Book Review

The Apprenticeship of a Mathematician— Autobiography of André Weil

Reviewed by V. S. Varadarajan

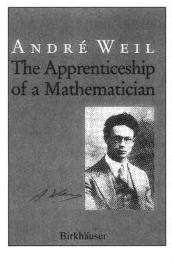
The Apprenticeship of a Mathematician— Autobiography of André Weil

Translation of Souvenirs d'apprentissage Translated by Jennifer Gage Birkhaüser ISBN 3-7643-2650-6 ISBN 0-8176-2650-6 197 pages

My life, or at least what deserves this name—a singularly happy life, its diverse vicissitudes withal is bounded by my birth on May 6, 1906, and the death on May 24, 1986, of my wife and companion. Eveline.... It is with these moving words that André Weil, one of the greatest mathematicians and mathematical personalities of this century, begins this extraordinary autobiographical account of his life from his childhood days until the fall of 1947, when he accepted a professorship in the department of mathematics of the University of Chicago offered by his friend Marshall Stone. In beautiful prose that at times becomes poetry and goes straight to the heart, Weil not only describes his life and its many ups and downs but also allows many deep glimpses into his heart and mind as he takes the reader along with him on his journey.

Weil describes his book as "an attempt to retrace the intellectual itinerary of a mathematician," and he has succeeded to a remarkable extent. It is as if he has detached himself mentally from his life and reviews it, giving the reader a faithful (but inevitably selective) recollection of what he saw and

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experienced, annotated with a commentary that is at various times trenchant, ironic, critical, moving, and yet ultimately very sincere and true to his own self. Some of the remarks he makes must be familiar to the readers of his "Commentaire" in his Collected Papers, and so this book is an attempt to reach a wider audience as well as to go beyond

the strictly mathematical aspects of his intellectual and emotional development.

Early Days and the École Normale

André Weil's grandfather was Abraham Weill, a respected member of the Jewish community in Alsace. His father was Bernard Weil, who was a physician in Paris. His mother's family came from Russia. André was born in 1906, and his early days, spent in the company of tutors (he was clearly a precocious child) and his younger sister, Simone, were, by his own account, very happy ones but very different from the lives of most other children because of the emphasis on intellectual activities. He was fortunate to come into contact with exceptional teachers whose influence stayed with him throughout his life. His education was a comprehensive one, encompassing the sciences, languages, and hu-

manities. His interest in and exceptional ability with languages were visible quite early, while his taste and passion for mathematics started from the time he was eight years old. When his mother expressed her fears to one of his early teachers that he may not be sufficiently well grounded in "arithmetic", the teacher replied, "No matter what I tell him on that subject, he seems to know it already."

The First World War touched the lives of the Weil family deeply, because his father was drafted into the medical corps of the army. The stupidity of the politicians and the generals and the relatively poor state of medical science contributed to huge casualties among the enlisted men, and Bernard Weil himself fell ill due to exhaustion and depression and was sent back from the front for rest and recuperation. There is a charming anecdote of how André and Simone planned to surprise their father with a birthday present, which was to be a reading of the newspaper for him by Simone, and how André was the fierce taskmaster during the secret preparations, making sure his sister pronounced all the subtle words in the correct way.

From the age of nine Weil started contributing to one of the journals which published mainly problems-essentially examination problems at the secondary school level-and the best solutions as well as the names of the successful solvers. Soon he found he could do some of these problems, and the day came when his solution was printed as the best. The years preceding his entry into the École Normale were spent in the Lycée Saint-Louis, which was acclaimed to be the best scientific lycée in France, although the humanities were by no means neglected. About a year before he left the Lycée Saint-Louis, when he was fourteen, he met Hadamard. Let me tell in Weil's own words his assessment of Hadamard at that time: "... The warmth with which he received me eliminated all distance between us. He seemed to me like a peer, infinitely more knowledgeable, but hardly any older; he needed no effort to make himself accessible to me...." When Weil received an endowment prize which allowed him to choose some books as a reward, Hadamard helped him make his choices. It was thus that he became acquainted with Jordan's Cours d'Analyse and Thompson and Tait's Treatise of Natural Philosophy.

