Franklin P. Peterson (1930–2000)

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Haynes Miller

Franklin Paul Peterson was born on August 27, 1930, in Aurora, Illinois. His father, Paul, died when Frank was around seven, and his mother, Mildred, soon married his father's brother, Conrad. Frank had a younger brother, Norman, who after graduating from MIT became a metallurgist at the Argonne National Laboratory.

After attending Northwestern University, Frank began his graduate studies in 1952 under Norman Steenrod at Princeton University, where he completed his Ph.D. in the nominally standard three years. He visited the University of Chicago on an NSF Postdoctoral Fellowship and then returned to Princeton as Higgins Lecturer for the period 1956–58, after which he moved to MIT. On August 8, 1959, Frank was married to Marilyn (née Rutz).

Frank was an inveterate traveler, and he maintained close friendships with the many contacts he made around the world. He participated in the Symposium on Algebraic Topology in 1956 in Mexico City and spent the summer of 1959 there. He spent the academic year 1960–61 at the University of Oxford as a Sloan Research Fellow. He visited Bucharest and Warsaw in 1961, Moscow and Tbilisi in 1967, and Tbilisi again in 1972. He spent the spring and summer of 1967 in Kyoto on a Fulbright Research Fellowship and took the overland route, stopping in Samarkand, Bokara, and Kathmandu. At the invitation of the Academia Sinica, Frank made a historic trip to China in May 1973, together with Donald Spencer, William Browder, and Marilyn. They visited mathematical institutions in Beijing, Shanghai, Suzhou, Hangzhou, and Canton. In 1990 he won the prestigious Humboldt Research Award for Senior U.S. Scientists and spent most of five summers between 1991 and 1998 in Heidelberg, enjoying a long friendship with the academic community there. Frank loved conferences and would usually arrange a gustatory tour with select friends, sometimes traveling considerable distances for the purpose.

A renowned bon vivant, Frank enjoyed fine foods and wines of many descriptions in the company of friends. His taste in wines was quite catholic, and he always kept in mind the ratio of quality to price. He and Marilyn studied cooking at the Cordon Bleu in Paris one summer. He played bridge, tennis, and table tennis with the same rare combination of competitiveness and unfailing fairness and good humor that marked his professional life.

Franklin Peterson suffered a fatal stroke while visiting friends in Washington, DC, on September 1, 2000.

Early Work

Frank Peterson was a prime representative of the dominant method of an era in algebraic topology: namely, the use of cohomology operations to analyze homotopy types, especially ones deriving from bundle theory and bordism. He contributed greatly to both the substance and the style of the period.

When Frank was a graduate student, Steenrod operations were young (first appearing in 1947, with successive clarifications by Cartan and by

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Ph.D. Students of Frank Peterson

Peter Anderson (1964)
Vincent Giambalvo (1966)
Gregory Brumfiel (1967)
David Segal (1967)
Carey Mann (1969)
Stephen Williams (1969)
Hans Salomonsen (1971)
E. Bruce Williams (1972)
W. Stephen Wilson (1972)
Henry Walker (1973)
Arthur Goldhammer (1973)
Manuel Moriera (1975)
James Krevitt (1977)
Kenneth Prevot (1977)
David Anick (1980)
Michael Hoffman (1981)
Paul Goerss (1983)
Ethan Devinatz (1985)
Raymond Coley (1985)
Thomas Hunter (1987)
Kathryn Lesh (1988)
Hal Sadofsky (1990)

Steenrod and his student José Adem continuing into the early 1950s). Serre's thesis (1951) extended the scope of methods involving fibrations, and it together with Borel's thesis (published in 1953) and the discovery by William Massey (an earlier student of Steenrod) of exact couples (1952) finally made Leray's theory of spectral sequences accessible. In an early paper Serre also introduced the first theory of localization in topology in the form of his "mod C" calculations.

