Mathematics and Mathematicians in World War II

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What is mathematics? I take the entirely pragmatic view that if a person's associates thought the problem he or she was solving was a mathematical problem, then it was. Many of you will disagree with this. Indeed, many of the mathematicians involved in such enterprises during the War privately did not accept this definition. The attitude of many with the problems they were asked to solve was that the given problem was not *really* mathematics but, since an answer was needed urgently and quickly, they got on with it.

And there was another aspect. Problems that purported to require mathematical treatment were often not clearly formulated. A discussion between the person with the problem and a mathematician could result in a major reformulation. This usually resulted in a simplification. I shall count this also as mathematics.

Somewhat between these two types is a case which I shall cite. An aerial survey was made of the environs of Ft. Monroe, Virginia, from which a scaled image of the ground was to be prepared. What appears on the film is not a scaled image of the ground unless the camera is pointing exactly straight down, which it seldom is. A standard textbook of the time, written by an engineer, described a method for solving the "resection problem," namely computing a genuine scaled image of the ground from the aerial photographs. This was a tedious method of successive approximations, that could become ill-conditioned or even diverge. This method was in use when Marston Morse happened to visit Ft. Monroe. He pointed out that the print on the film is a projection of the ground. So here was a problem in solid projective geometry. Since a projective transformation is described by a quotient of two linear forms, one can get the solution of the "resection problem" *exactly* in only *one* step by solving an appropriate set of simultaneous linear equations.

Few people, even among the mathematicians, realize what a towering structure the mathematical edifice is. The majority of people are decidedly non-mathematical, and indeed have no notion what mathematics is all about. For them, a mathematician is a person who is good at adding up bridge scores. However, even among non-professional mathematicians, there can be found various people who have mathematical capabilities to some degree. Engineers usually are fairly competent in calculus, some going beyond that a way. Theoretical physicists usually know a lot more, anywhere from the equivalent of an undergraduate major up to very comprehensive knowledge of mathematics and outstanding talent therein. Strangely, many mathematicians seem afflicted with a snobbishness that leads them to classify anything below the level of their current research as not *really* mathematics. This is very common, although it is obviously preposterous, as I now show by an example. Take the content of a junior year course in mathematics. It certainly is not chemistry, or animal husbandry, or high fashion. It is genuinely mathematics, and nothing else but.

Except in cryptanalysis, hardly any of the mathematics done for the War effort was of a higher level than this, and much was at lower levels. As I said, some did not go beyond getting the problem properly formulated. Although we had a six-day week during the War, several *hundred* mathematicians spent two to three years working diligently at such problems. Mathematically, this was not very satisfying. However, answers to these problems were crucial to the progress of the War. Without a person with competence to supply an answer by mathematics, the person with the problem would have had to resort to some scheme of experimental trial and error. This could be very expensive. Worse still, it could be very time-consuming, and everybody wished to get the War over as quickly as possible. So, though mathematicians turned up their noses at most of the problems brought to them, they did so privately, and labored enthusiastically to produce answers.

I have written to practically every mathematician still living who did mathematics for the War effort (there are still close to two hundred)

This article presents the text of Professor Rosser's address given at the Toronto meeting during the first segment of the AMS-MAA Joint Session on the History of Mathematics. The other address given that afternoon, *Homotopy theory: the first twenty-five years* by George W. Whitehead, will appear in the January 1983 issue of the *Bulletin*. The text of Jean Dieudonné's lecture on Bourbaki given in the second segment of the Joint Session will appear in the November issue of the *Notices*.

and I asked for an account of their mathematical activities during the War. Many did not answer. And many who answered said they did not really do any mathematics. I had a one-sentence answer from a man who said that he did not do a thing that was publishable. If we equate being mathematics to being publishable, then indeed very little mathematics was done for the War effort. But, without the unpublishable answers supplied by several hundred mathematicians over a period of two or three years, the War would have cost a great deal more and would have lasted appreciably longer.

