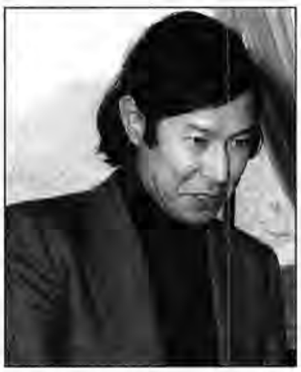
Interview with Heisuke Hironaka



Heisuke Hironaka is one of the premier algebraic geometers of the twentieth century. He is best known for his 1964 work on the resolution of singularities of algebraic varieties over a field of characteristic zero, for which he received the Fields Medal in 1970. The fundamental nature of this problem was apparent to many mathematicians in the first part of the twentieth century, notably Oscar Zariski, who solved the problem for curves and surfaces and had a profound influence on Hironaka. Taking a strikingly original approach, Hironaka created new algebraic tools and adapted existing ones suited to the problem. These tools have proved useful for attacking many other problems quite far removed from the resolution of singularities. Another major influence was Alexandre Grothendieck, who in 1959 invited the young Hironaka for the first of his many visits to the Institut des Hautes Études Scientifiques (IHÉS) in Paris. In a 2002 news release about the establishment of the Heisuke Hironaka Fund to promote ties between the IHÉS and Japanese mathematicians, IHÉS professor Mikhael Gromov is quoted as saying that Hironaka's resolution of singularities "is unique in the history of mathematics. It is one of the most difficult in the world that has not, to this day, been surpassed or simplified."

Heisuke Hironaka was born on April 9, 1931, in Yamaguchi-ken, Japan. As a student at Kyoto University, he was a member of the school around Yasuo Akizuki, who was a pioneer of modern algebra in Japan. Hironaka received his Ph.D. in 1960 from Harvard University, under the direction of Zariski. After positions at Brandeis University and Columbia University, he became a professor at Harvard in 1968, and from 1975 to 1988 he jointly held a professorship at Kyoto University. He served as director of the Research Institute of Mathematical Sciences at Kyoto University from 1983 to 1985. He received the Japan Academy Award in 1970 and the Order of Culture of the Japanese government in 1975. The fame and esteem Hironaka is accorded in Japan would strike Westerners as quite remarkable. An average Japanese person without much background in mathematics or science is likely to know his name.

Hironaka has contributed much time and effort to encouraging young people interested in mathematics. In 1980, he started a summer seminar for Japanese high school students and later added one for Japanese and American college students; the seminars ran for more than two decades under his direction and continue to this day. To support the seminars he established a philanthropic foundation in 1984 called the Japan Association for Mathematical Sciences. The association also provides fellowships for Japanese students to pursue doctoral studies abroad. From 1996 to 2002, Hironaka served as the president of Yamaguchi University, which is in the prefecture where he was born. Nowadays he continues his educational activities, particularly at the local level, and does mathematics research.

What follows is the edited text of an interview with Heisuke Hironaka, conducted in December 2004 by *Notices* senior writer and deputy editor Allyn Jackson.

—A. J.

Family and Childhood

Notices: *You were born in 1931 in Yamaguchi prefecture. I'm interested to hear about your life growing up there and what your family was like.*

Hironaka: The town I was born in is a very small town. One of my brothers is still living there. Yamaguchi prefecture is not far from Hiroshima. The town was on the east side of Yamaguchi prefecture and facing the inland sea. The population was something like 3,000. About half the people fished, and the other half farmed. But my father was a merchant, selling clothes.

Notices: *How many children in your family?*

Hironaka: I had many sisters and brothers. Altogether fifteen. But fifteen is actually four plus one plus ten. My father's first wife died, and his second wife died too after having four children. My mother had one child before marrying my father, and her husband died. So when my father and mother married, they had five children. My parents then had ten children, and I am the oldest boy among them. I have one elder sister. During World War II the government promoted having many children, particularly boys. But after the war, raising all the children was quite difficult. One brother, also a son of my father, died in New Guinea fighting against the Americans. He was twenty-three. Another brother was in the war with China. He was injured and died in a hospital in Beijing.

Incidentally, the mother of the four children, who my father married before he married my mother, was my mother's elder sister.

Notices: *I see—your father married two sisters.*

Hironaka: Yes. I think that my mother was trying to help her sister, who was sick. I like that I was in a big family. When you are small, it's good to have many people around of the same age or somebody older whom you can depend on or learn something from. And with the younger children, you feel that you have some responsibility to take care of them. My father was not poor, but the war made many things difficult. Before the war he had a textile factory. He sent his first son to an engineering school in Nagoya to learn about textiles. He sent his second son to study commerce. So my father had big hopes for these two children. When he lost those two sons, he became completely disappointed. He sold the factory, and he stopped working. After the war, of course, there were big changes. The economy was bad, and they changed the currency so that the old currency became useless. There was also a big cultural reform. My father had some peasants working for him, and he had to give all the land to them. In any place, when the war ends, things go crazy!

