Fulkerson Prizes in Discrete Mathematics

The Delbert Ray Fulkerson Prize in Discrete Mathematics was established in 1977 by friends of the late Ray Fulkerson. It is sponsored jointly by the Mathematical Programming Society and the American Mathematical Society and is awarded for outstanding papers in the area of discrete mathematics.

The guidelines for the Fulkerson Prize require that in order to be eligible, a paper must have been published during the six calendar years preceding the International Mathematical Programming Symposium at which the prize is awarded. They also specify that up to three prizes may be awarded in each three-year period between consecutive Symposia and, in the case of joint authorship, the prize will be shared.

The Fulkerson Prize Committee, consisting of Richard A. Karp, Victor L. Klee, Jr., and Ronald L. Graham (Chairman) elected to award three Fulkerson Prizes at the 11th Mathematical Programming Society Symposium, held in Bonn during August 23–27, 1982. The recipients and the respective papers cited for the awards are listed below in chronological order of publication.

One prize is awarded jointly to

D. B. Judin and A. S. Nemirovskiĭ

for their paper, *Informational complexity and effective methods of solution for convex extremal problems*, Ekonomika i Matematicheskie Metody **12** (1976), 357–369, and to

L. G. Khachiyan

for his paper, *A polynomial algorithm in linear programming*, Akademiia Nauk SSSR. Doklady **244** (1979), 1093–1096.

One prize is awarded jointly to

G. P. Egorychev

for his paper, *The solution of van der Waerden's problem for permanents*, Akademiia Nauk SSSR. Doklady **258** (1981), 1041–1044, and to

D. I. Falikman

for his paper, *A proof of the van der Waerden conjecture on the permanent of a doubly stochastic matrix*, Matematicheskie Zametki **29** (1981), 931–938.

One prize is awarded to

M. Grötschel, L. Lovász and A. Schrijver

for their paper, *The ellipsoid method and its consequences in combinatorial optimization*, Combinatorica **1** (1981), 169–197.

It is appropriate here to make a few remarks concerning the work for which the awards were made. Among the many discrete optimization problems in operations research, none has been more thoroughly investigated than that of linear programming. Yet in spite of this effort, until 1979 the computational complexity of linear programming had defied classification into either being solvable in polynomial-time or being NP-complete. Indeed, there were some researchers who felt that linear programming was a good candidate for an example of a problem which was not in P (the class of problems solvable by polynomial-time algorithms) and not NP-complete. (The reader unfamiliar with these concepts can find an excellent discussion in the book by M. R. Garey and D. S. Johnson, *Computers and intractability: A guide to the theory of NP-completeness*, W. H. Freeman and Company, San Francisco, 1979). It is known that such problems must exist if $P \neq NP$. What Khachiyan's paper provided was a proof that linear programming in fact belonged to P, thus resolving an important theoretical issue that had plagued researchers for years. His results were based on an adaptation of the ellipsoid method for convex optimization first described in the cited paper of Judin and Nemirovskiĭ. This algorithm for linear programming is radically different from the standard (and highly effective) simplex algorithm, and does not depend directly on the linearity of either the constraints or the objective function. It should be pointed out that the theory developed by Judin, Nemirovskiĭ and Khachiyan, rests firmly on the pioneering work done by the Soviet mathematician N. Z. Shor in the late 1960s and early 1970s.

Although the ellipsoid method is not (yet) a serious competitor to the simplex algorithm for the solution of everyday linear programming problems (it was not designed to be), it has proved to be an extremely potent tool in the arsenal of combinatorial optimizers. In the cited paper of Grötschel, Lovász and Schrijver, the ellipsoid method is applied to provide for the first time polynomial algorithms for the basic problems of minimizing submodular functions and finding maximum independent sets in perfect graphs, as well as providing relatively "easy" unified proofs of the polynomial solvability of

This report was prepared by Ronald L. Graham, chairman of the Fulkerson Prize Committee, for publication in the *Notices* and in *Optima*, the newsletter of the Mathematical Programming Society.

a number of other combinatorial optimization problems. Equally important, Grötschel, Lovász and Schrijver also show that a wide variety of optimization problems and their corresponding (apparently simpler) "separation" problems are actually computationally equivalent. While there is not space here to go into these important ideas any further, it seems clear that their power is just beginning to be tapped. For an excellent in-depth survey of all aspects of the ellipsoid method the reader should consult: R. G. Bland, D. Goldfarb, and M. J. Todd, *The ellipsoid method: A survey*, Operations Research **29** (1981), 1039–1091.

The papers of Egorychev and Falikman each present elegant and surprisingly simple proofs of the notorious "permanent" conjecture of van der Waerden. This conjecture, which had resisted the efforts of mathematicians for nearly 50 years, asserted that the permanent of any n by n doubly stochastic matrix (i.e., having nonnegative entries and all row and column sums equal to 1) is always at least as large as $n!/n^n$. The strong form of the conjecture (also proved by Egorychev) was that the unique matrix achieving this value has all entries equal to $1/n$. In a sense, both proofs have their roots in an earlier paper of Marcus and Newman, *On the minimum of the permanent of a doubly stochastic matrix*, Duke Mathematical Journal **26** (1959), 61–72, and both rely on a key lemma which turns out to be a special case of the Alexandrov inequalities for mixed discriminants of quadratic forms (although Falikman's proof is completely self-contained). An excellent overview of the two proofs has recently appeared in: J. C. Lagarias, *The van der Waerden conjecture: Two Soviet solutions*, Notices of the AMS **29** (1982), 130–133.

The presentations of the Fulkerson Prizes took place during the opening day ceremonies of the Symposium. Unfortunately, due to last minute circumstances beyond their control, none of the recipients from the Soviet Union were able to be present.

EDITOR'S NOTE. Of the five articles cited by the Fulkerson prize committee, one was written in English and four in Russian. Of the latter, three have been translated into English and may be found in the following places (*Mathematical Reviews* numbers in brackets):

Egorychev: Soviet Mathematics Doklady (American Mathematical Society, Providence) **23** (1982), 619–622. **[MR 82e:15006, 83b:15002a, 83b:15002b, 83b:15003]**

Falikman: Mathematical Notes of the Academy of Science of the USSR (Consultants Bureau, New York) **29** (1981), 475–479. **[MR 82k:15007]**

Khachiyan: Soviet Mathematics Doklady (American Mathematical Society, Providence) **20** (1979), 191–194. **[MR 80g:90071]**

The paper by **Judin & Nemirovskiĭ** was reviewed in *Mathematical Reviews* **[MR 56#10965]**, but has not appeared in translation.

The paper by **Grötschel, Lovás and Schrijver** is in English but has not yet been reviewed.

Queries

Edited by Hans Samelson

QUESTIONS WELCOMED from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published or unpublished conjectures.

REPLIES from readers will be edited, when appropriate, into a composite answer and published in a subsequent column. All answers received will ultimately be forwarded to the questioner.

QUERIES and RESPONSES should be typewritten if at all possible and sent to Professor Hans Samelson, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940.

Query

265. A. Wilansky (Department of Mathematics, Lehigh University, Bethlehem, Pennsylvania 18015). Let X be a BK space containing all finite sequences. Write (a) X^β is closed in X', (b) X^β is closed in X^f, (c) X has bounded sections. Does (a) imply (c)? Does (b) imply (c)? Does (a) imply (b)?