Mustererkennung/Machine Learning - Assignment 7 - Bonus

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Load the spam dataset:

In [1]:

```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
from sklearn.model_selection import train_test_split
from skimage.data import lfw_subset, astronaut
from sklearn.datasets import fetch_openml
from sklearn.metrics import confusion_matrix
import random
from tqdm import tqdm
from PIL import Image
import requests
```

The Decision Tree implementation, which will be used in AdaBoost

In [2]:

```
def accuracy(y_true, y_pred):
    return np.mean(y_true == y_pred)

def gini(y_true, c):
    """
    For simplicity reasons this assumes that there are only 2 classes
    """
    y_true = np.array(y_true)
    p_mk = np.mean(y_true == c)

    return 2 * p_mk * (1 - p_mk)

class LeafNode():
    def fit(self, c):
        self.c = c

    def predict(self, x):
        return self.c

class InternalNode():
    """
    Node of decision tree, which accepts tabular data
    """
    def fit(self, x, y, depth, max_depth):
        m, n = x.shape
        # columns are j, split_index, loss_total
        split_infos = []

        for j in range(n):
            # sort rows by feature j in ascending order
            sorted_indices = x[:,j].argsort()
            x, y = x[sorted_indices], y[sorted_indices]
            for split_index in self.get_unique_indices(x[:, j])[:-1]:
                y_top_split = y[:split_index + 1]
                y_bottom_split = y[split_index + 1:]

                if y_top_split.shape[0] == 0:
                    raise Exception("Error 1")

                if y_bottom_split.shape[0] == 0:
                    raise Exception("Error 2")

                c1 = self.find_c(y_top_split)
                c2 = self.find_c(y_bottom_split)

                loss_1 = gini(y_top_split, c1)
                loss_2 = gini(y_bottom_split, c2)

                # use weighted average which had better results
                loss_total = (y_top_split.shape[0] / m) * loss_1  + (y_bottom_split

                row = np.array([j, split_index, loss_total])
                split_infos.append(row)

        split_infos = np.array(split_infos)
        best_split_idx = np.argmin(split_infos[:,-1], axis=0)
        best_split = split_infos[best_split_idx]
        self.j = int(best_split[0])
        split_index = int(best_split[1])
```

```
        sorted_indices = x[:,j].argsort()
        x, y = x[sorted_indices], y[sorted_indices]

        x_top_split, y_top_split = x[:split_index + 1], y[:split_index + 1]
        x_bottom_split, y_bottom_split = x[split_index + 1:], y[split_index + 1:]

        self.z = (x_top_split[-1, self.j] + x_bottom_split[0, self.j]) / 2

        if self.is_pure(y_top_split) or x_top_split.shape[0] <= 2 or depth >= max_d
            self.left_child = LeafNode()
            c = self.find_c(y_top_split)
            self.left_child.fit(c)
        else:
            self.left_child = InternalNode()
            self.left_child.fit(x_top_split, y_top_split, depth + 1, max_depth)

        if self.is_pure(y_bottom_split) or x_bottom_split.shape[0] <= 2 or depth >=
            self.right_child = LeafNode()
            c = self.find_c(y_bottom_split)
            self.right_child.fit(c)
        else:
            self.right_child = InternalNode()
            self.right_child.fit(x_bottom_split, y_bottom_split, depth + 1, max_dep

    def predict(self, x):
        if x[self.j] <= self.z:
            return self.left_child.predict(x)
        return self.right_child.predict(x)

    def get_unique_indices(self, arr):
        idx = []
        arr_len = len(arr)
        for i in range(len(arr)):
            if i == arr_len - 1:
                idx.append(i)
            elif arr[i] != arr[i + 1]:
                idx.append(i)
        return idx

    def find_c(self, y):
        """
        For simplicity reasons this assumes that there are only 2 classes
        """
        y = np.array(y)
        zeros = np.sum(y == 0)
        ones = np.sum(y == 1)
        if ones > zeros:
            return 1
        return 0

    def is_pure(self, y):
        y = np.array(y)
        if np.sum(y == 0) == y.shape[0] or np.sum(y == 1) == y.shape[0]:
            return True
        return False

class DecisionTreeClassifier():
    """
    Basically just holds the root node of the tree which starts the recursion
    """
```

```
    def __init__(self, max_depth):
        self.max_depth = max_depth

    def fit(self, x, y, features):
        x = np.copy(x)
        y = np.copy(y)
        self.root = InternalNode()
        self.root.fit(x, y, 1, self.max_depth)

    def predict(self, x):
        y_preds = []
        for sample in x:
            y_pred = self.root.predict(sample)
            y_preds.append(y_pred)
        return np.array(y_preds)
```

