1994 Steele Prizes

Three Leroy P. Steele Prizes were awarded at the Minneapolis Mathfest.

These prizes, established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein, are endowed under the terms of a bequest from Leroy P. Steele.

Three Steele Prizes are awarded each year: one for expository mathematical writing, one for a research paper of fundamental and lasting importance, and one in recognition of cumulative influence extending over a career, including the education of doctoral students. The current award is $4,000 in each of these categories.

The recipients of the Steele Prizes for 1993 are INGRID DAUBECHIES for the expository award, LOUIS DE BRANGES for research work of fundamental importance, and LOUIS NIRENBERG for the career award. The prizes were presented at the AMS-MAA opening banquet on August 14, 1994, at the Minneapolis Mathfest.

The Steele Prizes are awarded by the Council of the Society acting through a selection committee whose members at the time of these selections were Eugenio Calabi (chair), Vaughan F. R. Jones, Robert P. Langlands, Barry Mazur, Paul Rabinowitz, Marina Ratner, Jane Cronin Scanlon, Jean E. Taylor, and William P. Thurston.

The text that follows contains the committee's citations for each award, the recipients' responses, and a brief biographical sketch of each recipient.

Citation for Ingrid Daubechies

The expository award goes to Ingrid Daubechies for her book *Ten Lectures on Wavelets* (CBMS 61, SIAM, 1992, ISBN 0-89871-274-2). The concept of wavelets has its origins in many fields, and part of the accomplishment of Daubechies is finding those places where the concept arose and showing how all the approaches relate to one another. The use of wavelets as an analytical tool is like Fourier analysis—simple and yet very powerful. In fact, wavelets are an extension of Fourier analysis to the case of localization in both frequency and space. And like Fourier analysis, it has both a theoretical side and practical importance.

Daubechies' lectures have been important in educating the mathematical community about wavelets; many of us first learned about wavelets through hearing her speak. But not that many people can be reached by any one lecture. The CBMS course format, with its week-long series of main lectures and its requirement that the lecturer produce a book, once again proves its worth, allowing a wider community to gain access. Daubechies' is an invaluable resource for the novice interested in learning about "The What, Why, and How of Wavelets", to borrow the title of the first chapter. It is entirely self-contained; if a desired result or application is not in the text, one is certain to find several references to where it can be found. It strikes an excellent balance between theory and application, effectively showing how each influenced the development and understanding of the other. The book also weaves in the history of wavelets, relating developments in disparate fields which converged to become wavelets.

Ingrid Daubechies

Daubechies has, of course, made major contributions to the subject herself. Haar wavelets (where the "mother wavelet" is the characteristic function on $[0, 1/2]$ minus the characteristic function on $[1/2, 1]$) have been known since 1910; they were thought to be a curiosity but not very useful. With Daubechies' work, Haar wavelets have been shown to be the first in a whole family of compactly supported nonsmooth wavelets: beautiful examples of functions with fractal higher derivatives. This book contains original results of hers as well as presents previous work by her and others.

Biographical Sketch

Ingrid Daubechies was born on August 17, 1954, in Houthalen, Belgium. She received her Ph.D. from Free University, Brussels (1980). Professor Daubechies is presently a member of AT&T Bell Laboratories' staff (until December 31, 1994). In January 1995 she will assume the position of professor of mathematics at Princeton University.

Professor Daubechies has served on the AMS Short Course Subcommittee and on the AMS Committee on Committees since 1993. She has given numerous addresses, including the following: Invited Address, SIAM (Chicago, 1990); Principal Lecturer, CBMS Regional Conference (Lowell, 1990); Invited Address, MAA (Baltimore, 1992); Invited Address, AMS (Bethlehem, April 1992); and Organizer and Speaker, AMS Short Course on Wavelets and Applications (San Antonio, January 1993). She presented a Plenary Lecture at the International Congress of Mathematicians in Zürich in August 1994.

In 1992 Professor Daubechies was awarded a five-year MacArthur Fellowship.

Response from Ingrid Daubechies

I feel greatly honored that the AMS has chosen to award a Steele Prize to my work. When I set out to organize my CBMS lectures and later *Ten Lectures on Wavelets*, I wanted to convey the many links that exist between this new mathematical development and ideas in physics, electrical engineering, computer vision, and, of course, other fields in mathematics. The interaction with applications has been a constant source of inspiration for my own work, and I find it deeply gratifying that this mix of mathematics and applications is so well received.

