

# 1981 VEBLEN PRIZES AWARDED

*at San Francisco Meeting*

Oswald Veblen (1880–1960), who served as President of the Society in 1923 and 1924, was well known for his mathematical work in geometry. In 1961 the Trustees of the Society established a fund in memory of Professor Veblen, contributed originally by former students and colleagues, and later doubled by his widow. Beginning in 1964 the fund has been used for the award of the Oswald Veblen Prize in Geometry. Subsequent awards were made two years later and, thereafter, at five year intervals. A total of eight awards have been made: Christos D. Papakyriakopoulos (1964), Raoul H. Bott (1964), Stephen Smale (1966), Morton Brown and Barry Mazur (1966), Robion C. Kirby (1971), Dennis P. Sullivan (1971), William P. Thurston (1976), and James Simons (1976). At present these awards are supplemented by money from the Steele Prize Fund, which brings the value of each to fifteen hundred dollars.

The 1981 recipients are Mikhael Gromov, SUNY, Stony Brook, and Shing-Tung Yau, Institute for Advanced Study. These prizes were awarded by the Council of the Society on the basis of recommendations made by a selection committee consisting of Raoul H. Bott, Eugene Calabi, and William P. Thurston, chairman. The awards were presented at the Annual Meeting of the Society in San Francisco on January 8, 1981. The committee's citations appear below, followed in each case by the recipient's response on receiving the award. Biographical and bibliographical information is included as well.

## Citation for Mikhael Gromov

Gromov's many beautiful insights have greatly enlarged our understanding of the relation between the topological and geometric properties of Riemannian manifolds.

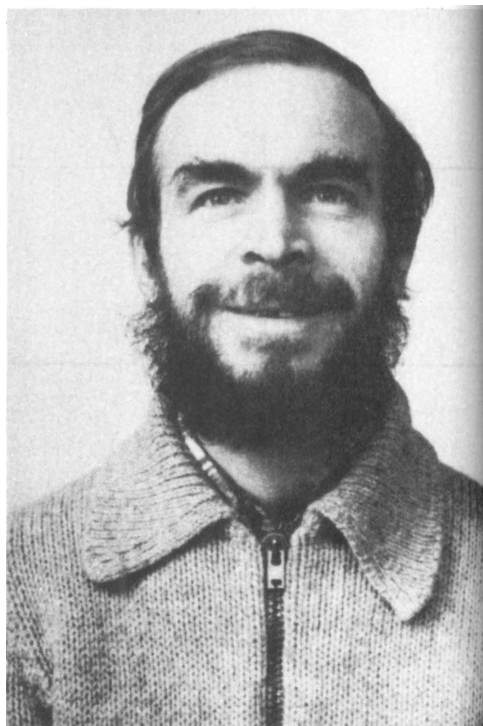
In particular, among many other contributions he has proven that a manifold with a bounded diameter and curvature near zero is a nilmanifold. He proved that there is a bound, depending only on the dimension, for the Betti numbers of a manifold of nonnegative curvature. He showed that in any dimension except three there are only finitely many manifolds with sectional curvature bounded between two negative constants, satisfying a given bound on volume. He defined a purely topological invariant of manifolds, nontrivial even in odd dimensions, which, for locally homogeneous manifolds, is proportional to the volume. He proved that every finitely generated group of polynomial growth contains a nilpotent group with finite index.

## MIKHAEL GROMOV

It is a great honor to receive a Veblen Prize. It is especially gratifying to have one's field of research recognized in this way.

The slow growth of global differential geometry can not match the "topological explosion" of the last decades. Topology has already taken a definite algebraic form, but we shall hardly ever achieve such structural clarity in geometry. Both topology and geometry reflect in their own ways the ancient vision of manifolds as global manifestations of more elementary local structures. Naive as it is, this idea has always been a source of inspiration in mathematics as well as in science.

