Tribute to André Lichnerowicz (1915–1998)

Yvette Kosmann-Schwarzbach

Thirty years ago: *Annals of Physics*, **111**. Deformation quantization was born. This sequence of two articles [BBFLS 1978] had been preceded by several papers written by Daniel Sternheimer, who has just addressed you,[1] by Moshé Flato whose memory Daniel has just recalled and whose personality and work in mathematics and physics were so remarkable, and by André Lichnerowicz.

Who was Lichnerowicz? To his students, he was "Lichné". To his friends, he was "André". As a mathematician, mathematical physicist, reformer of the French educational system and its mathematical curriculum, and as a philosopher, he was known to the public as "Lichnerowicz", a man of vast culture, an affable person, a great scientist.

There are many ways to approach André Lichnerowicz. One would be to read all his more than three hundred sixty articles and books that have been reviewed in *Mathematical Reviews* (MathSciNet), another would be to read only his personal choices among them, those that were published by the Éditions Hermann in 1982 as a 633-page book, *Choix d'œuvres mathématiques* [L 1982]. You can read the summaries of his work up to 1986 that were published by Yvonne Choquet-Bruhat, Marcel Berger, and Charles-Michel Marle in the proceedings of a conference held in his honor on the occasion of his seventieth birthday [PQG 1988], or his portrait and an interview that appeared in a handsome, illustrated volume describing the careers of twenty-eight of the most important French scientists of the twentieth century, *Hommes de Science* [HdeS 1990]. Finally, and sadly, scientific obituaries appeared in the *Gazette des Mathématiciens* [Gazette 1999], and they were quickly translated for the *Notices of the AMS* [Notices 1999] because Lichnerowicz was as famous outside France as he was within it. Other obituaries appeared in the *Journal of Geometry and Physics*, of which he had been one of the founders, and in many other journals.

Lichnerowicz had an exceptional career. He was a student from 1933 to 1936 at the École Normale Supérieure in Paris where he studied under Élie Cartan, who had a lasting influence on his mathematics. He completed his thesis, written under the direction of Georges Darmois, in 1939, and was named professor of mechanics at the University of Strasbourg in 1941. Because of the war, the faculty of the University of Strasbourg had already moved to Clermont-Ferrand in order to avoid functioning under the German occupation. However in 1943, the Germans occupied Clermont-Ferrand as well, and there was a wave of arrests in which Lichnerowicz was taken, but he was fortunate enough, or daring enough, to escape. In those days, he did what he could to help those who were in mortal danger, in particular Jewish colleagues.[2] After the Liberation, the University of Strasbourg returned to Strasbourg. In 1949 he was named professor at the University of Paris, and then in 1952 he was

Yvette Kosmann-Schwarzbach is professor of mathematics, retired from Université de Lille and Centre de Mathématiques Laurent Schwartz, École Polytechnique, France. Her email address is `yks@math.polytechnique.fr`.

[1] *This tribute was delivered at the conference "Poisson 2008, Poisson Geometry in Mathematics and Physics" at the École Polytechnique Fédérale, Lausanne, in July 2008. It was preceded by tributes to Stanisław Zakrzewski, founder of the series of international conferences on Poisson geometry, who died in April 1998 (by Alan Weinstein), to Paulette Libermann on the first anniversary of her death (by Charles-Michel Marle), and to Moshé Flato, co-author and friend of Lichnerowicz, who died in November 1998 (by Daniel Sternheimer). This tribute was followed by the award of the "André Lichnerowicz Prize in Poisson Geometry" to two prominent young mathematicians. The October 2008 issue of the* Gazette des Mathématiciens, *in addition to carrying a French version of the present article, has an announcement of the prize; an announcement also appeared in the November 2008* Notices, *page 1285.*

[2] *In his autobiography [S 1997, p. 200], Laurent Schwartz wrote that one day Lichnerowicz contrived to be in a police station and, while an officer was inattentive, he managed to borrow a stamp and apply it to a false identity card which he, himself, did not need, but which could save the life of a colleague or a student. To survive the war years in France with some honor was itself a great achievement.*

elected to a chair at the Collège de France, the most prestigious position in French higher education.[3]

When Lichnerowicz was elected to the Académie des Sciences de Paris—he was only 48, exceptionally young for a member of the Académie in those days—his students, as was customary, collected money to offer him his Academician's sword. (The sword is the only part of the Academician's very elaborate uniform that reflects his or her personality and accomplishments.) But two years later, for his fiftieth birthday, they contributed nearly as much to offer him something more to his taste, …a pipe! Indeed we could not imagine him without his pipe at any time …except during his lectures when he would fill the blackboard with equations in his dense handwriting, equations almost always comprising many tensorial indices. It is a fact that he can be seen in every photograph…with his pipe.

