

thornado-Hydro (xCFC)

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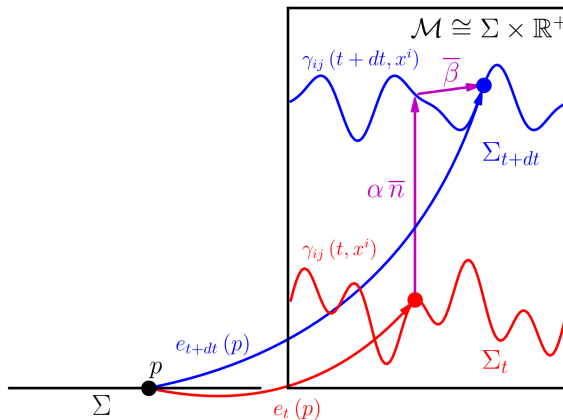
toolkit for high-order neutrino-radiation hydrodynamics

- DG
- SSPRK/IMEX
- GR (xCFC)
- Hydro^a (Valencia)
- Neutrino transport^b (M1)
- Interfaces to tabulated EoS/Opacities (weaklib: <https://github.com/starkiller-astro/weaklib>)
- Fluid self-gravity via Poseidon: <https://github.com/jrober50/Poseidon>
- GPUs via OpenACC or OpenMP pragmas
- MPI parallelism and AMR via AMReX: <https://github.com/AMReX-Codes/amrex>

^aEndeve et al. (2019); Dunham et al. (2020); Pochik et al. (2021)

^bLaiu et al. (2021)

3+1 Decomposition



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt) (dx^j + \beta^j dt)$$

Conformally-Flat Condition

Developed by Wilson et al. (1996),
extended by Cordero-Carrión et al.
(2009)

Special case: Schwarzschild spacetime
in isotropic coordinates ($G = c = 1$)

$$\gamma_{ij}(x) = \psi^4(x) \bar{\gamma}_{ij}(x^i)$$

$$K = 0, \quad \partial_t K = 0$$

(Always and everywhere)

$$\alpha = \left(1 + \frac{1}{2}\Phi\right) \left(1 - \frac{1}{2}\Phi\right)^{-1}$$

$$\psi = 1 - \frac{1}{2}\Phi$$

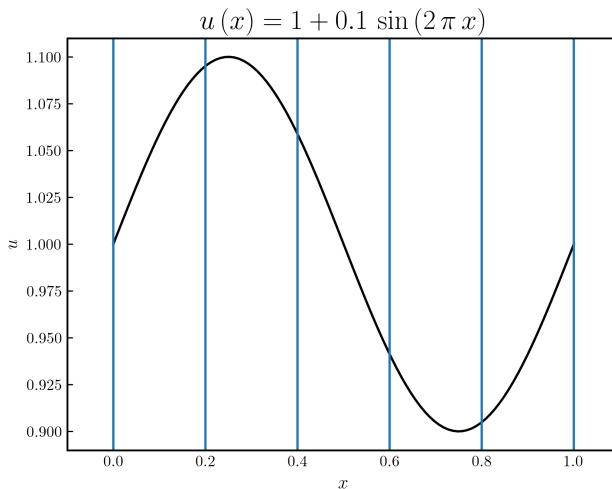
$$\beta^i = 0,$$

- Exact in spherical symmetry!
- Hyperbolic \rightarrow Elliptic equations
- Good for long-time simulations

