

Mary Cartwright (1900–1998)

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Dame Mary Cartwright, 1950.

Mary Cartwright died in Cambridge, England, on April 3, 1998, at the age of ninety-seven. Cartwright led a distinguished career in mathematics and university administration. During her career Cartwright authored over ninety articles, making important contributions to the theory of functions and differential equations. Her early work led to significant results concerning Dirichlet series, Abel summation, directions of Borel spreads, analytic functions regular on the unit disk, the zeros of integral functions, maximum and minimum moduli, and functions of finite order in an angle. She is particularly well known for her work with J. E. Littlewood on van der Pol's equation, nonlinear oscillators, and the development of topological techniques to prove existence theorems. Her later work includes contributions to the theory of cluster sets in collaboration with Edward Collingwood.

Cartwright and Littlewood were among the first mathematicians to recognize that topological and analytical methods could be combined to effi-

ciently obtain results for various problems in differential equations, and their results helped inspire the construction of Smale's horseshoe diffeomorphism. Their names can be included with those mathematicians, including Levinson, Minorsky, Liapounov, Kryloff, Bogolieuboff, Denjoy, Birkhoff, and Poincaré, whose work provided an impetus to the development of modern dynamical theory.

In addition to her research, Cartwright took a keen interest in mathematical education at all levels. She was an effective administrator at Cambridge University and ambassador for several mathematical and scientific organizations. She is remembered as a person who had a gift for going to the heart of a matter and for seeing the important point, both in mathematics and human affairs.

Cartwright received many honors. In particular, she was elected a Fellow of the Royal Society of London in 1947. Until Dusa McDuff was elected in 1994, Cartwright remained the only woman mathematician in the Royal Society. She received honorary doctorates from the universities of Edinburgh, Leeds, Hull, Wales, and Oxford. In 1969 Queen Elizabeth II elevated her to Dame Commander of the British Empire.

Mary Cartwright was born in Aynho, Northamptonshire, England, on December 17, 1900, into a family with a tradition of public service. She matriculated at St. Hugh's, Oxford, in the fall of 1919 and decided to pursue the honors mathematics program instead of her long-time favorite subject, history. Lectures were difficult to get into due to the flood of men recently released from the army. Cartwright went to the lectures she could and obtained notes from the others. After two years she received a second class on the Mathematical Moderations Examination. Although very few firsts were given that year, Cartwright was disappointed and seriously considered changing her focus to his-

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tory. After considerable deliberation she concluded that she was hooked on the theory of residues. She joked that history would entail longer hours of work anyway. Her decision to remain in mathematics did not diminish her interest in history. Many of her mathematical papers include historical perspectives that add an interesting dimension to her work. She also wrote several biographical memoirs that portray her exceptional sense of history.

Near the end of Cartwright's third year at Oxford, an important event in her mathematical career occurred at a well-chaperoned party. She was introduced to V. C. Morton, who told her that if she was really serious about mathematics she should read Whittaker and Watson's *Modern Analysis* and attend the evening classes of G. H. Hardy, then Savilian Professor of Geometry. Taking Morton's advice, she read *Modern Analysis* that summer and received special permission to attend Hardy's class. She found Hardy's lectures inspiring. With Hardy serving as an examiner, she received a first class on the Final Honours School Examination in Mathematics, obtaining her degree from Oxford in 1923.

During the next four years, Cartwright taught mathematics, first at the Alice Ottley School in Worcester and then at the Wycombe Abbey School in Buckinghamshire, where she also served as assistant mistress. Cartwright began to feel side-tracked by her mounting administrative duties. In addition, teaching and method content were strictly dictated at the school. Having no room to experiment led Cartwright to feel discontent with her career. She felt she had to return to mathematics research, and in January 1928 she arranged to join Hardy's group of research students at Oxford. Hardy's class consisted of an hour lecture followed by tea, biscuits, and talk about mathematics and mathematicians.

Cartwright's mathematical talent blossomed in the seminar. One evening Hardy gave a list of problems in his seminar, one of which referred to an application of Abel's method of summation to Dirichlet series. Hardy was amazed when Cartwright completely solved the problem by contour integration. Her work on generalized Abel summability with applications to Fourier series was published that year and also appears in the index of Hardy's book on divergent series.