Notices: *Was your area affected by the dropping of the atomic bomb?*

Hironaka: Actually, my father saw it, but it was some distance away, so he wasn't affected. My cousin was in school in Hiroshima and got radiation burns, but he recovered and survived. Somehow, people in our area were lucky. But people were afraid that we would be bombed. Whenever we heard a plane coming, people started spreading out. At that time, they didn't say it was an atomic bomb. They said it's a "special bomb." At that time, the American air force had complete control of the air—not the ground yet, but the air was completely controlled. We would see planes coming, and a rumor would start: "Maybe that's a 'special bomb'."

Heisuke Hironaka (second from right in front) with parents and siblings, Japan 1938.

In that kind of situation, grownups are desperate about the future. But children are not. Children don't know! We were quite happy, actually. The Americans put a lot of mines in the seabed, and after the war the Japanese brought them up and detonated them. It was like huge, beautiful fireworks. We enjoyed it!

Harvard graduate dorm, 1957.

Notices: *Were you interested in mathematics from a very young age?*

Hironaka: When we are children, we think of doing many, many things. In elementary school I wanted to be a *naniwabushi*, which is a kind of storyteller. He might tell a *yakuza* story, or samurai story, or a sad story, and the story is partly sung. I liked that, and I wanted to be that! But then in middle school I think I started liking mathematics. Actually, I was confident with mathematics from the first grade.

Notices: *Did you have a mathematics teacher at some point when you were young whom you found inspiring?*

Hironaka: It's kind of silly, but if some teacher praises you, for instance, for doing well on an exam, you become very proud of it. I had experiences like that.

Notices: *Did you study music also?*

Hironaka: When I was in junior high I wanted to be a pianist. But at that time only the school had a piano. Our school was in Yanai, which was then about thirty minutes' ride by train. I would take an early train to play the piano before school started. I played just as was written in the book. From time to time the music teacher taught me something and I would play, but not in any serious way. At some point they asked me to play in a joint concert with other schools. They asked me to play Chopin's

Impromptu—that's pretty difficult. And I made a mess! The girls played much better. But in the boys' school, nobody but me was playing the piano, so they asked me to play. The teacher of the girls' school said terrible things about my performance. That was quite a shock. At some point I asked one of my teachers about this. The teacher said I shouldn't be a musician because musicians, particularly pianists, start playing at age three and usually are taught by a special teacher. In my case, I started only in junior high, with no real teacher. So this teacher said I should forget about it. So I said, "Okay, I will forget about it."

Still, I liked piano music and also classics, and I was interested in musicians. When I was in Paris, from 1959 to 1960, I met Seiji Ozawa, and I became a very close friend of his. Seiji Ozawa now conducts the Vienna Opera. He comes to Japan every summer for about two months and participates in the Saito-Kinen Festival in Matsumoto. Saito was the teacher of Ozawa. For two or three weeks Ozawa conducts the orchestra and also teaches youngsters. There is a foundation that supports this festival, and Ozawa asked me to be the president of the foundation.

When I met him in Paris, we knew each other for only six months or so. We became good friends, and then I forgot about him, because I had to come back to Harvard to get my Ph.D. After I got my Ph.D., I started working in that part of Massachusetts. He suddenly wrote me a letter saying he is coming to the United States for the first time. So I went to the airport and picked him up, and he participated in a competition in Tanglewood. Charles Munch was the head of the jury. Ozawa won the first prize in conducting. I went to Tanglewood and listened to him conduct. Then he went back to Paris. Later, in 1964, I became a professor at Columbia University, and then he was in New York, working with Leonard Bernstein. So I saw Ozawa there and listened to his concerts. Then in 1968 I went back to Harvard as a professor, and again I forgot about him. Then he came to the Boston Symphony as conductor. So the relationship has continued. Maybe he likes me because I don't know much about music!

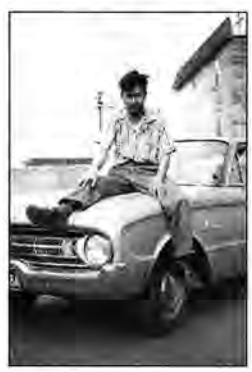
Notices: *Going back to things more mathematical, when did you get seriously interested in the subject?*

Hironaka: I think one of the times when I seriously started thinking about the possibility of becoming a mathematician was in senior high school, when a mathematics professor from Hiroshima University came to my school. He gave a general lecture to the students. It was a bit technical, so I couldn't understand everything. But he said at the beginning of his talk something like, "Mathematics is a mirror in which you can project everything in the world." I was very puzzled by that, but also very impressed. I applied to Hiroshima University because I wanted to study with him. But I didn't study at all for the entrance exam, and I failed it!