Frank's thesis exploited these new tools to extend known results about homotopy classes of maps from a complex K into the n-sphere S^n. Hopf had shown that if $\dim K \leq n$, then cohomology classifies such classes. The description of $[K, S^n]$ in case $\dim K = n+1$ was the problem that led Steenrod to discover the squares in the first place, and Frank now went further to prove classification results localized at a prime, using the mod C theory. Following a suggestion of John Moore, he also introduced coefficients into the picture by mapping into a "Moore space" $M(G, n)$ (a simply connected space whose only nonzero reduced homology group is $H_n(M(G,n)) = G$) in place of a sphere. (Much later Moore and his collaborators were to use the notation $P^{n+1}(p^r)$ in case G is a cyclic group of order p^r to denote spaces of this type.) Frank used to talk about Steenrod's demanding standards, claiming that he wrote from scratch seven drafts of his thesis before Steenrod was satisfied.

Very early on (1949) Steenrod had defined functional primary operations. Adem (1952) had made a start at defining secondary operations, and Massey had reported on his triple products in Mexico City in 1956, but these things were still very mysterious. Among the objectives Frank set himself was the task of establishing the general properties of higher-order operations. Postnikov's work on k-invariants appeared in Russian in 1955 and gave a context for studying higher-order operations (as well as clarifying the contents of Frank's thesis), and Frank became an expert, perhaps the leading expert, at operating this machinery. Some of his joint work with Norman Stein set out the basic definitions and proved the fundamental "Peterson-Stein formulas". See [6] for a modern perspective on these operations and an indication of their importance today.

Frank was not by nature a machine builder, however. He loved to compute and looked to geometric questions to guide him in his choice of computational project. Characteristic classes of vector bundles provided him with one such area. He applied the same machinery he had employed in his thesis to give simple conditions under which the triviality of characteristic classes implies triviality of the vector bundle. In the opposite direction, as a complement to the construction by Massey of secondary characteristic classes of a bundle with trivial Euler class, Peterson and Stein computed the cohomology of the universal example and showed that Massey's classes are algebraically independent.

These calculations led to collaboration with Bill Massey on a study of the cohomology of fiber spaces. The key idea was to study a fiber bundle $p : E \to B$ by means of a pair (E_T, E), where E_T is the fiberwise cone of E. This idea becomes effective only under rather strong geometric and algebraic hypotheses on the fibration, but ones that are satisfied in many important situations. In modern terms their method amounts to considering the low homological dimension portion of the Eilenberg-Moore spectral sequence, and the hypotheses are designed to guarantee that this portion generates the E^2-term and that the spectral sequence collapses. Their work grew into an AMS Memoir [7].

The Adams spectral sequence (1956) was another foundation stone of the edifice of algebraic topology in the second half of the twentieth century. It provided a mechanical way of studying the implications of higher-order operations and offered an alternative to the Postnikov tower, using homological algebra over the mod p Steenrod algebra $\mathcal{A}_p$ to begin a direct computation of homotopy classes of maps. Adams restricted himself to the stable range, but in their Memoir Massey and Peterson constructed, under appropriate conditions on a space X, an "unstable Adams tower" for X. Massey has written: "My memory is that most of these papers were genuine joint collaborations. Two exceptions: the unstable Adams spectral sequence was almost entirely Frank's work, while the Noetherian property of unstable modules over the mod 2 Steenrod algebra was my contribution."

Although the simplicial methods of Bousfield and Kan have since provided a much more general construction, the Massey-Peterson tower has proved easy to use and of surprising power and versatility. John Harper, Richard Kane, and others have used it to analyze H-spaces, and they are fundamental to Mark Mahowald's perspective on unstable homotopy theory. The category $\mathcal{U}$ of unstable modules over the Steenrod algebra, as explored by Massey and Peterson, turns out to relate intimately to modern objects such as Mac Lane homology and topological Hochschild homology.

