1982 Cole Prizes in Number Theory Awarded at Annual Meeting in Cincinnati

Two Cole Prizes in Number Theory were awarded in Cincinnati on January 14. The recipients were Robert P. Langlands of the Institute for Advanced Study and Barry Mazur of Harvard University.

Professor Langlands received his prize for pioneering work on automorphic forms, Eisenstein series and product formulas. The selection committee noted that he had proved Artin's conjecture that $L(s,\chi)$ is holomorphic for all characters χ of 2-dimensional representations of tetrahedral type (*Base change for* GL(2), Annals of Mathematics Studies, volume 96, Princeton University Press, 1980).

Professor Mazur received his prize for outstanding work on elliptic curves and Abelian varieties, especially on rational points of finite order. The committee cited his paper *Modular curves and the Eisenstein ideal*, Publications Mathématiques de l'Institut des Hautes Études Scientifiques, volume 47 (1977), pages 33 to 186.

The prizes were awarded by the Council of the American Mathematical Society on the recommendation of a selection committee consisting of J. W. S. Cassells, Wolfgang M. Schmidt (chairman), and John T. Tate.

Frank Nelson Cole served the Society for twenty-five years as Secretary and for twenty-one years as Editor-in-Chief of the *Bulletin*. When he retired from these positions, his friends collected a sum of money in his honor, which he in turn offered to the Society. The Council, on accepting the gift, created the "Cole Fund" which has been used for more than fifty years for the award of the Cole Prizes in Algebra and Number Theory. The names of the recipients of these prizes will be found in the November 1981 *Notices* on pages 650 and 651.

The original fund grew both from earnings and from additional gifts, including a gift made in 1929 by Charles A. Cole, Professor Cole's son, which more than doubled the size of the fund. In recent years the Cole Prizes have been augmented by awards from the Leroy P. Steele Fund, and currently amount to $1500 each.

At the presentation ceremony in Cincinnati, Professor Langlands expressed his appreciation *ex tempore*. Professor Mazur prepared a more formal response, which is reproduced below.

Barry Mazur

I should like to convey my warm thanks to the American Mathematical Society and the Cole Prize Committee. One of the great joys of writing the paper to which the Cole Prize Committee referred was the continual correspondence and conversation with many friends, on the subject of modular curves, for which I am also thankful.

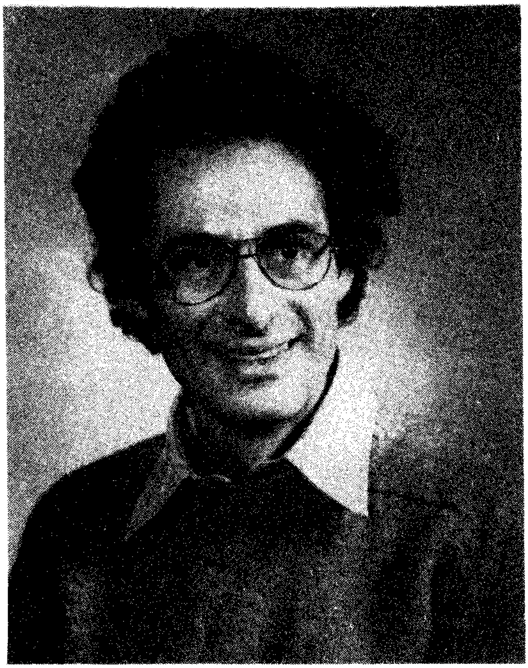
Modular curves provide felicitous meeting grounds for a multitude of diverse problems in Arithmetic. Rather than discuss my own paper specifically, I should like to take this opportunity to hint at some of these problems, to enumerate some facets of modular curves which serve to throw light on them, and finally, to suggest that the adjective "diverse," which I used in the previous sentence, is not terribly apt: There seems to be an overarching unity to this cluster of problems, so that, surprisingly often, consideration of one of these problems reveals the secrets of another.

Rational points of modular curves are essentially in one-to-one correspondence with elliptic curves endowed with specific structures,[1] defined over the field of rational numbers. This is the simplest instance of a fundamental theory of Shimura (the canonical models). There is, then, a novel twist to the venerable diophantine problem of studying these points.[2] For, firstly, every time you find a rational point you get something quite interesting; secondly, you can apply the formidable techniques of the arithmetic theory of elliptic curves to analyze what it is you have gotten. Moreover, there is the classical theory of complex multiplication at your disposal to produce a further bunch of points (*Birch-Heegner* points) which are defined over fields of low degree, and which play an important, but still mysterious, role. And then, of course, there are the cusps (studied by Ogg, Manin-Drinfeld, Kubert-Lang).

