WATERMAN AWARD FOR WILLIAM P. THURSTON

The fourth Alan T. Waterman Award was presented to William P. Thurston of Princeton University at a ceremony on May 17 at the National Academy of Sciences. Professor Thurston was selected from among 87 nominees and was the second mathematician to receive this award. Charles L. Fefferman also of Princeton received the initial award in 1976. The other recipients were: in 1977, J. William Schopf, a paleontologist at the University of California, Los Angeles, and in 1978, Richard A. Muller, a Physicist at the Lawrence Berkeley Laboratory, University of California, Berkeley.

The Award Committee consisted of sixteen members whose chairman was Frederick Seitz, President Emeritus of Rockefeller University. Twelve of the members, including Raoul Bott of Harvard University (the only mathematician), were appointed by the Director of the National Science Foundation; the remaining four ex officio members were the Director of the National Science Foundation, the Chairman of the National Science Board, and the Presidents of the National Academies of Sciences and Engineering.

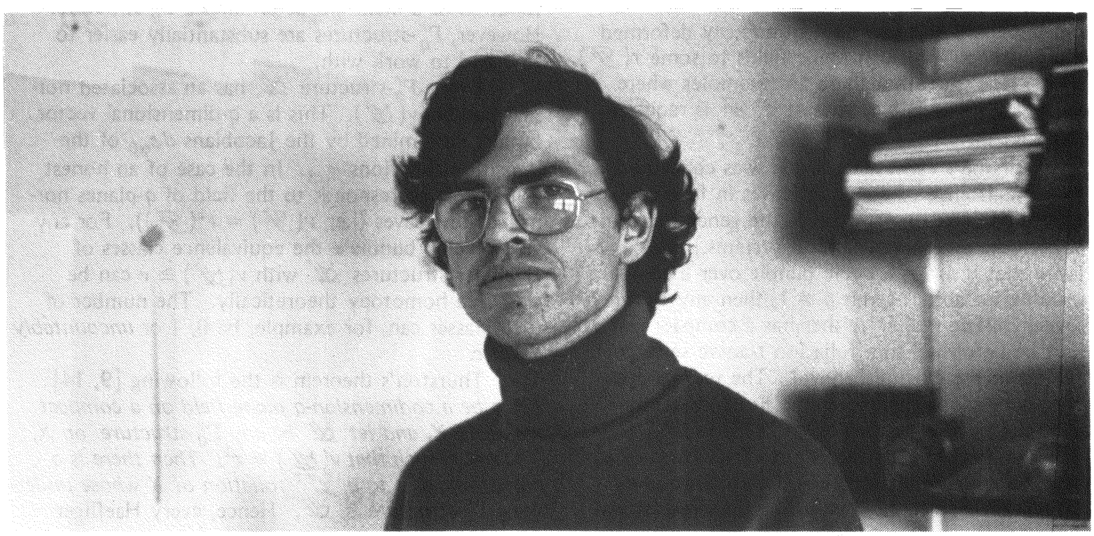
The Alan T. Waterman Award was established by Congress in 1975 in commemoration of the 25th anniversary of the creation of the National Science Foundation and in honor of Alan T. Waterman, first Director of the Foundation, who served for twelve years.

The Waterman Award was established for the purpose of recognizing and supporting talented young research scientists. The award is made each year to a scientist under the age of 35 who shows "outstanding capability and exceptional promise for significant future achievement." Each recipient is granted $150,000 for research at an institution of his or her choice. The Waterman Award is unique in that it carries no restrictions of the type usually associated with Government grants. The award was intended to encourage young scientists to progress beyond the bounds of their usual research in the hope that such work might establish a basis for future discoveries.

William P. Thurston was born October 30, 1946, in Washington, D. C. He received a B. A. in 1967 from New College, Sarasota, Florida, and a Ph. D. in 1972 from the University of California, Berkeley. His doctoral thesis is item [3] in the bibliography below; his advisor was Morris Hirsch. Professor Thurston spent the year 1972–1973 at the Institute for Advanced Study. He became assistant professor at the Massachusetts Institute of Technology in 1973, and in 1974 was named professor of mathematics at Princeton University. He held an Alfred P. Sloan Foundation Fellowship in 1974 and 1975, and was awarded the American Mathematical Society's Oswald Veblen Prize in Geometry in 1976 for his work on foliations.