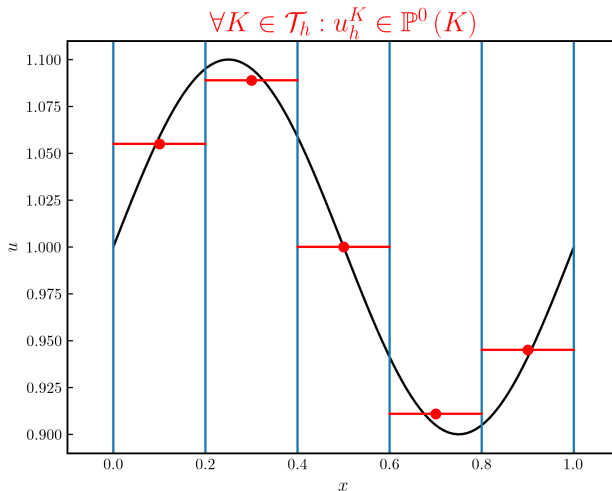
with

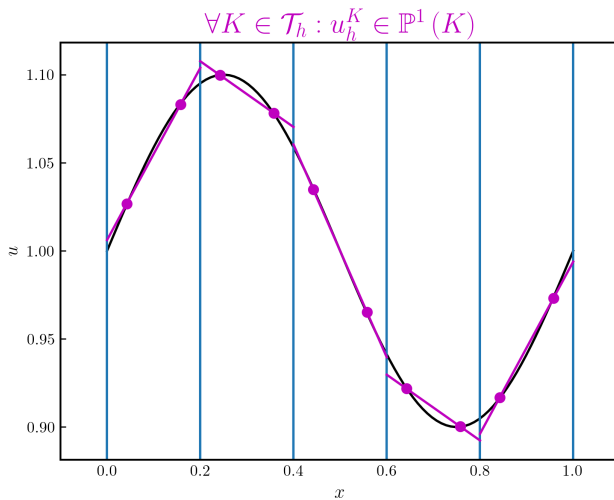
$$\Phi(r) := -\frac{M}{r}$$

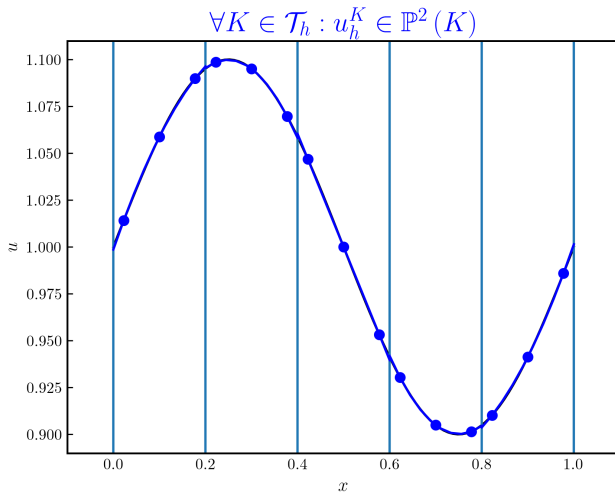
Discontinuous Galerkin (DG) Method



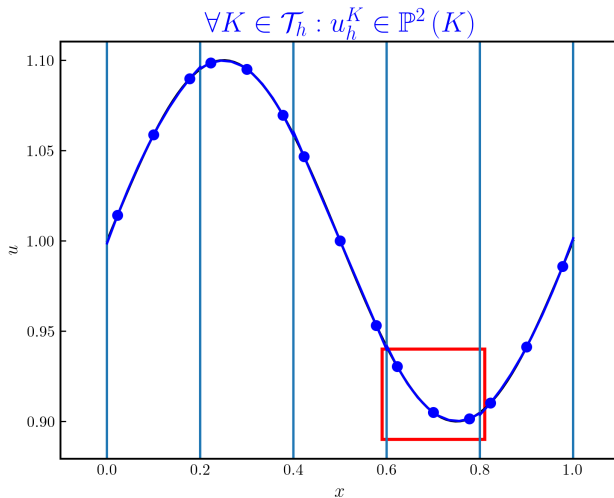
Finite-volume/DG(0)