The final building block of the edifice of algebraic topology of this era was René Thom's work on cobordism (1954). This opened the way to many geometric applications of homotopy theory, in which Frank was an active participant. He was delighted by Arunas Liulevicius's application of the Hopf algebra methods of Milnor and Moore. Frank's great contributions to our understanding of cobordism frequently occurred in joint work with Ed Brown, who has more to say about this below. This was truly one of the great collaborations in the history of topology.

Frank's perspective on basic homotopy theory is captured by the book *Cohomology Operations and Applications in Homotopy Theory*, by Robert Mosher (whom Frank regarded as his first student, though George Whitehead signed the thesis) and Martin Tangora. Mosher's lectures, upon which this book is based, closely followed lectures by Frank at MIT.

In later years Frank's interest focused increasingly on the algebraic structure imposed by an action of the mod 2 Steenrod algebra. An early manifestation of this interest was his work in the early 1970s with John Moore. They established easily checked conditions on a bounded-below module M over $\mathcal{A}_p$, at an odd prime p, guaranteeing that M is free, in analogy with a theorem of Adams and Margolis at the prime 2. They also gave general algebraic conditions on a graded algebra guaranteeing that among bounded-below modules the properties of being flat, projective, and injective coincide.

Frank was always quick to appreciate the potential of new ideas in the subject. A good example of this is his work with Vince Giambalvo that uses a new basis for the Steenrod algebra recently discovered by Dan Arnon to express the ideal of operations annihilating all elements of $H^*(\mathbb{R}P^\infty)$.

Service

Frank identified deeply with "Tech", as he called MIT. He served as chairman of the omnibus Pure Mathematics Committee for the years 1972–75, 1979–82, and 1984–87. In addition, he served as the graduate admissions chairman a number of times and expressed great pride at how well the students he had chosen turned out.

Franklin P. Peterson

Frank signed the theses of twenty-two Ph.D. students at MIT (see the sidebar). Several of these theses became fundamental to later developments in topology. Frank was a supportive and open-minded advisor. He was the de facto advisor also of Robert Mosher (formally George Whitehead's student) and of Peter Landweber (formally Raoul Bott's).

Frank taught his share of service courses. He would dress for the occasion of a large lecture by donning his legendary bow tie. At some point in the early 1980s Frank decided that his students by and large just did not seem as prepared as they had a decade or so earlier. Rather than leave it at that, with a complaint, Frank agreed the next year to read admission folders. He ranked the group he was assigned, kept a record of his rankings, and then compared his ranking with the choices made by the MIT admissions department. He was outraged to find that the MIT choices were close to the reverse of his own. He let his dissatisfaction be known, and a faculty movement was born, one which ultimately refocused the admissions criteria used by MIT.

One of the first things Frank did on arriving at MIT was to found (in collaboration with Ed Brown) the Monday MIT Topology Seminar, in imitation of Lefschetz's Thursday Topology Seminar at Princeton. This became the central meeting point for topologists of all persuasions in the Cambridge

Peterson with Cathleen Morawetz at his AMS retirement party in February 1999.

area. In the early 1960s he would make taking notes at the seminar and mimeographing them for distribution a part of the paid Research Assistantship duties of a graduate student in topology. An important part of the process for Frank was the choice of restaurant for dinner after the seminar. He liked to discover restaurants that did not yet have a liquor license and arrange with them to bring his own wine, whose price was then tabulated separately. He would try to negotiate an arrangement of arguable legality by which the restaurant would overlook his specially designed wine-bottle carrier, even after obtaining a license. Frank's warmth and inclusiveness was tremendously important to the creation of the topology community around MIT.

