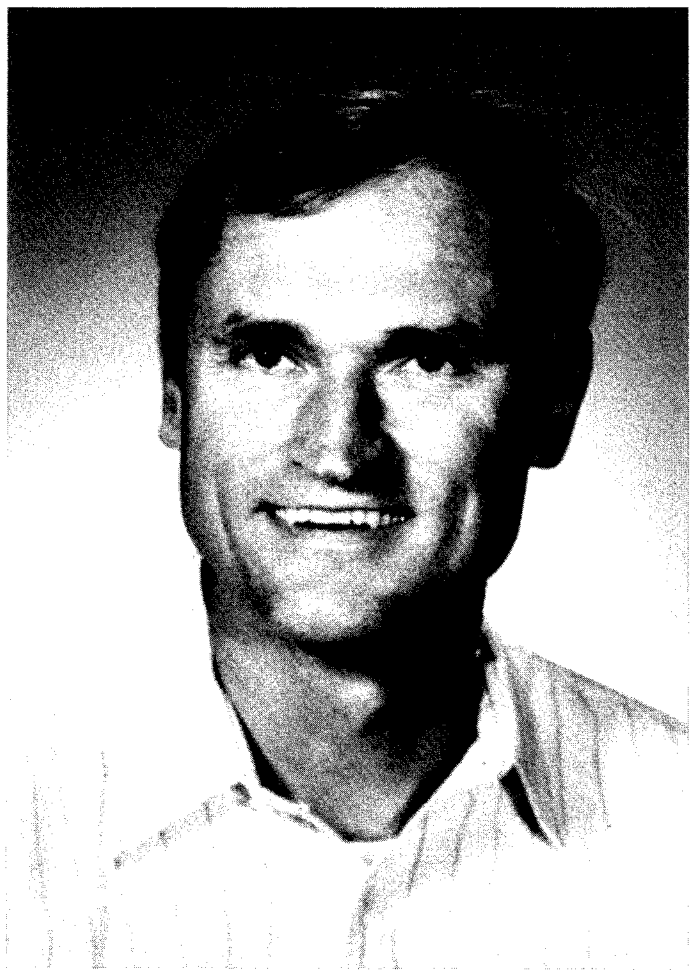
Richard M. Schoen Awarded 1989 Bôcher Prize

The Bôcher Prize is awarded every five years for a notable research memoir in analysis which has appeared in the previous five years. The prize honors Maxime Bôcher (1867–1918) who was the Society's second Colloquium Lecturer (1896) and tenth President (1909, 1910), and one of the founding editors of the *Transactions*. The sixteenth award was made at the Society's ninety-fifth Annual Meeting, in Phoenix, Arizona, on January 12, 1989.

The 1989 recipient is Richard M. Schoen of Stanford University. The Bôcher Prize is augmented by awards from the Leroy P. Steele Fund and currently amounts to $4,000.

The Prize was awarded by the Council of the Society acting on the recommendation of the Committee to Select the 1989 Recipient of the Bôcher Prize, consisting of Paul J. Cohen, Richard B. Melrose, Chairman, and Louis Nirenberg.

The text below includes the Committee's citation, the recipient's response on presentation of the award and a brief biographical sketch of the recipient.



Citation

The 1989 Bôcher Prize is awarded to Richard M. Schoen of Stanford University for his work on the application of partial differential equations to differential geometry, in particular his completion of the solution to the Yamabe Problem in "Conformal deformation of a Riemannian metric to constant scalar curvature". *Journal of Differential Geometry* **20** (1984) pages 479-495.