E. C. Titchmarsh, who succeeded Hardy as Savilian Professor at Oxford, became Cartwright's supervisor while Hardy was at Princeton during the 1928-29 academic year. During an interview in 1990, Cartwright recalled that Titchmarsh was a good supervisor and his suggestions suited her well. Both Hardy and Titchmarsh inspired Cartwright's interest in the theory of functions of complex variables. Cartwright completed her D.Phil. when Hardy returned to Oxford. Fortunately, J. E. Littlewood served as an external ex-

aminer. Littlewood recalled that the first question by the other examiner was so silly and unreal as to make her blush, but he thought he helped to restore her nerve with a wink. Her thesis on the zeros of integral functions generated a series of papers and eventually led to her book on integral functions.

In October of 1930, financed by a Yarrow Research Fellowship, Cartwright continued her work in the theory of functions at Girton College, Cambridge. During this time she attended several of Littlewood's courses and seminars. She drew Littlewood's attention when she obtained the right order of magnitude for the maximum modulus of multivalent functions, a problem that Littlewood had presented in his theory of functions class. Her result is:

Cartwright's Theorem. Suppose $w(z) = \sum_{n=0}^{\infty} a_n z^n$ is regular for $|z| < 1$. Let $r < 1$. If $w(z)$ takes no value more than p times in $|z| < 1$, then $|w(z)| < A(p)\mu(1-r)^{-2p}$ for all $|z| \leq r$, where $\mu = \max(|a_0|, |a_1|, |a_2|, \dots, |a_p|)$ and $A(p)$ is a constant depending only on p .

Cartwright was the first to obtain significant results for p -valent functions, and she did so using some rather unconventional methods. She used the conformal mapping technique pioneered by Ahlfors to prove the theorem and show that an entire function of order ρ has at most 2ρ asymptotic values. She had learned about the technique in her lectures from E. F. Collingwood. W. K. Hayman described the technique in a talk given on the occasion of Cartwright's ninetieth birthday. According to Hayman, Ahlfors mapped a strip-like domain S onto a strip $R = \{|\eta| < \frac{\pi}{2}\}$ in the $\zeta = \xi + i\eta$ plane. If S meets the line $\sigma = t$ in one or more segments of length at most $\theta(t)$ and if $s = \sigma + it$ corresponds to $\zeta = \xi + i\eta$, then Ahlfors showed that the map grows at least as fast as in the case when S is itself a strip. Suppose now that $f(z)$ is p -valent in the unit disk $\Delta = \{|z| < 1\}$, and assume for simplicity that $f(z) \neq 0$. Those who have had dealings with the zeta function may recall that zeros can give rise to problems. However, in this case the damage can be limited, since $f(z)$ can have at most p zeros. Map Δ onto the strip R by $\zeta = \log(\frac{1+z}{1-z})$, and consider $s = \log f(z(\zeta))$ as a function of ζ . Unfortunately this is not a 1-1 map if $p > 1$. However, if we consider a level curve $\sigma = t$, then the variation of the argument τ on such a curve cannot be more than $2\pi p$. Thus $\theta(t) \leq 2\pi p$.

Cartwright was able to adapt Ahlfors's technique to her situation and deduce that

$$\sigma = \log |f| \leq 2p\xi + O(1) \leq 2p \log \left(\frac{1}{1-|z|} \right) + O(1),$$

which yields her theorem.

Littlewood was impressed with Cartwright's creativity in adapting Ahlfors's Distortion Theorem to such a different situation. Her polished proof was published in *Mathematische Annalen* in 1935, and her results are referred to in Littlewood's *Lectures on the Theory of Functions*. Her paper inspired a great deal of work and was later improved by D. C. Spencer. Cartwright's Theorem is still frequently quoted and applied in signal processing and is perhaps her most important work in the theory of functions.