Edgar Brown

Frank and I first met in 1956 at the University of Chicago, where he was visiting for the year and I was an instructor. The mathematics department was a very lively place for topology and a great experience for both of us. Ed Spanier ran a topology seminar in which Frank and I gave talks on our Ph.D. theses, René Thom gave a talk on transverse regularity of smooth maps, and Steve Smale spoke about immersions of the circle into the plane. We also had a seminar to learn some Spanish in preparation for the 1956 International Topology Conference in Mexico City, another great experience. At the end of the summer we went our separate ways, rejoining in 1958, Frank at MIT and me at Brandeis. In 1959 Frank and I started a topology seminar, which in a few years became the MIT Monday topology seminar and which is still going. Initially it met each week, alternating between Brandeis and MIT, with participants George Whitehead, Arnold Shapiro, Frank, me, graduate students from MIT and Brandeis, and visitors to the area. George Whitehead gave the first talk on his work on homology theories. Arnold Shapiro spoke on the S-dual of a manifold and the Thom space of its normal bundle. For a good many years Frank would give the first talk of the new academic year.

Sam Gitler visited Brandeis during 1960–61 and in our mathematical discussions explained the "relations among characteristic classes" problem. For example, it was known that for the Stiefel-Whitney characteristic classes of a manifold, $w_1^3 + w_1 w_2 = 0$ on all 5-manifolds. The problem was to find all such relations. The close relationship between Stiefel-Whitney classes (hereafter SWCs) and the Steenrod squaring operations, $\mathrm{Sq}^i : H^q(X) \longrightarrow H^{q+i}(X)$, was well known ($H^q$ denotes cohomology with $\mathbb{Z}/2$ coefficients). Using this, Frank made some low-dimensional calculations, producing some new relations between SWCs. We began meeting from time to time in our homes, discussing how to find relations between SWCs, and drifted into a collaboration which was to last, on and off, for the next thirty years. Our first success was to find a very neat description of the ideal I_m of all polynomials in SWCs that vanish on all m-manifolds (manifold will mean compact, closed, smooth manifold).

We next extended our SWC results to Pontrjagin and Chern classes reduced mod p for odd primes p. Thom's cobordism machinery deals with $\Omega_*(G)$, the cobordism ring of manifolds whose stable tangent bundles have reductions to one of the classical families of Lie groups, $G = O, SO, U, SU, Spin, \ldots$. The general theory gives $\Omega_*(G) \approx \pi_*(MG)$, where MG is, more or less, the one-point compactification of the universal G vector bundle. The serious calculations occur in determining $H^*(MG;\mathbb{Z}/p)$ as a module over $\mathcal{A}_p$. For Pontrjagin and Chern classes one studies $H^*(MSO;\mathbb{Z}/p)$ and $H^*(MU;\mathbb{Z}/p)$, which Milnor had shown are free modules over $\mathcal{A}_p/(Q_0)$, where (Q_0) is the two-sided ideal generated by the Bockstein operation. ($\mathcal{A}_p/(Q_0)$ is the algebra of Steenrod reduced pth powers.) To carry over the I_m methods, we first thought of expressing MSO and MU, at a prime p, as a product of spaces X_r (actually spectra), where $H^*(X_r;\mathbb{Z}/p)$ is a free module over $\mathcal{A}_p/(Q_0)$ on one generator in dimension r. By an elaborate higher-order Bockstein operations–Postnikov tower construction, we succeeded in showing that X_r exists [3]. This spectrum came to be known as BP. Subsequently Quillen's canonical splitting of the localization MU at a prime p into a wedge of suspensions of BP opened the way to the "chromatic" approach to stable homotopy theory, first by Miller, Ravenel, and Wilson, and later by Devinatz, Hopkins, and Smith. Ironically, we did not need the X_r's to figure out $I_m(SO,p)$ and $I_m(U,p)$.

Using the techniques we had acquired in studying "relations", we turned to calculating cobordism groups. It turns out that $H^*(MG;\mathbb{Z}/p)$, G as above, is exceptionally simple as a module over $\mathcal{A}_p$. It gets progressively more complicated as G progresses from O to $Spin$, but remains a sum of cyclic modules over $\mathcal{A}_p$. Don Anderson joined our research efforts, bringing his knowledge of KO characteristic classes, which inspired us to tackle $\Omega_*(SU)$. Using the Adams spectral sequence, we succeeded

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in showing that $[M] \in \Omega_*(SU)$ is zero if and only if all its Stiefel-Whitney numbers and KO numbers are zero (number = characteristic class evaluated on an orientation). Among additional results we gave a basis for the torsion subgroups.