In [3]:

```
class AdaBoost():
    def __init__(self, num_trees, max_depth):
        self.num_trees = num_trees
        self.max_depth = max_depth

    def fit(self, x, y, features=None):
        x = np.copy(x)
        y = np.copy(y)
        self.trees = []
        self.says = []
        for i in tqdm(range(self.num_trees)):
            # sample weights will always add up to one
            sample_weights = np.ones((y.shape[0], )) / y.shape[0]
            tree = DecisionTreeClassifier(max_depth=self.max_depth)
            tree.fit(x, y, features)
            y_pred = tree.predict(x)
            error = self.calculate_error(y, y_pred, sample_weights)
            say = self.error_to_say(error)
            sample_weights = self.update_sample_weights(y, y_pred, say, sample_weig
            x, y = self.weighted_dataset(x, y, sample_weights)
            self.trees.append(tree)
            self.says.append(say)

        self.says = np.array(self.says)

    def weighted_dataset(self, x, y, sample_weights):
        x_new, y_new = [], []
        sample_weights_cum = np.cumsum(sample_weights)
        rand = np.random.uniform(low=0.0, high=1.0, size=(x.shape[0], ))
        for rand_el in rand:
            for i, cum_weight in enumerate(sample_weights_cum):
                if cum_weight >= rand_el:
                    x_new.append(x[i])
                    y_new.append(y[i])
                    break
        return np.array(x_new), np.array(y_new)



    def calculate_error(self, y_true, y_pred, sample_weights):
        """
        How much say a stump has is calculated by it's error, which is just the
        sum of the sample_weights for the missclassified samples.
        The error is always between 0 and 1 because the sample weights add up to on
        0 is the lowest possible error and 1 is the highest.
        """
        error_idx = y_true != y_pred
        return np.sum(sample_weights[error_idx])

    def error_to_say(self, error):
        """
        Transforms the error a stump has into it's say which will be used
        to weight the importance of one stumps prediction in the final prediction.
        The say is ~ between 3.5 and -3.5 which means a stumps prediction can actua
        be weighted negaively in the final prediction if it error is high.
        If error is 0 we will have division by 0, if error is 1, we will have log(0
        which is also not possible. So a small eps is added / subtracted from the
        error to keep calculations stable.
        """
```

```
        eps = 10 ** -10
        if error == 0:
            error = error + eps
        elif error == 1:
            error = error - eps
        return 0.5 * np.log((1 - error) / error)

    def update_sample_weights(self, y_true, y_pred, say, sample_weights):
        """
        Updates the sample weights by scaling them based on the amount of say the s
        has and wether it properly classified the sample.
        If say is high and the sample was missclassified, the sample weight will go
        If say is high and the sample was propely classified, the sample weight wil
        If say is low and the sample was missclassified, the sample weight will go
        If say is low and the sample was properly classified, the sample weight wil
        After updating the sample weights will still sum up to one.
        """
        sample_weights = np.where(y_true == y_pred,
                                  sample_weights * np.exp(-say),
                                  sample_weights * np.exp(say))
        # normalization so sample_weights add up to 1 again
        sample_weights = sample_weights / np.sum(sample_weights)
        return sample_weights

    def predict(self, x, use_cascade=False):
        y_preds = []
        if use_cascade:
            for sample in x:
                votes = np.array([])
                made_prediction = False
                for tree in self.trees:
                    # decision tree expects matrix as input
                    sample = sample.reshape((1, -1))
                    prediction = tree.predict(sample)
                    votes = np.concatenate((votes, prediction))
                    yes_say = np.sum(self.says[:len(votes)][votes == 1])
                    no_say = np.sum(self.says[:len(votes)][votes == 0])
                    if no_say >= yes_say:
                        y_preds.append(0)
                        made_prediction = True
                        break
                if not made_prediction:
                    y_preds.append(1)
        else:
            for sample in x:
                votes = np.array([])
                for tree in self.trees:
                    # decision tree expects matrix as input
                    sample = sample.reshape((1, -1))
                    prediction = tree.predict(sample)
                    votes = np.concatenate((votes, prediction))
                yes_say = np.sum(self.says[votes == 1])
                no_say = np.sum(self.says[votes == 0])
                if yes_say > no_say:
                    y_preds.append(1)
                else:
                    y_preds.append(0)

        return np.array(y_preds)
```

