Interview with Henri Cartan

The following interview was conducted on March 19–20, 1999, in Paris by *Notices* senior writer and deputy editor Allyn Jackson.

Photograph by Sophie Caretta.

Henri Cartan, 1996.

Biographical Sketch

Henri Cartan is one of the first-rank mathematicians of the twentieth century. He has had a lasting impact through his research, which spans a wide variety of areas, as well as through his teaching, his students, and the famed Séminaire Cartan. He is one of the founding members of Bourbaki. His book *Homological Algebra*, written with Samuel Eilenberg and first published in 1956, is still in print and remains a standard reference.

The son of Élie Cartan, who is considered to be the founder of modern differential geometry, Henri Cartan was born on July 8, 1904, in Nancy, France. He attended the École Normale Supérieure and received his *Docteur ès Sciences mathématiques* in 1928. After holding positions in Lille and Strasbourg, he returned to Paris and taught at the École Normale from 1940 until 1965. Later he moved to the Université de Paris-Sud at Orsay, and he retired in 1975. Cartan is a member of the Académie des Sciences of Paris and of twelve other academies in Europe, the United States, and Japan. He has received honorary doctorates from several universities, and also the Wolf Prize in Mathematics in 1980.

Henri Cartan's father, Élie Cartan.

Early Years

Notices: *Let's start at the beginning of your life. What are your earliest memories of mathematical interest?*

Cartan: I have always been interested in mathematics. But I don't think it was because my father was a mathematician. I had no doubt that I could become a mathematician. I had many teachers, good or not so good. I don't think it was because of a particular teacher. Of course, I had some conversations with my father. I was quite surprised when he told me that Euclid's Postulate was not a necessity.

Notices: *How old were you when he told you that?*

Cartan: I don't know, perhaps 14.

Notices: *Do you have other memories of mathematical discussions with your father?*

Cartan: My father was a discreet man, you know. He never tried to influence me—it was always possible to ask him questions, but which questions I don't know. Much later we worked together on some problems. For instance, he knew more than I did about Lie groups, and it was necessary to use this knowledge for the determination of all

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Photographs for this article, except for that of Élie Cartan, are courtesy of Henri Cartan.

bounded circled domains which admit a transitive group. So we wrote an article on the subject together ["Les transformations des domaines cerclés bornés", *C. R. Acad. Sci. Paris* **192** (1931), 709–712]. But in general my father worked in his corner, and I worked in mine.

Notices: You were interested in music.

Cartan: Yes. I had a brother two years younger than myself, who became a composer. But he died at age 25 of tuberculosis. It was a great loss. Of course I played much music, but I can't now because I can't see.

Notices: What about your other siblings? Did you have other brothers?

Cartan: Yes, a younger brother who became a physicist. But he was killed during the war by the Germans, because he was in the *Résistance*. He was deported to Germany in February 1943, sentenced to death in August, and beheaded in December of the same year. From February 1943 right until the end of May 1945, we never heard about him. Under the circumstances, one could not expect to get any news. Some German colleagues were so good as to try to find out about him, but they didn't succeed.

Notices: Who were the German colleagues?

Cartan: One of them was Heinrich Behnke. Though a little older, Behnke was a friend of mine. My first invitation to go to Germany occurred in May 1931. Behnke was teaching in Münster-in-Westfalen, and he had a lot of students, about 40. I was invited because I had published a note in the *Comptes Rendus de l'Académie des Sciences* about circled domains ["Les transformations analytiques des domaines cerclés les uns dans les autres", *C. R. Acad. Sci. Paris* **190** (1930), 718–720], where I had proved quite easily a theorem which had been proved earlier by Behnke, but under certain conditions, in a particular case. So I was invited to give several speeches in Münster. That was in 1931, and Behnke invited me again in 1937. It was the time of Hitler.

Before the war, since November 1931, I had been teaching at the University of Strasbourg. But in September 1939, the inhabitants of Strasbourg had to be evacuated. The university was displaced to Clermont-Ferrand, where I taught for a year before I was appointed professor at the Sorbonne in Paris, in November 1940 (in fact, I was to be in charge of the mathematics students at the École Normale).