Cartwright's mathematics flourished during the 1930s, and she published several papers on integral functions, meromorphic functions, and analytic functions with essential singularities. Cartwright's work included descriptions of the subtle phenomena that can appear near fractal boundaries and has found some new applications in this field. Collingwood provided a great deal of assistance with her work on entire integral functions and refereed nearly all of her papers published in the *Proceedings of the London Mathematical Society*. Her work on directions of Borel spreads in *Comptes Rendus* was noted by G. Valiron in his lectures at the Zürich International Congress in 1932. Both Valiron and Cartwright are known for some of the earliest results concerning meromorphic functions sharing a level curve or tract. She collaborated with L. S. Bosanquet in the early 1930s, and they coauthored two papers, the first on the Hölder and Cesàro means of an analytic function and the second on Tauberian theorems. In the former paper the authors obtain results on the limiting behavior of Cesàro means $C^{(k)}(z)$ as $z \rightarrow 0$ and show that the results still hold when the $C^{(k)}(z)$ are replaced by Hölder means. The authors also obtain results of the type of Montel's theorem for bounded means. Two typical results of their work on Tauberian theorems are the following. In both, let $f(z) = \sum_{n=0}^{\infty} a_n z^n$.

Theorem. Fix $\alpha \geq -1$ and a positive integer λ , and let $f_\lambda(z) = (1-z)^{-1} \int_z^1 f_{\lambda-1}(u) du$. If the (C, α) sums of $\sum_0^\infty a_n$ are bounded and if $f_\lambda(z) \rightarrow A$ as $z \rightarrow 1$ along some arc $\arg(1-z) = \gamma$ with $0 \leq |\gamma| < \pi/2$, then the series $\sum a_n$ is $(C, \alpha + \delta)$ summable for every $\delta > 0$.

Theorem. Fix α with $0 < \alpha < \pi/2$ and γ with $0 \leq |\gamma| < \alpha$. Suppose all a_n are real and $f(z) = O(\exp |(1-|z|)|^{-\pi/(2\alpha)})$ as $z \rightarrow 1$ in $|z| < 1$. If $f(z) \rightarrow A$ as $z \rightarrow 1$ along the arc $\arg(1-z) = \gamma$, then $f(z) \rightarrow A$ when $z \rightarrow 1$ within the set $|\arg(1-z)| \leq \gamma$.

Cartwright's work prompted Hardy, who had moved to Cambridge in 1931, and Littlewood to recommend her for an Assistant Lectureship in Mathematics. In 1934 she was appointed to a College Lectureship in Mathematics and a Staff Fellowship. Cartwright became a part-time University Lecturer in Mathematics in 1935. At Girton,

Cartwright also came under the influence of the director of studies, Frances E. Cave-Brown-Cave, who had finished just behind Hardy on the 1898 Cambridge Mathematical Tripos. In 1936 she succeeded Cave-Brown-Cave as director of studies in mathematics. She served as director until 1949.

In January of 1938 the Radio Research Board of the Department of Scientific and Industrial Research issued a memorandum requesting the expert guidance of pure mathematicians with nonlinear differential equations used in the modeling of radio and radar technology. Some of the problems presented in the literature intrigued Cartwright. At the time it seemed curious that such problems would appeal to her, since her interests lay with complex analysis and not dynamics. In hindsight, she believed that she intuitively recognized the topological implications of the problems.

Cartwright sought Littlewood's assistance in helping her understand the dynamical aspects of radio research, and in so doing they embarked on a ten-year collaboration. Their work was influenced by the mathematics of Poincaré, Birkhoff, and Bendixon, among others. Cartwright considered the area of nonlinear oscillations "a curious branch of mathematics developed by different people from different standpoints—straight mechanics, radio oscillations, pure mathematics and servo-mechanisms of automatic control theory." At the time, much of the nonlinear theory was concerned with equations of the form

$$(1) \quad \ddot{x} + f(x)\dot{x} + g(x) = p(t),$$

where $p(t)$ has period $2\pi/\lambda$, $g(x)$ behaves in a somewhat similar manner to a restoring force, and the damping $f(x)$, sometimes $f(x, \dot{x})$, is usually positive for large $|x|$. For such equations the principle of superposition does not hold. One of the most famous of such equations is that introduced by van der Pol in 1920:

$$(2) \quad \ddot{x} - k(1-x^2)\dot{x} + x = bk\lambda \cos \lambda t.$$

Even when the forcing term $bk\lambda \cos \lambda t$ is zero, the general solution to van der Pol's equation cannot be obtained by the combination of two linearly independent solutions.