I worked for three years during the War with a group that was charged with developing and producing rockets. I had a co-worker, R. B. Kershner, who was a very able mathematician. We were responsible for getting answers to the problems that arose that seemed too mathematical for the other people in the group. After a while some younger mathematicians were hired to help us. Kershner insisted to his dying day (which was fairly recent) that he never did an iota of mathematics during the War. True enough, the problems were mostly very pedestrian stuff, as mathematics. I was never required to appeal to the Gödel incompleteness theorem, or use the ergodic theorem, or any other key results in that league. One time the tedium was relieved when I had to do something with orthogonal polynomials, and I was glad to get out the Szegö tome [**26**] and bone up a bit. But mostly I was working out how fast our rockets would go, and where. On a good day, some problem would be up to the level of a junior course in mathematics.

Is OR (operations research) mathematics? Nowadays, the practitioners insist that it is a separate discipline, and I guess by now it is. It is certainly not now taught in departments of mathematics. But, it grew out of mathematics. At the beginning of OR, during the War, it was mathematics according to my definition above, although some of the very good operators were physicists and chemists. The Air Force Generals and Navy Admirals thought it was wonderful stuff. You could not have convinced one of them that it was not mathematics. Indeed, the Generals made special arrangements with the Applied Mathematics Group (AMG) at Columbia to recruit more mathematicians, teach them OR, and send them out to the field. There, though they remained civilians, they were attached directly to combat units.

I bring this up because I wish to give special attention to the steps taken to help bombers defend themselves against German fighter aircraft (and later Japanese). This was a very important endeavor because when Britain first tried sending fleets of bombers against German targets, the German fighters would sometimes shoot down more than half of a fleet of bombers on one sortie.

I first summarize a report by Edwin Hewitt [**10**]. He was in an OR group attached to the Eighth Bomber Command. Hewitt has worked in topology, measure theory, functional analysis, and harmonic analysis. So he is a highly qualified mathematician. Of course, he was not so well qualified during the War, but it did not matter because none of those specialties would have been of any use for the mathematical problems that he had to solve for his OR duties.

For defense, the B-17 bomber had about a dozen machine guns, and gunners, aboard. Later bombers had considerably more. The theory was that if a German fighter appeared, all gunners on that side of the bomber would start shooting at it. It was hoped that such a concentration of firepower would finish off the fighter quite promptly. But it did not work that way at first.

The British had OR before we got there, and had found what the trouble was. When a person on the ground shoots at a bird in flight, he aims in front of where the bird is at the time he pulls the trigger, hoping that by the time the bullet gets up there the bird will have advanced to the point he aimed at. So, the gunners manning the machine guns in the bombers were all aiming ahead of the attacking fighters. Because the bombers were flying at high speed, that was the wrong place to aim. To show this is utterly trivial, merely a matter of vector addition. But it must have been mathematics. At least, none of the generals, colonels, majors, etc., had thought of it. To figure out where you should have aimed was harder, though Kershner (and I fear many in the audience) would scorn to call it mathematics either. Just look up the ballistics of machine gun bullets, and then any mathematician can do it without much trouble. But the gunners could not be expected to.

To help the gunners aim right, the following scheme had been adopted. The window through which a gunner looks was divided into zones. If a gunner sees a fighter through a particular zone, he is supposed to aim a certain amount off from where he sees the fighter, the distance off and direction depending on which zone he sees the fighter through. These distances and directions were printed on mimeographed "poop sheets," and were supposed to be memorized by the gunner.

This system had been adopted by the British. When the Americans got their bombers into the combat area, they adopted it too. In fact, near the end of the War, I visited a Texas airfield where a similar system for aiming rockets from a plane was being taught.

Of course, the zones for one type of bomber have to be different from those of another type. Hewitt undertook the calculations for both the B-17 and the B-24. Not only did the zones have to be devised, but the instructions on the "poop sheet" for where to fire for each zone had to be calculated. Although these calculations were absolutely indispensable and crucial, it turned out that a major part of Hewitt's duties was lecturing to the newly arriving Americans on how to use the "poop sheet" and emphasizing the overriding importance of learning what was on it. In arranging these lectures, and many other matters, Hewitt was much helped by the head of his group. This was a lawyer named John M. Harlan. He could not provide any mathematical assistance at all, but he later became a Justice of the Supreme Court, and was very well qualified at arranging things.

This zone system improved the situation quite a bit, but was obviously far from perfect. So the people in the Applied Mathematics Group (AMG) at Columbia tried to think of something better.

