
1991 Steele Prizes Awarded in Orono

Three Leroy P. Steele Prizes were awarded at the Society's ninety-fourth Summer Meeting in Orono, Maine.

The Steele Prizes are made possible by a bequest to the Society by Mr. Steele, a graduate of Harvard College, Class of 1923, in memory of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein.

Three Steele Prizes are awarded each Summer: One for expository mathematical writing, one for a research paper of fundamental and lasting importance, and one in recognition of cumulative influence extending over a career, including the education of doctoral students. The current award is \$4,000 for each of these categories.

The recipients of the Steele Prizes for 1991 are JEAN-FRANÇOIS TREVES for the expository award; EUGENIO CALABI for research work of fundamental importance; and ARMAND BOREL for the career award.

The Steele Prizes are awarded by the Council of the Society, acting through a selection committee whose members at the time of these selections were Sylvain E. Cappell, Alexander J. Chorin, Charles L. Fefferman, William J. Haboush, Jun-ichi Igusa, Arthur M. Jaffe, Harry Kesten, Joseph J. Kohn, George Lusztig, and Mark Mahowald (chair).

The text that follows contains the Committee's citations for each award, the recipients' responses to the award, and a brief biographical sketch of each of the recipients. The biographical sketches were written by the recipients or were based on information provided by them. Professors Borel and Treves were unable to attend the Summer Meeting to receive the award in person. They did, however, send written responses to the award.

Expository Writing

Jean-François Treves

Citation

The 1991 Steele prize for an outstanding exposition of mathematics is awarded to Jean-François Treves for *Pseudodifferential and Fourier Integral Operators* (Volumes 1 and 2), Plenum Press, 1980. The theory of pseudodifferential operators arose from the study of the Hilbert transform in one variable and its generalization to singular integral operators in higher dimensions. Roughly speaking, the algebra

of pseudodifferential operators is an algebra generated by: Real powers of the Laplacian, singular integral operators, and linear partial differential operators; under the following operations: Composition, addition, and taking adjoints. The study of these operators has had many applications: Uniqueness of the Cauchy problem, regularity theorems in several complex variables and on CR manifolds, hypoellipticity, study of the oblique derivative problem, subelliptic operators, the index theorem, etc. Fourier integral operators arise naturally from the study of pseudodifferential operators, especially in connection with propagation of singularities, canonical changes of coordinates, and hyperbolic equations. The theory of Fourier integral operators serves as a fundamental tool in analysis on manifolds.



Jean-François Treves

Treves presents the theory with great elegance and lucidity, he gives many examples and a closely reasoned motivation, which enables him to develop the concepts in

full generality and at the same time make them accessible to the non-specialized reader. In fact, one of the great merits of this work is its accessibility, an individual, after mastering a standard first year course in analysis, should be able to follow these volumes. Perhaps the greatest virtue of these volumes is that they fully reflect the author's insight and expertise as a leading researcher in the field as well as an extremely prolific and successful expositor of analysis.

Response

The progress of PDE theory in the second half of the twentieth century has been propelled by a number of powerful new analytic tools. The first wave of new concepts appeared in the late 1940s and in the 1950s, emerging mainly from the success of the Sobolev spaces and, more generally, from Hilbert space theory, as well as from the brand of functional analysis created by S. Banach. Today we take for granted the availability and the multiple uses of weak solutions and generalized functions. Presenting or handling Fourier transform without distributions does not strike one, any more, as a viable option; it is more likely to appear as a provocative *tour-de-force*.

