Optimal (Feedback) Control as a model for movement selection

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Learning goals

After this lecture you should:

1. Understand which computational problems the brain has to solve in order to successfully execute a movement.
2. Understand the optimal control model, including each of the elements and how they are linked to each other and its features and assumptions/limitations
3. Understand the state estimation (Kalman filter) model, including each of the elements and how they are linked to each other and its features and assumptions/limitations
4. Understand Optimal Feedback Control Theory, including each of the elements and how they are linked to each other and its features and assumptions/limitations
5. Be able to formalize an Optimal (Feedback) Control model by formulating the state equation, the measurement equation, the cost function, the state estimation and the control policy based on a Linear Quadratic Gaussian (LQG) controller
6. Be able to explain and interpret the differences between optimal control and Optimal Feedback Control (OFC)
7. Be able to make predictions for experiments that address neural responses or human behaviour from the three covered theories (Optimal control, state estimation, and Optimal Feedback Control)
8. Be able to design behavioural or (single-)neuron experiments to test predictions from the three covered theories (Optimal control, state estimation, and Optimal Feedback Control)
9. Be able to explain which brain areas are thought to play an important role in each of the components of the three covered theories (Optimal control, state estimation, and Optimal Feedback Control)

Motor control

Example reaching movement

Input:

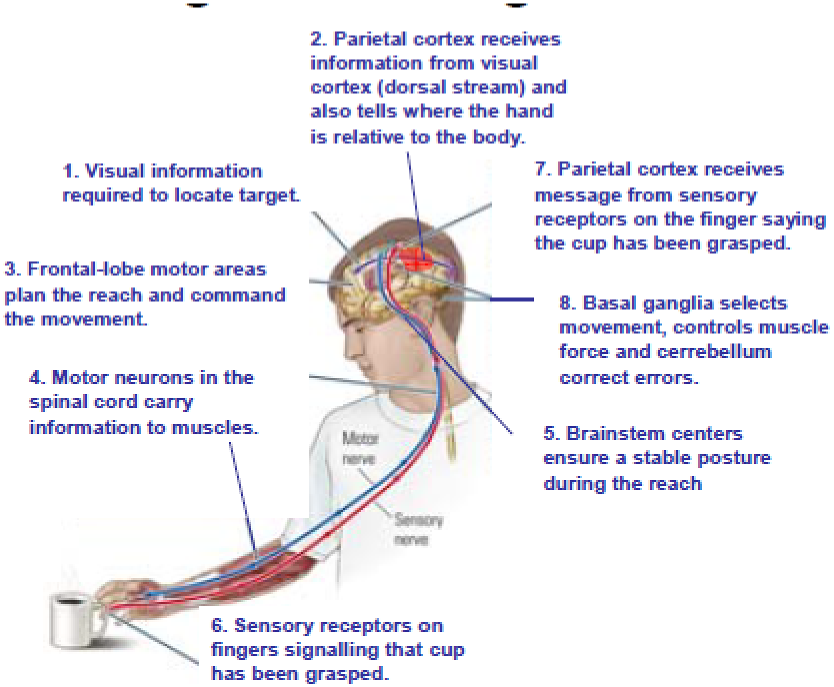
- Task goal
- Sensory information
  (proprioception, vision)

Estimate states, **x**:

- Arm position/velocity

Output:

- Motor command/movements, **u**
- Movement corrections



Kolb & Wishaw (2002). *An introduction to brain and behavior*

Motor control

Motor control is a difficult computational problem to solve, because of:

- Redundancy (degrees of freedom problem)
- Noise
- Delays
- Nonstationarity
- Nonlinearity

Central question: How does the brain overcome these difficulties to select and produce the stable and precise movements human usually make?

Motor control

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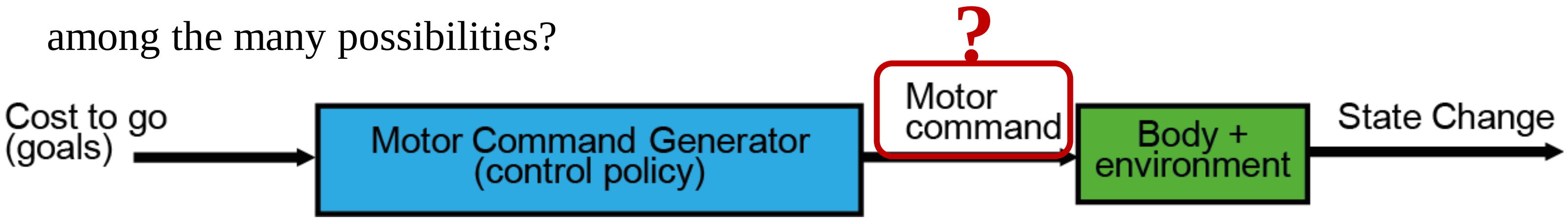
How do human select a movement strategy from among the many possibilities?



Balance the need for reward and the cost/effort (in terms of motor commands) → combined in a cost function → the motor command is selected that minimizes the cost function

Cost function = performance measure you want to minimize

Motor control

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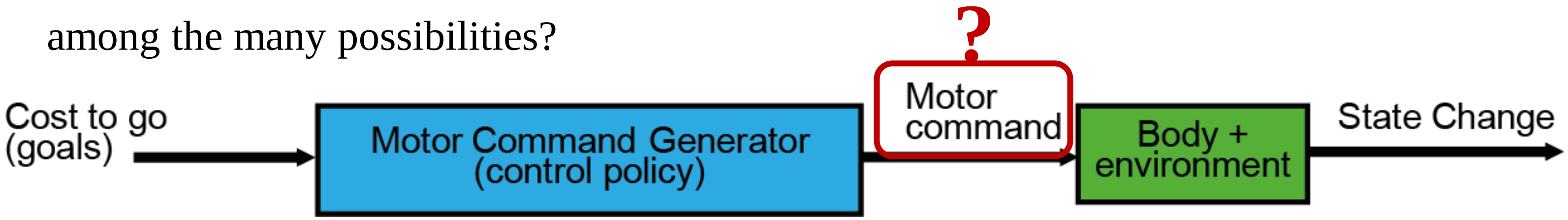

Examples of proposed cost functions:

- Minimum jerk (rate of change of acceleration)
- Minimum torque change

Motor control

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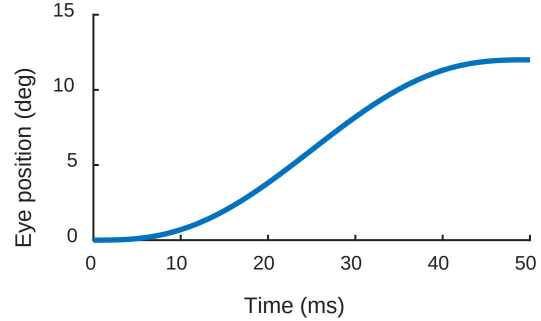

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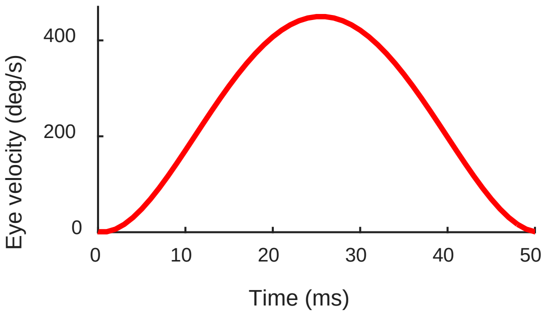
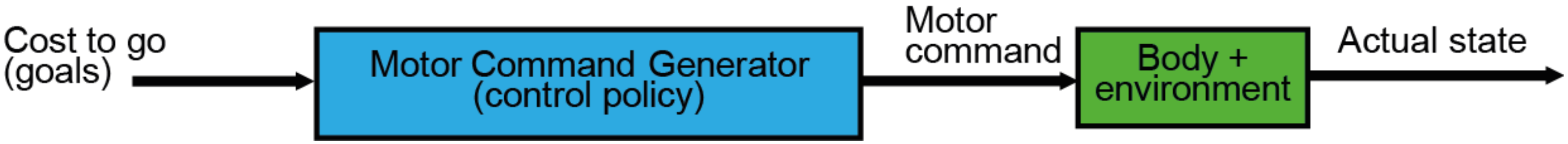


Optimal control theory (open-loop control)



Goal: find the motor commands that minimize the cost function J

J = f(states,motor commands) = f($\mathbf{x}$,$\mathbf{u}$)

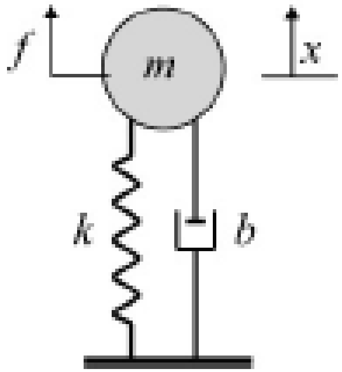
Example: look at the corner of the goal in the periphery

Optimal control theory (open-loop control)

Example: look at the corner of the goal in the periphery

Dynamic model of the eye: $x_{t+1} = f(x_t, u_t)$

(state equation)



$$m\ddot{x} = -kx - b\dot{x} + f$$

$$\alpha_1 \frac{df}{dt} + \alpha_2 = u$$

$$x_{t+1} = Ax_t + Bu_t$$

$$x_1 \equiv x, x_2 \equiv \dot{x}, x_3 \equiv f$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{1}{m} \\ 0 & 0 & -\frac{\alpha_2}{\alpha_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\alpha_1} \end{bmatrix} u$$

$$\dot{x} = Ax + Bu$$

$$x_{t+1} = Ax_t + Bu_t$$

States **x** :

- x_1 = Eye position, p
- x_2 = Eye velocity, v
- x_3 = force produced by muscle, f

State vector: **x** = [p ; v ; f]

Motor command/control input **u**:

- 'Activation' sent from higher (cortical) areas

Optimal control theory (open-loop control)

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States **x**:
- Eye position, p
- Eye velocity, v
- Force produced by muscle, f

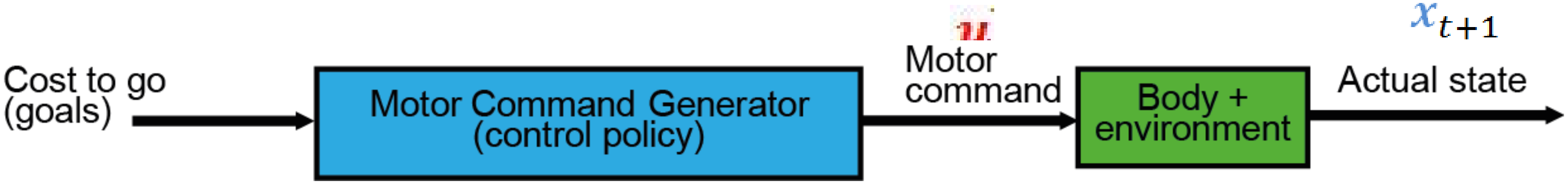
Motor command/control input **u**:
- ‘Activation’ sent from higher (cortical) areas

Often used cost function: minimization endpoint deviation/variation and control-effort

J = f(**x**,**u**) = end-cost + total running cost (effort) during the movement

$$J = x_{t_f}{}^T Q x_{t_f} + \sum_t u_t{}^T R u_t$$

$$u = \min_u(J(x,u)) = \min_u(x_{t_f}{}^T Q x_{t_f} + \sum_t u_t{}^T R u_t)$$



Optimal control theory (open-loop control)

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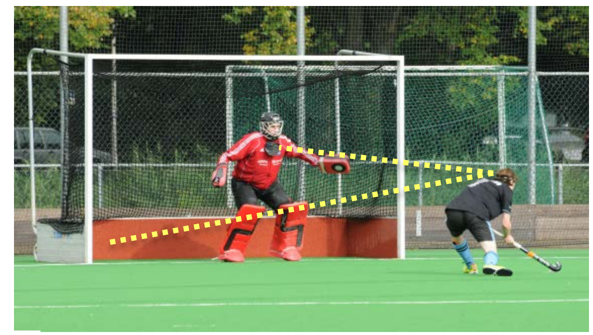