A geometer's manifold is built of infinitesimal elements that appear at first glance to be plain Euclidean spaces. Then, with second order magnification, one sees Riemann's curvatures which determine relative positions of neighboring infinitesimal elements and one seeks to visualize the manifold as it is being shaped by these curvatures. In particular, guided by topology, one looks at manifolds whose curvatures are restricted in certain ways—for example, required to be everywhere positive—and then one tries to relate these local data to global algebraic invariants,



Mikhael Gromov

such as Betti numbers. This line of thought goes a long way back to the Gauss-Bonnet theorem, but general structural problems have not been brought to light until recently. I see, in coming years, the whole magnificent picture of Riemannian manifolds emerging before our eyes.

### *Biographical Sketch*

Mikhael Gromov was born December 23, 1943, in Boksitogorsk, USSR. He received three degrees from the University of Leningrad: Master in 1965, Ph.D. in 1969, and Doctor of Science in 1973. He received the Moscow Mathematical Society Award in 1971.

From 1968 to 1973 he held a research position at Leningrad University. Since 1974 he has been at the State University of New York Center at Stony Brook, where he is professor of mathematics.

Professor Gromov gave addresses in the Special Session on Riemannian Geometry at the Summer Meeting in Kalamazoo, Michigan (August 1975), and at the International Congresses of Mathematicians in 1970 and 1978.

His fields of special interest in mathematics are geometry, topology, and analysis on manifolds.

### *Bibliography*

1. *Spin and scalar curvature in the presence of a fundamental group*. I (with Lawson, B.), *Ann. of Math.* (2) **111** (1980), 209-230.
2. *The classification of simply connected manifolds of positive scalar curvature* (with Lawson, B.), *Ann. of Math.* (2) **111** (1980), 423-434.
3. *Hyperbolic manifolds, groups and actions*, *Proc. Conf. on Kleinian Groups* (Stony Brook) (to appear).
4. *Volume and bounded cohomology*, *Inst. Hautes Études Sci. Publ. Math.* (to appear).
5. *Isometrical immersions, hyperbolic geometry and Burago's isoperimetric inequality*, SUNY, Stony Brook (preprint).
6. *On the entropy of holomorphic maps*, *Inst. Hautes Études Sci. Publ. Math.* (to appear).
7. *Synthetic geometry in Riemannian manifolds*, *Proc. Internat. Congr. Math.* (Helsinki, 1978) **1**, 415-419.
8. *Three remarks on geodesic dynamics and fundamental groups*, SUNY, Stony Brook, 1976 (preprint).
9. *Manifolds of negative curvature*, *J. Differential Geometry* **13** (1978), 223-230.
10. *Almost flat manifolds*, *J. Differential Geometry* **13** (1978), 231-241.
11. *Homotopical effects of dilatation*, *J. Differential Geometry* **13** (1978), 303-310.
12. *Topology of Riemannian manifolds with small curvature and diameter* (preliminary report), *Notices Amer. Math. Soc.* **22** (1975), A-592. (Abstract 726-58-1)

