# Michael H. Freedman Awarded 1986 Veblen Prize

Oswald Veblen (1880 - 1960), who served as President of the Society in 1923 and 1924, was well known for his mathematical work in geometry and topology. In 1961 the Trustees of the Society established a fund in memory of Professor Veblen, contributed originally by former students and colleagues, and later doubled by his widow. Beginning in 1964 the fund has been used for the award of the Oswald Veblen Prize in Geometry. Subsequent awards were made two years later and, thereafter, at five-year intervals. A total of ten awards have been made: Christos D. Papakyriakopoulos (1964), Raoul H. Bott (1964), Stephen Smale (1966), Morton Brown and Barry Mazur (1966), Robion C. Kirby (1971), Dennis P. Sullivan (1971), William P. Thurston (1976), James Simons (1976), Mikhael Gromov (1981), and Shing-Tung Yau (1981). At present the award is supplemented from the Steele Prize Fund, which brings the value to four thousand dollars.

The 1986 recipient is Michael H. Freedman, University of California, San Diego. The prize was awarded by the Council of the Society on the basis of a recommendation made by a selection committee consisting of R. H. Bing, Richard K. Lashof, and Shing-Tung Yau, chairman. The award was presented at the Annual Meeting of the Society in New Orleans on January 8, 1986.

The committee's citation appears below, followed by the recipient's response on receiving the award and a brief biographical sketch of the recipient.

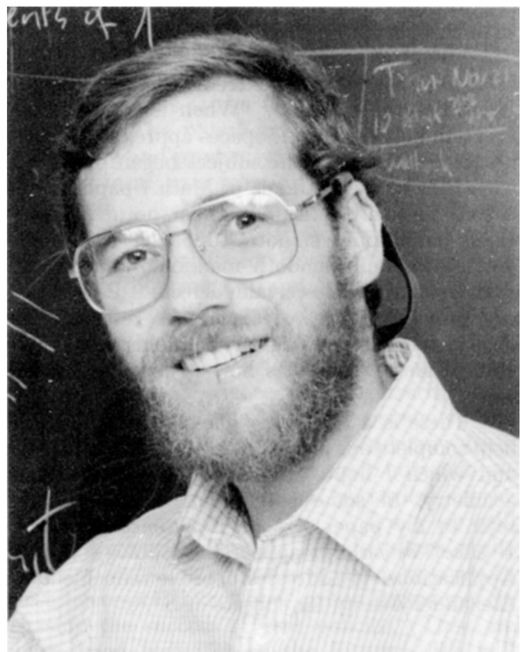
## Citation for Michael H. Freedman

After the discovery in the early 60s of a proof for the Poincaré conjecture and other properties of simply connected manifolds of dimension greater than four, one of the biggest open problems, besides the three dimensional Poincaré conjecture, was the classification of closed simply connected four manifolds. In his paper, *The topology of four-dimensional manifolds*, published in the Journal of Differential Geometry **17** (1982), 357-454, Freedman solved this problem, and in particular, the four-dimensional Poincaré conjecture. The major innovation was the solution of the simply connected surgery problem by proving a homotopy theoretic condition suggested by Casson for embedding a 2-handle, i.e., a thickened disc in a four manifold with boundary.

Besides these results about closed simply connected four manifolds, Freedman also proved:

a) Any four manifold properly homotopy equivalent to $R^4$ is homeomorphic to $R^4$; a related result holds for $S^3 \times R$.
b) There is a nonsmoothable closed four manifold.
c) The four-dimensional Hauptvermutung is false; i.e., there are four manifolds with inequivalent combinatorial triangulations.

Finally, we note that the results of the above-mentioned paper, together with Donaldson's work, produced the startling example of an exotic smoothing of $R^4$.



## Response

I thank the AMS and its Veblen Committee for this honor. I have had many teachers in mathematics; in recent years my students have been among them. I thank all of them, students and mentors. But I owe a special debt to Bob Edwards, who taught me the branch of geometry, "Bing topology," which plays a central role in the work which the AMS has recognized with this award.

My primary interest in geometry is for the light it sheds on the topology of manifolds. Here it seems important to be open to the entire spectrum of geometry, from formal to concrete. By spectrum, I mean the variety of ways in which we can think about mathematical structures. At one extreme the intuition for problems arises almost entirely from mental pictures. At the other extreme the geometric burden is shifted to symbolic and algebraic thinking. Of course this extreme is only a middle ground from the viewpoint of algebra, which is prepared to go much farther in the direction of formal operations and abandon geometric intuition altogether. A pair of contemporary examples should illustrate the two poles of geometric thinking. Thurston's work on hyperbolic geometry is concrete, you might almost say *visible*, you can see it if you have the strength, I believe. Hamilton's work on Ricci curvature has a large admixture of algebra—he does not, I think, fire up every neuron in an effort to picture $g_{ij}$ moving toward $R_{ij}$—rather, he counts indices and gets (2, 0) in both cases. (His full proof is, of course, considerably deeper than this sketch.)

"Bing topology" or decomposition theory resides near the concrete extreme of the spectrum. It studies the question, "When is a continuous function between metric spaces approximable by a homeomorphism?" The subject began in earnest with Bing's 1952 (Annals of Math.) paper about a construction on the Alexander horned sphere. Using traditional smooth topology, it is often possible to build smooth manifolds and smooth isomorphisms with certain defects. Bing's theory provides a way of sometimes resolving the defects to yield topological manifolds and homeomorphisms. That was how it entered into my work.

There is a remaining question which should help complete our picture of topological manifolds, and which I hope will reduce to a problem in geometry, of some sort. For a statement, I refer to my paper *Are the Borromean rings A-B slice?* which will appear in Topology and Its Applications. It addresses possible refinements of Alexander duality for subsets of $S^4$.

In the nineteenth century there was a movement, of which Steiner was a principal exponent, to keep geometry pure and ward off the depredations of algebra. Today I think we feel that much of the power of mathematics comes from combining insights from seemingly distant branches of the discipline. Mathematics is not so much a collection of different subjects as a way of thinking. As such, it may be applied to any branch of knowledge. I want to applaud the efforts now being made by mathematicians to publish ideas on education, energy, economics, defense, and world peace. Experience inside mathematics shows that it isn't necessary to be an old hand in an area to make a contribution. Outside mathematics the situation is less clear, but I can't help feeling that there, too, it is a mistake to leave important issues entirely to the experts.

## Biographical Sketch

Michael H. Freedman was born April 21, 1951 in Los Angeles, California. He received his Ph.D. from Princeton University in 1973.

Professor Freedman gave addresses in AMS Special Sessions on Algebraic and geometric topology at the Far Western Sectional Meeting in Hayward, California (April 1977) and on Dehn's lemma at the Annual Meeting in San Francisco, California (January 1981). He also gave Invited Addresses on Bing topology, infinite procedures, and the Poincaré conjecture in dimension four at the AMS Far Western Sectional Meeting in Bellingham, Washington (June 1982) and on four-dimensional manifolds at the AMS Annual Meeting in Denver, Colorado (January 1983).

Professor Freedman was awarded a Sloan Fellowship in 1980. In 1984, he was named the California Scientist of the Year and was elected to the National Academy of Sciences. Freedman received a MacArthur Fellowship in 1985.

Professor Freedman is currently a professor of mathematics at the University of California, San Diego.