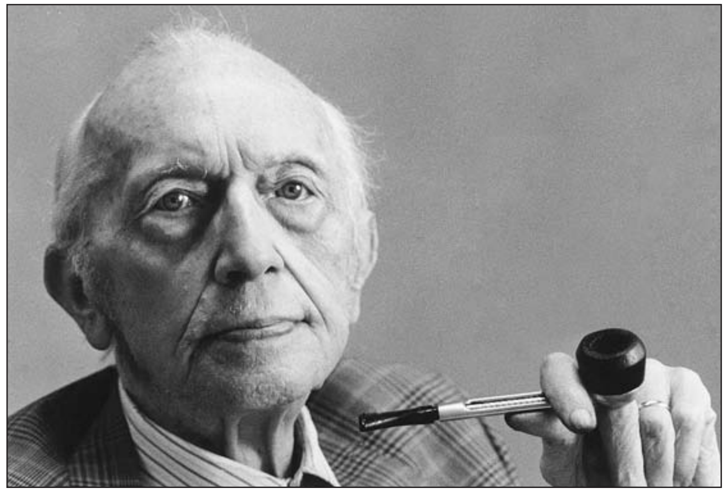
Lichnerowicz's work in general relativity began with his thesis, in which he gave the first global treatment of Einstein's theory of general relativity and determined necessary and sufficient conditions for a metric of hyperbolic signature on a differentiable manifold to be a global solution for the Einstein equations. He proved that there cannot exist any gravitational solitons; he established the "Lichnerowicz equation", an elliptic semi-linear equation used in the solution of the constraint equations to be satisfied by the initial conditions for Einstein's equations. He pursued this area of study throughout his research career. "Differential geometry and global analysis on manifolds", "the relations between mathematics and physics", "the mathematical treatment of Einstein's theory of gravitation", this is how he, himself, described his main interests and achievement in the interview published in *Hommes de Science* [HdeS 1990].

His work in Riemannian geometry remains particularly important. He was among the first geometers to establish a relation between the spectrum of the Laplacian and the curvature of the metric; he proved the now classical equivalence of the various definitions of Kähler manifolds; he showed, together with Armand Borel, that the restricted holonomy group of a Riemannian manifold is compact, and of course many other important results.

In the early 1960s Lichnerowicz established Cartan's and Weyl's theory of spinors in a rigorous differential geometric framework, on a pseudo-Riemannian manifold with a hyperbolic (Lorentzian) metric. Using this geometric approach in his courses at the Collège de France in 1962–1964, he developed Dirac's theory of the electron and that of Rarita-Schwinger for spin $\frac{3}{2}$, and then the Petiau-Duffin-Kemmer theory as well as the theory of the CPT transformations, while also in 1963 he published the landmark *Comptes rendus* note on harmonic spinors [L 1963], in which he proved that, for any spinor field, ψ,

$$\Delta\psi = -\nabla^{\rho}\nabla_{\rho}\psi + \tfrac{1}{4}R\psi\ ,$$

where $\Delta = P^2$ is the Laplacian on spinors, the square of the Dirac operator P, ∇ is the covariant derivative, and R is the scalar curvature. And he continued working on spinors to his last days.



André Lichnerowicz.

Photograph courtesy of the Collège de France Archives.

At the beginning of the 1970s Lichnerowicz's interest turned to the geometric theory of dynamical systems. Symplectic geometry had been studied for some time.[4] In January 1973 a conference, "Geometria simplettica e fisica matematica", was held in Rome that was, I believe, the first international meeting on this topic. As a young researcher I attended the congress, and heard and met many of the founders of symplectic geometry among whom were Jean Leray, Irving Segal, Bertram Kostant, Shlomo Sternberg, Włodzimierz Tulczyjew, Jean-Marie Souriau as well as the young Alan Weinstein and Jerry Marsden. And Lichnerowicz was one of the organizers of the meeting and delivered the opening lecture.

The main reason that we pay tribute to Lichnerowicz's memory here, today, at this conference on Poisson geometry, is that he founded it. This was a few years before the publication of the deformation quantification paper I recalled at the beginning of this tribute.