I shall cite details sent me by Daniel Zelinsky [32]. He was an algebraist, and after the War did a thesis under A. A. Albert on the arithmetic of some nonassociative algebras. None of this training helped him specifically in calculating where to aim machine guns from bombers.

The sights on the guns were just fixed reticules aligned in the direction of the gun barrel. For a start, one sight was made movable, and a simple linkage attached. The inputs to the linkage were the speed of the bomber (set manually after reading a dial installed in the bomber) and the angle between the gun and the axis of the bomber (set mechanically as part of the linkage). The linkage then was supposed to move the reticule so that if you look through the reticule and see the fighter, the gun is aimed (approximately) correctly. Zelinsky says it didn't take any real mathematical talent to figure how to put that linkage together. I will not say it did, but somebody had to use something resembling mathematics somewhere in the process.

Zelinsky doesn't know if the linkage ever got to the battle front. At the end of the War, they were getting around to moving the reticule by an elementary analog computer. Zelinsky says the design of this made for more interesting mathematics.

Let us look at a third attempt to help the gunners aim correctly. An outfit called the Jam Handy Organization constructed movie films depicting what the gunner would see, and where he should aim. The prospective gunner would study these films enough times to learn to aim correctly.

To simulate the fighter, they had a small scale replica. A movable camera would take still pictures of this. The camera and replica were repositioned between each picture so that when the pictures were run through in sequence a movie was produced showing the fighter in action.

To calculate where the camera and replica should be for each picture is not merely an application of spherical trigonometry. If the fighter was in a turn, you needed differential equations and elementary differential geometry to tell where it would be heading, and at what inclination. And then, of course, you had to calculate where the gunner should be aiming, and mark it on each picture.

Regardless of how simple Kershner or some of the more snobbish mathematicians might think this to be, the Jam Handy Organization thought they had better hire two mathematicians. They were William M. Borgman and Edwin W. Paxson. These belong to an earlier generation, and are probably not known to most of the audience. However, they were very capable mathematicians, and accustomed to much more sophisticated problems. Naturally, they knocked off the Jam Handy problems in a breeze. Indeed, they wrote comprehensive reports on how to solve them, with formulas for the key quantities, and all that. These reports are still on file at the Jam Handy Organization [12], in case they should ever have to do a similar enterprise. At the time, they were classified SECRET, and there has never been a question of publishing them.

On page 613 of [21] are described some studies made by the Applied Mathematics Panel of the defense of B-29s against fighters. I do not know the extent to which these studies were affected by, or integrated with, any of the three projects I have just described. Wartime security greatly hampered intercommunication of results.

I might point out that the Navy similarly had OR groups helping them with anti-submarine tactics, and other matters. See [19]. Here, at least for airplanes attacking submarines, the problem was not one of defense of the plane, but of tactics. Incidentally, for the OR groups attached to bomber outfits, a very important consideration was tactics. OR could tell the best number of planes to send against a target, the best spacing for dropping the bombs, and such [20]. This could make very considerable differences in the effectiveness of bombing.

I had better leave the details of bombing, and get to the general picture. Not only do we have to decide what mathematics is, but what time span we should cover, and what nations to consider. We really have to start in the thirties, and run until about the mid fifties, when OR and computer science actually separated off from mathematics proper. We restrict attention to the USA effort.

The services have contrived to keep going similar types of support since the War. The RAND Corporation and the Center for Naval Analyses receive all sorts of problems directly from the services, to which they try to give answers. Congress was persuaded to pass a special act authorizing the services to support basic research. They now maintain the Office of Naval Research, the Army Research Office at Durham, and the Air Force Office of Scientific Research, under which they give grants to universities, and that sort of thing.

Very importantly, modern computers did not really get into action until the War was over. For several years after the War, the military poured a lot of money into computer development. At first, the software for this was largely in the hands of mathematicians, but gradually computer science evolved as a separate discipline.

Before the War, Hitler made things so unpleasant for the Jews that many left. Although the USA was in a depression, perhaps 150 very good mathematicians were able to find support in the USA during the thirties. See [5] and [22]. This was quite a help, as the demand for mathematicians ran very high during the War. An incidental result was the founding of *Mathematical Reviews*, just before the War.