States **x**:
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Motor command/control input **u**:
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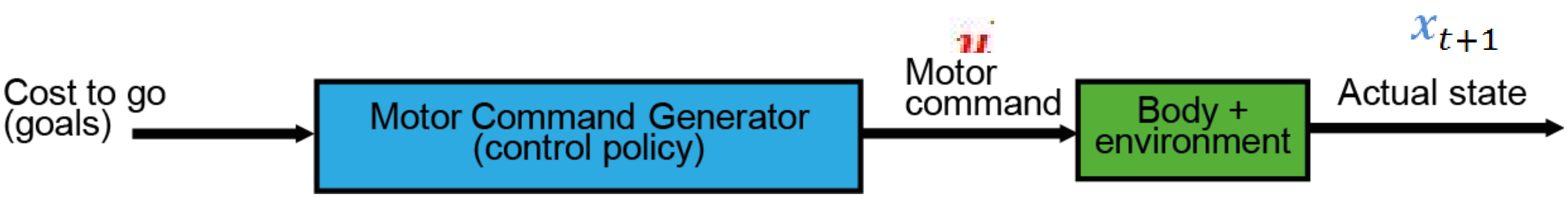
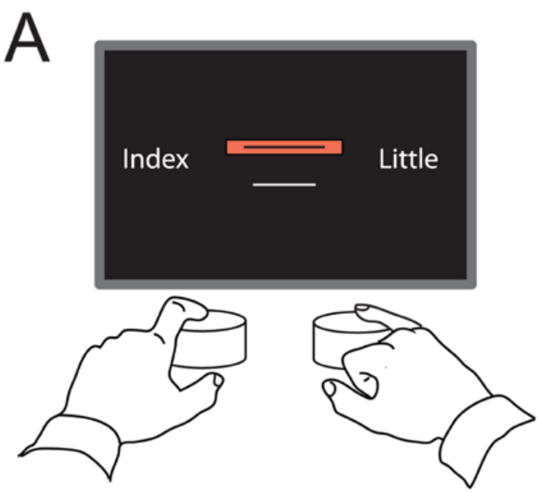
$$u = \min_u (J(x, \boldsymbol{u}) = \min_u (x_{t_f}{}^T Q x_{t_f} + \sum_t \boldsymbol{u}_t{}^T R \boldsymbol{u}_t)$$

$$x_{t+1} = f(x_t, \boldsymbol{u}_t) = A x_t + B \boldsymbol{u}_t$$



Optimal control theory (open-loop control)

Example:

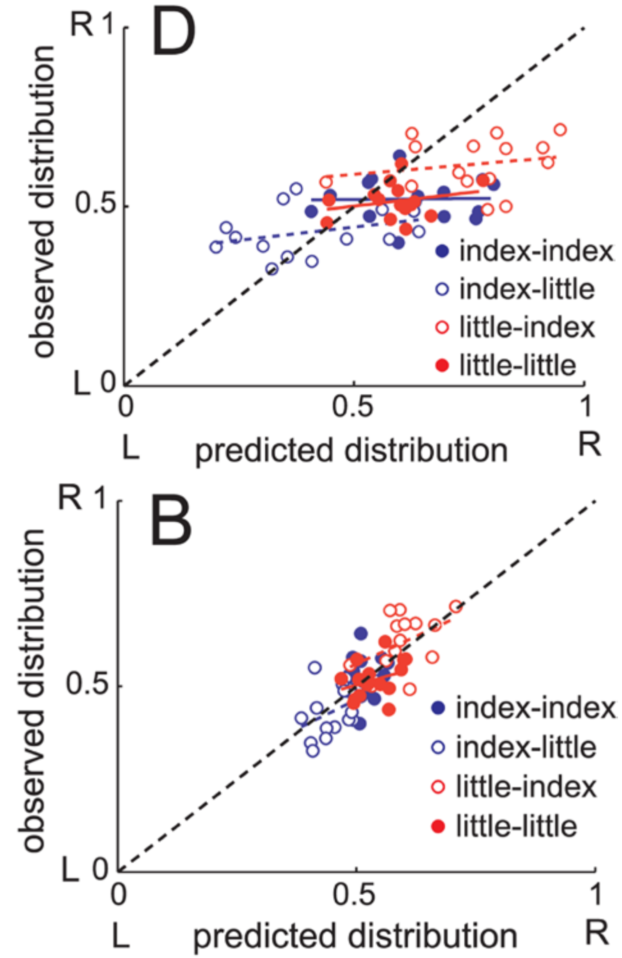


Penalize only inaccuracy:

$$J = E\left[\left(x_i + x_j - g\right)^2\right]$$

Penalize inaccuracy and motor cost:

$$J = vE\left[\left(x_i + x_j - g\right)^2\right] + \lambda\left(u_i^2 + u_j^2\right) + \mu\left(u_i^2 / MVC_i^2 + u_j^2 / MVC_j^2\right)$$



O'Sullivan et al. (2009). *Plos Comp Biol*

Motor control

Motor control is a difficult computational problem to solve, because of:

- Redundancy
- Noise
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For example noise in the motor system/ uncertain about consequences of motor command:

$$x_{t+1} = Ax_t + B\boldsymbol{u}_t + \boldsymbol{\varepsilon}_x \quad \boldsymbol{\varepsilon}_x \sim N(0, \sigma_x)$$

Minimize the expected value of the cost function:

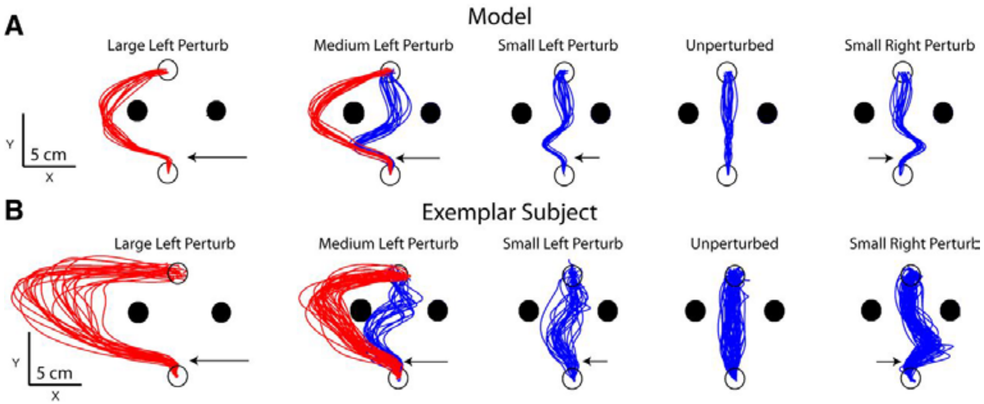
$$u = \min_u(E[J(x, \boldsymbol{u})]) = \min_u(\boldsymbol{E}[x_{t_f}{}^T Q x_{t_f} + \sum_t \boldsymbol{u}_t{}^T R \boldsymbol{u}_t])$$

Optimal control theory (open-loop control)

Problem of Optimal Control:

- Perturbations (online control)

We need to know the state of the system online, but:
- Noise
- Delays in sensory system



Nashed et al. (2014). *J Neurosci*

Optimal Feedback Control (OFC) theory (closed-loop control)

Optimal Feedback Control Theory (stochastic optimal control):

- Optimal control (previous part)

- Optimal state estimation (Kalman filter)

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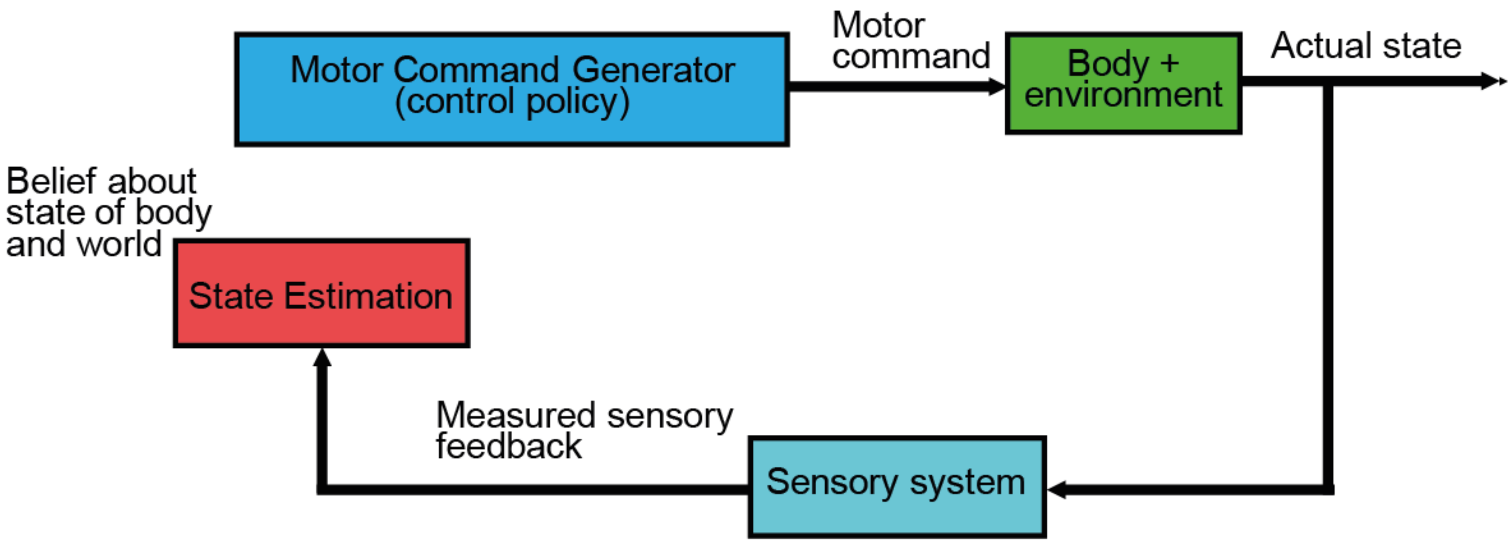
Optimal state estimation (Kalman filtering)

Optimal state estimation – find the most likely state of the world, e.g. hand position, target location, etc.



Input:

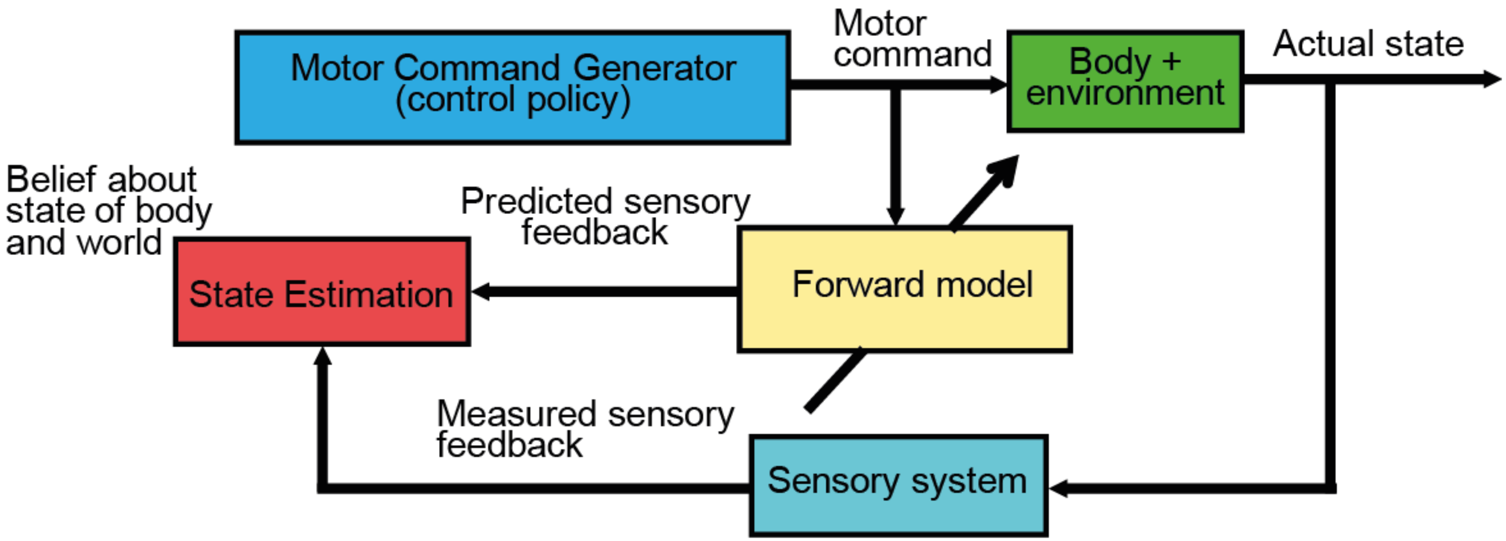
- Noisy and delayed measurements
- Predicted state using a forward model

Output/goal:

- Belief about the state of the body and the world

Optimal state estimation (Kalman filtering)

Optimal state estimation – find the most likely state of the world, e.g. hand position, target location, etc.