Citation for Louis de Branges

It was observed by R. L. E. Schwarzenberger that mathematicians strive to be original but seldom in an original way. Louis de Branges is a courageous exception; his originality is his own.

The Bieberbach conjecture, formulated in 1916 and the object of heroic efforts over the years by many outstanding mathematicians, was proved by de Branges in 1984. The Steele prize is awarded to him for the paper "A proof of the Bieberbach conjecture" published in *Acta Mathematica*, **154** (1985), 137–152. The conjecture itself is simply stated. If

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

converges for $|z| < 1$ and takes distinct values at distinct points of the unit disk, then $|a_n| \leq n$ for all n. Equality is achieved only for the Koebe functions $z/(1+\omega z)^2$ where ω is a constant of absolute value 1.

The classical ingredients of the proof, the Löwner differential equation and the inequalities conjectured by Robertson and Milin, as well as the Askey-Gasper inequalities from the theory of special functions, are clearly described in the volume *The Bieberbach Conjecture* (published by the Society). So is the generous reception of the Leningrad mathematicians to the efforts of de Branges to explain it and their help in the composition of the eminently readable *Acta* paper.

The Milin inequality was known to imply the Bieberbach conjecture, and Löwner had used his technique in the 1920s to deal with the third coefficient. For de Branges it was of capital importance that, in contrast to the Bieberbach conjecture itself, the Milin and Robertson conjectures were quadratic and thus statements about spaces of square-integrable analytic functions. The key was to find norms for which the necessary inequalities could be propagated by the Löwner equation. de Branges constructed the necessary coefficients from scratch, solely on the basis of their required properties, thereby reducing the verification of the Milin conjecture (and thus of the Bieberbach conjecture) for a given integer n to a statement that was almost immediate for very small n, that could be verified numerically for small n, yielding many new cases of the conjecture, and that ultimately revealed itself to be an inequality established several years earlier by Askey and Gasper. The entire construction required a thorough mastery of the literature, formidable analytic imagination, and great tenacity of purpose.

Louis de Branges

The proof is now available in a form that can be verified by any experienced mathematician (either in de Branges' own paper or in that of Fitzgerald and Pommerenke [*TAMS*, **290** (1985), 683–690]) as analysis that is "hard" in the original aesthetic sense of Hardy—simple algebraic manipulations linked by difficult inequalities. Although the mathematical community does not attach the same importance to the general functional-analytic principles that led to them as their author does, it is well to remember when recognizing his achievement in proving the Bieberbach conjecture that for de Branges its appeal, like that of other conjectures from classical function theory, is as a touchstone for his contributions to interpolation theory and spaces of square-summable analytic functions. Without anticipating the future in any way, the Society expresses its appreciation and admiration of past success and wishes him continuing prosperity and good fortune.

Biographical Sketch

Louis de Branges was born on August 21, 1932, in Paris, France. He received his B.S. degree from the Massachusetts Institute of Technology (1953) and his Ph.D. from Cornell University (1957).

Professor de Branges began his academic career as an assistant professor at Lafayette College (1957–1959). He was a member of the Institute for Advanced Study (1959–1960) and then a lecturer at Bryn Mawr College (1960–1961). He was also a member of the Courant Institute of Mathematical Sciences (1961–1962). In 1962 he became associate professor of mathematics at Purdue University. He advanced to professor of mathematics the following year and has been a faculty member at Purdue University since that time.

Professor de Branges has been a member of the Committee to Select Hour Speakers for Western Sectional Meetings (1968–1969). He gave an Invited Address (Madison, November 1963) and spoke at the Summer Institute on Entire Functions and Related Parts of Analysis (San Diego, June 1966).

Professor de Branges was an Alfred P. Sloan Foundation Fellow (1963–1966) and a Guggenheim Fellow (1967–1968). In 1992 he was awarded the first Ostrowski Prize for mathematical achievement by the University of Basel. His fields of research interest are functional analysis, operator theory (including applications to system theory), polynomial approximation, special function theory (including related group representations), geometric function theory, analytic number theory (including representation theory and modular forms), set theory, and quantum mechanics. He has also been a reviewer for *Mathematical Reviews* and a translator of Russian.