Excercise 2 (Bonus). Viola-Jones Face Detection

Implement the Viola-Jones algorithm (without the cascade mechanism) and use it on a LFW-Face-subset to classify faces.

In [4]:

```
def generate_dataset():
    """
    Generates dataset of 250 images of faces with labels 1 and 250 images of digits
    each are 25x25 pixels. If add_negative_samples is set, it will add an additiona
    samples, sampled from a uniform distribution.
    """
    x = lfw_subset()
    face_labels = np.ones((100, ))
    non_face_labels = np.zeros((100, ))
    y = np.hstack((face_labels, non_face_labels))
    assert(x.shape[0] == y.shape[0])
    return x, y
```

In [5]:

```
class RectBase():
    def calculate_area(self, integral_img,  ul_pixel_cord, lr_pixel_cord):
        """ Calculates the of the integral_img given by two pixel coordinates.
        ul_pixel_cord should be outside of the calculated area.
        lr_pixel_cord is inside of the calculated area.
        """
        if ul_pixel_cord[0] < 0 or ul_pixel_cord[1] < 0:
            A = 0
        else:
            A = integral_img[ul_pixel_cord[0], ul_pixel_cord[1]]

        if ul_pixel_cord[0] < 0:
            B = 0
        else:
            B = integral_img[ul_pixel_cord[0], lr_pixel_cord[1]]

        if ul_pixel_cord[1] < 0:
            C = 0
        else:
            C = integral_img[ul_pixel_cord[1], lr_pixel_cord[0]]

        D = integral_img[lr_pixel_cord[0], lr_pixel_cord[1]]
        return A - B - C + D


class TwoRect(RectBase):
    """ Feature for Viola-Jones, which looks like:
    -----
    --B--
    --W--
    -----
    """
    def __init__(self, x, y, scale_x, scale_y, img_shape):
        self.x = x
        self.y = y
        self.scale_x = scale_x
        self.scale_y = scale_y
        self.img_shape = img_shape

    def is_valid(self):
        """
        Returns wether the feature fits on the image
        """
        lowest_pixel = self.x + (self.scale_x * 2) - 1
        if lowest_pixel > self.img_shape[0] - 1:
            return False
        most_right_pixel = self.y + self.scale_y - 1
        if most_right_pixel > self.img_shape[1] - 1:
            return False
        return True

    def get_pixel_cords(self):
        black_area_ul = [self.x - 1, self.y - 1]
        black_area_lr = [self.x + self.scale_x - 1, self.y + self.scale_y - 1]
        white_area_ul = [self.x + self.scale_x - 1, self.y - 1]
        white_area_lr = [self.x + (self.scale_x * 2) - 1, self.y + self.scale_y - 1
        return {
            "black_area_ul": black_area_ul,
            "black_area_lr": black_area_lr,
```

```
            "white_area_ul": white_area_ul,
            "white_area_lr": white_area_lr
        }

    def apply_feature(self, integral_img):
        pixel_cords = self.get_pixel_cords()
        black_area_sum = self.calculate_area(integral_img, pixel_cords["black_area_
                                             pixel_cords["black_area_lr"])
        white_area_sum = self.calculate_area(integral_img, pixel_cords["white_area_
                                             pixel_cords["white_area_lr"])
        return white_area_sum - black_area_sum

    def visualise_feature(self):
        img = np.ones((self.img_shape[0], self.img_shape[1])) * 0.5
        pixel_cords = self.get_pixel_cords()
        for x in range(pixel_cords["black_area_ul"][0] + 1, pixel_cords["black_area
            for y  in range(pixel_cords["black_area_ul"][1] + 1, pixel_cords["black
