1991 Oswald Veblen Prize in Geometry Awarded in San Francisco

The 1991 Oswald Veblen Prize in Geometry was awarded at the Joint Mathematics Meetings in San Francisco to ANDREW J. CASSON of the University of California at Berkeley and to CLIFFORD H. TAUBES of Harvard University.

Oswald Veblen (1880-1960), who served as President of the Society in 1923 and 1924, was well known for his mathematical work in geometry and topology. In 1961, the Trustees of the Society established a fund in memory of Professor Veblen, contributed originally by former students and colleagues, and later doubled by his widow. Since 1964, the fund has been used for the award of the Oswald Veblen Prize in Geometry. Subsequent awards were made at five-year intervals. A total of ten awards have been made: Christos D. Papakyriakopolous (1964), Raoul H. Bott (1964), Stephen Smale (1966), Morton Brown and Barry Mazur (1966), Robion C. Kirby (1971), Dennis P. Sullivan (1971), William P. Thurston (1976), James Simons (1976), Mikhael Gromov (1981), Shing-Tung Yau (1981), and Michael H. Freedman (1986). At present, the award is supplemented from the Steele Prize Fund, bringing the value of the Veblen Prize to $4000, divided equally between this year's recipients.

The 1991 Veblen Prize was awarded by the AMS Council on the basis of a recommendation by a selection committee consisting of Edgar H. Brown, Jr., Michael H. Freedman (chair), and Mikhael Gromov.

The text that follows contains the committee's citation for each award, the recipients' responses at the prize session in San Francisco, and a brief biographical sketch of each recipient.

Andrew J. Casson

Citation

For his work on the topology of low dimensional manifolds and specifically for the discovery of an integer valued invariant of homology three spheres whose reduction mod(2) is the invariant of Rohlin. The mod(2) Rohlin invariant was extremely important in high dimensional topology. It is essentially the Kirby-Siebenmann obstruction to triangulating topological manifolds. Casson's integer lifting of this classical invariant has intimate connections with gauge theory, symplectic geometry, and representation varieties and has blossomed into a rich subject. This achievement stands among his many results and joint works in low dimensional topology.

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Response

I am very grateful to the American Mathematical Society and its Veblen Prize Committee for this honor. I also want to thank the many mathematicians who guided me over the years, in particular Terry Wall, who introduced me to topology, and Cameron Gordon, who worked with me on knot cobordism and other questions.

My work in geometric topology has been concerned, directly or indirectly, with the classification problem for manifolds. Of course, spectacular progress has been made on this problem, particularly by previous winners of the Veblen Prize. In the past two decades, interest has centered on the classification of "low dimensional" manifolds, meaning manifolds of dimension three or four. For reasons which seem at first sight purely technical, but turn out to be fundamental, low dimensional manifolds are radically different from the high dimensional manifold classified by Smale and others in the 1960s. In 1974 I introduced objects that I called "flexible handles" in an attempt to adapt Smale's methods to four-dimensional manifolds. I was fortunate indeed when Michael Freedman, by a superhuman effort, showed in 1981 that they do in fact play an important role in the topological classification of four-dimensional manifolds. Almost simultaneously, remarkable developments in gauge theory showed that the differentiable classification of four-manifolds is very different.

More recently, I have worked mainly on three-dimensional manifolds. Again this turns out to be a new subject, having surprising connections with classical geometry and analysis, especially Kleinian groups. These connections are made explicit in Thurston's famous "Geometrization Conjecture", which also holds out the exciting possibility of a classification of three-dimensional manifolds that could be implemented on a computer. Partly in an attempt to test special cases of Thurston's conjecture, I was led in 1985 to introduce a new invariant for (certain) three-dimensional manifolds. It was a pleasant surprise to me how many mathematicians and physicists have taken an interest in this work.

Geometric topologists are sometimes accused of failing to give rigorous proofs. Certainly, many major advances in geometric topology do not fit the popular notion of mathematics as a subject which proceeds by a sequence of small, logical steps. Rather, they depend on new ideas which are conceived geometrically, and not easily translated into words on a page. The function of the words is to communicate the geometrical pictures, so that the reader or listener can mentally recreate the complete argument. When one considers that most of the objects being studied cannot be pictured in full detail in three-dimensional space, it is remarkable that topologists are in fact able to reach agreement about the correctness of geometric proofs.

