

14-Moment-Inspired Visco-Resistive MHD

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July 28, 2025

Baryon number, momentum, and energy are conserved:

$$\nabla_{\mu} N^{\mu} = 0, \quad (1)$$

$$\nabla_{\mu} T^{\mu\nu} = 0, \quad (2)$$

where

$$T^{\mu\nu} = T_{\text{hydro}}^{\mu\nu} + T_{\text{dissipative}}^{\mu\nu} + T_{\text{EM}}^{\mu\nu}. \quad (3)$$

How to define N^{μ} and $T_{\text{hydro}}^{\mu\nu}$, etc.?

Viscous Hydrodynamics

Fluid-frame projector: $\Delta^{\mu\nu} := g^{\mu\nu} + u^\mu u^\nu$

Decompose momentum into components parallel and perpendicular to four-velocity: $p^\mu = E_p u^\mu + p^{\langle\mu\rangle}$, with $p^{\langle\mu\rangle} := \Delta^\mu_\nu p^\nu$

$$N^\mu = n u^\mu + \textcolor{red}{n}^\mu \quad (4)$$

$$\begin{aligned} T^{\mu\nu} = & (\rho + \rho \epsilon) u^\mu u^\nu + P \Delta^{\mu\nu} \\ & + \textcolor{red}{\Pi} \Delta^{\mu\nu} + \textcolor{red}{q}^\mu u^\nu + \textcolor{red}{q}^\nu u^\mu + \textcolor{red}{\pi}^{\mu\nu} \\ & + T_{\text{EM}}^{\mu\nu}, \end{aligned} \quad (5)$$

where

$$T_{\text{EM}}^{\mu\nu} := \frac{1}{4\pi} \left(F^{\mu\alpha} F_{\alpha}{}^{\nu} - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right) \quad (6)$$

$$\begin{aligned} &= \frac{1}{2} (u^\mu u^\nu + \Delta^{\mu\nu}) (e_\alpha e^\alpha + b_\alpha b^\alpha) - 2 u^{(\mu} b^{\nu)\alpha} e_\alpha - (e^\mu e^\nu + b^\mu b^\nu) \end{aligned} \quad (7)$$

$$\nabla_\mu \mathcal{J}^\mu = 0 \quad (8)$$

$$\mathcal{J}^\mu := \rho_q u^\mu + \textcolor{red}{J}^\mu \quad (9)$$

$$\nabla_\nu (J^\mu u^\nu) = -\frac{1}{\tau} (J^\mu - \eta^{-1} e^\mu) + \frac{\delta_{\text{VB}}}{\tau} \sqrt{b^2} b^{\mu\nu} J_\nu \quad (10)$$

$$\nabla_\mu \left[\left(\Pi + \frac{\zeta}{\tau_\Pi} \right) u^\mu \right] = S_\Pi \quad (11)$$

$$\nabla_\mu \left[q_\nu u^\mu + \frac{\kappa}{\tau_q} T \Delta^\mu{}_\nu \right] = (S_q)_\nu \quad (12)$$

$$\nabla_\mu \left[\pi^{\alpha\beta} u^\mu + 2 \frac{\eta}{\tau_\pi} g^{\mu(\alpha} u^{\beta)} \right] = (S_\pi)^{\alpha\beta} \quad (13)$$

$$\nabla_\mu [J_\nu u^\mu] = (S_J)_\nu \quad (14)$$

Primitive Variable Recovery

$$V = (\rho, W v^i, e, B^i, E^i, J_\nu, q^\mu, \pi^{\mu\nu}, \Pi)^T \quad (15)$$

$$U(V) = \begin{pmatrix} \rho W \\ \rho h W^2 v_j + \epsilon^{ijk} E_j B_k + \dots \\ \rho h W^2 - p - \rho W + \frac{1}{2} (E^2 + B^2) + \dots \\ B^i \\ E^i \\ J_\nu u^0 \\ q_\nu u^0 + \frac{\kappa}{\tau_q} T \Delta^0_\nu \\ \pi^{\mu\nu} u^0 + 2 \frac{\eta}{\tau_\pi} g^{0(\mu} u^{\nu)} \\ \left(\Pi + \frac{\zeta}{\tau_\Pi} \right) u^0 \end{pmatrix} \quad (16)$$

Problem: How to compute $V(U)$?