The first big problem studied by Cartwright and Littlewood concerned the amplitude of the stable periodic solution of van der Pol's equation without forcing term. Van der Pol used this equation as the basic model for a triode oscillator, since it retained the essential features of the situation being experimented with by radio engineers. Van der Pol's research seemed to imply convergence to a periodic solution with amplitude 2. Cartwright and Littlewood proved that when k is large, all non-trivial solutions converge to a periodic solution whose amplitude tends to 2 as $k \rightarrow \infty$.

When the nonlinear term of equation (2) is large—that is, when k is large—van der Pol's equation represents a relaxation oscillator. The next problem considered by Cartwright and Littlewood was whether this equation could have two stable periodic solutions with different periods in this situation. Such a phenomenon had been suggested by experiments. During their investigations Cartwright and Littlewood were considerably influenced by Levinson's 1944 paper "Transformation theory of nonlinear differential equations of the second order". This paper laid the foundations of a general topological approach to nonautonomous periodic second-order differential equations. Cartwright and Littlewood expanded their research on equation (2) to the generalized form of van der Pol's equation, equation (1), and showed the existence of two stable sets of subharmonics of different orders. Their results led to some rather interesting mappings, and their topological approach to the problem led to several unsolved problems in topology. The inner and outer edges of the boundary between the domains of attraction of the two sets had different rotation numbers under the diffeomorphism corresponding to advancing the solutions of (2) through one period, leading to the conclusion that the boundary could not be a Jordan curve and was most likely to be an indecomposable continuum. Cartwright and Littlewood's fine structures are recognized by contemporary mathematics as typical manifestations of the "butterfly effect".

In their work Cartwright and Littlewood made use of an analytic transformation, developed by Birkhoff, of the plane into itself with complicated invariant curves. They were among the earliest mathematicians to associate such transformations with a simple differential equation. The diffeomorphism indicated that the nonstable periodic solutions and subharmonics of the equation corresponded to fixed points of the set. This led Cartwright and Littlewood to develop some original fixed-point theorems for continua invariant under a diffeomorphism of the plane. In particular, they needed a fixed-point theorem that would apply to a bounded invariant continuum whose complement is a single simply connected domain that may not have a Jordan curve tending to it. The theorem they developed is stated below. This theorem is more general than Brouwer's fixed-point theorem, since I need not be locally connected; however, the conditions on T are more restrictive. One application of their theorem is to an area-preserving twist homeomorphism of the annulus.

Cartwright & Littlewood Fixed-Point Theorem.

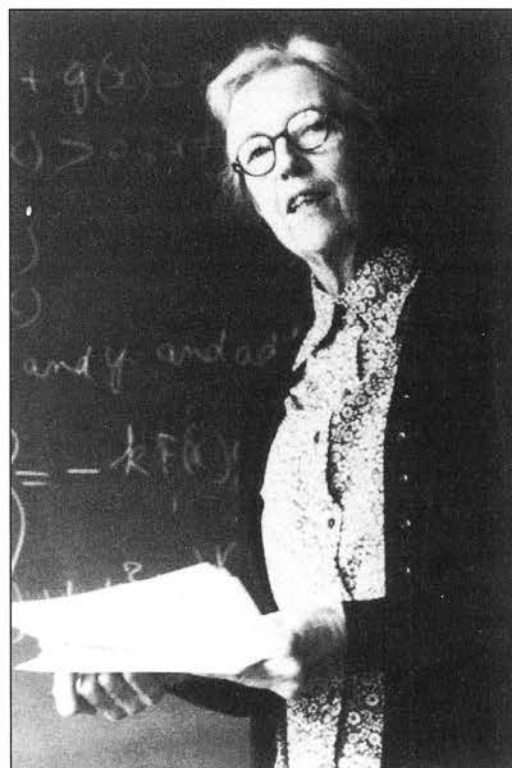
If T is a 1-1 continuous and orientation-preserving transformation of the whole plane into itself that leaves a bounded continuum I invariant and if $T(I)$ is a single simply connected domain, then I contains a fixed point.

It eventually became evident that the van der Pol oscillator provided an example of a stable chaotic system. Dissipation in the van der Pol oscillator meant that the system's shape in phase space would contract. A complex combination of transformations was necessary to fully represent the system in phase space, inspiring Smale's construction of the horseshoe diffeomorphism, the cornerstone of the modern theory of chaos.