The second wave arrived around 1960 and was born of the conjunction of Fourier transform and singular integral operators. Its fairly easy acceptance was certainly due to the spectacular success of its first applications: By A. Calderón to the uniqueness in the Cauchy problem and soon after by M. Atiyah and I. Singer to the proof of the index theorem. Of course, both developments—generalized functions, pseudodifferential operators—had strong roots in earlier mathematics. Several of their concepts and results had been anticipated by mathematicians in the first half of the century. Heaviside calculus and Hadamard's finite parts were precursors of distribution theory. The invariance of singular integral operators under diffeomorphism was proved by G. Giraud in 1934. The language of distributions made possible the passage from kernels to symbols (although special types of symbols showed up already in the work of S. G. Mikhlin in the late 1930s). The use of symbols allowed the analyst to move from one differential operator to another one by a path through the territory of operators that are not differential, that are pseudodifferential—like deforming one polynomial into another one through smooth functions. It was a great gain, crucial to the proof of the index theorem—in which deformations were an especially attractive option, thanks to the homotopy invariance of the index of an elliptic operator.

Singular integral operators led, circa 1964, to pseudodifferential operators, of which, in their standard form, they constitute the zero-order "level". A limp article of J. J. Kohn and L. Nirenberg sold them to the "public" (an independent and simultaneous presentation by J. Bokobza and A. Unterberger was less accessible, based as it was on topological tensor products). Soon after, a completely local theory was proposed by L. Hörmander. In the same spirit, but using amplitudes rather than symbols, M. Kuranishi gave an extremely short and elegant proof of the invariance of pseudodifferential operators under a diffeomorphism, in-

variance which is essential to their transfer onto a smooth manifold. At that stage, the core of the theory had found its natural shape—and was ready for a presentation in an expository text. But still more progress ensued, e.g. the adaptation to boundary value problems by L. Boutet de Monvel and, especially, the emergence of Fourier integral operators in the work of V. P. Maslov and, in a more systematic, and at the same time more accessible, guise, in that of L. Hörmander. Fourier integral operators force one to reset the problems in the framework of the symplectic geometry of the cotangent bundle. Through them the link with the classical (old) quantum mechanics becomes apparent. Among their many successes, one ought to mention the asymptotic estimate of the eigenvalue distributions for elliptic equations, the generalized Poisson formula, the applications to hyperbolic (diffraction) theory, and their use in tomography. They have played a crucial role in an endeavor in which I was fortunate to participate, in joint work with L. Nirenberg: The description of the linear partial differential equations (with simple real characteristics) that admit local solutions.

Surely it would be inappropriate for me to argue with the Committee that decided the attribution of the Steele Prize for Expository Writing and to point out the many, glaring shortcomings of the book for which the Prize was given. I believe that the great honor bestowed upon me is really a tribute to the magnificent collective effort of which my book is but an imperfect reflection. It was my great luck to have been present at the creation of the tools of microlocal analysis and to have been the contemporary of brilliant minds.

Biographical Sketch

Jean-François Treves was born in Brussels, Belgium, on April 23, 1930, of Italian parents. Part of his childhood and adolescence was spent in Belgium and part in Italy. In 1947 the family emigrated to France. In 1948 J.-F. Treves passed the first Baccalaureat examination, in the Latin-Greek section; he was almost completely ignorant of mathematics and remembers having to learn then what a vector was. In 1949 he obtained the second Baccalaureat, in the Philosophy section, but by now he was studying heat transfer in an engineering school in Paris. There he was taught Laplace's method for solving the mixed problem for the heat equation in a metal bar of finite length, using the Gaussian function. Laplace's method made such a strong impression on him that he resolved to study partial differential equations (PDE), which he has been doing ever since. At the time, however, he was thinking of studying them as a physicist. He joined a group of nuclear physicists at the Atomic Energy Agency in France; they directed him towards the Fourier transform and the Boltzman equation. Relief from the Boltzman equation came when Treves, as a resident alien in France, was forbidden to enter the grounds of the atomic reactor at Saclay. Another turn of good luck occurred at the end of his "Licence" studies at the Sorbonne, when a mathematics tutor of his engineering days, met by chance, offered to

introduce him to Laurent Schwartz, who agreed to supervise Treves' thesis work. His doctoral dissertation was defended in June 1958. At that time, it was not possible for an Italian citizen to obtain a permanent position in a French university. On the recommendation of L. Schwartz, Treves was offered a job, to start in September, at the University of California, Berkeley—a place he could not locate on the map, where people spoke a language he could only read in texts that consisted mostly of mathematical formulas. Since then he has lived in the United States; he became a U.S. citizen in 1972.

