

Passive Low Pass Filter

Basically, an electrical filter is a circuit that can be designed to modify, reshape or reject all unwanted frequencies of an electrical signal and accept or pass only those signals wanted by the circuits designer.

In other words they “filter-out” unwanted signals and an ideal filter will separate and pass sinusoidal input signals based upon their frequency. In low frequency applications (up to 100kHz), passive filters are generally constructed using simple RC (Resistor-Capacitor) networks, while higher frequency filters (above 100kHz) are usually made from RLC (Resistor-Inductor-Capacitor) components.

Passive filters are made up of passive components such as resistors, capacitors and inductors and have no amplifying elements (transistors, op-amps, etc) so have no signal gain, therefore their output level is always less than the input.

Filters are so named according to the frequency range of signals that they allow to pass through them, while blocking or “attenuating” the rest. The most commonly used filter designs are the:

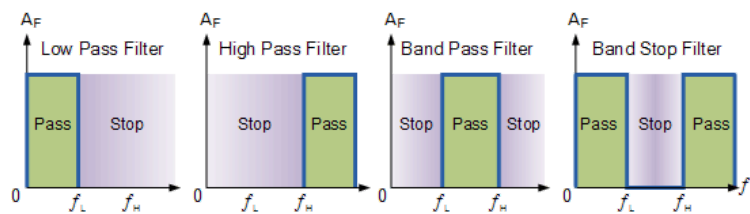
- The Low Pass Filter – the low pass filter only allows low frequency signals from 0Hz to its cut-off frequency, f_c point to pass while blocking those any higher.
- The High Pass Filter – the high pass filter only allows high frequency signals from its cut-off frequency, f_c point and higher to infinity to pass through while blocking those any lower.
- The Band Pass Filter – the band pass filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies either side of this frequency band.

Simple First-order passive filters (1st order) can be made by connecting together a single resistor and a single capacitor in series across an input signal, (V_{in}) with the output of the filter, (V_{out}) taken from the junction of these two components.

Depending on which way around we connect the resistor and the capacitor with regards to the output signal determines the type of filter construction resulting in either a **Low Pass Filter** or a **High Pass Filter**.

As the function of any filter is to allow signals of a given band of frequencies to pass unaltered while attenuating or weakening all others that are not wanted, we can define the amplitude response characteristics of an ideal filter by using an ideal frequency response curve of the four basic filter types as shown.

Ideal Filter Response Curves



Filters can be divided into two distinct types: active filters and passive filters. Active filters contain amplifying devices to increase signal strength while passive do not contain amplifying devices to strengthen the signal. As there are two passive components within a passive filter design the output signal has a smaller amplitude than its corresponding input signal, therefore passive RC filters attenuate the signal and have a gain of less than one, (unity).

A Low Pass Filter can be a combination of capacitance, inductance or resistance intended to produce high attenuation above a specified frequency and little or no attenuation below that frequency. The frequency at which the transition occurs is called the “cut-off” or “corner” frequency.

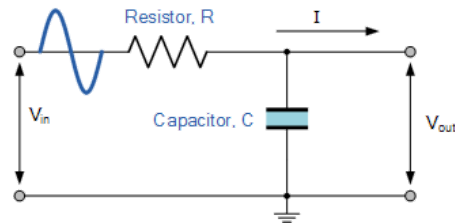
The simplest low pass filters consist of a resistor and capacitor but more sophisticated low pass filters have a combination of series inductors and parallel capacitors. In this tutorial we will look at the simplest type, a passive two component RC low pass filter.

The Low Pass Filter

A simple passive **RC Low Pass Filter** or **LPF**, can be easily made by connecting together in series a single Resistor with a single Capacitor as shown below. In this type of filter arrangement the input signal (V_{in}) is applied to the series combination (both the Resistor and Capacitor together) but the output signal (V_{out}) is taken across the capacitor only.

This type of filter is known generally as a “first-order filter” or “one-pole filter”, why first-order or single-pole?, because it has only “one” reactive component, the capacitor, in the circuit.

RC Low Pass Filter Circuit



As mentioned previously in the **Capacitive Reactance** tutorial, the reactance of a capacitor varies inversely with frequency, while the value of the resistor remains constant as the frequency changes. At low frequencies the capacitive reactance, (X_C) of the capacitor will be very large compared to the resistive value of the resistor, R .