Throughout the war I was not permitted to go to my apartment in Strasbourg. One day, Behnke

Traité d'Analyse. Réunion du 14 Janvier 1935.

Présents : ~~WEIL~~ (M. le Doyen) - DELSARTE - MANDELBROJT - de POSSEL - CARTAN - CHEVALLEY - LERAY.

Sur une question de DELSARTE, on fixe la composition du Comité rédacteur. Après un rapide échange d'idées on établit une liste maximum de neuf membres, qui sont :
WEIL - DELSARTE - MANDELBROJT - CARTAN - DUBREIL - DIEUDONNÉ - de POSSEL - CHEVALLEY - LERAY.

Il est entendu que la liste définitive, extraite de la précédente sera composée des noms des membres présents à la réunion plénière d'Août ou Septembre prochain, réunion dans laquelle sera dressé le plan définitif et précis du traité.

Après quelque temps le calme renait, WEIL prend la parole et expose les idées suivantes.

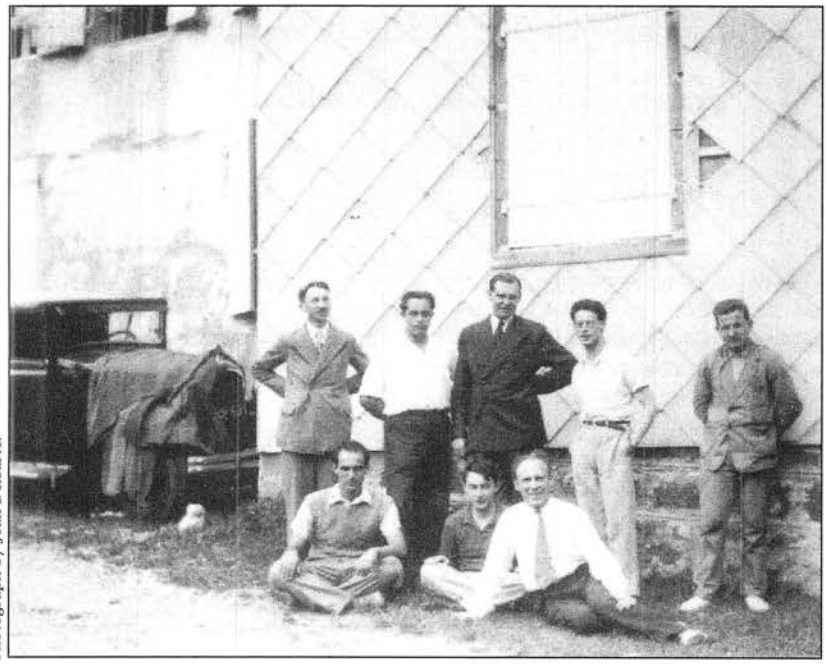
Il faut faire un traité utile à tous : aux chercheurs (patentés ou non), aux " trouveurs ", aux candidats aux fonctions de l'enseignement public, aux physiciens et à tous les techniciens. Comme critérium, il faut qu'on puisse, sans mercantilisme, conseiller la fréquentation du traité, ou tout au moins de ses fascicules essentiels, à un étudiant obligé de travailler seul, présumé d'ailleurs d'intelligence médiocre.

revenir sur tous ces théorèmes. Il importe donc de donner aux usagers une collection d'outils, ces outils devant être aussi robustes et aussi universels que possible. C'est le principe d'utilité et de commodité qui doit servir de guide. Il va sans dire que le comité est seul juge de ce qui est utile aux gens et de ce qui leur est commode. Comme le dit CARTAN dans une formule saisissante, c'est le principe du " despotisme éclairé ".

DELSARTE termine en critiquant la désinvolture avec laquelle chacun est arrivé à la présente séance. Il faut préparer ce qui a été demandé, et ne pas se fier à ses facultés d'improvisation, si brillantes qu'on les estime.
Chacun doit préparer et envoyer à WEIL, 3 jours au moins avant la prochaine réunion, un plan complet soigneusement rédigé. WEIL décantera ensuite.