After leaving Berkeley in 1961, J.-F. Treves taught at Yeshiva University and Purdue University. In 1970, he joined the mathematics department at Rutgers University, where he has been teaching ever since. He has been a member of the American Mathematical Society since 1959 and, in 1971, he received the Mathematical Association of America's Chauvenet Prize. He gave an Invited Address at the International Congress of Mathematicians in Nice in 1970. He is the author of several books on the subject of PDE and functional analysis and of many technical articles. He has lectured extensively in Europe, the Americas, and in Asia. When traveling in Latin America and in Asia, he collects butterflies.

Fundamental Paper

Eugenio Calabi

Citation

Calabi's work on global differential geometry, especially complex differential geometry, has profoundly changed the landscape of the field. His fundamental contributions heralded in important new directions in diverse areas. His construction of compact non-Kaehler manifolds with Eckmann, now known as the Calabi-Eckmann manifolds, and his construction of 6-dimensional almost complex manifolds shed light on the theory of Kaehler manifolds and the theory of integrability of almost complex structures and serve as important examples for concrete understanding of those theories. His computation with Vesentini of the cohomology groups with coefficients in the tangent bundle for compact locally Hermitian symmetric spaces is an important step in the theory of rigidity of compact complex manifolds. The way the curvature term was analyzed in the computation is instrumental in the later theory of strong rigidity for Kaehler manifolds with nonpositive curvature conditions. The first example of the important twistor program of Penrose in relativistic quantum field theory is due to Calabi who constructed it before the work of Penrose. Calabi constructed all harmonic maps from the standard 2-sphere to the standard n -sphere by projecting onto the n -sphere certain holomorphic maps from the Riemann sphere to a complex manifold over the n -sphere. This complex manifold constructed by Calabi is now known as the twistor space of the n -sphere.

Among the numerous important contributions of Calabi, the jewel on the crown is his work on the Monge-Ampere equation and the Kaehler-Einstein metric. This general area is now under the umbrella name of the Calabi conjecture. The problem is to construct Kaehler-Einstein metrics on certain class of compact Kaehler manifolds. The sign of the first Chern class is an important condition of the existence of a Kaehler-Einstein metric. For the case of zero first Chern class, Calabi formulated the problem in the form of the complex Monge-Ampere equation. He proved the uniqueness of the solution and set down the program of proving the existence by the method of continuity and pointed out the openness and the need of *a priori* estimates up to order three. By studying the geometry of improper affine hypersurfaces of convex type, he obtained the third-order *a priori* estimate for a real Monge-Ampere equation in terms of a second-order *a priori* estimate. His method for this third-order estimate is the foundation for the third-order estimate for the complex Monge-Ampere equation. Later, the second-order estimate was obtained by Aubin and Yau, yielding the existence of Kaehler-Einstein metrics for the case of the negative Chern class. Coupled with the work of Guggenheimer, this negative Chern class case of the Calabi conjecture gives the conclusion for the equality case of the Chern number inequality due in the algebraic geometric setting to Bogomolov and Miyaoka for surfaces of general type, leading to a characterization of compact quotients of the complex 2-ball (and in general the complex n -ball) as a compact complex manifold with positive canonical line bundle satisfying that Chern class inequality. This characterization removed the additional condition for the even dimensional case in the result of Hirzebruch-Kodaira on topological characterization of the complex structure of the projective space. It also made possible the construction of 2-ball quotients by branch covers over the complex projective plane by Hirzebruch and his school and the construction of non locally symmetric compact complex manifolds of negative curvature of Mostow's method of complex reflection groups. Finally the zeroth order estimate by Yau completed the solution of the Calabi conjecture for the zero first Chern class case. After that, the need, in string theory, of the construction of compact Ricci-flat Kaehler threefolds with certain topological structures gave rise to a great deal of activities in the explicit construction of such threefolds which are now called Calabi-Yau manifolds. The zero Chern class case of the Calabi conjecture provided the means to finally confirm the missing case of Kodaira's conjecture that a compact complex surface is Kaehler precisely when its first Betti number is even. It also made possible the proof of the smoothness of the local moduli of compact Ricci-flat Kaehler manifolds. Recently the positive Chern class case of the Calabi conjecture is also an active area of research with a number of significant results.