So I began to study the year after, and I applied to Kyoto University and went there. At that time I wanted to be a physicist, for a naive reason—nothing serious or philosophical. It was because the physicist Yukawa was the first Nobel laureate from Japan, and he was at Kyoto University. I was lucky that my sister had married somebody in Kyoto so I could stay there. I studied physics fairly seriously for the first year, also chemistry and some biology. But by the second year it became quite clear that I am suited to mathematics rather than science. As I studied, when it came to a mathematical question, I was always sort of excited. So it was quite clear in maybe the third year that mathematics would be my future profession. After that—just mathematics.

Notices: *How were the mathematics professors at Kyoto University at that time?*

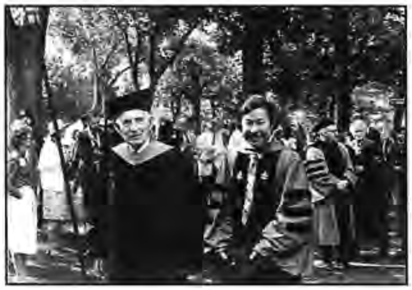
Hironaka: We were young, so we didn't know much, but, still, we had the sense that Japan was far behind in mathematics and science, compared to the United States and Europe. So we wanted to go to Europe or America if possible. But there was one strong point in Japan at that time, which was lucky for me, and in some sense it determined my style of mathematics. That was abstract algebra. Japanese mathematics tried to be on top of whatever was the trend in mathematics, and abstract algebra at least looked like the top end of mathematics at the time. Other mathematics that engineers or physicists use—that was "earthly mathematics". My teachers and their colleagues were much more interested in abstract algebra; number theory too, but of an abstract kind. So when I had a chance to go to the United States, at least in one thing I was confident, that was abstract algebra. So it helped me avoid an inferiority complex!



On hood of Ford Falcon, 1961. First car bought in U.S.

Notices: *How did you end up going to Harvard and studying with Zariski?*

Hironaka: I entered Kyoto University in 1949 and was in the college for four years. Then I entered the graduate school at Kyoto University. At that time



Oscar Zariski (left) receiving honorary degree at Harvard, 1981, with Hironaka as escort.

Akizuki was a professor there. He did not do much work in algebraic geometry, but he was very interested in it and in establishing algebraic geometry in Japan, particularly in Kyoto. He was the first to invite the young, good people in algebraic geometry. At that time Japanese universities were hiring their own students. But he didn't hire any of his own students. He took the top four or five people—one from Nagoya, one from Osaka City University, one from Tokyo University, and so on—and formed a group. Also, he became quite popular because his lectures were very up to date. Other lectures were sort of boring—classical complex analysis and things like that, things written in books. But Akizuki was quite different. He tried to introduce new things and invited many people. I joined that seminar group, and I was the youngest member.

At some point Akizuki invited Zariski to Kyoto. Zariski was at the time one of the top people making algebraic geometry very algebraic—he was doing *algebraic* algebraic geometry. His philosophy was that when you base geometry on algebra, you can avoid being misled by geometric intuition. He said that when he writes algebraic geometry based on algebra, the rigor is automatic; it's unquestionably there. Algebra can become sort of "abstract nonsense"—playing with symbols without knowing what it's for. But Zariski and also Akizuki had the idea that we should do geometry with algebra. When Zariski visited, I tried to tell him what I was doing. I have never been good in English! But my colleagues and teachers helped me to explain to him what I was doing. At some point Zariski said, "Maybe you can come to Harvard and study." And I said, "Okay."

Notices: *When did you go to Harvard?*

Hironaka: 1957, the summer after Zariski went back. I tell youngsters, if you go abroad or even if you study in Japan, choose the best scholar in the field. But don't expect you can learn from him! The amazing thing is that with that kind of person, there are many talented young people around, and you learn a lot from them. Anyway, that was my case. I learned a great deal from Zariski's papers, and sometimes he made suggestions, but he was so busy. Among the other students were Michael Artin, Steve Kleiman, and David Mumford. Sometimes we four decided to have our own seminar. We were students, so we had a lot of time to talk about mathematics. No formal duties—it's good to be a student!