Response from Louis de Branges

The Proof of the Bieberbach Conjecture in Retrospect*

This report begins with two acknowledgements. One is made to the American Mathematical Society for its continued endorsement of research related to the Bieberbach conjecture. The Steele Prize is only the latest expression of its interest. It should be unnecessary to say that fundamental research cannot be sustained for long periods without the support of learned societies. The American Mathematical Society has earned a reputation as the world's foremost leader in fundamental scientific research.

Another acknowledgement is due to Ludwig Bieberbach as a founder of that branch of twentieth century mathematics which has come to be known as functional analysis. This mathematical contribution has been obscured by his political allegiance to National Socialism, which caused the mass emigration of German mathematical talent, including many of the other great founders of functional analysis. The issue which divided Bieberbach from these illustrious colleagues is relevant to the present day because it concerns the teaching of mathematics. Bieberbach originated the widely held current view that mathematical teaching is not second to mathematical research. As a research mathematician he exhibited intuitive talent which surpassed his more precise colleagues. Yet the proof of his conjecture is a vindication of their more logical methods [12].

The proof of the Bieberbach conjecture is difficult to motivate because it is part of a larger research program whose aim is a proof of the Riemann hypothesis. This research effort begins with the theory [1] of Hilbert spaces whose elements are entire functions and which have these properties:

(H1) Whenever $F(z)$ belongs to the space and has a nonreal zero w, the function $F(z)(z-\bar{w})/(z-w)$ belongs to the space and has the same norm as $F(z)$.

(H2) For each nonreal number w a continuous linear functional is defined on the space by taking $F(z)$ into $F(w)$.

(H3) The function $F^*(z) = \bar{F}(\bar{z})$ belongs to the space whenever $F(z)$ belongs to the space, and it always has the same norm as $F(z)$.

The theory of these spaces is related to the theory of entire functions $E(z)$ which satisfy the inequality

$$|E(x-iy)| < |E(x+iy)|$$

for $y > 0$. If $E(z)$ is any such function, write

$$E(z) = A(z) - iB(z)$$

where $A(z)$ and $B(z)$ are entire functions which are real for real z and

$$K(w,z) = [B(z)\overline{A}(w) - A(z)\overline{B}(w)]/[\pi(z-\overline{w})].$$

Then the set $\mathcal{H}(\mathcal{E})$ of entire functions $F(z)$ such that

$$||F||^2 = \int_{-\infty}^{\infty} |F(t)/E(t)|^2 dt$$

is finite and such that

$$|F(z)|^2 \leq ||F||^2 K(z,z)$$

*Research supported by the National Science Foundation.

for all complex z is a Hilbert space of entire functions which satisfies the axioms (H1), (H2), and (H3). For each complex number w the expression $K(w,z)$ belongs to the space as a function of z, and the identity

$$F(w) = \langle F(t), K(w,t) \rangle$$

holds for every element $F(z)$ of the space. A Hilbert space, whose elements are entire functions, which satisfies the axioms (H1), (H2), and (H3), and which contains a nonzero element, is isometrically equal to a space $\mathcal{H}(\mathcal{E})$.

The aspect of these spaces which is now relevant is the remarkable interplay which exists between particular analytic functions and related spaces of analytic functions. When information about an analytic function is wanted, it is often obtainable from properties of a related space of analytic functions. This technique is applied in the proof of the Bieberbach conjecture and is expected to be applicable in a proof of the Riemann hypothesis.

Another example of such a relationship between isolated analytic functions and spaces of analytic functions appears in the factorization theory for functions analytic in the unit disk. Engineering language has displaced the original mathematical terminology in this field.

