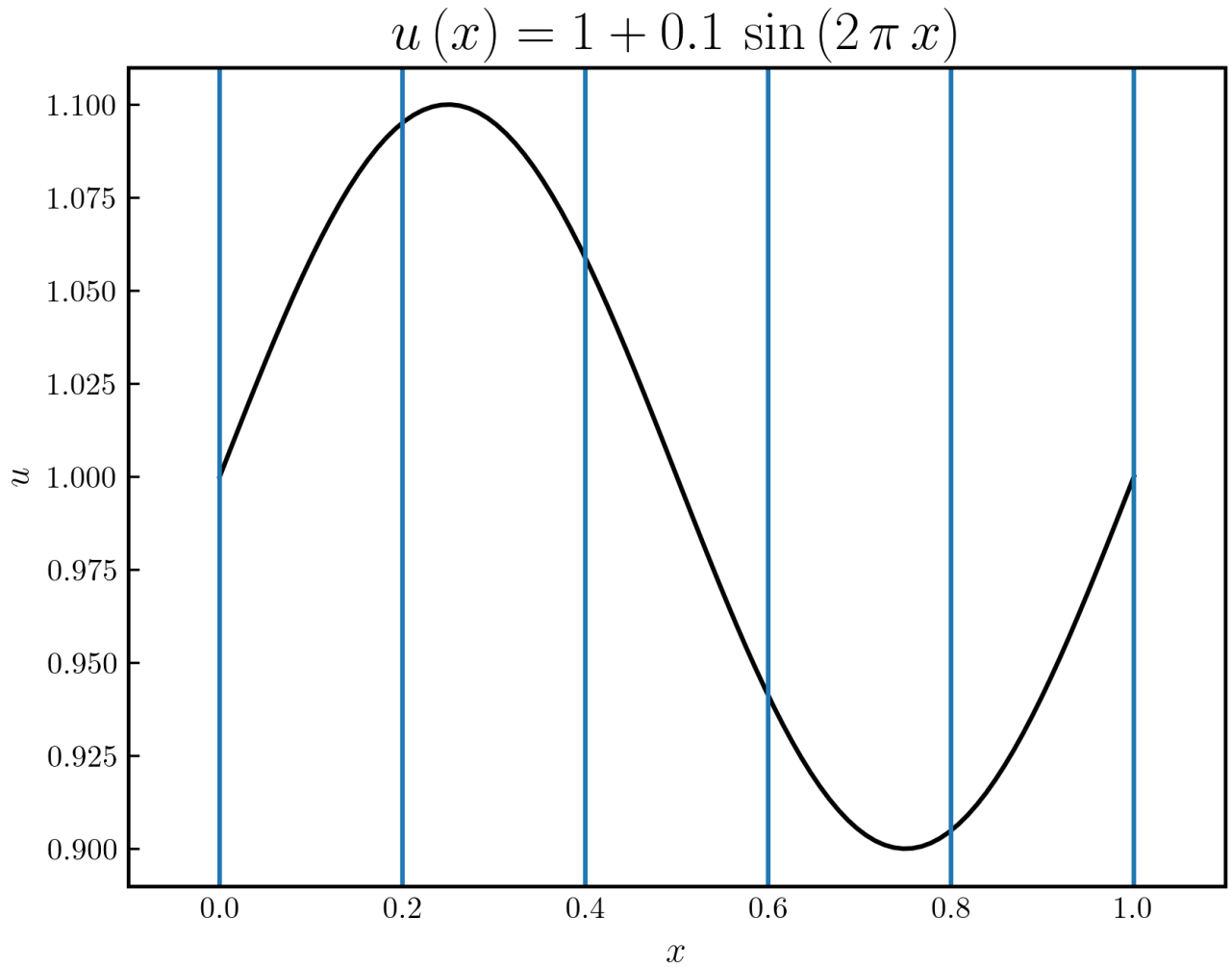
Special case: Schwarzchild spacetime in isotropic coordinates ($G = c = 1$)

$$\alpha = \left(1 + \frac{1}{2}\Phi\right)\left(1 - \frac{1}{2}\Phi\right)^{-1}$$
$$\psi = 1 - \frac{1}{2}\Phi$$
$$\beta^i = 0,$$

with

$$\Phi(r) := -\frac{M}{r}$$

# Discontinuous Galerkin (DG) Method



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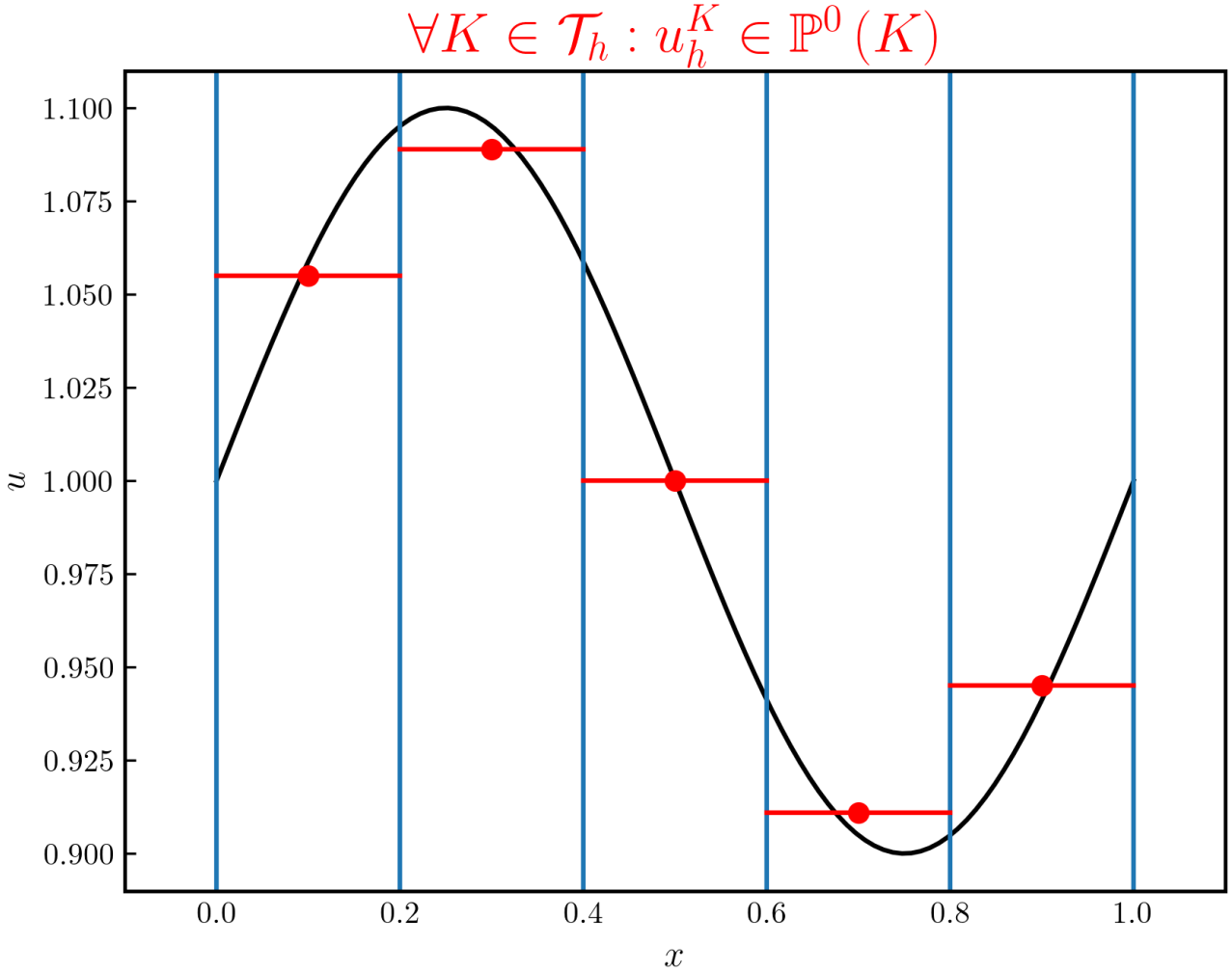