                img[x, y] = 0
        for x in range(pixel_cords["white_area_ul"][0] + 1, pixel_cords["white_area
            for y  in range(pixel_cords["white_area_ul"][1] + 1, pixel_cords["white
                img[x, y] = 1
        return img

class ThreeRect(RectBase):
    """ Feature for Viola-Jones, which looks like:
    -----
    -BWB-
    -----
    """
    def __init__(self, x, y, scale_x, scale_y, img_shape):
        self.x = x
        self.y = y
        self.scale_x = scale_x
        self.scale_y = scale_y
        self.img_shape = img_shape

    def is_valid(self):
        """
        Returns wether the feature fits on the image
        """
        lowest_pixel = self.x + self.scale_x - 1
        if lowest_pixel > self.img_shape[0] - 1:
            return False
        most_right_pixel = self.y + (self.scale_y * 3) - 1
        if most_right_pixel > self.img_shape[1] - 1:
            return False
        return True

    def get_pixel_cords(self):
        left_black_area_ul = [self.x - 1, self.y - 1]
        left_black_area_lr = [self.x + self.scale_x - 1, self.y + self.scale_y - 1]
        white_area_ul = [self.x - 1, self.y + self.scale_y - 1]
        white_area_lr = [self.x + self.scale_x - 1, self.y + self.scale_y * 2 - 1]
        right_black_area_ul = [self.x - 1, self.y + self.scale_y * 2 - 1]
        right_black_area_lr = [self.x + self.scale_x - 1, self.y + (self.scale_y *
        return {
            "left_black_area_ul": left_black_area_ul,
            "left_black_area_lr": left_black_area_lr,
            "white_area_ul": white_area_ul,
            "white_area_lr": white_area_lr,
            "right_black_area_ul": right_black_area_ul,
```

```
            "right_black_area_lr": right_black_area_lr
        }

    def apply_feature(self, integral_img):
        pixel_cords = self.get_pixel_cords()
        left_black_area_sum = self.calculate_area(integral_img, pixel_cords["left_b
                                                  pixel_cords["left_black_area_lr"])
        white_area_sum = self.calculate_area(integral_img, pixel_cords["white_area_
                                                  pixel_cords["white_area_lr"])
        right_black_area_sum = self.calculate_area(integral_img, pixel_cords["right
                                                  pixel_cords["right_black_area_lr"])
        return white_area_sum - (left_black_area_sum + right_black_area_sum)

    def visualise_feature(self):
        img = np.ones((self.img_shape[0], self.img_shape[1])) * 0.5
        pixel_cords = self.get_pixel_cords()
        for x in range(pixel_cords["left_black_area_ul"][0] + 1, pixel_cords["left_
            for y  in range(pixel_cords["left_black_area_ul"][1] + 1, pixel_cords["
                img[x, y] = 0
        for x in range(pixel_cords["white_area_ul"][0] + 1, pixel_cords["white_area
            for y  in range(pixel_cords["white_area_ul"][1] + 1, pixel_cords["white
                img[x, y] = 1
        for x in range(pixel_cords["right_black_area_ul"][0] + 1, pixel_cords["righ
            for y  in range(pixel_cords["right_black_area_ul"][1] + 1, pixel_cords[
                img[x, y] = 0
        return img

class FourRect(RectBase):
    """ Feature for Viola-Jones, which looks like:
    ------
    --WB--
    --BW--
    ------
    """
    def __init__(self, x, y, scale_x, scale_y, img_shape):
        self.x = x