His son, Jérôme Lichnerowicz, speaking of his father's collaboration with Moshé Flato, said: "There was no master and no student but an incredible synergy between friends. I saw Moshé encourage André when, ageing, he doubted his own strength," and he added: "I heard Moshé tell me: 'It is unbelievable, he [Lichnerowicz] had an arid period, but now he is back doing mathematics as

[3] *A chair in mathematics was established by François I at the founding of the Collège in 1531!*

[4] *As Hermann Weyl explained at the beginning of Chapter VI of his book* The Classical Groups *[W 1939], he had coined the adjective "symplectic" after the Greek as an alternative to the adjective "complex".*

before!'." [CMF 2000] Starting in 1974, working with Moshé Flato and Daniel Sternheimer, Lichnerowicz formulated the definition of a Poisson manifold in terms of a bivector, i.e., the contravariant 2-tensor advocated by Lie, Carathéodory, and Tulczyjew, which is the counterpart of the 2-form of symplectic geometry. In his article published in *Topics in Differential Geometry* [L 1976] he defined the canonical manifolds, and one can already find in that paper a formula for the bracket of 1-forms associated to a Poisson bracket of functions, although still only for exact forms,

$$[df, dg] = d\{f, g\}.$$

(Later he showed that the canonical manifolds are those Poisson manifolds whose symplectic foliation is everywhere of co-dimension one.) In his 1977 article in the *Journal of Differential Geometry*, "Les variétés de Poisson et leurs algèbres de Lie associées" [L 1977], Lichnerowicz introduced the cohomology operator that is now called the "Poisson cohomology operator" but really should be called the "Lichnerowicz-Poisson cohomology operator", a profound discovery. We read there as well as in [BFFLS, 1978] that, in the particular case of a symplectic manifold,

$$\mu([G, A]) = d\mu(A),$$

where A is a field of multivectors. (The notations are G for the Poisson bivector and μ for the prolongation to multivectors of the isomorphism from the tangent bundle to the cotangent bundle defined by the symplectic form, the bracket is the Schouten-Nijenhuis bracket, and d is the de Rham differential.) This formula, which we rewrite in a more familiar notation,

$$\omega^\flat([\pi, A]) = d(\omega^\flat(A)) \quad \text{or} \quad \pi^\sharp(d\alpha) = d_\pi(\pi^\sharp\alpha)$$

(here G is replaced by π, and μ by $\omega^\flat$, with inverse $\pi^\sharp$, while α is a differential form and d_π is the Lichnerowicz-Poisson differential, $d_\pi = [\pi, \cdot]$, acting on multivectors) is the precursor of the chain map property of the Poisson map, mapping the de Rham complex to the Lichnerowicz-Poisson complex, and, more generally, this article is the point of departure for the great development of Poisson geometry that we have witnessed and in which we are participating here. Together with his earlier articles written jointly with Flato and Sternheimer [FLS 1974, 1975, 1976] and with the article in the *Annals of Physics* [BBFLS 1978], solving quantization problems by a deformation of the commutative multiplication of the classical observables when given a Poisson structure, this article established the foundation of what has become a vast field of mathematical research.

It was a privilege for Lichnerowicz's many doctoral students, of whom I was one in the late 1960s, to be received by him in his small office under the roof of the Collège de France, or in the study of his apartment on the Avenue Paul Appell, on the southern edge of Paris. Surrounded by collections of journals and piles of papers, Lichné, with his pipe, would offer encouragement and invaluable hints as to how to make progress on a difficult research problem. I knew then, we all knew, that we were talking to a great mathematician. But I did not even guess that I was talking to the creator of a theory which would develop into a field in its own right, one with ramifications in a very large number of areas of mathematics and physics.

References

[BBFLS 1978] François Bayen, Moshé Flato, Chris Fronsdal, André Lichnerowicz, Daniel Sternheimer, Deformation theory and quantization, *Annals of Physics* **111** (1978); I. Deformations of symplectic structures, 61–110; II. Physical applications, 111–151.

[L 1982] André Lichnerowicz, *Choix d'œuvres mathématiques*, Hermann, Paris, 1982.

[PQG 1988] *Physique quantique et géométrie*, Colloque Géométrie et Physique de 1986 en l'honneur de André Lichnerowicz (Daniel Bernard and Yvonne Choquet-Bruhat, eds.), Hermann, Paris, 1988.

- Yvonne Choquet-Bruhat, L'œuvre de André Lichnerowicz en relativité générale, pp. 1–10.
- Marcel Berger, L'œuvre d'André Lichnerowicz en géométrie riemannienne, pp. 11–24.
- Charles-Michel Marle, L'œuvre d'André Lichnerowicz en géométrie symplectique, pp. 25–42.