Early in the thirties, the WPA, to help relieve unemployment, set up a project to compute mathematical tables [16]. This employed a number of mathematicians. As the War came nearer, and then during the War, the need for computations increased, so that the project grew, and was eventually taken over by the National Bureau of Standards. Finally, after the War, when large computers appeared in some numbers, the project became obsolete, and was discontinued.

By about two years before the War, preparations were being made for our entry. A broad overall description of the scientific activities during the War can be found in [2]. It scarcely mentions any mathematical activity.

A reason for this is that, except in cryptanalysis, which is still cloaked in secrecy, there was not any sensational breakthrough in mathematics comparable to the atomic bomb in physics, or radar, or the proximity fuze. Although mathematics pervaded all the scientific studies, and was often indispensable for progress, the problems, considered as mathematics, were seldom very formidable. As we noted earlier, most could have been solved by theoretical physicists, and many by smart engineers. But theoretical physicists and smart engineers were even more critically needed for many other things. So some hundreds of mathematicians were pressed into service, mostly on leave from their schools. Reasonable, though sketchy, accounts of the mathematical activities can be found in [21] and [30]; the latter is primarily an account of statistical activities. As far as that goes, the present account is more sketchy than complete.

Actually, the most sensational achievements of mathematics during the War were probably in ciphers and code breaking. This is still heavily covered with secrecy, and little can be told. [13] tells a lot, but doesn't really get to the heart of the matter. One incident has been publicized in [15]. A cryptanalytic breakthrough enabled the USA to win a major naval battle at Midway Island. The Japanese later pinpointed this as the turning point of the naval war between Japan and the USA [6]. Note the title of [6]. The British have relaxed the secrecy on their work with ciphers and the like. A flood of books has appeared, each "telling all." You could start with [31] and [14].

With hundreds of mathematicians on leave from their schools to work on military-related problems, the schools were in short supply, even with the 150 or so mathematicians who had immigrated from Germany. Of course, enrollments were way down, with most men being drafted. However, because of the high technology of the War, the military wished special mathematical training for many in the services. This seldom went above algebra and trigonometry, but the schools were hard pressed to supply the needed teaching. During the War, I heard that Agnew, then chairman at Cornell, was seen one Saturday afternoon at the intersection of the two main streets of Ithaca, accosting passersby. He would ask, "Do you know the difference between algebra and trigonometry?" If the answer was "Yes", he said, "You're hired." Agnew says he did not really do this, but he was tempted. However, he scrounged around, and found faculty members, say from the music department, or wives of such, who, on a whim, had taken calculus and so could teach algebra or trigonometry. Thereby, he managed to get all his classes taught. See [33].

How did those hundreds of mathematicians get dispersed into all sorts of wartime activities? During World War I, Aberdeen Proving Ground had chanced to hire a number of mathematicians and had found them very helpful. Hence, as World War II came near, they got Oswald Veblen to join the staff, primarily to recruit mathematicians. Altogether, they got somewhere over twenty, plus assorted astronomers, physicists, and what have you. This collection of talent more or less rewrote the science of gun ballistics. [17] pretty much covers what evolved.

The Office of the Chief of Ordnance enlisted Marston Morse, who did a similar thing on a much smaller scale with OCO. They had considerable rivalry with Aberdeen, but managed to cooperate sufficiently that they were somewhat helpful to each other. With the tight security there was during the War, such cooperation was not easy.

If you think this does not sound very systematic, you are right. Before the War, there was set up the NDRC (National Defense Research Committee). It had divisions devoted to research in various areas; there was not one for mathematics, nor was there any provision for getting mathematicians into any of the divisions. Later, an umbrella was thrown over NDRC, namely OSRD (Office of Scientific Research and Development), but still no provision for mathematics.

I got into Division 3 of NDRC, devoted to rockets, because a chemist friend of mine told them I might be of some use. They interviewed me and offered me a job, which I took. I wrote [24] and [25], mostly while there, but published afterward. That steered me into computer software. There I could use my early training in symbolic logic and I am still involved. I also consulted on rocket work, up to helping with the Apollo (man on the moon) Project. My training as a logician did not help with rocketry.

