

Who Is Alexander Grothendieck?

Winfried Scharlau

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Scharlau is writing a three-volume biography, *Wer ist Alexander Grothendieck?: Anarchie, Mathematik, Spiritualität*. The first volume, which primarily treats the lives of Grothendieck's parents, has appeared as a self-published book and is available on the Web at http://www.scharlau-online.de/ag_1.html.

For a mathematician, it is not hard to give an answer to the question posed in the title of this lecture: Grothendieck is one of the most important mathematicians of the second half of the twentieth century, to whom we owe in particular a complete rebuilding of algebraic geometry. This systematic rebuilding permitted the solution of deep number-theoretic problems, among them the final step in the proof of the Weil Conjectures by Deligne, the proof of the Mordell Conjecture by Faltings, and the solution of Fermat's Last Problem by Wiles. However, this lecture is concerned not with Grothendieck's mathematics but with his very unusual life on the fringes of human society. In particular, there is, on the one hand, the question of why at the age of forty-two Grothendieck first of all resigned his professorship at the Institut des Hautes Etudes Scientifiques (IHES); then withdrew from mathematics completely; and finally broke off all connections to his colleagues, students, acquaintances, friends, as well as his own family, to live as a hermit in an unknown place. On the other hand, one would like to know what has occupied this restless and creative spirit since his withdrawal from mathematics. I will try to pursue both questions, even

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though an exhaustive and satisfactory answer is surely impossible.

Grothendieck's Parents

One can only understand the life of Grothendieck—if one can understand it at all—if one knows about the life of his parents. I report briefly on the life of his father.

He was from a Jewish family, was (probably) called Alexander Schapiro, and was born in 1890 in Novozybkov in the border area of Russia, White Russia, and Ukraine. At the age of fifteen he was recruited by anarchist groups that were fighting against the tsarist regime; in 1905 Russia was in uproar. After two years of fierce battles, he and all of his comrades were taken as prisoners. All were sentenced to death, and all but Schapiro were executed; he was led to the execution plaza every day for three weeks before being pardoned because of his youth and sentenced to life in prison, where he spent the next ten years. In the confusion of the October Revolution and the First World War, he escaped and immediately joined the anarchist peasant army of the Ukrainian General Machno. He married a Jewish woman called Rachil and with her fathered a son named Dodek, but carried on a busy love life outside of marriage. Again, after fierce battles, he was taken prisoner by the Bolsheviks and sentenced to death. Probably during an attempt to escape (or in an assassination attempt?), he lost his left arm. With the help of various women and

comrades in arms, he managed to flee to western Europe. He went into hiding first in Berlin, then in Paris. From this time on he lived with forged documents under the name of Alexander Tanaroff. For many years he earned his living as a street photographer. Around the year 1924 he returned to Berlin, where he met Hanka Grothendieck. He introduced himself to her husband, Alf Raddatz, with the words, "I will steal your wife."

And so it happened. In March 1928 Alexander Grothendieck, the son of Alexander Tanaroff and Hanka Grothendieck, was born. For five years the "family", consisting of these three people, together with Hanka's daughter Maidi (Frode Raddatz) from her marriage, lived in the so-called "Scheunenviertel" in Berlin, where for some time they operated a photography studio. After the National Socialists came to power, the situation in Germany became too dangerous for the Jewish Tanaroff, and he moved back to Paris. Hanka Grothendieck decided to follow her companion as soon as possible. Around New Year's 1933-34 she placed her five-year-old son in a foster home with the family of the Hamburg pastor Wilhelm Heydorn. (Like all people close to Grothendieck, Heydorn was a very remarkable personality about whom a 450-page biography was published.¹) Hanka then went to France as well. Both she and Tanaroff took part in the Spanish Civil War, not fighting actively but in supporting roles. After the defeat of the Republicans, both returned to France. Certainly with the start of the Second World War, Tanaroff was in danger in France as well—as an alumnus of the Spanish War, as a Jew, and as an illegal alien. He was interned in the infamous camp Le Vernet, extradited to the Germans in 1942, and transported to Auschwitz. Under the name Alexandre Tanaroff, he appears on the list of victims of the Shoah. Throughout his adventurous life he had known only one goal: the fight for freedom and self-determination of all people. For that he would put his whole existence on the line at any time.

The life of Hanka Grothendieck was similarly dramatic, although the drama is more internal than external. Her great goal was to be a writer. Although she had remarkable talent, she ultimately failed. She too lived a life on the fringe. For reasons of space and time, I will not go into any detail in this lecture.

I now come to Alexander Grothendieck himself. I first would like to report on the outline of his biography. For further information, I refer to the very informative article by Allyn Jackson.²

¹Wilhelm Heydorn, Nur Mensch Sein!: Lebenserrinerungen, I. Groschek and R. Hering, editors, Dölling und Galitz Verlag, Hamburg, 1999.

²Allyn Jackson, "Comme appelé du néant—As if summoned from the void: The life of Alexandre Grothendieck", Notices, October 2004 and November 2004.



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Grothendieck around 1936 in the garden of the Heydorn's house in Hamburg-Blankenese.

Child to Mathematician to Hermit

As mentioned before, Alexander Grothendieck was born on March 28, 1928, as Alexander Raddatz in Berlin and lived there with his parents and his half-sister, Maidi, for the first six years of his life. From early 1934 to the end of April 1939 he lived together with other foster children in the home of Wilhelm and Dagmar Heydorn in Hamburg-Blankenese, where he initially attended elementary school and then the *Gymnasium*. Except for the years at the IHES, this may have been the only period in his life when he lived in "normal circumstances". In early 1939 his situation in Germany became too dangerous, particularly since his foster parents opposed the Nazi regime and had to contend with the possibility that their foster children would be taken away from them. In such a situation, his Jewish heritage would have come to light. So at the end of April 1939, Alexander was sent to his parents in France. It is unknown where he spent the next few months; he was probably with his mother in Nîmes. After the start of the war, Hanka, as a citizen of an enemy nation, was interned together with her son in the camp Rieucros near Mende. Alexander was able to attend school there and sometimes had private tutoring as well. Around 1942 Alexander somehow arrived in Le Chambon sur Lignon. This small town in the Massif Central was a center of resistance against the Nazis; thousands of refugees were hidden there, given false papers and food vouchers, and then smuggled across the Swiss border. Thousands were saved from deportation to German death camps. The crucial person in this collective resistance was the Protestant clergyman André Trocmé, who systematically traveled to French camps and tried in particular to get out as many children as possible. Perhaps this is how Grothendieck came to



Grothendieck's house in Villecun, where he lived from 1973 to 1979.