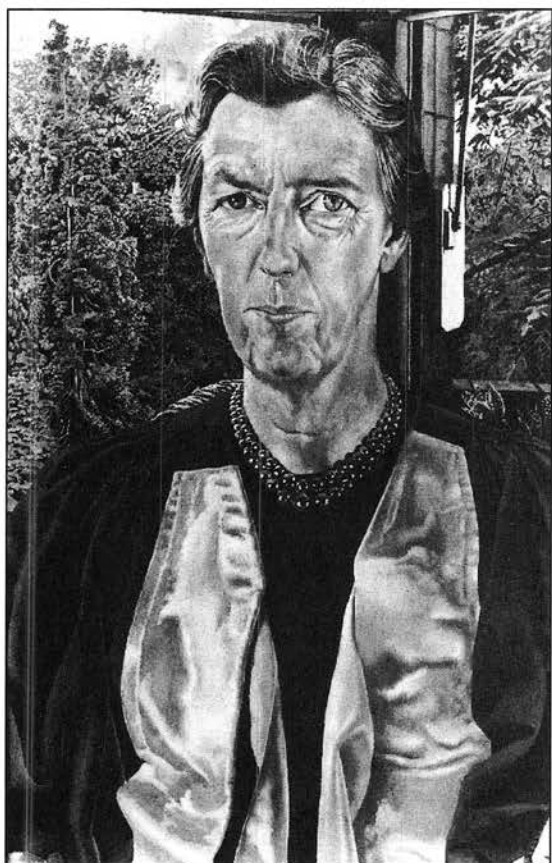
Cartwright and Littlewood's collaboration produced

some of the earliest fully rigorous work in relaxation oscillations. According to J. Guckenheimer, none of Levinson, Cartwright, and Littlewood carried through an analysis of the strange asymptotic behavior exhibited by some solutions, since adequate tools to do so were not then available. However, the results they obtained on the periodicity and stability of solutions of nonlinear differential equations influenced the development of the modern theory of dynamical systems, and there has been an enormous expansion of the subject since their work was completed. Modern techniques have enabled contemporary mathematicians to carry through a more thorough analysis of the exotic behavior of solutions to the forced van der Pol equation. For example, N. G. Lloyd proved the Cartwright-Littlewood conjecture that there is only one stable oscillation for $b > 2/3$. Also, Guckenheimer and Levi have each used modern methods from dynamical systems to give a more complete analysis of the phenomena that appear in the forced van der Pol equation.

The bulk of Cartwright and Littlewood's collaborative work was completed in the mid-1940s and contributed to her election as a Fellow of the Royal Society of London in 1947. When asked if she felt there was any prejudice toward women at Cambridge or within the Royal Society, Cartwright responded that during her time she never felt as if there was. She even felt favored and preferred to men available on many occasions, for example, when representing the London Mathematical Society, the Cambridge Philosophical Society, and



Dame Mary teaching, around 1970.



Stanley Spencer's portrait of Cartwright in 1958 conveys her as a scholar and administrator, but misses the warm sense of humor and sympathy that her friends, colleagues, and students knew.

Cambridge University at the Centenary of Henri Poincaré. She explained that although very few women had been elected to the Royal Society, there were also few women combining research and teaching. In addition, she noted that "hordes of men are put up for election and never get in."

In the years during and following the Second World War, Cartwright carried out a very full program of teaching and research at Girton. She collaborated with Littlewood, she was commandant of the college's Red Cross detach-

ment from 1940 to 1944, and she worked with Copson and Greig to publish a thorough expository article on nonlinear vibrations. Her Girton students were at first apt to find her rather frail and timid. Closer acquaintance quickly dispelled that impression. Cartwright was a no-nonsense person with a wry sense of humor. It was said that even Hardy ducked when she threw her hardball.

In July of 1948 Cartwright was preelected mistress of Girton. Before taking up office, she spent the early part of 1949 lecturing on nonlinear differential equations in the United States at the invitation of John Curtis, Solomon Lefschetz, and Mina Rees. She spent three weeks at Stanford and one week at UCLA as a visiting professor. She spent several more weeks lecturing at Princeton as a consultant under the Office of Naval Research, since Princeton did not allow for women faculty. Lecture notes from her Princeton seminar appear in a 1950 volume in the series *Annals of Mathematics Studies*.