We next turned to $\Omega_*(Spin)$. Here again we used the technique of attempting to split the Thom spectrum into a wedge of simpler factors. By low-dimensional calculations Frank developed a theory of modules over a Hopf algebra which enabled us to say a good deal about $\Omega_*(Spin)$, including a complete determination of its additive structure [1].

In 1963, when we had essentially completed our analysis of I_m, a preprint of *Groups of Homotopy Spheres* by Kervaire and Milnor was in circulation. I thought homotopy theory could resolve a problem left open in this paper, namely, the Arf invariant problem, which asks whether or not a particular easily constructed differentiable structure on a $(4k+1)$-sphere is an exotic structure. Kervaire had shown that it was exotic for dimension 10. For each k he gave necessary and sufficient conditions consisting of the vanishing of the Arf invariant (the mod 2 analog of the signature of a real quadratic form) of a naturally defined quadratic form $\phi : H^{2k+1}(M) \longrightarrow \mathbb{Z}/2$ when M is stably parallelizable. Frank and I began collaborating on the Arf invariant problem, and our first result was to show that for $2k+2 \neq 2^i$, $i > 3$, the decomposition of Sq^{2k+2} gives a secondary cohomology operation on cohomology classes of dimension $2k+1$, yielding a ϕ with the desired properties and providing an $\mathrm{Arf} : \Omega_{8k+2}(Spin) \longrightarrow \mathbb{Z}/2$. Results of Conner and Floyd about $\Omega_*(SU)$ and a product formula for Arf gave an affirmative answer for dimensions $(8k+2)$ [4]. Bill Browder, by similar but considerably more delicate arguments, extended this to all dimensions except $2^i - 2$, which remains open for $i > 5$.

From our perspective on relations between characteristic classes, Frank and I thought about the immersion conjecture: Any m-manifold immerses in $\mathbb{R}^{2m-\alpha(m)}$, where $\alpha(m)$ is the number of ones in the dyadic expansion of m. Our results on I_m show that SWCs will not contradict this conjecture. By work of Smale and Hirsch on immersions, the immersion conjecture is equivalent to the conjecture that for any M the classifying map for the normal bundle of M embedded in a high-dimensional Euclidean space, $M \longrightarrow BO$, lifts to $M \longrightarrow BO_{m-\alpha(m)}$. Using "Brown-Gitler technology", an extension of "relations among SWCs", Frank and I constructed a space BO/I_m over BO such that $H^*(BO) \longrightarrow H^*(BO/I_m)$ is an epimorphism and has kernel I_m. Furthermore, for any m-manifold the classifying map of its normal bundle lifts to BO/I_m. This in turn provided the foundation for Ralph Cohen's work on the immersion conjecture.

All in all, Frank and I had a very enjoyable and productive collaboration in which our talents very successfully complemented each other. With varying regularity we would meet for an hour or two, exchange thoughts about the subject under investigation, usually only mildly comprehensible to each other. After a while this produced some proofs and theorems which, within days, Frank would write up, producing a first draft requiring at most minor changes.

Fred Cohen

On Frank's Later Work, and a Personal Recollection

Frank Peterson's joy in doing mathematics as an individual as well as with other people had a profound positive impact on the lives of many of his friends. I have taken the liberty of describing one such personal experience with Frank which is surely typical. A brief discussion of where a small part of Frank's later mathematics fits is given afterwards.

I was visiting the Institute for Advanced Study during one of the occasions when Frank gave a lecture at Princeton University. Our first conversation occurred that evening during a party at Bill Browder's house and concerned a recipe for caneton à l'orange using Tang, an instant orange juice mix. Frank was unfamiliar with the recipe, but he listened patiently.