[HdeS 1990] *Hommes de Science*, 28 portraits, Entretiens et photographie de Marian Schmidt, Hermann, Paris, 1990.

[Gazette 1999] *Gazette des Mathématiciens* **81** (1999), 94–95, and **82** (1999), 90–108.

[Notices 1999] Marcel Berger, Jean-Pierre Bourguignon, Yvonne Choquet-Bruhat, Charles-Michel Marle, and André Revuz, André Lichnerowicz (1915–1998), *Notices Amer. Math. Soc.* **46**(11) (1999), 1387–1396.

[S 1997] Laurent Schwartz, *Un Mathématicien aux Prises avec le Siècle*, Odile Jacob, Paris, 1997.

[L 1963] André Lichnerowicz, Spineurs harmoniques, *C. R. Acad. Sci. Paris* **257** (1963), 7–9.

[W 1939] Hermann Weyl, *The Classical Groups*, Princeton University Press, Princeton, NJ, 1939; second edition, 1946.

[CMF 2000] Jérôme Lichnerowicz, in *Conférence Moshé Flato 1999*, vol. I (Giuseppe Dito and Daniel Sternheimer, eds.), Kluwer, Dordrecht, 2000, pp. 7–8.

[L 1976] André Lichnerowicz, Variétés symplectiques, variétés canoniques et systèmes dynamiques, in *Topics in Differential Geometry, in memory of E. T. Davies*, Academic Press, New York, 1976, pp. 57–85.

[L 1977] ______ , Les variétés de Poisson et leurs algèbres de Lie associées, *J. Differential Geometry* **12** (1977), 253–300.

[FLS 1974] Moshé Flato, André Lichnerowicz, Daniel Sternheimer, Déformations 1-différentiables d'algèbres de Lie attachées à une variété symplectique ou de contact, *C. R. Acad. Sci. Paris Sér. A* **279** (1974), 877–881.

Presidential Views: Interview with James Glimm

Every other year, when a new AMS president takes office, the *Notices* publishes interviews with the outgoing and incoming presidents. What follows is an edited version of an interview with James G. Glimm, whose two-year term as president ends on January 31, 2009. The interview was conducted in fall 2008 by *Notices* senior writer and deputy editor Allyn Jackson. Glimm is a Distinguished Professor of Applied Mathematics and Statistics at Stony Brook University.

An interview with president-elect George Andrews is scheduled to appear in the March 2009 issue of the *Notices*.

Notices: *As AMS president, you have attended many meetings and talked to many mathematicians. What kinds of issues or challenges did you hear about in these conversations?*

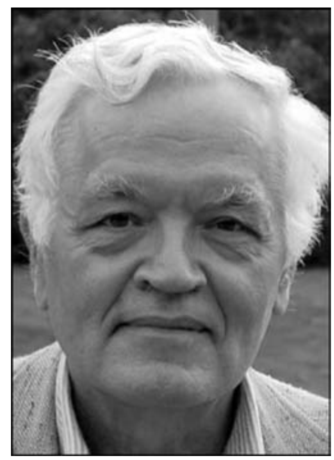
Glimm: Of course, the math community is quite diverse, and the challenges are reflected by the diversity of the community. There are concerns about getting bright students into majors, because many of them are placing out of calculus courses, which is a traditional way of recruiting students. They take calculus in high school instead. There is a set of pedagogical issues of that type. In terms of research, I think there is the usual concern about the poor level of funding in mathematics.

I actually would turn the question around a little bit and argue that there is a lot of vitality in the math community. I think there are many positive things going on.

Notices: *Can you give some examples?*

Glimm: Intellectually, the triumph with the Poincaré conjecture is certainly something that we are all delighted about. The continued progress in the connections between geometry and string theory always seems to be very surprising. The continued blossoming of new applications requiring new types of mathematics is always very heartening. Sometimes the math community thinks they've missed the boat. People say, "That already happened twenty years ago and now it's too late." But there are always new opportunities and new chances.