Input:

- Noisy and delayed measurements
- Predicted state using a forward model

Output/goal:

- Belief about the state of the body and the world

Wolpert et al. (1995). *Science*

Optimal state estimation (Kalman filtering)

Optimal state estimation (Kalman filter) in equations:

Dynamics of the system:

State noise

$$x_{t+1} = Ax_t + B\boldsymbol{u}_t + \boldsymbol{\varepsilon}_x \quad \boldsymbol{\varepsilon}_x \sim N(0, \sigma_x)$$

$$y_t = Hx_t + \boldsymbol{\varepsilon}_y \quad \boldsymbol{\varepsilon}_y \sim N(0, \sigma_y)$$

(Measurement equation)

Sensory noise

Two steps:

1. Prediction step:

$$\hat{x}_{t+1} = A\hat{x}_t + B\boldsymbol{u}_t$$

$$\hat{y}_t = H\hat{x}_t$$

2. Update step:

Kalman gain

Sensory prediction error

$$\hat{x}_{t+1} = \hat{x}_{t+1} + K_t(y_t - \hat{y}_t)$$

Wolpert et al. (1995). *Science*

Optimal state estimation (Kalman filtering)

Optimal state estimation (Kalman filter) in equations:

Dynamics of the system:

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(Measurement equation)

Two steps:

1. Prediction step:

$$\hat{x}_{t+1} = A\hat{x}_t + B\boldsymbol{u}_t$$

$$\hat{y}_t = H\hat{x}_t$$

Prior estimate

2. Update step:

Posterior estimate

$$\hat{\boldsymbol{x}}_{t+1} = \hat{x}_{t+1} + K_t(y_t - \hat{y}_t)$$

Wolpert et al. (1995). *Science*

Optimal state estimation (Kalman filtering)

Optimal state estimation (Kalman filter) in equations:

Two steps:

1. Prediction step:

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Prior estimate

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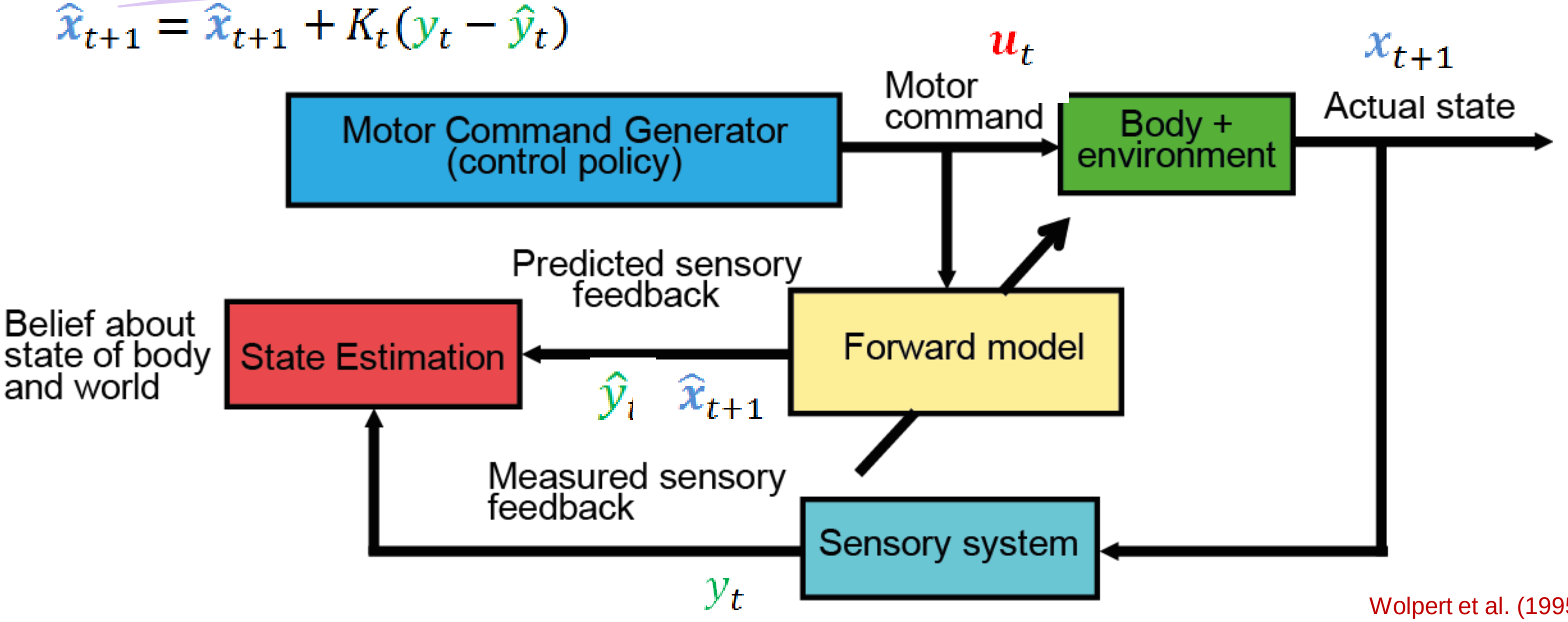
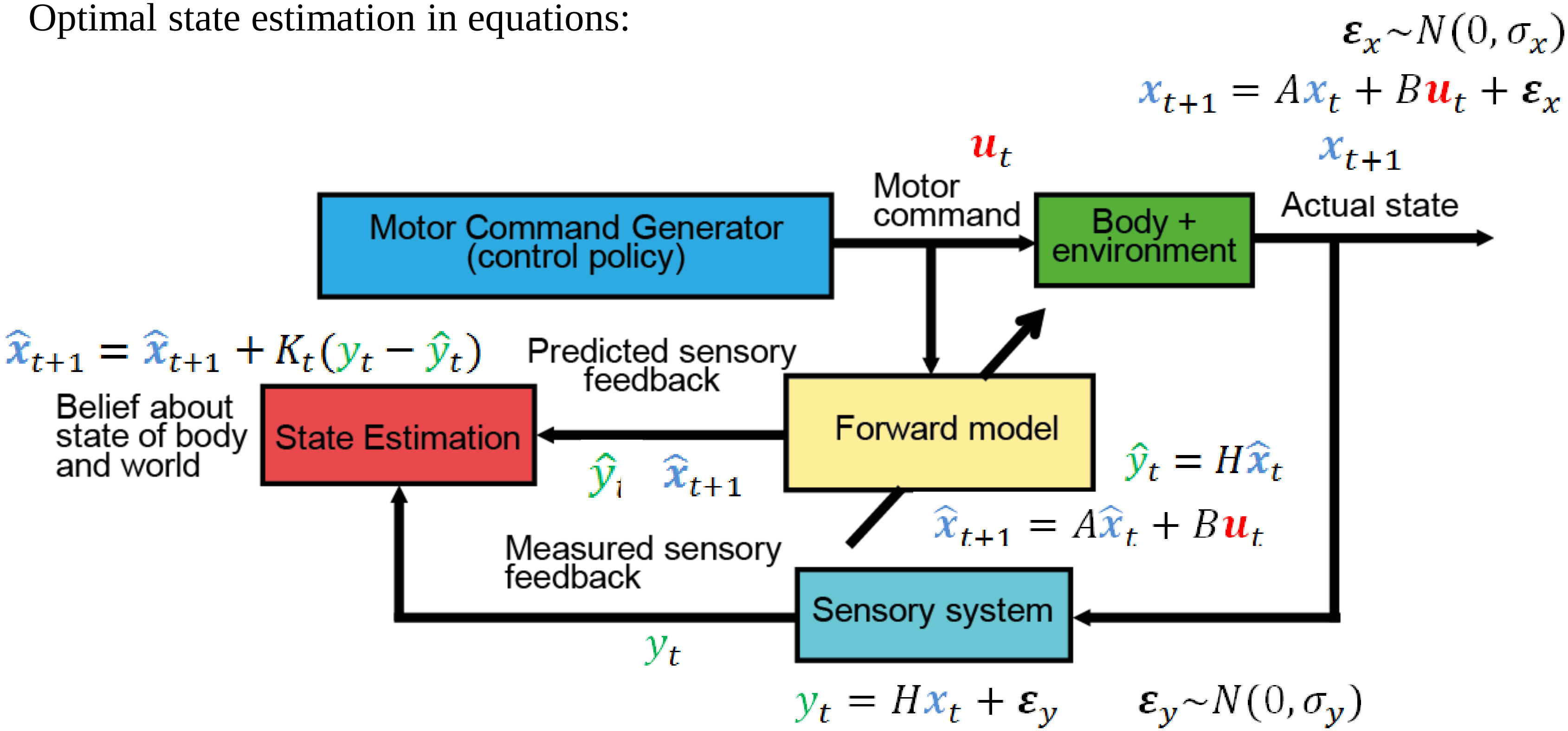


Wolpert et al. (1995). *Science*

Optimal state estimation (Kalman filtering)

Optimal state estimation in equations:



Wolpert et al. (1995). *Science*

Optimal state estimation (Kalman filtering)

Optimal state estimation:

$$\hat{x}_{t+1} = \hat{x}_{t+1} + K_t(y_t - \hat{y}_t)$$

Kalman gain K = optimal sensitivity to prediction error → minimize the variance of the posterior estimate $\hat{x}_{t+1}$

$$K_t = \frac{var(prior\ estimate\ of\ the\ state)}{var(measurement)} * H^T = \frac{var(\hat{x}_{t+1})}{var(y_t)} * H^T$$

Assumptions:
- Assumes linear dynamics
- Only optimal (minimize posterior variance) if all noise terms are Gaussian

Wolpert et al. (1995). *Science*

Optimal state estimation (Kalman filtering)

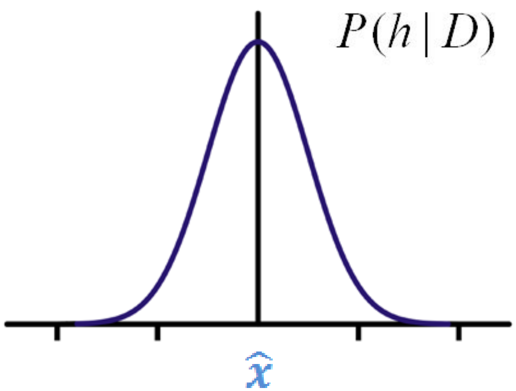
Kalman filter estimated states $\hat{x}$ are the MAP estimates of the posterior distributions over the states at each time point (Bayes theorem)

posterior

likelihood

prior

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}$$

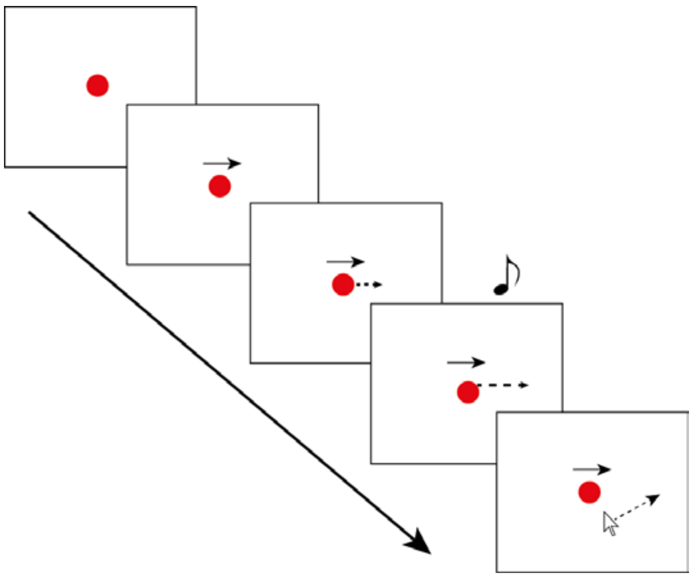


MAP estimate

Suggested reading:
Faragher, R. (2012). Understanding the basis of the Kalman filter via a simple and intuitive derivation. *IEEE Signal Processing Magazine*, *29*(5), 128-132.

