1999 Bôcher Prize

The Maxime Bôcher Memorial Prize is awarded every five years for a notable research memoir in analysis which has appeared in the previous five years. The prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer (1896) and tenth president (1909–1910) and was also one of the founding editors of *Transactions of the AMS*. The recipient must be a member of the Society, or the memoir for which the prize is given must be published in a recognized North American journal. The prize carries a cash award of $4,000.

The eighteenth Bôcher Prize was awarded to Demetrios Christodoulou, Sergiu Klainerman, and Thomas Wolff at the 105th Annual Meeting of the AMS in January 1999 in San Antonio.

The prize was awarded by the AMS Council acting on the recommendation of a selection committeee, whose members at the time the 1999 prize was awarded consisted of James Glimm (chair), Peter Sarnak, and Leon Simon.

The text that follows contains, for each prize recipient, the committee's citation, a brief biographical sketch, and the recipient's response upon receiving the award.

Demetrios Christodoulou

Citation

The Bôcher Prize is awarded to Demetrios Christodoulou for his contributions to the mathematical theory of general relativity. In particular, the prize is awarded for his remarkable memoir with S. Klainerman, *The Global Nonlinear Stability of the Minkowski Space*, Princeton Math Ser., vol. 41, Princeton University Press, 1993, which establishes the nonlinear stability of the Minkowski metric; and for his fundamental papers "Examples of naked singularity formation in gravitational collapse of a scalar field", *Ann. Math.* **140** (1994), 607–665, and "The instability of naked singularities in the gravitational collapse of a scalar field", *Ann. Math.* **149** (Jan. 1999), 183–217, which show, contrary to the widely held view, that naked singularities occur in the gravitational collapse of a scalar field; and for his analysis of the instability of these singularities.

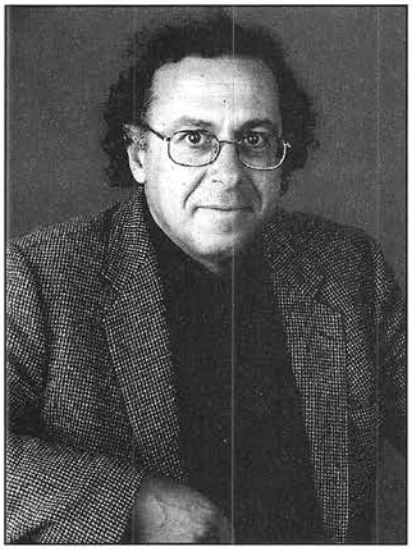
Biographical Sketch

Demetrios Christodoulou was born October 19, 1951, in Athens, Greece. He received his Ph.D. in physics from Princeton University in 1971. He held positions as a Humboldt fellow at the Max Planck Institute (1976–81), as a visiting member at the Courant Institute (1981–83), as an associate professor (1983–85) and professor (1985–87) at Syracuse University, and as professor of mathematics (1988–92) at the Courant Institute. Since 1992 he has been professor of mathematics at Princeton University.

Christodoulou was awarded the Otto Hahn Medal in mathematical physics in 1981, the Xanthopoulos Award in relativity in 1991, and a MacArthur Fellowship in 1993. He also received the Excellence in the Sciences Award from the Academy of Athens in 1996, an Honorary Doctorate in the Sciences from the University of Athens in 1996, and a Guggenheim Fellowship in 1998.

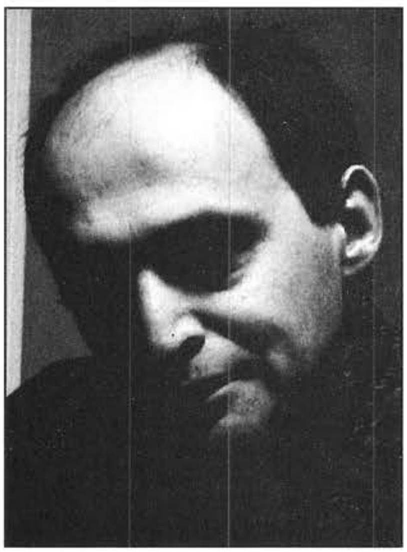
Response

It is a great honor for me to be awarded the 1999 Bôcher Prize. I would like to express my gratitude



Pryde Brown Photographs

Demetrios Christodoulou



Photograph courtesy of Anca Niculin.