Notices: *One of these new areas of opportunity was the subject of a special session you organized at the 2008 Joint Mathematics Meetings called "Mathematics of Knowledge and Information", and you will organize another special session on this topic at the January 2009 meetings. Can you describe what this field is about and why it is important?*



James Glimm

Glimm: Certainly for several centuries, mathematics, in its applications, has been dominated by following in the footsteps of physics: taking equations from the physical sciences, figuring out what could be said about the solution spaces, finding numerical methods for evaluating solutions, making estimates and approximations, taking asymptotic limits. There are all sorts of ways that a mathematician can extract information related to a set of equations. Going forward, that process is maturing, and some of the more innovative developments are in the past. If you look into the future, there will be a big emphasis on doing the same thing, where the input is not the laws of physics but observed data. Just as there was an explosion of computer knowledge that allowed the solution of equations in physics, there is an explosion of sensor data that allows meaningful analysis of experimental facts independently of any laws of physics. Sometimes there isn't even the prospect of an intermediate law of physics, for example, if you are doing pattern recognition of photographs in a newspaper or something like that. Sometimes people run conservation laws to sharpen up boundaries in images, but basically the problem is not directly connected to physical equations in any obvious way. The modeling is pretty much independent of the equations.

Notices: *So this area offers new ways for mathematicians to contribute?*

Glimm: Absolutely. If you think that we have had several centuries squeezing juice out of the equations of physics, we might have some number of centuries with data. I think it's a big issue. It's a century-long issue.

Notices: *Are scientists clamoring for mathematicians' help with this?*

Glimm: Yes, in part, and partly they just do it without us. But that is no reason for us to stand back and not join in. If we do stand back, we will find that other people will step forward, and we will once again find our subject diminished by having lost a very vital component. It was for this reason that I thought this was a strategic choice to emphasize this particular application. It has nothing to do with my own research, so I have no personal interest in it. I just believe that it has a huge role in the future of mathematics.

Notices: *Can you give examples of work in this area?*

Glimm: The topics in the special session were very diverse. One talk was about trying to understand the traffic patterns in the Internet. Financial mathematics wasn't included in the session but might well have been, and that is another example. People use this method for analyzing genome data, medical data, and protein expression data. The biomedical world is part of this explosion of data. For instance, even in structural biology, when you are trying to predict a protein, there are two basic approaches. One is based on solving the classical equations of physics for the interaction of atoms that lead to the formation of molecules. But you can just as well do it by pattern recognition, where you look at different bonds between the proteins and how they attach. They attach first of all on a linear chain, and then they have to fold up in certain characteristic patterns. There is a wealth of data about the geometry of proteins whose structure has been solved. It's like a library of solutions. You look through this library and pick a solution that is very similar to your current problem or little piece of a current problem, and you figure out that a certain bond is likely to fold in a certain way. That's basically pattern recognition. There are many cases in biomedical research where there are laws of physics that apply, but it is more efficient and effective to solve problems on the level of pattern recognition. The same happens in weather forecasting. Global weather forecasts are achieved by solving differential equations numerically. For local weather forecasting, usually they see what the global forecast says is coming in and then look up the local history and find an event that most closely matches the regional part of the global forecast. They look up weather for those days and get a range of what could happen as the forecast. So it's basically pattern recognition.

Notices: *So this really goes across all areas of science.*

Glimm: Absolutely, all of science, but also areas that were not previously considered to be science. For instance, there are ways of recognizing handwriting, or of automatically translating between languages, that are based on computer models that solve Markov processes. There is something called a hidden Markov model that drives the statistics of language recognition and of speech recognition.

One of the themes that I have been pushing, especially for AMS meetings, is what I call the "big tent" philosophy of mathematics. This has several aspects. One is that lively areas that happen to be somewhat at the fringe of whatever people think the center is, should not be competing against things in the center. They both have their own validity. Neither should compete against the other, but each one should compete against some absolute standard of what we want in our meetings. In the big tent philosophy, the way the tent gets big is by emphasizing the edges. It has to be done by enlarging the space, so that includes enlarging the meetings. I have been continually urging the AMS to enlarge its meetings. Now, that can't be done instantly because there are long-term contracts and meetings are planned out years in advance. It's a little bit like trying to steer a battleship. But you can always change the direction, achieve incremental progress, and move towards something that will reach these goals over some period of years. In the Washington meeting I think the "big tent" spirit has been reflected in the program.

Notices: *You also have been interested in undergraduate education during your time as president. Can you tell me about this?*

Glimm: We have a particular project that I think is probably going to be promising, to look into computer grading of homework. Now, the practices for grading of homework are all over the map. Sometimes homework is graded assiduously, sometimes it is not graded at all, sometimes it is graded by an undergraduate who didn't take much care, sometimes it is graded but returned two weeks later. On the average, there is a lot of room for improvement.