Upon her return to Cambridge, Cartwright's life changed dramatically when she took up the responsibilities of mistress of Girton. The women's colleges had just become full members of the University during the previous year. Cartwright's quiet,

unassuming, and clear-headed leadership helped the college to adapt and take its place in the University. However, becoming mistress did put many administrative duties on her agenda. Demands from various University committees were very heavy. Her commitments included service as chairman of the Cambridge University Women's Appointments Board, service on the Education Syndicate, and two terms on the Board Council of the Cambridge University Senate.

Although Cartwright found it impossible to refuse to be elected mistress of Girton, she did refuse to take on all but a few research students and "those for some special reason." She believed that perhaps being head of a college and trying to avoid too many research students stopped her promotion to full professor. Her research students included Sheila Scott (Macintyre); Hilary Shuard, who did not take a D.Phil. but did become a prominent figure in the Teaching Training World; Barbara Maitland, who became a Lecturer at Liverpool; Marc Noble, who became a professor of mathematics at the University of Canterbury; James Ejeilo, who became a professor, and vice-chancellor for a time, at Nsukka University of Nigeria; Carl Lindon, who taught at the University College of Swansea; W. K. Hayman, who is a Fellow of the Royal Society and was awarded the Berwick Prize from the London Mathematical Society; and Chike Obi, a self-taught Nigerian, who was taken over by Littlewood and became a professor at Layos University. Cartwright was reportedly an excellent supervisor who gave due encouragement and took great care in reading and criticizing her students' work, both in content and form. Hayman recalls that Cartwright was a marvelous supervisor who was never discouraging. She corrected, but did not put down. Her advice was always available and pointed, and her conversation interesting and varied. She was noted for being meticulous about English grammar and syntax. In addition, Hayman says, "Mary taught me that proofs must be supplied by the writer and not the reader and that we had to refer to previous authors and what they had done."

After completing her collaborative work with Littlewood and becoming mistress of Girton, Cartwright claimed that her burden of administrative duties prevented her from devoting herself wholeheartedly to mathematics. However, in the years between 1950 and 1989 she still managed to publish several papers in differential equations, most using topological methods, as well as some research in theory of functions, some expository articles, memorial articles, and papers with historical and biographical interest, including historical research on the Hardy-Littlewood-Paley-Riesz circle from the 1920s and 1930s. Her book *Integral Functions* was published in 1956 and exceeded in depth and precision anything that had gone before. The book was heavily based on work

that she had almost finished when the war began in 1939. In 1960 she proved some interesting results about solutions to a third-order differential equation using Lyapunov functions. In the mid-1960s she wrote papers that addressed the importance of linking topological results more closely with the theory of differential equations. Even though she was not a topologist by training, she believed that mathematicians ought to make fuller use of the topological way of thinking in formulating problems. In the 1970s she collaborated with H. P. F. Swinnerton-Dyer, cowriting three papers on boundedness theorems for second-order differential equations. And as late as 1987, with G. E. H. Reuter she coauthored a paper on periodic solutions of van der Pol's equation with large parameters. However, regarding her research after 1950, she is probably best known for her work with Collingwood in the theory of cluster sets.

Cartwright and Collingwood had worked together before the war. They resumed their collaboration around 1950 when Collingwood pointed out that in her 1936 paper "On the behaviour of an analytic function in the neighborhood of its essential singularities" she had not taken into account the possibility that a function analytic in the unit disk might tend to a limit along a spiral touching the boundary or along an oscillating curve. Recognition of the too-strong hypothesis led them to redevelop the theory from a more general point of view, culminating in the famous Collingwood-Cartwright theory of cluster sets, in which they developed boundary uniqueness theorems for meromorphic functions on $|z| < 1$. Results in the first part of the paper belonged to what they called boundary theory in the large. In the second part of their paper they establish a system of boundary theorems in the small corresponding to their previously proved boundary theorems in the large. Here they study the behavior of $f(z)$ near a selected point $z = e^{i\theta}$ of the boundary. In their introduction they say that the central idea of their method was derived from Iversen's theory of the inverse function. It consists in the continuation of an ordinary or algebraic element of the inverse function along an appropriate path free from nonalgebraic singularities. They define the expressions $C(f)$, $R(f)$, and $\Gamma(f)$ as follows:

1. $C(f)$ denotes the set of cluster values of f . A point a is a cluster value of $f(z)$ if there is a point z_0 of the boundary $|z| = 1$ and a sequence $\{z_n\}$, $|z_n| < 1$, such that $\lim_{n \rightarrow \infty} z_n = z_0$ and $\lim_{n \rightarrow \infty} f(z_n) = a$.