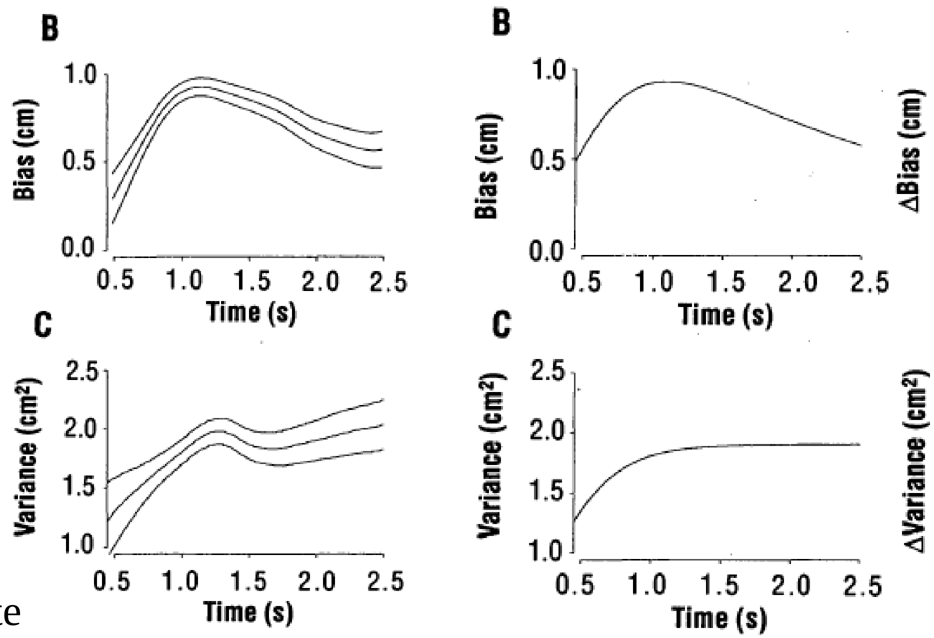
Wolpert et al. (1995). *Science*

Optimal state estimation (Kalman filtering) - Example



Human behaviour

Kalman filter predictions



Kalman filter: combine prior state estimate with (weighted) proprioceptive information

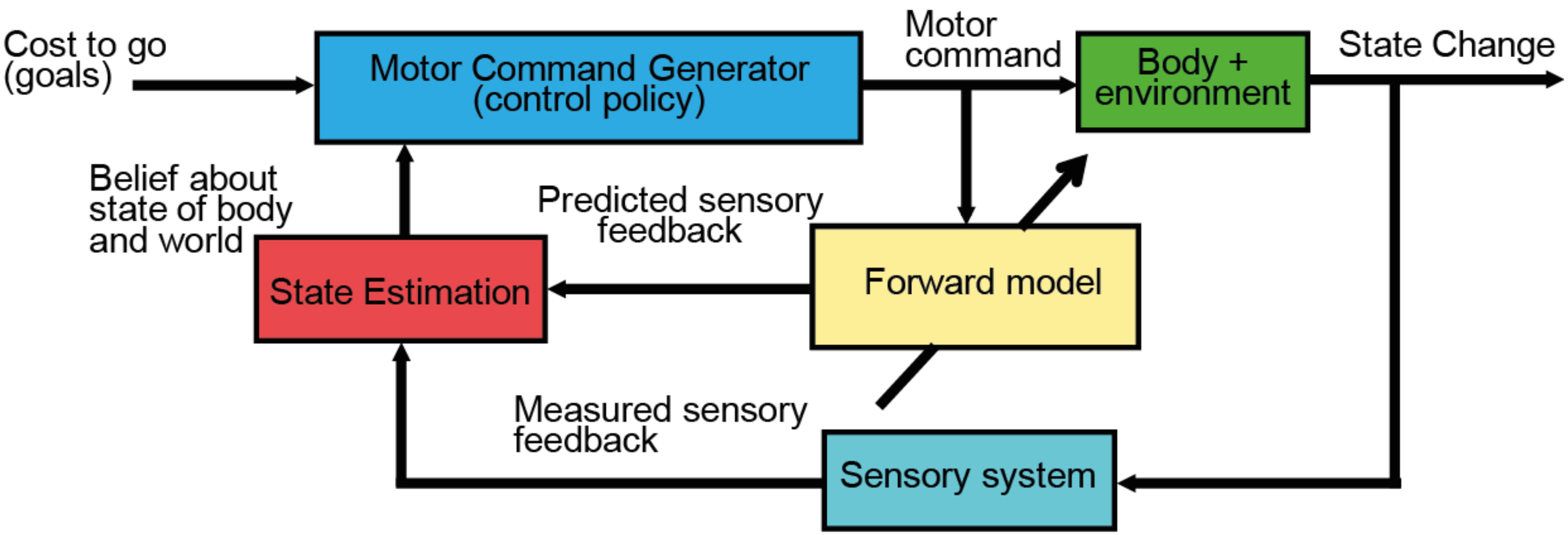
Human behaviour corresponds to predictions from a Kalman filter

Wolpert et al. (1995). *Science*

Optimal Feedback Control (OFC)

Optimal Feedback Control Theory:

- Optimal control (previous part)
- Optimal state estimation (Kalman filter)



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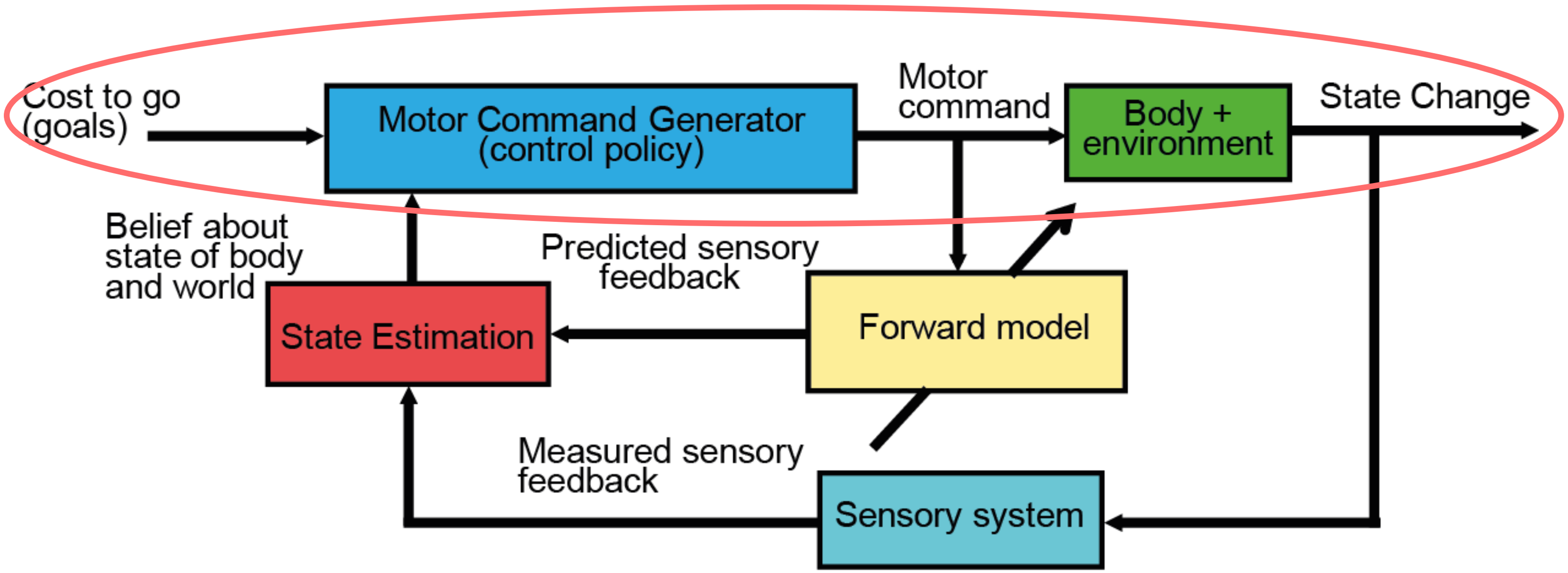

Optimal Feedback Control (OFC)

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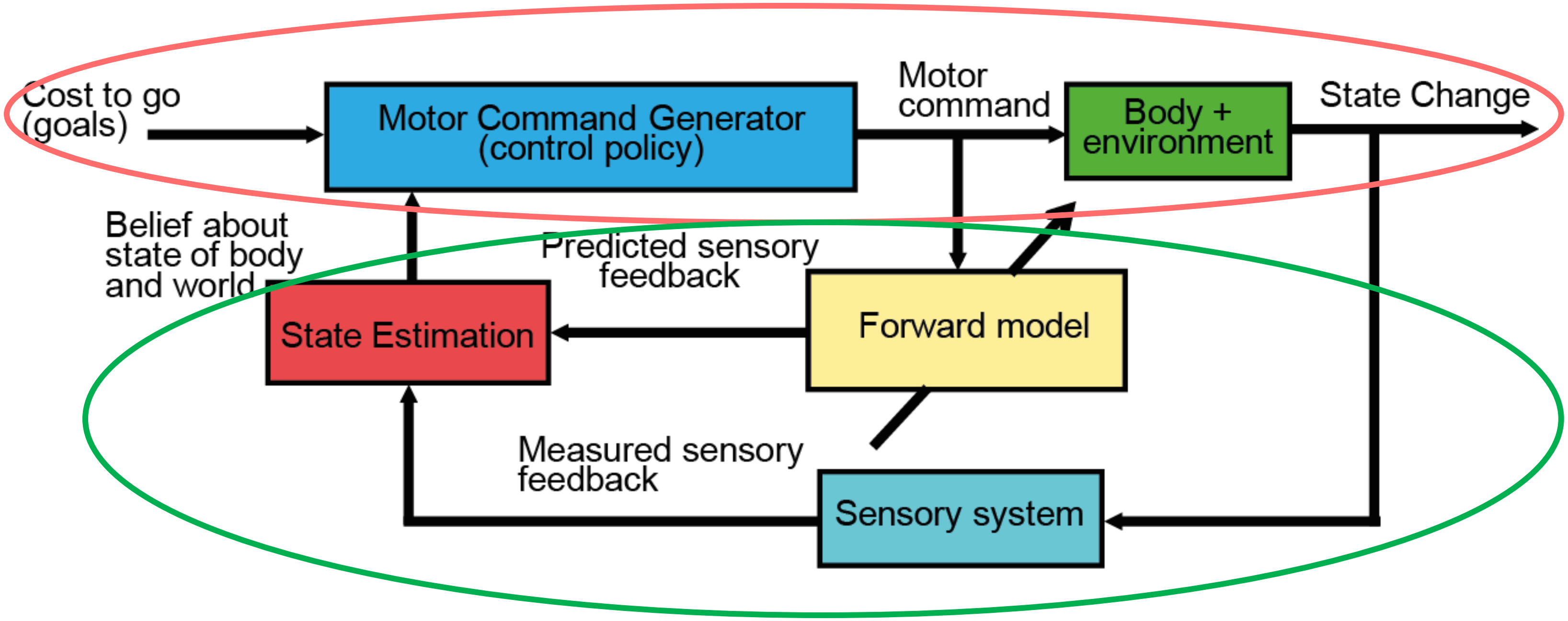