Tout le monde approuve ces fortes paroles, il est même entendu que de graves sanctions pénaliseront les contrevenants.

Excerpts from the minutes by Jean Delsarte of the first preparatory meeting of the group that was later to become Bourbaki. These excerpts are taken from the only extant original copy of the minutes, in the possession of Henri Cartan. The nine people listed in the top excerpt were the members of the group. The first meeting was January 14, 1935. Later Dubreil and Leray dropped out, and the seven others became Bourbaki. The second and third excerpts give André Weil's ideas concerning readership and style, and the fourth discusses assignments for the next meeting.



Photograph by Jean Delsarte.

The first Bourbaki congress, at Besse-en-Chandesse, July 1935. Left to right, standing: Henri Cartan, René de Possel, Jean Dieudonné, André Weil, and a university laboratory technician; seated: Mirlès ("guinea pig"), Claude Chevalley, and Szolem Mandelbrojt. The "guinea pig" was a person being considered for membership in Bourbaki.

offered to try and retrieve some mathematical papers I had left there. He actually went to Strasbourg, but to no avail. He tried again and succeeded. He managed to get hold of some documents, which he left with the library of the University of Freiburg. In 1945, some members of the French Forces in Germany happened to find them there and returned them to me. Among those papers were the *comptes rendus*, minutes, of the first meetings of certain people who later became the Bourbaki group. That was the very beginning of Bourbaki's work. I don't think there exists another copy of these *comptes rendus*.

I returned to the University of Strasbourg after the war, at the end of 1945, for two years. In November 1946, I went to the Research Institute in Oberwolfach. It was very cold; there was snow and ice. I saw Professor Süß (the founder of Oberwolfach) and Frau Süß, and also Heinrich Behnke. I remember they asked me to play the piano. It was a beautiful piano—there were two pianos there. The old *château* at Oberwolfach doesn't exist anymore. I visited Oberwolfach several times after that. I was very grateful to my German colleagues for what they had done during the war.

Notices: *Your relations with your German colleagues did not suffer at all during the war?*

Cartan: No—with certain German colleagues, not all of them.

Notices: *When you first went to visit Behnke, were there a lot of differences between the mathematical climates in France and in Germany?*

Cartan: It is difficult to answer the question, because it depends. Of course, there were not so many mathematicians in France who were interested in analytic functions of several complex variables.

Notices: *But there was a big tradition in France for function theory of one variable.*

Cartan: Yes. It was perhaps the thing which was studied in France by most mathematicians—my father's work was an exception.

Notices: *But you were the one who started to work on several variables in France.*

Cartan: I believe it was André Weil who suggested that it could be interesting. He told me about the work of Carathéodory on circled domains. That was the beginning of my interest.

Behnke's assistant in 1931, namely Peter Thullen, was to become one of my best friends. We collaborated and wrote a paper together for the *Mathematische Annalen* ["Zur Theorie der Singularitäten der Funktionen mehrerer Komplexen Veränderlichen", *Math. Ann.* **106** (1932), 617–647]. I always had a good relationship with Thullen. At the beginning of 1933 or 1934, he left Germany—not because he was Jewish, he was Catholic, a deeply convinced Catholic. Thanks to Courant, he got a position in Ecuador. Thullen founded the Social Security system in Ecuador, and later in Colombia. In 1951, I think, he came back to Europe. I was very glad he did. He went to Geneva to work with the Bureau International du Travail (B.I.T.), where he was in charge of Social Security systems in Latin America. When he retired from that position, he was appointed at the University of Zürich, thanks to Van der Waerden, and later he became professor of mathematics in Fribourg, Switzerland. I have always had a very good relationship with his eldest son, who worked with the United Nations in Geneva, and is now retired. He visits us every year.