Primitive Variable Recovery

Never been solved before, so no literature to follow

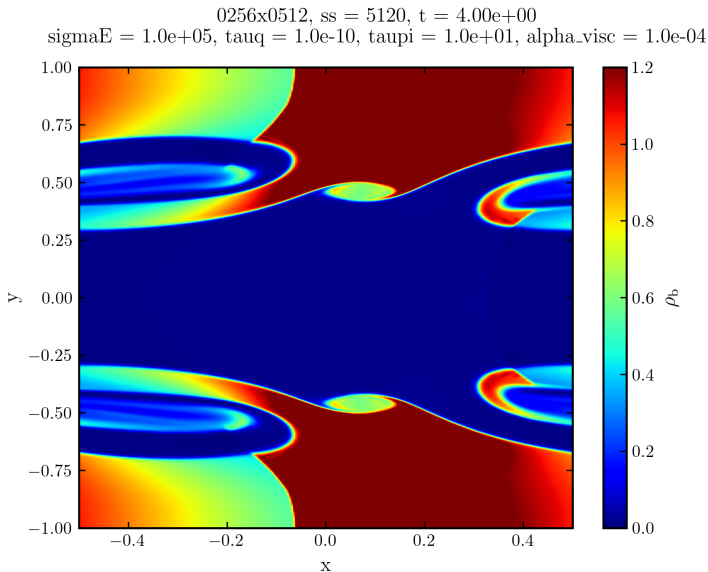
Strategy: Combine previous (known) methods for smaller systems and invent methods as needed

Issues:

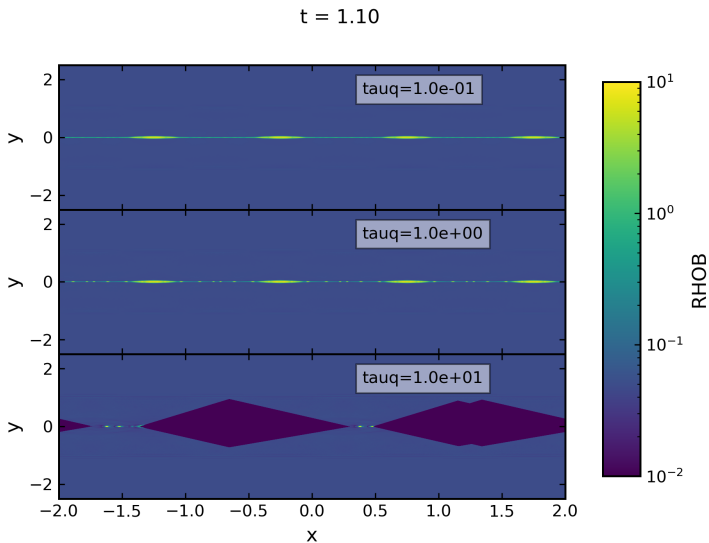
- Root-finder converges to incorrect solution
- Wrong temperature recovered for heat flux

- ➊ Guess v^i and $\rho h W$ (e.g. $v^i = 0$, $h W = 1$)
- ➋ Use these to compute guesses for thermo quantities (e , p , T , etc.)
- ➌ Use thermo to compute dissipation coefficients (ζ , κ , etc.)
- ➍ With these, recover all primitives (incl. updated v^i and $\rho h W$)
- ➎ Iterate steps 1-4 within Newton–Raphson algorithm
- ➏ If 5 fails to converge, repeat 1-5 using entropy to compute e (more stable, but invalid near shocks)
- ➐ If 6 fails to converge, use ideal (non-viscous, non-resistive) solution

What Works: Viscous, Non-Resistive, No heat-flux



What Doesn't Work: Viscous, Resistive



Tried So Far

- Setting floors
- Use better initial guess (e.g., ideal solution for v^i)
- Iterate over different variables (e.g., Poynting flux, pressure, $1/(h W)$)

What to Try Next?

- Check if $e < 0$ with zero heat flux; if so, add energy
- If $e > 0$ without heat flux and $e < 0$ with heat flux, then check T
- If T is bad, define $q^\nu = q^\nu(q^*, e)$; else, limit heat flux

- Developed solver for visco-resistive MHD equations
- Shown that the model works and shown the influence of visco-resistivity
- Make con2prim algorithm more robust

Boltzmann–Vlasov Equation

Four-momentum vector: $p^\mu = (E_{\mathbf{p}}, \mathbf{p})^\mu$, with $E_{\mathbf{p}} := -u \cdot p$

Faraday tensor: $F^{\mu\nu} := u^\mu e^\nu - u^\nu e^\mu + \epsilon^{\mu\nu\alpha\beta} b_\alpha u_\beta$

Distribution function: $f = f(x, p)$

$$p^\mu \partial_\mu f + \left(q F^{\mu\nu} p_\nu + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \right) \partial_{p^\mu} f = C[f] \quad (17)$$

[?]

Moments of Boltzmann–Vlasov equation lead to definitions of N^μ and $T_{\text{hydro}}^{\mu\nu} + T_{\text{dissipative}}^{\mu\nu}$:

$$N^\mu := \int \frac{g d^3p}{(2\pi)^3 E_p} p^\mu f \quad (18)$$

$$T_{\text{hydro}}^{\mu\nu} + T_{\text{dissipative}}^{\mu\nu} := \int \frac{g d^3p}{(2\pi)^3 E_p} p^\mu p^\nu f \quad (19)$$

[?]

With $p^\mu = E_p u^\mu$ and $f = (\exp((E_p - \mu)/T))^{-1}$, these become the hydro part of the ideal magnetohydrodynamics equations and $T_{\text{dissipative}}^{\mu\nu} = 0$



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