Resolution of Singularities

Notices: *When did you first get interested in working on the resolution of singularities?*

Hironaka: That was I think my third year at Kyoto University. There were around ten people in the Akizuki School, as we called it, always talking together and doing seminars. I was almost always the listener, because I was the youngest in the group. One time one of the people in the group talked about the question of resolution of singularities. That was the first time I heard the name of Oscar Zariski. I thought that problem was very interesting, so I decided to do it. I had no techniques, nothing, but I thought the problem was interesting. So the problem stayed in my mind, although I didn't work on it directly. I was still reading introductions to algebraic geometry and papers on the subject by Jean-Pierre Serre or André Weil or Zariski. But I think that was the first time that I got interested in that problem. It helped me decide to get into algebraic geometry.

Notices: *Why did that problem seem significant to you?*

Hironaka with wife Wakako and daughter Eriko, 1977.

Hironaka: I don't know. It's like a boy falling in love with a girl. It's hard to say why. Afterward you can make all sorts of reasons. For instance, I studied quite a bit of abstract algebra, so anything that could be expressed in terms of algebra was interesting. But algebra itself is too abstract—it doesn't catch your heart. This was a geometry problem, but not geometry per se. It was quite clear to me that you could not solve that kind of problem by geometric intuition. Oscar Zariski had already solved it for curves in one dimension, two dimensions, and even partly in three dimensions. So it was a question of higher dimensions. In the higher dimensions you cannot see everything, so you must have something, some tool, to guess or formulate things. And the tool was algebra, unquestionably algebra. That's one reason the problem hit me. Also, I like basic things. Very clever people tend to jump to the new techniques: something is developing very fast, and you want to be on top of it; and if you are smart, you can be a top runner. But I am not so smart, so it is better that I start something where there are no techniques for the problem, and then I can just build step by step. But actually, it was not so hard. It turned out to be easier than I thought.

Notices: *Is that so? I have read statements saying that your proof of the resolution of singularities is reputed to be one of the most difficult proofs in mathematics.*

Trieste Center for International Physics, summer 1991. Hironaka in center of second row from front.

Fields medalists, Nice, 1970 (left to right) Alan Baker, Heisuke Hironaka, John G. Thompson.

Michael Atiyah and Hironaka, Berkeley, 1986.

Hironaka: It was not so hard, because I learned quite a bit from many people. For instance, Kyoto University is close to Osaka University, and the president of Osaka University was a mathematician named Shoda. In Japan he was the father of modern algebra. He is related to the imperial family; his niece is the wife of the emperor now. Also, in Nagoya there was Nakayama, also in abstract algebra, and one of his best students, Nagata, came to Kyoto at the invitation of Akizuki. So I was lucky to be in Kyoto and to have contact with these people. Then I met Zariski. Zariski was really a geometer. When he was in Italy, he found the Italian geometers were very intuitive, so quite often they made statements where the proof was wrong but the statement was right. Zariski wanted to have a solid foundation for such results, and he chose algebra to be the foundation. I think that was just the general trend in algebraic geometry. André Weil, in connection with number theory, also wanted to make algebra the basis of algebraic geometry. When I was at Harvard, I learned those algebraic techniques strongly connected to geometry, not just abstract algebra. I was very used to everything being abstract, and I wasn't afraid of making things abstract. Also, Zariski's students, like Mumford and Artin, had good intuition in geometry. I remember that I felt they were much more geometers than I am. They are really geniuses. But luckily I didn't feel so inferior, because I was good with algebra! Also, I had good luck in that I went to Paris. Grothendieck came to Harvard from 1958 to 1959, when I was a student there. I became friends with him, and he said I should come to France.

Let me explain a little bit about geometry. Geometry has global problems and local problems. Local problems are usually done by very concrete calculations. For instance, if you have an equation, then you can write down the equation, take its Taylor expansion, look at the terms, play with them. But then when you go back to the global problem, the local solutions do not fit each other. That is one of the problems that Zariski had. He had extremely local techniques: you have some geometric object, you modify it, and you localize it. If you localize it, then you can do many tricks, but then later you cannot connect it to have a global solution. With the resolution of singularities, Zariski had a hard time even in dimension 3, and finally he gave up. Generally speaking, it's easy when you have one equation. But if you have many equations, then it's difficult, or people had the impression that it's difficult. But I observed that one can use induction to handle many equations. So I started from dimension 1, but with many equations. Then I noticed that the next dimension might have many equations, but it's the same style. It's a very simple observation, but that helped my local theory. Still, the global problem was there. You can't have global coordinates; only locally do you have coordinates and equations. So I had a problem there, but Grothendieck—Grothendieck is an amazing fellow! He doesn't look at the equations. He just looks at everything globally from the beginning. So his technique was very useful for me.