Response

It is a great honor for me to receive the Bôcher prize of 1989, and I thank the A.M.S. and the Bôcher Committee for recognizing my work in this way. There is a long tradition of interaction between the cited areas of Differential Geometry and Partial Differential Equations. I will discuss the interaction which centers around *nonlinear* problems. An early example which illustrates this tradition is H. Weyl's formulation, in 1916, of the isometric embedding problem for closed surfaces of positive curvature. The geometric problem was reduced to certain estimates which were later obtained by Nirenberg and Pogorelov for smooth metrics. In the meantime Lewy had solved the problem for analytic metrics. This example is typical in that it often happens that a geometric problem can be formulated as a specific problem in P.D.E.; thus from the geometer's point of view P.D.E. often provides the framework necessary to attempt a solution as well as a body of knowledge (estimates, regularity results, and such) and more importantly methods of attack. The interaction benefits both areas because

Differential Geometry provides a rich class of nonlinear problems which serve as model problems from which a body of knowledge may emerge. The geometric meaning of the solution may lead one toward the correct analytical estimates and theorems.

The Yamabe Problem should be viewed as a part of the variational theory for the Einstein-Hilbert variational problem; the elliptic version of the action principle governing the motion of the gravitational field in General Relativity. Yamabe viewed this general problem as an analytic approach to solving the three dimensional Poincaré conjecture. He formulated the problem which bears his name as a semilinear scalar P.D.E. around 1960 and it was solved in special cases by Trudinger and Aubin over the next 15 years. In its strongest form the solution asserts that critical points of the variational problem which are predicted by Morse theory do, in fact, exist. The proof of this general result involves consideration of large energy solutions, and, in fact, one must consider singular weak solutions of the P.D.E. The structure of weak solutions is an intricate topic about which we have limited knowledge. Additionally, an important role in the derivation of the necessary estimates is played by results from General Relativity involving gravitational energy. Thus we return full circle to the origins of the problem.

Another problem which arises from Differential Geometry and has had substantial impact on the development of P.D.E. is the Plateau problem, or the study of minimal submanifolds. The nonparametric minimal surface equation provided a strong impetus for the development of the theory of quasi-linear elliptic equations in the fifties and sixties. A successful higher dimensional parametric theory was begun approximately 30 years ago through the work of DeGiorgi, Federer, and Fleming. A major outgrowth of this work was the introduction of partial regularity theory into P.D.E. The partial regularity method has become a powerful tool applicable to a variety of nonlinear problems. Finally, the theory of minimal hypersurfaces has been applied to obtain results about manifolds of positive scalar curvatures by S.T. Yau and myself. This application uses the analytic theory (existence and regularity) in an important way. Thus the theory of minimal submanifolds is of substantial importance for both geometry and P.D.E.

A common feature of both the fields of nonlinear P.D.E. and global Differential Geometry is that our knowledge in both areas is very primitive, and there are many more unsolved problems than there are theorems. Specific areas in which new results would be important are quasi-linear elliptic systems (hopefully including the elliptic Einstein equations), nonlinear parabolic and hyperbolic evolution equations, and singularities of nonlinear equations. The problems involving singularities which are important include the study of singular sets for elliptic problems such as those of least area submanifolds as well as singularity development in nonlinear evolution equations. I am confident that the long tradition of interaction between Differential Geometry and nonlinear P.D.E. will continue to be fruitful to both fields for many years to come.

Biographical Sketch

Richard M. Schoen was born October 23, 1950, in Celina, Ohio. He received his B.S. degree from the University of Dayton in 1972 and his Ph.D. from Stanford University in 1976 under the direction of Leon Simon and Shing-Tung Yau. He was a lecturer at the University of California, Berkeley from 1976–1978 and an assistant professor at the Courant Institute from 1978–1980. From 1980–1987 he was a professor at the University of California; in Berkeley from 1980–1985 and in San Diego from 1985–1987. Since 1987 he has been Professor of Mathematics at Stanford University.

Professor Schoen has been a visiting member at the Institute for Advanced Study on two occasions, during the 1979–1980 academic year and during the spring semester of 1984. He was a visiting professor at the University of Melbourne in Australia during the fall of 1980 and a visiting professor at Stanford University during the fall of 1983. In the fall of 1985 he was a visitor at the Mittag-Leffler Institute in Sweden and at the Institut des Hautes Études Scientifiques in France.

Professor Schoen was awarded an NSF Graduate Fellowship in 1972, a Sloan Foundation Fellowship in 1979, and a MacArthur Prize Fellowship in 1983. He was elected to the American Academy of Arts and Sciences in 1988.