This means that the voltage potential, V_C across the capacitor will be much larger than the voltage drop, V_R developed across the resistor. At high frequencies the reverse is true with V_C being small and V_R being large due to the change in the capacitive reactance value.

While the circuit above is that of an RC Low Pass Filter circuit, it can also be thought of as a frequency dependant variable potential divider circuit similar to the one we looked at in the **Resistors** tutorial. In that tutorial we used the following equation to calculate the output voltage for two single resistors connected in series.

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

where: $R_1 + R_2 = R_T$, the total resistance of the circuit

We also know that the capacitive reactance of a capacitor in an AC circuit is given as:

$$X_C = \frac{1}{2\pi fC} \text{ in Ohm's}$$

Opposition to current flow in an AC circuit is called **impedance**, symbol Z and for a series circuit consisting of a single resistor in series with a single capacitor, the circuit impedance is calculated as:

$$Z = \sqrt{R^2 + X_C^2}$$

Then by substituting our equation for impedance above into the resistive potential divider equation gives us:

RC Potential Divider Equation

$$V_{out} = V_{in} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = V_{in} \frac{X_C}{Z}$$

So, by using the potential divider equation of two resistors in series and substituting for impedance we can calculate the output voltage of an RC Filter for any given frequency.

Low Pass Filter Example No1

A **Low Pass Filter** circuit consisting of a resistor of $4k7\Omega$ in series with a capacitor of $47nF$ is connected across a $10v$ sinusoidal supply. Calculate the output voltage (V_{out}) at a frequency of $100Hz$ and again at frequency of $10,000Hz$ or $10kHz$.

Voltage Output at a Frequency of $100Hz$.

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 47 \times 10^{-9}} = 33,863\Omega$$

$$V_{OUT} = V_{IN} \times \frac{X_c}{\sqrt{R^2 + X_c^2}} = 10 \times \frac{33863}{\sqrt{4700^2 + 33863^2}} = 9.9v$$

Voltage Output at a Frequency of $10,000Hz$ ($10kHz$).

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10,000 \times 47 \times 10^{-9}} = 338.6\Omega$$

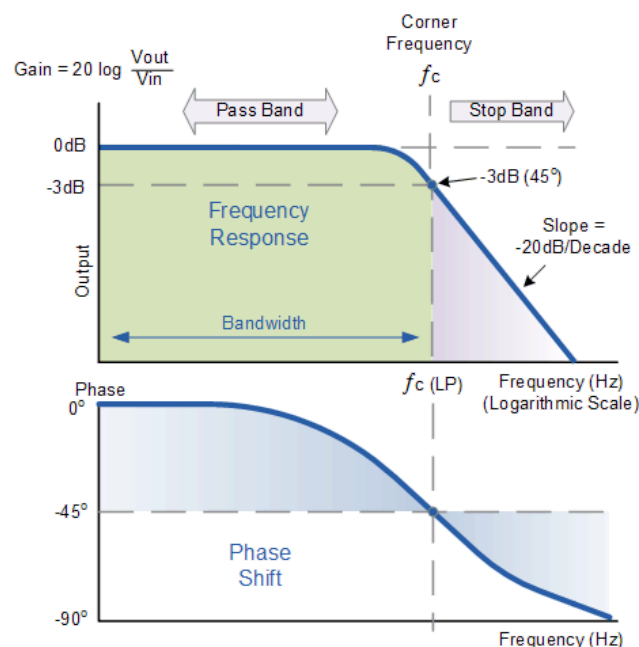
$$V_{OUT} = V_{IN} \times \frac{X_c}{\sqrt{R^2 + X_c^2}} = 10 \times \frac{338.6}{\sqrt{4700^2 + 338.6^2}} = 0.718v$$

Frequency Response

We can see from the results above, that as the frequency applied to the RC network increases from $100Hz$ to $10kHz$, the voltage dropped across the capacitor and therefore the output voltage (V_{out}) from the circuit decreases from $9.9v$ to $0.718v$.

By plotting the networks output voltage against different values of input frequency, the **Frequency Response Curve** or **Bode Plot** function of the low pass filter circuit can be found, as shown below.