For a holomorphic vector bundle the analog of the Kaehler-Einstein metric is the Hermitian-Einstein metric, alias the Yang-Mills field, with the Chern class condition replaced by the notion of stability. There the work of

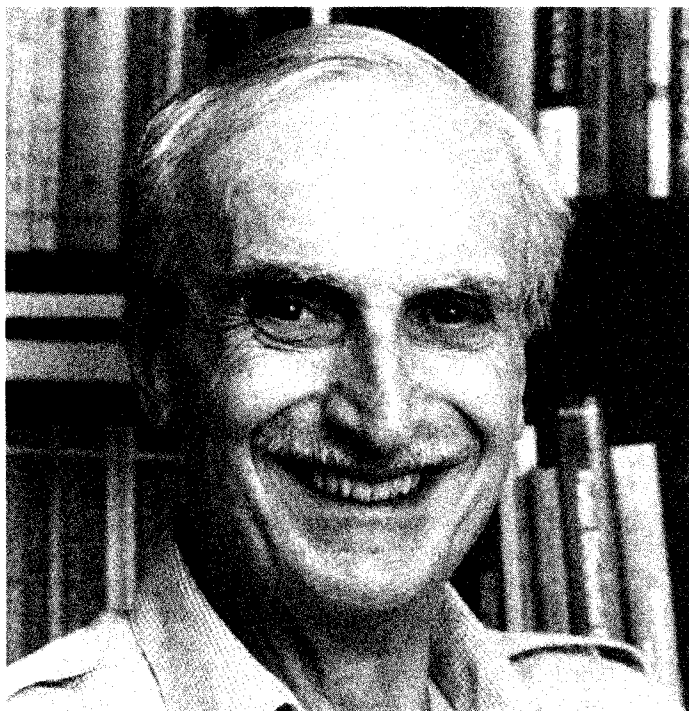
Donaldson, Uhlenbeck-Yau, and other people established an active and very fruitful area of research.

Other settings for the complex Monge-Ampere equations are pseudoconvex domains and quasi-projective manifolds, in which there are results by Bedford-Taylor, Chang-Yau, Lee-Melrose, Cafarelli-Kohn-Nirenberg-Spruch, and others.

Most recently, Demailly used the complex Monge-Ampere equation with some point measures on the right-hand side to obtain results on the Fujiki conjecture which gives the power needed to raise an ample bundle to get a very ample bundle after tensoring with the canonical line bundle. Such results so far have eluded the reach of methods of algebraic geometry.

Calabi also started the program of investigating Kaehler-Einstein metrics as extremal Kaehler metrics and introduced the method of constructing Kaehler-Einstein metrics for special complex manifolds by solving certain ordinary differential equations.

Over the years, there may have been improvements on the technical aspects of *a priori* estimates for the complex Monge-Ampere equation. For example, Evans introduced a simpler method to dispense with the third-order estimate for the solution of the Monge-Ampere equation. Yet the unusually penetrating geometric insight and the brilliant idea of Calabi in the geometric use of the complex Monge-Ampere equation will always remain one of the most important powerful fundamental tools in complex differential geometry and algebraic geometry.



Eugenio Calabi

Response

There is no instance in my professional life where I can recall a greater sense of surprise, and elation, than when I

first received notice of the great honor being bestowed on me tonight.