13. *Convex integration of differential relations*. I. (Russian), *Izv. Akad. Nauk SSSR Ser. Mat.* **37** (1973), 329-343. MR **54** #1323.
14. *Degenerate smooth mappings*. (Russian), *Mat. Zametki* **14** (1973), 509-516. MR **49** #3972.
15. *Construction of a smooth mapping with a prescribed Jacobian*. I. (Russian) (with Eliasberg, Ja. M.), *Funkcional. Anal. i Prilozen.* **7** (1973), no. 1, 33-40. MR **50** #5841.
16. *Smoothing and inversion of differential operators*. (Russian), *Mat. Sb. (N.S.)* **88** (130) (1972), 382-441. MR **46** #10022.
17. *Isometric embeddings in multidimensional geometry*, Thesis abstract, Leningrad University, 1972.
18. *Construction of nonsingular isoperimetric films*. (Russian) (with Eliasberg, Ja. M.), Boundary value problems of mathematical physics, 7. *Trudy Mat. Inst. Steklov.* **116** (1971), 18-33, 235. MR **52** #9281.
19. *A topological technique for the construction of solutions of differential equations and inequalities*, *Actes du Congrès International des Mathématiciens* (Nice, 1970), **2**, 221-225. Gauthier-Villars, Paris, 1971. MR **54** #8709.
20. *Elimination of singularities of smooth mappings*. (Russian) (with Eliasberg, Ja. M.), *Izv. Akad. Nauk SSSR Ser. Mat.* **35** (1971), 600-626. MR **46** #903.
21. *Nonsingular mappings of Stein manifolds*. (Russian) (with Eliasberg, Ja. M.), *Funkcional. Anal. i Prilozen.*, **5** (1971), no. 2, 82-83. MR **46** #394.
22. *Imbeddings and immersions in Riemannian geometry*. (Russian) (with Rohlin, V. A.), *Uspehi Mat. Nauk* **25** (1970), no. 5 (155), 3-62. MR **44** #7571.
23. *Isometric imbeddings and immersions*. (Russian), *Dokl. Akad. Nauk SSSR* **192** (1970), 1206-1209. MR **43** #1212.
24. *Simplexes inscribed on a hypersurface*. (Russian), *Mat. Zametki* **5** (1969), 81-89. MR **39** #6220.
25. *Stable mappings of foliations into manifolds*. (Russian), *Izv. Akad. Nauk SSSR Ser. Mat.* **33** (1969), 707-734. MR **41** #7708.
26. *Mappings of foliations into manifolds which are equipped with supplementary structure*. (Russian), *Vestnik Leningrad. Univ.* **23** (1968), no. 19, 167. MR **40** #3565.
27. *The number of simplexes in subdivisions of finite complexes*. (Russian), *Mat. Zametki* **3** (1968), 511-522. MR **37** #3555.
28. *Transversal mappings of foliations*. (Russian), *Dokl. Akad. Nauk SSSR* **182** (1968), 255-258. MR **38** #6628.
29. *On a geometric hypothesis of Banach*. (Russian), *Izv. Akad. Nauk SSSR Ser. Mat.* **31** (1967), 1105-1114. MR **36** #655.

## Citation for Shing-Tung Yau

We have rarely had the opportunity to witness the spectacle of the work of one mathematician affecting, in a short span of years, the direction of whole areas of research. In the field of geometry, one of the most remarkable instances of such an occurrence during the last decade is given by the contributions of Shing-Tung Yau. Among the numerous areas that have been decisively influenced by his research, we will cite only three, with no inference intended that contributions to other areas may be of lesser importance: the theory of nonlinear elliptic partial differential equations, the topology of low-dimensional manifolds, and the analytic geometry of complex manifolds.

In the domain of nonlinear partial differential equations, Yau's best known work is contained in the estimates for elliptic solutions of  $n$ -dimensional Monge-Ampère equations (in collaboration with S. Y. Cheng) with notable applications, in particular to the Minkowski problem, and in his joint research with R. Schoen on the Einstein field equations, leading to the proof of the positive mass conjecture.

Yau's contributions to the topology of differentiable manifolds stems from his studies on minimal surfaces. Starting with the result of R. Osserman that any area-minimizing surface in euclidean 3-space has no local singularities, and working jointly with W. H. Meeks, he deduced a new analytic proof of Dehn's lemma, and applied it to solve the part of P. A. Smith's conjecture concerning differentiable actions of finite groups on 3-dimensional manifolds.

Finally, Yau's work on the complex Monge-Ampère equation on compact complex manifolds has been essential in settling all at once several outstanding questions in algebraic and in complex analytic geometry, among which we shall mention only the following ones: the uniqueness of the complex structure of Kähler manifolds of the homotopy type of the complex projective plane, and the existence of Kähler-Einstein metrics in a large class of compact complex manifolds (a question studied concurrently by T. Aubin); in addition, this work has opened an essentially new vista in studying the structure of the analytic moduli space for the so-called  $K3$  surfaces, by the use of the Ricci-flat metrics.

Few mathematicians can match Yau's achievements in depth, in impact, and in the diversity of methods and applications.