Frequency Response of a 1st-order Low Pass Filter



The Bode Plot shows the **Frequency Response** of the filter to be nearly flat for low frequencies and all of the input signal is passed directly to the output, resulting in a gain of nearly 1, called unity, until it reaches its **Cut-off Frequency** point (f_c). This is because the reactance of the capacitor is high at low frequencies and blocks any current flow through the capacitor.

After this cut-off frequency point the response of the circuit decreases to zero at a slope of -20dB/Decade or (-6dB/Octave) “roll-off”. Note that the angle of the slope, this -20dB/Decade roll-off will always be the same for any RC combination.

Any high frequency signals applied to the low pass filter circuit above this cut-off frequency point will become greatly attenuated, that is they rapidly decrease. This happens because at very high frequencies the reactance of the capacitor becomes so low that it gives the effect of a short circuit condition on the output terminals resulting in zero output.

Then by carefully selecting the correct resistor-capacitor combination, we can create a RC circuit that allows a range of frequencies below a certain value to pass through the circuit unaffected while any frequencies applied to the circuit above this cut-off point to be attenuated, creating what is commonly called a **Low Pass Filter**.

For this type of “Low Pass Filter” circuit, all the frequencies below this cut-off, f_c point that are unaltered with little or no attenuation and are said to be in the filters **Pass band** zone. This pass band zone also represents the **Bandwidth** of the filter. Any signal frequencies above this point cut-off point are generally said to be in the filters **Stop band** zone and they will be greatly attenuated.

This “Cut-off”, “Corner” or “Breakpoint” frequency is defined as being the frequency point where the capacitive reactance and resistance are equal, $R = X_c = 4k7\Omega$. When this occurs the output signal is attenuated to 70.7% of the input signal value or -3dB ($20 \log (V_{out}/V_{in})$) of the input. Although $R = X_c$, the output is **not** half of the input signal. This is because it is equal to the vector sum of the two and is therefore 0.707 of the input.

As the filter contains a capacitor, the Phase Angle (Φ) of the output signal **Lags** behind that of the input and at the -3dB cut-off frequency (f_c) is -45° out of phase. This is due to the time taken to charge the plates of the capacitor as the input voltage changes, resulting in the output voltage (the voltage across the capacitor) “lagging” behind that of the input signal. The higher the input frequency applied to the filter the more the capacitor lags and the circuit becomes more and more “out of phase”.

The cut-off frequency point and phase shift angle can be found by using the following equation:

Cut-off Frequency and Phase Shift

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 4700 \times 47 \times 10^{-9}} = 720\text{Hz}$$

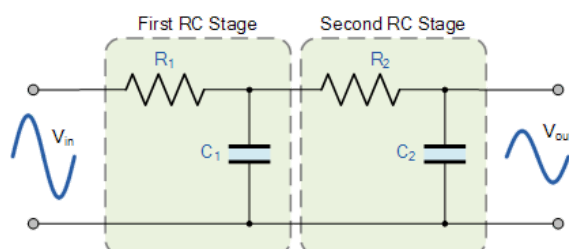
$$\text{Phase Shift } \phi = -\arctan (2\pi fRC)$$

Then for our simple example of a “**Low Pass Filter**” circuit above, the cut-off frequency (f_c) is given as 720Hz with an output voltage of 70.7% of the input voltage value and a phase shift angle of -45° .

Second-order Low Pass Filter

Thus far we have seen that simple first-order RC low pass filters can be made by connecting a single resistor in series with a single capacitor. This single-pole arrangement gives us a roll-off slope of -20dB/decade attenuation of frequencies above the cut-off point at $f_{-3\text{dB}}$. However, sometimes in filter circuits this -20dB/decade (-6dB/octave) angle of the slope may not be enough to remove an unwanted signal then two stages of filtering can be used as shown.

Second-order Low Pass Filter



The above circuit uses two passive first-order low pass filters connected or “cascaded” together to form a second-order or two-pole filter network. Therefore we can see that a first-order low pass filter can be converted into a second-order type by simply adding an additional RC network to it and the more RC stages we add the higher becomes the order of the filter.

If a number (n) of such RC stages are cascaded together, the resulting RC filter circuit would be known as an “ n^{th} -order” filter with a roll-off slope of “ $n \times -20\text{dB/decade}$ ”.

