Fred Almgren (1933–1997)

Lover of Mathematics, Family, and Life's Adventures

Dana Mackenzie

One of my father's mottoes was "Anything that's worth doing is worth doing badly." It took me a long time to figure out what he meant, but I think it was simply that you shouldn't be afraid of doing what you want to do, regardless of how you compare with anyone else.

—Robert Almgren, Fred Almgren's son

Frederick J. Almgren Jr., a mathematician of many facets and a man of still more, died on February 5, 1997, at the age of sixty-three. Though he had known since August that he had myelodysplasia, the same disease that ultimately claimed the life of Carl Sagan, Fred's zest for life, together with a transplant of bone marrow from his sister in October, had given everyone hope that he would recover.

Fred Almgren took an indirect route into mathematics, graduating in 1955 from Princeton University with a degree in engineering and then flying jets for three years in the Navy. Even in his Navy years, Fred showed his cool analytical mind. Jean Taylor, his wife, tells one of his favorite stories, about the time he flew into a thunderstorm: "When he looked at his instruments, they showed he was flying upside down, but he was sure he was not. Nevertheless, he 'went by the book'. He rolled the airplane until the instruments said he was right side up (although he said he could then feel himself hanging by the straps that pilots belt themselves in with) and turned around to fly back out the direction he'd been coming in.... After a while he emerged from the storm—and there were the lights of Galveston *below* him. So, yes, the instruments had been correct. But it took him a while to reorient his sense of up and down and accept that he wasn't hanging by his straps." Maybe it was the same willingness to suspend disbelief that led him to be such a successful mathematician.

Though he was perfectly at home with the abstractions of pure mathematics, such as surfaces in spaces of arbitrarily high dimension and the nearly impenetrable language of geometric measure theory, Fred's interest in the phenomena of the real world was never far beneath the surface. From the growth of a snowflake to the rolling of a hula hoop, whenever nature had an interesting surprise, Fred wanted to know about it.

In the month of Fred's death the *American Mathematical Monthly* published a short article about the rolling of an ideal, massless wheel with a point mass attached to the top. Surprisingly, a simple mechanical argument predicts that the wheel will hop off the floor after rotating a quarter turn. The author of the article, Tadashi Tokieda, who was an admirer of Fred's expository writing, showed Fred an early draft. Fred asked if he had actually witnessed the phenomenon; Tokieda admitted that he had not. "Next day he appeared in Fine Hall with a hula hoop, batteries, and tape," Tokieda says. They taped the battery to the hoop and rolled it

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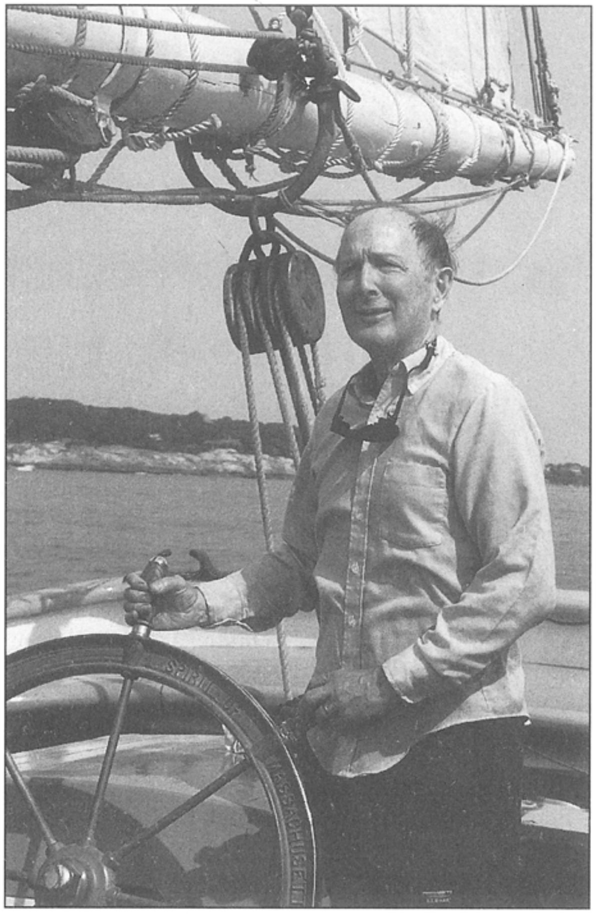
A survey of Almgren's work, by Brian White, will appear in an upcoming issue of the Notices.