While it is usual and fitting, on such a happy occasion, to express one's thanks to teachers, colleagues, assistants, and students, acknowledging their contributions to the work cited in the award, I want to be especially emphatic in doing so now, for the following reason. Having been all my life a slow reader and an awkward writer ("negligent" and "lazy" were some of the more charitable epithets I sometimes got from teachers in school), I have always relied primarily on the spoken word for most of my education. In mathematics too, I think that most of what I know or understand has come through lectures, seminars, conferences, or from the patient attention of many friends when I approached them aggressively with my unsolved problems.

I want to recall, first of all, the early encouragement to pursue study in mathematics during my adolescence from teachers and family, and later from Guido Fubini, Dirk J. Struik, and Norman Levinson. During my graduate training, my natural attraction to geometric subjects was reinforced by the inspired influences of Harry Levy, Don C. Spencer, and my thesis advisor, Salomon Bochner. An especially important role was played, then and since then, in preparing me for the core areas of my subsequent work, by the legendary seminar of deRham and Kodaira of 1949-50 at the Institute for Advanced Study and by the "Nothing" seminars of Kodaira and Spencer in the 1950s. Among the people to whom I feel especially indebted for helping me broaden my views and techniques over the years, I want to recall Emil Artin, Louis Nirenberg, Arthur Erdélyi, Paul C. Rosenbloom, Arthur N. Milgram, René Thom, and Marcel Berger. And then there is S.-T. Yau: Without his contributions, I probably would not be speaking tonight.

For most of my adult life I have often wavered, as I suspect many colleagues do, between positive and negative feelings about the significance of our work, or, more generally, of mathematics as a whole within the framework of human cultural evolution. To explain the negative feelings, we need only to browse through mathematical periodicals of, say, the 1930s or 1940s to notice how many of the topics treated then, that seemed "hot" at the time, are now largely ignored or forgotten: Does this mean that much of what we do today may look, say by the year 2050, as "sonnets in the Etruscan language"? With tonight's activities, I feel far more inclined toward the polar view: it is precisely the constant, unpredictable shift of what are current interests and concerns that makes up the essence of progress; every article published contributes, however imperceptibly, to such a shift, leading to the delimitations of what will turn out to be the "hot" topics in the mid-twenty-first century.

I would like to recall here a cautionary essay of nearly twenty years ago that made a lasting impression on me: It is the 1972 Gibbs Lecture delivered by Freeman J. Dyson, at the annual meeting of the A.M.S., with the title "Missed Opportunities". The written article contains a wealth of information and inspiring ideas, but its main theme is to decry what the author perceived as a "divorce"

between mathematics and theoretical physics, contributing to a “shabby” state of the latter (in 1972!). To back his point, he advanced some examples of current advances in his own area of physics, that could have been made decades earlier, had the physicists only been more aware of developments in mathematics. It may be partly as a result of Dyson’s scolding lecture that we have seen in the years since then a conspicuous turnaround (a “reconciliation”?) between the two fields. Indeed, we have witnessed during the last score of years not only how theoretical physicists are “eating up” such traditionally “pure” subjects as differential topology, algebraic geometry (not to mention functional analysis and calculus of variations), in the development of gauge field theory, string theory, and their outgrowths; we have in fact also seen the reverse flow, whereby mathematicians, working in quantum groups, knot theory, low-dimensional topology, and noncommutative differential geometry, look at ideas from theoretical physics as a heuristic source of intuition. It is not a coincidence that the concept of functional integration, for instance, was first devised by physicists. I am still puzzled by the reason for an advice that Fubini gave me in 1940, “If you want to learn mathematics, you’d better study first a lot of physics!”.