Optimal Feedback Control (OFC)

Optimal Feedback Control Theory (stochastic optimal control):

- Optimal control
- Optimal state estimation (Kalman filter)

Continuous state estimation requires a **feedback control law / policy**: $\boldsymbol{u}_t = \pi(\hat{x}_t)$

In case of linear dynamics, additive Gaussian noise, and quadratic costs (**LQG**):

$$\boldsymbol{u}_t = -L_t \hat{x}_t$$

$$L_t = (R + B^T S_{t+1} B)^{-1} B^T S_{t+1} A$$

$$S_t = Q_t + A^T S_{t+1}(A - BL_t)$$

Dynamics:

$$x_{t+1} = Ax_t + B\boldsymbol{u}_t + \boldsymbol{\varepsilon}_x \qquad \boldsymbol{\varepsilon}_x \sim N(0, \sigma_x)$$

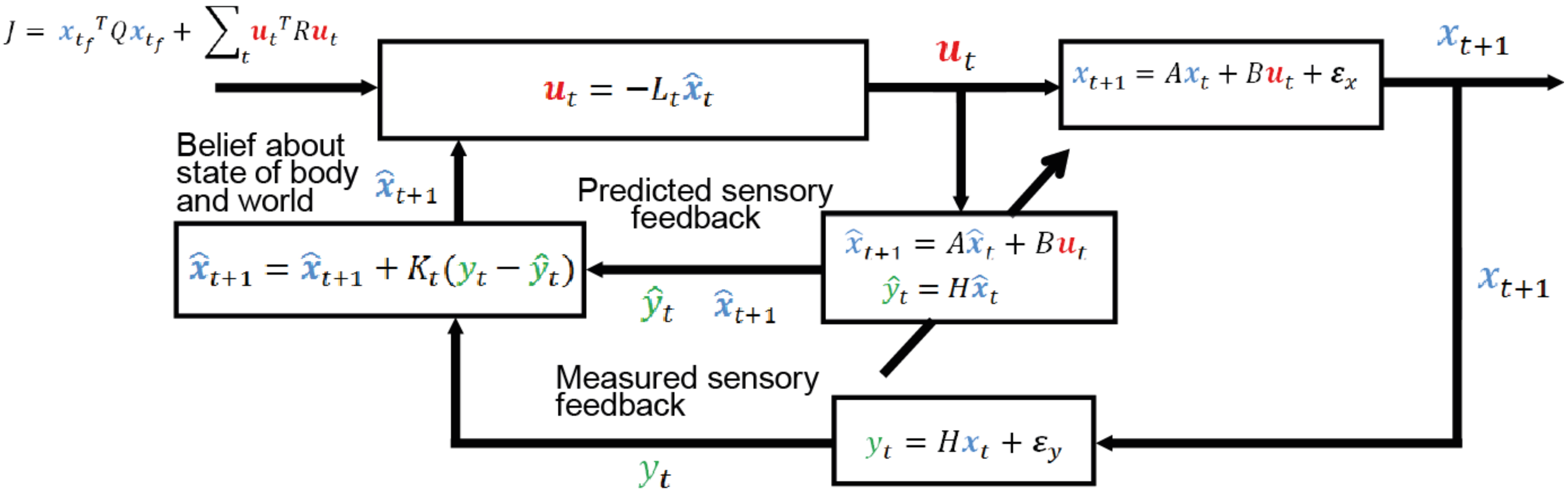
$$y_t = Hx_t + \boldsymbol{\varepsilon}_y \qquad \boldsymbol{\varepsilon}_y \sim N(0, \sigma_y)$$

Cost function:

$$J = x_{t_f}{}^T Q x_{t_f} + \sum_t \boldsymbol{u}_t{}^T R \boldsymbol{u}_t$$

Optimal Feedback Control (OFC)

Large texts



$$L_t = (R + B^T S_{t+1} B)^{-1} B^T S_{t+1} A$$

$$S_t = Q_t + A^T S_{t+1}(A - BL_t)$$

Optimal Feedback Control (OFC)

Features:

- Feedback control law (control policy) depends on the dynamics of the system
- Feedback control law depends on Q (penalizes deviation from goal)
- Feedback control law is inversely proportional to R (penalizes effort)
- Control policy: choose the best action given the current situation
- No distinction between trajectory planning and movement execution
- Errors are only corrected if they influence the task goal (‘minimal intervention’ principle)

Todorov & Jordan (2002). *Nat Neurosci*

Motor control

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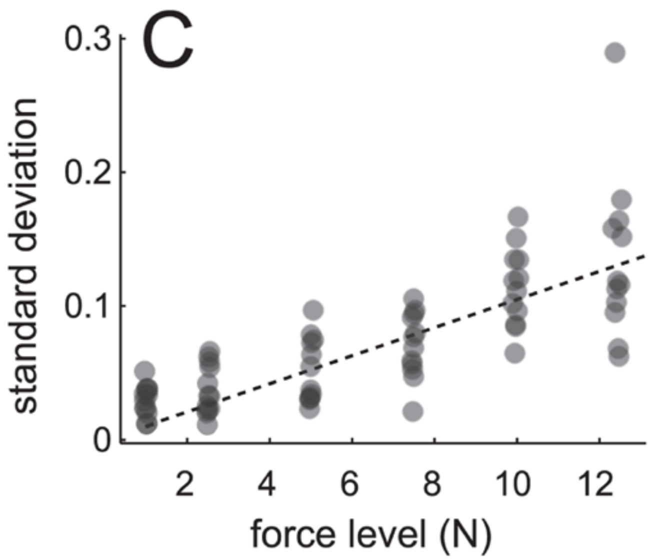
Noise is signal-dependent!

Signal-dependent motor noise

OFC solution:

Todorov (2005). *Neural Comp*



O'Sullivan et al. (2009). *Plos Comp Biol*

Harris & Wolpert (1998). *Nature*

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Enormous other field in motor control research named motor learning and adaptation. → suggests that the forward model adapts when errors are repeatedly experienced.

Suggested literature:

Wolpert, D. M., Diedrichsen, J., & Flanagan, J. R. (2011). Principles of sensorimotor learning. *Nature Reviews Neuroscience*, *12*(12), 739–751.

Motor control

Motor control is a difficult computational problem to solve, because of:

- Redundancy
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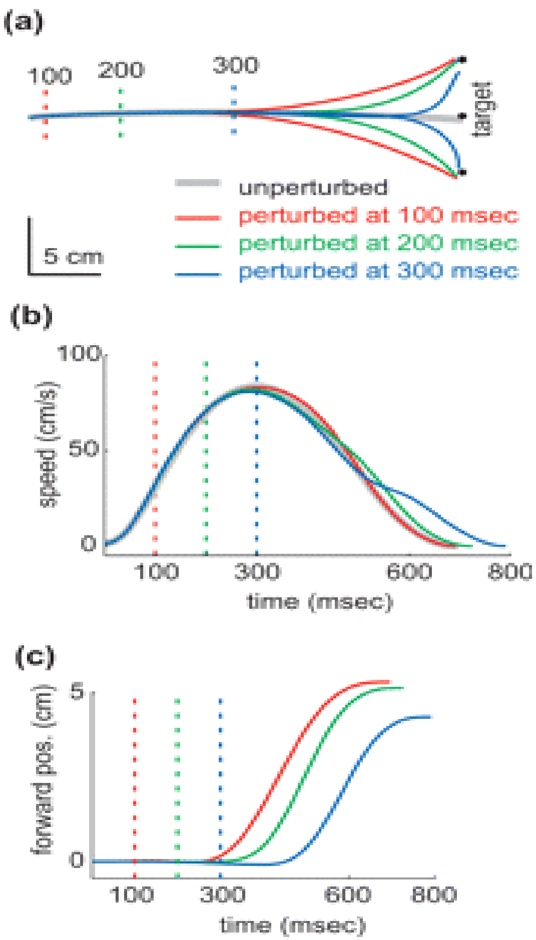
Problem for the current framework; no analytical solution.

(Current) solution: linearization as approximation

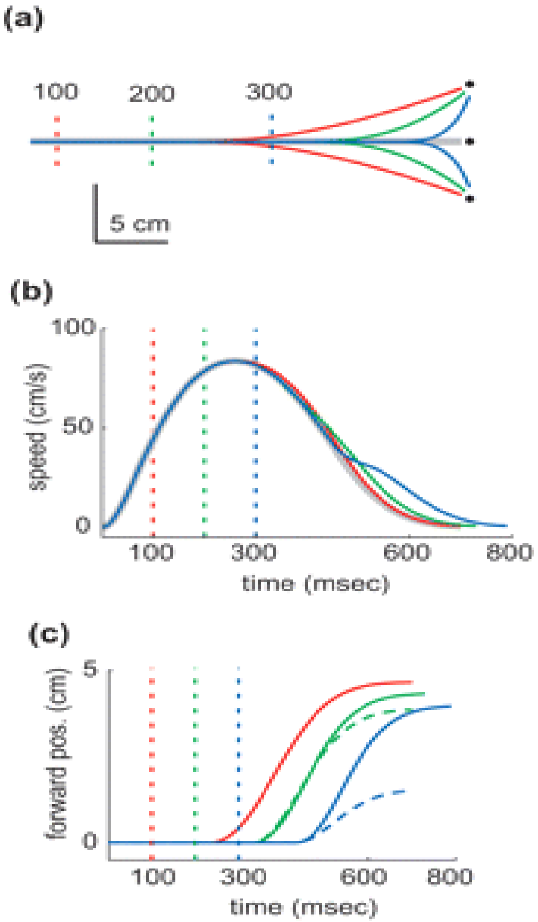
What behavioural/neural data can the model explain?