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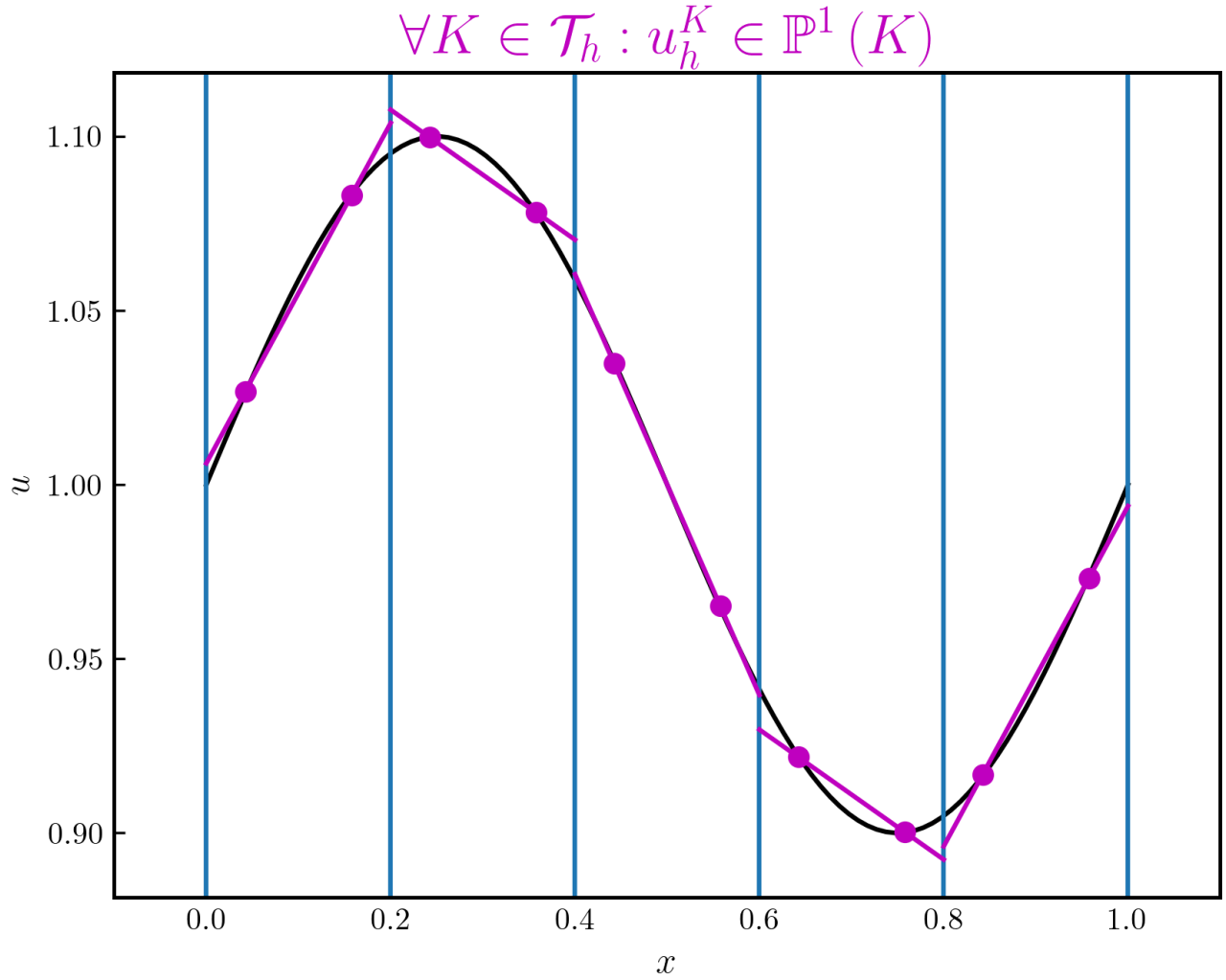