A few years later he suggested that we have lunch together during an AMS annual meeting in Atlanta. Friends had regaled me with stories about Frank, one of which concerned his personal bottle of mustard. As predicted, that bottle made an appearance during our lunch in Atlanta. These events had a very settling effect on a somewhat nervous recent Ph.D. and were the beginning of a friendship I have treasured. Frank and I got together many times after that. We proved theorems, sometimes cooked, and had a joyful time.

John Moore had taught a course on classical homotopy theory in which he had proven early global theorems due to Ioan James and Hirosi Toda that addressed bounds for the order of the torsion in the homotopy groups of spheres. That course pointed the way toward subsequent work of Paul Selick and of Frank, as well as joint work of John Moore, Joe Neisendorfer, and me. Paul Selick had proven a beautiful theorem which solved a problem in Moore's course (a conjecture of Michael Barratt's). Paul's theorem states that if p is an odd prime, then p times every element in the p-primary component of any homotopy group of the 3-sphere is zero. His proof used an ingenious choice of map $s : \Omega^2(S^3\langle 3\rangle) \to \mathrm{map}_*(P^2(p), \Omega S^{2p+1})$, which was an H-map. However, the nonexistence of such a

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F. P. Peterson

map when spaces are localized at the prime 2 led to the notion of an "atomic" space as follows. I had shown that no hypotheses at all were required for the map above to yield a proof of Paul's theorem as any self-map of $\Omega^2(S^3\langle 3\rangle)$ that induces an isomorphism on the first nonvanishing homotopy group modulo p must be an equivalence (after localization at p). This structure concerning self-maps was subsequently formalized in the notion of an "atomic" space in joint work of John Moore, Joe Neisendorfer, and myself.

Frank became interested in this feature almost at once. During one of Frank's early visits, we considered the "atomicity" of the space of continuous functions from the real projective plane to loop spaces of spheres. Frank, Paul, Eddie Campbell, and I showed that the space of pointed maps from the real projective plane to the loop space of the n-sphere is atomic if $n \neq 2, 4, 5, 8, 9$, or 17. All the other cases admit nontrivial product decompositions given in terms of loop spaces of spheres as well as the classical fibre of the double suspension. These decompositions are extensions to the prime 2 of a decomposition given by Paul Selick for odd primes.

Frank, Paul, and Eddie addressed the atomicity properties of other related spaces [5]. One beautiful result of theirs is as follows. The localization at 2 of the $(n-1)$-st loop space of the n-connected cover of the n-sphere is atomic if n is not 2, 4, or 8. Thus, if $n \neq 2, 4, 8$, then any self-map of $\Omega^{n-1}S^n\langle n\rangle$ that is nontrivial when restricted to the second homotopy group of $\Omega^{n-1}S^n\langle n\rangle$ is a 2-local homotopy equivalence. They also proved an analogous result in case spaces are localized at odd primes p and n is odd.

The spaces in question are huge. That they are atomic is striking. One immediate application is the Kahn-Priddy theorem (which asserts that the stable homotopy groups of $K(\mathbb{Z}/p\mathbb{Z}, 1)$ surjects to the p-torsion of the stable homotopy groups of spheres). Their proof of atomicity of $\Omega^{n-1}S^n\langle n\rangle$ was another beautiful and difficult computation using the classical Dickson algebra, an algebra which would also appear in later work of Frank and his collaborators. One question left open by this work is whether there is an analogue of the Kahn-Priddy theorem for the spaces $\Omega^{2n}(S^{2n+1}\langle 2n+1\rangle)$.

The Dickson algebra D_k is given by the algebra of invariant elements obtained from the natural action of $GL(k, \mathbb{F}_p)$ on the polynomial ring over $\mathbb{F}_p$ with k indeterminates, the tautological representation of $GL(k, \mathbb{F}_p)$), as introduced by L. E. Dickson. One way in which the Dickson algebra arises in algebraic topology is as the dual of the Araki-Kudo-Dyer-Lashof algebra. The Dickson algebra had played an important role in earlier work of Madsen, Madsen-Milgram, May, Mitchell, Mui, Priddy, and Wilkerson.

