MARSHALL HARVEY STONE
1903-1989

Marshall Stone was born in New York City on April 8, 1903. He was the son of Harlan Fiske Stone and Agnes Harvey Stone. Harlan Stone was a prominent jurist who served on the U.S. Supreme Court from 1925 to 1946 and as chief justice from 1942 to 1946. The family lived in Englewood, New Jersey, during Stone's school days and he attended the local public schools. In 1919 at the age of only 16 he entered Harvard University from which he was graduated, summa cum laude, in 1922.

Although it had been assumed that Marshall would follow his father into the law, a growing fascination with mathematics led to an extraordinary arrangement in which he spent the academic year 1922-1923 as a part time instructor at Harvard to find out whether he liked teaching. It turned out that he did and proceeded quickly to write a Ph.D. thesis under the direction of G.D. Birkhoff. The degree was awarded in 1926 but the work was completed rather earlier. The very distinguished mathematical career of Marshall Stone was under way.

Before settling down at Harvard for the thirteen year period 1933–1946 Stone held a variety of positions. He was at Columbia from 1925 to 1927, at Harvard from 1927–1931, at Yale from 1931 to 1933 and at Stanford for the summer of 1933. He became a full professor at Harvard in 1937. These early years were enormously fruitful ones for Stone's career as a research mathematician—so much so that he was elected a member of the National Academy of Sciences in 1938 at the unusually early age of 35.

His first paper was a short note on normal orthogonal sets of functions published in 1925 and by 1928 had published ten more papers on various aspects of the theory of orthogonal expansions—special emphasis being placed on expansions in terms of eigenfunctions of linear differential operators. This was one of the principal interests of G. D. Birkhoff and Stone's work was in the same tradition. Then in 1929 he began to work on the abstract theory of possibly unbounded self adjoint operators in Hilbert space, announcing his results with three notes published in the *Proceedings of the National Academy of Sciences* in 1929 and 1930. This work culminated in a six hundred page book which is now one of the great classics of twentieth century mathematics. It was entitled "Linear transformations in Hilbert space and their applications to analysis". This comprehensive and beautifully written book has been enormously influential. Modern functional (or abstract) analysis began with the ideas of Volterra on "functionals" in the late nineteenth century and was transformed and given considerable impetus in the first two decades of the twentieth century by the work of Hilbert and F. Riesz. The very different books of Banach (1922) and Stone set the stage for the extensive developments of the past half century.

In his introduction Stone freely acknowledges his scientific debt to J. von Neumann. Von Neumann published a long paper on the same subject in 1929 and it is not easy to disentangle their respective contributions. What is clear is that Stone was originally stimulated by preliminary work of von Neumann but had many key ideas quite independently. Moreover the whole last half of the book, including the chapter on spectral multiplicity theory and the extensive applications to differential and integral operators, has no counterpart in von Neumann's writings. The central point of the work of both men was the extension of Hilbert's spectral theorem from bounded to unbounded operators. This extension was made necessary by the problem of making mathematically coherent sense of the newly discovered refinement of classical mechanics known as quantum mechanics. Here an important part of the problem was discovering the "correct" definition of self adjointness for unbounded operators. This correct definition is rather delicate and the extension of the older theory of Hilbert and others was a major task.

The last of the three notes mentioned above was entitled "Linear transformations in Hilbert space III. Operational methods and group theory". The material it summarized was originally meant to be included as an extra chapter of the book but was omitted for reasons of space. The two theorems it announces are of sufficient importance to be discussed here in some detail.

Three years earlier, in 1927, Hermann Weyl and Eugene Wigner had introduced group theoretical methods into the new quantum mechanics in quite different ways. Weyl's idea was to use group theory to help clarify the foundations. His paper, written in physicists' language, implicitly conjectured two theorems about one parameter groups of unitary operators in Hilbert space. Stone's note states these conjectures as carefully formulated theorems, announces that he is in possession of proofs and gives some indication of their nature. Detailed proofs of both theorems were published in 1932—one by Stone and one by von Neumann. Both theorems were not only important for quantum mechanics, in the manner indicated by Weyl, but were also highly significant early steps in the then nascent unitary representation theory of non compact locally compact groups. One of them also played an important role in the chain of events leading through a note of B.O. Koopman to the ergodic theorems of von Neumann and Birkhoff and on to modern ergodic theory.

At this point it is useful to distinguish between two versions of one of Stone's theorems. The version suggested by Weyl's paper (and which stimulated Koopman) asserts that for every one parameter unitary group $t \to U_t$ there is a unique self adjoint operator H such that $U_t \equiv e^{iHt}$. The version emphasized in Stone's note is an analogue of the spectral theorem for one parameter unitary groups. This version has the great advantage that it can be generalized almost verbatim to arbitrary (separable) locally compact commutative groups.

The other theorem—the celebrated Stone-von Neumann uniqueness theorem—states the uniqueness of the irreducible solutions of the Heisenberg commutation relations in integrated form. It may be interpreted as giving a complete determination of all unitary irreducible representation of a certain non compact non commutative locally compact group—now well known as the Heisenberg group. So interpreted, it is the first example of such a determination by about a decade. Finally a series of natural generalizations of the Stone-von Neumann uniqueness theorem culminated in the imprimitivity theorem and the extension of the notion of induced representation from finite groups to general locally compact groups.