So for example, a second-order filter would have a slope of -40dB/decade (-12dB/octave), a fourth-order filter would have a slope of -80dB/decade (-24dB/octave) and so on. This means that, as the order of the filter is increased, the roll-off slope becomes steeper and the actual stop band response of the filter approaches its ideal stop band characteristics.

Second-order filters are important and widely used in filter designs because when combined with first-order filters any higher-order n^{th} -value filters can be designed using them. For example, a third order low-pass filter is formed by connecting in series or cascading together a first and a second-order low pass filter.

But there is a downside too cascading together RC filter stages. Although there is no limit to the order of the filter that can be formed, as the order increases, the gain and accuracy of the final filter declines.

When identical RC filter stages are cascaded together, the output gain at the required cut-off frequency (f_c) is reduced (attenuated) by an amount in relation to the number of filter stages used as the roll-off slope increases. We can define the amount of attenuation at the selected cut-off frequency using the following formula.

Passive Low Pass Filter Gain at f_c

$$\left(\frac{1}{\sqrt{2}}\right)^n$$

where "n" is the number of filter stages.

So for a second-order passive low pass filter the gain at the corner frequency f_c will be equal to $0.7071 \times 0.7071 = 0.5V_{in}$ (-6dB), a third-order passive low pass filter will be equal to $0.353V_{in}$ (-9dB), fourth-order will be $0.25V_{in}$ (-12dB) and so on. The corner frequency, f_c for a second-order passive low pass filter is determined by the resistor/capacitor (RC) combination and is given as.

2nd-Order Filter Corner Frequency

$$f_c = \frac{1}{2\pi\sqrt{R_1C_1R_2C_2}} \text{ Hz}$$

In reality as the filter stage and therefore its roll-off slope increases, the low pass filters -3dB corner frequency point and therefore its pass band frequency changes from its original calculated value above by an amount determined by the following equation.

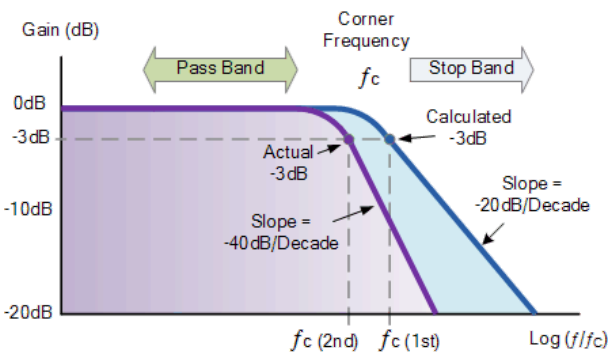
2nd-Order Low Pass Filter -3dB Frequency

$$f_{(-3\text{dB})} = f_c \sqrt{2^{\left(\frac{1}{n}\right)} - 1}$$

where f_c is the calculated cut-off frequency, n is the filter order and $f_{-3\text{dB}}$ is the new -3dB pass band frequency as a result in the increase of the filters order.

Then the frequency response (bode plot) for a second-order low pass filter assuming the same -3dB cut-off point would look like:

Frequency Response of a 2nd-order Low Pass Filter



In practice, cascading passive filters together to produce larger-order filters is difficult to implement accurately as the dynamic impedance of each filter order affects its neighbouring network. However, to reduce the loading effect we can make the impedance of each following stage 10x the previous stage, so $R_2 = 10 \times R_1$ and $C_2 = 1/10 \times C_1$. Second-order and above filter networks are generally used

in the feedback circuits of op-amps, making what are commonly known as **Active Filters** or as a phase-shift network in **RC Oscillator** circuits.

Low Pass Filter Summary

So to summarize, the **Low Pass Filter** has a constant output voltage from D.C. (0Hz), up to a specified Cut-off frequency, (f_c) point. This cut-off frequency point is 0.707 or **-3dB** ($\text{dB} = -20 \log V_{\text{out}}/V_{\text{in}}$) of the voltage gain allowed to pass.

The frequency range “below” this cut-off point f_c is generally known as the **Pass Band** as the input signal is allowed to pass through the filter. The frequency range “above” this cut-off point is generally known as the **Stop Band** as the input signal is blocked or stopped from passing through.