While he was preparing to enter the École Normale, Weil met another person whose influence was decisive in his intellectual growth. This was Sylvain Lévi, leading scholar in the field of Indian studies in France at that time. Already, in his very early teens, Weil had formed a vague resolution to learn Sanskrit so as to be able to read Indian epic poetry, and the acquaintance with Lévi sparked this latent wish and started him on a path that would prove fateful in its implications later on. It would create an intense desire to go to India, and when

the opportunity came later, he would grasp it. But more about this later.

He entered the École in 1922, when he was sixteen years old. He joined the famous Hadamard Seminar, where the participants would report on topics of current interest in mathematics. In its reach and diversity the Hadamard Seminar was almost universal and reflected the universality of Hadamard's mind. It is quite probable that it was this seminar, at least intellectually if not personally, that served as the model for Weil's own famous current literature seminar, which he conducted jointly with Princeton University for many years while at the Institute for Advanced Study. He attended Lebesgue's lectures. He also started two things which would be of tremendous importance for him later. He had already become convinced by his reading of the Greek poets that the only way to deeper knowledge was through the study of the works of the truly great minds. So he began to study Riemann, often seeking help from Felix Klein's mimeographed lecture notes on Riemann's work, which were available at the École library. In addition, he went to Sylvain Lévi to seek advice for some vacation reading (!) in Sanskrit. Lévi gave him a copy of the Bhagavad Gita1 with the comment: "Read this. First of all, you cannot understand anything about India if you haven't read it," and here Lévi's face lit up, and he added, "and besides, it is beautiful." Weil read the Gita from cover to cover and was affected by its beauty immediately. The thought behind it impressed him profoundly and was, in his own words, the only form of religious thought that could satisfy his mind. As we shall see later in greater detail, the Gita is not a dry system of philosophy, but a prescription for action for a man confronted by conflicting choices. Weil's love and knowledge of the Gita was at the heart of many of the decisions and actions that would be critical later in his life: his decision to go to India

¹The Bhagavad Gita, or Gita for brevity, which literally means "The Song of the Lord", is a long poem imbedded in the Indian epic Mahabharata. It consists of the instruction and advice given by Krishna, an incarnation of the god Vishnu, to Arjuna, the great warrior king, on the eve of the battle of Kurukshetra, a central episode in the Mahabharata. The Mahabharata is an extraordinary but long and complex epic. It has become a part of the cultural heritage not only of India but of many of the countries of South and Southeast Asia. There is a very good translation by J. A. B. van Buitenen, published by the University of Chicago Press. It is, however, incomplete and in particular does not include the Gita, although van Buitenen has a very nice description of the Gita in the Encyclopedia Brittannica, 15th edition, Macropedia, 8:937. Peter Brooks's nine-hour dramatization of the Mahabharata is a wonderful evocation in television of this great epic suitable for Western audiences. The influence of the Bhagavad Gita has been profound, and it has shaped the lives of many famous Indians, including Mahatma Gandhi.



André Weil and his sister Simone vacationing in Knokke le Zoute, Belgium, summer 1922.

and immerse himself in Indian life and culture for a couple of years, his decision to refrain from joining the army when the Second World War broke out, and the fortitude with which he endured the traumatic consequences that flowed out of this decision. He felt that it was his understanding of the ideas in the *Gita* that made him comprehend his sister's way of thinking, which would often appear mystical in her later years.

He attended Jules Bloch's course on the *Veda*, Meillet's lectures on Indo-European linguistics, and Sylvain Lévi's course on *Meghaduta*, a beautiful poetic fantasy by one of ancient India's greatest poets, Kalidasa. He lovingly recalls the gentle voice of Lévi intoning Kalidasa's beautiful cadences. These years at the École, when his mind was at its most eager and receptive, were the seeds from which his life grew to its fullest stature in later years. In some sense everything that he did or that happened to him, either in mathematics or in his personal life, could be traced to these years of gestation at the École.

Travels and Thesis

In 1925, with the years at the École behind him, Weil started to travel. Normally he would have had to do military service for a year, but in his case, because he was very young, this was postponed, which gave him an opportunity to travel. He went to Italy, Germany, Scandinavia, and England, enriching his life not only by meeting the major mathematicians in these places but by immersing himself in the rich cultural ambiance that these countries provided. But before he started these travels, he had some time on his hands, and he slowly started making his future plans in mathematics. It was during this time that he began thinking about diophantine geometry, combining the ideas of Riemann with those of Fermat, and began his lifelong love affair (to borrow the felicitous phrase by which he himself described Emil Artin's work²) with the arithmetic of algebraic varieties.