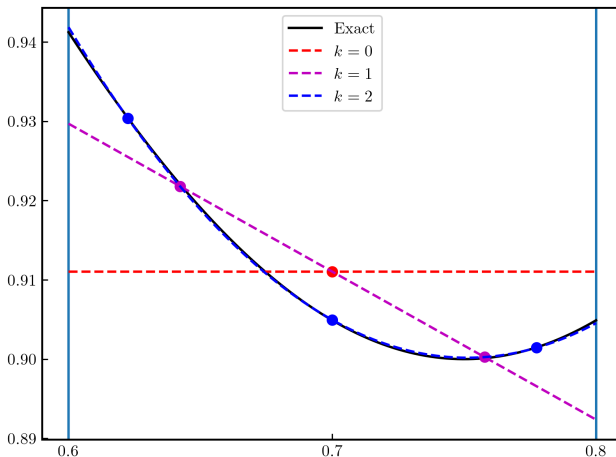


Discontinuous Galerkin (DG) Method

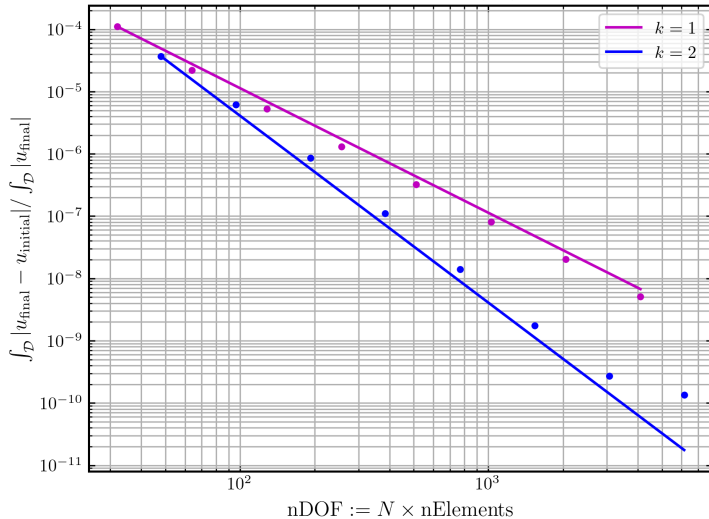


Discontinuous Galerkin (DG) Method

$$u_h(x, t) := \sum_{i=1}^{k+1} u_i(t) \ell_i(x)$$

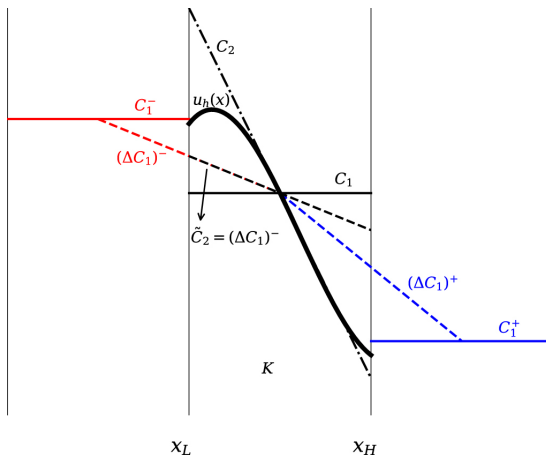


Convergence Rates for Sine Wave Advection (1D)

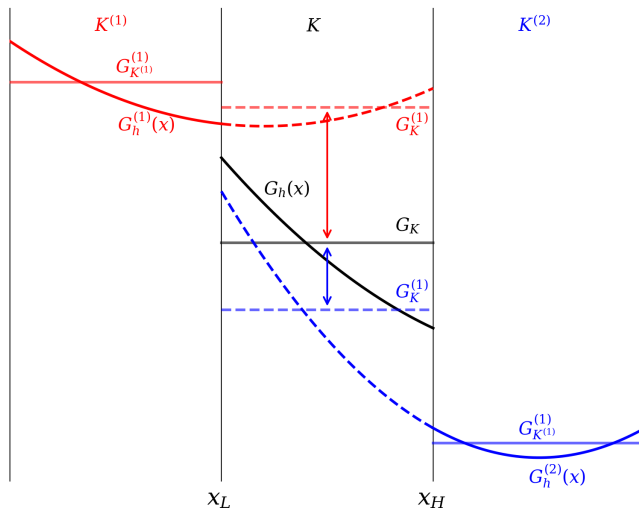


Slope Limiter (MinMod)

$$u_h(x, t) = \sum_{n=1}^N C_n(t) P_n(x) \implies \tilde{u}_h(x, t) = C_1(t) P_1(x) + \tilde{C}_2(t) P_2(x)$$