Other divisions of NDRC acquired mathematicians in a similarly haphazard way. Some never did.

The Naval Research Laboratory, Frankford Arsenal, and various other outfits, did like Aberdeen and OCO, and recruited on their own. Commercial outfits did likewise.

Finally, in spite of considerable opposition from somebody high in NDRC, it was decided that NDRC would establish an Applied Mathematics Panel (AMP) [**1**]. This was fragmented all over the place, but mostly at universities through contracts with AMP. There were Applied Mathematics Groups, Statistics Research Groups, at least one Bombing Research Group (BRG), and I don't know what else.

The theory was that the various Groups of the AMP would recruit able mathematicians. People in the military with mathematical problems would submit them to AMP, which would assign them to the appropriate Group. But there were deviations from this. Stewart Cairns was reassigned from the BRG individually as consultant to the Army Air Forces Board in Orlando, Florida. There he remained as the only mathematician throughout most of the War. A special letter from General Eubank commended him for his help. And recall that the AMG at Columbia was asked to recruit mathematicians and train them in OR for assignment to the Air Force.

However, there is no question that AMP recruited a lot of mathematicians and solved a lot of problems. The collection of their reports, in the National Archives, takes up 45 feet of shelf space.

There were various special cases. Some were cases in which a mathematician either enlisted or was drafted. When his talents were found out, he was usually transferred to a suitable laboratory. S. C. Kleene and J. H. Curtiss are examples. The Bureau of Ordnance happened already to have a mathematics division under R. S. Burington when the War broke out. It was simply expanded. See also [7] for another case.

During the War, Bell Aircraft Corporation developed the first airplane to exceed the speed of sound. It was much helped in this by a group of seven mathematicians. Maybe one or two were primarily aerodynamicists, and all became fairly competent at aerodynamics before the War ended. They were William H. Pell, Wilhelm S. Ericksen, John Giese, Paco Lagerstrom, V. M. Morkovin, Wilbur L. Mitchell, and John van Lonkhuyzen. They seemed to work as a team in a way that is not too common among mathematicians.

While we are on the subject of aircraft, you might note [**23**].

As recounted in [**30**], admonitions and training by statisticians resulted in significant improvements in the quality of manufactured goods.

The War produced a big surge in numerical analysis. Everybody wished to have numbers. All existing texts were carefully studied, and people began to invent new methods. There began to be great pressure to build mechanical calculators which would be faster than the desktop models which had been in existence for many years. Incidentally, in the thirties Vannevar Bush invented the analog computer, which was very good for many types of problems. For a while, analog computers were much in vogue. Two were installed at Aberdeen during the War to help with computation of ballistic tables.

A start on the development of digital computers was made as early as 1937 by Stibitz at Bell Laboratories, using phone relays. Some of his later models were actually used in War-related problems. See the essay by Stibitz in [**18**].

George David Birkhoff appreciated the role that computing might have, and by using a bequest that Harvard had and a lot of help from IBM, he financed the construction of a large calculator, MARK I, at Harvard by Howard Aiken, which was dedicated in 1944. The Navy was much impressed by this calculator, and ordered three more improved models for installation at Naval laboratories. A very few details are given in the essay by Garrett Birkhoff in [**18**].

However, it is the electronic digital computer which has utterly transcended all these early attempts. In 1935, Alan Turing described how to build a computing machine, the so-called "Turing machine." John von Neumann got into the act with proposals for how to go about building such a machine using electronic components. At that point, electronics had not quite evolved enough to build one, but the Army poured money into electronic development. See two essays, one by Eckert and one by Mauchly in [**18**]. Finally, just about at the end of the War, the ENIAC was completed and installed at Aberdeen, to compute firing tables. This was not quite a "Turing machine," because the computer could not change the instructions for a program. However, by 1950 the very first "Turing machines" appeared in the USA. About that time, with the influence of Turing, the English managed to complete one. John von Neumann finally managed to get his operating in 1952. See [**8**] and [**18**].

At first, the people who knew enough to operate the computers were mostly mathematicians, preponderantly numerical analysts. As there got to be more computers, and the rules for software began to develop, there began to be computer scientists. Probably what marked the real beginning of computer science as a separate discipline was the realization that computers could be used for information manipulation and storage, and not solely as "number crunchers." By the mid fifties, computer science had broken off from mathematics proper. And now we have PAC-MAN!