In a lighter vein, it may well be that the time has come to think of a new essay, countering Dyson’s thoughts of 1972, with a title like “Lucky Breaks” or “Narrow Escapes”. We should back it with examples, which I’d like to collect, where the actual timing of the formulation or dissemination of some specific ideas has had a serendipitous effect, whereas if they had occurred earlier the outcome might have been worse. Here is one for a start. Suppose that in the years between the time of Kepler and that of Newton, some enterprising physicist, perhaps trying to estimate a speed of light or maybe whether light transmission was instantaneous, had devised an interference screen (not an inconceivable idea in the seventeenth century) and performed the Michelson-Morley experiment some 250 years ahead of its time. Would the outcome of the experiment not have been a perfect vindication of the ptolemaic view of the universe, confining Copernicus to oblivion for generations to come?

Yes, the development of all sciences is a long history of accidents, a few with negative effect, but the overwhelming majority being “lucky breaks”. In reviewing my own life as a mathematician, I see there too a corresponding preponderance of lucky breaks, more than I am able to recount, and of narrow escapes, from foolish or glaring mistakes caught just in time. For all of these, I am forever thankful to a benevolent Fate that has guided me over the years and to all those people who have so ably and effectively aided Him (or Her?) in that endeavor. Especially to my dear wife Giuliana, who in the process has put up with my swinging moods.

Biographical Sketch

Eugenio Calabi was born on May 11, 1923, in Milano, Italy to a middle-class Italian Jewish family. They left Italy in 1938 to escape racially discriminatory laws of the Fascist government and, in 1939, immigrated to the United States.

Calabi became a U.S. citizen in 1943. He received his B.S. in chemical engineering from Massachusetts Institute of Technology (1946), his M.A. in mathematics from the University of Illinois (1947), and his Ph.D. from Princeton University (1950).

Professor Calabi was a part-time instructor at Princeton University (1947–1951) and an assistant professor at Louisiana State University (1951–1955). In 1955, he joined the faculty of the University of Minnesota as an assistant professor and progressed to associate professor (1957) and to full professor (1960). Since 1964, he has been a professor at the University of Pennsylvania and served as chairman of the department of mathematics from 1971–1973.

In 1951–1952, Professor Calabi was a Research Fellow at Princeton University and, in 1962–1963, was a J.S. Guggenheim Fellow. He has had visiting positions at the California Institute of Technology (1954–1955), the Institut des Hautes Etudes Scientifiques (1968–1969), Stanford University (spring, 1977), University of Paris-VII (1983), and Princeton University (1988–1989). He has also been a member of the Institute for Advanced Study (1958–1959, fall 1979, and spring 1983). In 1982, he was elected a member of the National Academy of Sciences.

Career Award

Armand Borel

Citation

The career of Armand Borel has so far spanned four decades and, for the course of that span, his work has been utterly fundamental to the development and formation of modern mathematics: In 1950, he proved with J.-P. Serre the impossibility of fibering a euclidean space with compact fibres, not reduced to a point. This was one of the first striking applications of Leray’s spectral sequence. His subsequent work on the cohomology of classifying spaces and homogeneous spaces demonstrated the effectiveness of spectral sequences in topology. It was a basis for further developments in the theory of characteristic classes, pursued in part in joint work with Serre and with F. Hirzebruch. His recasting of the Smith theory in that framework led to further fundamental developments in transformation group and fixed point theory, based on the “Borel construction”, a first example of “equivariant cohomology”, and, jointly with J.C. Moore, to a new homology theory.

One of his most astonishing achievements at that time was laying the foundations of a theory of linear algebraic groups valid in all characteristics. Together with the work of C. Chevalley, this was the fundamental breakthrough which rendered the theory of algebraic groups susceptible to analysis by the methods of algebraic geometry. Together with J. Tits, he established later the basic properties of the group of rational points of semi-simple groups and determined the representations or homomorphisms of such groups, viewed as abstract groups.