Sergiu Klainerman

Thomas Wolff

to the Bôcher Prize Committee and the AMS for recognizing my work.

My mathematical research has centered on the study of global problems associated to nonlinear systems of partial differential equations of hyperbolic type, i.e., the global existence and regularity of solutions to the initial value problem, the formation and structure of singularities, and the long-time asymptotic behavior. I have given particular attention to the Einstein equations of general relativity where the solution of problems requires a combination of purely analytic and differential-geometric methods.

The joint work with Sergiu Klainerman demonstrated that any asymptotically flat initial data for the vacuum Einstein equations which is suitably close to trivial data gives rise to a global solution, a geodesically complete spacetime tending to flatness at infinity along any geodesic, thus establishing the stability of the Minkowski space of special relativity in the framework of the general theory and providing the basis for a rigorous theory of gravitational radiation.

Another work, involving several steps completed over the years, concerns the initial value problem in the large under the assumption of spherical symmetry, for the Einstein equations with matter, the energy-momentum-stress tensor being that corresponding to a scalar field. One of the principal results of that work was that if the initial data verifies a certain largeness condition, then catastrophic gravitational collapse must occur. This is signaled by the formation of a trapped region, that is, a spacetime region where the future light cones have cross-sectional areas decreasing with time. The formation of a trapped region is preceded by that of an event horizon, namely, of a future boundary of the set of points which are causally connected to infinity. The trapped region ends at a singular boundary, whose structure was analyzed in detail. Another, unexpected result of the work was that naked singularities, namely, singular points which are not preceded by a trapped region and which are causally connected to infinity, also occur. The work culminated in a paper which shows that in the space of initial data the subset leading to the formation of naked singularities has positive codimension, hence must be viewed as being exceptional, and initial data belonging to the complement of this subset leads either to a complete regular solution or to the formation of a trapped region.

Despite the fact that some progress has been achieved, our subject is still very much in its infancy. The global stability of the Kerr family of nontrivial stationary solutions of the vacuum Einstein equations has not yet been investigated. The simultaneous removal of symmetry assumptions as well as restrictions on the size of the initial data leads us to a territory which is at present almost completely unexplored. Moreover, the general framework of nonlinear systems of partial differential equations of hyperbolic type offers many additional problems of fundamental significance, such as the development of a general theory of shock formation and evolution in 3-dimensional continuous media.

Sergiu Klainerman

Citation

The Bôcher Prize is awarded to Sergiu Klainerman for his contributions to nonlinear hyperbolic equations. In particular, the prize is awarded for his remarkable memoir, joint with D. Christodoulou, *The Global Nonlinear Stability of the Minkowski Space,* Princeton Math Ser., vol. 41, Princeton University Press, 1993, which establishes the nonlinear stability of the Minkowski metric; and for his fundamental papers, joint with M. Machedon, "Space-time estimates for null forms and the local existence theorem", *Comm. Pure Appl. Math.* **46** (1993), 1221–1268, and "Smoothing estimates for null forms and applications", *Duke Math. J.* **81** (1995), 99–133, on well posedness and global existence for nonlinear wave equations and the crucial "null form" condition central to these works.

Biographical Sketch

Sergiu Klainerman was born in Bucharest, Romania, in 1950. He received his Ph.D. from New York University and was a Miller Fellow at Berkeley from 1978 to 1980. He was assistant, associate, and full professor at New York University from 1980 to 1987. Since 1987 he has been a full professor at Princeton University.