In Rome he met Vito Volterra and became a close friend of Volterra's son, Edoardo. He met Severi and attended his lectures on algebraic surfaces. He also came to know of Mordell's famous 1922 paper on rational points on elliptic curves over the field of rational numbers. He then went to Frankfurt and met a remarkable and tightly knit group of mathematicians: Dehn, Hellinger, Epstein, Szász, and Siegel. Their knowledge, their attitude toward the philosophy of mathematics, and their insistence on seeing mathematics as a whole and not as a splintered collection of subdisciplines made a profound impression on him. Weil mentions a seminar session devoted to Cavalieri where Dehn showed how one should read Cavalieri's text, taking into account what was known at that time and contrasting this knowledge with the new ideas that Cavalieri was trying to introduce. He met Erhard Schmidt and, in Stockholm, Mittag-Leffler. Mittag-Leffler promised Weil that the Acta Mathematica would publish Weil's thesis, which was as yet unwritten. He then returned to Göttingen, where it suddenly occurred to him that his ideas on diophantine geometry would allow him to prove a far-reaching extension of Mordell's theorem. Although it would take him a year to turn this sudden insight into a rigorous proof, there is not much doubt that it was during this visit to Göttingen that the "phase transition" took place in his understanding of the arithmetic of algebraic varieties.

Upon his return to Paris, his immediate task was to write up his thesis and get it accepted. He had refined his ideas on diophantine equations and succeeded in proving the finite generation of the rational points of the Jacobian of a curve of arbitrary genus defined over a number field. He approached Hadamard and asked his advice about submitting his thesis, adding that he thought he might also be able to prove the Mordell conjecture. Hadamard advised him to wait till he settled the Mordell conjecture also (!) with the words: "Weil, several of us think highly of you; you owe it to yourself, when presenting your thesis, not to stop halfway through. What you say shows that your work is not yet mature." But Weil decided to present his thesis as it was, a decision that was wise, because the Mordell conjecture would not be settled for another fifty years and then only after the birth of a new vision of algebraic geometry due to Grothendieck.

Writing up the thesis, which he did in the summer of 1927 and the year following, was one thing, but getting it accepted was another entirely different proposition because of the bureaucratic requirements of the French university system. He

²In his review of Artin's Collected Papers he says: "Perhaps the best part of (Artin's) career may be described as a love affair with the zeta function."

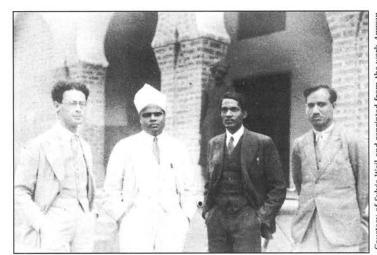
managed to persuade Picard to chair his thesis committee, but Picard did not want to write the report. Weil had already discussed his results with Siegel and knew that Siegel thought highly of his work. So all he had to do was to find another person to be a member of the committee and write a favorable report. Weil's account of how he succeeded in getting this done is both hilarious and charming. As mentioned already, the thesis would appear in the *Acta Mathematica*.

Once the question of the thesis was disposed of, Weil could turn to other matters. First of all there was the question of military service, which should have been done when he left the École and which was postponed because he was too young at that time. He joined the 31st Infantry regiment and endured battalion life for a year, which he succeeded in cutting short by a couple of months by some ingenious maneuvering and excuses. It was then time to see about getting a job. A position was about to open up in Strasbourg, but that was to go to his friend Henri Cartan. Slowly the idea that he should go to India began to take shape in Weil's mind. He mentioned this to Sylvain Lévi, obviously hoping that Lévi's contacts might lead to something in that direction.

