James Eells
1926–2007

Domingo Toledo

Photo courtesy of the Archives of the Mathematisches Forschungsinstitut Oberwolfach, photographer D. Ferus.

James Eells, around 1980.

James Eells was born on October 25, 1926, in Cleveland, Ohio, and died on February 14, 2007, in Cambridge, England. He married Nan Munsell in June 1950. Their children Mary, Betsey, and John are musicians; Emily is a professor of English.

Jim graduated from Bowdoin College in 1947. He was instructor in mathematics at Robert College in Istanbul (1947–48), at Amherst College (1948–50) and at Tufts University (1953–54). He received his Ph.D. from Harvard in 1954, where he wrote a thesis in geometric integration theory under the direction of Hassler Whitney. During the next two years he was Whitney's assistant at the Institute for Advanced Study. He went on to positions at the University of California at Berkeley (1956–58) and Columbia University (1958–63). Then he accepted a professorship at Cornell University, with intermediate stays in Cambridge as Fellow of Churchill College. In 1969 he became Professor of Analysis at the University of Warwick, a position that he held until his retirement in 1992, at which point he and Nan moved back to Cambridge. In 1986 he was the initiator and the first director of the Mathematics Division at the International Centre for Theoretical Physics in Trieste, a position that he continued until his retirement. More details on his career and personality can be found in [13].

From the beginning of his career Jim worked in the interface between topology and analysis. His focus soon became global analysis, in particular the interaction between the calculus of variations and the topology of function spaces, where he played a pioneering role and wrote a foundational survey [2]. Global analysis could be described as the philosophy that topology and analysis should strengthen each other and the mixture of methods from both disciplines should go much farther than either discipline could go by itself. Important among Jim's sources of inspiration was Marston Morse's work on the calculus of variations in the large. For example, in Chapter VII of [16] Morse used geodesics in a particular metric on a sphere to compute the Betti numbers of the loop space of the sphere (long before topological methods were available for this computation), then used this topological information to conclude the existence of infinitely many geodesics in arbitrary metrics on the sphere.

Jim's most famous work is undoubtedly his joint paper with J. H. Sampson "Harmonic Mappings of Riemannian Manifolds" [11], published in 1964, which founded the theory of harmonic maps and provided the seeds for many further developments. It proves that every continuous map between compact Riemannian manifolds, with target of non-positive curvature, can be deformed to a harmonic map. This existence theorem has

Domingo Toledo is professor of mathematics at the University of Utah. His email address is `toledo@math.utah.edu`.

become a standard tool of geometers. It has been extended in many directions. It has found, and continues to find, a great number of applications to geometric problems. Particularly fruitful have been its applications to rigidity questions and to arithmeticity of lattices in Lie groups, see [1] for an excellent survey of these developments, up to the thirtieth anniversary of the paper. The technique of nonlinear heat flow introduced by Eells and Sampson has found other applications. Most notable is its clear influence on R. S. Hamilton's work on the Ricci flow, starting in [15], and thus on the subsequent work by G. Perelman on the Poincaré and Geometrization Conjectures [17, 18, 19]. These works represent one of the most spectacular applications to date of the philosophy of global analysis.

In spite of the great success of his paper with Sampson, Jim's main interest always was in harmonic maps to other targets, in particular to positively curved ones, where no general existence theorem is available, in fact, where often there are no harmonic maps at all. I think he would have said something like this: "Now that we understand negatively curved targets, let's tackle the *real* problem." The basic problem for him was to understand which homotopy classes contain harmonic representatives. Two results that help put this problem in perspective and that he particularly liked were his theorem with J. C. Wood [12] that there is no harmonic degree one map from a 2-torus to a 2-sphere, no matter which metrics are chosen in the domain and target, and his paper with M. J. Ferreira [4] showing that for domains of dimension at least 3 and for any homotopy class, a conformal change of metric in the domain produces a harmonic map.

In this context, one of Jim's passions was the study of harmonic mappings of spheres and ellipsoids. His monograph with A. Ratto [10] gave many new methods of construction of harmonic maps between spheres. One specific hope of these endeavors was to derive, by analytic means, some topological conclusions about the spaces of maps between spheres, in the spirit of Morse's work on geodesics. Jim also wanted to understand the formation of singularities in the heat flow, to have a clear picture of the obstructions to existence of harmonic maps. His two reports on harmonic maps, written with L. Lemaire [7, 8], collected in [9], have been very influential in giving a broad and clear picture of the subject. His last work on harmonic maps was his monograph, written with B. Fuglede [5] and published in 2001, on harmonic theory with singular domains and targets. His papers on harmonic maps, not including the reports and monographs, are collected in the volume *Harmonic Maps* [3], which I highly recommend to the reader who would like to get a fuller picture of the topics that I can only briefly describe here.