Thurston is a member of the Committee on Postdoctoral Fellowships of the American Mathematical Society (1977–1980). He spoke at the 1972 Annual Meeting of the Society in Las Vegas (special session on foliations and stable manifolds) and at the AMS Summer Institute on differential geometry (1973). He gave an invited hour address entitled *The existence of foliations* at the April 1974 AMS meeting in New York, and spoke at the International Congress in Vancouver (1974), the NSF Regional Conference on foliations is St. Louis (1975), the special session on foliations at the AMS Annual Meeting in Atlanta (1978), and at the International Congress in Helsinki (1978).

His work is described in the accompanying reports.



THURSTON'S WORK ON FOLIATIONS

Blaine Lawson

It is fair to say that in the course of three years Bill Thurston revolutionized our understanding of foliations. Recall that a codimension-q foliation of a manifold X is a geometric structure on X defined by an atlas of coordinate charts $\{f_\alpha: U_\alpha \longrightarrow X\}$ with $U_\alpha \subseteq \mathbf{R}^{n-q} \times \mathbf{R}^q$ such that the coordinate transformations have the form

$$F_{\alpha\beta}(x, y) = (\psi_{\alpha\beta}(x, y), \varphi_{\alpha\beta}(y)).$$

The local equations $y =$ constant, then determine a decomposition of X into disjoint, connected submanifolds, called the *leaves* of the foliation. If the coordinate transformations are all of differentiability class $\mathcal{C}^r$, then the foliation is said to be of class $\mathcal{C}^r$. Note that the leaves fit together locally in a $\mathcal{C}^r$-fashion like the pages of a book. Globally, however, this structure can be enormously complicated. Consider, for example, a discrete group Γ of Möbius transformations $(w = (z - a)/(1 - \bar{a}z)$ for $|a| < 1)$ acting on the unit disk $\Delta \subset \mathbf{C}$ so that $\Delta/\Gamma = \Sigma$, a compact Riemann surface of genus $g > 1$. Then Γ acts diagonally on $\Delta \times \partial\Delta$, and the foliation whose leaves are $\{\Delta \times \text{point}\}$ descends to a foliation of $M^3 = \Delta \times \partial\Delta/\Gamma$. Here M^3 is the set of unit tangent vectors to Σ; so M^3 is a "bundle of circles" over Σ. The foliation above is transverse to these circles and is invariant under the geodesic flow on M^3 determined by the Poincaré metric.

Every foliation $\mathcal{F}$ has an associated field $\tau(\mathcal{F})$ of tangent $(n - p)$-planes, the tangent planes to the leaves. Given any smooth field τ of tangent $(n - p)$-planes on X, one could ask whether $\tau = \tau(\mathcal{F})$ for $\mathcal{F}$ a foliation on X. When τ has dimension 1, the answer is always "yes" since the problem can be reduced to solving a system of ordinary differential equations. However, in higher dimensions the corresponding system of partial differential equations is overdetermined, and can only be solved if the classical Frobenius integrability conditions are satisfied. This is almost never true. The most one could hope is that τ can be continuously deformed or "homotoped" through plane fields to some $\tau(\mathcal{F})$. R. Bott has shown that there are examples where even this is not possible, at least if $\mathcal{F}$ is required to be smooth.

Thurston's earliest work [**3**] was concerned with the existence of compact leaves in foliations of 3-manifolds. (A compact leaf is the generalization of a periodic orbit in dynamical systems.) He showed that if M^3 is a circle bundle over a compact orientable surface of genus $g \neq 1$, then any codimension-one foliation of M^3 either has a compact leaf or can be deformed to a foliation transverse to the fibres (as in the example above). The second possibility is ruled out whenever the Euler characteristic of the bundle χ satisfies $|\chi| > |2g - 2|$.

Thurston's most sweeping results, however, are concerned with the existence and classification of foliations on manifolds. One of his first results was a construction of a continuum of codimension-one foliations $\{\mathcal{F}_t\}_{t\in\mathbf{R}}$ of the 3-sphere which are radically different from one other [**4**]. (A certain characteristic number gv, sensitive only to the "cobordism class" of $\mathcal{F}_t$, satisfies $\mathrm{gv}(\mathcal{F}_t) = t$.) This construction has a generalization to higher dimensions. It was clear at this point that the space of foliations of a manifold is enormously complicated.