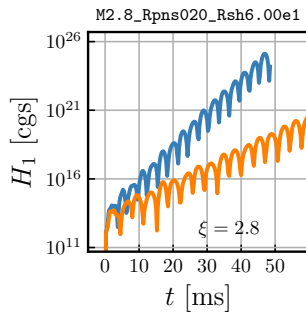
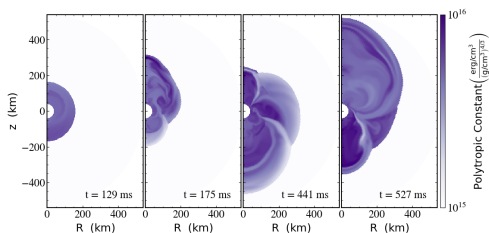


Troubled Cell Indicator



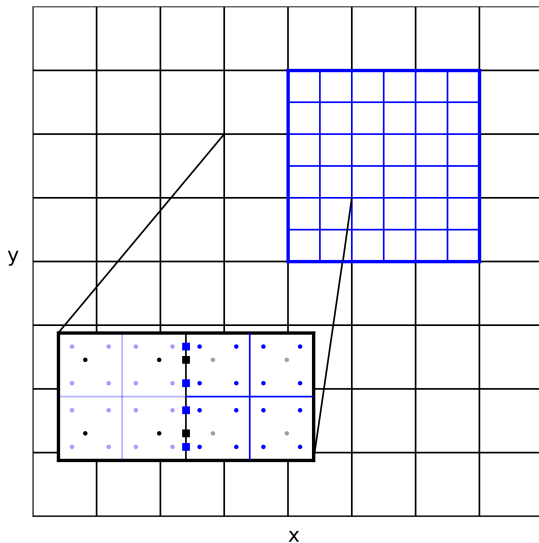
Standing Accretion Shock Instability

Used thornado to investigate the role of GR on the SASI¹

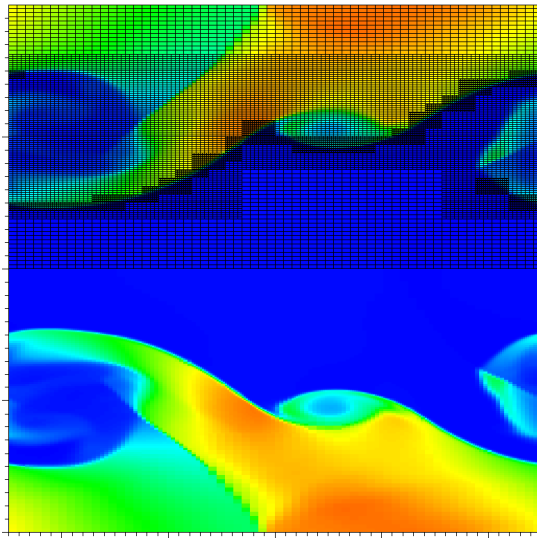


¹Dunham et al. (2020, 2023)