A simple 1st order low pass filter can be made using a single resistor in series with a single non-polarized capacitor (or any single reactive component) across an input signal V_{in} , whilst the output signal V_{out} is taken from across the capacitor.

The cut-off frequency or -3dB point, can be found using the standard formula, $f_c = 1/(2\pi RC)$. The phase angle of the output signal at f_c and is -45° for a Low Pass Filter.

The gain of the filter or any filter for that matter, is generally expressed in **Decibels** and is a function of the output value divided by its corresponding input value and is given as:

$$\text{Gain in dB} = 20 \log \frac{V_{\text{out}}}{V_{\text{in}}}$$

Applications of passive Low Pass Filters are in audio amplifiers and speaker systems to direct the lower frequency bass signals to the larger bass speakers or to reduce any high frequency noise or “hiss” type distortion. When used like this in audio applications the low pass filter is sometimes called a “high-cut”, or “treble cut” filter.

If we were to reverse the positions of the resistor and capacitor in the circuit so that the output voltage is now taken from across the resistor, we would have a circuit that produces an output frequency response curve similar to that of a **High Pass Filter**, and this is discussed in the next tutorial.

Time Constant

Until now we have been interested in the frequency response of a low pass filter when subjected to sinusoidal waveform. We have also seen that the filters cut-off frequency (f_c) is the product of the resistance (R) and the capacitance (C) in the circuit with respect to some specified frequency point and that by altering any one of the two components alters this cut-off frequency point by either increasing it or decreasing it.

We also know that the phase shift of the circuit lags behind that of the input signal due to the time required to charge and then discharge the capacitor as the sine wave changes. This combination of R and C produces a charging and discharging effect on the capacitor known as its **Time Constant** (τ) of the circuit as seen in the **RC Circuit** tutorials giving the filter a response in the time domain.

The time constant, **tau** (τ), is related to the cut-off frequency f_c as.

$$\tau = RC = \frac{1}{2\pi f_c}$$

or expressed in terms of the cut-off frequency, f_c as.

$$f_c = \frac{1}{2\pi RC} \text{ or } \frac{1}{2\pi \tau}$$

The output voltage, V_{out} depends upon the time constant and the frequency of the input signal. With a sinusoidal signal that changes smoothly over time, the circuit behaves as a simple 1st order low pass filter as we have seen above.

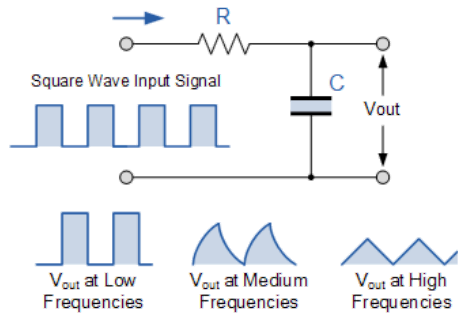
But what if we were to change the input signal to that of a “square wave” shaped “ON/OFF” type signal that has an almost vertical step input, what would happen to our filter circuit now. The output response of the circuit would change dramatically and produce another type of circuit known commonly as an **Integrator**.

The RC Integrator

The **Integrator** is basically a low pass filter circuit operating in the time domain that converts a square wave “step” response input signal into a triangular shaped waveform output as the capacitor charges and discharges. A **Triangular** waveform consists of alternate but equal, positive and negative ramps.

As seen below, if the RC time constant is long compared to the time period of the input waveform the resultant output waveform will be triangular in shape and the higher the input frequency the lower will be the output amplitude compared to that of the input.

The RC Integrator Circuit



This then makes this type of circuit ideal for converting one type of electronic signal to another for use in wave-generating or wave-shaping circuits.

89 Comments

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SUBMIT

A Ahmad Nawaz

hi every one.... kindly give me answer of my question???..
Question...if cutoff frequency of low pass filter is 400hz then how to find its gain in dbs..
a. 200hz
b.400hz
c.800hz
d.1600hz.. plz any one... email....

Posted on July 02nd 2017 | 8:54 am

Reply



Wayne Storr

Really, Its a Low Pass Filter, then any frequency below the cut-off frequency is in the pass band, you work it out.

Posted on July 02nd 2017 | 2:55 pm