        self.y = y
        self.scale_x = scale_x
        self.scale_y = scale_y
        self.img_shape = img_shape

    def is_valid(self):
        """
        Returns wether the feature fits on the image
        """
        lowest_pixel = self.x + self.scale_x * 2 - 1
        if lowest_pixel > self.img_shape[0] - 1:
            return False
        most_right_pixel = self.y + self.scale_y * 2 - 1
        if most_right_pixel > self.img_shape[1] - 1:
            return False
        return True

    def get_pixel_cords(self):
        left_white_area_ul = [self.x - 1, self.y - 1]
        left_white_area_lr = [self.x + self.scale_x - 1, self.y + self.scale_y - 1]
        left_black_area_ul = [self.x + self.scale_x - 1, self.y - 1]
        left_black_area_lr = [self.x + self.scale_x * 2 - 1, self.y + self.scale_y
        right_black_area_ul = [self.x - 1, self.y + self.scale_y - 1]
        right_black_area_lr = [self.x + self.scale_x - 1, self.y + self.scale_y * 2
        right_white_area_ul = [self.x + self.scale_x - 1, self.y + self.scale_y - 1
```

```
        right_white_area_lr = [self.x + self.scale_x * 2 - 1, self.y + self.scale_y
        return {
            "left_white_area_ul": left_white_area_ul,
            "left_white_area_lr": left_white_area_lr,
            "left_black_area_ul": left_black_area_ul,
            "left_black_area_lr": left_black_area_lr,
            "right_black_area_ul": right_black_area_ul,
            "right_black_area_lr": right_black_area_lr,
            "right_white_area_ul": right_white_area_ul,
            "right_white_area_lr": right_white_area_lr
        }

    def apply_feature(self, integral_img):
        pixel_cords = self.get_pixel_cords()
        left_white_area_sum = self.calculate_area(integral_img, pixel_cords["left_w
                                                  pixel_cords["left_white_area_lr"])
        left_black_area_sum = self.calculate_area(integral_img, pixel_cords["left_b
                                                  pixel_cords["left_black_area_lr"])
        right_black_area_sum = self.calculate_area(integral_img, pixel_cords["right
                                                  pixel_cords["right_black_area_lr"])
        right_white_area_sum = self.calculate_area(integral_img, pixel_cords["right
                                                  pixel_cords["right_white_area_lr"])
        return left_white_area_sum - left_black_area_sum - right_black_area_sum + r

    def visualise_feature(self):
        img = np.ones((self.img_shape[0], self.img_shape[1])) * 0.5
        pixel_cords = self.get_pixel_cords()
        for x in range(pixel_cords["left_white_area_ul"][0] + 1, pixel_cords["left_
            for y  in range(pixel_cords["left_white_area_ul"][1] + 1, pixel_cords["
                img[x, y] = 1
        for x in range(pixel_cords["left_black_area_ul"][0] + 1, pixel_cords["left_
            for y  in range(pixel_cords["left_black_area_ul"][1] + 1, pixel_cords["
                img[x, y] = 0
        for x in range(pixel_cords["right_black_area_ul"][0] + 1, pixel_cords["righ
            for y  in range(pixel_cords["right_black_area_ul"][1] + 1, pixel_cords[
                img[x, y] = 0
        for x in range(pixel_cords["right_white_area_ul"][0] + 1, pixel_cords["righ
            for y  in range(pixel_cords["right_white_area_ul"][1] + 1, pixel_cords[
                img[x, y] = 1
        return img
```

