3.2.3 WeakLib Opacities

Opacity tables with all electron-type neutrino and antineutrino interaction rates based on (Bruenn, 1985) had been created using `WeakLib`. (And bremsstrahlung) Following Bruenn using a multigroup flux-limited diffusion approximation (MGFLDA), the Boltzmann transport equations of the 0th-order moment and the 1st-order moment are (Equation (A12) in Bruenn (1985))

$$\begin{aligned}
&\frac{1}{c}\frac{\partial}{\partial t}\psi^{(0)} + \frac{1}{3r^2}\frac{\partial}{\partial r}\left(r^2\psi^{(1)}\right) + \frac{1}{3c}\frac{\partial \ln\rho}{\partial t}\left(\omega\frac{\partial}{\partial\omega}\psi^{(0)}\right) \\
&= j(\omega)\left(1-\psi^{(0)}\right) - \Psi^{(0)}/\lambda^{(a)}(\omega) + A^{(0)}(\omega)\psi^{(0)} + B^{(0)}(\omega)\psi^{(1)} + C^{(0)}(\omega),
\end{aligned} \tag{3.36}$$

and (A13)

$$\begin{aligned}
&\frac{\partial}{\partial r}\psi^{(0)} + \left[-\frac{2}{c}\frac{\partial\ln\rho}{\partial t}\psi^{(1)} - \frac{2v}{cr}\psi^{(1)} + \frac{3}{5c}\frac{\partial\ln\rho}{\partial t}\frac{1}{\omega^3}\frac{\partial}{\partial\omega}\left(\omega^4\psi^{(1)}\right) + \frac{4}{5}\frac{v}{cr}\frac{1}{\omega^3}\frac{\partial}{\partial\omega}\left(\omega^4\psi^{(1)}\right)\right] \\
&= -\left[j(\omega) + 1/\lambda^{(a)}(\omega)\right]\psi^{(1)} + A^{(1)}(\omega)\psi^{(0)} + B^{(1)}(\omega)\psi^{(1)} + C^{(1)}(\omega),
\end{aligned} \tag{3.37}$$

respectively.

Neutrino Absorption on Nucleons and Nuclei (AbEm)

Neutrino absorptivity on nucleons and nuclei, which is $\left[j(\omega)+1/\lambda^{(a)}(\omega)\right]$ on the right-hand-side of Equation (3.37), is tabulated in `WeakLib` AbEm table in unit of cm^{-1}. It's also knons as inverse mean free path on free nucleona and nuclei. AbEm tables are four-dimensional table with (ω, ρ, T, Y_e). To interpolate the AbEm opacity with a given ω array for (ρ_i, T_i, Y_{ei}) state, one can

```
CALL LogInterpolateSingleVariable_1D3D_Custom
```

The interpolated value, $\left[j(\omega)+1/\lambda^{(a)}(\omega)\right]$, is ploted directly in example plot Figure 3.1.

Neutrino Isoenergetic Scattering on Nucleons and Nuclei (Iso)

The 0-th and 1-st order of Legendre coefficients of neutrino isoenergetic scattering kernel on nucleons and nuclei, which are $\Phi_{0,\mathrm{IS}}$ and $\Phi_{1,\mathrm{IS}}$ with $\frac{4\pi}{c(2\pi\hbar c)^3}\omega^2$ and the coefficents 1/2 and 3/2, respectively, are tabulated in `WeakLib` Iso table in unit of cm^{-1}. So that (Equation (A41) in Bruenn (1985))

$$B^{(1)}_{\mathrm{IS}}(\omega) = -\frac{2\pi}{c(2\pi\hbar c)^3}\omega^2\left[\Phi_{1,\mathrm{IS}}(\omega) - \Phi_{0,\mathrm{IS}}(\omega)\right], \tag{3.38}$$

which is the only non-zero term of $A^{(\alpha)}_{\mathrm{IS}}(\omega), B^{(\alpha)}_{\mathrm{IS}}(\omega),$ and $C^{(\alpha)}_{\mathrm{IS}}(\omega)$, is given by

$$\mathrm{TableValue}_0 - \frac{\mathrm{TableValue}_1}{3}.$$

Iso tables are five-dimensional table with $(\omega, l, \rho, T, Y_e)$, where l is the order index of the Legendre coefficients: $l=1$ for 0-th order ($\mathrm{TableValue}_0$) and $l=2$ for 1-st order ($\mathrm{TableValue}_1$). To interpolate the Iso opacity with a given ω array for (ρ_i, T_i, Y_{ei}) state, one can

```
CALL LogInterpolateSingleVariable_1D3D_Custom
```

once if only needs 0-th order, and twice if needs both 0-th and 1-st order Legendre coefficients of neutrino isoenergetic scattering kernel. $-B^{(1)}_{\mathrm{IS}}(\omega)$ is plotted in example plot Figure 3.1.