Behavioural

OFC model predictions:

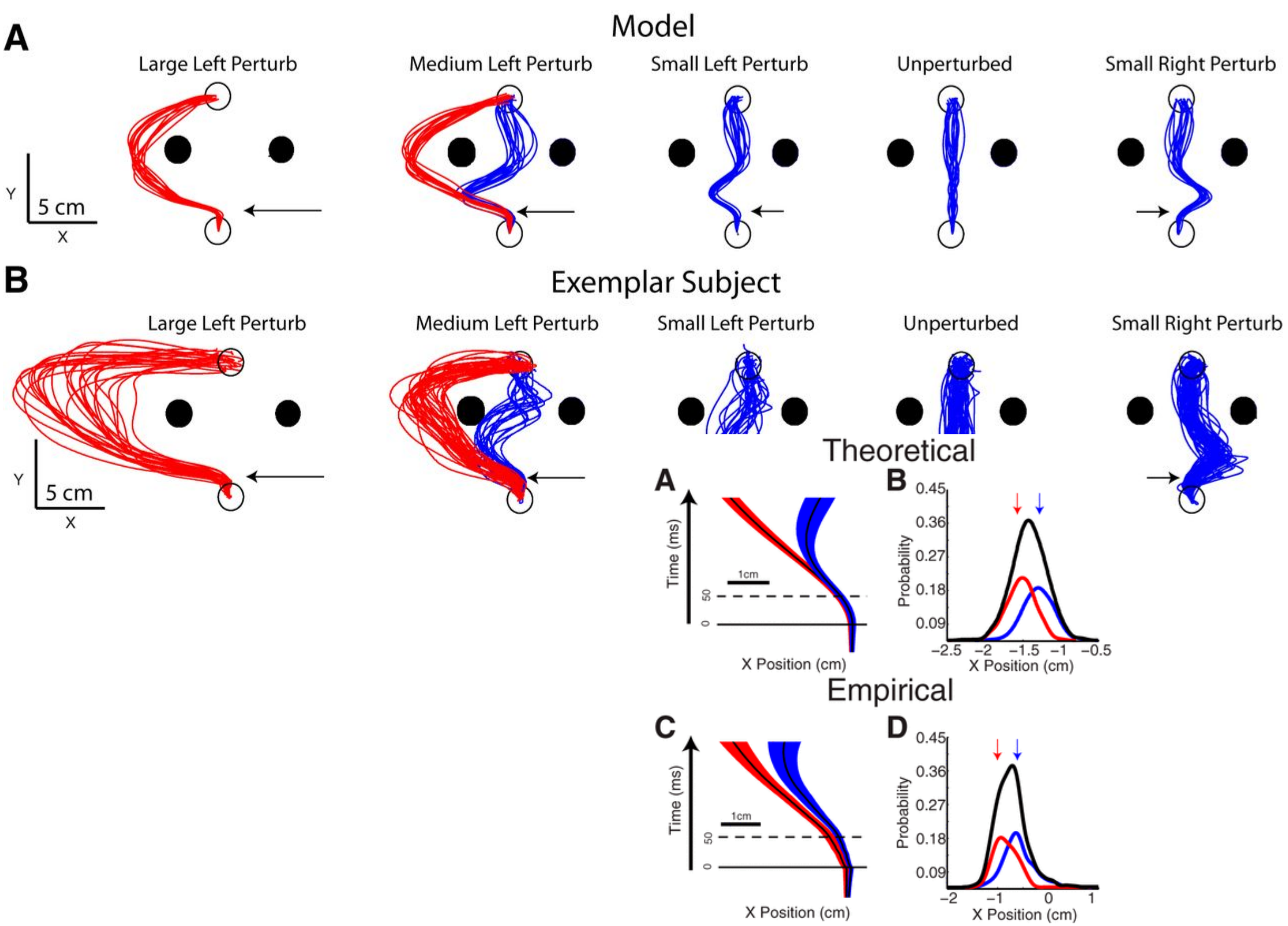


Human data:



Liu & Todorov (2007). *J Neurosci*

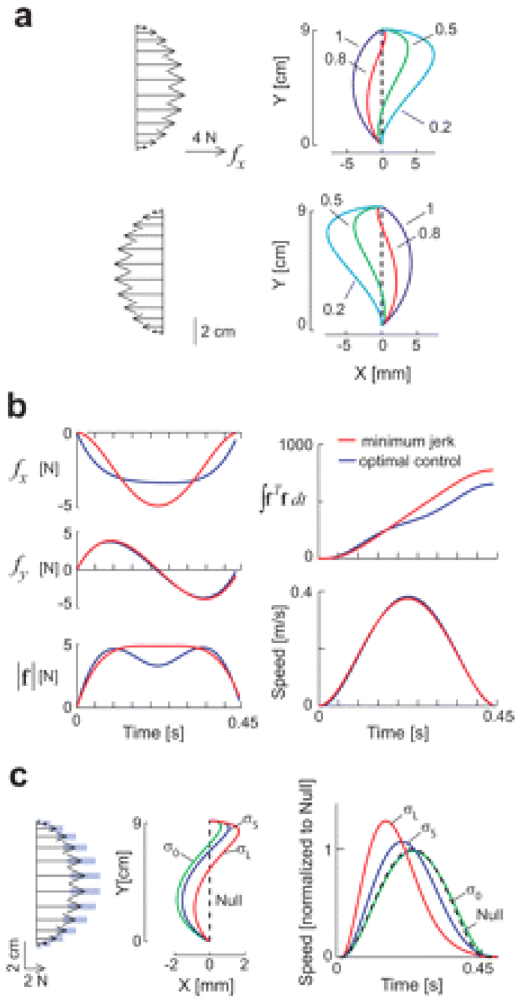
Behavioural



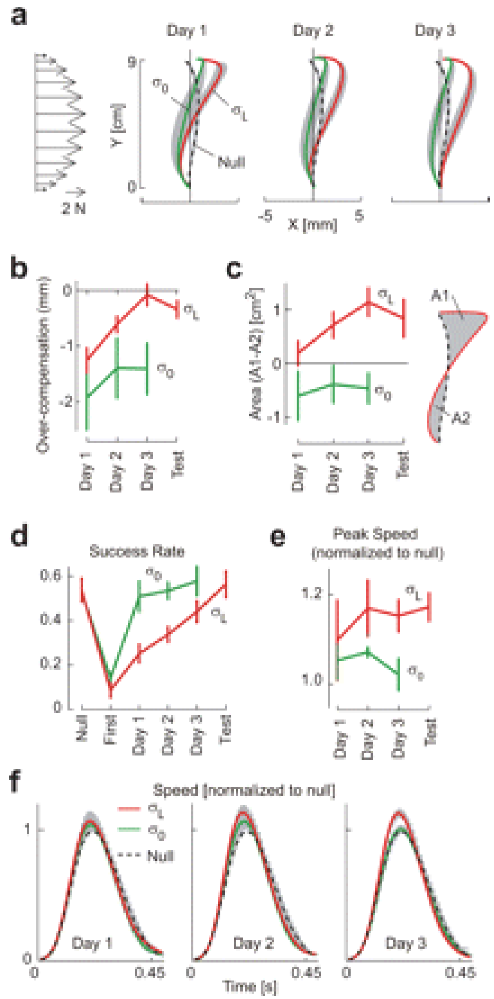
Nashed et al. (2014). *J Neurosci*

Behaviour

Model predictions:



Human data:

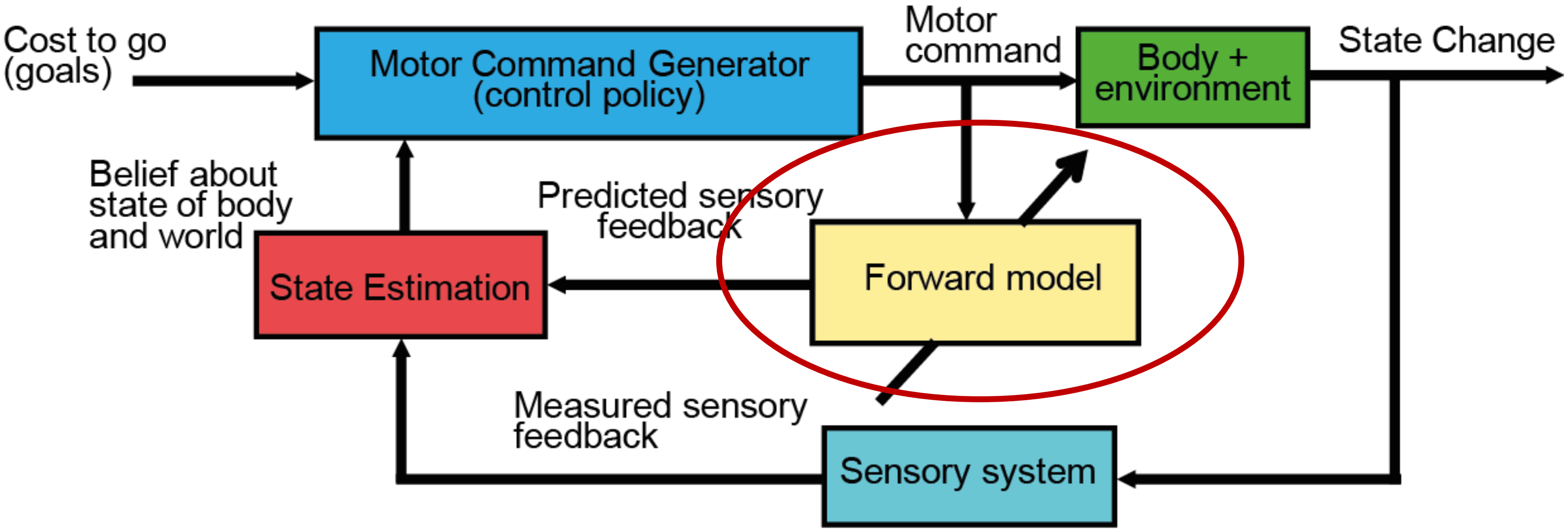


Izawa & Shadmehr (2008). *J Neurosci*

OFC and the brain

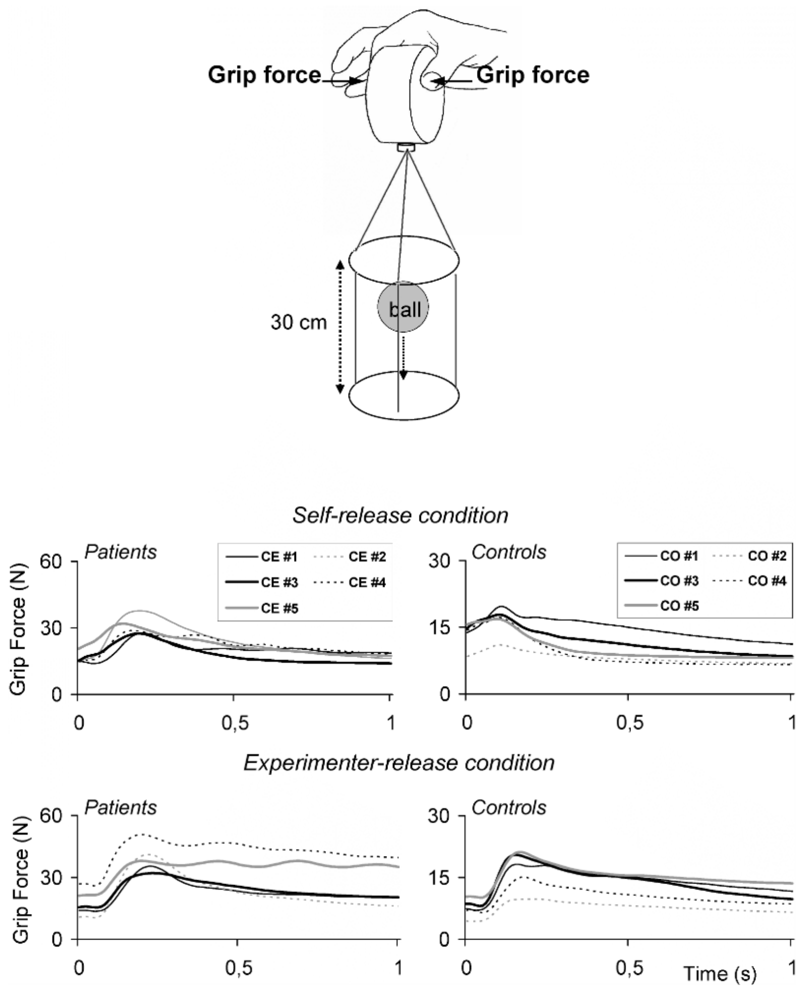
“Clearly, the brain does not use the formalisms of OFC to solve control problems.” (Scott (2012). *Trends Cogn Sci)*