When I came back from Paris in 1960 I had written a paper that did not yet solve the resolution of singularities problem but is related to it. I was just getting ready to solve it. You know, once you finish your Ph.D. and you get a job, you are much more relaxed. Particularly when you are a foreigner, not having a job is a terrible thing. So I got a job at Brandeis University, and I felt, "Well, now what should I do?" And then I said, "My God, if I put Kyoto, Cambridge, and Paris together, the whole problem is solved!" I was very lucky.

***Notices:** One person told me that you went into a trancelike state and emerged with an incredibly complicated proof. He compared it to Andrew Wiles's proof of Fermat, where he sat in his attic and worked for seven years. Was it like that for you?*

Hironaka: Well, any problem that has many aspects and many facets that you must put together requires really immersing yourself into thinking about the whole problem. I remember that when I first called Zariski to say that I had solved the problem, he said, "You must have strong teeth."

Notices: *What does that mean?*

Hironaka: I think he meant that I needed teeth for biting—the problem was tough, so you must really bite into it. But he was very kind. He was always encouraging me. He somehow had confidence in me. So I started writing and rewriting, writing and rewriting, and finally I finished.

But I don't know if I can compare it with Andrew Wiles's work on Fermat's Last Theorem. His theory is much harder. Mine is easier. The resolution of singularities was done by hand. It doesn't use large theory or techniques. It's done just by hand. Andrew Wiles put many things together for his proof. I was just making new definitions and working case by case.

Notices: *So you were mostly making up your own theory as you went along, rather than using existing theories.*

Hironaka: Yes, that's my style, actually.

Notices: *You must have had to explain the problem of resolution of singularities to nonmathematicians. How do you explain it?*

Hironaka: Singularities are all over the place. Without singularities, you cannot talk about shapes. When you write a signature, if there is no crossing, no sharp point, it's just a squiggle. It doesn't make a signature. Many phenomena are interesting, or sometimes disastrous, because they have singularities. A singularity might be a crossing or something suddenly changing direction. There are many things like that in the world, and that's why the world is interesting. Otherwise it would be completely flat. If everything were smooth, then there would be no novels or movies. The world is interesting because of the singularities. Sometimes people say resolving the singularities is a useless thing to do—it makes the world uninteresting! But technically it is quite useful, because when you have singularities, computation of change becomes very complicated. If I can make some model that has no singularities but that can be used as a computation for the singularity itself, then that's very useful. It's like a magnifying glass. For smooth things, you can look from a distance and recognize the shape. But when there is a singularity, you must come closer and closer. If you have a magnifying glass, you can see better. Resolution of singularities is like a magnifying glass. Actually, it's better than a magnifying glass.

A very simple example is a roller coaster. A roller coaster does not have singularities—if it did, you would have a problem! But if you look at the shadow that the roller coaster makes on the ground, you might see cusps and crossings. If you can explain a singularity as being the projection of a smooth object, then computations become easier. Namely, when you have a problem with singularities in evaluation or differentiation or whatever, you can pull back to the smooth thing, and there the calculation is much easier. So you pull back to the smooth object, you do the computation or analysis, and then pull back to the original object to see what it means in the original geometry.

Notices: *It's a beautiful idea.*

Hironaka: Yes, I think that's a good idea.

Notices: *But to prove it in general must be difficult.*

Hironaka: Well, I wasn't scared by having many simultaneous equations. It was my own contribution that I was most proud of. Common sense was that a single equation is much easier to handle than a system of many simultaneous equations. It is certainly true in many instances when you are working with problems with a fixed number of variables. But when you want to build a proof for all dimensions by induction on the number of variables, it becomes easier to formulate the problem in terms of many simultaneous equations from the beginning. It may sound paradoxical, but it isn't. Think of trying to prove a problem in $(n+1)$ variables by making use of results in n variables and less. A "single" polynomial in $(n+1)$ variables is written as a linear combination of powers of the last variable in that the coefficients are "many" polynomials in the first n variables. So you need results in many simultaneous equations in n variables. If the problem is formulated in many simultaneous equations to begin with, the inductive proof goes on smoothly in any number of variables. It's a very simple observation, but when it came up, I felt the whole proof was there around the corner. That was the only problem at the local level. The global techniques are just from Grothendieck.

Hironaka with wife, after receiving Order of Culture, 1975.

Notices: *Can you explain how your work on the resolution of singularities connects to more recent work, like Shigefumi Mori's work on the minimal model problem?*

Hironaka: By the way, Mori is a genius. I am not. So that is a big difference! Mori was a student when I was a visiting professor at Kyoto University. I gave lectures in Kyoto, and Mori wrote notes, which

Back row, left to right: Hironaka, David Mumford, Steven Klieman, Michael Artin. Front row: Oscar and Mrs. Zariski.

were published in a book. He was really amazing. My lectures were terrible, but when I looked at his notes, it was all there! Mori is a discoverer. He finds new things that people never imagined.