In 1934 and 1935 Stone published two more notes in the *Proceedings of the National Academy* which seemed at first to represent a completely new departure. They were entitled "Boolean algebras and their applications to topology" and "Subsumption of Boolean algebras under the theory of rings". Actually, just as Stone's work on spectral theory may be regarded as a natural outgrowth of his earlier work on concrete eigenfuntion expansions, so can his work on Boolean algebras be regarded as a natural outgrowth of his work on spectral theory. This is because of the role played in spectral theory by Boolean algebras of projections. In an entirely characteristic attempt to get to the bottom of things Stone undertook a thoroughgoing study of Boolean algebras and made a number of far reaching discoveries relating Boolean algebras to general topology on the one hand and to the theory of rings and ideals on the other.

The discovery of these connections has had significant consequences for all three subjects. One beautiful result is the celebrated Stone-Weierstrass theorem vastly generalizing the theorem of Weierstrass concerning the uniform approximability of arbitrary continuous functions on a finite interval by polynomials. Another is the natural one to one correspondence between all compact Hausdorff spaces on the one hand and certain rings on the other. Stone's studies of the relationship between compact spaces and rings of continuous funtions anticipated important elements in the modern theory of commutative Banach algebras. The detailed development of Stone's ideas on Boolean algebras, general topology, etc. were published in three lengthy papers in 1935, 1936 and 1937 respectively. The first appeared in the *American Journal of Mathematics* and the second and third in the *Transactions of the American Mathematical Society*. Applications of the new ideas to spectral theory were announced in notes published in 1940 and 1941.

Soon after the entry of the United States into World War II the character of Stone's work underwent a considerable change. For several years he was engaged in secret work for the U.S. government. A year after the end of the war he resigned his position at Harvard to take on the chairmanship of the mathematics department at the University of Chicago. This once great department had been declining in quality and Stone's mission was to strengthen it and bring it up to its former stature. In this he succeeded admirably. Before very long it was regarded by many as the best mathematics department in the country and, while a position like that is hard to keep indefinitely, it has remained one of the strongest departments ever since. He brought in André Weil, S.S Chern, Saunders Mac Lane and a number of promising younger men. Moreover in the words of one of the latter "Marshall devoted himself with both intensity and breadth,—from the largest issues to the smallest details—to the Department's welfare and development".

In 1952, Stone turned the chairmanship of the Chicago department over to Saunders Mac Lane but continued to be occupied with administrative matters. He was a strong force in reestablishing the International Mathematical Union—was much involved in the drafting of its constitution and served as its president from 1952 to 1954. He also interested himself actively in the problems of mathematical and scientific teaching—especially at the international level—and served on various boards and commissions.

While his various administrative concerns and activities prevented him from working on mathematical problems with his former intensity Stone continued to work and to publish results rather steadily until the early 1960's. At the same time his mathematical interests tended more and more toward the elucidation of questions of great generality and and profundity about the true nature of mathematics and mathematical concepts. There is reason to believe that his publications of the last thirty years give a very incomplete picture of his mathematical activity. At a conference in honor of his second retirement (see below) Stone gave a remarkable two hour lecture outlining his rather unusual and original views on the nature and structure of mathematics.

Stone remained at the University of Chicago until he retired as Professor Emeritus in 1968. At this time there was a week long conference in his honor the proceedings of which were published in 1970 by Springer-Verlag. Felix Browder was the editor. He did not wish to stop teaching however and forthwith began a new career as George David Birkhoff professor of mathematics at the University of Massachusetts in Amherst. No doubt the fact that Amherst, Massachusetts had been his father's childhood home added to the attractiveness of this move. He taught there for the next twelve years and among other activities supervised two Ph.D. theses. During his final year he was honored with a second retirement conference.

One of the many striking accomplishments of Marshall Stone was a truly extraordinary command of the English language. This gave a special flavor to his book, his mathematical papers and his many writings on other subjects.

His skill with the English language also manifested itself in his lectures which were models of clarity and organization. Stone was only moderately active in supervising Ph.D. theses. Indeed there are anecdotes about his reluctance to do this sort of teaching. On the other hand he did turn out a respectable number of new Ph.D.s and influenced many other young mathematicians by his writings and through informal personal contact.

Stone married young (in 1927) and he and his first wife Emmy raised three daughters. Reports have it that he was serious about fatherhood in a rather old fashioned way. His daughters had regular chores to do and in financial matters were kept on strict allowances. On the other hand he also believed in family fun and one of his many side interests was gourmet cookery. This marriage dissolved in divorce in 1962 but Stone soon remarried. His second marriage, to Ravijojla Kostic, lasted the rest of his life.

Of all Stones's many interests his love of travel was surely dominant. He began to travel when he was quite young and was on a trip to India when he died. He travelled frequently and extensively and was interested in seeing *all* parts of the globe. For example he visited the Pacific islands and (while travelling with Ravijojla) was shipwrecked in Antartica. It is very hard to think of a place that he has not come fairly close to at some time or another.

Stone, of course, was the recipient of many honors. We have mentioned his early election to the National Academy of Sciences. He was also elected, at an early age to the American Academy of Arts and Sciences and to the American Philosophical Society. He was president of the American Mathematical Society (1943-1944) and received many honorary doctorates, both domestic and foreign. In 1982 he was awarded the National Medal of Science. According to Edwin Hewitt, two extra honors that gave him special pleasure were his election to an Honorary Professorship at Columbia Teachers College and to membership in the Explorers Club of New York City.

Marshall Stone was a man with a very broad outlook and wide range of interests who seems to have thought rather deeply about a number of issues. One had only to talk to him at length or read his non mathematical writings to come away with the impression that here was an unusually thoughtful man with a high degree of penetration and insight. More than most he seemed well endowed with a quality which I can only describe as wisdom.

While on a visit to Madras, India he died quickly of a sudden illness on January 9, 1989.

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