1996 Oswald Veblen Prize

The 1996 Oswald Veblen Prize in Geometry was awarded at the Joint Mathematics Meetings in Orlando in January 1996 to Richard Hamilton of the University of California, San Diego, and to Gang Tian of the Massachusetts Institute of Technology.

Oswald Veblen (1880–1960), who served as president of the Society in 1923 and 1924, was well known for his mathematical work in geometry and topology. In 1961 the trustees of the Society established a fund in memory of Professor Veblen, contributed originally by former students and colleagues and later doubled by his widow. Since 1964 the fund has been used for the award of the Oswald Veblen Prize in Geometry. Subsequent awards were made at five-year intervals. A total of ten awards have been made: Christos D. Papakyriakopolous (1964), Raoul H. Bott (1964), Stephen Smale (1966), Morton Brown and Barry Mazur (1966), Robion C. Kirby (1971), Dennis P. Sullivan (1971), William P. Thurston (1976), James Simons (1976), Mikhael Gromov (1981), Shing-Tung Yau (1981), Michael H. Freedman (1986), and Andrew Casson and Clifford H. Taubes (1991). At present, the award is supplemented from the Steele Prize Fund, bringing the value of the Veblen Prize to $4,000, divided equally between this year's recipients.

The 1996 Veblen Prize was awarded by the AMS Council on the basis of a recommendation by a selection committee consisting of Jeff Cheeger, Peter Li, and Clifford Taubes (chair).

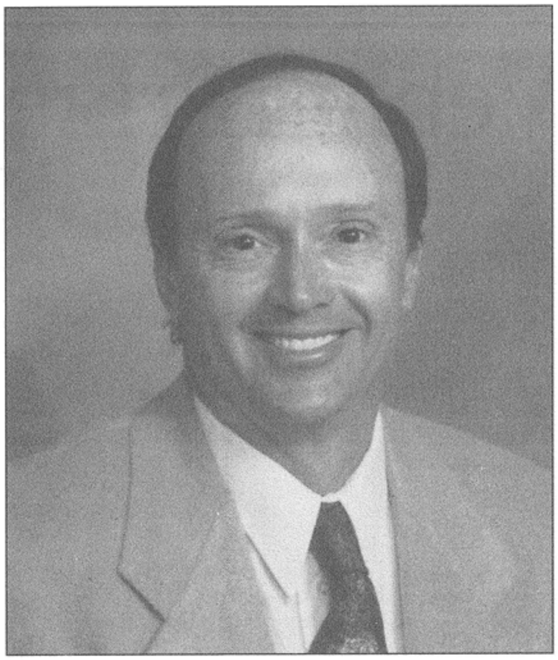
The text that follows contains the committee's citation for each award, the recipients' responses upon receiving the prizes, and a brief biographical sketch of each recipient.

Richard Hamilton

Citation

Richard Hamilton is cited for his continuing study of the Ricci flow and related parabolic equations for a Riemannian metric and he is cited in particular for his analysis of the singularities which develop along these flows.

The Ricci flow equations were introduced to geometers by Hamilton in 1982 ("Three manifolds with positive Ricci curvature", *J. Differential Geometry* **17** (1982), 255–306). These equations form a very nonlinear system of differential equations (of essentially parabolic type) for the time evolution of a Riemannian metric on a smooth manifold. The equations assert simply that the time derivative of the metric is equal to minus twice the Ricci curvature tensor. (The Ricci curvature tensor is a symmetric, rank two tensor which is obtained by a natural average of the sectional curvatures.) This flow equation can be thought of as a nonlinear heat equation for the Riemannian metric. After an appropriate, time-dependent rescaling, the static solutions are simply the Einstein metrics. In introducing the Ricci flow equations, Hamilton proved that compact, three-dimensional manifolds with positive definite Ricci curvature are diffeomorphic to spherical space forms. (These are quotients of the three-dimensional sphere by free, finite group actions.)