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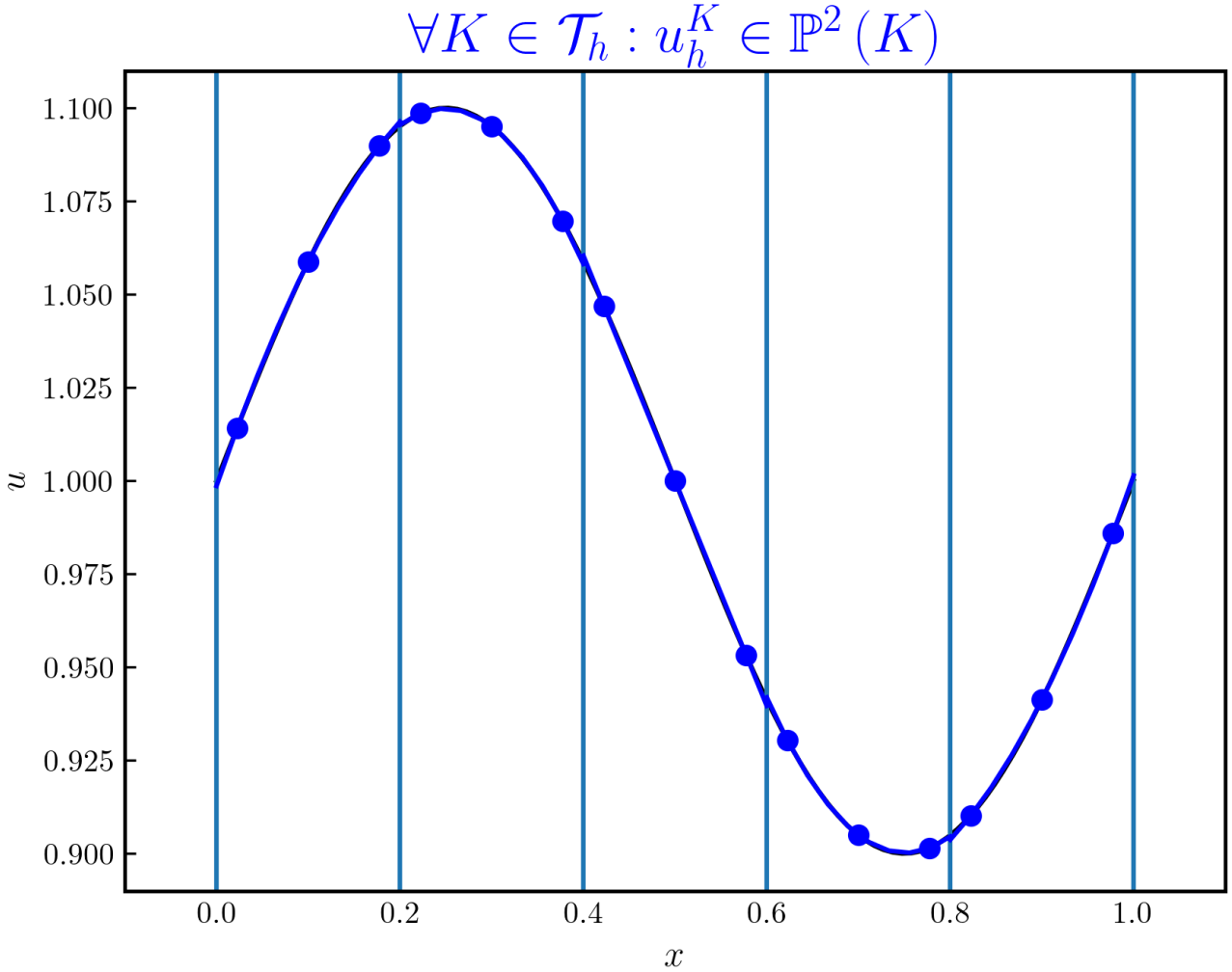