## SHING-TUNG YAU

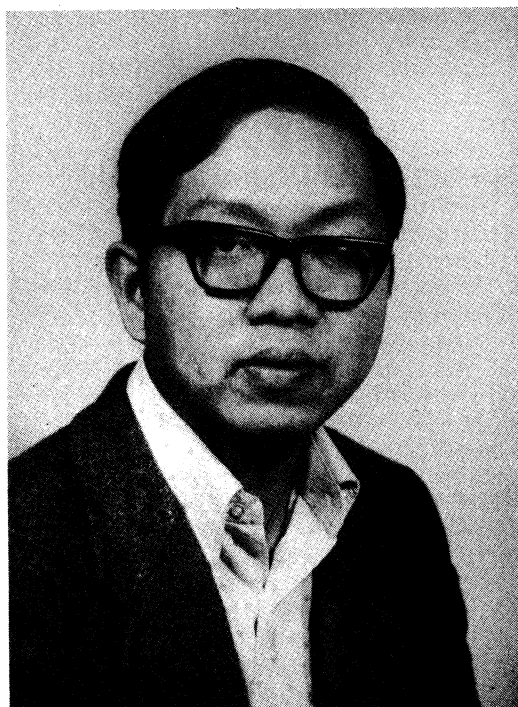
It is an honor for me to be chosen by the Committee to receive the Veblen Prize. It was said that my award is based on my work on nonlinear partial differential equations which gives contribu-

tions to Kähler geometry and the theory of minimal surfaces.

Before I explain my work in more detail, I would like to thank my coworkers S. Y. Cheng, W. Meeks, III, R. Schoen, and Y. T. Siu. If I deserve any honor at all, I hope they will share the same honor with me because their collaborations with me are indispensable to all the work that I have done.

My interest in Chern's work on the representation of the Chern classes by curvature forms started when I was a graduate student under the direction of Chern. For compact Kähler manifolds, the first Chern class is the simplest Chern class and it is represented by the Ricci form. Since the Chern class is an invariant depending only on the complex structure, this fact gives a simple integrability condition for a tensor to be a Ricci tensor of some Kähler metric. Many years ago, Calabi believed that this is the only integrability condition and he conjectured that, given a closed  $(1, 1)$  form which represents the first Chern class for a compact Kähler manifold, there is a Kähler metric whose Ricci form is the given  $(1, 1)$  form. It turns out that if one fixes the Kähler class of the metric, then the metric is unique.

Thus Calabi's conjecture solves the inverse problem for the Ricci curvature in the category of Kähler manifolds. It also provides canonical Kähler metrics for algebraic manifolds. They are rather strong invariants of the manifolds and can be used to solve some difficult problems in al-



Shing-Tung Yau

gebraic geometry. As algebraic geometry is a rich source for interesting manifolds, one also finds a new variety of examples of manifolds that differential geometers sought for a long time. Among those examples, one finds a compact simply-connected manifold with zero Ricci curvature. This is also an example of interest to physicists who work in general relativity. Despite some partial progress made by T. Aubin, very few geometers believed in Calabi's conjecture until my solution appeared in 1976.

The second topic is the application of minimal surfaces in geometry and topology. For a long time, geometers have been using geodesics to study the global structure of manifolds. It is therefore natural to use minimal surfaces, which are higher dimensional analogues of geodesics, to study some more delicate aspects of geometry and topology. Thus, Schoen and I studied compact manifolds with positive scalar curvature, Siu and I studied compact Kähler manifolds with positive bisectional curvature, Meeks and I applied the embedding properties of minimal surfaces to study finite group actions on three-dimensional manifolds. Instead of going into details, I should say that the idea of applying minimal surfaces to low-dimensional topology seems to lead to fruitful results. In this connection, one should also mention the works of Sacks-Uhlenbeck and Freeman-Hass-Scott.

#### *Biographical Sketch*

Shing-Tung Yau was born April 4, 1949, in Kwungtung, China. He received his Ph.D. from the University of California, Berkeley, in 1971. In 1971-1972 he was a fellow at the Institute for Advanced Study in Princeton. In 1972 he became assistant professor of mathematics at the State University of New York, Stony Brook. He spent the year 1973-1974 as visiting assistant professor at Stanford University. He became associate professor at Stanford in 1974, and professor of mathematics in 1977. He is currently a permanent member at the Institute for Advanced Study.

