THE 1979 STEELE PRIZES

Leroy Powell Steele, a graduate of Harvard College (B.A., 1923), died January 7, 1968, and bequeathed the bulk of his estate to the American Mathematical Society to be used for the award from time to time of prizes in honor of George David Birkhoff, William Fogg Osgood and William Caspar Graustein. The sixteenth, seventeenth and eighteenth Steele Prizes were awarded to Antoni Zygmund, University of Chicago, to Robin Hartshorne, University of California, Berkeley, and to Joseph J. Kohn, Princeton University.

Professor Zygmund's award is for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students.

Professor Hartshorne's award is for mathematical exposition. The two works cited are:

Equivalence relations on algebraic cycles and subvarieties of small codimension, Proceedings of Symposia in Pure Mathematics, volume 29, American Mathematical Society, Providence, RI, 1975, pp. 129–164.

Algebraic geometry, Springer-Verlag, Berlin and New York, 1977.

Professor Kohn's award is for a paper which has proved to be of fundamental or lasting importance in its field, or a model of important research. The paper cited is:

Harmonic integrals on strongly convex domains. I, II, Annals of Mathematics (2) **78** (1963), 112–148; ibid., **79** (1964), 450–472.

The prizes were awarded by the Council of the American Mathematical Society, based on recommendations made by the Committee on Steele Prizes consisting of Edward B. Curtis, Irving Kaplansky, H. Blaine Lawson, Henry O. Pollak, Gian-Carlo Rota, Hans Samelson, Stephen S. Shatz, Joseph L. Taylor, Raymond O. Wells, Jr., and Hans F. Weinberger (chairman).

The three prizes, currently totaling fifteen hundred dollars each, were presented at the prize session held at the Summer Meeting of the Society in Duluth, Minnesota, August 24, 1979, on which occasion the recipients were given the opportunity to speak in response to the awards. Professor Zygmund was out of the country and unable to attend the prize session to respond in person. Instead, he submitted the written response printed below. Professors Hartshorne and Kohn were present. Their remarks follow Professor Zygmund's response.

Antoni Zygmund

Antoni Zygmund was born in Warsaw, Poland on December 26, 1900. He received his Ph.D. from the University of Warsaw in 1923, and honorary D.Sc. degrees from Washington University (1972), the University of Toruń in Poland (1973), the University of Paris (1974), and the University of Uppsala in Sweden (1976). From 1922 to 1930 he was instructor of mathematics at Warsaw Polytechnical School, from 1926 to 1930 Privatdocent of the University of Warsaw, and during 1930 to 1939 professor at Wilno University. He spent the year 1939-1940 as a visiting lecturer at the Massachusetts Institute of Technology, then became assistant professor at Mt. Holyoke College, where he advanced to associate professor by 1945. He was a professor at the University of Pennsylvania from 1945 to 1947, then became professor at The University of Chicago, where he has been Swift Distinguished Service Professor of Mathematics since 1967.

Professor Zygmund served as member-at-large of the Council of the American Mathematical Society (1954 to 1956), and as its vice president (1954, 1955). He has been a member of many committees, including the Transactions Editorial Committee (1944 to 1949, Associate Editor 1944 to 1946); the Committee on Known Results (1947); the Committee to Nominate a Representative of the Society on the Policy Committee for Mathematics (1954); the Executive Committee of the Council (1956, 1957); the Colloquium Editorial Committee (1961 to 1966); the Committee to Select Hour Speakers for Western Sectional Meetings (1963, 1964); the Organizing Committee for the Symposium on Singular Integrals (April 1966); the Committee to Select the Winner of the Bôcher Prize (1974); and co-chairman of the Committee for the Summer Institute on Harmonic Analysis in Euclidean Spaces and Related Topics (July 1978).

Professor Zygmund gave an invited address at the 1943 summer meeting in New Brunswick, New Jersey. He delivered the Colloquium Lectures at the 1953 summer meeting in Kingston, Ontario, gave a 30-minute address at the 1954 International Congress of Mathematicians in Amsterdam, and spoke at the Symposium on Singular Integrals in Chicago in April 1966. He was also an organizer of the special session on Differentiation Theory in Evanston, Illinois, in November 1968.