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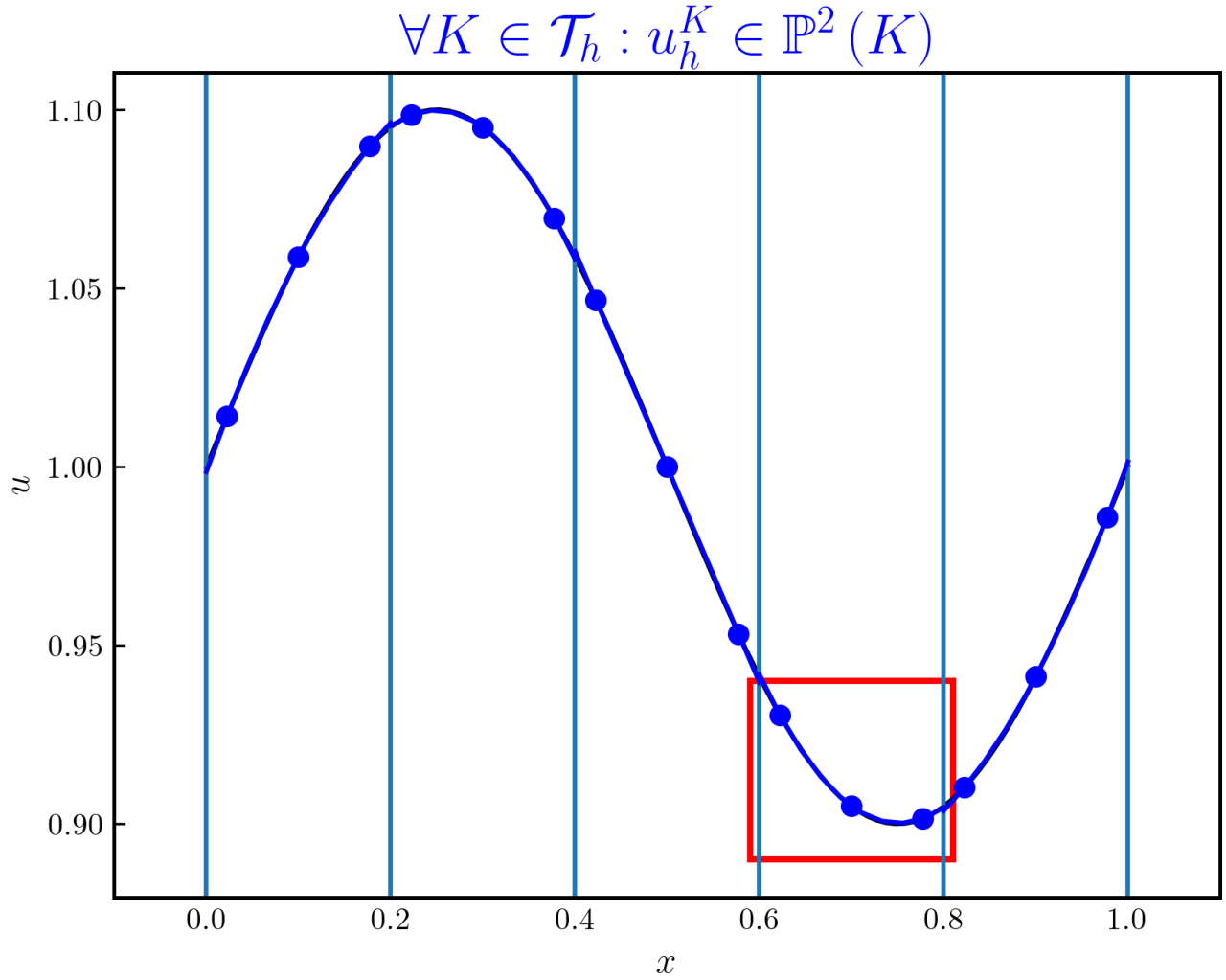
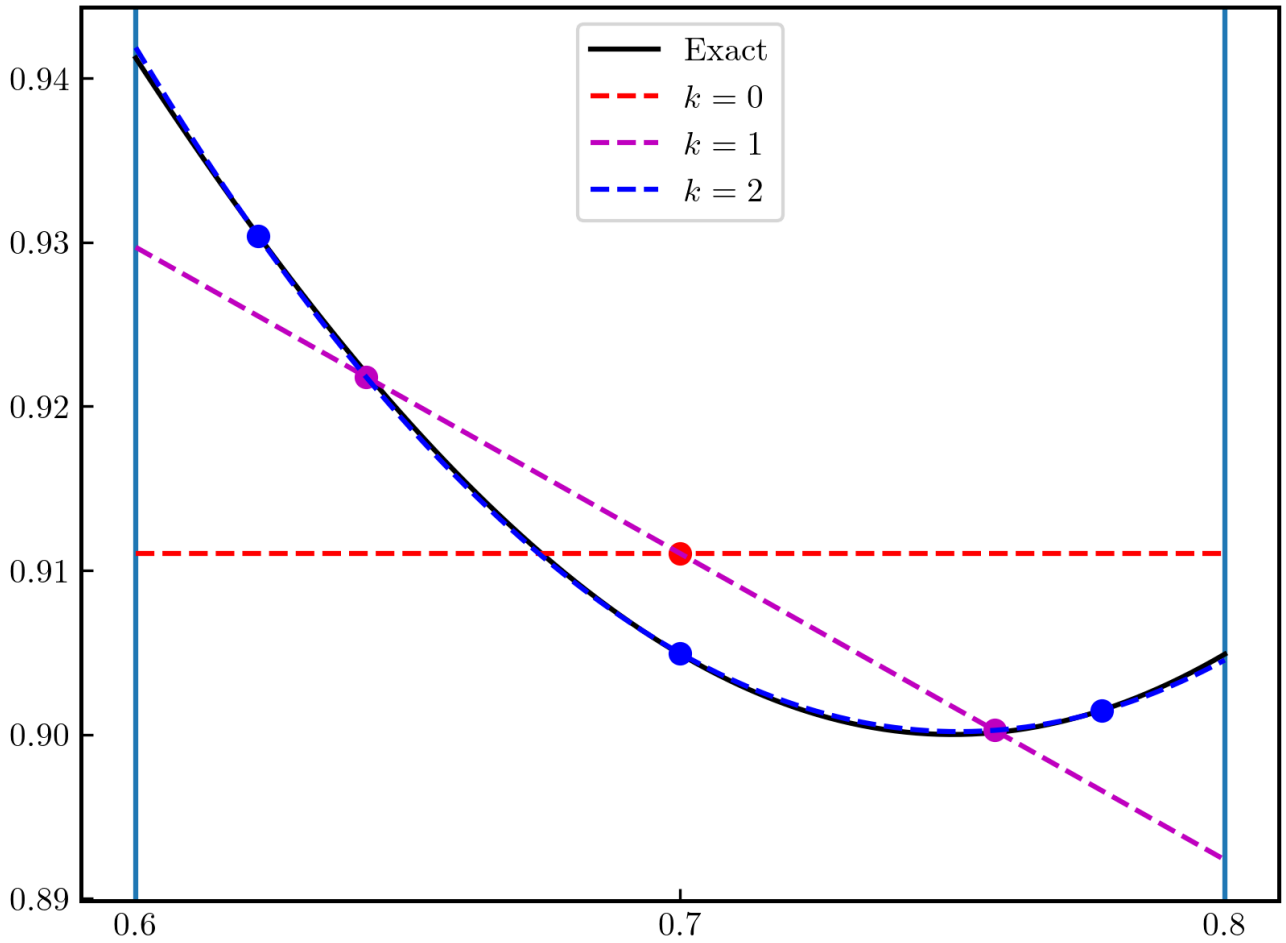
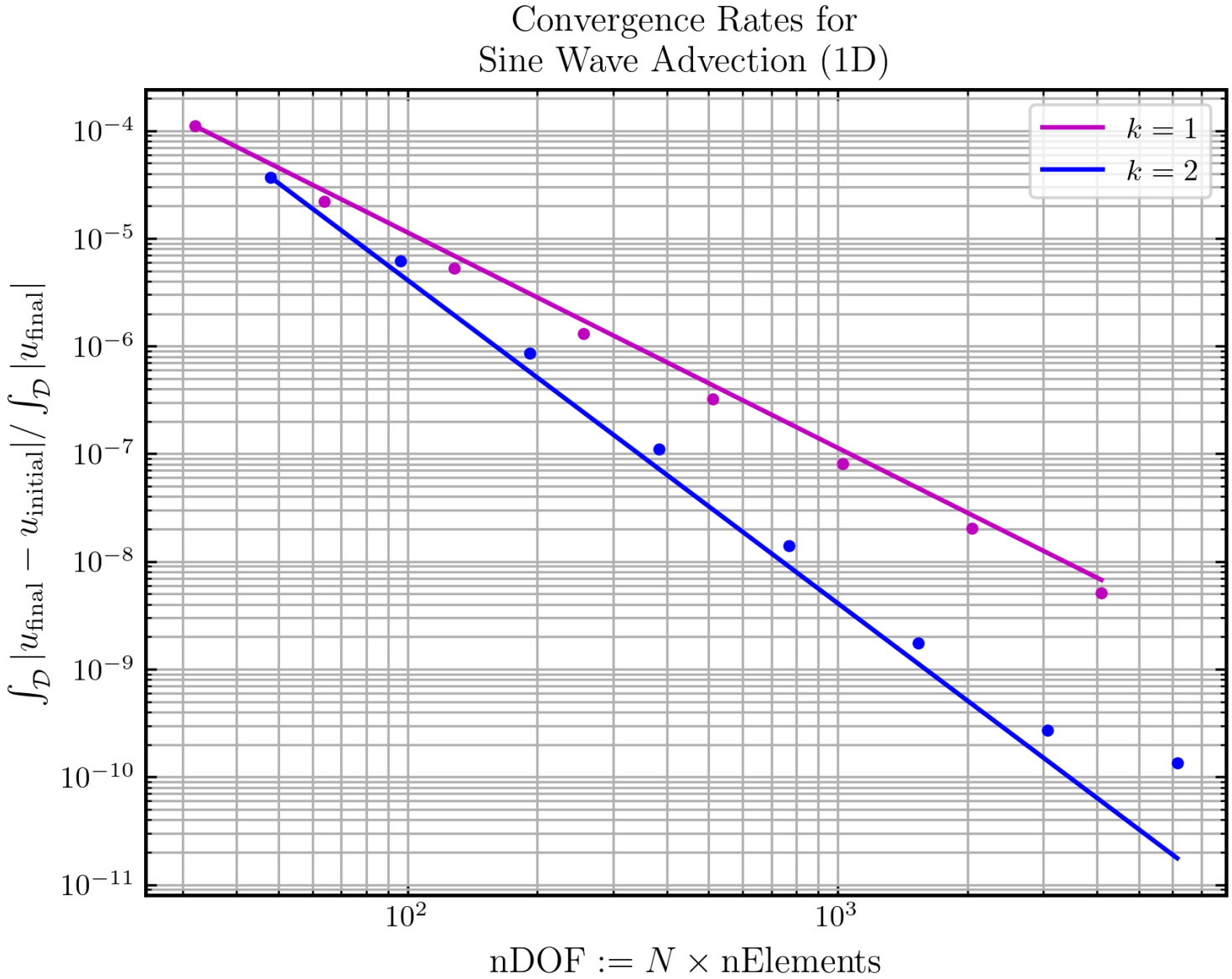