A linear system is a matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

whose entries are continuous linear transformations. The matrix acts on the Cartesian product of a vector space $\mathcal{H}$ with scalar product, called the state space, and a vector space $\mathcal{C}$ with scalar product, called the coefficient space. The Cartesian product is realized as column vectors with upper entry in the state space and lower entry in the coefficient space. The main transformation A maps the state space $\mathcal{H}$ into itself. The input transformation B maps the coefficient space $\mathcal{C}$ into the state space $\mathcal{H}$. The output transformation C maps the state space $\mathcal{H}$ into the coefficient space $\mathcal{C}$. The external operator D maps the coefficient space into itself. The transfer function of the linear system is the power series

$$W(z) = \sum W_n z^n$$

whose entries are the operators defined on the coefficient space $\mathcal{C}$ by

$$W_0 = D$$

and

$$W_{n+1} = CA^n B$$

for every nonnegative integer n.

The state space and the coefficient space are considered in the weak topology induced by self-duality. The linear system is said to be unitary if its matrix is unitary. And it is said to be conjugate isometric if its matrix has an isometric adjoint.

A canonical conjugate isometric linear system with transfer functions $W(z)$ is a conjugate isometric linear system whose state space consists of power series with coefficients in $\mathcal{C}$ and which has these properties: The main transformation takes $f(z)$ into $[f(z) - f(0)]/z$. The input transformation takes c into $[W(z) - W(0)]c/z$. The output transformation takes $f(z)$ into $f(0)$. The external operator is $W(0)$.

A canonical unitary linear system with transfer function $W(z)$ is a linear system whose state space consists of pairs $(f(z), g(z))$ of power series with coefficients in $\mathcal{C}$ and which has these properties: The conjugate power series is defined by

$$W^*(z) = \sum W_n^- z^n$$

if

$$W(z) = \sum W_n z^n$$

where the bar denotes the adjoint of an operator on $\mathcal{C}$. The main transformation takes $(f(z), g(z))$ into

$$([f(z) - f(0)]/z, zg(z) - W^*(z)f(0)).$$

The adjoint of the main transformation takes $(f(z), g(z))$ into

$$(zf(z) - W(z)g(0), [g(z) - g(0)]/z).$$

The input transformation takes c into

$$([W(z) - W(0)]c/z, [1 - W^*(z)W(0)]c).$$

The adjoint of the input transformation takes $(f(z), g(z))$ into $g(0)$. The output transformation takes $(f(z), g(z))$ into $f(0)$. The adjoint of the output transformation takes c into

$$([1 - W(z)W(0)^-]c, [W^*(z) - W^*(0)]c/z).$$

The external operator is $W(0)$. Its adjoint is $W(0)^-$. Such a linear system is always unitary.

The theory of linear systems with finite-dimensional state space and coefficient space is a fundamental tool of the engineering community in this last half of the twentieth century. The mathematical community needs to revise its graduate teaching to include this fundamental concept. The mathematical treatment of the concept should however include infinite-dimensional state spaces and coefficient spaces. The required topological methods are due to Mark Krein.

The anti-space of a vector space with scalar product is the same vector space considered with the negative of the given scalar product. A Krein space is a vector space with scalar product which is the orthogonal sum of a Hilbert space and the anti-space of a Hilbert space. Krein spaces are the natural choice of state space and coefficient space for conjugate isometric and unitary linear systems.

An important special case of the theory of canonical conjugate isometric linear systems occurs when the coefficient space is the one-dimensional Hilbert space of complex numbers considered with absolute value as norm. If the state space is a Hilbert space, then the transfer function is analytic and bounded by one on the unit disk. A construction of such spaces is due to James Rovnyak and the author [18]. They also obtain a related construction of canonical unitary linear systems whose state space is a Hilbert space [19]. These results recapture the starting situation in which a space of analytic functions is associated with a particular analytic function.

These constructions associate a Hilbert space, whose elements are functions analytic in the unit disk, with any given function $W(z)$ which is analytic and bounded by one in the unit disk. The function $W(z)$ is the transfer function of a canonical conjugate isometric linear system whose state space is the Hilbert space of analytic functions. There is also a related Hilbert space whose elements are pairs of functions analytic in the unit disk. The function $W(z)$ is also the transfer function of a canonical unitary linear system whose state space is the Hilbert space of pairs of analytic functions.

