

1. [20 marks] Digital Logic.

$$\frac{(17+t)}{\frac{17}{2} + \frac{t}{1024}} = 77 \quad 17+t = 77 \cdot \frac{11}{2} + \frac{77}{1024} \cdot t$$

$$t - \frac{77}{1024}t = 637.5 \Rightarrow t = 689.3 \Rightarrow 690$$

a) Let 'X' be the ternary connective such that 'Xpqr' is equivalent to '(p /\ q) + (q /\ r)'. '+' is exclusive or. 'F' and 'T' denote the 0-ary connectives 'false' and 'true', respectively.

Using {'X', 'T'}, synthesize:

$$\sim p \mid = \mid X \quad P \quad P \quad T$$

Using {'X', 'F'}, synthesize:

$$p \setminus / q \mid = \mid X \quad F \quad P \quad q \quad \checkmark$$

b) 'M' is the ternary 'minority' connective. 'Mpqr' is true iff a minority of its arguments is true. 'F' denotes the 0-ary connective 'false'.

Using {'M', 'F'}, synthesize:

$$\sim p \mid = \mid M \quad P \quad P \quad F \quad \checkmark$$

Using {'M', '~', 'F'}, synthesize:

$$p \setminus / q \mid = \mid M \quad \sim P \quad \sim q \quad F \quad \checkmark$$

2. [20 marks] Amdahl's Law.

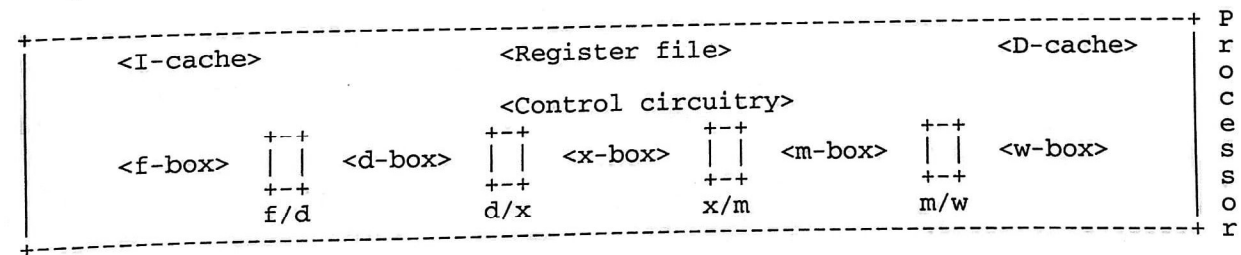
a) On a uniprocessor, moderately serial portion A of program P consumes 27 s, while perfectly parallel portion B consumes 143 s. On a parallel computer, portion A speeds up 2x, while portion B speeds up by the number of processors. Given 1,024 processors, what is the speedup on program P?

$$(27+143) \div (\frac{27}{2} + \frac{143}{1024}) = 12.46 \quad su = 12.46 \text{ times } \checkmark$$

b) On a uniprocessor, moderately serial portion A of program P consumes 17 s, while perfectly parallel portion B consumes 't' s. On a parallel computer, portion A speeds up 2x, while portion B speeds up by the number of processors. How big must 't' be so that using 1,024 processors gives a speedup of at least 77 times? Round any nonintegral 't' to the next highest integer.

$$(17+t) \div (\frac{17}{2} + \frac{t}{1024}) = 77 \quad t = 690 \text{ s } \checkmark$$

5. [20 marks] Pipeline Boxes and Pipeline Latches



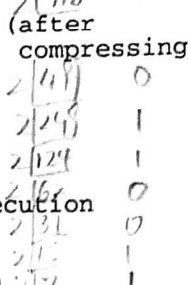
Memrefs have 2 m-boxes, and floating-point multiplies have 4 x-boxes. Integer add is denoted xi. Consider the following space-time diagram:

		1	2	3	4	5	6	7	8	9	0	1	2	3
1.d	f0,0(r1)	f	d	xi	m1	m2	w							
1.d	f2,0(r2)		f	d	xi	m1	m2	w						
mul.d	f4,f0,f2			f	d	s	s	x1	x2	x3	x4	n	w	
s.d	f4,0(r1)				f	s	s	d	xi	s	s	m1	m2	

(before compressing)

But the actual physical behavior of the pipeline is better represented by the following space-time diagram:

2	1938	
2	3884	0
2	1371	0
2	008	1
(after compressing		
2	41	0
2	24	1
2	124	1
2	16	0
cution		
2	31	0
2	11	1
2	7	1



mul.d f4, f0, f2

State

this is the first

box completes.

address  $r_1 + 0$  w  
 $r_2 + 0$ , and put

value of  $f_0, f_2$ ,  
and Instruction 4

10 11 12 13 14 15

d) Mt, 2 Ml at the same time.