# Discontinuous Galerkin (DG) Method

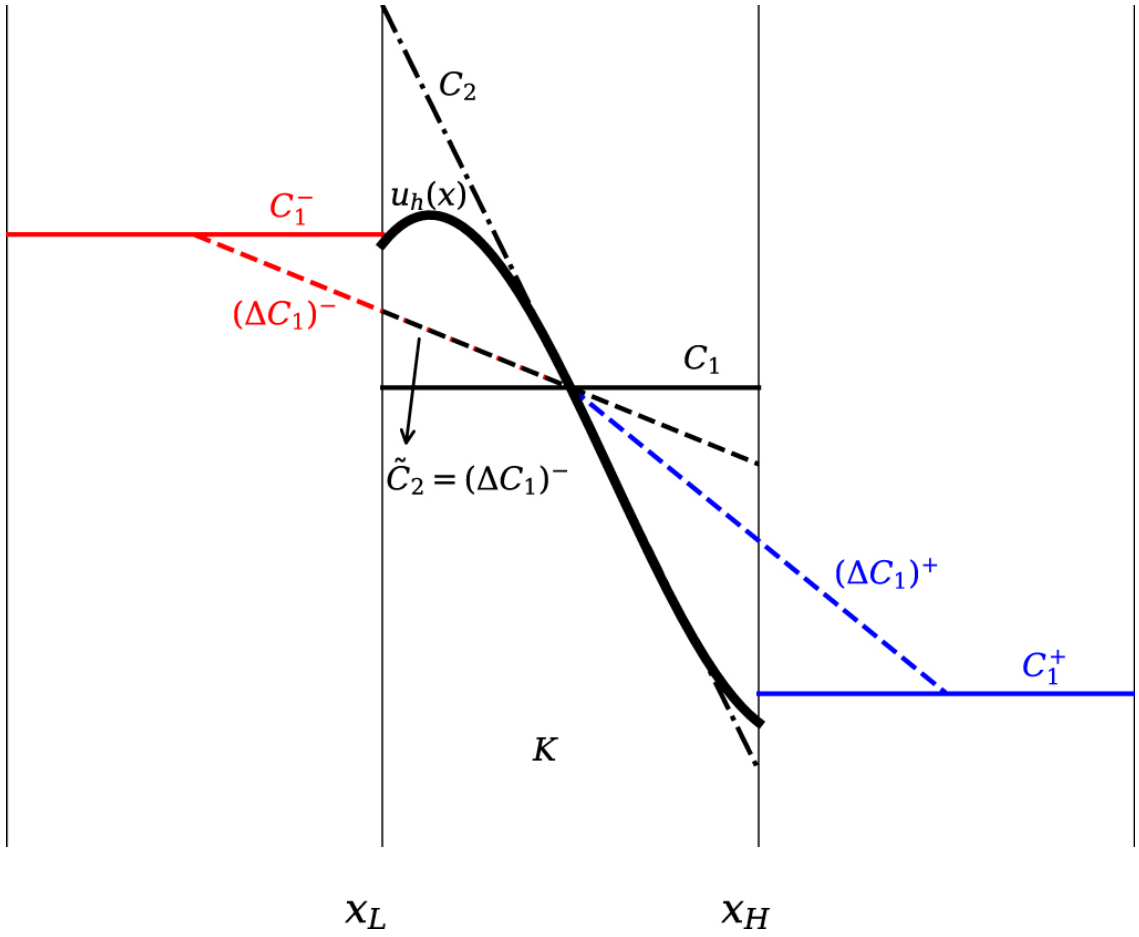
$$u_h(x,t) := \sum_{i=1}^{k+1} u_i(t)\, \ell_i(x)$$




Convergence Rates for
Sine Wave Advection (1D)
$k = 1$
$k = 2$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
$10^{-8}$
$10^{-9}$
$10^{-10}$
$10^{-11}$
$10^{2}$
$10^{3}$
$\int_{\mathcal{D}} |u_{\text{final}} - u_{\text{initial}}| / \int_{\mathcal{D}} |u_{\text{final}}|$
nDOF := $N \times$ nElements

