Interview with Ennio De Giorgi

Michele Emmer

Q. *How did you become a mathematician?*

I think that every mathematician has a different story to tell. As for me, as a child I had a special taste for puzzling out solutions to little problems, but I also had a certain passion for experimenting with little gadgets—experiments, if not of physics, of "pre-physics". After Liceo I was placed in the first year of engineering. In those days the courses of study in mathematics, engineering, and physics were the same for the first two years. It was in that first year that I realized that my natural aptitude was, above all, in mathematics. I think it has been a big mistake to separate out the study of these different disciplines. In the first place, first-year students do not so carefully differentiate their interests, so there results the possibility of losing a certain number of people with great mathematical potential. Secondly, those who are initially disposed toward mathematics and choose that course of study miss the direct contact with those sciences which have significant links to mathematics, and potentially end up in mathematical isolation from those disciplines with which mathematics must maintain a constant active connection.

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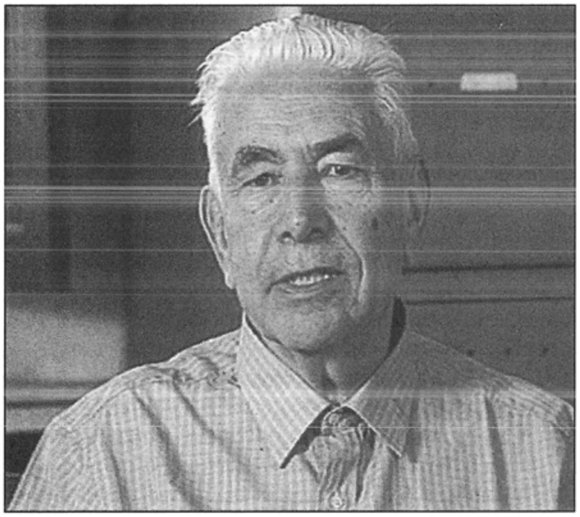
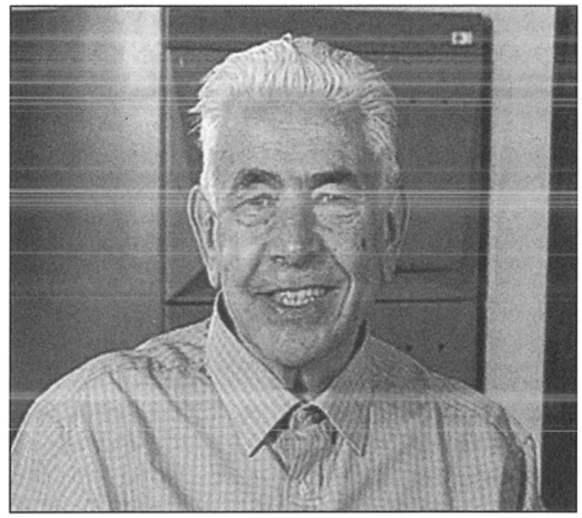
Q. *What are the links between mathematics and physical reality—"the unreasonable utility of mathematics"?*

I think that the reason for the usefulness of mathematics in reality—not just physical, but also biological, economic, etc.—is a mystery. For me the most suggestive indicator is in the Book of Proverbs, one of the most ancient books of the Bible, which at a certain point says that wisdom (which is wider than mathematics) was with God when He created the world and that this wisdom is to be found by men who search for it and adore it. Mathematics is one of the most significant manifestations of the love of wisdom. On the one hand, there are no boundaries in mathematical thought and imagination, but on the other hand there is the reality that the world is made of things both visible and invisible and that mathematics is the unique science with the capacity to pass from the observations of visible things to the imagination of things invisible. This is perhaps the secret of the strength of mathematics.

Another aspect of mathematics which is one of the secrets of its strength is its liberty and conviviality. The mathematician has a freedom which other scientists do not have: to think of things solely because they are the most interesting; to select the problems which are the most beautiful and the ways of attacking them which are the most beautiful; finally, to freely set the axioms from which the theory follows. On the other hand, the mathematician feeds on dialogue with others: to solve a mathematical problem without having a friend to whom to expound the solution and with whom to discuss also the nature of the problem





and its importance would be to lose a good part of the flavor of mathematics.

So I think that the basis for the strength of mathematics is precisely this: the knowledge of how to unite freedom of initiative, this capacity of working alone, with the gregarious side, the ability to exchange ideas and interests. In Liceo already I was pleased if there was a theorem for which I could construct a proof different from that of the book. That pleasure was incomplete until I could share it with more informed colleagues, even less informed, so long as they were disposed to listen and comment.