is illegal,  $m_1$  &  $m_2$  overlaps

memory addresses is OK.  
n. 4 must be in It is  
exit 'x' repeated  
---in every part, pipeline  
to the answer.

mediate>(r5)'. In  
nd added to the  
times X

00 000000 a b = (0\*) 8al

immediate>'. In  
and added to the  
1F34

~~1 F 3 4~~

7 = 4 + 2 + 1 l.d. do not need arithmetic

c) Machine C has 64-bit instructions. Consider 'l.d f8, <immediate> (r5)'. In hex, the 12-bit immediate is '7cd'. What is the 64-bit addend added to the base register 'r5'?   
 1997 7cd is positive   
 (0\*) 7cd

ans: 00007cd

d) Machine D has 64-bit instructions. Consider 'bne r3, r5, <immediate>'. In hex, the 12-bit immediate is '8ab'. What is the 64-bit addend added to the base register 'PC'?   
 2219 x 8 = 17752   
 17752 = 109010101000   
 8 ab 32 bits left shift 3 bits

ans: 0004558   
 (F\*) C558

5. [20 marks] Fractional-Number Formats.

a) [math] Display the infinite binary expansion of  $7 \frac{1}{20}$ . Fact:  $20 = 5 \times 4$ . Use the overbar to indicate repetition of a bitstring. Show work.

7 = 111000011

ans: 111.000011

b) [math] Normalize the infinite binary expansion of  $7 \frac{1}{20}$ , adding a scale factor, and preparing to move to a floating-point format. Show work.

1.11000011 x 2<sup>2</sup>

ans: 1.11000011 x 2<sup>2</sup>

PFP is a slight variant of IEEE floating point. In particular, i) there is no sign bit, ii) the exponent is 8-bit two's complement, iii) the fractional field is 40 bits, and iv) exponents are unbiased and no exponents have been removed to encode special values.

48 ÷ 4 = 12 40 ÷ 4 = 10

c) Show the register contents of  $7 \frac{1}{20}$  in PFP in hexadecimal. You may use the repetition convention of Qn. 4. Show work.

00000010 1100 0011 0011

ans: 02C333333333   
 9 times = 02C333333333

d) Here is the register contents of a PFP number in hexadecimal: '06f0'. Square this number and express the answer in the same way. Hint: Consider scaling (floating). Show work.

1.1110000 = 1.9375 x 2<sup>0</sup> = 1.9375   
 0.0000110 = 124   
 124<sup>2</sup> = 15376 ⇒ 11100000000000000000000000000000

ans: 0de080000000   
 = 0de080000000

e) What is the difference between the largest PFP number and the next largest PFP number? Express your answer using powers of 2. Show work.

exponent: 01111111 = 2<sup>7</sup> - 1 = 127   
 2<sup>127</sup> x 2<sup>-40</sup> = 2<sup>87</sup>

ans: 2<sup>87</sup>

2 | 17752

2 | 8876 0  
2 | 4438 0  
2 | 2219 0  
2 | 1109 1  
2 | 554 1  
2 | 277 0  
2 | 138 1  
2 | 69 0  
2 | 34 1  
2 | 17 0

2 | 8  
2 | 4  
2 | 2  
2 | 1  
0  
0  
0  
0  
0  
0

2<sup>127</sup> x 2<sup>-40</sup> = 2<sup>87</sup>

$\frac{1}{20} \times 2 = \frac{2}{20} 0$   
 $\frac{2}{20} \times 2 = \frac{4}{20} 0$   
 $\frac{4}{20} \times 2 = \frac{8}{20} 0$   
 $\frac{8}{20} \times 2 = \frac{16}{20} 0$   
 $\frac{16}{20} \times 2 = \frac{32}{20} 1$   
 $\frac{32}{20} \times 2 = \frac{64}{20} 1$   
 $\frac{64}{20} \times 2 = \frac{128}{20} 1$   
 $\frac{128}{20} \times 2 = \frac{256}{20} 1$

$\frac{4}{20} \times 2 = \frac{8}{20} 0$   
 $\frac{8}{20} \times 2 = \frac{16}{20} 0$   
 $\frac{16}{20} \times 2 = \frac{32}{20} 1$   
 $\frac{32}{20} \times 2 = \frac{64}{20} 1$   
 $\frac{64}{20} \times 2 = \frac{128}{20} 1$   
 $\frac{128}{20} \times 2 = \frac{256}{20} 1$

11 = 1.1 x 2<sup>1</sup>

3 12 1 0

16<sup>3</sup> 16<sup>2</sup> 16<sup>1</sup> 16<sup>0</sup>

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