In [6]:

```
class ViolaJonesClassifier():
    def __init__(self, num_trees, max_depth, max_features=None):
        self.adaboost = AdaBoost(num_trees, max_depth)
        self.max_features = max_features

    def fit(self, x, y):
        x = np.copy(x)
        y = np.copy(y)
        features = self.generate_features(x[0].shape)
        if self.max_features != None:
            random.shuffle(features)
            features = features[:self.max_features]
        x = self.transform_x(x, features)
        self.adaboost.fit(x, y)
        self.features = features

    def predict(self, x, use_cascade=False):
        x = np.copy(x)
        x = self.transform_x(x, self.features)
        return self.adaboost.predict(x, use_cascade)

    def transform_x(self, x, features):
        imgs = []
        for sample in x:
            imgs.append(self.to_integral_img(sample))
        x = np.array(imgs)
        x = self.apply_features(x, features)
        return x

    def apply_features(self, integral_imgs, features):
        """
        Transforms the integral images into a tabular dataset of shape (num_imgs, n
        where [i, j] corresponds to the j-th feature applied to the i-th img
        """
        x_new = np.zeros((integral_imgs.shape[0], len(features)))
        for row in range(x_new.shape[0]):
            for feature in range(len(features)):
                x_new[row, feature] = features[feature].apply_feature(integral_imgs
        return x_new

    def to_integral_img(self, img):
        """
        Transforms a batch of images into integral images
        """
        img = img.copy()
        for row in range(img.shape[0]):
            for col in range(img.shape[1]):
                new_val = img[row, col]
                if row != 0:
                    new_val += img[row - 1, col]
                if col != 0:
                    new_val += img[row, col - 1]
                if row != 0 and col != 0:
                    new_val -= img[row - 1, col - 1]
                img[row, col] = new_val
        return img

    def generate_features(self, img_shape):
        """
```

```
        Generates a list of haar features
        """
        features = []
        for x in range(img_shape[0]):
            for y in range(img_shape[1]):
                for scale_x in range(1, img_shape[0]):
                    for scale_y in range(1, img_shape[1]):
                        feat = TwoRect(x, y, scale_x, scale_y, img_shape)
                        if feat.is_valid():
                            features.append(feat)
                        feat = ThreeRect(x, y, scale_x, scale_y, img_shape)
                        if feat.is_valid():
                            features.append(feat)
                        feat = FourRect(x, y, scale_x, scale_y, img_shape)
                        if feat.is_valid():
                            features.append(feat)
        return features

    def slide_over_img(self, img, slide_shape, stride, use_cascade):
        marked_img = img.copy()
        for x in range(0, img.shape[0] - slide_shape[0], stride):
            for y in range(0, img.shape[1] - slide_shape[1], stride):
                cur_window = img[x:x + slide_shape[0], y:y + slide_shape[1]]
                cur_window = cur_window.reshape((1, cur_window.shape[0], cur_window
                prediction = self.predict(cur_window, use_cascade=use_cascade)
                if prediction[0] == 1:
                    #left line
                    marked_img[x:x + slide_shape[0], y] = np.ones((1, slide_shape[0
                    #right line
                    marked_img[x:x + slide_shape[0], y + slide_shape[1]] = np.ones(
                    #top line
                    marked_img[x, y:y + slide_shape[1]] = np.ones((slide_shape[1],
                    # bottom line
                    marked_img[x + slide_shape[0], y:y + slide_shape[1]] = np.ones(
        return marked_img
```