This combination of free-flying imagination within the confines of one's study with the activity of exchanging these flights with others—scientists, thinkers in other disciplines—philosophy, art, letters—is the strength of mathematics. This double aspect is, according to me, the reason for its fascination and possibly also the secret of its strength. This is at its basis one of the strongest manifestations of the love of that knowledge from which science is born and the resulting human capacity to partially understand the world, without forgetting the famous words of Shakespeare: "There are more things in heaven and earth than are dreamt of in all your philosophy" [Hamlet to Horatio].

This also explains why, in mathematics, there is no conflict between innovation and tradition, the two sources of everything great and beautiful which mathematicians have done. In mathematics these are in harmony. To illustrate: one finally understands the force of Pythagoras's theorem when one comes to infinite-dimensional Hilbert spaces and discovers that even there there is the equivalent of the Pythagorean theorem.

This is part of a vision—more precisely, of a mystique: the concept that science is a part of wisdom and, further, that there is a direct link between science and human rights. For example, there is the beautiful article (of the declaration of human rights) about the school which recommends not only tolerance but also understanding and friendship between the various nations and the various religious groups. These, comprehension and friendship, are two notions which are often forgotten when one talks of tolerance. Pure and sentimental tolerance is insufficient; only when united with understanding and friendship does it truly allow human activity to progress. In particular, the sciences cannot move forward without understanding and friendship among all scientists. Understanding among religious groups also supposes that each explains with great simplicity and much naturality their ideas, those religious principles in which they truly believe. As for me, for example, what particularly interests me is the proposition of resurrection. The idea of the resurrection— that life does not end in the brief arc of years which we have here, that even our loved ones who have passed away still live in some way—is one of the fundamental elements of my life and even of my research activity. I am able to continue to study, to imagine new things even at an age which one could say is the end of my academic career, because I see this as a journey throughout which, until the end, one must love knowledge completely, expecting that this love will continue in another form even after death.

Q. *Is your academic and scholastic environment important to you?*

Mathematics calls for freedom and the opportunity for personal reflection on the one hand and on the other searches out discourse with others. Thus, having a stimulating environment—professors, students, friends disposed to friendly discussion of mathematics, science and philosophy—is essential to the formation of a mathematician, as well as for that of any other philosopher. For example, at the Institute for Mathematics in Rome I studied with and received my degree from Professor Picone, who was, as an academic, faithful to the style of those days, that of the so-called "Baron", but who, in discussions of scientific problems, was completely open. I remember that while I was still a student, he said, "Mind that when we speak of scientific problems you are completely free to tell me that I am mistaken, because we are equal in front of science." So he was extremely liberal in scientific dialog but fully respectful of the discipline and the academic customs of the day. For these reasons Picone became one of the great teachers and had many very diverse students, like Fichera, Caccioppoli, and many others, with greatly differing personalities and interests, even in mathematics. Everyone was attracted to Picone because of his accessibility, his interest in all problems, whether those which he personally had studied and resolved or those which interested whoever came to speak with him.

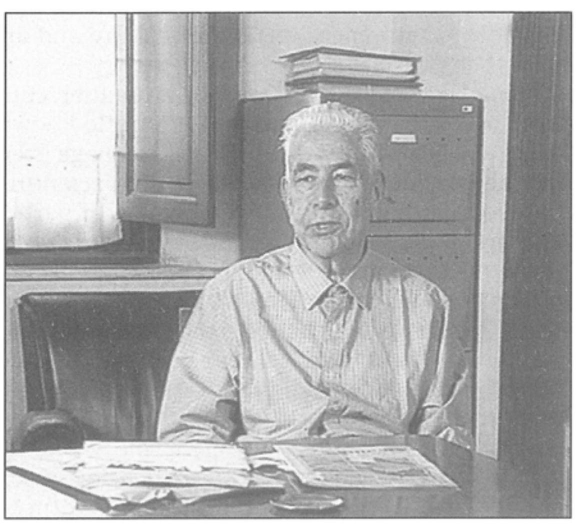
Q. *What is the role of creativity in mathematics, compared to that of other disciplines of human activity?*