The proof of the Bieberbach conjecture is concerned with a special case of the theory of such canonical conjugate isometric and canonical unitary linear systems in which the transfer function W defines an injective mapping of the unit disk into itself having the origin as a fixed point. These properties of the transfer function are characterized by the existence of a new Hilbert space whose elements are analytic functions and a new Hilbert space whose elements are pairs of analytic functions. For every point w in the unit disk the expression

$$\log \frac{1 - W(z)W(w)^-}{1 - z\bar{w}}$$

is a function of z which is analytic in the unit disk and has value zero at the origin. The expression belongs to the space of analytic functions, and the identity

$$f(w) = \left\langle f(z), \log \frac{1 - W(z)W(w)^-}{1 - z\bar{w}} \right\rangle$$

holds for every element $f(z)$ of the space. For every point w in the unit disk the expression

$$\log \frac{1 - W(w)^-/W^*(z)}{1 - \overline{w}/z}$$

is a function of z which is analytic in the unit disk and which has value zero at the origin. The pair

$$\left(\log \frac{1 - W(z)W(w)^-}{1 - z\overline{w}}, \log \frac{1 - W(w)^-/W^*(z)}{1 - \overline{w}/z} \right)$$

belongs to the Hilbert space of pairs of analytic functions and the identity

$$f(w) = \Bigg\langle (f(z), g(z)), \left(\log \frac{1 - W(z)W(w)^-}{1 - z\overline{w}}, \log \frac{1 - W(w)^-/W^*(z)}{1 - \overline{w}/z} \right) \Bigg\rangle$$

holds for every element $(f(z), g(z))$ of the space. This characterization of the injective property is essentially due to Helmut Grunsky [7].

When $f(z)$ belongs to the Grunsky space of analytic functions, then

$$\exp f(z)$$

belongs to the state space of the canonical conjugate isometric linear system, and the inequality

$$\| \exp f(z) \|^2 \leq \exp \| f(z) \|^2$$

is satisfied. If equality holds, the identity

$$\langle \exp f(z), \exp u(z) \rangle = \exp \langle f(z), u(z) \rangle$$

holds for every element $u(z)$ of the Grunsky space of analytic functions. Whenever $(f(z), g(z))$ belongs to the Grunsky space of pairs of analytic functions, then

$$(\exp f(z), W^*(z)/z \ \exp g(z))$$

belongs to the state space of the canonical unitary linear system, and the inequality

$$\| (\exp f(z), W^*(z)/z \ \exp g(z)) \|^2 \leq \exp \| (f(z), g(z)) \|^2$$

is satisfied. If equality holds, then the identity

$$\begin{aligned} &\langle (\exp f(z), W^*(z)/z \ \exp g(z)), \\ &(\exp u(z), W^*(z)/z \ \exp v(z)) \rangle \\ &\quad = \exp \langle (f(z), g(z)), (u(z), v(z)) \rangle \end{aligned}$$

holds for every element $(u(z), v(z))$ of the Grunsky space of pairs of analytic functions.

The proof of the Bieberbach conjecture makes indirect use of these exponential relations to obtain information about the coefficients of $W(z)$. The underlying problem is to characterize an initial segment of coefficients of the power series. The proof of the Bieberbach conjecture is complicated by the fact that the underlying problem remains unsolved. A direct use of the exponential relations is desired.

Difficult problems were left unsolved by the proof of the Bieberbach conjecture. Some of these have since been clarified. Progress has been made, for example, in the structure theory of canonical unitary linear systems and its applications to analytic function theory. Of particular interest is a generalization of the Beurling inner-outer factorization [17]. This result is the culmination of a series of publications on canonical unitary linear systems whose state space is a Krein space. They supplement a previous series on canonical unitary linear systems whose state space is a Hilbert space.

Progress has also been made towards the initial objective of a proof of the Riemann hypothesis [14]. The results are conjectured to be also relevant to the proof of the Bieberbach conjecture. A positivity condition has been found for Hilbert spaces of entire functions which is suggested by the theory of the gamma function [16]. The condition appears, for example, in the structure theory of plane measures with respect to which the Newton polynomials form an orthogonal set [22].

Results which are related to the present work have been obtained by Xian-Jin Li in his thesis [23]. An axiomatic treatment is given of the Szegö theory of polynomials which are orthogonal on the unit circle. A variant of the axioms (H1),(H2), and (H3) for Hilbert spaces of entire functions is used for this purpose. The results obtained are an application of the factorization theory for canonical conjugate isometric linear systems whose state space is a Hilbert space and whose coefficient space is a two-dimensional vector space with indefinite scalar product. A quantum positivity condition is introduced which adapts the theory of the gamma function to the unit disk. The ultimate objective of his work is a new proof of the Riemann hypothesis for function fields. A proof by the methods of factorization theory seems to be as difficult as it is for the classical Riemann hypothesis.