In [7]:

```
x, y = generate_dataset()
X_train, X_test, y_train, y_test = train_test_split(x, y, random_state=0, shuffle=T
```

In [8]:

```
%%time

clf = ViolaJonesClassifier(num_trees=20, max_depth=1, max_features=3000)
clf.fit(X_train, y_train)
```

```
100%|██████████| 20/20 [01:54<00:00,  5.70s/it]

CPU times: user 1min 56s, sys: 375 ms, total: 1min 57s
Wall time: 1min 57s
```

In [9]:

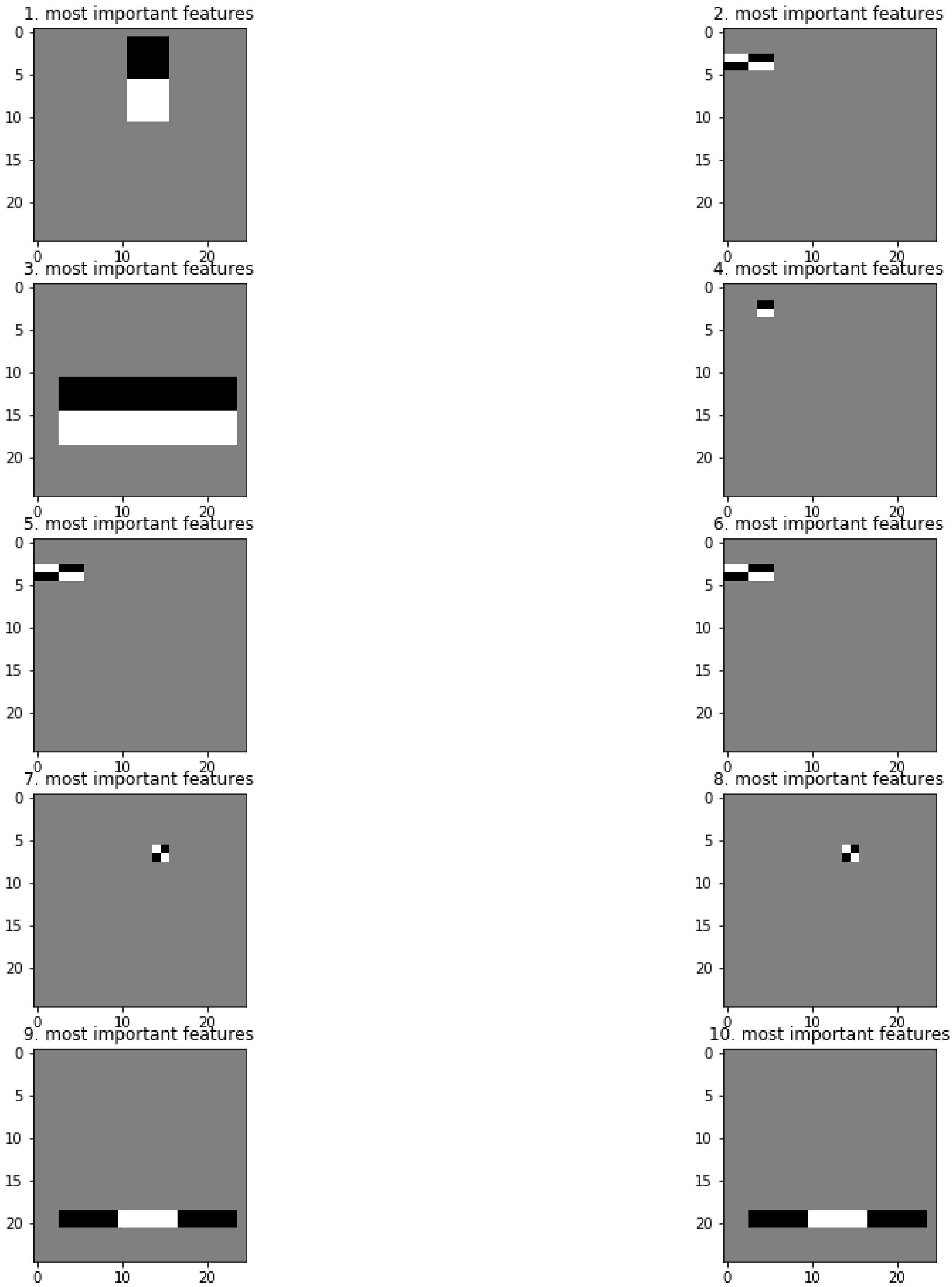
```
%%time
y_pred = clf.predict(X_test)
acc = accuracy(y_test, y_pred)
print(f"Accuracy of {round(100 * acc, 4)}%")
```

```
Accuracy of 90.0%
CPU times: user 895 ms, sys: 72 µs, total: 895 ms
Wall time: 894 ms
```

2.a) Visualize the top ten face classifiers.