Algebraic groups supplied a framework to the study of arithmetic groups. In 1960, jointly with Harish-Chandra, he generalized to arbitrary groups over number fields and refined the reduction theory known for classical groups. This and infinite dimensional representations supplied the basis of the modern understanding of the theory of automorphic forms. Together with W. Baily, he showed that a specific Satake compactification of the quotient of a bounded symmetric domain by an arithmetic group carries a natural structure of projective variety, a result which is basic in the study of Shimura varieties. Later, Serre and he introduced a compactification of locally symmetric spaces as a manifold with corners and used it to prove basic results on the cohomology of S -arithmetic groups. It allowed him to determine the stable cohomology of classical groups, the rational higher K -groups of rings of integers of number fields, further information on "Borel regulators" and to realize the Steinberg representation of a reductive p -adic group in the space of square integrable harmonic forms on a Bruhat-Tits building, which led to an important contribution to the theory of square integrable, non cuspidal, representations of p -adic groups. A monograph written jointly with N. Wallach presents a systematic exposition of known and new results on relative Lie-algebra cohomology, continuous cohomology, and cohomology of cocompact discrete subgroups. In 1983-1984, he organized seminars which led to notes on the Goresky-MacPherson intersection homology and on the Beilinson-Bernstein theory of algebraic D -modules. More recently, jointly with G. Prasad, following Margulis' breakthrough, he proved a generalization in an S -arithmetic framework of the Oppenheim conjecture.

In each decade, he has played a major role in several of the most important developments of the era and it is reasonable to assume that this will continue. Each of these contributions has been of major importance in shaping the mathematical life that came afterwards. His work provided the empirical base for a great swath of modern mathematics and his observations pointed out the structures and mechanisms that became central concerns of mathematical activity.

In the course of amassing these astounding achievements, he placed the facilities of the Institute for Advanced Study at the service of mathematics and mathematicians, using them to foster talent, share his ideas, and facilitate access to recent developments through seminars and lectures. It is just simply not possible to cite a career more accomplished or fruitful or one more meaningful to the contemporary mathematical community.

Response

To receive a "career award" brings home the fact that more lies in the past than in the future and makes one look back, notwithstanding the lingering hope that it is not quite all over yet. Doing so gives me first of all the impression of having been very fortunate. For instance, when I went to Paris in 1949, with some trepidation, and some hopes, no doubt, I could hardly anticipate the tremendous benefits I would gain

from Cartan's Seminar, Leray's lectures, my contacts with them and especially with Serre. Combined with a knowledge of Lie groups acquired earlier in Zürich, it provided the framework for my thesis and subsequent work in topology. Similarly, my *Annals* paper on linear algebraic groups could hardly have come about without my stay in Chicago (1954-1955) and the influence of A. Weil, which helped me to get familiar with the algebraic-geometric setting which I eventually used, influenced moreover by earlier results of E. Kolchin and an outlook on compact Lie groups I had learned from H. Hopf and E. Stiefel in Zürich. Then there has been (and is) the privilege of spending the major part of my career under the ideal set up of the Institute. In fact, I have had so many "opportunities" that I cannot help feeling I "missed" too many. But rather than pursuing this melancholy thought, I prefer, without worrying as to whether this honor is really deserved, to express my appreciation to the Steele Prize Committee for this testimony of scientific esteem.



Armand Borel

Biographical Sketch

Armand Borel was born May 21, 1923 in La Chaux-de-Fonds, Switzerland. He received his *diplôme* from the Federal Institute of Technology in Zürich in 1947 and his *doctorat d'état* from the University of Paris in 1952. He served as an assistant (1947-1949) and professor (1955-1957; 1983-1986) at the Federal Institute. He went to the Centre National de Recherche Scientifique in Paris in 1949, and then to the University of Geneva as a Supplying Professor for Algebra in 1950. From 1952 to 1954, he was a member at the Institute for Advanced Study in Princeton and, in 1957, he was appointed to his current

position of professor at the Institute. He also served as a visiting lecturer at the University of Chicago (1954-1955), and as a visiting professor at the Massachusetts Institute of Technology (1958, 1969), the Tata Institute of Fundamental Research in Bombay (1961, 1983, 1990), the University of Paris (1964), the University of California at Berkeley (1975), the University of Chicago (1976), Yale University (1978), and Tôhoku University in Sendai, Japan (1990).