Jim had many other mathematical interests, and a great number of collaborators. Besides the ones already mentioned, he had long term collaborations with N. H. Kuiper in topology, with C. J. Earle in Teichmüller theory, and with K. D. Elworthy in infinite dimensional topology and in stochastic dif-

Photo courtesy of the Archives of the Mathematisches Forschungsinstitut Oberwolfach.

James Eells (left) and Nicolas Kuiper, around 1979.

ferential geometry. These works have formed the basis of further developments, sometimes many years later. For example, the "Eells–Kuiper invariant" of smooth manifolds [6], introduced in 1962, has been used in work starting about forty years later to identify Milnor spheres that carry metrics of non-negative curvature [14]. Jim had many other shorter term collaborations and much other work. His publications span a period of approximately fifty years.

Jim's influence on mathematics extended well beyond his published work. The Mathematics Genealogy Project lists 38 students and (as of this writing) 147 descendants. I am very fortunate to have been one of his students. I was always amazed at the breadth of his knowledge and of his interests, and how this was reflected in the great variety of research topics pursued by his students. Above all, I was always impressed by the vibrancy of his personality and his contagious enthusiasm for mathematics, which he clearly communicated to his students and to all who came in contact with him. He was a frequent organizer of symposia at Warwick and at the ICTP in Trieste, the latter being focused on developing countries. At the common room in Warwick he would direct the teatime conversation to the latest mathematical developments, he would insist that all present communicate with each other on their work, and he would masterfully facilitate that communication. He was a mover and organizer who helped many mathematicians in their careers. The warm and generous hospitality extended by Jim and Nan to the mathematical communities and visitors at Warwick and Trieste made their home an important meeting place for these communities, and a source of inspiration for many of us.

References

[1] K. Corlette, Harmonic maps, rigidity and Hodge theory, *Proceedings of the International Congress of Mathematicians*, Zürich, 1994, Vol I, 465–471, Birkhäuser, Basel, 1995.

[2] J. Eells, A setting for global analysis, *Bull. Amer. Math. Soc.* **72** (1966), 751–807.

[3] _____ , Harmonic Maps, *Selected Papers of James Eells and Collaborators*, World Scientific Publishing Co., Inc., River Edge, NJ, 1992.

[4] J. Eells and M. J. Ferreira, On representing homotopy classes by harmonic maps, *Bull. London Math. Soc.* **23** (1991), 160–162.

[5] J. Eells and B. Fuglede, *Harmonic Maps Between Riemannian Polyhedra*, with a preface by M. Gromov, Cambridge Tracts in Mathematics, **142**, Cambridge University Press, Cambridge, 2001.

[6] J. Eells and N. H. Kuiper, An invariant for certain smooth manifolds, *Ann. Mat. Pura Appl.* (4) **60** (1962), 93–110.

[7] J. Eells and L. Lemaire, A report on harmonic maps, *Bull. London Math. Soc.* **10** (1978), 1–68.

[8] _____ , Another report on harmonic maps, *Bull. London Math. Soc.* **20** (1988), 385–524.

[9] _____ , *Two Reports on Harmonic Maps*, World Scientific Publishing Co., Inc., River Edge, NJ, 1995.

[10] J. Eells and A. Ratto, *Harmonic Maps and Minimal Immersions with Symmetries*, Methods of Ordinary Differential Equations Applied to Elliptic Variational Problems, Annals of Mathematics Studies **130**, Princeton University Press, Princeton, NJ, 1993.

[11] J. Eells and J. H. Sampson, Harmonic mappings of Riemannian manifolds, *Amer. J. Math.* **86** (1964), 109–160.

[12] J. Eells and J. C. Wood, Restrictions on harmonic maps of surfaces, *Topology* **15** (1976), 263–266.

[13] D. Elworthy, James Eells, innovative mathematician, *The Independent*, 17 April 2007, available at `http://www.independent.co.uk/news/obituaries/james-eells-445040.html`.

[14] K. Grove and W. Ziller, Curvature and symmetry of Milnor spheres, *Ann. of Math.* (2) **152** (2000), 331–367.

[15] R. S. Hamilton, Three-manifolds with positive Ricci curvature, *Jour. Differential Geom.* **17** (1982), 255–306.

[16] M. Morse, *The Calculus of Variations in the Large*, Amer. Math. Soc. Colloq. Publ., vol. 18, Amer. Math. Soc., New York, 1934.

[17] G. Perelman, The entropy formula for the Ricci flow and its geometric applications, `arXiv:math/0211159`.

[18] _____ , Ricci flow with surgery on three-manifolds, `arXiv:math/0303109`.

[19] _____ , Finite extinction time for the solutions to the Ricci flow on certain three-manifolds, `arXiv:math/0307245`.