1992 Cole Prize in Algebra

The Frank Nelson Cole Prize in Algebra is awarded every five years for a notable research memoir in algebra which has appeared during the previous five years. This prize, as well as the Frank Nelson Cole Prize in Number Theory, was founded in honor of Professor Frank Nelson Cole on the occasion of his retirement as Secretary of the American Mathematical Society after twenty-five years and as Editor-in-Chief of the *Bulletin* for twenty-one years. The original fund was donated by Professor Cole from moneys presented to him on his retirement. It has been augmented by contributions from members of the Society, including a gift made in 1929 by Charles A. Cole, Professor Cole's son, which more than doubled the size of the fund. In recent years, the Cole Prizes have been augmented by awards from the Leroy P. Steele Fund and currently amount to $4,000.

The Twenty-Fourth Cole Prize was awarded jointly to KARL RUBIN of Ohio State University and PAUL VOJTA of the University of California at Berkeley. The prize was awarded at the Society's ninety-eighth Annual Meeting in Baltimore. The Cole Prize was awarded by the Council of the American Mathematical Society, acting through a selection committee consisting of Gerd Faltings, Wilfried Schmid, and Harold Stark (Chair).

The text below includes the Committee's citations, the recipients' responses to the award, and a brief biographical sketch of each of the recipients.

Karl Rubin

Karl Rubin

Citation

To Karl Rubin for his work in the area of elliptic curves and Iwasawa Theory with particular reference to his papers "Tate-Shafarevich groups and L-functions of elliptic curves with complex multiplication" and "The 'main conjectures' of Iwasawa theory for imaginary quadratic fields."

Response

I would like to thank the American Mathematical Society for this award. It is also a great pleasure on this occasion to acknowledge the work of Francisco Thaine and of Victor Kolyvagin. Without their wonderful new ideas, the two papers of mine which were cited by the Cole Prize Committee could not have come about.

During the past few decades, there has been a great deal of research on elliptic curves defined over the rational numbers. One of the fundamental questions in this area is the conjecture of Birch and Swinnerton-Dyer. This conjecture, which grew out of computer calculations in the late 1950s and early 1960s, relates the arithmetic of an elliptic curve with the behavior of its L-function at the point 1. The first important breakthrough in the direction of this conjecture was made by Coates and Wiles in 1977. They proved that if an elliptic curve over the rational numbers has complex multiplication, and its L-function does not vanish at 1, then the curve has only finitely many rational points. My work is a natural outgrowth of theirs, obtained by combining their techniques with the methods of Thaine and Kolyvagin. Under the same hypotheses, I showed that the Tate-Shafarevich group is finite and has the order predicted by the Birch and Swinnerton-Dyer conjecture, to within powers of 2 and 3. Further, if the L-function instead has a simple zero at 1, then the group of rational points has rank one as predicted by Birch and Swinnerton-Dyer. Most of these results have now been proved by Kolyvagin for the larger class of modular elliptic curves, without the assumption of complex multiplication. Combined with work of Gross and Zagier, these results go a long way toward settling the Birch and Swinnerton-Dyer conjecture when the L-function of the elliptic curve has a zero of order at most one at the point 1. The current great mystery of this subject is that almost nothing is known about the cases where the L-function has a zero of order greater than one.

In closing, I would also like to thank my colleagues in the Ohio State University mathematics department for the support they have shown for my research and for going to great lengths to provide me with a productive research environment.

Biographical Sketch

Karl Rubin was born on January 27, 1956 in Urbana, Illinois. After attending Washington, D.C. public schools, he received an A.B. degree from Princeton University in 1976 and M.A. (1977) and Ph.D. (1981) degrees from Harvard University. He has been on the faculty of Ohio State University since 1984, first as an Assistant Professor and, since 1987, as a Professor. In the academic year 1988–1989 he was a Professor at Columbia University.

In 1990 and 1991 Professor Rubin was a member of the AMS Centennial Fellowship Committee. He gave invited hour addresses at AMS meetings in East Lansing (March 1988) and Worcester (April 1989) and he spoke in the Special Session on Algebraic Geometry and Number Theory in Muncie (October 1989).

Professor Rubin was a National Science Foundation Postdoctoral Fellow (1981–1984), a Sloan Fellow (1985–1987), and an Ohio State University Distinguished Scholar (1987–1990), and is a Presidential Young Investigator (1988–1993). He has held one-year visiting positions at the Institute for Advanced Study in Princeton and the Mathematical Sciences Research Institute in Berkeley, and has visited for shorter periods at the Institut des Hautes Études Scientifiques (Paris), the Max-Planck-Institut für Mathematik (Bonn), and the Nankai Institute (Tianjin).

Paul Vojta

Citation

To Paul Vojta for his work on Diophantine problems with particular reference to his paper "Siegel's theorem in the compact case."

Response

I would like to express my warmest thanks to the American Mathematical Society, and to the Cole Prize Committee in particular, for granting me this honor. I also thank numerous colleagues and former teachers for their assistance and encouragement.

The diophantine problems considered in my paper are systems of polynomial equations for which one searches for solutions in either rational numbers or integers. In the case of integral solutions, Siegel proved in 1929 that certain one dimensional systems of equations have only finitely many solutions. Siegel's proof was based on work of Thue in 1909: he constructed an auxiliary polynomial and used properties of that polynomial to derive a contradiction if there were too many solutions.

Paul Vojta

More recently, Faltings proved the Mordell conjecture in 1983, which established finiteness for *rational* points on certain one-dimensional systems of equations. He did this using some very high-powered results on moduli spaces of abelian varieties, together with some ideas of the Russian mathematician Arakelov. What Arakelov and others did was to generalize the methods of algebraic geometry to allow diophantine problems to be discussed in much the same framework as equations in finite extensions of the field $k(T)$. Arakelov theory is sufficiently general to describe the notions of both integral and rational solutions in the same framework.

One might wonder, then, whether Siegel's and Faltings' theorems could both be proved by similar methods. This is what my paper did: it gave a second proof of Faltings' theorem using a translation of Thue's method into the language of Arakelov theory, as refined more recently by Gillet and Soulé. Thus, neither the theorem nor the method were new. But on the other hand, it did show how to apply the methods of diophantine approximations to the study of rational points. This led to some recent and beautiful work by Faltings, in which he proved a more general finiteness theorem for rational points on subvarieties of abelian varieties.

Biographical Sketch

Paul Vojta was born on September 30, 1957 in Minneapolis, Minnesota. He received his B.Math. in 1978 from the University of Minnesota and his Ph.D. from Harvard University in 1983. He wrote his thesis, "Integral points on varieties," under the direction of Barry Mazur.

Professor Vojta held the position of Gibbs Instructor at Yale University from 1983 to 1986, and spent the following year at the Mathematical Sciences Research Institute in Berkeley. During the years 1984–1987, he also held a National Science Foundation Postdoctoral Fellowship. He spent the next two years at the Berkeley campus of the University of California as a Fellow, supported by the Miller Institute for Basic Research in Science. Since 1989, he has been an Associate Professor of Mathematics at the University of California, Berkeley, during which time he spent 1989–1990 at the Institute for Advanced Study in Princeton.

Vojta participated in the 1975 International Mathematical Olympiad held in Burgas, Bulgaria, and also placed in the top five in the William Lowell Putnam Intercollegiate Mathematical Competition in 1977. He gave an Invited Address at the International Congress of Mathematicians in Kyoto, 1990, and presented the C.I.M.E. Lecture Series in Trento, Italy in 1991. He is an editor of the *International Mathematics Research Notices.*