Le Chambon. (The great story of Le Chambon has been the subject of many documentaries, novels, and movies.³) In Le Chambon, Grothendieck was able to attend the Collège Cévénol, an international private school founded by Trocmé, which from the beginning was dedicated to nonviolence and the solidarity of all people—not popular ideas in a time of war. In 1945 Alexander completed there his rather chaotic schooling with the *baccalauréat*.

It is probably by accident that Grothendieck ended up in Montpellier after the war. Perhaps his mother had found work there. He received a modest scholarship and started his studies of mathematics. It soon turned out that the university did not have much to offer him, and he had to rely largely on self-study. Since the time he was in school, he had planned to find out what concepts like length and volume really mean, and according to his own reports, he basically developed the theory of the Lebesgue integral. In the fall of 1948 he went to Paris for a year, where he met the most important French mathematicians of the day, both the active middle generation of Henri Cartan, André Weil, Jean Leray, Laurent Schwartz, and Claude Chevalley, as well as the younger generation, his contemporaries Jean-Pierre Serre, Pierre Cartier, François Bruhat, and Armand Borel. Originally, Grothendieck had hoped to be able to quickly get a Ph.D. for his work on the “Lebesgue integral”. Of course, he now found out that to a large extent he had simply rediscovered known things. Nevertheless, he wanted to stick with this subject, so, following the advice of Cartan and Weil, on June 20, 1949, he wrote a letter to Jean

Dieudonné, who like Schwartz was teaching in Nancy. From this time on, Grothendieck came into the mathematical mainstream, and it is generally known what he did and achieved during the next twenty years. So that I can keep my account short, I refer for details to Jackson and the literature quoted there.

To begin with, Schwartz gave Grothendieck a paper to read that he had just written with Dieudonné, which ended with a list of fourteen unsolved problems. After a few months, Grothendieck had solved all of them. Try to visualize the situation: On one side, Schwartz, who had just received a Fields Medal and was at the top of his scientific career, and on the other side the unknown student from the provinces, who had a rather inadequate and unorthodox education. Grothendieck was awarded a Ph.D. for his work on topological vector spaces and stuck with that field for a while. He went to Brazil for two years and then to Kansas. Largely under the influence of Serre, he turned to algebraic geometry beginning in 1954. The most spectacular new result in the field was the theorem of Riemann-Roch-Hirzebruch. Within two years of the awakening of his interest in algebraic geometry, Grothendieck found a far-reaching generalization and a completely new proof, which has remained possibly his most significant single achievement in mathematics.

The next fifteen years of Grothendieck's scientific work were dedicated to the rebuilding of algebraic geometry. In 1958 he was appointed to the IHES, which had just been founded by the businessman Léon Motchane. Together with Dieudonné, his former teacher and now colleague at the IHES, Grothendieck began working on the *Éléments de Géométrie Algébrique* (EGA) and held the legendary *Séminaire de Géométrie Algébrique* (SGA). Many mathematicians who were close to him in those days emphasize that his way of doing mathematics was completely singular: He was not interested in the solving of difficult or famous problems, especially if it had to be done “by force”, but his goal was to achieve such a deep and complete understanding of the underlying structures that the solutions of such problems would fall out “on their own”.

During his twelve years at the IHES, Grothendieck led an outwardly bourgeois life: He married Mireille Dufour and had three children with her, born in 1959, 1961, and 1965. Earlier he had had a son from a previous relationship. However, the education of his children was unconventional; at least temporarily, they did not attend public schools. Grothendieck thought that finding one's own way was more important than a formal education. His home was hospitable, and he sometimes took in people in need for weeks at a time.

In his IHES seminar, Grothendieck surrounded himself with a group of outstanding students to

³ See for example Philip P. Hallie, *Lest Innocent Blood Be Shed: The Story of the Village of Le Chambon and How Goodness Happened There*. This book has been published in several editions by Harper & Row, New York.

whom he generously gave his ideas for them to pursue. At the same time, more and more conflicts developed with Motchane, the founder and director of the institute. Grothendieck's relationship with his colleague René Thom was not without complications either. At the 1966 International Congress of Mathematicians, Grothendieck was awarded the Fields Medal. He was at the pinnacle of his fame. In May 1968 the student revolution erupted in Paris and made a deep impression on Grothendieck that would change his life decisively. I will return to this later on.

In the year 1970 an event occurred that Grothendieck later would often call "the great turning point" (*"le grand tournant"*). He gave up his job at the IHES and started turning away from mathematics, although he did for a few years more have positions at the Collège de France and at the University of Paris, Orsay. He turned to the problems of environmental protection and ecology, he supported the antinuclear power movement, and he fought against military buildup, especially of nuclear weapons, and the military-industrial complex. To pursue these goals actively, he and a number of comrades founded the group *Survivre*, which later was also known as *Survivre et Vivre*. For about three years he devoted all his energy to this movement.

At the same time, his family life dissolved. On a "propaganda trip" for *Survivre* through America, he met Justine Skalba, with whom he lived in France in a commune he founded and with whom he had a son. For a time, his children from his first marriage also lived in this commune. In 1973 there was another decisive change: he left Paris and moved to the tiny village of Villecun, on the southern edge of the Cévennes, about sixty kilometers northwest of Montpellier. Since that time Grothendieck has lived only in small villages or hamlets. More and more he broke off contact with former colleagues, students, acquaintances, friends, and his own family; his relationship with Justine Skalba also ended after two years.

Soon after his move to Villecun he took a job as a professor at the University of Montpellier, although he taught there only irregularly. For months or even years on end he stopped doing mathematics altogether before starting to write down obsessively his mathematical "meditations" of hundreds or even thousands of pages. In the last few years before his retirement in 1988, he again held a research position in the CNRS (Centre National de la Recherche Scientifique), though he did research only sporadically.