Professor Yau has given addresses at the Special Session on Partial Differential Equations and Geometry on Complex Manifolds (San Francisco, April 1978), and at Symposia on the Geometry of the Laplace Operator (Hawaii, March 1979) and on the Mathematical Heritage of Henri Poincaré (Bloomington, April 1980).

Professor Yau is the recipient of a Guggenheim Foundation Fellowship for 1980. His major area of research interest is differential geometry.

#### *Bibliography*

1. *On univalent harmonic maps between surfaces* (with Schoen, Richard), *Invent. Math.* **44** (1978), no. 3, 265-278. MR **57** #17706.

2. *Remarks on the group of isometries of a Riemannian manifold*, *Topology* **16** (1977), no. 3, 239-247. MR **56** #6686.

3. *Calabi's conjecture and some new results in algebraic geometry*, *Proc. Nat. Acad. Sci. U.S.A.* **74** (1977), no. 5, 1798-1799. MR **56** #9467.

4. *Hypersurfaces with constant scalar curvature* (with Cheng, Shiu Yuen), *Math. Ann.* **225** (1977), no. 3, 195-204. MR **55** #4045.

5. *On the regularity of the Monge-Ampère equation  $\det(\partial^2 u / \partial x_i \partial x_j) = F(x, u)$* , *Comm. Pure Appl. Math.* **30** (1977), no. 1, 41-68. MR **55** #10727.

6. *Complete Kähler manifolds with nonpositive curvature of faster than quadratic decay* (with Siu, Yum Tong), *Ann. of Math. (2)* **105** (1977), no. 2, 225-264. MR **55** #10719.

7. *Harmonic maps and the topology of stable hypersurfaces and manifolds with non-negative Ricci curvature* (with Schoen, Richard), *Comment. Math. Helv.* **51** (1976), no. 3, 333-341. MR **55** #11302.

8. *Maximal space-like hypersurfaces in the Lorentz-Minkowski spaces* (with Cheng, Shiu Yuen), *Ann. of Math. (2)*, **104** (1976), no. 3, 407-419. MR **55** #4063.

9. *Harmonic functions on complete Riemannian manifolds*, *Comm. Pure Appl. Math.* **28** (1975), 201-228. MR **55** #4042.

10. *Parallelizable manifolds without complex structure*, *Topology* **15** (1976), no. 1, 51-53. MR **53** #890.

11. *On the structure of complete simply-connected Kähler manifolds with nonpositive curvature* (with Siu, Yum Tong), *Proc. Nat. Acad. Sci. U.S.A.* **73** (1976), no. 4, 1008. MR **53** #8476.

12. *Some function-theoretic properties of complete Riemannian manifolds and their applications to geometry*, *Indiana Univ. Math. J.* **25** (1976), no. 7, 659-670. MR **54** #5502.

13. *On the regularity of the solution of the  $n$ -dimensional Minkowski problem* (with Cheng, Shiu Yuen), *Comm. Pure Appl. Math.* **29** (1976), no. 5, 495-516. MR **54** #11247.

14. *Isoperimetric constants and the first eigenvalue of a compact Riemannian manifold*, *Ann. Sci. Ecole Norm. Sup. (4)* **8** (1975), no. 4, 487-507. MR **53** #1478.

15. *Curvature estimates for minimal hypersurfaces* (with Schoen, R. and Simon, Leon), *Acta Math.* **134** (1975), no. 3-4, 275-288. MR **54** #11243.

16. *Differential equations on Riemannian manifolds and their geometric applications* (with Cheng, S. Y.), *Comm. Pure Appl. Math.* **28** (1975), no. 3, 333-354. MR **52** #6608.

17. *Submanifolds with constant mean curvature. I, II*, *Amer. J. Math.* **96** (1974), 346-366; *ibid.* **97** (1975), 76-100. MR **51** #6670.

18. *On the curvature of compact Hermitian manifolds*, *Invent. Math.* **25** (1974), 213-239. MR **52** #3588.

19. *Intrinsic measures of compact complex manifolds*, Math. Ann. **212** (1975), 317-329. MR **51** #3503.

20. *Non-existence of continuous convex functions on certain Riemannian manifolds*, Math. Ann. **207** (1974), 269-270. MR **49** #3771.