Despite this complexity, Thurston then succeeded in actually classifying the codimension-q, $\mathcal{C}^r$ foliations of any compact manifold. One consequence of this work was the theorem that a compact manifold X admits a smooth, codimension-one foliation if and only if the Euler characteristic of X is zero. In fact, he proved that every codimension-one plane field on X can be deformed to the tangent plane field of a smooth foliation. (A few years prior to this it was not even known that S^5 supported a codimension-one foliation.) Another consequence is that any 2-plane field or parallelizable p-plane field on a compact manifold is homotopic to a smooth foliation. Particularly striking is his result that *any* plane field on a compact manifold is homotopic to a $\mathcal{C}^0$ foliation with $\mathcal{C}^\infty$ leaves. By the theorem of Bott mentioned above, there are cases where these foliations are not smoothable.

A statement of the full existence theorem requires the notion of a Haefliger Γ_q^r-structure. This is a generalized notion of a codimension-q, $\mathcal{C}^r$ foliation which makes sense on any topological space X. Essentially, one retains only the "coordinates" y and the transition functions $\varphi_{\alpha\beta}(y)$ from the definition of a foliation given above. These structures enjoy enough "naturality" properties so that there exists a classifying space for them. This is a homotopy theoretic object $B\Gamma_q^r$ equipped with a universal Γ_q^r-structure. The equivalence classes of structures on any paracompact X are then induced from the universal one by unique homotopy classes of maps $X \longrightarrow B\Gamma_q^r$. In particular, the study of these structures is amenable to the methods of algebraic topology . Needless to say, any codimension-q, $\mathcal{C}^r$ foliation of a manifold determines a Γ_q^r-structure. However, Γ_q^r-structures are substantially easier to find and to work with.

Every Γ_q^r-structure $\mathcal{A}$ has an associated normal bundle, $\nu(\mathcal{A})$. This is a q-dimensional vector bundle determined by the Jacobians $d\varphi_{\alpha\beta}$ of the transition functions $\varphi_{\alpha\beta}$. In the case of an honest foliation it corresponds to the field of q-planes normal to the leaves (i.e., $\nu(\mathcal{F}) = \tau^\perp(\mathcal{F})$). For any given vector bundle ν the equivalence classes of Haefliger structures $\mathcal{A}$ with $\nu(\mathcal{A}) \cong \nu$ can be classified homotopy theoretically. The number of such classes can, for example, be 0, 1 or *uncountably infinite.*

Thurston's theorem is the following [**9, 14**]. *Let τ be a codimension-q plane field on a compact manifold X, and let $\mathcal{A}$ be any Γ_q^r-structure on X, $0 \leqslant r \leqslant \infty$ such that $\nu(\mathcal{A}) \cong \tau^\perp$. Then there is a homotopy of τ to a $\mathcal{C}^r$ foliation of X whose underlying Γ_q^r-structure is $\mathcal{A}$.* Hence, every Haefliger structure sitting above $\tau^{\perp}$ can be realized by a foliation. There is also a relative version of this theorem which is tricky in codimension-one due to the Reeb stability phenomenon. The proof there uses Thurston's beautiful and sharp improvement of the classical Reeb stability theorem [**8**].

A consequence of these results is that concordance classes of codimension-q, $\mathscr{C}^r$ foliations on a compact manifold X are in one-to-one correspondence with homotopy classes of Haefliger structures $\mathscr{H}$ together with concordance classes of injections of ν into the tangent bundle of X. (Two foliations $\mathscr{F}_0$ and $\mathscr{F}_1$ of X are said to be *concordant* if there is a foliation of $X \times [0, 1]$ transverse to the boundary and inducing $\mathscr{F}_k$ on $X \times \{k\}$ for $k = 0, 1$.)

It should be mentioned that similar results were known for open manifolds. This followed from work of Haefliger, Phillips and Gromov. (See Bull. Amer. Math. Soc. **80** (1974), 369–418, for example.) The compact case, however, is substantially more difficult. A good analogy is found in the question of existence of metrics of positive sectional curvature. Gromov's general theorem shows that any connected, noncompact manifold has such a metric. However, compact positively curved manifolds are still largely hidden from our view.

Understanding the complexity of the space of foliations on a compact manifold is now reduced to understanding the topology of Haefliger's classifying space $B\Gamma_q^r$. In this area Thurston made another major contribution. Building on work of John Mather, he established a deep relationship, at the homology level, of $B\Gamma_q^r$ with the group of $\mathscr{C}^r$ diffeomorphisms of $\mathbf{R}^q$ with compact support. One consequence of this is that the map $B\Gamma_q^r \longrightarrow BGL_q$ classifying the universal normal bundle is $(q + 2)$-connected if $r = \infty$ and is a weak homotopy equivalence if $r = 0$. The importance of this lies in the fact that the equivalence classes of Haefliger structures $\mathscr{H}$ with $\nu(\mathscr{H}) \cong \nu$ correspond one-to-one with homotopy classes of liftings

$$\begin{tikzcd} & B\Gamma_q^r \arrow[d] \\ X \arrow[ur, dashed] \arrow[r, "f_\nu"'] & BGL_q \end{tikzcd}$$

where f_ν classifies ν.