Dana N. Mackenzie (formerly Dana Nance) was tenth in the family of Fred Almgren's doctoral students, receiving his Ph.D. in 1983. After teaching for thirteen years at Duke University and Kenyon College, he is now finishing a year of study in the Science Communication Program at the University of California at Santa Cruz. He plans to remain in Santa Cruz as a freelance science writer. His e-mail address is mackenzie@amsci.org.

down the twelfth-floor hallway in Fine Tower. The hoop hopped, just as predicted! (But only when Tokieda rolled it. Perhaps it was just as well that Fred did not become an engineer.)

Of all the mathematicians at Princeton, Fred was the one most likely to get excited about a seemingly trivial fact like the hopping of a hula hoop—and also the only one you could count on to have one handy. After all, this was a man whose bedside table when he died contained a gyroscope, two magnets, four colored balls from Cheerios boxes, two magnifying glasses, a Star Trek communicator button, and five pretty rocks. It was "the drawer of a six-year-old kid, not a sixty-three-year-old man," says Jean Taylor.



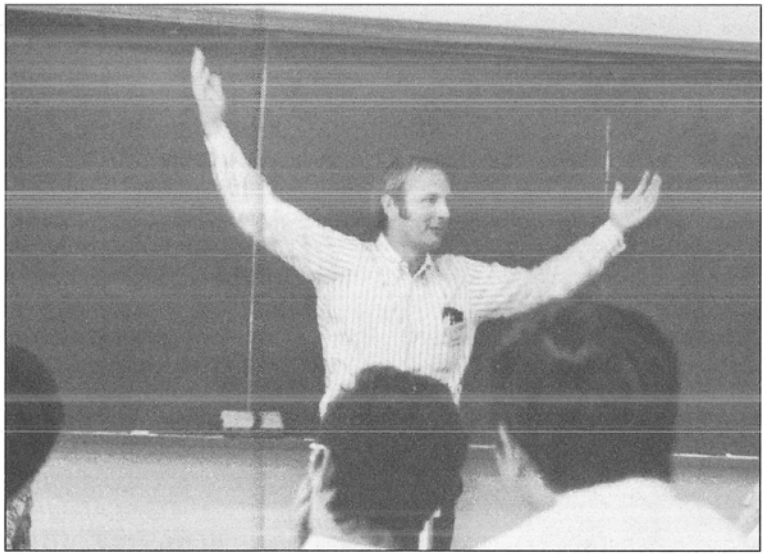
Almgren sailing on the schooner *The Spirit of Massachusetts* in the Esperanto Cup in Gloucester, Massachusetts, Labor Day, 1996. "The image that will linger in my mind is him standing on the deck of the *Spirit*, sea breeze on his face, laughing in the sunshine, and exuberant in the expectation of life's next great adventure."
—Linda Almgren Kime, Fred's sister

During his mathematical career, Fred's interest in nature expressed itself mostly through the study of minimal surfaces—surfaces or configurations that minimize energy. For a soap film stretched across a wire frame, the surface energy is directly proportional to the area. Thus the configuration that uses the least energy is called an area-minimizing surface. In some cases the soap film will find only a relative minimum (a stable minimal surface) rather than an absolute minimum. But does a minimum always exist? Could there be a wire loop so bizarre as to admit no soap film of least area, so that the soap could cascade through an infinite series of more and more efficient configurations without ever finding one that was best? Even if an area-minimizing surface exists, is it necessarily smooth and gently curved? Could there be a wild, irregular surface, perhaps with self-intersections or fractal convolutions, that improved upon nature? And what happens if the constraints of the problem are changed—for example, if one talks about a cluster of soap bubbles constrained to contain a certain volume of air instead of a soap film constrained to lie across a given boundary?