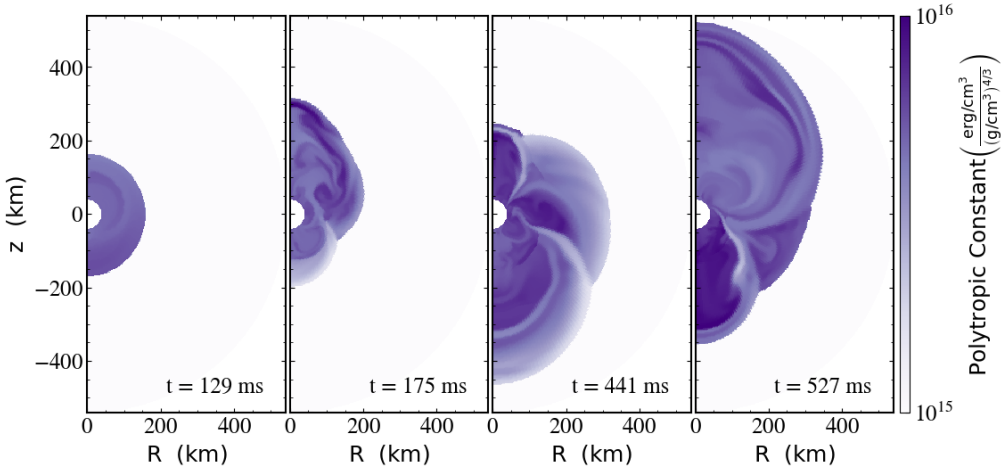
# Slope Limiter

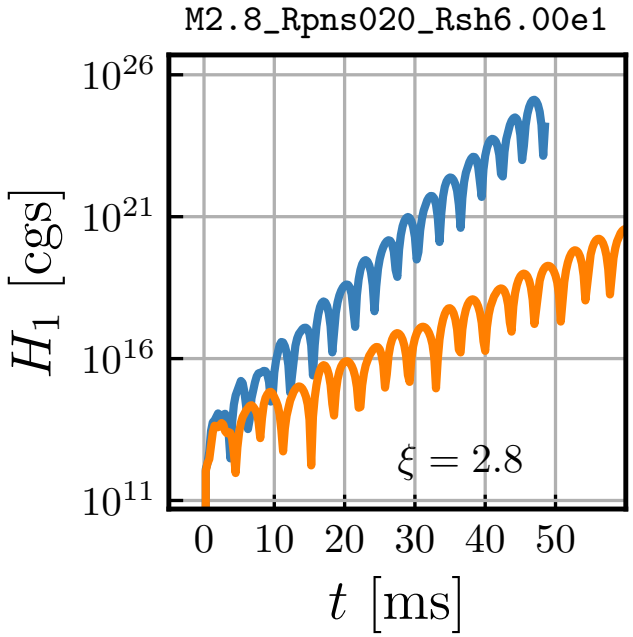
$$u_h(x,t) = \sum_{n=1}^{N} C_n(t)\ P_n(x) \implies \tilde{u}_h(x,t) = C_1(t)\ P_1(x) + \tilde{C}_2(t)\ P_2(x)$$



# Standing Accretion Shock Instability

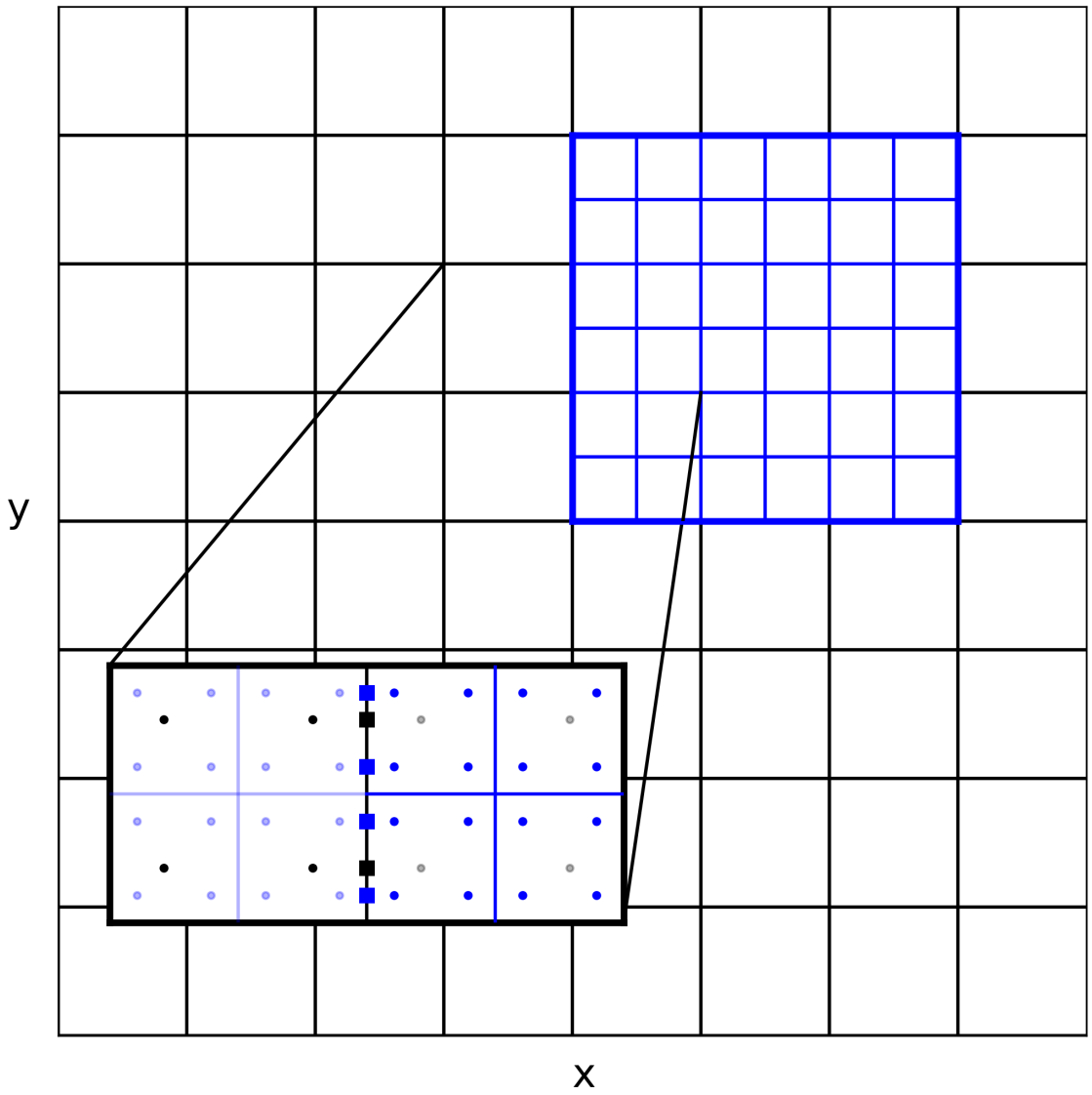
Used `thornado` to investigate the role of GR on the SASI[1]