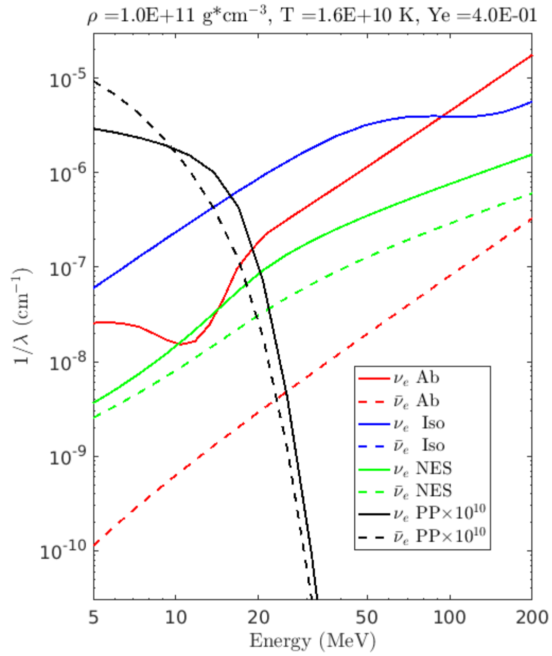

Figure 3.1: Example of `WeakLib` opacities. Adapted from Bruenn (1985) Figure 36(a).

Neutrino-Electron Scattering (NES)

The coefficient function H_l^I and H_l^{II} with $l = 0, 1$ of the first two Legendre coefficients of neutrino-electron scattering kernel with $\frac{2\pi}{c(2\pi\hbar c)^3}$ and $\int dE_e F_e(E_e)\left[1 - F_e(E_e + \omega - \omega')\right]$ are tabulated in `WeakLib` NES table. Following Bruenn (1985) Equation (C50),

$$\Phi_{l,\mathrm{NES}}^{\left\{\begin{array}{c}\mathrm{in}\\\mathrm{out}\end{array}\right\}} = \frac{G^2}{\pi\omega^2\omega'^2}\int dE_e F_e(E_e)\left[1 - F_e(E_e + \omega - \omega')\right]\left\{\begin{array}{c}\exp\left[-\beta(\omega - \omega')\right]\\ 1\end{array}\right\}$$
$$\times\left[(C_V + C_A)^2 H_l^{\mathrm{I}}(\omega, \omega', E_e) + (C_V - C_A)^2 H_l^{\mathrm{II}}(\omega, \omega', E_e)\right], \tag{3.39}$$

and Equation (A38)

$$B_{\mathrm{NES}}^{(1)}(\omega) = -\frac{2\pi}{c(2\pi\hbar c)^3}\int_0^\infty \omega'^2 d\omega' \left\{\Phi_{0,\mathrm{NES}}^{\mathrm{in}}(\omega, \omega')\,\psi^{(0)}(\omega') + \Phi_{0,\mathrm{NES}}^{\mathrm{out}}(\omega, \omega')\left[1 - \psi^{(0)}(\omega')\right]\right\}. \tag{3.40}$$

NES tabls are five-dimensional table with $(\omega', \omega, l, T, \eta)$, where $\eta = \mu_e/k_B T$. To interpolate the NES opacity with a same given energy array for ω and ω' and state (ρ_i, T_i, Y_{ei}), one needs to interpolate for η first. Then interpolate for $H_l^{\mathrm{I/II}}$ by

```
CALL LogInterpolateSingleVariable_2D2D_Custom
```

Next, he/she needs to assemble $\Phi_{l,\mathrm{NES}}^{\mathrm{In/Out}}$ using the corresponding coefficients C_V and C_A and integrate over electron energy E_e. $-B_{\mathrm{NES}}^{(1)}(\omega)$ is plotted with $\psi^{(0)}(\omega') = 0$ in example plot Figure 3.1.

Neutrino Production from Pair Process (Pair)

The coefficient function J_l^I and J_l^{II} with $l = 0, 1$ of the first two Legendre coefficients of pair process kernel with $\frac{2\pi}{c(2\pi\hbar c)^3}$ and $\int_0^{\omega+\omega^+} dE_e F_e(E_e)[1 - F_e(E_e)][1 - F_{e^+}(\omega + \omega' - E_e)]$ (annihilation coefficient) are tabulated in `WeakLib` Pair table. Following Bruenn (1985) (A62), the Legendre coefficients of the pair process kernels are

$$\Phi_{l,\mathrm{TP}}^{\left\{\begin{array}{c} p \\ a \end{array}\right\}} = \frac{2G^2}{2\pi} \int_0^{\omega+\omega^+} dE_e \left\{ \begin{array}{c} F_e(E_e) F_{e^+}(\omega + \omega' - E_e) \\ [1 - F_e(E_e)][1 - F_{e^+}(\omega + \omega' - E_e)] \end{array} \right\}$$
$$\times (C_V + C_A)^2 J_l^{\mathrm{I}}(\omega, \omega', E_e) + (C_V - C_A)^2 J_l^{\mathrm{II}}(\omega, \omega', E_e) \tag{3.41}$$

and (A47)

$$B_{\mathrm{TP}}^{(1)}(\omega) = -\frac{2\pi}{c(2\pi\hbar c)^3} \int_0^{\infty} \omega'^2 d\omega' \left\{ \Phi_{0,\mathrm{TP}}^{p}(\omega, \omega') \left[1 - \bar{\psi}^{(0)}(\omega')\right] + \Phi_{0,\mathrm{TP}}^{a}(\omega, \omega') \bar{\psi}^{(0)}(\omega') \right\}. \tag{3.42}$$

Same as NES table, Pair tabls are five-dimensional table with $(\omega', \omega, l, T, \eta)$. And the interpolation is also the same as NES: find the η first,

```
CALL LogInterpolateSingleVariable_2D2D_Custom
```

then assemble and integral. $-B_{\mathrm{TP}}^{(1)}(\omega)$ is plotted with $\bar{\psi}^{(0)}(\omega') = 0$ in example plot Figure 3.1.

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