Notices: *When did you go to the United States for the first time?*

Cartan: I went to the United States for the first time in 1948. In fact, during the war—I think it was in 1942—I was invited to the United States, because they wanted to protect French people from German aggression. But it was not possible for me to move there, because of my family, and because my father was getting old. I was invited by Harvard University to visit from the beginning of February till the end of May 1948. But prior to that I was invited to Chicago by André Weil in January. So before I left I had to learn some English, because in order to be able to give lectures in English, it was necessary for me to know some words! My first lectures I had to write out in English beforehand. Later I wrote only the *résumés*. When I arrived at the airport in New York in December 1947, I was met by Samuel Eilenberg. It was my first encounter

with Sammy. Staying in the States was very important for me. I learned a lot.

Beginnings of Bourbaki

Notices: *You first met André Weil when you were students together at the École Normale in Paris.*

Cartan: Yes. We were there together, but he had been admitted the year before, at age 16.

Notices: *You and Weil happened to be students at the École Normale together with such people as Jean Dieudonné, Claude Chevalley, Jean Delsarte, Jean Leray ....*

Cartan: Dieudonné of course came after me. I was no longer at the École Normale when Leray entered. He entered the same year as Chevalley.

Notices: *These were some of the people who went on to form Bourbaki. Was there something at the École Normale, or in your common background, that later made you form Bourbaki?*

Cartan: You see, after the First World War, there were not so many scientists, I mean good scientists, in France, because most of them had been killed. We were the first generation after the war. Before us there was a *vide*, a vacuum, and it was necessary to make everything new. Some of my friends went abroad, notably to Germany, and observed what was being done there. This was the beginning of a mathematical *renouveau*, renewal. It was due to such people as Weil, Chevalley, de Possel .... The same people, responding to André Weil's initiative, came together to form the Bourbaki group. In Bourbaki I learned very much. Almost all I know in mathematics I learned from and with the Bourbaki group.

Notices: *Leray was not in Bourbaki.*

Cartan: Not quite. In fact, Leray was in the first group, at the beginning: a group which met from January to June, 1935. At that time we were considering writing a *Traité d'Analyse*. But two out of the group left before the summer; they didn't wish to continue. Leray was one of them. The final group was fixed in July 1935, in the so-called *Congrès de Besse-en-Chandesse*, and this group made decisions as to what they were going to treat.

Notices: *It was the* comptes rendus *of these meetings in the first half of 1935 that were left in Strasbourg in your apartment?*

Cartan: Yes.

Notices: *Who wrote the* comptes rendus?

Cartan: They were always written by Delsarte. Because of his position in the University of Nancy, he could have them typed by his secretary. Of course, typing was not so easy as it is now.

Notices: *In those early meetings, was there a certain person who was dominant in the group?*

Cartan: My answer is "André Weil", but you see, in Bourbaki most members had strong personalities. We often disagreed, we often had big arguments—but we remained good friends. For each subject, a "rédacteur" was appointed. Later, his *rédaction* was read aloud and thoroughly examined. The next "rédacteur" was given the appropriate instructions, and so on. For each chapter there could be up to nine *rédactions*. But in the end, everybody was *fatigué*—tired. And Dieudonné would say, "It is finished now. ***I*** shall write the last *rédaction*." Which he did. And eventually, although it seemed to be impossible to reach a complete agreement, there was an agreement. But it took time. It is perhaps not the best way in terms of teamwork, but that was the way we took.

Münster, December 20, 1963. Front row, left to right: Heinrich Behnke and Henri Cartan.

Notices: *Do you think that the Bourbaki style is dominant in France now?*

Cartan: I don't think so. You see, it's a way of presenting some mathematical theories, but it does not claim to be the only way of doing mathematics. And each member of Bourbaki actually did things on his own and in his own way. When I wrote papers, I was not making *rédactions* for Bourbaki.