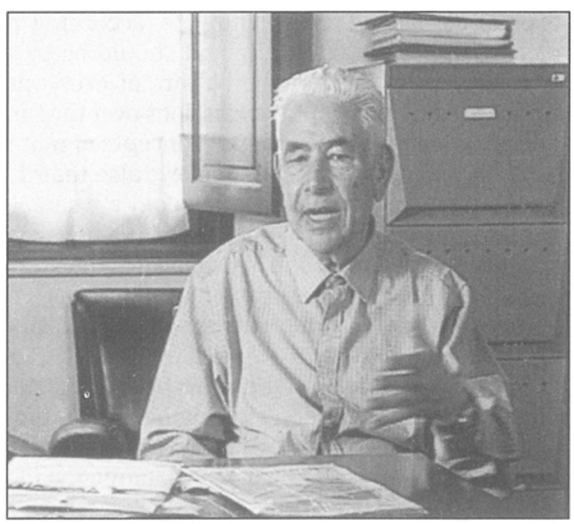
I think that the origin of creativity in all fields is that which I call the capacity or disposition to dream: to imagine different worlds, different things, and to seek to combine them in one's imagination in various ways. To this ability—very similar in all the disciplines—one must add the ability to communicate those dreams unambiguously, requiring knowledge of the language and internal rules of the various disciplines. I believe this must be an ability to dream in an uncompartmentalized way, in the way called philosophy in antiquity. So, for the love of knowledge and confidence in communicating one's dreams unambiguously we must study the various languages, the differing theories of the various disciplines, and even of the arts—all the forms of human knowledge. This ability, initially to communicate to others in various ways, is in the end the way to clarify things for oneself. For when one successfully communicates something, in reality one must make it clear for oneself; every person who has the experience of teaching knows that after teaching a subject one understands it more deeply than before. Through communication with others and by means of that communication and listening to the reactions of others, we deepen our own thoughts.

What I would like to make most clear in this interview is that, with the passage of time, I have come to see clearly that the fundamental idea common to all sciences and arts is that love of knowledge has many faces.

We must recognize that human nature and human language need, in order to be clear and unambiguous, to withdraw from time to time in particular specialized points of reference. At the same time, we must beware of closing ourselves in a specialization, a narrow branch of mathematics, lest our creativity in this field wither away. We must know and respect the language, methods, and criteria particular to several disciplines and avoid the reductionism which attempts to constrain the methods and the language of all disciplines to the methods and language of a single discipline. Such strict adherence to a branch of a discipline renounces every minimal rule of coherence and precision, without which discussions are free-wheeling discussions from which no one can extract any precise meaning. The advice which I give to everyone is this: think expansively, with great freedom, but also force yourself to communicate to friends and others your thoughts in a way which is comprehensible, clear, and unambiguous. In this way you will see effectively if you have found the right form for your thoughts.

Q. *In recent years the answer one hears to the question, What does a mathematician do? often is, Works with the computer. Do you think that today there is a connection between mathematical research and the use of the calculator? Will it have a future development, or do you think that its use will be marginal?*





I think that the computer is a useful aid for those who know how to use it with a certain amount of security and freedom, or at least for those who have friends who know how to so use it, also with imagination. Clearly, the computer can become dangerous if one thinks of it as a substitute for fantasy. Freedom to fantasize in one's own mind must be retained. We should see in the computer a means of testing hypotheses or as a source of suggestions showing us strange phenomena for which we can try to imagine an interpretation. The computer is also a means for organizing our expositions. It is certainly a useful and important aid as a source of problems, to find out what is computable and what is not, and sometimes simply to execute a certain operation. Undoubtedly from this point of view the very existence of computers presents a problem and also a source of new ideas for mathematics. One could talk about an abstract theory of computable functions which can be developed even in the absence of computers; the basic idea of a calculating machine is sufficiently old. Even Pascal and Leibniz thought for many years about mechanizing mathematical calculations in some way; there has long been the stimulus to think about automating things, ideally as well as practically. Finally, there is the study of the computer other than as a useful instrument for mathematical work—as a useful object for mathematical reflection.

Q. *What do you think is the importance, from the cultural point of view, of clarifying all this?*

I think that reflection on what the fundamental concepts of mathematics are, which axioms to try to represent clearly and unambiguously, is one of the most important cultural aspects of mathematics, both ancient and modern. This is not just of our times; the continuing discussion over the postulates of Euclid began already in antiquity. There also has been a long dispute, not yet completely resolved, on the foundation of the infinitesimal calculus. These arguments, according to me, are important culturally and should be part, at least superficially, of the culture of everyone. When I could, I sought out discussions over the fundamental axioms, fundamental concepts of mathematics, and other sciences. I believe also that the axiomatizations of physics and biology are equally necessary, so as to put in order what the physicist or the biologist thinks of life, organisms, the brain, etc.

The attempt to axiomatize is, so to speak, simply an attempt to say with the greatest possible clarity and simplicity what seems to be the starting point for our discussions of mathematics, biology, physics, economics, etc. I found a great interest in such questions and discussions among the faculty of economics in Rome.