In dimensions higher than 3, if you want to classify what kind of manifold or geometry is there, it helps to make a model that is smooth, because a smooth model has many techniques that apply to it. Smooth means that local problems disappear and there are only global problems.

Here is how you can make a singularity. You take some manifold, grab some part of it, crush it to a point, and that's a singularity. So the singularity itself has a geometry. Stephen Hawking has said that in a black hole there is another universe. A singularity is like that: if you really look inside it, then you see a big universe. So the problem of dealing with singularities is that the singularity is just one point, but it has many, many things in it. Now, to see what is in it, you must blow it up, magnify it, and make it smooth, and then you can see the whole picture. That's resolution of singularities. What Mori does is he creates a singularity by collapsing something.

Notices: *So he starts with a smooth model, then creates a singularity.*

Hironaka: Yes, a singularity of a very good nature. That was work he did in his early thirties. It was absolutely new. To classify geometry, it is helpful to have a minimal model for each class. Then the other objects in the class are made from the minimal model. In dimension 2, Zariski had a minimal model theory. Of course, the Italians had a minimal model theory, but Zariski made it rigorous. There the minimal model is smooth—no singularities. That was the best minimal model. But in dimension 3, if you insist on smoothness, then you may not get minimal models.

Mori says, "Okay, if we allow a small, good-natured singularity, then we get minimal models." This was completely unexpected. We didn't expect that there were such small minimal models. I knew that a smooth minimal model doesn't exist in dimension 3 and higher, so when he was doing this work, I thought he was doing something wrong: it's not there, so why is he looking for it? But he said, "No, if you admit certain singularities that we understand completely, then there *is* a minimal model, and everything else can be made out of it."

From the U.S. Back to Japan

Notices: *While you were at Harvard, you started spending part of your time there and part of your time in Kyoto.*

Hironaka: Yes. Being away twenty years or so from the country and coming back, at least in the first few years, I had some difficulties. For instance, at one time I was chairman of the mathematics department at Harvard. When we discussed, for example, new appointments, everybody had a different opinion. One person would strongly recommend somebody, somebody else would recommend some other person, and we would discuss it. But in Japan it's different. If you make a recommendation, then nobody says anything.

Notices: *Nobody contradicts it?*

Hironaka: No. In Japan it goes like this. Suppose Professor A recommends a young man, Dr. B. It is a fact that he recommended Dr. B: he presented the recommendation in a document. Why should he now insist on it? He thinks, "Let other people talk about it." Then Professor A says something good about Professor C's recommendation. If I take that seriously, then I have the wrong idea, and Professor A gets mad after the meeting. So I must listen very carefully and realize that he actually wanted support for *his* recommendation.

A Japanese person will insist on what he recommended or what he wants. But he doesn't express it. Because if he expresses it and if it doesn't come out in his way, then he has some kind of dishonor or disgrace. So you must be careful not to disgrace him and to guess what he really wants indirectly.

Notices: *So you had to figure out again how to do this when you first moved back to Japan.*

Hironaka: I couldn't stand it; I was so used to the American way! In the first two years or so, I made enemies, which was completely unexpected. But now I am used to it. Mind you that I am speaking

Hironaka, left, with Akizuki, Nakai, Nishi on Lake Biwa for a conference in 1963.

of some decades ago and of my generation and older. The younger generation in Japan is much more Americanized or globalized, even more than myself, though I lived abroad for much longer than most of them.

Notices: *Is there also a Japanese style of doing mathematics?*

Hironaka: That's hard to say. Mathematics is of course a science, but it also depends on personality. But certainly you see a difference in how people behave at conferences. If it's only Japanese mathematicians, instead of making propaganda about their own ideas, usually they praise the ideas of others—and quite often without meaning it! You must get used to that kind of thing.

There is a cultural feature of Japanese people that affects not the product of doing mathematics but the way of doing mathematics. In some sense, it is similar to the Russian way. For example, Kyoshi Oka graduated from Kyoto University, and he didn't publish for about ten years afterward, so he couldn't get a job in a good university. Finally he got a job at Nara Women's College. He was a bit crazy, but he was very original. I can see the same style and very high creativity in Mikio Sato and also to some extent in Kunihiko Kodaira. Kodaira went to the United States, so he became much more Western-style, but nonetheless his nature is like that. It is something to do with Japanese culture. This is a simplistic way to describe it, but usually in the Western world you try to express yourself, to show off in some way, to appear to be more than you are, and by doing so, you get more motivation and drive. And thanks to that, you reach a higher level of productivity and originality. That's one way. But the Japanese way, at least the traditional way, is not like that. You don't show off. You wait until somebody starts recognizing you. Even then, staying modest is considered a good, respectable feature. So not writing any papers for ten years—that's nothing. The mathematician must believe in what he is doing, without showing off.