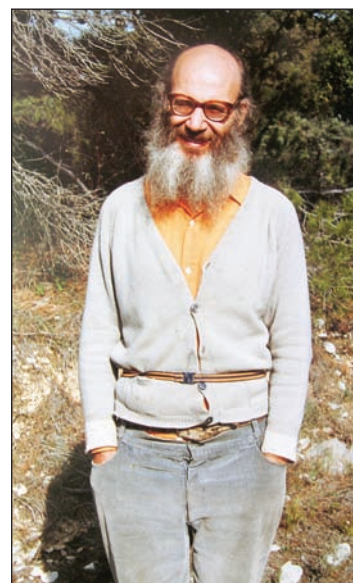
From 1974 Grothendieck turned to Buddhism; several times he was visited by Japanese monks from the order Nipponzan Myohoji (in English the name translates roughly as "Japanese community of the wonderful lotus sutra"), which preaches strict nonviolence and erects peace pagodas

throughout the world. But his attachment to Buddhism did not last. From around 1980 Grothendieck gravitated toward Christian mystical and esoteric ideas. More and more often there were periods of serious psychological problems; presumably things were surfacing in Grothendieck that had always been there inside him. For a while he identified with the stigmatized Catholic nun Marthe Robin, who claimed to have lived for thirty years on the Eucharist alone. A kind of angel, whom he calls Flora or Lucifera depending on whether he wants to emphasize her divine or her devilish side, plays an important role in his thoughts. For nights on end Grothendieck played chorals on the piano and sang along. Finally in 1988 a period of excessive fasting almost cost him his life. Apparently he wanted to force God to reveal Himself. He wanted to consciously experience the moment of death and outdo Jesus's forty-day fast. In 1999 he predicted that the Final Judgment was imminent and that a golden age would start thereafter. Later on, these delusions extended to nonreligious areas, including, for example, questions of cosmology. There can be no doubt that, at least since the end of the 1980s, his life has been dominated for long periods by delusions and that he would have needed urgent medical and psychiatric help.

In the summer of 1991 Grothendieck suddenly left his residence in Les Aumettes and withdrew to a place that remained unknown for a long time. He refuses almost all contact and seems to be occupied daily with writing down his meditations.

The Great Turning Point

Certainly there is a multitude of reasons that contributed to the "great turning point" of 1970. They are complementary and contradictory. Some seem obvious, while others are buried in the depths of Grothendieck's existence and his past and can hardly be brought to light. Much—in fact a great deal—remains a riddle. One does not have the impression that one understands or that one can understand his radical actions. Grothendieck's colleagues, pupils, and friends must all have asked themselves what the causes of this step could have been. I want to emphasize that the following attempt at an explanation is based on my personal views. Another person might well interpret the facts at hand in a different way.



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Grothendieck in the 1980s.

It has often been said that the decisive reason for Grothendieck's break with the IHES was the fact that a part of the IHES budget (about 5 percent) came from the French defense ministry. This could not be reconciled with Grothendieck's pacifist, anarchist, and radical leftist political convictions. Grothendieck himself has often confirmed this version. But I think this explanation is not the whole truth and is not particularly plausible. It is no doubt correct that the financial support by the defense ministry was not acceptable to Grothendieck. But there had been many discussions of this topic between the leadership of the IHES and the faculty, in which the permanent professors had largely supported Grothendieck. With good will surely the problem could have been solved. In fact, the relationship between the founder and director, Motchane, and Grothendieck was already completely dysfunctional at this time. The reasons that led to the break with the IHES and in particular with Motchane have been analyzed by David Aubin in his Ph.D. thesis. We refer to his investigation for the details of this conflict.⁴ In this conflict, it seems much more plausible that cause and effect were reversed: The dispute over the budget gave Motchane the possibility (finally) to get rid of Grothendieck, whom he regarded as a paranoid troublemaker. Perhaps Motchane had no choice, for if Grothendieck had stayed, probably two of the other permanent professors, Thom and Louis Michel, would have left.

That the conflict over the IHES budget is insufficient as grounds for Grothendieck's departure also follows from the fact that the conflict does not explain why he turned away from mathematics and from the mathematical community. Throughout the world he could have found places to work that were consonant with his moral and political convictions. He would have been welcome everywhere; he could have continued his research, and many of his students would have followed him.

In his commentary on the meditation *Récoltes et Semailles*, Jean-Pierre Serre talks about the decisive point. He says that Grothendieck never had the urge to do what perhaps the whole world expected of him, namely, to give a coherent explanation in the 1,600 pages of this treatise. Serre says:

But you do not ask the most obvious question, the one every reader expects you to answer: why did you yourself abandon the work in question?

A few lines later Serre attempts to answer his own question:

⁴David Aubin, "A cultural history of catastrophes and chaos: Around the Institut des Hautes Etudes Scientifiques, France, 1958-1980", Ph.D. thesis, Princeton University, 1998.

I have the impression that, despite your well known energy, you were quite simply tired of the enormous work you had undertaken.

By letter and in conversations, Serre later confirmed this view. Considering that, as a colleague put it, Grothendieck had done mathematics twelve hours a day, seven days a week, and twelve months a year for twenty years, one can only agree. But a question remains. Many scholars (or artists) give up a project they have started because their creativity and strength dissipate. But they do remain respected members of the community.

Serre speaks simply of "tiredness". Similar views have been expressed by others, who, however, see deeper reasons, including "disappointment". According to an oral communication from Helmut Koch, Igor Shafarevich thought that it was a disastrous decision on the part of Grothendieck to begin working on the *Eléments de Géométrie Algébrique*. He should have used his creativity on the "great problems" and not on a complete construction of a gigantic theory. Comments in this direction have also been made by the physicist David Ruelle, a colleague of Grothendieck's at the IHES: After a superhuman effort, Grothendieck had to admit that he would never be able to complete the oeuvre he had begun. It was as if he had set his mind on building a cathedral with his own hands. When the walls were two meters high, he had to stop.