In [10]:

```
trees = clf.adaboost.trees[:10]
features = clf.features
most_imp_features = [features[tree.root.j] for tree in trees]

fig=plt.figure(figsize=(16, 16))
columns = 2
rows = 5
for i in range(1, columns*rows +1):
    visualization = most_imp_features[i - 1].visualise_feature()
    fig.add_subplot(rows, columns, i)
    plt.imshow(visualization, cmap='gray')
    plt.title(f"{i}. most important features")
plt.show()
```



Excercise 3 (Bonus). Cascade-Classification

Implement a cascade algorithm to classify faces in a picture of your choice (there should bemore than a face on your image, e.g. skimage.data.astronaut())

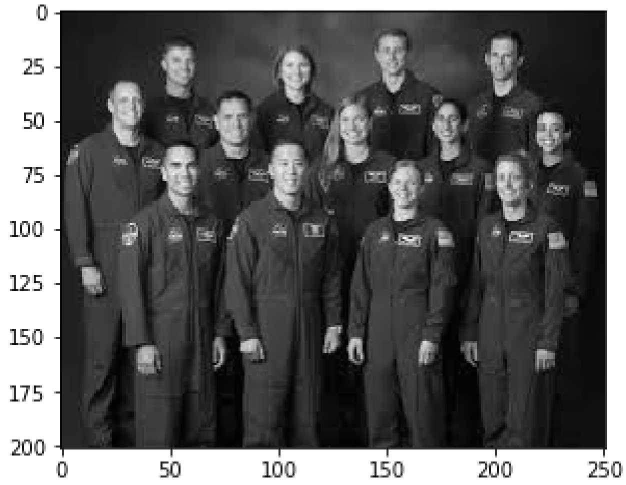
In [11]:

```
%%time

y_pred = clf.predict(X_test, use_cascade=True)
acc = accuracy(y_test, y_pred)
print(f"Accuracy of {round(100 * acc, 4)}%")
```

```
Accuracy of 94.0%
CPU times: user 915 ms, sys: 61 µs, total: 915 ms
Wall time: 913 ms
```

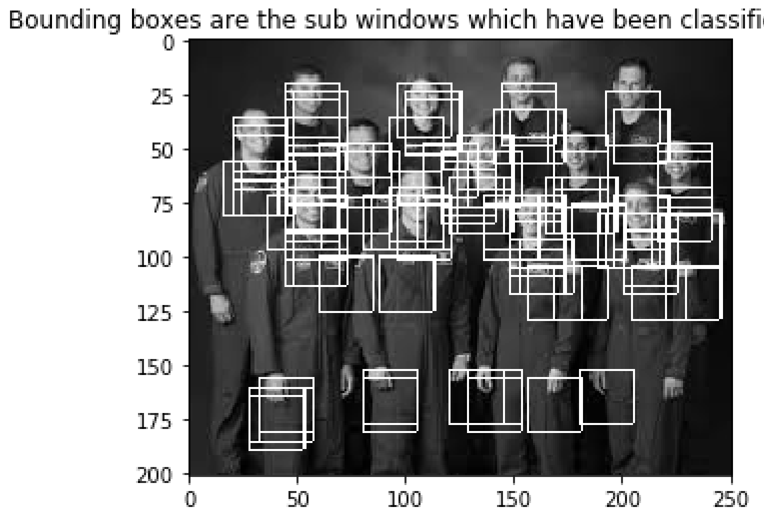
In [12]:

```
url = "https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcR-szxRx86psEWuHGQe0oWQ
astronauts = np.array(Image.open(requests.get(url, stream=True).raw).convert('L'))

plt.imshow(astronauts, cmap='gray')
plt.show()
```



In [13]:

```
%%time
marked_img = clf.slide_over_img(astronauts, slide_shape=(25, 25), stride=4, use_cas


plt.imshow(marked_img, cmap="gray")
plt.title("Bounding boxes are the sub windows which have been classified as a face"
plt.show()
```




```
CPU times: user 43.7 s, sys: 4.54 ms, total: 43.7 s
Wall time: 43.8 s
```