21. *Scalar curvature, non-abelian group actions, and the degree of symmetry of exotic spheres* (with Lawson, H. Blaine, Jr.), Comment. Math. Helv. **49** (1974), 232-244. MR **50** #11300.

22. *Curvature preserving diffeomorphisms*, Ann. of Math. (2) **100** (1974), 121-130. MR **50** #8376.

23. *Remarks on conformal transformations*, J. Differential Geometry **8** (1973), 369-381. MR **49** #3770.

24. *Some global theorems on non-complete surfaces*, Comment. Math. Helv. **48** (1973), 177-187. MR **49** #6108.

25. *Sur les métriques riemanniennes à courbure de Ricci nulle sur le quotient d'une surface  $K^3$*  (with Bourguignon, Jean-Pierre), C. R. Acad. Sci. Paris Ser. A-B **277** (1973), A1175-A1177. MR **48** #12398.

26. *Compact manifolds of nonpositive curvature* (with Lawson, H. Blaine, Jr.), J. Differential Geometry **7** (1972), 211-228. MR **48** #12402.

27. *Compact flat Riemannian manifolds*, J. Differential Geometry **6** (1971/1972), 395-402. MR **46** #4439.

28. *On the fundamental group of manifolds of non-positive curvature*, Proc. Nat. Acad. Sci. U.S.A. **67** (1970), 509. MR **42** #8423

29. *On the fundamental group of compact manifolds of non-positive curvature*, Ann. of Math. (2) **93** (1971), 570-585. MR **44** #956.

## LETTERS TO THE EDITOR

---

### South Africa

Early in 1979 I received a preliminary inquiry from the Secretary of the South African Mathematical Society, asking me if I might be interested in receiving an invitation to be the SAMS Visiting Lecturer for 1981. It was explained that such a role would, in principle, involve me in visiting each of the sixteen institutions of higher education in South Africa.

I have received previous invitations to visit South Africa, but have declined them. I decided, on this occasion to test the goodwill and intentions of my would-be hosts and thus entered into what has proved an extremely thorough and searching correspondence, in which I have endeavored to establish conditions for my visit which would be compatible with my opposition to racism and my desire to help mathematicians and students anywhere in the world. I asked, in particular, for specific guarantees that I would be able to communicate freely and informally with colleagues and students of all races and colors. I made it plain that I did not wish to go to South Africa simply as a topologist, giving specialized lectures at the faculty or research level; my prime interest was to meet South Africans of every race and persuasion, and I would want to discuss with them matters of educational and social concern as well as mathematics.

Throughout these protracted negotiations I have received from the Secretary of SAMS and others of his South African colleagues not only very great courtesy, but total understanding of my position and aspirations and full cooperation in the planning of my itinerary in accordance with my wishes.

I have therefore now decided to accept the invitation. I hope that I may thereby be able to help, in some small way, the causes of freedom and education. I intend, on my return, to publish an article

giving my impressions of the points of view and attitudes of academics and students in South Africa to their present problems—a matter of great interest about which I, for one, am woefully ignorant.

Peter Hilton  
Case Western Reserve University

### Women in Mathematics

Readers of the Notices will surely be interested in the following items. The first is a statement passed by the executive committee of the Association for Women in Mathematics on January 8, 1981. The second is an editorial (*Sex and Mathematics*), written by two members of the AMS-MAA-NCTM-SIAM Committee on Women in Mathematics, which appeared in the 16 January 1981 issue of *Science*. The third item consists of two references to the literature on the subject of women in mathematics.

Bhama Srinivasan  
University of Illinois,  
Chicago Circle  
Mary W. Gray  
American University  
Alice T. Schafer  
Wellesley College  
Lida K. Barrett  
Northern Illinois University

### AWM Resolution

January 8, 1981

The Association for Women in Mathematics is outraged by the irresponsible coverage in the December 12 issue of *Science* of a study of dubious validity on sex differences in mathematical ability. We strongly support the views of the Joint Committee on Women in Mathematics as expressed in an editorial to appear in the January 16 issue of *Science*.