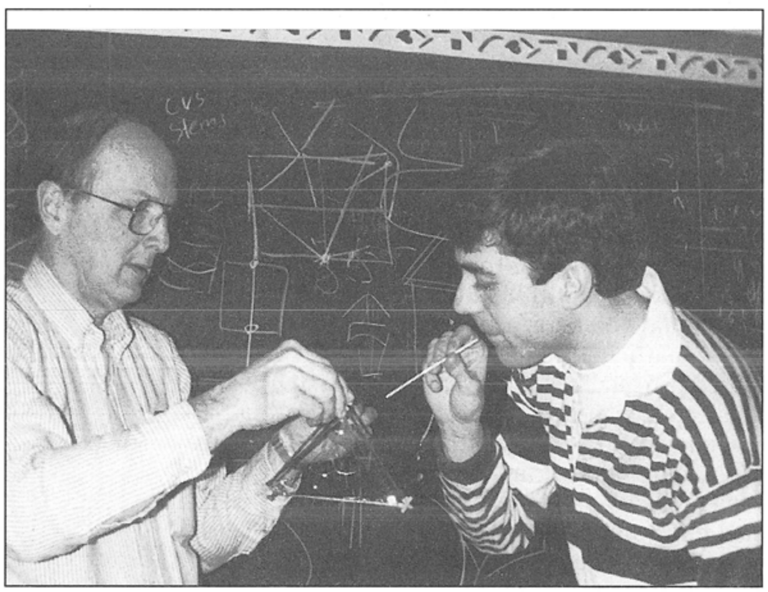
More than anyone else, Almgren tamed the wilderness of bizarre possibilities. Soap bubble clusters do have self-intersections, but Almgren showed that the self-intersections, both for bubbles and for films, have negligible area compared with the surface itself. For the soap-bubble problem (though not for the soap-film problem) he showed that an area-minimizing solution must exist; the infinite cascade of better and better solutions cannot happen. Perhaps even more fundamentally, he was the first mathematician to tackle these problems in the context of a realistic model for soap films and bubbles. Earlier minimal surface theorists had relied on the theory of complex analysis, which assumed a high degree of smoothness and was completely silent on the question of why real soap bubble clusters come together in threes and fours to form a crisp, clean network of edges. Almgren's model for soap films made it possible for Taylor to put together the definitive explanation of this phenomenon, and the two of them beautifully described their results in a *Scientific American* article in 1976.

The particular kind of research Fred chose required two qualities in abundance: persistence and intellectual integrity, an unwillingness to accept the obvious or to take shortcuts. "His theory of soap films, for example, unlike the earlier, more convenient theory of minimal surfaces, faithfully modeled physical reality, with all of its daunting complexities," writes Frank Morgan, one of Fred's doctoral students. "He would not compromise, and he would not give up."

Almgren's insistence on rigor, unlike some mathematicians', was never pedantic, but arose from an acute insight into the crux of a problem. Elliot Lieb, who coauthored about twelve papers with Fred, experienced this firsthand. "Fred took the sketch of a proof that we had and started asking embarrassing questions about some of the little cracks that eventually turned out to be fault lines. Not only that, he was able to use our errors to turn matters upside down, to find out what the true answer ought to be."



Almgren in the classroom at Princeton, circa 1978. "In his geometric measure theory class, Almgren sometimes taught from dry, technical yellowed notes, like Federer's text. Just when it was becoming unbearable, he would put the notes down and speak extemporaneously of the beautiful underlying geometric ideas. Every such minute was a lesson of a lifetime."
—Frank Morgan, graduate student '77



Fred's interest in the phenomena of the real world was never far below the surface. Almgren with John Sullivan, graduate student '90.