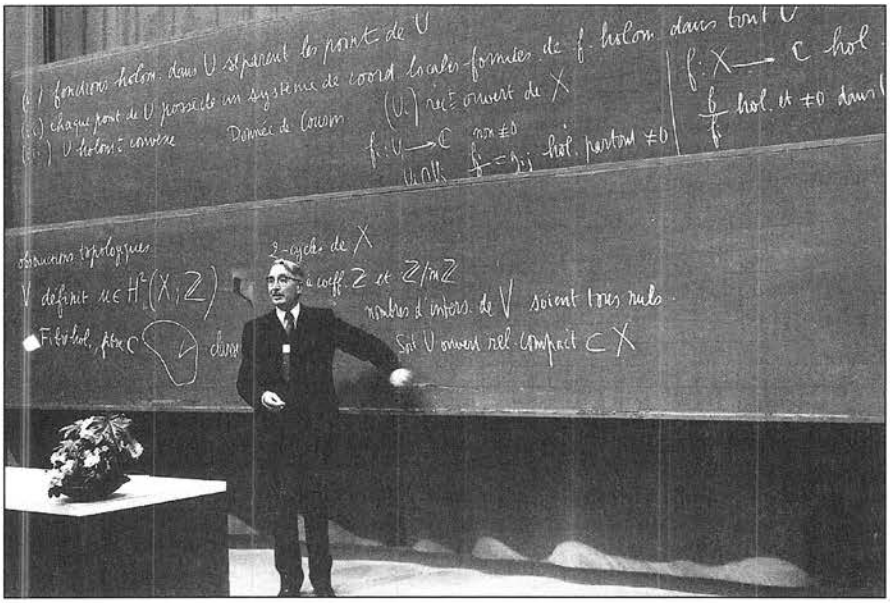
Notices: *And yet, Bourbaki had an enormous influence on French mathematics.*

Cartan: Not only in France. Outside France also, certain great mathematicians were influenced. For instance, Sir Michael Atiyah did some very beautiful mathematics, and being familiar with the Bourbaki way of thinking certainly helped him. Today, things are completely different. I don't think Bourbaki has a big influence on today's mathematicians. But it was certainly of great importance in the development of mathematics over several decades.

Notices: *So you are saying the influence of Bourbaki now is not as strong as it used to be.*

Cartan: What Bourbaki had to do is done now. Bourbaki is not eternal. But there still is the Séminaire Bourbaki, organized by some present members of Bourbaki. It is difficult for me to judge its influence, because I cannot have a good look at the mathematics of today.

Notices: *Your work stretches over many parts of mathematics. It is quite unusual, because nowadays people are much more specialized. You seem*



Cartan speaking at a colloquium in honor of the 60th birthday of Karl Stein, Munich, January 12, 1973.

to have covered most of pure mathematics, or touched upon all the areas.

Cartan: I can't agree with this statement. I was interested in several fields, for instance, homological algebra, because of Eilenberg. We discovered together the generality of this notion. In fact, we had to find a title for our book, so we said: "It is algebra, but it is *homological* algebra. So we will call the book *Homological Algebra*." What is surprising is that this book is still printed and sold today. I think it is quite remarkable, because it is now 43 years since the book first appeared. All of it was written by Sammy—I say "Sammy" because that's the way everybody called him. Sammy wrote everything; I wrote nothing. Of course, we had discussions, but after that Sammy wrote. And I was in charge of correcting the spelling mistakes—in English! I don't know much English, but I can spell. It was very easy and pleasant to work and discuss with Sammy.

Teaching at the École Normale and Learning There and Elsewhere

Notices: *How did you choose problems to work on?*

Cartan: They just came. Some people got me interested in some questions, for instance, Marcel Brelot. He was a great specialist in potential theory. He posed me some questions and problems, and I could solve them. It was during the war.

After the war, when I began to give my Séminaire at the École Normale, Jean-Pierre Serre asked me very many questions. He helped me discover many things by his questions. It was very important for me to know Serre and to hear his questions. When he prepared his thesis, he kept asking me questions, so I was forced to think about them. I learned a lot from my students at the École Normale. Many of them prepared theses under my direction—one normally says "direction", but my "direction" consisted in understanding what they had in mind. So I learned very much. I was able to collaborate with Godement, for instance, who was one of my students at the École Normale. He entered the École Normale in 1940, when I began to teach there. In that *promotion*, there was Godement and there was Koszul, who later prepared his thesis "under my direction". But he knew what he wanted to do. The students had to be helped of course, but they had their own ideas. Each one has his own personality, and you have to respect this personality, to help him find his own personality, and certainly not to impose somebody else's ideas.