Professor Zygmund held a Rockefeller Foundation Fellowship at Oxford and Cambridge Universities (1929-1930), and a Guggenheim Foundation Fellowship (1953-1954). He received the prize of the Polish Academy of Sciences in 1939. He is a member of the National Academy of Sciences, the American Association for the Advancement of Science, the Polish Academy of Sciences, the Argentinian National Academy, the Spanish National Academy in Madrid, and the National Academy of Sciences in Palermo (Italy). His research interests are Fourier series and real variables.


Antoni Zygmund

During his student days, Professor Zygmund was influenced primarily by Professors W. Sierpiński and A. Rajchman, and by S. Saks, his older colleague. In those days, the work of Sierpiński and Saks dealt in general with real variables, and in particular with the differentiability of functions and integrals, while that of Rajchman was in the direction of trigonometric series and the calculus of probability. Zygmund's doctoral thesis dealt with extensions of Riemann's theorems on trigonometric series to the case of coefficients not tending to 0, but perhaps his best result during that period was his proof of the existence of sets of uniqueness of positive measure. More precisely, he showed that given *any* sequence of positive numbers $\epsilon_n \to 0$ and any number $\delta > 0$, there is a set $E \subset [0, 2\pi]$ of measure greater than $2\pi - \delta$ and a trigonometric series (S):

$$\frac{1}{2}a_0 + \sum_1^\infty (a_n \cos nx + b_n \sin nx)$$

not identically zero such that S converges to 0 in E with $|a_n| < \epsilon_n$, $|b_n| < \epsilon_n$ for all n. He tried hard to prove the existence of a set E of measure 2π with analogous properties; however, that that is actually so was proved only recently by Kahane and Katznelson.

In a classic paper in 1927, M. Riesz showed (among other things) that if (S):

$$\frac{1}{2}a_0 + \sum_1^\infty (a_n \cos nx + b_n \sin nx)$$

is the Fourier series of a function $f \in L^p$, $1 < p < \infty$, then the conjugate series $(\widetilde{S})$:

$$\sum_1^\infty (a_n \sin nx - b_n \cos nx)$$

is the Fourier series of a function $\widetilde{f} \in L^p$ and $\|\widetilde{f}\| \leqslant A_p \, \|f\|_p$. It was natural to consider the case $p = 1$, and Zygmund showed that if $f \in L \log^+ L$, then $\widetilde{S}$ is the Fourier series of a function $\widetilde{f} \in L$ with

$$\|\widetilde{f}\|_1 \leqslant A \int_0^{2\pi} |f| \log^+ |f| dx + B,$$

and that the class $L \log^+ L$ is best possible in this context. Since then Zygmund has searched systematically for theorems where the class $L \log^+ L$ (or its generalizations) would play such a decisive role.

Such situations are not infrequent and perhaps one result (obtained jointly with Jessen and Marcinkiewicz) deserves mention here. Let $x \in \mathbf{R}^m$ ($m = 1, 2, \ldots$) and let f be a function such that $|f| (\log^+ |f|)^{m-1}$ is locally integrable. Then the indefinite integral $F(E) = \int_E f$ is differentiable almost everywhere, to the value $f(x)$, in the sense that for almost every point $x \in \mathbf{R}^m$ if R_x is any parallelopiped with sides parallel to the coordinate axes and containing x, then

$$(1/|R_x|) \int_{R_x} f \longrightarrow f(x) \text{ as } R_x \text{ shrinks to } x.$$

There again the class $L(\log^+ L)^{m-1}$ is best possible.