Every graduate student of Almgren was familiar with these "embarrassing questions". It was not Fred's style to say a student was wrong or needed to work on something more. Aaron Nung Kwan Yip, one of Almgren's most recent doctoral students, recalls, "When I came to him with complicated ideas and unpolished proofs, within minutes he would ask, 'How do you do this step?', already understanding the most crucial obstacle of the proof. Through such encounters I began to learn how to simplify problems and focus on the essential difficulties." In a similar vein, Gary Lawlor, a student of Frank Morgan's (and hence Fred's academic "grandchild"), says, "I remember when I told him about a proof for a minimal Möbius band. He wondered whether the geometric configuration I was using would work topologically and asked if that might be a problem. I said, no, it's correct, and he amiably acquiesced. It wasn't till later that I realized on my own that he was right."

But the reverse was also true: Fred could tell a student was making progress even when the student had no idea of it. His first Ph.D. student, Jean Taylor, says her mood almost always reversed when she came to talk with him: "When I'd come in feeling I was making zero progress, I always came out feeling much better, because lo and behold, the thing I was stuck on and the progress I was making on it were something worth doing." Working with Fred, a student learned that "getting stuck" was something to be welcomed, not dreaded, because it meant you had gotten to the heart of the problem. Brian White, Fred's eighth doctoral student, says that Fred gave him the following advice: "When you're working on a problem, you should focus most of your effort on the part where things are least likely to work out."

Fred's persistence and commitment to getting all the details right reached their zenith in a legendary 1,720-page paper, the culmination of his work on the singularities of area-minimizing surfaces over an eleven-year span, from 1971 to 1982. The paper, in Lieb's words, has only been circulated "in samizdat", being far too long for any journal to print. Sadly, this paper represents a journey on which Fred had very few followers; its length and technicality are just too forbidding. Perhaps it will yet be interpreted in a more accessible form. At present Vladimir Scheffer, one of Fred's doctoral students, is retyping the entire manuscript in TeX and hopes to distribute it on floppy disks.

After Fred completed his magnum opus, a new era in his career began, when he began to stretch mathematicians' ideas of how mathematics ought to be done. In 1985 he became one of the founders of the Geometry Supercomputer Project, later the Geometry Center, in Minneapolis. As Albert Marden, the first director, explains it, the center was founded "to do something that had never been done in math—to try to use visualization as a means for doing research and for communicating research mathematics to the world." To mathematicians at that time the idea of doing mathematics with pictures had a touchy-feely sound to it; Marden says he was often asked, "What theorems have come out of this?" But no one could question Fred's commitment to the highest standards of scholarship.

One theorem that came out of the work of the minimal surfaces team at the Geometry Center, albeit in an indirect way, was the recent disproof of the century-old Kelvin Conjecture by Denis Weaire and Robert Phelan. Kelvin had conjectured a plausible solution to the problem of dividing space up into cells in such a way that the interfaces have the least possible area—a three-dimensional version of the planar problem that bees apparently solve with their honeycombs. John Sullivan, a graduate student of Fred's in the late 1980s, credits him with being the first mathematician to consider seriously the possibility that Kelvin's solution was wrong. "The Kelvin problem is mentioned in Hermann Weyl's *Symmetry*, and in D'Arcy Thompson's *On Growth and Form*," Sullivan writes. "I think most people assumed that his partition was best.... But Fred alone was convinced it could be beaten." Five years later Weaire and Phelan's work proved that Fred's intuition was right: a less symmetric foam could divide up space more efficiently. Though the honor of finding a better way of dividing space than Lord Kelvin eluded Fred's team, the Surface Evolver software developed at the Geometry Center by Kenneth Brakke (one of Almgren's earliest graduate students) was used by Weaire and Phelan in constructing their example. The Evolver has been used also for the design of fuel tanks for spacecraft (which rely on surface tension, not gravity, to get the fuel from the tank to the combustion chamber) and for modeling the behavior of solder in assembly of computer components to avoid unwanted short circuits. Thanks to the computer and to Fred, the abstract machinery of minimal surfaces has finally begun solving engineering problems.