In 1929 exactly what he was hoping for happened. Sylvain Lévi phoned him one day and asked him if he was prepared to go to India to teach French civilization in an Indian university. Weil told Lévi that he would do anything to go to India. Then Lévi asked him to take a taxi and come at once to his home. There Weil met Syed Ross Masood, vice chancellor of the Muslim University of Aligarh (a small town near Delhi). It was Masood's idea that French culture should find a place alongside English culture in Indian universities, and with the (unlimited) power given him, he was taking some of the first steps towards realizing that goal. Weil was very impressed with Masood, whom he describes as "a tall man who filled the room with his broad frame, stentorian voice and ringing laughter," with a "presence that makes itself felt anywhere." Nothing happened for a while after this meeting. But one day Weil received a cable from India: "Impossible create chair French civilization. Mathematics chair open. Cable reply." The answer was obvious.

India

He was in India for more than two years, from the beginning of 1930 to the early months of 1932, after which he returned to Paris. This was a stay whose memories and effects stayed with him forever afterwards. He went everywhere, met everyone who was anyone, dipped deeply into Indian culture and books with the help of his newly acquired Indian friends. His account of his travels and experiences in India communicate wonderfully his thrill and excitement of coming into deep contact



Weil (left) in Aligarh, India, with Vijayaraghavan (second from left) and two students, 1931.

with a culture that was totally different from anything he had ever experienced. It is a tribute to his genius and the strength of his character that he absorbed most of the good elements of this ancient civilization and incorporated them into his personality at a deep level. Mathematically it was here that he had the ideas on functions of several complex variables that led him to a generalization of Cauchy's formulae in the higher-dimensional case and to results on representing a holomorphic function as a polynomial series on pseudoconvex domains.

His immediate task, however, as required by the University, was to assess the quality of the mathematics faculty there and report on it to the authorities. He had a virtual carte blanche in framing his report. This report would then serve as the basis for actions of dismissals and hirings that the University administration would undertake. One should imagine the circumstances: he, a young man only twenty-three years old, abruptly dropped into a world he knew only from books, dealing with a culture that had existed for several thousand years, and given unlimited power over the professional careers of the members of the faculty around him. The faculty was small and consisted of people who had no idea of what mathematics really was, the library was very inadequate, and there were no traditions to speak of. No wonder Weil blundered many times, especially in decisions involving human beings.

However, Weil was very lucky in one thing. He was able to get appointed to the faculty a young man, Vijayaraghavan, who was a pupil of Hardy's. Vijayaraghavan had written several papers, but unfortunately did not have a degree and so was overlooked by the administration. But Weil, not trusting the administrators (a trait that he would never lose throughout his life), insisted on looking at the qualifications of all the applicants before any selection was made. It was thus that he came across Vijayaraghavan's name and succeeded

in getting him selected. Weil took an immediate liking to Vij (as he was called), and they became very close friends. Weil says that in the time they were both together in Aligarh, he almost never left Vijayaraghayan's side. He became virtually a member of Vij's family and even ate in their house often, although Vij's mother perhaps had reservations about feeding a foreigner who was outside the brahmin caste. It was Vijayaraghavan who helped Weil to go deeper into the Indian classics like the Mahabharata and the Chandogya Upanishad.³ From Weil's description of his friendship with Vijayaraghavan it is clear that there was a true harmony between the two. Weil's account of Vijayaraghavan's death, learned from his son, is brief but poignant.

Weil's attempts to carry out his commission went awry because he simply did not understand the human issues involved. Intrigues began to develop, and after he came back from a vacation he learnt that Vijayaraghavan had been eased out and had gone to take up a position at Dacca University. Weil himself was then fired on trumpedup charges of travelling without permission, although this was not to be attributed to conditions in India, since he had a similar experience later in Strasbourg. These events may perhaps be explained as the consequence of the natural reaction of entrenched forces to an outsider who came on a white horse with a mandate to sweep away the rubble and establish a new era.

The time of Weil's stay in India coincided with one of the most dramatic periods in her long struggle for independence from British rule. Mahatma Gandhi launched one of his greatest campaigns of satyagraha⁴ against the British during the years 1930-32. Gandhi wanted it to be a movement that would involve the entire nation, which meant that the rationale had to be framed in a manner that would resonate with the masses. For this purpose Gandhi focussed on the salt tax which Britain had imposed so it could sell English salt at exorbitant prices. A natural consequence of this British policy of protection for their salt was a law that made it illegal for any Indian to make salt out of saltwater. The fact that a universal necessity like salt was controlled by the British in such a callous