It seems to me that all three—Serre, Shafarevich, and Ruelle—are making important points but missing the decisive one. They might explain why Grothendieck gave up mathematics, but not why he changed his whole life, why he withdrew from human society. No doubt this event, originating in the core of his personality, is much more deep-seated and more emotional than giving up one's job or withdrawing from scientific research. Again it seems natural to think that cause and effect might have been reversed: Because, for whatever reason, Grothendieck could no longer live in the society he had lived in from 1950 to 1970, he had to leave mathematics as well.

Grothendieck's old friend and colleague Cartier⁵ has undertaken a less superficial attempt to explain Grothendieck's decision. He does not discount the importance of the financing of the IHES, and he sees the crises in Grothendieck's mathematical work, but he also sees that the rupture in Grothendieck's life had deeper reasons:

I would like to try to analyze the reasons for this abrupt end to a career so astonishing and fertile at the age of 42. The reason given was that he had

⁵Pierre Cartier, "A mad day's work: From Grothendieck to Connes and Kontsevich", *Bulletin of the AMS (N.S.)* **38** (2001), 389-408.

discovered that the Ministry of Defense had been subsidizing the institute....In order to understand the vehemence of Grothendieck's reaction, one must take account of his past and the political situation of the time. He is the son of a militant anarchist who had devoted his life to revolution. This was a father of whom he had very little direct knowledge; he knew him mostly through his mother's adulation. He lived as an outcast throughout his entire childhood and was a "displaced person" for many years....He had always been uncomfortable frequenting the "better" places and felt more at ease among the poor, even the impoverished. The solidarity of outcasts had created in him a strong feeling of compassion. He lived his principles, and his home was always wide open to "stray cats". In the end he came to consider Bures a gilded cage that kept him away from real life. To this reason he added a failure of nerve, a doubt as to the value of scientific activity. Starting in 1957 at a Bourbaki Congress, he confided his doubts to me and told me that he was considering activities other than mathematics.⁶ One should perhaps add the effect of a well-known "Nobel syndrome". [After being awarded the Fields Medal in 1966], when he was laboring over the last (decisive) stages of the proof of the Weil conjectures and perhaps beginning to perceive that Deligne would be needed to complete in 1974 the program he had set for himself, and perhaps yielding to the pernicious view that sets 40 as the age when mathematical creativity ceases, he may have believed that he had passed his peak and that thenceforth he would be able only to repeat himself with less effectiveness.

The mood of the time also had a strong influence. The disaster that had been the second Vietnam War from 1963 to 1972 had awakened many consciences.

When it was said above that the dispute over the budget was not the really decisive point, this was not meant to imply that political, or more precisely socio-political, reasons played no role. On the contrary, they were of great importance to

⁶In a short note about the spiritual landmarks of his life, Grothendieck mentions for 1957 "calling and unfaithfulness" (footnote of W. S.).

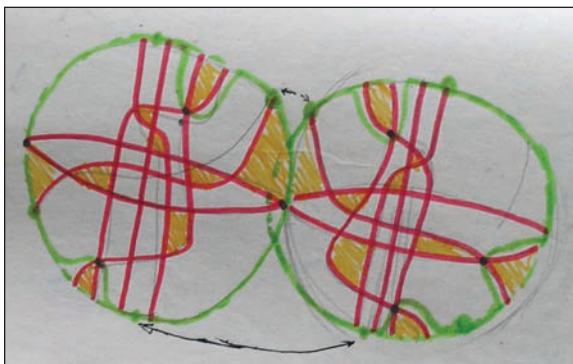
Grothendieck. To explain this, we must recall some of his political activities.

These activities have to be seen against the backdrop of his own life and the lives of his parents. Cartier is certainly right when he emphasizes that Grothendieck was always conscious of the example of his parents. His father had fought all his life for freedom and self-determination and against the powerful in this world. Grothendieck's sympathy was always with the poor, the persecuted, the oppressed, those in the shadows, and he always held leftist, liberal, and possibly even anarchist political convictions. But for many years these convictions were not expressed in political actions. In the late 1950s and early 1960s, he opposed the French war in Algeria as a matter of course, but in contrast with many of his closest colleagues such as Schwartz, Chevalley, Samuel, or Cartier, he did not participate in public protests. At least he took the matter seriously enough to consider emigrating to the United States.

Grothendieck's political commitment became publicly visible in the summer of 1966, when he refused to travel to Moscow to receive the Fields Medal at the International Congress of Mathematicians (ICM). This was his protest against the persecution and imprisonment of the Russian writers Yuri Daniel and Andrei Siniavsky. This action attracted a lot of attention. Some years later it was held very much against Grothendieck by orthodox communists and socialists who played a big role in the student movement.

His next political action was a trip, made at his own initiative, to Hanoi and North Vietnam during the last three weeks of November 1967 in the middle of the Vietnam War. He gave a series of lectures about this trip, in Paris on December 20, 1967, and later in other places. In addition to reporting on scientific and personal contacts and on the lectures he had given, he described the destruction the war had wrought, the bomb attacks, material deprivations, and the faith of the Vietnamese people in their own future. Though he cautiously criticized the indoctrination in dialectical materialism and the overwhelming regimentation of public life, every sentence of his report spoke of his deep sympathy for the struggles of the Vietnamese people to build a new society under difficult circumstances and to support public education and scholarship.

Grothendieck's spontaneous trip to Vietnam was probably typical of him in that it was an "individualistic" action. Many French intellectuals, including well-known figures such as Jean-Paul Sartre, had long taken an interest in Indochina. Many mathematicians were also committed to this cause, none more than Grothendieck's advisor, Laurent Schwartz. In his autobiography Schwartz talked at length about the fight for an independent Vietnam and about his love for that country and its



An example of a handwritten mathematical diagram drawn by Grothendieck.

inhabitants.⁷ He negotiated with many influential politicians, among them the Vietnamese prime minister Phan Van Dong and Ho Chi Minh himself.

Schwartz was also one of the initiators of the Russel tribunals, held in 1967 in Stockholm and Roskilde. He mentions as his comrades in arms many well-known French and other mathematicians, among them Jean-Pierre Kahane, Bernard Malgrange, Pierre Cartier, André Martineau, and Stephen Smale, but Grothendieck's name is mentioned only in passing. In those years Grothendieck had no interest in actions organized together with other people; he did not participate, perhaps he was even indifferent. All those who observed his political actions testified to his good will and his serious and honest intentions but at the same time ascribed to him an unbelievable naiveté and even ignorance. (I can barely believe the reports that Grothendieck at the time did not know what NATO really was.)