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Citation for Louis Nirenberg

The career award goes to Louis Nirenberg for his numerous basic contributions to linear and nonlinear partial differential equations and their application to complex analysis and differential geometry.

Louis Nirenberg

His first major work was his thesis on global differential geometry, in which he solved the long open Weyl problem of isometric embedding of surfaces of positive curvature in $\mathbb{R}^3$.

Nirenberg is a master of the art and science of obtaining and applying a priori estimates in all fields of analysis. A minor such gem is the useful set of Gagliardo-Nirenberg inequalities. A high point is his joint research with Agmon and Douglis on a priori estimates for general linear elliptic systems, one of the most widely quoted results in analysis. Another is his fundamental paper with F. John on functions of bounded mean oscillation which was crucial for the later work of C. Fefferman on this function space.

Nirenberg has been at the center of many major developments. His theorem with his student, Newlander, on almost complex structures has become a classic. In a paper building on earlier estimates of Calderón and Zygmund, he and J. Kohn introduced the notion of a pseudo-differential operator which helped to generate an enormous amount of later work. His research with Treves was an important contribution to the solvability of general linear PDEs. Some other highlights are his research on the regularity of free boundary problems with Kinderlehrer and Spruck, existence of smooth solutions of equations of Monge-Ampère type with Caffarelli and Spruck, and singular sets for the Navier-Stokes equations with Caffarelli and R. Kohn. His study of symmetry of solutions of nonlinear elliptic equations using moving plane methods with Gidas and Ni, and later with Berestycki, is an ingenious application of the maximum principle.

In addition to his own research, Nirenberg has had over forty Ph.D. students. His boundless enthusiasm and encouragement have served as an inspiration to several generations of younger mathematicians, both at the Courant Institute and worldwide.

Biographical Sketch

Louis Nirenberg was born on February 28, 1925, in Hamilton, Ontario, Canada. He received his B.Sc. from McGill University (1945), and his M.S. (1947) and Ph.D. (1949) from New York University. He began his academic career at New York University, where he advanced from research assistant to research associate (1945–1951) and then from assistant professor to associate professor (1951–1957). Since 1957 he has been a professor at NYU.

A member of the American Mathematical Society for forty-six years, Professor Nirenberg has served as a member-at-large of the Council (1963–1965), as vice-president (1976–1977), and on the following AMS committees: Program Committee (1960), Executive Committee (1965), *Transactions* and *Memoirs* Editorial Committee (1965–1967), Nominating Committee (1967–1973), Committee on Summer Institutes (1968–1973), Committee to Select the Winner of the Bôcher Prize (1969–1970, 1989), Committee on Steele Prizes (1981–1983), Committee on 1983 AMS-SIAM Summer Institute on Nonlinear Functional Analysis and Its Applications, Colloquium Editorial Committee (1983–1985; chair, 1985), Committee on National Awards and Public Representation (1990–1991).

Professor Nirenberg has given the following addresses: AMS Invited Address (New York, February 1958), Symposium on Differential Geometry (Tucson, February 1960), Symposium on Partial Differential Equations (Berkeley, April 1960), International Congress of Mathematicians (Stockholm, 1962), Symposium on Singular Integrals (Chicago, April 1966), Summer Institute on Global Analysis (Berkeley, July 1968), AMS Colloquium Lectures (San Francisco, January 1974), AMS-MAA Invited Address (Orono, August 1991).

Professor Nirenberg received the Bôcher Prize (1959), was a Sloan Fellow (1958–1960), and was a Guggenheim Fellow (1966–1967 and 1975–1976). In 1982 he received the Crafoord Prize. He has been awarded an Honorary Doctor of Science from McGill University (1986), University of Pisa (1990), and Université de Paris IX, Paris-Dauphine (1990). He was awarded Honorary Professorship at Nankai University (1987) and Zhejiang University (1988). He is a member of the National Academy of Sciences, the American Philosophical Society, and the American Academy of Arts and Sciences. He is a foreign member of Accademia dei Lincei, Académie des Sciences de France, Accademia Mediterranea Delle Scienze, Istituto Lombardo, Accademia Scienze e Lettere, and Academy of Science of Ukraine. His research interests include partial differential equations, fluid dynamics, differential geometry, and complex analysis.