He has held visiting positions at various institutions, including the Institute for Advanced Study in Princeton, the Institut des Hautes Études Scientifiques in Bures-sur-Yvette, France, the Eidgenössische Technische Hochschule in Zürich, and Stanford University, as well as in Jerusalem, Bonn, and Kyoto. Last year he held the position of Blaise Pascal International Chair at Université de Paris VI.

Klainerman's honors include a MacArthur Fellowship; Miller, Sloan and Guggenheim Fellowships; and the Le Conte Prize of the French Academy of Sciences. He is a Fellow of the American Academy of Arts and Sciences. His research interests are partial differential equations of mathematical physics and their connections to Fourier analysis and geometry.

Response

I am greatly honored to share the 1998 Bôcher Prize with my colleagues D. Christodoulou and T. Wolff and grateful to the selection committee for their kind recognition of my work and the field I represent. Before commenting on the citation, I would like to thank various people for the special role they have played in my scientific career. I will mention only a select few. On a personal level my parents, brother, uncle Hers, and my wife, Anca, have given me unconditional love and support. O. Liess was my enthusiastic mentor during my undergraduate education in Romania; I owe him a lot. I am also greatly indebted to my former thesis advisor, L. Nirenberg, whose insight and generosity helped me to find my own research path. F. John was a great source of inspiration, a role model, and a personal friend. As a graduate student I was also influenced, in a broad scientific sense, by many stimulating conversations with J. Moser. A. Majda influenced my scientific tastes a great deal during the two-year period of my Miller Fellowship at Berkeley. I am especially indebted to my collaborators, foremost among them D. Christodoulou and M. Machedon.

As mentioned in the citation, most of my work has to do with nonlinear hyperbolic equations. This is a difficult subject, with its roots in the many examples of such systems appearing in continuum mechanics, general relativity, relativistic field theory, and geometry. The modern theory of hyperbolic equations, with its emphasis on qualitative features rather then explicit solutions, goes back to the beginning of the century. To Hadamard we owe the very distinction between well-posed and ill-posed problems and thus the first meaningful characterization of hyperbolicity. The linear theory was later developed by people like H. Lewy, K. Friedrichs, I. G. Petrowsky, S. Sobolev, J. Leray, etc. They were also responsible for developing the methods to prove the classical theorem of, local in time, existence and uniqueness for large classes of nonlinear hyperbolic equations. The one-dimensional theory for systems of hyperbolic laws goes back to Riemann. It had a very productive period from the mid 1940s to the late 1960s due to the eminent efforts of people like R. Courant, K. Friedrichs, J. von Neumann, P. Lax, S. Kruzkov, J. Glimm, and others. It has recently made important advances due to the work of Bressan.

The progress on higher-dimensional nonlinear equations was very slow, however, and by the early 1960s almost all results, which went beyond local in time existence and uniqueness, were restricted to one space dimension. The situation started to change in the 1960s and 1970s through the pioneering works of K. Jorgens, F. John, I. E. Segal, C. Morawetz, and W. Strauss. Today the subject of nonlinear hyperbolic equations is going through an exciting period of both broadening of scope and deepening of its methods and results. I can mention two developments which I consider most significant for higher-dimensional problems: the first is the emergence of methods tied to the geometric structure of nonlinear wave equations, and the second is the development of spacetime estimates based on Fourier analysis.

Geometric methods, by which I mean the use of the special symmetries of the equations in physical space, have allowed us to prove small data global regularity results for large classes of nonlinear wave equations, including quasilinear. They have played a fundamental role in my work in collaboration with D. Christodoulou on the *The Global Nonlinear Stability of the Minkowski Space*, mentioned in the citation. Geometric methods, in a somewhat different way, also played an important role in the beautiful global regularity result for the Yang-Mills equations in $\mathbb{R}^{3+1}$ due to Eardley and Moncrief.