In May of 1968 the student revolution that was soon to envelop the whole Western world broke out in Paris. There were strikes and demonstrations that sometimes bordered on riots; there were demands for radical changes in university curricula, for abolishment of examinations, for self-determined learning, and for equal representation for faculty, staff, and students; in extreme cases, there were even demands for the destruction of computing centers and academic departments that were suspected of conducting military research. It was a true "cultural revolution" (which today seems like a distant past that has already faded). In several places in Grothendieck's meditations written in later times, he alludes to the deep impression these events made on him. He was convinced of the seriousness of the young people's revolution and certain that Western civilization and capitalism were headed for a deep crisis; he

developed doubts about whether his own scholarly occupation was the right path and even wondered whether it was irresponsible to engage in such activity. This happened to many academics and intellectuals at the time, particularly in France; it was simply the "zeitgeist" (which is stronger than everything else). But Grothendieck reacted to this with his characteristic forcefulness, rigor, and recklessness against others and himself, and perhaps also with obstinacy and with a sense of mission (though maybe he was just more clairvoyant than those around him).

Considering this background, there can be no doubt that Grothendieck was incapable of compromising on the question of the IHES receiving a part of its budget from the French defense ministry. In his meditations (especially in *Récoltes et Semailles*), he has said repeatedly that this issue had to lead to a break with the IHES, and his closest colleagues, such as Serre and Deligne, have confirmed how decisive this issue was. Nevertheless, it must have happened as I explained above: Motchane had plenty of reasons to try to get rid of Grothendieck, and Grothendieck had already reached the great turning point internally, even if he was not yet conscious of this.

In trying to understand how the "great turning point" happened, one has to take into account Grothendieck's mental state, which already by then must have been unstable and perhaps sometimes out of control. This would not have been apparent in his interactions with his colleagues and students, even though Cartier hints at it. But in closer interaction, a deep personality disorder is clearly visible. This is not the place to go further into this subject.

We now come to the second aspect of the "great turning point", the departure from mathematics. It seems to me that this process has a "negative" and a "positive" aspect. The "negative" has already been mentioned: fatigue and disappointment, as seen and described by Serre, Shafarevich, Ruelle, and also Cartier. The "positive" aspect is that Grothendieck found an occupation that seemed to him to be more important than mathematics and to which he devoted himself for the next two or three years with the same energy and drive that he had previously lavished on mathematics. This occupation was environmental protection in the widest sense of the term, the rising ecological movement (the word "ecology" existed at that time only as the name of a subdiscipline of biology), resistance to atomic energy, the struggle against militarism and the arms race, support for a new society and a "cultural revolution"—all in all, a movement that followed many ideals of the generation of 1968. Apparently this movement, the new goals, and the new ideals impressed Grothendieck so much that he became a committed follower. At this time he did not yet consciously abandon mathematics; he

⁷ Laurent Schwartz, *Un Mathématicien aux Prises avec le Siècle*, Odile Jacob, Paris, 1997; English translation, *A Mathematician Grappling with His Century*, Birkhäuser, Basel, 2001.

did not yet speak about it being for him like “a trip through the desert”, as he would often say later on. But he had found something that, at least for the moment, was more important to him than mathematics.

Grothendieck’s main activity with regard to these goals was the foundation of a group initially called *Survivre* and later on *Survivre et Vivre*:

SURVIVRE

*Mouvement international pour la survie
de l’espèce humaine*

An international and interprofessional
movement for the survival of humanity

The goals of this movement are summarized in its first bulletin of August 1970 (this quotation is from the English edition of the bulletin, as is the slogan just above):

To fight for the survival of the human species and of life in general, threatened as they are by the ecological disequilibrium created by contemporary industrial society (pollution, waste, devastation of natural resources), as well as by military conflicts and the threat of military conflicts.

On balance, it seems that Grothendieck’s involvement in this movement, in complete contrast with his work in mathematics, has been without lasting effect and ended in defeat. That is how it would have seemed to him, but perhaps that point of view is too superficial: It is certain that during his *Survivre et Vivre* period, Grothendieck deeply impressed some young people and completely changed their lives. And maybe the group did make a contribution to the establishment of the “green” movement that has firmly taken root in society and politics in Europe.

Shortly after his official letter of resignation, on June 26, 1970, Grothendieck gave a lecture to hundreds of listeners at the University of Paris in Orsay in which he talked about all that had become important to him: the spread of nuclear weapons, the arms race, the threat to humanity posed by technological progress. He went so far as to call mathematical research dangerous because it is part of this technological progress. The content of this lecture was later circulated in various unofficial manuscripts under titles such as “Responsabilité du savant dans le monde d’aujourd’hui: Le savant et l’appareil militaire” (“The Responsibility of Scientists in Today’s World: The Scientist and the Military Establishment”).

The foundation of the group *Survivre* came about during a summer school on algebraic geometry in July and August 1970 in Montreal. Grothendieck had been invited to this meeting to deliver lectures on crystalline cohomology. He agreed to go under

three conditions: In addition to his mathematical lecture, he wanted to give a lecture of equal duration about his ecological goals, and this additional lecture was to be advertised and published in the same way as the scientific lectures. The organizers of the meeting accepted these conditions, and so the participants found among the materials that they received at the beginning of the meeting a text based on the earlier lecture in Orsay. Apparently Grothendieck’s charismatic personality impressed a whole group of mostly young mathematicians so much that *Survivre* was founded spontaneously by the group. One of the most active members was Gordon Edwards, who was then a Ph.D. student under Grothendieck’s friend from student days, Paulo Ribenboim, and who later became a leader in the antinuclear movement in Canada.

The first bulletin of the group contains a list of members running to twenty-five people, eighteen of whom were mathematicians. One has to suspect that most of them were “recruited” by Grothendieck. His mother-in-law, Julienne Dufour, was among them, as was his son Serge, then seventeen years old. Grothendieck soon succeeded in winning over other prominent mathematicians, particularly those who had always been part of leftist movements. For the second issue of the bulletin, Claude Chevalley was the *directeur de publication* and a member of the editorial committee. About a year later, Pierre Samuel joined the French editorial committee.