Richard Hamilton

Over the subsequent years, Hamilton has continued his study of the Ricci flow equations and related equations, delving ever deeper to understand the nature of the singularities which arise under the flow. (Hamilton proved that singularities do not arise in three dimensions when the Ricci curvature starts out positive.) Hamilton has come to understand the geometric constraints on the singularities which arise under the Ricci flow on a compact, three-dimensional Riemannian manifold and under a related flow equation (for the "isotropic curvature tensor") on a compact, four-dimensional manifold. This understanding has allowed him, in many cases, to classify all possible singularities of the flow.

In the four-dimensional case, Hamilton was recently able to give a topological characterization of the possible singularities which arise from the isotropic curvature tensor flow if the starting metric has positive isotropic curvature tensor. The conclusion is as follows: If a singularity arises, then it can be described as a lengthening neck in the manifold whose cross-section is an embedded spherical space form with injective fundamental group. Hamilton deduced from this fact that simply connected manifolds with positive isotropic curvature are diffeomorphic to the four-dimensional sphere.

For the compact 3-manifold case, Hamilton, in a recent paper, analyzed the development of singularities in the Ricci flow by studying the evolution of stable, closed geodesics and stable, minimal surfaces under their own, compatible, geometric flows. This analysis of the flows of stable geodesics and minimal surfaces leads to a characterization of the developing singularities in terms of Ricci soliton solutions to the flow equations along degenerating, geometric subsets of the original manifold. (A Ricci soliton is a solution whose motion in time is generated by a 1-parameter group of diffeomorphisms of the underlying manifold.)

The Oswald Veblen Prize in Geometry is awarded to Richard Hamilton in recognition of his recent and continuing work to uncover the geometric and analytic properties of singularities of the Ricci flow equation and related systems of differential equations.

Response

It is a great honor to receive the Oswald Veblen Prize from the AMS. This award recognizes the tremendous growth of the whole field of nonlinear parabolic partial differential equations in geometry, of which my own work is but a small part. Especial thanks are due to my parents, Dr. and Mrs. Selden Hamilton, who provided me with every conceivable head start in education; my high school geometry teacher, Mrs. Becker, for an enduring love of three-dimensional geometry; my mentor, James Eells, Jr., whose work with Joseph Sampson on the Harmonic Map Heat Flow originated and inspired the field; and my colleagues S.-T. Yau and Richard Schoen, who suggested the neck-pinching phenomenon and encouraged me to study the formation of singularities.

It is a pleasure to share the prize with Gang Tian, whose work on Kähler manifolds is outstanding.

Biographical Sketch

Professor Hamilton was born in Cincinnati, Ohio, in 1943. He received his B.A. from Yale University in 1963 and his Ph.D. from Princeton University in 1966 under the direction of Robert Gunning. He has held professorships at Cornell University and the University of California at Berkeley and visiting positions at the University of Warwick, the Courant Institute, the Institute for Advanced Study in Princeton, and the University of Hawaii. He is currently professor of mathematics at the University of California, San Diego.

Gang Tian

Citation

Gang Tian is cited for his contributions to geometric analysis and, in particular, for his work on the question of existence and obstructions for Kähler-Einstein metrics on complex manifolds with positive first Chern class.

The basic Kähler-Einstein problem is to find necessary and sufficient conditions for the existence of a Kähler metric on a given complex manifold whose Ricci curvature is a constant multiple of the metric itself. The sign of the constant is determined by the degree of the manifold's first Chern class. The case where the sign is negative was solved independently by Aubin and Yau, while the sign zero case (where the first Chern class vanishes) was solved by Yau in his celebrated solution to the Calabi Conjecture. Applications of the zero (and non-positive) first Chern class results have been legion, and so progress on the positive first Chern class cases has been eagerly sought after. However, the case of positive first Chern class has remained mostly mysterious until the recent work of Tian (and others).