2. $R(f)$ denotes the range of values of f . A point a is an element of $R(f)$ if there is a sequence $\{z_n\}$, $|z_n| < 1$, such that $\lim_{n \rightarrow \infty} |z_n| = 1$ and $f(z_n) = a$ for all values of n .

3. $\Gamma(f)$ denotes the set of asymptotic values of f . A point a is an asymptotic value of $f(z)$ if there is a continuous simple path $z = z(t)$, $\alpha < t < 1$,

such that $|z(t)| < 1$, $\lim_{t \rightarrow 1} |z(t)| = 1$, and $\lim_{t \rightarrow 1} f(z(t)) = a$.

With these definitions, the main theorem of Cartwright and Collingwood is as follows.

Theorem. If $f(z)$ is meromorphic in $|z| < 1$, then the following relations are satisfied:

- (i) If $\Gamma(f)$ is unrestricted, then the union of the frontiers of $C(f)$ and $R(f)$ is equal to the intersection of $C(f)$ and the closure of the complement of $R(f)$. In addition, this set is contained in the closure of $\Gamma(f)$.
- (ii) If $\Gamma(f)$ is of linear measure zero, then it contains the complement of $R(f)$.

In each of the above, the complement is with respect to the closed complex plane.

Cartwright and Collingwood's work on cluster sets led to some of Collingwood's best work, culminating in his presidency of the London Mathematical Society. Their other paper was published in 1961 and contains an extension of various results on asymptotic values and ranges of values to the derivatives of the function considered.

Cartwright's colleagues recall that she never seemed harried. During her entire tenure as mistress of Girton she exhibited a readiness to look ahead and to respond to new needs by adopting new practices. It meant a great deal to the College that their mistress contributed to the advance of knowledge in her own field as well as devoting time and energy to the College's concerns. Cartwright served as president of the Mathematical Association from 1951 to 1952. In 1956 she was appointed a member of the Royal Society's delegation to the Soviet Union and Poland. Cartwright attended conferences, and she visited Moscow University and the Polish Academy of Sciences at Warsaw. In 1959 she was promoted to Reader in the Theory of Functions at Cambridge. She served as president of the London Mathematical Society from 1961 to 1963. After her retirement from Girton in 1968 she held visiting professorships at universities in England, America, and Poland before returning to Cambridge, where she was one of the editors of *The Collected Papers of G. H. Hardy*.

Cartwright fondly remembered her academic travels to the United States and Europe, and in later years she enjoyed telling stories of her travels over lunch at Girton. She spent the academic year 1968-69 as a resident fellow at Brown University. During her stay, she was particularly impressed by the students at Brown who were putting a great deal of pressure on the administration to change the curriculum. She spent the next academic year as a visiting professor at the Claremont Graduate College, Professor of the Royal Society to Poland, and Visiting Professorial Fellow at the University of Wales. Both she and some of her colleagues at the Claremont Colleges clearly remembered that



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she broke her hip in a bicycle accident. The following academic year she was a visiting professor at Case Western Reserve. She was elected an honorary member of the Institute of Mathematics and received a medal from the University of Finland.

Cartwright was a pioneer in many senses. She was the first woman to achieve a first class on the Oxford Finals, one of the first women to be elected to the Royal Society, the first woman to sit on its Council, and the first woman to receive its Sylvester Medal (1964)—for her “distinguished contributions to analysis and the theory of functions of a real and complex variable.” She was, as of this writing, the only woman elected president of the London Mathematical Society (1961–63) and the only woman to receive its De Morgan Medal (1968).

Cartwright obtained immense satisfaction from doing mathematics. She believed that a number of major developments in pure mathematics were first thought out in terms of real-world situations. When asked what her favorite paper was, she gave the ultimate mathematician's reply: “...the one I was working on at the moment.”

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