In the development of the atomic bomb, there was such a concentration of distinguished physicists, many of them theoretical, that there was not much need to call for mathematicians [11]. However, there were a few mathematicians involved, specifically John von Neumann and Stan M. Ulam [28]. However, the atomic bomb was finished with very little help from professional mathematicians.

After a bit, work began on the hydrogen bomb. It was far harder to develop the hydrogen bomb than it had been for the atomic bomb. By 1949, a possible method of construction had been thought of. But, would it work? Ulam, with help from another mathematician, Cornelius Everett, undertook to find out by a hand computation. Others undertook to find out by computing on the ENIAC, then the fastest computer available. Ulam and Everett finished their hand calculations before answers were available from the ENIAC. They said it would not work. Of course, nobody believed them. But finally the ENIAC gave the same answer!

Teller, on page 272 of [27], says of Ulam's calculations: "In a real emergency the mathematician still wins—if he is really good."

After a while, a better idea for making a hydrogen bomb was thought of. Ulam's calculations showed that it should work. By now, a better computer than the ENIAC was available, the SEAC at the National Bureau of Standards. It confirmed Ulam. See page 273 of [27]. By the time the hydrogen bomb was actually built, a computer called the MANIAC had been built at Los Alamos and von Neumann had his computer at the Institute for Advanced Study in Princeton. They all got into the act. However, let us not forget that a human mathematician was able to beat an electronic computer two different times.

I have related a few points of how mathematicians affected the War effort. How did the War effort affect mathematicians? As I have related above, two new branches of the mathematical sciences, OR and computer science, grew out of mathematics proper in about ten years, and have now split off from mathematics proper.

How about changes in mathematics itself? In talking about acceptance tests, a Navy Captain asked the following. Suppose acceptance tests are to be performed on a hundred items chosen from a large shipment. If six items are defective, the shipment is to be rejected. The Captain pointed out that if six defectives turn up in the first fifty tests, there is no need to make the other fifty tests. He asked if it was not possible to make something like this part of the statistical theory? Starting from this suggestion, Abraham Wald worked out the theory of sequential analysis. See [29] and [30]. Not only did this greatly improve the conduct of acceptance tests, but there were many other useful consequences, so that it is now an important branch of statistics.

George B. Dantzig worked during the War as Chief of the Combat Analysis Branch of the Air Force. As military operations became more complex, planning became more difficult. At the end of the War, one program required seven months of study to be sure it did not contain contradictory instructions. After the War, the Air Force funded a study to try to improve planning methods. In 1947, Dantzig invented what is now called linear programming. See [4]. This is based on a generalization of the Leontief "input-output" matrix, and can cope with problems that were formerly almost intractable. The first test of linear programming was done by the old WPA computing group. It had not yet been dissolved, and was then at the National Bureau of Standards. It took 120 man days of calculation on desk calculators. With modern electronic "Turing machines," such a calculation requires a matter of minutes. As all large organizations have complex planning requirements, linear programming is now much used, and is an important technique in mathematics.

In order to be able to use the ENIAC efficiently after it was delivered to Aberdeen, I. J. Schoenberg invented a way of smoothing functions. This was based on a mathematical analysis of the shapes assumed by splines; splines were flexible strips which were forced into curves for designing the hulls of ships. Now known as "spline functions," generalizations of the theory of splines have assumed great importance in many branches of numerical analysis. See [3] and [9].

With the advent of the electronic calculator, numerical analysts now accomplish feats that could hardly have been imagined forty years ago. The solution to the four color problem, and verification that the first 170,000,000 zeros of the Riemann zeta function off the real axis have real part equal to 0.5 are particularly striking cases.

References

If the number of the document is followed by A, as 7.A, this means that a copy of the document is on file in the Archives of American Mathematics at the Humanities Research Center, P.O. Box 7219, The University of Texas, Austin, Texas 78712.