Frank's interests in the Dickson algebra and the cohomology of iterated loop spaces fitted closely with another area in which he worked. The mod-p cohomology of a space is an algebra over the mod-p Steenrod algebra $\mathcal{A}_p$. Frank asked for the dimension of a minimal set of generators for the underlying $\mathcal{A}_p$ module in the case of $p = 2$ when the space in question is an n-fold product of infinite-dimensional real projective spaces. He formulated the conjecture that the generators are all in degree d, where $\mu(d) \leq n$ and $\mu(d)$ is defined to be the smallest integer k such that d is the sum of k integers, each of which is one less than a power of 2.

This guess, which became known as the "Peterson Conjecture", attracted many people. For example, the case of $n = 3$ was solved in the thesis of M. Kameko. In addition, D. Anick, M. Boardman, D. Carlisle, M. Crabb, V. Giambalvo, J. Hubbuck, N. H. V. Hung, M. Kameko, D. Pengelley, J. Repka, P. Selick, J. Silverman, W. Singer, G. Walker, F. Williams, and Reg Wood were some of the people who worked on this problem or closely related problems.

The Peterson Conjecture was eventually solved by Reg Wood, whose paper was followed immediately by a paper of Frank's [9] in which he used the solution of the Peterson Conjecture to prove a beautiful theorem concerning the unoriented bordism class of a smooth manifold. Namely, let M denote a closed C^∞ manifold of dimension d, for which n denotes the cup-length for the mod-2 cohomology ring of M. Frank proved that if $\alpha(n) > d$, then the manifold M is an unoriented boundary. This theorem complements work of Harpreet Singh, which had occasioned Frank's conjecture in the first place.

With N. H. V. Hung, Frank pursued an analogous problem to describe a minimal set of $\mathcal{A}_2$-module

Left: Frank Peterson receiving a commemorative chair from AMS Executive Director John Ewing and making remarks, during his retirement party in 1999.

generators for the mod-2 Dickson algebra D_k, $D_k \otimes_{\mathcal{A}_2} \mathbb{F}_2$, and extended Bill Singer's computation for $k = 2$ to cover $k = 3$ and 4. They investigated a construction of Lannes and Zarati and proposed a conjecture, still open, which would imply that the elements in the Hurewicz image for the mod-2 homology of $\Omega_0^\infty S^\infty$ are limited to elements arising in homotopy with either Hopf invariant one or Arf invariant one.

These mathematical directions touched on relationships of topology, homotopy theory, modular representation theory, Lie algebras, and the structure of modules over the Steenrod algebra. This mathematics represents a small but illustrative taste of Frank's interests, together with some of the mathematics which those interests spawned.

D. Pengelley, V. Giambalvo, Frank Williams, and I were working with Frank at the time of his death. These mathematicians are finishing their joint projects in memory of a good friend and colleague.

Fred Gehring and Al Taylor

Franklin Peterson and the AMS

The current healthy and stable financial condition of the American Mathematical Society is due in no small part to the vision and influence of Franklin Peterson throughout his twenty-five years of service as treasurer. We both had the opportunity to work with Frank, the first of us through the late 1970s and 1980s and the second in the 1990s. This period encompassed both good years and lean years for the Society. We were always impressed by Frank's good judgment, his careful stewardship of the Society's assets, and his concern that the Society remain dedicated to its mission of supporting mathematical research and scholarship.

Frank had considerable power as treasurer of the AMS, and he was careful to use it almost exclusively in connection with financial issues facing the Society. One story told us by Steve Armentrout, associate treasurer with Frank for over fifteen years, recalls a legendary incident that illustrates Frank's diplomatic skills.

In the 1980s the Society had two codirectors, the executive director of the Society in Providence and the executive editor of *Mathematical Reviews* in Ann Arbor. At a meeting during this period the codirectors could not reach agreement on a financial issue that affected both units. Frank suggested that the codirectors, the chair of the Board of Trustees, and the associate treasurer meet in a few days and have lunch at Frank's house in Boston to discuss the matter further.