manner, when pointed out by Gandhi, galvanized the people. With characteristic brilliance and insight, Gandhi announced that he would march a few hundred kilometers from Ahmedabad, his normal place of residence, to a seaside village in the south and make salt there, daring the government to arrest him. This was the famous Salt March, arguably the single greatest campaign of satyagraha waged by Gandhi against the British, and the events springing out of it shook the British rule to its foundations and almost brought it to an end.5 Weil was a full participant, emotionally and intellectually, in the events of this period in Indian history. He met almost all of the leading figures of the Indian independence movement, including Gandhi himself, and made deep friendships with some of them, like Zakir Husain, who was to become the president of India. This exposure to Gandhi's ideas was to prove fateful later when the time came for him to decide whether he was going to take part in a war that he did not feel was his war. But this is getting ahead of the story.

Strasbourg, Bourbaki, and More Mathematics

Weil returned to Paris and almost immediately got a position in the University of Marseilles as a lecturer. He held the position for a short time before going to Strasbourg, where his friend Henri Cartan was already in the faculty. From the fall of 1933 until 1939 he taught there, and these were happy and productive years for him.

There were at that time about ten young mathematicians, including Weil and Cartan, scattered in diverse parts of France, who were taking their teaching duties seriously and thinking about various pedagogical questions. Cartan himself often came to Weil and raised foundational questions, such as how best to formulate and prove a general form of Stokes' theorem to handle all applications. It was under the impulse of these questions from his friends that Weil had the idea that they should all get together and thrash out these problems once and for all. In retrospect this would turn out to be an important idea and mark the moment of conception of Bourbaki, which would dominate

³The Upanishads are original texts containing Hindu speculative thought. There are 108 in all, dating back to between 1000 B.C. and 500 B.C., although some of them are thought to go back to even earlier times. Among the most famous ones are the Chandogya Upanishad, and the Katha Upanishad. The Chandogya Upansihad introduces the supreme reality that is the Universe (Brahman) and discusses the relation of the individual self (Atman) to this reality.

⁴Satyagraha literally means "insistence on truth". In Gandhi's hands it was transformed from a well-intentioned code of ethics into a dynamic instrument that energized a whole nation and led it to independence.

⁵The Salt March was from Ahmedabad to Dandi, a village on the west coast of India, several hundred miles to the south. It took Gandhi and his followers almost a month (from March 12, 1930, to April 6, 1930) to walk all the way. The march, with speculation (about British intentions toward the Mahatma) and the consequent tension mounting every day, attracted international attention. At the end of the march Gandhi and hundreds of his followers took sea water and distilled it to get salt. He was arrested a few days after this, but the satyagraha movement continued, paralyzing the country for months afterward. For a wonderful account see On the Salt March, by Thomas Weber, HarperCollins Publishers India, 1997, ISBN 81-7223-263-2.

the mathematical instructional scene at its most advanced levels from then on. The founding members were Weil himself, Cartan, Delsarte, Chevalley, and Dieudonné. There were a few more who dropped out, but these five founders were the ones who continued till they were fifty, which was the age of retirement. Nowadays most mathematicians know the rituals and the history of the Bourbaki congresses and what became of them, so that there is no need to repeat the story here.⁶

However, there is one point that must be mentioned: it is not enough to create an author; one must have a publisher also. Bourbaki was fortunate to have the continued and unqualified support of Enrique Freymann, who ran the Hermann publishing company at that time. Mercurial, imaginative, and audacious, Freymann supported the young group wholeheartedly and gave them the financial support their efforts needed until they established themselves. Weil mentions many anecdotes in his brilliant sketch of Freymann's inimitable personality.

The War Years

Weil continued to travel, and among his travels was a visit to Russia, where he made the acquaintance of the principal Soviet mathematicians Alexandrov, Kolmogorov, Pontryagin, and others. But the times were slowly becoming ominous. War clouds were gathering in Europe. Moreover, these political developments were beginning to have an effect on his personal life. As a reserve army officer he was bound to serve in the French army when the draft call went up, and he was not willing to do it. Thus began the question within his own mind that tormented him all the time. Eventually he came to the decision that he would flee France and go to a neutral country if and when war broke out. He was aware that in all but technicalities this was desertion and that he would have to face grave consequences.