When I taught the first-year students, they had to solve some problems at home, and I corrected them. But they would also come to the blackboard, and I would help them discover various mathematical theories.

Notices: *So this was a very individual teaching; you had students one on one.*

Cartan: Yes, so it was possible to see what the students were able to understand. For the second year, I gave a regular course on a subject which changed every year. For the third year, there was the *Agrégation*. The students had to give lectures and to be criticized by the others—and by myself, of course. The fourth-year students had to choose a field of research. I had some discussions with each of them. In the first year, there were about twenty students. But in the second year, there were not so many, because some chose mathematics while others chose physics, so there were ten or twelve, and the same number for the third year.

Notices: *Were there differences between the way you were taught as a student at the École Normale, and the way you taught when you were a professor there?*

Cartan: Oh, yes. It was necessary to change that way of teaching. I did change it.

Notices: *Was there one among your professors whose style you found particularly agreeable?*

Cartan: I liked some professors, yes, of course. Gaston Julia for one, or my father, who gave some lessons at the École Normale, so I was his student. The students at the École Normale also had to attend general courses at the Sorbonne.

Notices: *What about your Séminaire? How was that related to your teaching?*

Cartan: I wanted to get several of my fourth-year students interested in some fields, notably in topology. Very soon Jean-Pierre Serre said, "But you must write down the *exposés*." So, I wrote down the *exposés*, or I sometimes asked some of the others to write them down. At the beginning I had no precise long-term plans, but somehow this turned out

to be an irreversible process, hence the Séminaire. Every year more and more people kept coming—not only students, but also mathematicians from France and abroad. Of course, there was no other Séminaire at the time. I typed the *exposés* myself, but later on, the job was done by the secretary of the Institut Henri Poincaré. And then, one thing leading to another, it even happened that for some parts of the Séminaire, I didn't do a thing—for instance, when Grothendieck spent half a year explaining and developing his ideas.

Notices: *What are your memories of Grothendieck?*

Cartan: He is an exceptional man, of course. He had a great influence on some parts of mathematics, notably algebraic geometry. His approach was something new—though not completely new, because at the beginning, for instance, he himself was influenced by Serre. Grothendieck is a very special man. Nobody knows where he is or what he is doing now.

Photo: Gerd Fischer.

Photograph by Klaus Peters.

Cartan at Oberwolfach in September 1971 (above) and October 5, 1984 (left).

Notices: *Who is the mathematician you admire most?*

Cartan: I admired quite a few when I could do and understand mathematics. Today ...!

Notices: *When you could do it, who was the person you admired most?*

Cartan: No, I don't wish to answer the question. I admire many mathematicians, but I don't wish to award prizes.

Notices: *What do you consider your most important achievement in mathematics?*

Cartan: It is for other people to say that.

Notices: *But maybe there is something you particularly liked, or something that was especially satisfying.*

Cartan: The connection between algebraic topology and analytic functions. I discovered general theorems which play a big role. But in this I was helped by Serre. By the way, this is one example of a mathematician for whom I have complete admiration: Serre.

Notices: *You have worked in many areas of mathematics. Do you feel equally at home in analysis, in algebra, in geometry ...?*

Cartan: Geometry—not exactly geometry. Topology, I would say. But I could also see the relations between them. One day I discovered that topological notions, and in particular sheaf theory, could be applied to analytic functions of several variables. This was very important. One can use results from topology in order to get some important results for analytic functions. I think that is interesting.

Notices: *You have always worked in pure mathematics, and today applied mathematics is very important. What do you think of this?*

Cartan: You see, which parts of mathematics are applicable—not applied but applicable—it is very hard to tell in advance. When I was invited to Münster for the first time in 1931, they gave a big dinner at the end of my stay, and there was a big discussion. There was a philosopher who spoke about which parts of mathematics can be applied. "Anyway," somebody said, "not analytic functions of several complex variables! That can't be applied." But it was actually applied later. So it is difficult to tell in advance. And why mathematics can be applied to other things, to physics—that's a mystery.