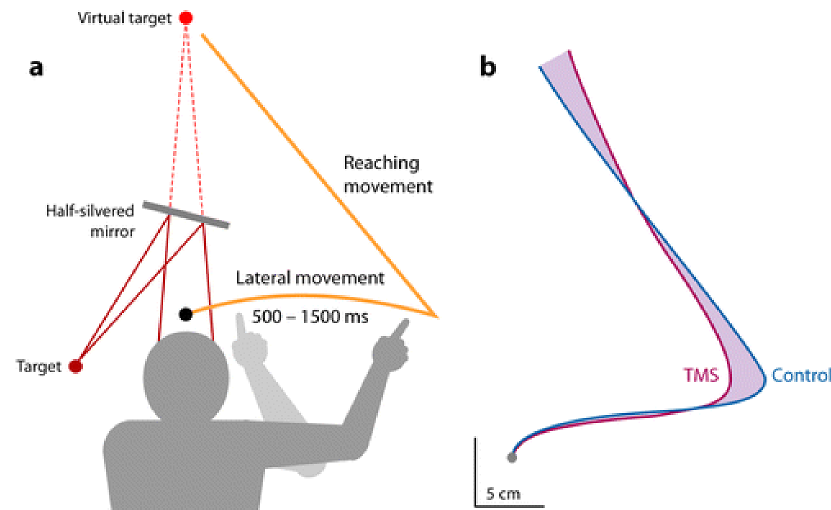
OFC and the brain



OFC and the brain

Forward model - Cerebellum





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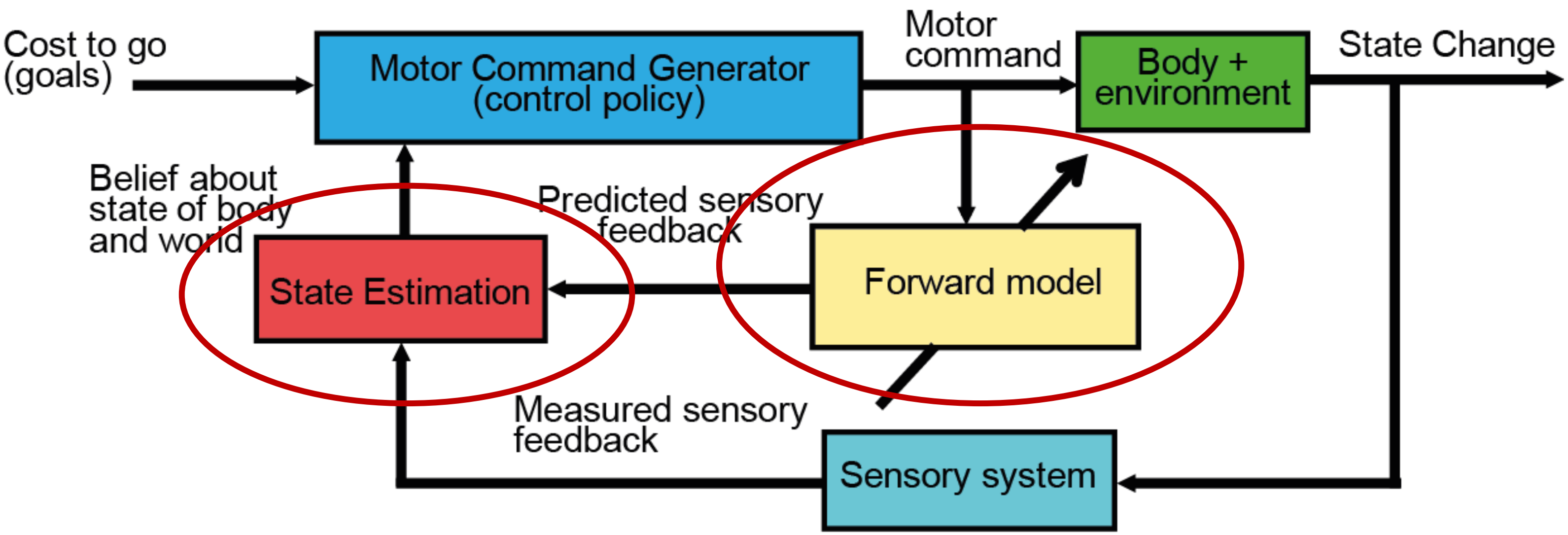
Nowak et al. (2004). *The Cerebellum*

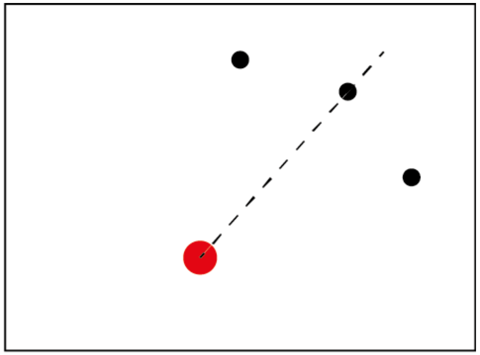
Miall et al. (2007) *Plos Biol*

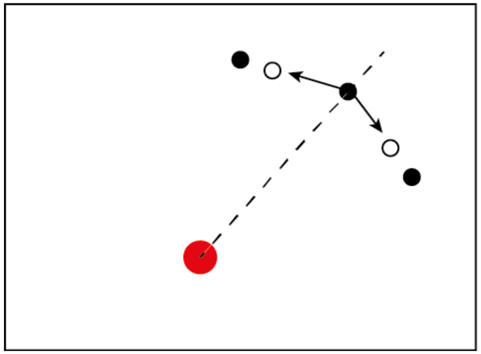
OFC and the brain

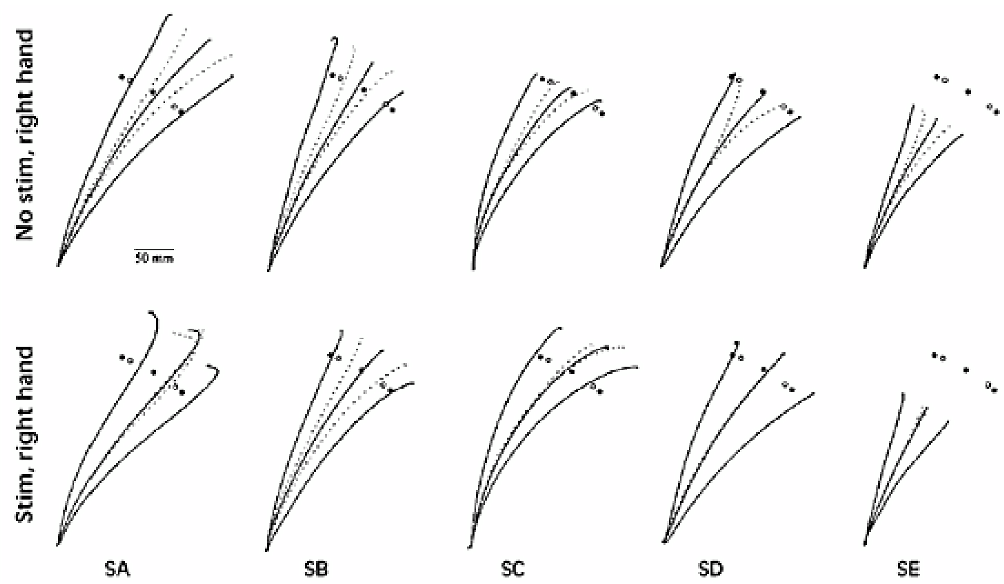


OFC and the brain

State estimation – Posterior Parietal Cortex







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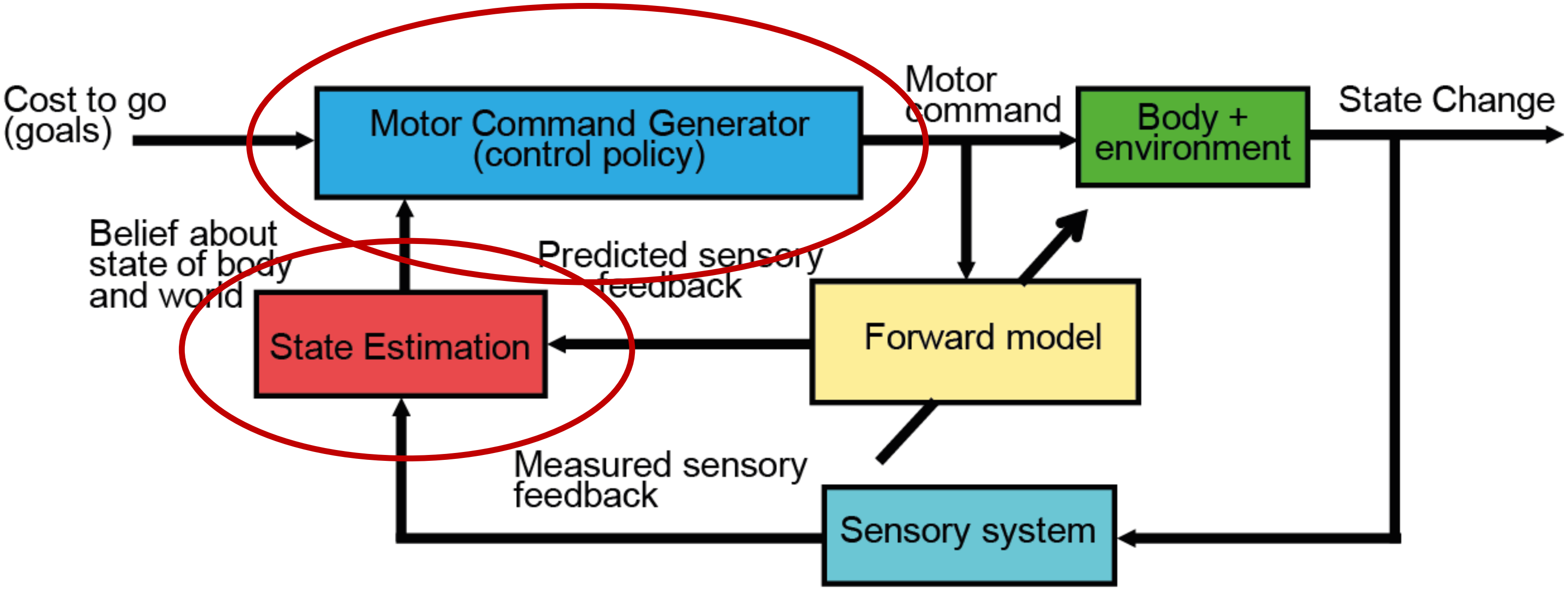
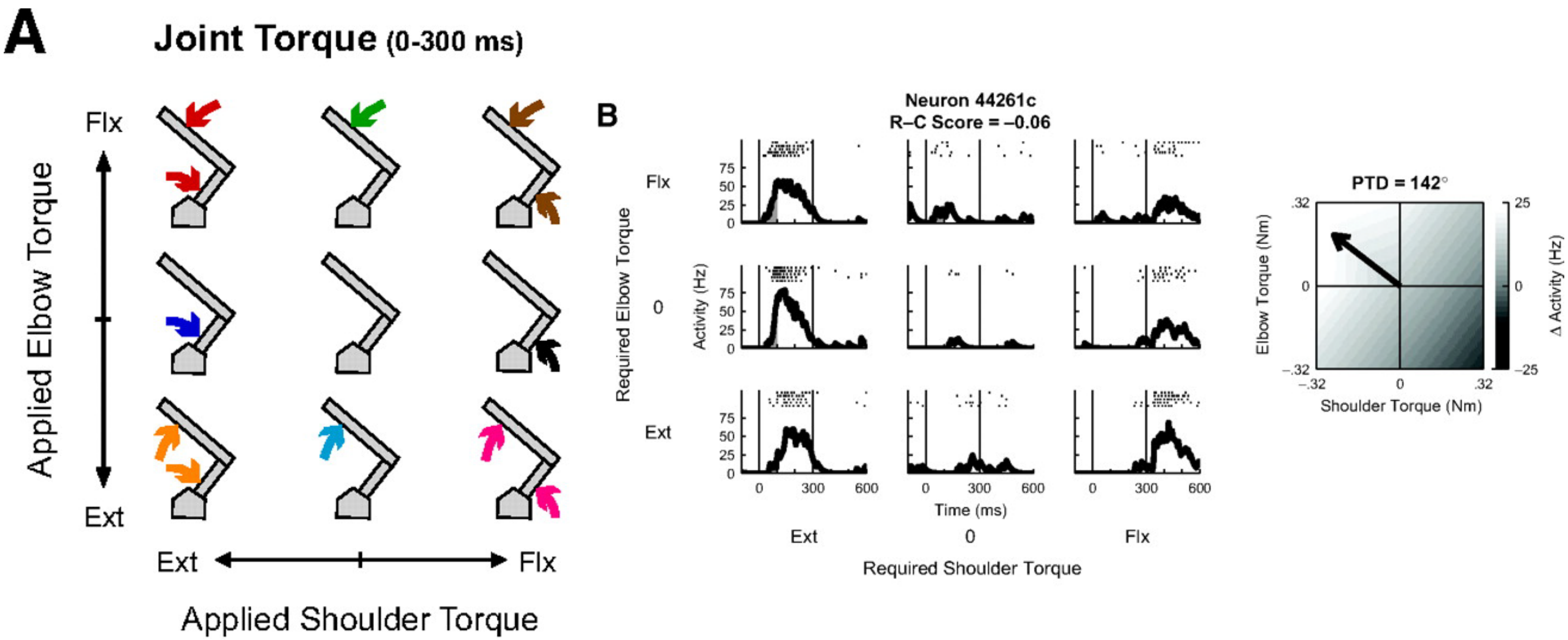
OFC and the brain