Biographical Sketch

Andrew John Casson was born January 22, 1943 in London, England. He received his B.A. degree from University of Cambridge in 1965. He was a research fellow (1967–1971), assistant lecturer, (1971–1976), and lecturer (1976–1981) at Trinity College, Cambridge University. From 1981 to 1986, he was a professor at the University of Texas, Austin before moving to his present position as professor, University of California at Berkeley.

Casson presented an address at the AMS Special Session on low dimensional topology at the Joint Mathematics Meetings in San Francisco (January 1981) and an AMS Invited Address at the Joint Mathematics Meetings in Pittsburgh (August 1981). He also presented invited addresses at the International Congress of Mathematicians in 1978 and in 1986.

Clifford H. Taubes

Citation

For his foundational work in Yang-Mills theory. Taubes, since the time of his Ph.D. thesis and book on vortices and monopoles (coauthored with Arthur Jaffe), has done as much as any individual to forge emerging physical concepts into powerful mathematical tools.

The harnessing of Yang-Mills theory by mathematicians began with Karen Uhlenbeck's work on the singularities of, and curvature estimates for, the solutions of these equations. From this beginning, Taubes laid a geometric and analytical foundation for the study of the Yang-Mills functional. His initial paper—"Self-dual Yang-Mills connections on non-self-dual 4-manifolds" (*J. Diff. Geom.*, **17**, 1982)—contained the technical basis for Simon Donaldson's first celebrated non-existence theorem. Similarly, antecedents of Andreas Floer's remarkable "homology theory" occur in Taubes' use of spectral flow to determine the signs in his paper, "Casson's invariant and gauge theories" (*J. Diff. Geom.*, **31**, 1990).

The understanding that solutions to the Yang-Mills equations could be constructed over metrically arbitrary 4-manifolds is due to Taubes. A fundamental tool in the analysis of Yang-Mills fields on manifolds is "neck-pulling," wherein the underlying manifold degenerates. The behavior of the Yang-Mills field is carefully tracked during the degeneration so that knowledge of the limit yields implications about the original fields. Taubes pioneered this method in his analysis of End periodic 4-manifolds ("Gauge theory on asymptotically periodic 4-manifolds," *J. Diff. Geom.*, **25**, 1987). Among the topological implications was a proof that there are uncountably many smooth structures on R^4.

Taubes' most recent work (unpublished) analyzes the behavior of Yang-Mills connections along ends which have nontrivial first homology. Important topological consequences of such connections have recently been discovered by his students Tom Mrowka and Bob Gompf.

Cliff Taubes laid much of the foundation for a remarkable decade of Yang-Mills theory.

Response

The muses of mathematics have been very good to me; I will try to repay them for their kindness. And, thank you to the American Mathematical Society for bestowing upon me such a great honor.

Biographical Sketch

Clifford Henry Taubes was born on February 21, 1954 in New York City. He received his B.A. from Cornell University (1975) and his M.S. (1978) and Ph.D. (1980) degrees from Harvard University. From 1980 to 1983, he was a junior fellow at Harvard University. After that, he held a position as acting associate professor at the University of California at Berkeley before returning to Harvard as professor of mathematics in 1985. He held a National Science Foundation (NSF) Mathematical Sciences Postdoctoral

Clifford H. Taubes

Fellowship (1984–1987). He is currently a member of the NSF Advisory Committee for the Mathematical Sciences. In 1990, he was elected to the American Academy of Arts and Sciences.

Taubes presented an address at the Special Session on Nonlinear Generalizations of Maxwell's Equations at the AMS Meeting at the University of Massachusetts at Amherst in October 1981; an AMS Invited Address at the Joint Mathematics Meetings in Eugene, Oregon in August 1984; and a 45-minute address at the International Congress of Mathematicians at Berkeley in 1986. He was also an invited speaker at the Symposium on the Mathematical Heritage of Hermann Weyl, held at Duke University in May 1987. His areas of research are differential geometry, nonlinear partial differential equations, and mathematical physics.