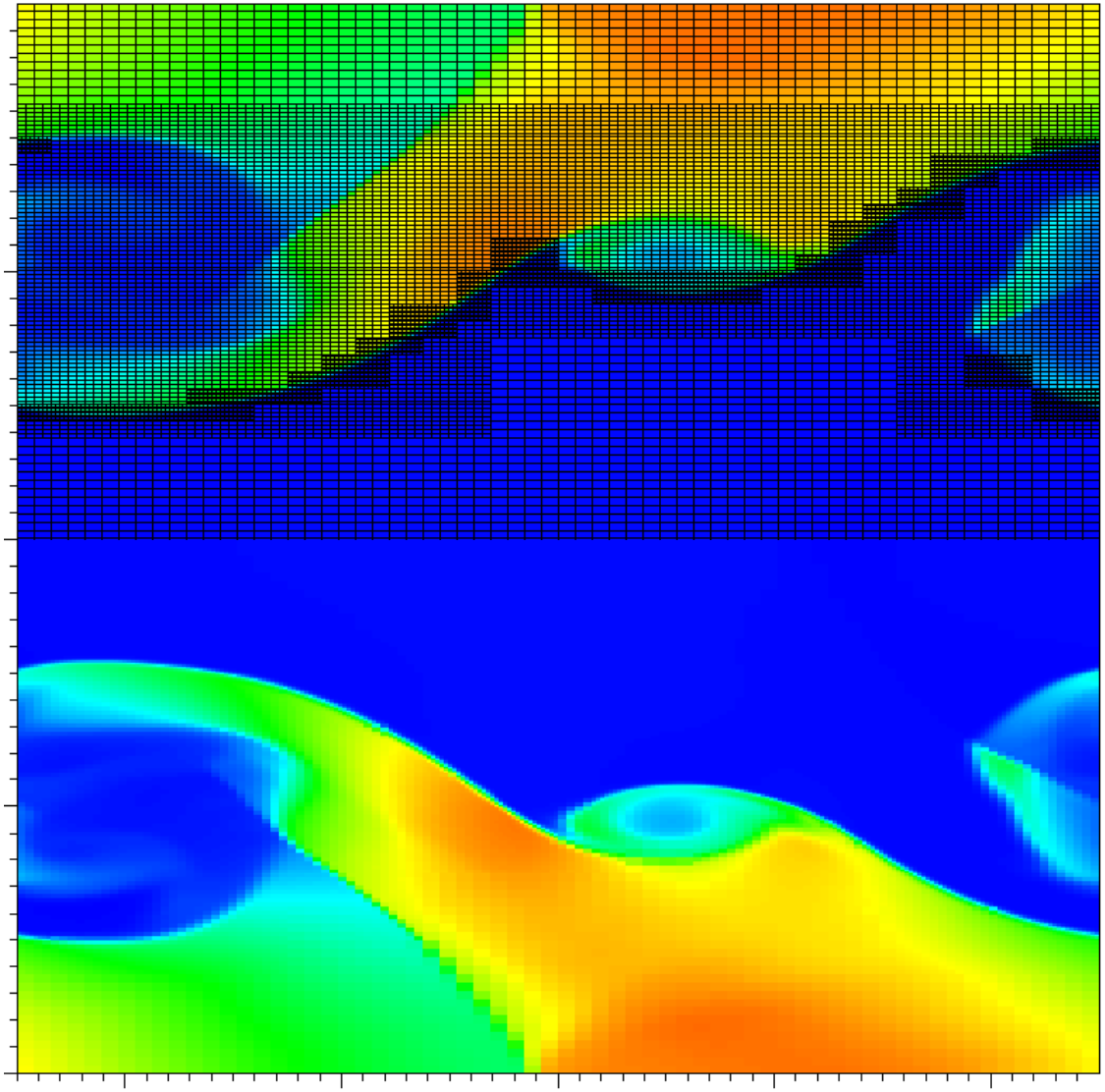




[1] Dunham et al. (2020, 2023)