The group met on a beautiful autumn day in Boston, Frank was a genial host, and lunch—with a couple of bottles from Frank's famous wine cellar—was very pleasant. Everyone relaxed and a short discussion after lunch was all that was needed to resolve the matter in question. All went away marveling at Frank's diplomacy.

During the next few years before the dual directorship was abolished, there were a couple of other occasions when the Providence and Ann Arbor units seemed to be at an impasse over some issue. Each time, after a couple of minutes of tense silence, Frank would smile gently and say softly, "Perhaps it's time to have luncheon at my house again." Needless to say, it was never necessary.

Another example of Frank's careful leadership occurred annually when the treasurer, associate treasurer, chair of the Board of Trustees, executive director, and chief financial officer met to discuss salary increases for AMS staff. Frank realized that a great strength of the Society is the dedication of its long-term employees. He also understood the importance of being able to work in pleasant surroundings and to have competitive salary and benefits. On the other hand, he recognized that

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personnel costs were the largest single component of the Society's expenses.

Frank's long view and sense of fairness gave a remarkable perspective and balance to these discussions. In lean years when there were great fiscal pressures on the AMS, he made sure that the AMS staff was not carrying an undue share of the budget-balancing burden. In good years he saw to it that the interests of the AMS members were not forgotten.

Frank oversaw tremendous growth in the Society's assets and impact during his years of service from 1973 through 1998. A few numbers illustrate this growth. Over its first eighty-five years of existence, the AMS accumulated a total of $660,000 in assets. During Frank's tenure as treasurer, the operating revenue increased by 500 percent, the total assets by 1400 percent, and the number of staff by 50 percent.

The increase in assets was not a continuous process, but one that involved many ups and downs. The most difficult of these was a three-year period in the early 1980s when large deficits began to threaten the very existence of the Society.

A basic strategy adopted at this time, advocated strongly by Frank and supported by others, was that the Society systematically build up a substantial reserve fund so that it could weather times when income and expenses fell out of balance. This economic stabilization fund has been in existence for almost two decades and has now achieved the goal of its founders.

Frank chaired the Investment Committee, which oversees the Society's endowment and long-term reserve funds. The vigorous investment environment of the 1990s allowed the reserve fund to increase more rapidly than anyone had hoped, and the goal was achieved more than five years ahead of schedule. We view the establishment of this fund as one of Frank's most significant accomplishments, one that will benefit the Society for decades to come.

There are, as one might expect, many personal stories about Frank after so many years as treasurer. One concerns his personal teapot at the AMS building in Providence. It was always present at every committee meeting he attended. In fact, on the occasion of Frank's retirement as treasurer, one AMS staff member playfully estimated that about 1,200 gallons of "Treasurer Tea" had been prepared at AMS meetings during Frank's term.

Another concerns his ever-present bow tie. Frank noticed that Archibald Cox wore bow ties while investigating the 1973 Watergate break-in. Shortly thereafter, President Nixon fired Cox. At that point, Frank said he started to wear bow ties in honor of Cox. He often carried one in his shirt pocket to provide "instant formality" when needed.

Frank was a terrific person to work with because he had such good financial sense and so much devotion to the Society. He was careful and cautious, always ready with a suitable reminder that times were not always good and that preparing for the bad times was everyone's obligation during the good. On the other hand, Frank was thoughtful, forward looking, and ready to support ideas he felt promoted mathematics. Two projects he enthusiastically endorsed were the Centennial Fellowships and the establishment of the Washington office, which has given so much more visibility to mathematics in the past few years.

Franklin Peterson dedicated a large part of his life to the American Mathematical Society. There are countless numbers of grateful members who appreciate all that he did for them, and there are others, yet to come, who will benefit from his caring stewardship of the Society's endowments and reserves.

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