This was the beginning of his war experiences, which he calls "a comedy in six acts: prelude, Finnish fugue, arctic intermezzo, under lock and key, serving the colors, and a farewell to arms." His decision, which he learned later was not technically a desertion since he was not in the army, was taken only after the most elaborate arguments and counterarguments in his mind. He went to Finland to spend a few weeks there. But the march of events forced his wife to return to France, with no clear assurance in their minds as to when and how they would be united again. Then the Finnish-Russian war broke out while he was still

in Helsinki. He was already under observation by the Finnish police because of his correspondence with Soviet mathematicians. As soon as war broke out between Finland and Russia, the Finnish police, certain that Weil was a spy for the Russians and acting with stupidity. scooped him up and put him in jail. He



Weil in Brittany, about 1960.

narrowly escaped being executed thanks to the fortuitous intervention of Nevanlinna. Weil recalls a conversation he had with Nevanlinna twenty years after these events in which Nevanlinna, recounting what took place, said that he (Nevanlinna) was at a state dinner where he met the chief of police of Helsinki and had the following exchange with him:

C[hief]: Tomorrow we are executing a spy who claims to know you. Ordinarily I wouldn't have troubled you with such trivia, but since we are both here anyway, I am glad to have the opportunity to consult you.

N: What is his name?

C: André Weil.

N (shocked): I know him. Is it really necessary to execute him?

C: Well, what do you want us to do with him?

N: Couldn't you just escort him to the border and deport him?

C: Well, there is an idea: I had not thought of it.

Weil was thus deported to a prison in Sweden and from there shipped to France via England. Once he entered France he was arrested and put in jail in Le Havre. After a little while he was transferred to the military prison in Rouen pending his trial.

He spent about three months in the prison at Rouen before his trial. It was a time of profound introspection for him. The loss of freedom and enforced isolation made him retreat to his inner self. It was in the prison at Rouen that he did what is arguably his greatest work, the proof of the

⁶See A. Borel's article in these Notices, Vol. 45, Number 3, 373–80, as well as an older article by P. R. Halmos, in the Scientific American, May 1957 (part of a book, Mathematics in the Modern World, published by W. H. Freeman and Company, 1968, pp. 77–81).

Riemann hypothesis for smooth projective curves over a finite field. It was also from here that he wrote his famous letter to his sister outlining for her his ideas about number theory and his perspective of viewing it in a way that unifies number theory and the theory of Riemann surfaces, a viewpoint that started with Hilbert, came to maturity with Weil, and eventually reached its full development with Grothendieck.

It was also from here that he wrote some very touching letters to his wife. These letters, which capture vividly his emotional and physical isolation and his deeply philosophical state of mind during his imprisonment, are among the most moving parts of the whole book. When he was not thinking about or doing mathematics, he was reading the *Gita* and the *Chandogya Upanishad*. In his mind he had a vision of Krishna to guide him through the ordeal. Here are some extracts from these letters.

(March 4) What can I say about myself? I am like the snail, I have withdrawn inside my shell; almost nothing can get through it, in either direction.

(March 30) ...I am reading the *Gita*, in small doses as one ought to read this book. The more detail one absorbs, the more one admires it.

(April 7) My mathematics work is proceeding beyond my wildest hopes, and I am even a bit worried—if it is only in prison that I work so well, will I have to arrange to spend two or three months locked up every year?

I am sending Papa Cartan a note for the *Comptes Rendus...* I am very pleased with it, especially because of where it was written (it must be a first in the history of mathematics), and because it is a fine way of letting all my mathematical friends around the world know that I exist. And I am thrilled by the beauty of my theorems.

Here are some lines from the *Gita* that I like very much: "A leaf, a flower, a fruit, some water, whoever dedicates it with love, this love offering I accept with the devotion of his soul."⁷

If I get started on this topic, I won't finish for a year, and you may not find it terribly interesting—but I can hardly amuse you by describing the walls of

my cell, which are the only landscape before my eyes now; and of everything in the *Gita*, all I have to offer Krishna is water, or now and then a fruit—an orange or banana that they give me for dessert; sometimes, these last few days, a young leaf, all crinkled up still, that the wind has blown onto the walk—but no flowers.

(April 22)...I would much rather be sitting on that bench surrounded by ivy, near the yellow flowers smelling of honey, where I would speak to you of Krishna...."Of all the seasons," says Krishna, "I am the season of flowers." But he does not tell us which flowers....

