COMP228 Assignment 2 Fall 2017

Issued: September 26, 2017 Due: October 10, 2017

Submit typed hardcopy in class. No extension will be granted.

1. [20 marks] Both {'~', '/\'} and {'~', '\/'} are _complete_ in the sense that
any Boolean function can be expressed using only connectives from the set.

'2!' and 'E' are ternary connectives. '2! pqr' is true iff precisely two of 'p', 'q', and 'r', are true. 'E pqr' is true iff an even number of 'p', 'q', and 'r', is true. 'T' is the nullary (0-ary) connective that is always true. 'F' is the nullary (0-ary) connective that is always false. Set A = {'2!', 'T', 'F'}. Set B = {'E', 'T', 'F'}. Below, complete the stated logical equivalences or write "impossible" if this cannot be done.

a) [4 marks] Synthesize '~' from set A. ans: ~p |= =| 2! ___ ___ ___

b) [4 marks] Synthesize '/\' from set A. ans: p /\ q |= =| 2! ___ ___ ___

c) [4 marks] Synthesize '~' from set B. ans: ~p |= =| E ___ ___ ___

d) [4 marks] Synthesize '/\' from set B. ans: p /\ q |= =| E ___ ___ ___

e) [4 marks] Fact: If you had been allowed to use '~' in b) or d), it would have
made no difference. We therefore conclude that set A is _____, while set B
is _____.

2. [20 marks] Convert each of the following six binary or hexadecimal natural numbers to decimal. Show work.

a) [10 marks] Binary numbers: 1011; 1011 0010; 1011 0010 1101 0001

b) [10 marks] Hexadecimal numbers: 8b; 8b4c; 8b4c51d0

3. [20 marks] Fixed-point numbers

a) [10 marks] A fixed-point binary number system has 'm' integer bits and 'n' fractional bits. What is the range of numbers represented, assuming an unsigned format? Does every binary rational in this range have a finite decimal expansion? ___ (yes/no) Justify your answer.

b) [10 marks] A fixed-point binary number system has 4 integer bits and 12 fractional bits, corresponding to a 20-bit register. Represent the decimal fractions 3.375, 6.7, and 1/20 in this format. Deal with the integer part separately. Now, take the fractional part, multiply by 2, record the integer part as one digit of your answer, and repeat until the fractional part is 0, or you run out of binary digits. Do not round. Show the answers in both binary and hexadecimal.

4. [10 marks] Multiply the following 4-bit unsigned binary numbers. Present your work in the following format: i) Show the initial value of the accumulator, and ii) Show each new distinct value of the accumulator up to and including the final answer. As before, show both binary and hexadecimal.

a) [5 marks] x = 1011 (multiplicand) and y = 1101 (multiplier)

* 0000 0000 = 00 <initial binary and hex>

b) [5 marks] x = 1111 (multiplicand) and y = 1011 (multiplier)

* 0000 0000 = 00 <initial binary and hex>

5. [30 marks] Integer Multiplication.

a) [10 marks] Redo the multiplication steps exactly as in question 4 a), but initialize the accumulator to 1001 instead of 0000. What are the successive distinct values of the accumulator? Use the format of Qn. 4.

b) [10 marks] Show that, regardless of the initial value of the accumulator, the fused multiply-add result of two n-bit natural-number operands is always representable in 2n bits.

c) [10 marks] A modular-adder device 'M' operates with 8-bit registers. You feed it two 8-bit natural numbers 'a' and 'b'. It adds them, divides by 2^8, keeps the quotient 'q' a secret, and publishes the remainder 'r'. Hint: Before answering, experiment with small addition tables.

i) [5 marks] If a = 158 and r = 40, what are 'b' and 'q'?

ii) [5 marks] If a = 118 and r = 196, what are 'b' and 'q'?