The role of spacetime estimates was first pointed out by I. E. Segal, and it was then discovered by R. S. Strichartz that such estimates are intimately tied to the restriction theorems in Fourier analysis pioneered by E. Stein. Strichartz type estimates, as later developed by H. Pecher, Ginibre-Velo, and Keel-Tao, have found important applications, the most notable among them being the global regularity result for critical, scalar, nonlinear wave equations due to M. Struwe and M. Grillakis.

My work with M. Machedon builds on the latter development, based on Fourier analysis, but attempts also to take advantage of the nonlinear structure of special geometric equations such as Yang-Mills and Wave-Maps. Our main achievement was to develop a class of bilinear and some multilinear estimates intimately tied to the specific "null form" structure of these equations. One can thus obtain a lot more regularity than is possible with Strichartz type estimates alone. In the case of the $3+1$ dimensional Yang-Mills equations, these new estimates have allowed us to prove global regularity in the energy class, thus extending and giving a new proof of the above-mentioned result of Eardley-Moncrief. In the case of Wave-Maps in $2+1$ dimensions, our new estimates go quite a long way but still fall short of proving the expected regularity result. New ideas are certainly needed.

Despite the remarkable progress made during this past century, the subject of hyperbolic equations is still in its infancy. Most of the fundamental questions concerning the regularity and asymptotic behavior of solutions to the important physical equations remain wide open. The methods which have been used so far are still crude by comparison to the powerful tools of elliptic theory, and this reflects in the very incomplete nature of our results. Yet there lies the great attraction of the field to me: the basic rules of the game are still to be uncovered.

Thomas Wolff

Citation

Tom Wolff is awarded the Bôcher Prize for his work in harmonic analysis, notably the work presented in his papers "A Kakeya type problem for circles", *Amer. J. Math.* **119** (1997), 985–1026, and "An improved bound for Kakeya type maximal functions", *Rev. Mat. Iberoamericana* **11** (1995), 651–674. The techniques presented there represent an important contribution to our understanding of the structure of subsets of Euclidean space, involving the interplay between geometric measure theory and harmonic analysis. The award is also made in acknowledgement of Wolff's work on harmonic measure and unique continuation, including in particular *Counterexamples with harmonic gradients in* R^3, Princeton Math. Ser., vol. 42, Princeton University Press, 1995, 321–384.

Biographical Sketch

Thomas Wolff was born in New York City, July 14, 1954, and received his A.B. degree from Harvard in 1975 and his Ph.D. from the University of California, Berkeley, in 1979, under Donald Sarason. He was a National Science Foundation Postdoctoral Fellow at the University of Chicago (1980–82) and has held teaching positions at the University of Washington; New York University; University of California, Berkeley; and California Institute of Technology, where he is now professor of mathematics.

Response

The Bôcher Prize is truly a great honor, and I am most grateful to the committee for recognizing my work in this manner. Many people have contributed in an essential way to the circle of ideas around the Kakeya problem and to the other topics mentioned in the citation, both before me and also recently, and I take this as a tribute to all of them.

Mathematics often develops in an incremental way, and most of the things I have done have been motivated by related results in the literature. For the two papers in geometric measure theory, I started from some of Bourgain's papers on the Kakeya problem and was then able to incorporate some ideas from the combinatorics literature, especially the paper of Clarkson, Edelsbrunner, Guibas, Sharir, and Welzl, "Combinatorial complexity bounds for arrangements of curves or spheres". Much remains to be done in this area, and my hope is that these ideas may continue to prove useful.

I first learned about the power of combinatorial ideas in analysis twenty years ago as a graduate student by reading the proof of the corona theorem and other related papers in function theory. The construction in my paper on harmonic gradients was suggested by the work that had been done on the analogous snowflake domains in the plane by Carleson, and by Kaufman and Wu.

Finally, it's never been easy for me, and I would like to thank several people for encouragement at the beginning of my career—Alice Chang, John Garnett, Jürgen Moser, Don Sarason, and Nick Varopoulos.