Perhaps it is not unnatural to mention in this context another class of functions whose roots go to Riemann and which seems to be of significance although one is not sure he understands the significance fully. Let f be periodic of period 2π. Denote by Λ_α (or, sometimes, Lip α) the classes of f such that $|f(x+h) - f(x)| \leqslant A\,|h|^\alpha$. Clearly, only the case $0 < \alpha \leqslant 1$ is of interest. This class is important in the problem of *best approximation* of f by trigonometric polynomials T_n of a fixed order n. Roughly speaking, one approximates an f by polynomials T_n with error $O(n^{-\alpha})$ if and only if $f \in \Lambda_\alpha$, *provided,* *however*, $0 < \alpha < 1$. This is a classic result of De la Vallée-Poussin and Serge Bernstein, and for a long time the problem remained open for the case $\alpha = 1$. It was Zygmund who showed that f has best approximation of order $O(1/n)$ if and only if $f(x + h) + f(x - h) - 2f(x) = O(|h|)$ uniformly in x. This class of functions already occurs in the work of Riemann, but even now we do not fully understand the properties of such functions.

The purpose of these last paragraphs is to suggest the flavor of Zygmund's work; it is not possible to give a detailed account of the full body of that work here. In particular, the whole theory of singular integrals and their applications to real variables and differential equations is omitted. The omission is doubly regrettable since that theory developed out of a collaboration between Zygmund and his students J. Marcinkiewicz and A. P. Calderón. Zygmund is fond of saying that this was one of the rare occasions in mathematics when the contribution of the students was decisive for the success of the collaboration.

Upon learning he was to receive a Steele Prize, Professor Zygmund expressed deep thanks to the Society for the recognition thus accorded his work in and for mathematics. He spoke also, and affectionately, of his students and colleagues here and abroad who over the years had been joined with him in this work. He remembered his university affiliations in Poland and the United States, which in the last analysis, he felt, had made his mathematical work possible; and in particular, he spoke warmly of his more than thirty pleasant and productive years at The University of Chicago.

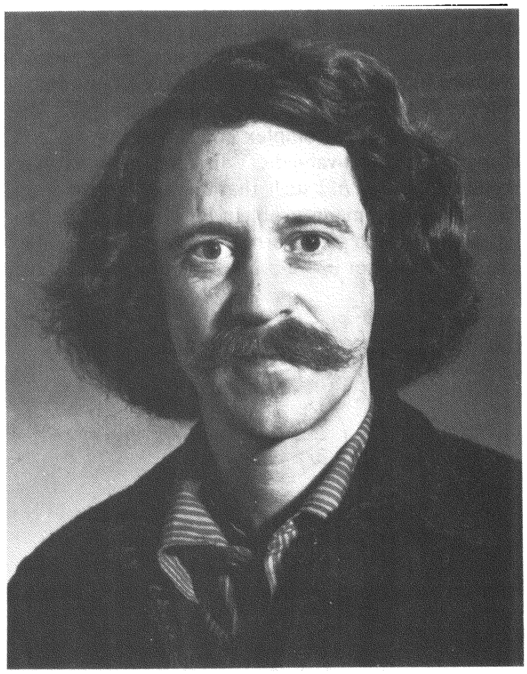
Robin Hartshorne

I am greatly honored to receive the Steele Prize. I would like to take this opportunity to say a few words about algebraic geometry and the problems of mathematical exposition.

Algebraic geometry has a long and varied history, at different times emphasizing different techniques, and bearing upon different related areas of mathematics. Perhaps because of these multiple roots, when I was a student in the early 1960s, algebraic geometry had an aura of mystery and erudition which made it seem unapproachable. The latest revolution initiated by Serre and Grothendieck, who introduced the methods of sheaf theory and cohomology of schemes, may have increased this image of inaccessibility. But I was fortunate to come along at a time when the published chapters of Grothendieck's *Elements* did not yet exceed one thousand pages, and there was a sense of freshness and eager anticipation of what the new methods would bring. Thus I accepted the "general nonsense" without question, and have been filling in the geometry ever since. What I wrote in my book reflects my own learning experience. I would call it "basic training for algebraic geometers." It is written in the new language, but always refers back to the underlying geometry.