Politics and Human Rights

Notices: *You have been active in politics. Can you say something about that?*

Cartan: This is not mathematics!

Notices: *It has no relationship to mathematics?*

Cartan: Maybe it has .... You see, a mathematician thinks: "What is this question? What happens exactly? Why is it so, and not so? What is the reason? What is the logical consequence of all this?" I am applying this to politics. I have tried to analyze situations and to draw logical consequences. This was the way I became a European Federalist, because I understood that there is no other way. For me, what is going on now, today, proves the necessity of Federalism—if the word Federalism is correctly understood, that is. Some people misunderstand the word. In France, everything is decided at the top; nobody—except the Government—is responsible for anything. Look at what is going

Photographs by Sophie Caretta.

Cartan at his desk (above) and with Madame Cartan, 1996.

on today in Kosovo. You hear people say, "Ah, Europe must have a common foreign policy." Those are empty words: how can this ever be achieved without an authority that can make decisions and implement them and that can also be under democratic control? Nothing can ever be achieved unless a European constitution gives birth to a federal authority and makes it fully responsible for clearly defined areas of common interest. Of course, this federal authority would in no way encroach on the respective spheres of responsibility of either the states, regions, or townships.

All this I believe only because I am a mathematician who thinks about the situation and the logical consequences of this situation.

Notices: *Would you agree that having lived in Strasbourg and gone through your experiences in the war may have influenced you in these views?*

Cartan: Yes, but I was not a Federalist at the end of the Second World War. I became a Federalist a few years later.

Talking about the European elections—the first one was in 1979. It was very difficult to obtain these elections. The European Parliament did not have much power. After those elections I went to Strasbourg for each session of the European Parliament. Five years later, for the second election in 1984, I was a candidate (in France, of course). In fact, I was at the head of a list. We didn't have a party. Our list was called "Pour les États-Unis d'Europe". Unfortunately, when it came to choosing a voting system for the European elections, the French Parliament decided in favor of choosing members out of national lists so that the French members of the European Parliament are chosen by political parties instead of grass-roots voters. Besides, one should not overlook the fact that such campaigns cost a tremendous amount of money, so that only the constituted parties (which are officially subsidized by the State) can afford them.

Notices: *You received an award from the New York Academy of Sciences, the Pagels Award. What was the award for?*

Cartan: This was for assisting dissidents. I was involved in the defense of dissidents in the USSR and other countries, especially mathematicians. The so-called *Comité des Mathématiciens* was created to defend dissidents. I was very active in this defense process. The most famous dissident mathematician at the time was Leonid Plyushch. There was a big to-do! Plyushch was in a "special" psychiatric hospital. This was back in 1973, and our attention had been called on his case by Andrei Sakharov. We began to call upon the Soviet Embassy in Paris—it was possible at the time, but it soon became impossible. When the International Congress of Mathematicians was held in Vancouver in 1974, we tried to stir up the participants about the case of Plyushch. We asked people to sign an appeal, and gathered a thousand signatures asking for his liberation. I was asked to send a telegram to the Soviet authorities. As a consequence, when we returned to Paris, we organized this *Comité des Mathématiciens*, and we had several meetings at the seat of the Human Rights League in Paris. We held a big meeting in the Salle de Mutualité with several thousand people. Eventually, the Soviet authorities decided to free Plyushch in January 1976. It was a big success.

Later on, we worked for other people. The Uruguayan mathematician José Luís Massera, who was a Communist, was a victim of the military dictatorship there. The *Comité des Mathématiciens*—with such people as Laurent Schwartz—worked on his case. Today we have a very active *Comité de Défense des Hommes de Science* (CODHOS) within the French Académie des Sciences. Its president is François Jacob, a Nobel Prize laureate in medicine. Similar committees in France, Sweden, Great Britain, Italy, and the United States are now connected together and work hand in hand for similar purposes.