Notices: *You established a foundation called the Japan Association of Mathematical Sciences, partly to support summer seminars for young people, which have been going for over twenty years now. How do these seminars work?*

Hironaka: There are two seminars: one is a U.S.-Japan seminar for college students, and the other is just for Japanese high school students. Coming from my experience as a mathematician, I think that what's interesting is talking and ideas, not well organized lectures where you sit and listen and take notes. When young people want to have a creative life, they should learn to enjoy talking about ideas, even if the ideas are not well formulated or keep changing. In fact, one of the most interesting and enjoyable parts of a creative activity is that ideas change. This is how the seminars are run.

I started the foundation to support the seminars. After running them for twenty years, I said, "I'm getting old, and I quit." Then the alumni started organizing the seminars, and they have continued. Nowadays I am teaching young people in two places in a very leisurely way. I give one lecture a week at a school of music, painting, and art. I also teach in a first-grade class; that's only two days a month, but it's very interesting.

Once I said that kids are like chimpanzees, and somebody got angry at my saying that! But a chimpanzee is really amazing. Kyoto University has a laboratory to study chimpanzees, and I watched them there. They are very intuitive. They judge everything instantly. For instance, the chimp might be trained to push a button to name a person who walks in, and if he gets the name right, he gets a reward, say a banana. The chimp pushes the button immediately and never thinks, even if he sees only half a face. But if you somehow test the chimpanzee to see why he decided this or that, the chimpanzee gets really irritated! Kids are like that. For instance, now I am teaching the first-graders Euler's formula, the relation between the number of faces and edges and vertices of a polygon. They are amazingly intuitive and can guess the answer.

When a person works, he must have knowledge or he will make terrible mistakes. But at the same time, knowledge alone doesn't do anything new. You must have instinct and somehow be conscious of making use of instinct. It is an interesting question how to give kids knowledge without having them lose their instinctive power. If you just keep pounding them with knowledge, most lose their instinct and try to depend on knowledge. This balance between knowledge and instinct is interesting.

Hironaka with family, 1972, daughter Eriko, son Jo, wife Wakako.

I have also been doing a little mathematics, which occupies me quite a bit in a very pleasant way. When you are young, you want to make an achievement or be recognized by your community. But I don't have that kind of drive now. The drive is much more internal. I want to enjoy the creative thinking. When you come to a certain age, you start finding out how to be friendly with the flow of time. When you are young, you sometimes don't know what to do with the time. I do mathematics fairly seriously, as seriously as when I was in my twenties or thirties, but in a much more enjoyable way. I don't worry that if I don't publish something in two months, somebody else might publish it. I don't do that kind of problem anymore. I don't have to. I want to do what nobody would think about! And I can just enjoy doing it.

Why Do Mathematics?

Notices: *A naive question: Why do you do mathematics?*

Hironaka: When I was young, I had some idea about why I should do mathematics. But the idea changed, and at some point I didn't think about it anymore. It was just my profession. But then later, when I was about fifty, I came back to Japan, and I gave a lecture at an elementary school in Kyoto. The teachers arranged a question-and-answer session with the kids. I told them they can ask me anything. One kid asked, "Why are you doing mathematics?" I don't remember what I said; I just made up some answers: it's an interesting subject, it's challenging, or something like that. But after I left the school, I started really wondering why I *was* doing mathematics! I thought I knew why, but then if somebody asks that question, I start really thinking about it.

Heisuke Hironaka, 1961.

I accumulate anything to do with numbers. For instance, I have more than 10,000 photos of flowers and leaves. I like to just count the numbers and compare them. I am so pleased to be a mathematician, because I can see the mathematical interest in things.

Notices: *Does the desire to do mathematics come from a curiosity about things?*

Hironaka: Yes. First of all, numbers are interesting. I think that number theory is actually the most important subject of mathematics. It's also very difficult. If you really think about the relation between addition and multiplication, it's amazingly strange. For instance, 5 is a prime number, but if you add 1, it immediately becomes 6, which is 2 times 3—two different numbers come out. It seems stupid to make a big fuss about it, but if you really think about why multiplication comes up in this strange way, it is very closely related to many questions in number theory and particularly the Riemann Hypothesis, the distribution of prime numbers. That's very difficult. That problem has been there for many years, and nobody has really made any good progress toward a solution.

Notices: *Have you tried to work on it?*

Hironaka: No, I am not good enough to do that! Nonetheless, it gives me a way to enjoy life. I think I can enjoy much more than nonmathematicians just by looking at nature from the point of view of numbers and additions and multiplications.