For an intellectual, any stay in prison with an uncertain future looming in front is always a time for reflection and introspection. Weil's musings from the prison remind me of Jawaharlal Nehru's description of his life in a British prison in India.⁸

Weil's incarceration was succeeded by a trial which was a farce. The punishment was already determined before a single word was spoken at the trial. He was sentenced to five years' imprisonment (the maximum), to be suspended if he would submit to serve in the army, which he did. But by then the military situation was becoming chaotic for France. Crushed and humiliated in the battlefield, France had neither the military nor the political will to carry on. The front was changing constantly as the French forces were in retreat in confusion and panic, and his regiment was shipped to England to join other French forces in a camp. The troops had a choice either to join de Gaulle's army or be deported out of England. By sheer chance Weil's regiment was delayed getting to the rendezvous point and so had to stay behind. Moving from camp to camp, he came to London during the time it was bombed daily by the Luftwaffe. Eventually he boarded a hospital ship sent by Pétain to return to France. Using a card he had fabricated in England, Weil managed to get himself discharged. But his future was still uncertain, because no one was sure whether he would be asked to serve the remainder of his suspended sentence. Somehow he and his wife and her young son succeeded in getting visas to the United States and arrived there in the beginning of 1941.

United States and Brazil

His initial experiences in the United States were unpleasant and humiliating. He was on a grant from the Rockefeller Foundation and then later on a

⁷Lines spoken by Krishna: Bhagavad Gita, IX: 26.

⁸See Nehru's book The Discovery of India, Meridian Books, London, 1956, especially the beginning.

grant from the Guggenheim Foundation. These grants were just enough to get by, and the small colleges where he was given teaching positions exploited his circumstances and the fact that he was already being paid by these grants to obtain his services for a pittance. It was during these very difficult times that he wrote his famous letter to Artin, outlining his proof of the Riemann hypothesis for curves over a finite field. His approach required him to build a substantial amount of machinery, and since he was not sure when these preliminaries would be completed, he decided to communicate his results in the form of this letter. The machinery itself would be completed in the following two years and would appear as his famous Foundations of Algebraic Geometry.

He received an offer from the University of São Paulo to take up the position which was previously occupied by Albanese. He arrived in January of 1945 and spent two years there. He had the company of Zariski for a year and Dieudonné for another year, but he really missed the excitement of being in a stimulating atmosphere. As he was pondering what to do, his friend Marshall Stone offered him a professorship at the University of Chicago in 1947, which he accepted. His torments and difficulties were over, and he could look forward to a normal and peaceful life as a mathematics professor.

To Serve or Not To Serve

This review would not be complete without a discussion of the pivotal decision in his life, namely, his decision not to report to the French army when the Second World War started, although as a reserve army officer he was bound by his oath to do so. He offers an elaborate rationale for his decision, although admitting that his reasoning would appear confused and unconvincing to an outsider. There are two great episodes from Indian life and culture that provided him with the inspiration that led to his decision—one of them concerns Arjuna, the great warrior king on the eve of the epic battle of Kurukshetra, and the other relates to Gandhi and his interpretation of *satyagraha*.

Let me set the scene on the eve of the battle of Kurukshetra, the climactic battle in the Indian epic *Mahabharatha*. The dispute between rival factions of the same ruling family has gone out of control and has led to the brink of war; every king in the land has had to choose sides, and brothers and uncles and nephews are arrayed against each other to fight to the end, for a retreat is unthinkable, disgraceful, and against the warrior code of behavior. Arjuna, the greatest warrior of them all, with right on his side, as it is he and his brothers who have been deprived of their kingdom in a stacked gambling episode, is about to start the battle. His charioteer is Krishna himself. Arjuna asks Krishna to

drive the chariot to the middle of the battlefield, from where he surveys the two great armies. Then, overwhelmed by compassion for all the people who will be killed and the devastation to families that will ensue, he falters and tells Krishna that it is not worth doing this for the mere recovery of a kingdom and the worldly pleasures of being the king. Indeed, the battle would result in universal destruction exactly as Arjuna fears. And Ariuna, invincible



The Weil family (Eveline, Sylvie, Nicolette, André) on the day of departure from Brazil, September 13, 1947, Santos, Brazil. The two men at the right are from the mathematics department at the Universidade de São Paolo.

because of the combination of his great prowess and the fact that the Lord himself is his charioteer, would emerge victorious in the end, but his triumph would be empty.