Elliptic curve factors of the jacobians of modular curves have been conjectured (Taniyama-Weil) to exhaust the class of *all* elliptic curves defined over the rational number field. This conjecture would establish a link between modular curves and elliptic curves, utterly different from that described in the previous paragraph, and indeed much deeper. Merely to exhibit an elliptic curve as such a factor often yields number-theoretic information otherwise unattainable, and sometimes even provides "conceptual methods" for generating its Mordell-Weil group (e.g., by specific images of Birch-Heegner points).

[1]Precisely which specific structure depends on which modular curve you are dealing with.

[2]For example, this diophantine problem (restricted to a certain well known class of modular curves) is essentially equivalent to the determination of all possible torsion subgroups of Mordell-Weil groups of elliptic curves defined over the rational number field.



Barry Mazur

The *classical geometry* of modular curves viewed as quotients of the Poincaré plane comes to the service of Arithmetic insofar as a close study of homology classes supported on various geodesics of the modular curves is an essential pre-requisite to understanding *special values* of a class of important Dirichlet series (among these are the *special values* which appear in the Birch-Swinnerton-Dyer conjecture).

The *function theory* of modular curves has been widely known for over a century to have rich consequences for number theory: The coefficients of Fourier expansions of functions and sections of line bundles over modular curves often encapsulate much coveted, and otherwise inaccessible arithmetic information. The work of Serre, Swinnerton-Dyer, and Katz establish an arithmetic theory of these Fourier coefficients. This ties in with the fascinating structure of the reductions of models of modular curves in prime characteristics (as studied by Igusa, Deligne-Rapaport, and Katz).

The *fields generated by division points* on the jacobians of modular curves are usually nonabelian extensions of the rational number field whose arithmetic can be studied in great depth by virtue of their connections with the modular curves (Shimura, Serre, Ribet, Wiles). Can one generate all class fields of abelian number fields in this manner? We can, if we wish, reverse direction and use the algebraic number theory of these fields to gain an understanding of the Selmer groups of the modular curve jacobians via classical and modern methods of descent.

To be sure, there is also the Langlands program which views a modular curve as a symmetric space and relates aspects of its arithmetic to the theory of automorphic representations of various reductive algebraic groups.

These are some distinct views and problems, fostered by the study of modular curves, each one having deep implications for the others. We have begun to suspect that there is a unity to all this. Much work lies ahead.

Biographical Sketch

Barry Mazur was born on December 19, 1937, in New York City. In 1958 and 1959 he was a Research Fellow at the Institute for Advanced Study in Princeton. He received his Ph.D. degree from Princeton University in 1959. He was a Junior Fellow in the Harvard Society of Fellows from 1959 to 1962. From 1962 to 1966 he was a fellow of the Alfred P. Sloan Foundation. In 1966 he was awarded the Society's Oswald Veblen Prize in Geometry. He is currently professor of mathematics at Harvard University and a frequent visitor to the Institut des Hautes Études Scientifiques in Bures-sur-Yvette.

Robert P. Langlands

Biographical Sketch

Robert P. Langlands was born in New Westminster, British Columbia, on October 6, 1936. He received a B.A. degree in 1957 and an M.A. in 1958, both from the University of British Columbia, and a Ph.D. from Yale University in 1960. He taught at Princeton from 1960 to 1967, and was professor at Yale from 1967 to 1972. He was a member of the Institute for Advanced Study in 1962-1963, and has been professor of mathematics there since 1972.

Professor Langlands has served on the AMS Organizing Committee for the Summer Institute on Automorphic Forms, Representations and L-Functions (1977), the Interim Editorial Committee for Research Announcements (1978), and the *Bulletin* Editorial Committee (Associate Editor, Research Announcements) (1979, 1980). He has given addresses at the Summer Research Institute on Algebraic Groups and Discontinuous Subgroups (Boulder, July 1965), the Symposium on Mathematical Developments Arising from the Hilbert Problems (DeKalb, Illinois, May 1974), the 1977 Summer Research Institute on Automorphic Forms, Representations and L-Functions, and at the International Congresses in Nice (1970) and Helsinki (1978).

Professor Langlands was a Miller Foundation Fellow at the University of California (Berkeley) in 1964-1965, and an Alfred P. Sloan Foundation Fellow, 1964 to 1966. His research interests include group representations and automorphic forms.

He was elected a Fellow of the Royal Society of Canada in 1972 and a Fellow of the Royal Society in 1981. In 1975 he received the Wilbur Cross Medal of Yale University. He is a member of the editorial boards of *Annals of Mathematics* and *Compositio Mathematica*, and has held visiting positions at Orta Doğu Teknik Universitesi in Ankara, 1967-1968, Universität Bonn in 1970-1971 and 1980-1981, and at École Normale Supérieure de Jeunes Filles, 1980.