Notices: *That's an aesthetic sense.*

Hironaka: Yes, so it's a matter of naive human interest and a way to make life interesting and enjoyable.

Notices: *Do you think mathematics is something that has an independent existence that mathematicians discover or that it is invented by humans?*

Hironaka: I am not a historian, but, roughly speaking, after World War II, up to the 1960s and 1970s, mathematics was really by itself. It had a very strong motive to develop by itself through internal motivations and internal interests. For instance, Grothendieck is one person who lived by this principle. In the 1950s and 1960s we mathematicians looked down on people who talked about applications to the real world. If a mathematician started talking about applications, we would say, "Oh, he stopped being a mathematician; he has become an engineer," even if he was doing important things. The first part of the twentieth century was a unique time, a phenomenal episode in the history of mathematics, with the field flourishing—at least we thought it was flourishing!—and being pure and independent of the world. This led to big progress, and mathematics changed quite a bit. Even then I remember some people saying that mathematicians are doing "abstract nonsense" or "pure nonsense". But mathematicians didn't think that way; they were doing pure mathematics. If somebody had asked, "How has your work helped the world or produced something you can use?" then I am sure around that time pure mathematicians would have said, "That's a very stupid question! A very lowly question!"

Poincaré was a good mathematician from the point of view of pure mathematics. But at the same time, he emphasized the point that mathematics is nurtured by trying to understand physical phenomena. The very fast development of fundamental physics changed the world view quite a bit and gave a big push to mathematics. Look at Einstein's work or quantum mechanics—mathematics was very useful there. But physics changed too. Nowadays, physics has changed from pure physics to much more applied. If you look at the Nobel Prize winners in physics, for instance, in the second half of the twentieth century, many are much more applied—they work on electricity, electronics, chemical applications, superconductivity, and that kind of thing. It's much more the real world. I remember in the earlier part of the twentieth century, at least among my friends and teachers, there were many physicists interested in theoretical and fundamental physics. But that changed quite a bit. I think that had an effect on mathematics too.

Notices: *If you look at the Fields Medalists, they are all as pure as ever.*

Hironaka: That's true! But I think the 1960s and 1970s were the peak of mathematics having an unquestionable raison d'être.

Notices: *Do you think there is a central core to mathematics? Are there certain very fundamental ideas that form the core and other things in outlying areas that are less fundamental?*

Hironaka: Well, I think that any theories deeply connected with numbers are the ideal of mathematicians. It's an idealistic part of mathematical activity.

Notices: *Number theory is the queen of mathematics.*

Hironaka: Queen may not be such a good term! Sometimes we have not such kind queens! But anyway, it is something that people think of as the ultimate objective of mathematics. Geometry is very playful—you can change shapes or expand or deform or add handles, things like that. You can play with it. Whether it is meaningful or important, independent of those criteria, it's fun. Then there are people working on dynamics and analysis. My crazy idea is that analysts are like samurai. They make a cut—shhh! And if it's a good cut, then they can throw a part away. Then they can find a very good formula!

Notices: *They cut away the unnecessary parts.*

Hironaka: Yes. At the very base of all these different kinds of mathematics, there is an eternal question or eternal mission of mathematics, and that is infinity. Consciously or unconsciously, what mathematicians do is a finitization of infinity. It is impossible to put infinite things into a computer. It doesn't matter how good a computer may be, how fast it becomes; it cannot compute infinite things. But there the mathematician has a job to do: to formulate a model. The model may not match exactly the original phenomenon, but it helps you to understand it. And the model is finite: you can put it in a computer, and the computer computes the exact answer, at least for the model. So mathematicians give infinity a finite shape or a finitely computable and understandable form.

This is quite an interesting feature of human nature. To my way of thinking, humans are different from other animals in that humans have a notion of infinity. They never see infinity, they never live infinitely, and even the universe may not last infinitely long. But humans cannot live without the idea of infinity.

Notices: *You think not?*

Hironaka: No. This is the reason that people create religions. Religions say that the world is much longer and the universe is much bigger than you can reach within a lifetime. So then you feel better. Infinity is like a belief. If you have a belief in infinity or eternity, you feel happier.

Notices: *There is something satisfying about it. It sort of completes the world.*

Hironaka: Right. I don't think other animals do that. I think many human cultural and intellectual activities that no other animals can do have something to do with the feature of infinity. That's one thing. But at the same time, when you come to understand something and actually can compute and put it to work to make an actual product, everything is finite. If it were infinite, you could not do anything. People cannot do or make or plan infinity. A human has two hands: one hand is in infinity, the other hand is in the finite real world. I think that the real task of the mathematician is to somehow connect these two.