I believe it is now safe to say that the new methods developed by Grothendieck and his students during the last twenty years have proved themselves. The turmoil necessarily created by rewriting the foundations of the subject has subsided, and we are entering a period of applications to older problems. Furthermore, the natural importance of algebraic geometry among other branches of mathematics has been highlighted by recent applications in number theory, complex manifolds, and mathematical physics. With the publication in the last few years of three basic algebraic geometry books besides my own, I hope the mystery will be dispelled as the subject becomes more accessible.

As to the problems of writing mathematics, they are often neglected in the rush to publish new results. Yet it is essential in a time of ever-increasing complexity and specialization that we take care in conveying our results to others. My own guiding principles have been not to shirk from developing general theory, but always to keep potential applications in mind, and to give lots of examples along the way.



Robin Hartshorne

I believe that clear writing promotes clear thinking. And if you have something to say, it is worth the trouble to say it well. Therefore, I am particularly pleased to see the importance of writing in mathematics recognized, and my own efforts rewarded, by the offering of the Steele Prize for Mathematical Exposition.

BIOGRAPHICAL SKETCH

Robin Hartshorne was born in Boston on March 15, 1938. Educated at the Shady Hill School, Phillips Exeter Academy, and Harvard College, he earned his A.B. in 1959. He spent one year at the École Normale Superieure in Paris, then went to Princeton, where he earned the Ph.D. in 1963. He was a Junior Fellow at Harvard from 1963 to 1966, then successively Assistant and Associate Professor at Harvard. In 1972, he moved to the University of California, Berkeley, where he is now Professor of Mathematics. He has been a visiting professor at the Tata Institute of Fundamental Research, the Collège de France, and Kyoto University.

Professor Hartshorne was a Sloan Fellow from 1970 to 1972. He gave an invited address at the 1973 summer meeting of the Society in Missoula, Montana. He has served on the organizing committee of the 1974 Summer Research Institute in Algebraic Geometry at Arcata, the panel to select speakers in algebraic geometry for the International Congress in Helsinki, and is presently a member of both the ad hoc Summer Conference Committee, and the AMS Committee on Postdoctoral Fellowships (1979-1981).

Besides his book on algebraic geometry mentioned above, he has written an undergraduate text on projective geometry, and numerous research articles. His main interests have been in the cohomology theory of algebraic varieties; he is now studying the classification of vector bundles on projective spaces.

Professor Hartshorne is married to Edie Churchill, educator and psychotherapist. He has two sons, Jonathan, 7, and Benjamin, 2. His hobbies include foreign languages, music, and mountaineering.

Joseph J. Kohn

It is a great honor to receive the Steele Prize.

To respond to this award I will describe briefly the work for which it was given.

The main result of the cited papers is the solution of the $\bar\partial$-Neumann problem on complex manifolds with strongly pseudo-convex boundaries. The $\bar\partial$-Neumann problem was formulated by D. C. Spencer in order to study several complex variables by means of partial differential equations.

I will restrict myself to describing the $\bar\partial$-Neumann problem in the case of (0,1)-forms on a bounded domain $\Omega \subset \mathbf{C}^n$ with a smooth boundary $b\Omega$. Let $z_1, \ldots, z_n$ denote complex coordinates on $\mathbf{C}^n$ and let r denote a real-valued C^∞ function defined in a neighborhood of $b\Omega$ such that $dr \neq 0$ and $r = 0$ on $b\Omega$. The $\bar\partial$-Neumann problem is formulated as follows. Given a (0,1)-form $\alpha = \Sigma\alpha_j d\bar z_j$ does there exist a (0,1)-form $\varphi = \Sigma\varphi_j d\bar z_j$ satisfying the following:

$$(1) \qquad -\sum_k \frac{\partial^2\varphi_j}{\partial z_k \partial \bar z_k} = \alpha_j, j = 1, \ldots, n,$$

with

$$(2) \qquad \sum_k \frac{\partial r}{\partial z_k}\left(\frac{\partial\varphi_k}{\partial z_j} - \frac{\partial\varphi_j}{\partial z_k}\right) = 0 \text{ and}$$