OFC and the brain

Feedback control – Motor cortex, M1

M1 neurons are tuned to mechanical perturbations



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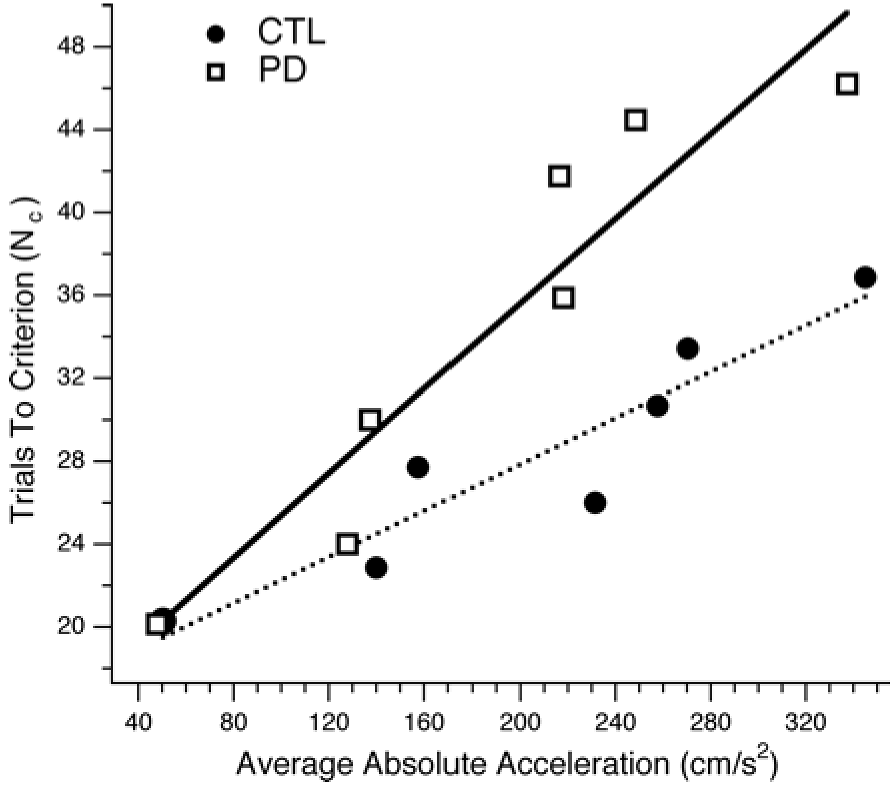
OFC and the brain

Task/Goal selection & effort/cost – Basal Ganglia



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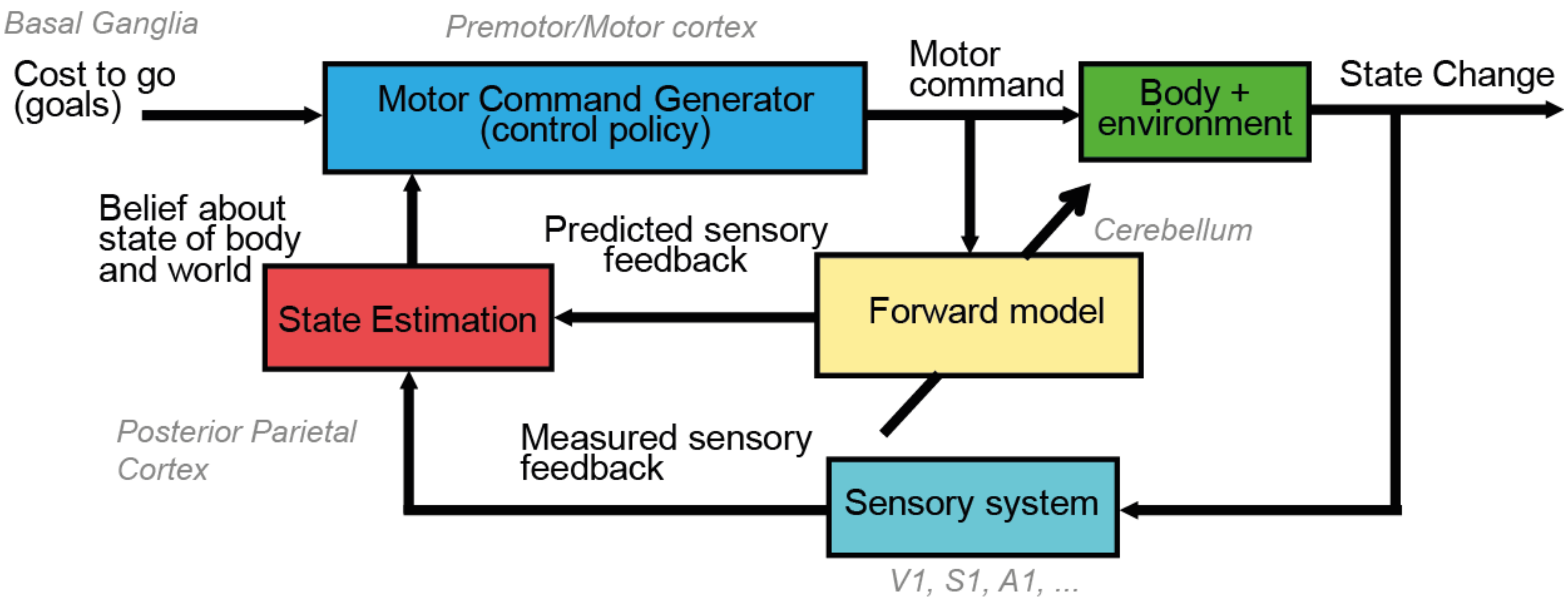
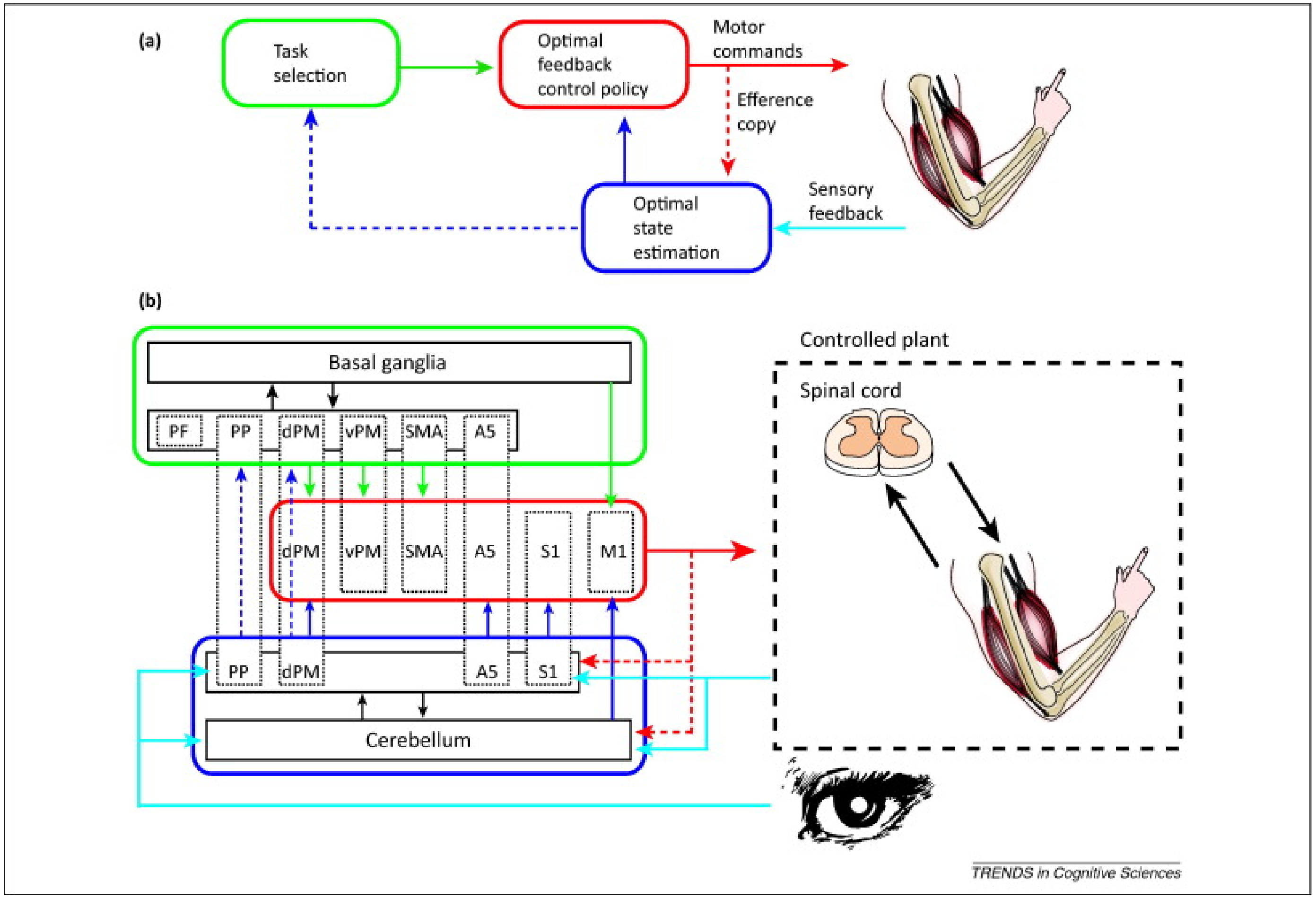
OFC and the brain

Large texts



OFC and neural implementation

Large texts



Scott (2012). *Trends Cogn Sci*

References

Required reading:
Franklin, D. W., & Wolpert, D. M. (2011). Computational Mechanisms of Sensorimotor Control. *Neuron*, *72*(3), 425–442.

Shadmehr, R., & Krakauer, J. W. (2008). A computational neuroanatomy for motor control. *Experimental Brain Research*, *185*(3), 359–381.

Further reading:
Book chapter:
Shadmehr, R., & Mussa-Ivaldi, S. (2012). Optimal feedback control (chapter 12). *Biological learning and control* (pp. 335-365). Cambridge: The MIT Press.
http://shadmehrlab.org/MMC/contents.htm

Full set of equations as a solution to OFC with linear dynamics and quadratic costs:
Todorov, E. (2005). Stochastic optimal control and estimation methods adapted to the noise characteristics of the sensorimotor system. *Neural Computation*, *17*, 1084-1108.
Also Matlab code accompanying this paper:
http://homes.cs.washington.edu/~todorov/papers.html

Questions?