A method of placing in the simplest possible schematic model the fundamental truths of a discipline, or at least to make a proposal for the fundamental concepts and methods, enunciating axioms, and also trying to make precise the sense in which these axioms are to be taken—I think that this is culturally valid and most important. Above all it is important to not think of this as a specialist's job. If one thinks that one should speak and listen to only the specialists for the foundations of mathematics, one has lost the cultural significance of research in the foundations of a discipline. I am not saying that specialized research on the theorem of Gödel or on undecidable questions are not important and should not be developed. However, it is also important to succeed in having every cultured person understand at least that the roots of these questions are in perfectly understandable problems. Of course, this requires will and work on the part of the cultured person. In antiquity there was a lively discussion over irrational numbers which was, at bottom, a philosophical discussion on foundations: are numbers only the integers, or can we conceive that there might be more? In this discussion, as in the discussion of the possibility, for example, of actual or potential division of a segment into infinitely many parts, one sees a great affinity between modern problems and those of antiquity. For example, the old paradox of the liar is at present, in various forms, one of the central themes of logical thought in mathematics. Finally, this interest, according to me, of the cultured person of the Greek world in the paradox of the liar or the problem of the existence of irrational numbers and analogous curiosities, could be—in fact, should be—central elements of debate and reflection in the modern culture. The cultured person should understand that even modern technical results, like the theorem of Gödel, cause us to reflect anew on fundamental questions about the natural numbers.

[De Giorgi muses over the possibility of presenting foundational issues in a manner comprehensible to the general person of culture and argues that specialists always must try.]

Language will evolve, and in a manner conforming to the development of scientific knowledge, to a beautiful and expressive language even for the exposition of the most modern scientific theories and the succeeding dispersion of particular problems. In such a language we can present to the public more profound problems: for example, on the significance in mathematics of the word "exist", on the relation between physical reality and mathematical reality. The entire range of passage from the particular to the general, from antiquity to the modern, can be presented to the curious public. Also, let's take note of the great role which the history of science has in the comprehension of scientific disciplines without falling into that form of reductionism which is historicism, which is to think that we know all there is to know of a certain argument once we know the history of that argument.

While studying the initial ideas of a particular person, let's keep in mind the implications of those ideas, the potential that they have, which was realized much later without attributing to that person ideas which did not exist in his time. One who has written certain axioms could not know all the theorems which follow, but our knowledge of these theorems without doubt helps us to grasp the initial intuition. Recognize the importance of history, but don't restrict attention just to historical accounts, and keep in mind the uses of history. While looking at those initial ideas in the context of the time, also look at how they have developed throughout time even if expressed in different ways.

Certain themes return again and again in widely separated epochs and in different ways; thus it is essential to have this double vision: of how things were at the time and how they have developed since. This is what I mean by the "meta-historical" perspective.

Q. *What are your ideas on discovery and invention in mathematics? Axioms are set, and then thousands of years later it is discovered that they have significant consequences. What is the role of intuition?*

This is the role of the existence or the reality of mathematical objects. Invention and discovery have much in common: both come from searching. By discovery we mean "pulling the cover off" something which is already there, bringing to light something which was hidden; by invention we mean a construction out of that which seems to be laying about. The issue of whether a particular new idea was uncovered or constructed often cannot be resolved. At the bottom is the eternal issue: what does it mean to recognize something, to know it? What is invention, what is discovery? This is a discussion circling around the mysterious course of knowledge. Further compounding this is the question of "existence" of assertions. For example, an axiom stated in one century has as a necessary logical consequence a theorem whose proof is found many centuries later. When did the theorem begin to exist? We speak of a theorem as "discovered" and a proof as "invented", but a "discovered" object has always been there, while an "invented" object is created on the spot. Or is it just the stumbling upon the particular one of many streets emanating from the axioms which leads to the theorem? I think that proof is an invention—a construction of a road leading to the theorem. It happens sufficiently frequently that two mathematicians prove in independent ways the same theorem as stated, and the proof is rarely the same proof. Thus a theorem is something discovered; its proof something invented.

Q. *Would you like to add anything else?*

An example from my personal experience. Nash and I proved the same theorem,[1] or, rather, two theorems very close to each other. From the theorem of Nash one can deduce more or less immediately my theorem, following a quite different line of proof. Thus, from my experiences, the discovery of a theorem can be made by different people, as if it were there waiting for someone to uncover it, and the statement of the theorem is always the same. However, the invented proof can vary greatly according to the mathematician who finds it. Very often the initial proof of a theorem is very complicated, but eventually, by thinking about it in different ways, we come to simplify it and render it more elegant, making it adaptable for proofs of more general theorems; to me this is the invention of more and more useful proofs of the discovered theorem. Alas, from the logical point of view one might say that a proof is no more than a chain of propositions, each one of which can be considered a theorem, so that invention is no more than a succession of discoveries.

[1] *De Giorgi is speaking of the well-known De Giorgi-Nash theorem on the regularity of elliptic differential equations. E. De Giorgi,* Sulla differenziabilità e l'analiticità delle estremali degli integrali multipli regolari, *Mem. Accad. Sci. Torino Cl. Sci. Fis. Mat. Nat.* ***3*** *(1957), 25–43; J. Nash,* Continuity of solutions of parabolic and elliptic equations, *Amer. J. Math.* ***80*** *(1958), 931–954.*