As far as I know, a total of nineteen issues of the bulletin of the movement, totaling approximately seven hundred pages, were published between 1970 and 1975. There is no doubt that in the early years the main burden of the editorial work rested on Grothendieck, who surely wrote many of the unsigned articles. After his move to Villecun in 1973, he must have been less involved, if at all. As is usual with such groups, a tendency to disintegrate started after only a short time; for example, Samuel left the group in 1973.

It is important to note that at the beginning of his period of ecological and antimilitarist activities, Grothendieck consciously tried to profit from his reputation as a scholar. He was deeply imbued with the truth of the goals of *Survivre et Vivre*, and without doubt he thought that anyone would have to come to the same conclusions when presented with appropriate enlightenment and information. He took it for granted that any rational, sensible person would have to agree with the views of the group *Survivre*, so it was natural for him to first try to convince other mathematicians. And at the beginning, he believed that his efforts would be successful.

In Montreal he had convinced some participants just through the momentum of the early enthusiasm. Perhaps it was not difficult to win over in personal conversations some acquaintances

who had always been left-wing activists, such as Chevalley or Samuel. Others, like Serre or Deligne, must have been more cautious. But the moment of truth came when he tried to attract new converts through public activism, for example at the 1970 ICM in Nice. There he set up an information booth and attracted attention through spectacular actions in the expectation that mathematicians would join the group *Survivre* in droves. As he himself summarized, this attempt was an utter failure and surely contributed to his estrangement from the community of mathematicians. After a few years of toiling in vain, he must have reached the conclusion that mathematicians and scientists are blind to the dangers threatening human society and do not think and behave rationally. And so Grothendieck withdrew more and more, not just from mathematics but from the community of mathematicians.

At this point I want to conclude the discussion of the reasons that might have led to the “great turning point”. But it seems to me that the decisive point has not been touched upon yet: Why has Grothendieck withdrawn from human society itself? Thinking about his whole personal life, one has the impression that, for whatever reason, it has been impossible for him to maintain a long-term personal relationship with anybody. Whenever such a relationship did not come to an end for purely external reasons, it inevitably led to deep conflicts and usually terrible reproaches, even imprecations, on the part of Grothendieck. He could not live long-term in human society as it exists, and therefore he also had to give up mathematics and activities connected to it.

No doubt this is a depressing tally for the life of a unique scientist and man.

The Meditations of Grothendieck

We now turn to the question of what Grothendieck did after his withdrawal from society. It goes without saying that such an active and creative mind could not remain idle. His main intellectual occupation clearly was, and still is, writing down his “Meditations”, which, as far as is known, cover biographical, religious, esoteric, and philosophical themes. I use here Grothendieck’s own word, “Meditations” (sometimes also “Reflections”), whereby he means, as he says on many occasions, “meditating” as well as writing. Since the 1960s, when he spent many hours daily at the typewriter, he has been used to putting down his thoughts in writing. (It is natural to conjecture that, from the 1980s onwards, this habit practically became a compulsion.)

To begin with, one has to observe that all his life Grothendieck felt a calling to be a writer and that he is without doubt a master of written expression. His linguistic and stylistic prowess, and above all his creativity in inventing words, would

be to the credit of any writer. He does of course have a “genetic predisposition”. His mother had the ambition to be a writer and left behind an important literary work, the autobiographical novel *Eine Frau*. His father too saw literature as his real calling, although his lifelong struggle for the anarchist movement prevented him from pursuing this vocation. Grothendieck himself first played with the idea of turning to poetry after the proof of the Riemann-Roch theorem. He wrote a good deal of occasional verse (in German, French, and English; most of it has been lost), and he has translated poetic texts from German into French. He undertook his first serious poetic attempt in 1979, when he wrote down the *Eloge*, about which more will be said later, and he had further plans at that time.

To create an overview, we now give a chronological survey of Grothendieck’s known “Meditations”, followed by a few comments about their content.

1979: *L’Eloge de l’Inceste (In Praise of Incest)* (January to July 1979, perhaps lost)

1981: *La Longue Marche à Travers la Théorie de Galois (The Long March through Galois Theory)* (January to June 1981, about 1,600 pages, plus about the same amount of commentary and supplementary material; unpublished, but since 2004 parts have been available on the Internet)

1983: *A la Poursuite des Champs (Pursuing Stacks)* (approximately 650 pages, started as a “letter” to D. Quillen, unpublished). Associated with this is an extensive correspondence with Ronnie Brown and Tim Porter.

1984: *Esquisse d’un Programme (Sketch of Program)* (January 1984)

1983–1985: *Récoltes et Semailles: Réflexions et Témoignage sur un Passé de Mathématicien (Reapings and Sowings: Reflections and Testimony on the Past of a Mathematician)* (1,252 pages plus approximately 200 pages of introduction, commentary, and summaries; produced as photocopies; available on the Internet)

1987: *La Clef des Songes (The Key to Dreams)* (315 pages, unpublished)

1987–88: *Notes pour la Clef des Songes (Notes on the Key to Dreams)* (691 pages, unpublished); includes a freestanding work, *Les Mutants*

1990: *Développements sur la Lettre de la Bonne Nouvelle (Developments on the Letter of Good News)* (82 + 2 pages, unpublished; written February 18–March 15, 1990)

1990: *Les Dérivateurs* (about 2,000 pages, unpublished, but parts available on the Internet)

Surely this considerable number of manuscripts is not all that Grothendieck wrote during those years. Various eyewitnesses have reported that one day (perhaps in 1990 or in early 1991) he burned many manuscripts and perhaps other documents, such as correspondence, in an old oil barrel. His only

work of fiction, *L'Eloge de l'Inceste*, possibly fell victim to this deed; it is also possible that a copy survives somewhere.

We now make a few comments on some of these “Meditations” only to show that a more extensive analysis and interpretation in a wider context would be desirable.