In particular, Tian completely settled the existence question for Kähler-Einstein metrics on complex surfaces, showing that they exist if and only if the group of holomorphic transformations is reductive. Later, Tian (generalizing work with W. Y. Ding) found the first obstructions to the existence of Kähler-Einstein metrics which do not require the existence of holomorphic vector fields. Subsequently, he was able to show that for hypersurfaces, the existence of a Kähler-Einstein metric implies that the hypersurface is stable in the geometric invariant theory sense. (This constitutes a first big step in Yau's program to characterize manifolds with Kähler-Einstein metrics in geometric invariant theory terms.) Tian had previously developed some general criteria for the existence of Kähler-Einstein metrics, which he applied to complex hypersurfaces in complex projective spaces.

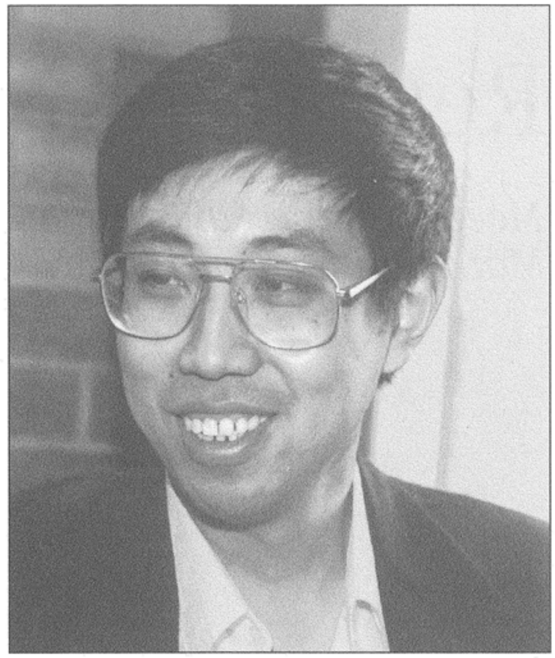
Tian has also proved various theorems which control the limiting behavior of sequences of Kähler-Einstein metrics with bounded L^n- norm on a complex n-dimensional manifold. And, he has classified the asymptotically locally Euclidean Kähler-Einstein manifolds which result as limits of such sequences.

On a different subject, Tian (with Y. Ruan) also published a sequence of fundamental papers on the new subject of quantum cohomology which prove, in particular, that the quantum cohomology ring is associative. (Quantum cohomology refers to a family of deformations of the cohomology ring of a symplectic manifold which is defined by an appropriate count of intersection numbers of cohomology classes with certain symplectic curves.)

For these contributions and others unnamed, Gang Tian is awarded the Oswald Veblen Prize in Geometry.

Response

I am highly honored to be one of the recipients of the Veblen Prize of the American Mathematical Society. First, I would like to express my gratitude to my thesis advisor, S.-T. Yau, for having initially suggested this problem to me: finding Kähler-Einstein metrics on manifolds with the first Chern class positive. Ten years ago he also shared with me his belief that the problem would be related to certain stability properties of the underlying manifolds. I would also like to thank my colleagues at the Courant Institute of Mathematical Sciences for providing me with an excellent environment for my research. It is surely one of the most stimulating places for mathematical research. Finally, I am very happy to share this prize with R. Hamilton.



Gang Tian

Biographical Sketch

Gang Tian was born on November 24, 1958, in the People's Republic of China. He received his B.S. from Nanking University (1982), his M.S. from Peking University (1984), and his Ph.D. from Harvard University (1988). After positions at Princeton University and the State University of New York at Stony Brook, he went to the Courant Institute of Mathematical Sciences at New York University in 1991. In 1995 he moved to the Massachusetts Institute of Technology. He also holds professorships at the Mathematics Institute of the Academia Sinica and at Peking University. He has held visiting positions at the Institute for Advanced Study in Princeton, the Institut des Hautes Études Scientifiques, and Stanford University. Tian received a doctoral dissertation fellowship (1987) and a research fellowship (1991–1993) from the Alfred P. Sloan Foundation. In 1990 he presented a 45-minute invited address at the International Congress of Mathematicians in Kyoto. He presented the Bergmann Memorial Lecture at Stanford University in 1994. That same year, he received the 19th Alan Waterman Award from the National Science Foundation.