Response from Louis Nirenberg

I am very honoured and enormously pleased to be awarded the Leroy P. Steele Prize for Lifetime Achievement. As the kind citation says, my work has centered around Partial Differential Equations (PDE), and I've been lucky to be at the right place at the right time. I had the good fortune to come to New York University as a graduate student after the Second World War. Richard Courant and Kurt Friedrichs began to establish a center for work in PDE, which eventually grew into the Courant Institute, one of the world centres in the subject.

There were some extremely talented fellow students: Harold Grad, Peter Lax, Joe Keller, Martin Kruskal, Cathleen Morawetz to name a few. Interaction with them was very important for me, particularly with Peter Lax, from whom, over the years, I have learned many things in mathematics. My thesis adviser was Jim Stoker, who was in turn a student of Heinz Hopf. But the teacher who influenced me the most was probably Friedrichs. His teacher was Courant, who was in turn a student of Hilbert—not a bad lineage. From Friedrichs I believe I acquired a certain point of view: developing general techniques and methods is often of greater interest than proving some particular result. In addition I acquired a love for inequalities.

The period since my graduate school days has been a golden one for PDE. There were masters working in the subject, whose work greatly affected mine: beside those mentioned, Fritz John, a colleague; Jean Leray in Paris; Hans Lewy and C. B. Morrey in Berkeley. In addition, during this golden age, there were (and are) wonderful contemporaries; to name just a few (not already named) whose work greatly influenced my own: Shmuel Agmon, Alberto Calderón, Ennio De Giorgi, Lars Gårding, Lars Hörmander, Jürgen Moser, John Nash, Paul Rabinowitz, S.-T. Yau, and many more, including some below and a number in the former Soviet Union. The citation mentions joint work. In fact, 90 percent of my papers are joint, and this is an opportunity to thank some of my coauthors; I can only name a few. Collaborating with others has been one of the greatest pleasures for me in doing mathematics. Agmon and I wrote several joint papers, some with Avron Douglis, and they played a big role in shaping my future research. The papers I wrote with Lipman Bers (from whom I learned a lot), the one with Morrey, the one with S. S. Chern and H. I. Levine, and especially the one with Fritz John, gave me great pleasure. A paper I wrote with Phil Hartman on geometry, not PDE, was rather elementary but great fun to do. Though Peter Lax and I have been colleagues all these years, we published only one joint paper, but I continue to learn things from him.

It was a great pleasure to work with François Treves on local solvability of linear PDEs and then with Joe Kohn on various things. Incidentally, Friedrichs suggested the name "pseudo-differential operators".

I would like to mention also a joint work with K. Kodaira and Don Spencer in complex analysis—I attended their "Nothing Seminar" in spring 1952—and one with Charles Loewner in geometry. Their names remind me that one of the joys of being a member of the family of mathematicians is that one gets to know so many lovely and interesting people.

For the last twenty-five years or so I have worked mainly on nonlinear problems. In recent years I have had the great good fortune to work with Haim Brezis, Luis Caffarelli, David Kinderlehrer, Bob Kohn, Joel Spruck, Vassilis Gidas, and Wei Ming Ni, and most recently with Henri Berestycki and S.R.S. Varadhan.

I have also been blessed by having many talented students and have learned much from them as well as from many of the postdocs we have had at Courant Institute. One semester I ran a seminar for postdocs and faculty in which people described problems on which they were stuck—I called it group therapy for analysts—such as a lemma or inequality they needed. Or else, they discussed some known result for which they thought there should be a better proof. It was a big success—a number of the problems posed were even solved—and I recommend to everyone to try it. Of course, you need a minimal list of problems on hand in order to get it started.

This award is for lifetime achievement; I'm still doing or trying to do mathematical research. I'm slower and make more mistakes, but it's still an enormous pleasure. My warmest thanks to the Steele Prize Committee for this lovely award.