Without doubt the best known of his “Meditations” is *Récoltes et Semailles*, which contains above all his reckoning with mathematics and the mathematical community. Mostly because of his attacks on many colleagues and former pupils, which seem more or less unjustified, this text has achieved a certain notoriety. The widespread idea that he is “crazy” and “paranoid” is based mostly on this text. By now there is an extensive “unofficial” literature about this work (which is easy to research on the Internet), so we will not comment further on it here. It is hard to say what this text really is: not an autobiography, not a work of fiction, but not a scientific work either; in a letter to German friends, Grothendieck once called it a “mathematical phantasmagoria”.

Grothendieck has said about *L'Eloge de l'Inceste* that it is on the one hand his first systematic reflection of a philosophical nature and on the other hand a work of fiction—he calls it a “song”. He mentions it occasionally in his other meditations and apologizes for the somewhat flamboyant (*un peu tapageur*) title. In his correspondence with his German friends, Grothendieck mentions the *Eloge* on several occasions, for the first time on August 17, 1979:

Since the beginning of June, I have withdrawn to a solitary hermitage in the Vaucluse, where nobody knows me—maybe I will stay here for a whole year to “turn in” quietly. At the end of July, I finished the first version of the first song “In Praise of Incest”. At the beginning of September, I want to go over it with a friend and then slowly type up a clean version—it will probably take two to three months, a few pages a day, after all I have other things to do as well.... And so in November or December, I will have it photocopied—but not at the university,... Two hundred copies to begin with—it’s going to be about 170 pages all in all.... I have not yet decided 100 percent whether I will publish the song. Probably yes. It is surely the most meaningful thing I have ever done—but then there isn’t much except mathematics. In any case, I will wait at least until the spring before I entrust a publisher with the first song. By then I assume that the substance of the next two songs will have ripened

and clarified and that their form will have emerged, at least in outline.

It seems best to discuss *La Longue Marche à Travers la Théorie de Galois* (LM) and *Esquisse d’un Programme* (EP) together, because EP is in a certain sense a summary of LM. With the EP, Grothendieck applied for a position in the CNRS. The text contains a summary of his mathematical thoughts since the early 1970s. It has now been published together with an English translation.⁸ The central objects of the investigations are the moduli spaces $M(g, n)$ of compact Riemann surfaces of genus g with n marked points that had been studied earlier by Pierre Deligne and David Mumford. Grothendieck makes a connection to arithmetic objects, in particular the absolute Galois group of the field of rational numbers \mathbb{Q} . To come to grips with the elementary geometric and combinatorial aspects of these questions, Grothendieck designs his theory of “children’s drawings” (*dessins d’enfants*). He also speculates on an “anabelian geometry”. Of all of Grothendieck’s mathematical “meditations”, these have certainly had the largest echo. There are many papers concerned with these questions, and in the 1990s several workshops were organized around these topics.

La Clef des Songes

I now come to the meditation *La Clef des Songes*, which is still largely unknown. As with the others, it is easier to say what it is not than to say what it is. It is not a scholarly work, because it has no clearly defined subject and the considerations follow no scholarly methodology. It is not an autobiography, although Grothendieck occasionally recounts episodes from his life. And it is certainly not a work of fiction in any form, for it has no narrative, no plot line, and no characters that could have carried a plot. However, at many points Grothendieck uses poetic language, and much of it can be understood only in the way in which one understands—or “absorbs”—poetry: not rationally but emotionally. (An example: *The only God is silent. And when He speaks, it is in such a low voice that nobody ever understands.*) And the text is not a systematic analysis of, say, the phenomenon of dreams, for it is not about concrete dreams. It is perhaps something like a confessional—but what would he be confessing? It is best to let Grothendieck speak for himself: “It is the record of a long meditation. A meditation that has no aim, in which the thoughts are left largely to themselves.”

⁸The following conference proceedings concern the circle of ideas around LM and EP, in particular the letter to Faltings and the text of EP: The Grothendieck Theory of Dessins d’Enfants, Leila Schneps, editor, *London Mathematical Society Lecture Notes* 200 (1994); Geometric Galois Actions 1 and 2, Leila Schneps and Pierre Lochak, editors, *London Mathematical Society Lecture Notes* 242 and 243 (1997).



Grothendieck in May 1988.

Many people who know Grothendieck report that he has “always” been very interested in dreams. But they became the central theme of his thinking only after the “great turning point” of 1970. It appears that he thoroughly worked through, for example, Freud’s *Traumdeutung* (*The Interpretation of Dreams*) and also read other relevant literature.⁹ On the other hand, he does not describe even a single one of his own dreams that had great importance for him, and he says nothing about how he analyzed them.

Instead of attempting a summary or a table of contents of *La Clef des Songes*, we will confine ourselves to mentioning a few key ideas. Grothendieck starts with the statement that there is an external being, the “dreamer”, who knows the people and sends them dreams so that they will recognize themselves. Among these dreams there are some that carry particularly important messages. Because of torpor and fear of change, many people do not understand these messages. The dreams are not the result of mental processes of people; rather, they come from outside. Next Grothendieck analyzes the nature of the “dreamer” and comes to the conclusion that God exists and is the dreamer (*Le reveur n’est autre que Dieu*). He then discusses the question of how he himself arrived at his belief in God. There is a rather detailed description of the biography of his parents—both were convinced atheists and anarchists—and he talks a bit about his childhood and youth. Surely he is trying to express that, given his biography, it was by no means obvious that he would find the “path to God”, but that this required an impulse from outside (namely, from God himself).

Grothendieck is convinced that each person has a “mission” and that an important part of this mission consists of finding one’s self, of recognizing one’s own self. Only through this search are a person’s creative powers liberated, for they are ordinarily suppressed in many ways through the constraints of society and through inner torpor,

which prevent their unfolding. He discusses the important role of “eros” as a decisive creative power. Furthermore, he discusses three levels at which a person develops both in general and with regard to creative powers: the physical, the mental-intellectual, and the spiritual level. Spirituality is a key concept in Grothendieck’s thoughts, not only in *La Clef des Songes*. He measures all people according to how far they have come in attaining a spiritual life. He also discusses the spiritual aspect of his mathematical work. Finally, he speaks about the many deformations of humanity that go hand in hand with a loss of spirituality and manifest themselves, for example, in a loss of the feeling for beauty in all areas of life.