In 1972, Professor Borel received an honorary doctorate from the University of Geneva and in 1978 was awarded the Brouwer Medal of the Dutch Mathematical Society. A member of a number of scientific societies, he was elected to the American Academy of Arts and Sciences in 1976 and to the National Academy of Sciences in 1987. He is also a foreign member of the Finish Academy of Sciences and Letters, the French Academy of Sciences, and the American Philosophical Society.

Professor Borel was an editor of *Annals of Mathematics* from 1962 to 1979. He has been an editor of *Inventiones mathematicae* since 1979 and of *Commentarii Mathematici Helvetici* since 1984. He presented a Plenary Address at

the International Congress of Mathematicians (ICM) in Stockholm (1962) and an Invited Address at the ICM in Vancouver (1974). In addition, he has given invited lectures at numerous conferences all over the world, including the Invited Address at the AMS Eastern Sectional Meeting in New York (February 1954) and the Colloquium Lectures at the AMS Summer Meeting in University Park (1971).

Professor Borel's professional activities include membership on the Consultative Committee of the ICM held in Moscow in 1966, and the chairmanship of the same committee for the ICM held in Helsinki in 1978. His AMS activities include serving as a Member-at-Large of the Council (1968-1970) and on the following committees: The Committee to Select Hour Speakers for Eastern Sectional Meetings (1962-1963); co-chair of the Invitations and Organizing Committees for the AMS Summer Institutes (1965, 1977); the Nominating Committees for the 1973 and 1976 (chair) elections; and as chair of the Committee on Progress in Mathematics, a newly-established AMS lecture series, for 1988-1990.

COMBINATORIAL GAMES

RICHARD K. GUY, EDITOR

PROCEEDINGS OF
SYMPOSIA IN
APPLIED MATHEMATICS
Volume 43



■ "The subject of combinatorics is only slowly acquiring respectability and combinatorial games will clearly take longer than the rest of combinatorics. Perhaps this partly stems from the puritanical view that anything amusing can't possibly involve any worthwhile mathematics."—from the Preface

■ Based on lectures presented at the AMS Short Course on Combinatorial Games, held at the Joint Mathematics Meetings in Columbus in August 1990, the ten papers in this volume will provide readers with insight into this exciting new field. Because the book requires very little background, it will likely find a wide audience that includes the amateur interested in playing games, the undergraduate looking for a new area of study, instructors seeking a refreshing area in which to give new courses at both the undergraduate and graduate levels, and graduate students looking for a variety of research topics.

■ In the opening paper, Guy contrasts combinatorial games, which have complete information and no chance moves, with those of classical game theory. Conway introduces a new theory of numbers, including infinitesimals and transfinite numbers, which has emerged as a special case of the theory of games. Guy describes impartial games, with the same options for both players, and the Sprague-Grundy theory. Conway discusses a variety of ways in which games can be played simultaneously. Berlekamp uses the theory of "hot" games to make remarkable progress in the analysis of Go Endgames. Pless demonstrates the close connection between several impartial games and error-correcting codes. Fraenkel explains the way in which complexity theory is very well illustrated by combinatorial games, which supply a plethora of examples of harder problems than most of those which have been considered in the past. Nowakowski outlines the theory of three particular games—Welter's Game, Sylver Coinage, and Dots-and-Boxes. A list of three dozen open problems and a bibliography of 400 items are appended.

■ 1980 *Mathematics Subject Classifications*: 90; 94
ISBN 0-8218-0166-X, LC 90-22771, ISSN 0160-7634
233 pages (hardcover), February 1991
Individual Member \$31, List Price \$52,
Institutional Member \$42
To order please specify PSAPM/43NA

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