In recent years Fred aided two notable journal startups, both dedicated to his geometric and visual style of doing mathematics. David Epstein, an early member of Fred's minimal surfaces team at the Geometry Center, founded and edited *Experimental Mathematics* and calls Fred a "tremendous help and support." Steven Krantz, the founding editor of the *Journal of Geometric Analysis*, calls Fred "one of the most active associate editors, who brought a number of excellent articles our way." Krantz and Harold Parks (also an Almgren student) dedicated their book *A Primer of Real Analytic Functions* to Fred.

Fred was unusual among his colleagues for the attention he gave to all the students he came in contact with, even if they were not "his" or not graduate students. Stephen Miller, a current Princeton graduate student who met Fred when he went to him to take a language exam, writes, "I quickly learned who to go to for advice. I'm embarrassed to mention how many stupid questions I asked him, but he was extremely patient with me. Going to his office was like entering a friend's home." Steven Strogatz, now a professor at Cornell, recalls this experience when he was having trouble with his undergraduate thesis:

Fred Almgren with his wife, Rutgers mathematician Jean Taylor, outside their home on Riverside Avenue in Princeton, a familiar gathering place for friends and family for over twenty years.

> One day in mid-January I screwed up my courage and entered Fred's office to ask his permission to give up the quest for a theory of tangling. I remember that my heart was beating fast. But instead of the disapproval I had feared, Fred was gentle and completely understanding. He told me that he now realized that tangling was much too hard for a senior thesis problem.... When I described some alternative problems that were both biologically important and solvable by an undergraduate, he beamed at me with that unforgettable Almgren smile. From then on, Fred became my cheerleader.... At a pivotal time in my career, Fred Almgren pointed me in the right direction, showed me great kindness, and taught

me about the joys of mathematical exploration.

Such joy was equally evident in the other activities that Fred pursued with passion. He was a founder of the annual Princeton-Rutgers Mathematicians' May Day Relay Race, which has now been renamed in his honor. At times he jestingly accused his competition of rigging the race by admitting graduate students on the basis of their times in the mile instead of their mathematical abilities. An avid sailor, kayaker, whitewater rafter, and windsurfer, he seemed to enjoy nearly any activity that involved adventure. His constant companions were Jean Taylor and the rest of his family, and he excelled at getting others involved and getting them to enjoy the experience. His sister, Linda Almgren Kime, recalls, "I remember Fred's look of exuberance as Jean excitedly covered his face with war paint and we whooped and plunged over Colorado's most fearsome rapid, Lava Falls."

Fred was also a wine connoisseur, who frequently offered a bottle as a prize for solving an especially important or challenging problem. Though he knew nothing about cooking when he married Jean in 1973, he took classes in cooking at the Princeton Adult School and became an expert chef. As always, he was eager to share his knowledge: John Sullivan and others recall lessons in cooking spaghetti carbonara as part of their graduate education. The Thanksgiving feasts at Fred's house became legendary among his family and friends: Jean would cook, Fred would carve, and extended-family members Bob Miner would bring wine and Bonnie Scott would bring pumpkin and Dutch apple pies. There were so many guests that the table would stretch out of the dining room and eight feet into the living room.

In his career at Princeton, which spanned thirty-five years from his original hiring as a non-tenure-track instructor, Almgren produced sixty-seven published papers—not including his mammoth 1,720-page paper or the two papers in preparation when he died—and served as mentor to nineteen Ph.D. students. In the Bernoulli tradition he even contributed his own progeny to mathematics: two of his three children, Robert and Ann Almgren, are mathematicians, and the third, Karen, is a math major at the Massachusetts Institute of Technology. According to Robert, who now teaches at the University of Chicago, "He was known to say, at least once, 'I don't put any pressure on my son to follow me in my work. He can do anything he wants to. He can be an algebraist, a topologist, a geometer....' But the funny part is that he actually put no pressure on his kids to follow him.... The reason [they did] is that it was obvious to all of us how much joy our father took in his work."