$$\sum_k \frac{\partial r}{\partial z_k}\varphi_k = 0 \text{ on the boundary.}$$

Here the operators $\frac{\partial}{\partial z_k}$ and $\frac{\partial}{\partial \bar z_k}$ are defined as usual by

$$(3) \qquad \frac{\partial}{\partial z_k} = \frac{1}{2}\left(\frac{\partial}{\partial x_k} - i\frac{\partial}{\partial y_k}\right) \text{ and}$$

$$\frac{\partial}{\partial \bar z_k} = \frac{1}{2}\left(\frac{\partial}{\partial x_k} + i\frac{\partial}{\partial y_k}\right),$$

when $x_k = \mathrm{Re}(z_k)$ and $y_k = \mathrm{Im}(z_k)$.

This problem can be solved on any domain in $\mathbf{C}^1$ (in fact, when $n = 1$ it is equivalent to the Dirichlet principle). In higher dimensions there are many domains for which the $\bar\partial$-Neumann problem is not solvable (those are the domains on which the range of the L_2 closure of $\bar\partial$ is not closed). For domains Ω on which the $\bar\partial$-Neumann problem is solvable, its solution (i.e. the operator sending α to φ) is a very useful tool for the study of the behavior of holomorphic functions. In particular it is used to prove existence and boundary regularity of holomorphic functions on complex manifolds. Other applications include the inversion of the $\bar\partial$-operator (i.e., solving the inhomogeneous Cauchy-Riemann equations) and the study of the Bergman projection operator.

Joseph J. Kohn

BIOGRAPHICAL SKETCH

Joseph J. Kohn was born May 18, 1932, in Prague, Czechoslovakia. He received a B.S. from the Massachusetts Institute of Technology in 1953, and his M.A. (1954) and Ph.D. (1956) from Princeton

University. He was an instructor at Princeton University during the academic year 1956-1957. Between 1958 and 1963 he advanced from assistant professor to professor at Brandeis University and remained on the Brandeis faculty until 1968. In 1968 he became professor of mathematics at Princeton University. He was a member of the Institute for Advanced Study during the academic years 1957-1958, 1961-1962 and 1976-1977. He was a visiting member of the Courant Institute during 1966-1967 and a visiting professor at the University of Florence during 1972-1973. He served as department chairman at Brandeis (1964 to 1966) and at Princeton (1973 to 1976). He has been a recipient of a Sloan Fellowship (1964-1966) and a Guggenheim Fellowship (1976). He has participated in the scientific exchange programs with the U.S.S.R. (1965) and with Czechoslovakia (1967 and 1977).

Professor Kohn is a member of the Board of Trustees of the American Mathematical Society (1978 to 1982) and is currently serving as Secretary of the Board. He has been a member of several Society committees, including the Transactions and Memoirs Editorial Committee (1968 to 1971); the Nominating Committee for the 1975 Election; the Program Committee (1976); the Bicentennial Program Committee (Chairman, 1976). He was a member of the American Pure and Applied Mathematics Delegation to the People's Republic of China which visited the People's Republic of China in May 1976.

Professor Kohn has given invited addresses at two AMS meetings (April 1964 and March 1970) and at the International Congress of Mathematicians in 1966. He gave a series of lectures at each of the following summer programs: at the Instituto Politécnico Nacional in Mexico City in 1963; at the Séminaire de Mathématiques Supérieures, University of Montreal, 1969; at the University of Buenos Aires sponsored by UNESCO in 1966; at the Latin American Mathematics Summer School, Mexico, in 1970; at CIME Conferences meeting in Bressanone: Several Complex Variables, 1973, and Pseudo-Differential Operators, 1977; at the AMS Summer Research Institute on Several Complex Variables in Williamstown, 1975. With Professor E. Vesentini, Professor Kohn has organized two international conferences on Several Complex Variables which were held in Cortona, Italy, in 1976 and 1977.

Professor Kohn is an editor of the *Annals of Mathematics* and serves on the editorial boards of the *Journal of Differential Geometry* and of *Advances in Mathematics*. He was elected to membership of the American Academy of Arts and Sciences in 1967. His major research interests are several complex variables and partial differential equations.