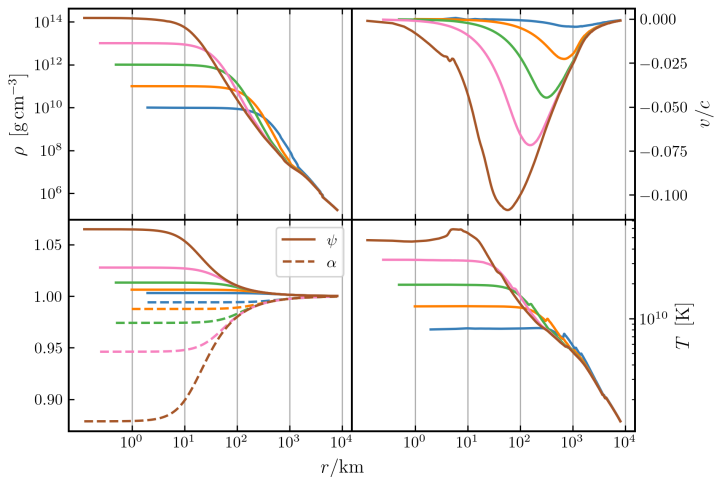
Mesh Refinement



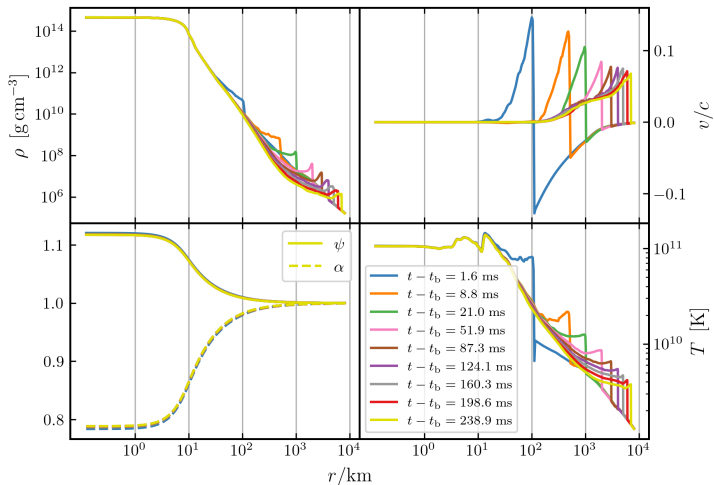
Kelvin–Helmholtz Instability



Adiabatic Collapse (AMR, Collapse Phase)



Adiabatic Collapse (AMR, Post-Bounce Phase)

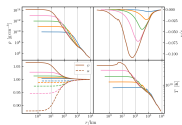
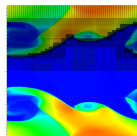


Bibliography

- Eirik Endeve, Jesse Buffaloe, Samuel J. Dunham, Nick Roberts, Kristopher Andrew, Brandon Barker, David Pochik, Juliana Pulsinelli, and Anthony Mezzacappa. thornado-hydro: towards discontinuous Galerkin methods for supernova hydrodynamics. In *Journal of Physics Conference Series*, volume 1225 of *Journal of Physics Conference Series*, page 012014, May 2019. doi: 10.1088/1742-6596/1225/1/012014.
- Samuel J. Dunham, E. Endeve, A. Mezzacappa, J. Buffaloe, and K. Holley-Bockelmann. A discontinuous Galerkin method for general relativistic hydrodynamics in thornado. In *Journal of Physics Conference Series*, volume 1623 of *Journal of Physics Conference Series*, page 012012, September 2020. doi: 10.1088/1742-6596/1623/1/012012.
- David Pochik, Brandon L. Barker, Eirik Endeve, Jesse Buffaloe, Samuel J. Dunham, Nick Roberts, and Anthony Mezzacappa. thornado-hydro: A Discontinuous Galerkin Method for Supernova Hydrodynamics with Nuclear Equations of State. *ApJS*, 253(1):21, March 2021. doi: 10.3847/1538-4365/abd700.
- M. Paul Laiu, Eirik Endeve, Ran Chu, J. Austin Harris, and O. E. Bronson Messer. A dg-imex method for two-moment neutrino transport: Nonlinear solvers for neutrino–matter coupling*. *The Astrophysical Journal Supplement Series*, 253(2):52, apr 2021. doi: 10.3847/1538-4365/abe2a8. URL <https://dx.doi.org/10.3847/1538-4365/abe2a8>.
- J. R. Wilson, G. J. Mathews, and P. Marronetti. Relativistic numerical model for close neutron-star binaries. *Phys. Rev. D*, 54(2):1317–1331, July 1996. doi: 10.1103/PhysRevD.54.1317.
- Isabel Cordero-Carrión, Pablo Cerdá-Durán, Harald Dimmelmeier, José Luis Jaramillo, Jérôme Novak, and Ericourgoulhon. Improved constrained scheme for the Einstein equations: An approach to the uniqueness issue. *Phys. Rev. D*, 79(2):024017, January 2009. doi: 10.1103/PhysRevD.79.024017.
- Samuel J. Dunham, Eirik Endeve, Anthony Mezzacappa, John M. Blondin, Jesse Buffaloe, and Kelly Holley-Bockelmann. A Parametric Study of the SASI Comparing General Relativistic and Non-Relativistic Treatments. *arXiv e-prints*, art. arXiv:2307.10904, July 2023. doi: 10.48550/arXiv.2307.10904.

Summary

Can run pure hydro problems in GR with AMR



Can run hydro+self-gravity problems in GR with AMR

Working on coupling GR transport to existing hydro+gravity modules