Computer-graded homework captures the students' attention. Apparently the students get excited about homework and spend more time on it because they get instant feedback. It's somewhat like a computer game. So they just sit there and do it until they have mastered it, and of course that's the way to learn. The problems are generated with random input, which discourages copying. The experience is that, with computer-graded homework, we probably have the same number of Ds and Fs that we always had, but some of the Cs have moved up to Bs and some of the Bs have moved up to As. The overall effect is roughly a grade point average in the scoring. That's what the rumor circuit says, and we are proposing to have a more careful evaluation and to discover the "lessons learned". We will find out what people have to say when they have used computer grading—whether they kept it, whether they didn't keep it, and why. Also, we will find out practical things, like what is needed for infrastructure, for computer resources or human resources, to run it. We will make this information available to the math community. If you are a freshman taking precalculus or calculus, what is the chance you would be in such a course? I would guess it would be in the single digits of percentages. But I think it is ready to jump, to 20 percent, then 40 percent, maybe 60 percent. At that point it would be freestanding, and people would do what they judge worthwhile with it.

Notices: *What would be the AMS role in this?*

Glimm: The AMS has a very important role to play in issues relating to education of undergraduates. Obviously, MAA will be in the lead for many aspects of these issues, but the AMS retains an important role. The faculty at research universities are responsible for the education of a very significant number of freshman and sophomores. Many of these faculty look primarily to the AMS for leadership. Generally, the faculty who teach effectively enjoy their teaching greatly, and so the rewards of good teaching are rather obvious. We want to promote ways to help bring this about, and, particularly for faculty at research universities, we want to do this without compromising the quality of their research. The computer grading of homework seems to be the type of low-hanging fruit that is ideally suited to this situation.

We have applied for a grant from the National Science Foundation—to the Division of Undergraduate Education, so the grant would not compete with research grants in the Division of Mathematical Sciences. We proposed that the AMS do the kind of study I just indicated. So that is one role. I have been pushing this through my office as president in any forum where I have a chance to make such statements. I think I have raised consciousness about these issues. Some of the products belong to MAA [Mathematical Association of America], and some belong to commercial publishers. I don't think there is any need for a new product, and there is no reason for the AMS to have its own product competing with others. I think if we just encourage our members to use one of these products, they can take their pick. So the real role is to facilitate a climate whereby people will evaluate computer homework-grading systems and presumably come to use them. If the systems work as promised, then there will be an uptick in the quality of education and the satisfaction of students.

Notices: *Many people are using clickers in mathematics classrooms. We will have an article on this topic in the Notices* [the article appears in this issue].

Glimm: There is a certain amount of ferment about these different technology tools. I'm not trying to advocate any particular one of them. I just think that we can make some progress by adopting some of these tools. While the tools are in an evaluation stage, different people will decide to do different things. Eventually the community will more or less make a collective decision and cluster around some set of technologies that are beneficial.

Notices: *Is there anything else you wanted to mention?*

Glimm: One thing I would like to note, with pleasure, is the fact that there are private donors for mathematics. I don't recall this at other points in my professional lifetime. We have a new privately funded institute at Stony Brook [the Simons Center for Geometry and Physics; see *Notices*, June/July 2008, page 685–686]. There is the American Institute of Mathematics in Palo Alto, which is also privately funded. There have been private donations to the Mathematical Sciences Research Institute in Berkeley. Another example is the Clay Mathematics Institute. We have been more successful with private donors than at any time I can recall in my professional lifetime.

Notices: *Why do you think that is?*

Glimm: Well, I don't have an answer to that. Maybe some mathematicians have gone into the private sector and become wealthy. I don't have a complete list of these things, but it seems to me to be a huge explosion compared to what used to be the case.

Notices: *What did you enjoy most about being president?*

Glimm: I've watched the math community over many years wring its hands over the "spilled milk" decisions of the past. There is no way to recover the past, but these decisions are being made all the time. Boundaries between subjects look very obvious when you are at the center of the subject looking out into the distance, where there is some boundary seemingly off at infinity. But when you go out to the boundary, these areas are very lively. We can take actions to promote mathematics and not repeat actions that will make people complain twenty years from now about decisions made at the present time.

The Society seems to me to be a particularly well-run organization. I don't want to say it runs on auto-pilot, but there is a structure for how things are done, and the structure seems to suit people and leads naturally to decisions that are constructive. I had a few more committee meetings than I would normally have volunteered for, but that goes with the game. You can't do this kind of thing without being immersed in the committees.