1. *Summary technical report,* Applied Mathematics Panel, NDRC, 1946. Deposited in National Archives. Volume 1. *Mathematical studies relating to military physical research,* edited by I. S. Sokolnikoff; Volume 2. *Analytical studies in aerial warfare,* edited by Mina Rees; Volume 3. *Probability and statistical studies in warfare analysis,* edited by S. S. Wilks. "Classification cancelled per memorandum, Acting Secretary of Defense, dated August 2, 1960."

These three volumes give the authors, titles, and identification numbers of all reports written by the members of the AMP. Not only are the three volumes in the National Archives, in NARS RG 227, but the reports as well, occupying 45 feet of shelf space.

2. James Phinney Baxter III, *Scientists against time,* Little, Brown and Company, Boston, 1946, xv + 473 pages. This book, an official Office of Scientific Research and Development history, contains "the revealing history of American scientists at war and the story of the death-dealing and lifesaving devices which they contributed to victory in World War II."

3. Carl de Boor, *A practical guide to splines*, Applied Mathematical Sciences, volume **27**, Springer-Verlag, Berlin and New York, 1978.

4. G. B. Dantzig, *Linear programming and extensions*, Princeton University Press, Princeton, NJ, 1963.

5. Arnold Dresden, *The migration of mathematicians*, American Mathematical Monthly, volume **49** (1942), pages 415–429.

6. Mitsuo Fuchida and Masatake Okumiya, *Midway, the battle that doomed Japan*, copyrighted by the U.S. Naval Institute, Annapolis, 1955. A Japanese version of the Battle of Midway. A paperback edition was put out by Ballantine Books, 1958.

7.A Leonard Gillman, Unpublished. This report describes work done for the Navy on the mathematical theory of pursuit curves at Tufts University during the War by Gillman and others.

8. Herman H. Goldstine, *The computer from Pascal to von Neumann*, Princeton University Press, Princeton, NJ, 1972.

9. T.N.E. Greville, *Theory and applications of spline functions*, Academic Press, New York, 1969.

10. Edwin Hewitt, *A sketch of gunnery activities in the Operational Research Section, Eighth Air Force, from June 1943 to August 1944*, unpublished. This is a report to Hewitt's superior. It may be available in the Maxwell Field Archives (see [**20**] of this bibliography).

11. R. G. Hewlett and O. E. Anderson, Jr., *A history of the U.S. Atomic Energy Commission*, volume 1, *The new world, 1939–1946*, volume 2, *Atomic shield, 1947–1952*, Pennsylvania State University Press, University Park, PA, 1962.

12. The Jam Handy Organization, 2900 East Grand Blvd., Detroit, Michigan 48202 has on file the reports: *Elements of the theory of aerial gunnery, Mathematical formulae for synthetic targets for 3A-2 and 3A-36*, and *Instructor's manual for 3A-2 gunnery training.*

13. David Kahn, *The code-breakers*, Macmillan, New York, 1967. This is a monumental work. However, due to secrecy restrictions still in effect, it is not specific as to what the exact accomplishments of the World War II mathematicians were.

14. Ronald Lewin, *Ultra goes to war*, McGraw-Hill, New York, 1978, 398 pages, plus several photographs. Also published in London in 1978 by Hutchinson. A paperback edition was put out by Pocket Books, New York, 1980. The first account of World War II's greatest secret based on official documents.

15. Walter Lord, *Incredible victory*, Harper and Row, New York, 1967. A paperback edition was put out by Harper and Row, 1968. This book refers to a cryptanalytic break-through by which the U.S. obtained advance warning of Japan's intent to attack Midway Island. Various U.S. Naval officers are mentioned by name (Commander Rochefort, Lt. Commander W. A. Wright, Lt. Commander Thomas Dyer). After retirement from the U.S. Navy, the then Captain Dyer became a mathematician at the University of Maryland. We understand he was given tenure status there–a rather unusual event for a non-Ph.D.-holder whose formal education was at the U.S. Naval Academy.

16. A. N. Lowan, *The computation laboratory of the National Bureau of Standards*, Scripta Mathematica, volume **15** (1949), pages 33–65. Mostly lists the tables prepared, with some reasons for choosing these to do. Hardly a word about the personalities involved.

17. E. J. McShane, J. L. Kelley, and F. V. Reno, *Exterior ballistics*, University of Denver Press, Denver, CO, 1953, xvi + 834 pages.

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