I personally consider the *Notes pour la Clef des Songes* to be the most interesting of Grothendieck’s meditations known so far. Originally it was really meant to be remarks about *La Clef des Songes*. But soon an independent text called *Les Mutants* developed out of it. The somewhat strange title “The Mutants” (a word that in French too comes from the vocabulary of science fiction) refers to people who differ from “mere mortals” in a spiritual way; in particular, they are ahead of their times. At one place in the text Grothendieck gives the following explanation of this concept (slightly shortened in translation):

There have been in this century (as doubtless in other centuries past) a certain number of isolated men who seem to my eyes to be “new men”—men who appear to be “mutants” and who already today, in one way or another, prefigure the “man of tomorrow” embodied in the present; the man in the full sense of the word, who undoubtedly will emerge in the generations to come, in the course of the “post-herd” age, of which the dawn is very close and which they tacitly herald.

For hundreds of pages Grothendieck describes and discusses the lives and works of a total of eighteen mutants. It becomes clear that he sees a personal connection between these mutants and himself; for example, he occasionally calls himself their heir, or he calls them his elders. We now give the list of these mutants, as he assembled it himself. No doubt their selection is rather arbitrary. A central (and not very original) theme in Grothendieck’s thinking is the spiritual decline of humanity, necessarily followed by an apocalypse and soon thereafter by the “new age”, the age of freedom and self-determination and of life in harmony with one’s own “soul”. The mutants are people who announce and anticipate this new age. This is the criterion by which he selected them. The list comprises the following people, all

⁹This is apparent from, for example, marginal notes in Grothendieck’s copy of Freud, *Die Traumdeutung*, Fischer Studienausgabe, Band II.

men. The remarks on their works are taken from Grothendieck:

C. F. S. Hahnemann: German medical doctor and scholar, renewed the medicine of his time

C. Darwin: English natural scientist, scholar

W. Whitman: Journalist, American writer, poet and teacher

B. Riemann: German mathematician, scholar

Râmakrishna: Indian (Hindu) preacher, teacher

R. M. Bucke: American doctor and psychiatrist, scholar and *annonciateur* (herald)

P. A. Kropotkin: Russian geographer and scholar, anarchistic revolutionary

E. Carpenter: Priest, farmer, English thinker and writer, teacher

S. Freud: Austrian doctor and psychiatrist; scholar and creator of psychoanalysis, key to a new scientific humanism

R. Steiner: German scholar, philosopher, writer, orator, pedagogue...; visionary teacher, creator of anthroposophy

M. K. Gandhi: Indian lawyer and politician, teacher, worked for the spread of *ahimsa* (non-violence)

P. Teilhard de Chardin: French (Jesuit) priest and paleontologist, Christian religious ecumenical thinker, mystical visionary, worked toward the reconciliation of religion and science

A. S. Neill: English teacher and pedagogue who championed freedom in education

N. Fujii (called Fujii Guruji): Japanese Buddhist monk, teacher

J. Krishnamurti: Orator, Indian religious thinker and writer, teacher

M. Legaut: University professor, farmer, French Christian religious thinker and writer, disciple of Jesus of Nazareth, worked for a spiritual renewal of Christianity

F. Carrasquer: Spanish elementary school-teacher and pedagogue; militant anarchist for a “self-determined” school and society

Slovik: American worker and low-ranking employee apparently without any particular vocation

I cannot do much more here than give the names and list the aspects under which these people are discussed; they are sex (*sexe*), war (*guerre*), self-knowledge (*connaissance de soi*), religion (followed by an extensive explanation of what is meant—certainly not the church as an institution and not liturgy either), science (*science*), culture (*la civilisation actuelle et ses valeurs*, “*culture*”), eschatology (*la question des destinées de l’humanité dans son ensemble*, “*eschatologie*”), social justice (*justice sociale*), education (*education*), spirituality (“*science de demain*” ou “*science spirituelle*”).

Perhaps this small bit of information gives a vague impression of what this meditation is about.

To complete the picture, I would like to mention that Felix Carrasquer and his wife, Matilde Escuder, were close friends of the Grothendieck family (the original acquaintance was through Grothendieck’s wife, Mireille Dufour) and that the writing of the *Notes pour la Clef des Songes*, including *Les Mutants*, was substantially inspired by Grothendieck’s reading of the books of M. Legaut. A more detailed discussion must be left for another occasion.

Among these texts, the philosophical ones (and in a certain way the mathematical ones as well) all follow a common presentation. Grothendieck recorded his thoughts section by section, as if in a diary, and later edited these sections very little if at all. When he had more to say on one of his sections, he usually did so through footnotes or addenda, which sometimes led to whole new sections. It also happened that he would meditate on parts of the written text; this generated remarks on remarks on...as well as numerous footnotes. Surely this presentation does not facilitate reading, but in my view the more important criticism is that many of these long manuscripts do not have a clear aim. Both in *Récoltes et Semailles* and in *La Clef des Songes* it is obvious that new points of view appeared after the writing had already begun. As the texts do not seem to have any clear aim, there is no clear structure either. They meander, unstreamlined and unchanneled, in loops that change direction through a wide swath of landscape, as if through the valley of a primordial stream. The author drifts without any will to focus. It is completely different from his earlier writings on mathematics: Both EGA and SGA go into the breadth and the details, but there is a very clear aim, the “correct” development of algebraic geometry or the “correct” cohomology theory in algebraic geometry.

Since his disappearance, Grothendieck has written down tens of thousands of pages of his meditations. Printed in their entirety, they would surely fill dozens of volumes. It seems almost impossible that all of this, or even a large part of it, could be of importance. One cannot imagine that truly important writings can come about in complete intellectual and human isolation.

But one has to remember that Grothendieck is a true master of language, that he certainly has unconventional ideas and thoughts, and that he sees the world in an unusual and even singular way. So one can imagine that in the countless pages of his manuscripts there might often be finished texts: poems, biographical episodes from his life and the lives of people who were close to him, commentaries on books he read, perhaps lyric texts that go beyond poetry, philosophical thoughts, apocalyptic visions. It seems necessary to begin before it is too late to preserve for future generations those parts of Grothendieck’s life’s work that may be important beyond his mathematics.