Krishna does not respond directly to Arjuna's reasoning for not fighting, but instructs him on the ethics of living, on the mystery of life and death, and what is the right course of action for an individual. Krishna's advice constitutes what is known as the *Bhagavad Gita*. Krishna's injunction is that the individual should always act according to his or her *dharma*, which in Arjuna's case, as he is a member of the warrior caste, means that he should fight. That the fight would certainly cause universal devastation and grief is not an excuse to stay away from the fight, because the fight and its consequences as far as human beings are concerned are mere illusions; the soul is invincible and neither slays nor is slain.¹⁰

Dharma is a universally applicable code of behavior for the individual. It is not easy to define precisely because it is not a microcosmic concept but a macrocosmic one and might depend on the evolutionary state of the society. In ancient times it was determined by the individual's caste, and for Arjuna, as a member of the warrior caste, his dharma was to fight. However, in modern times where caste and other subdivisions have eroded,

⁹Arjuna is also known as Partha, and so one of Krishna's names is Parthasarathy, Partha's charioteer.

¹⁰Bhagavad Gita, *II: 19.*



Weil in family apartment at 3 Rue Auguste Comte, Paris, in June 1993, with Nobuko Inaba, who did the Japanese translation of his autobiography.

it is more difficult to see clearly what one's *dharma* is. Weil chose in 1938 to define his *dharma* as the duty to be a scholar and teacher of mathematics. One might argue that this arbitrariness in defining one's *dharma* might lead to chaos. In the *Gita*, Krishna, who exists outside the *dharma* as divinity incarnate, says that if in the course of time the balance between right and wrong gets distorted, he will incarnate himself to redress the balance.¹¹

The second inspiration for Weil came from his understanding of Gandhian *satyagraha*, gathered through his experiences in India during 1930–32. He was deeply impressed by Gandhi's insistence that when the laws governing a person's conduct are unjust, it is not merely the person's right but his duty to rise against these laws, regardless of the consequences.

Weil was also inspired by the example of Siegel, who deserted in 1918 when he decided that the war was not his war ("Dieser Krieg war nicht mein Kreig"). It is clear that Weil felt that no Frenchman could accept the war as his war, given the stupidity and shortsightedness of the French military and political bodies. Of course, he paid dearly for his decisions and actions.

The *Gita* is much more than what my brief remarks above can hope to convey. Its vision, its beauty, and its timeless relevance make it one of the great documents created by man. The doctrine of *satyagraha*, at least as can be understood from Gandhi's use of it, is a complex one, and this is not the place to go into it in any depth. That Weil, in the greatest crisis of his life, fell back on the *Gita* and the teachings of Gandhi to show him the light in the darkness is an indication of how profoundly he had been influenced by Indian culture and thought as well as by his long stay in India.

This elaborate explanation of his actions by Weil may perhaps appear self-serving to some, an unconvincing argument that cannot explain away the failure to do his duty, even raising questions of cowardice. Such an interpretation is too simplistic; the issues involved are emotional and complicated. Further, as he himself ruefully remarks, things turned out to be entirely different from what he had foreseen, and his life took completely unanticipated turns. We have already seen what these were.

Clearly there was no need for him to explain or justify his behavior—after all, he had faced the consequences of his actions, some of which were very traumatic. In my opinion this discussion by Weil should be seen as an attempt by him to throw open his mind and to invite the reader into its innermost chambers.

Concluding Remarks

Weil is a luminous figure in twentieth-century mathematics. The beauty of his discoveries and the clarity and consistency of his vision have been the sources of continued inspiration for two generations of mathematicians. But there is much more to him. His autobiography gives some wonderful insights into the mind of a profound thinker who was supremely creative, even when everything around him was collapsing, who had a full understanding of the world and yet stayed aloof from it. In his mind everything fitted perfectly-mathematics, philosophy, and politics. In this book he presents this world view with great forthrightness and eloquence. It will be a worthy addition to the personal library of every mathematician whose interests go beyond the merely mathematical.

¹¹Bhagavad Gita, IV: 7.