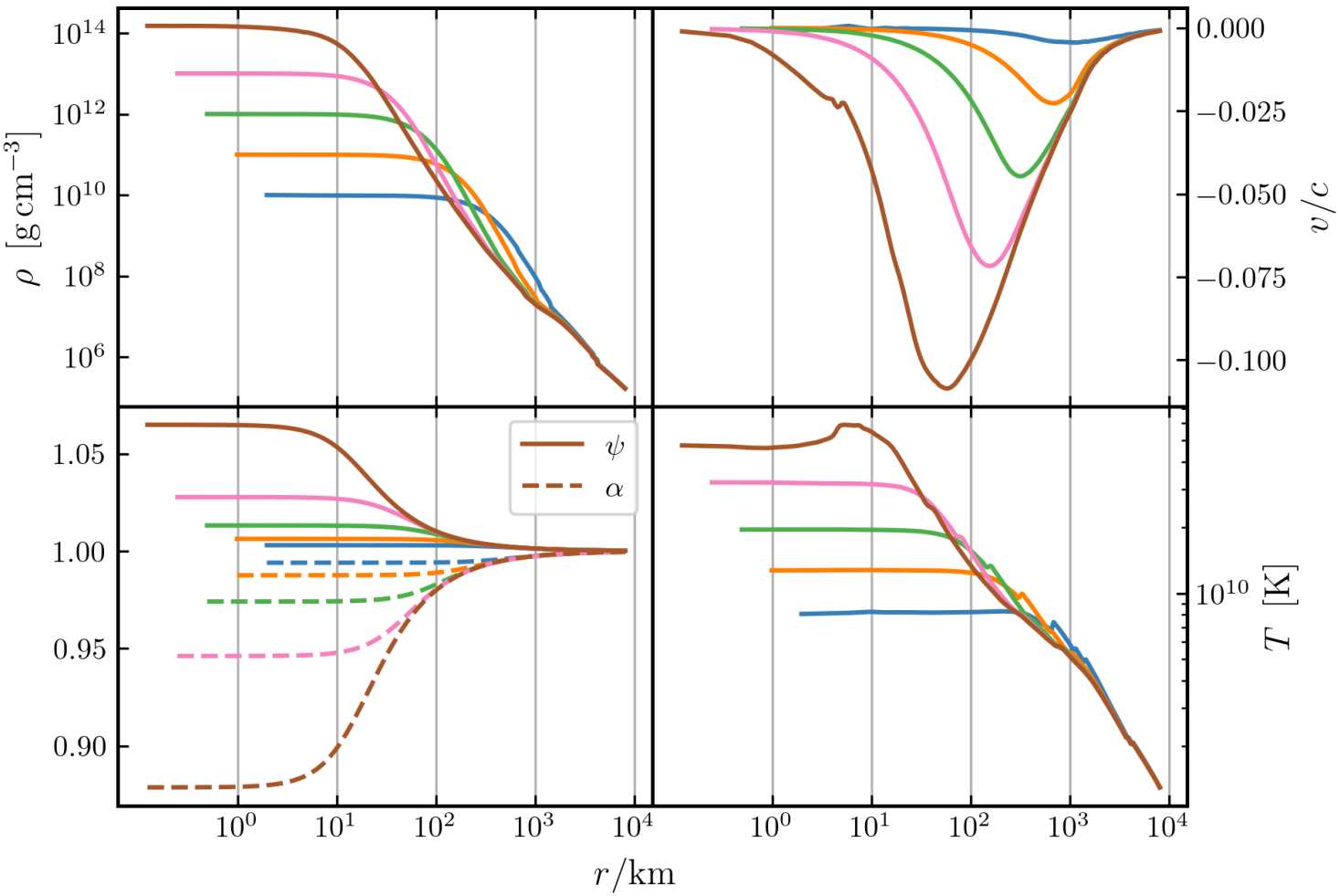
# Mesh Refinement



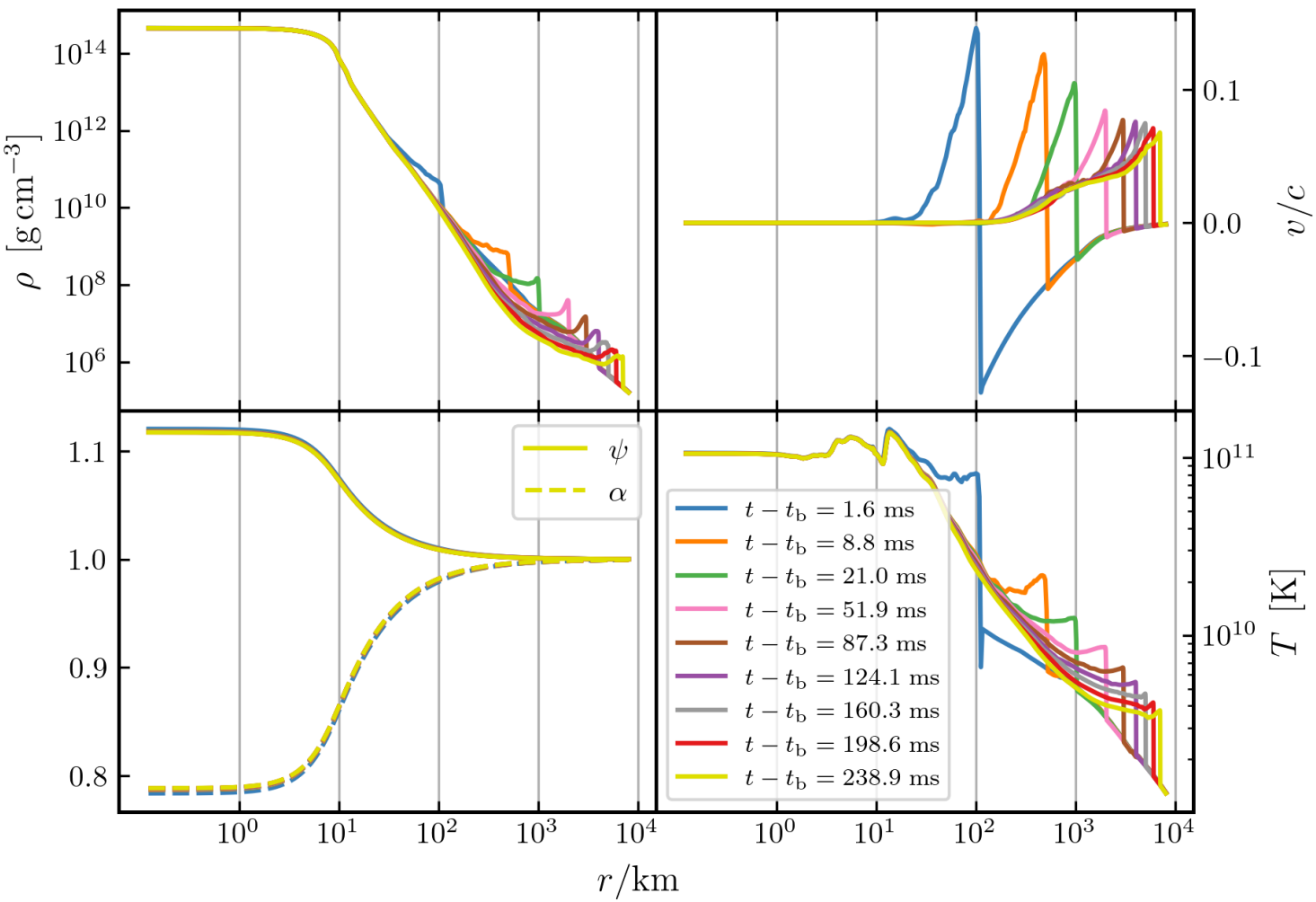
# Kelvin–Helmholtz Instability



# Adiabatic Collapse (AMR, Collapse Phase)



# Adiabatic Collapse (AMR, Post-Bounce Phase)



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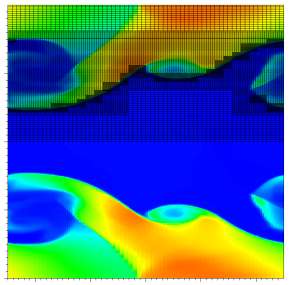
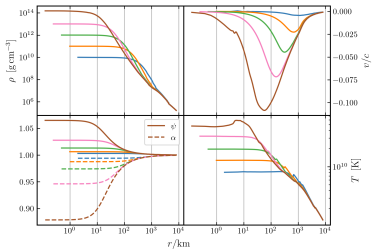
# Summary

Can run pure hydro problems in GR with AMR





Can run hydro+self-gravity problems in GR with AMR

Working on coupling GR transport to existing hydro+gravity modules