Using his work on $B\Gamma_q^r$ and results of M. Herman, Thurston went on to prove the following important theorem [**6**]. *The identity component of the group of diffeomorphisms of any compact manifold is a simple group.* The relationship he established between $\mathrm{Diff}(X)$ and $B\Gamma$ was totally unexpected and certainly nontrivial.

It is evident that Thurston's contributions to the field of foliations are of considerable depth. However, what sets them apart is their marvelous originality. This is also true of his subsequent work on Teichmüller space and the theory of 3-manifolds.

THE NEW GEOMETRY OF THURSTON

Dennis Sullivan

Imagine on a compact surface of constant negative curvature an infinitely long geodesic $\mathscr{L}$ which does not intersect itself. The closure of $\mathscr{L}$ is locally a Cantor set of disjoint geodesic segments. Adding a natural measure on the strands, Thurston defines a new geometrical object generalizing a closed or periodic geodesic. He employs these *laminar geodesics* in a variety of ways.

Using these objects in a purely topological form (measured foliations) like elements in a completed fundamental group, Thurston develops a beautiful classification of surface transformations by canonical models. The general type of transformation determines two of these transversal laminar geodesics or measured foliations and the picture is analogous to the two eigen-foliations of a general 2×2 unimodular matrix (in the torus case). To arrive at this picture, Thurston studied the sequence of closed curves obtained by iterating an approximate transformation many times and noticed a simple pattern among the complicated successive images.

Thurston classified the limiting patterns. They trun out to be measured foliations on the surface and form a sphere of dimension $6g - 7$ (g equals the number of handles on the surface). This sphere of patterns can be added as a natural equivariant boundary for the Teichmüller space of all constant negative curvature metrics on the surface.

In particular, from one point in Teichmüller space (a specific metric of constant negative curvature on the surface) one moves geometrically to any other points by choosing a direction (thought of as a particular laminar geodesic) and then performing an "earthquake" of some strength in that direction (namely by slicing the surface along the geodesic lamination, sliding the complementary regions rigidly an amount proportional to the measure, and re-gluing isometrically).

Moving on to dimension 3, Thurston proved a sweeping result, that many 3-dimensional manifolds admit a canonical complete metric of constant negative curvature -1 (called a hyperbolic structure), providing a profound substitute in dimension 3 for the classical uniformization theorem for surfaces. For example, most knot or link complements in the 3-sphere can be obtained from hyperbolic 3-space by dividing by a discrete finite-volume subgroup of isometries. There is one cusp for each component of the link, and the groups are often arithmetically defined. More generally, Thurston develops a theme that deep and rich topological phenomena of 3-manifolds may be discovered by imposing natural and canonical geometrical structure on the problems.

Again in dimension 3, laminar geodesics appear in several connections. Firstly, by bending hyperbolic planes in 3-dimensional space along laminar geodesics an angular amount proportional to the given measure, Thurston obtains a class of useful surfaces in space (*uncrumpled surfaces*). These surfaces include the local picture of the boundary of a convex region in hyperbolic 3-space.

Secondly, the bending picture of Thurston allows one to visualize Poincaré's projective structures on a surface, analytically described as all the multi-valued solutions of $Su = f$, where S is the Schwarzian derivative and f is any quadratic differential on the surface.

Finally, the laminar geodesics and the uncrumpled surfaces are used in a long geometrical analysis whose purpose is to chart the unknown regions in basic noncompact hyperbolic 3-manifolds. By studying the intersection with these surfaces, Thurston manages to control lengths of infinite families of curves to establish a fundamental bound. This bound is perhaps the most difficult part of his 3-dimensional topological uniformization theorem.

Similar geometric bounds give a powerful new tool to the study of Kleinian groups. For example, Thurston obtained significant progress on Ahlfors' measure problem.

Thurston's results are surprising and beautiful. The method is a new level of geometrical analysis—in the sense of powerful geometrical estimation on the one hand, and spatial visualization and imagination on the other, which are truly remarkable.

Thurston's Publications

1. (with G. Sandri) *Classical hard sphere three-body problem*, Bull. Amer. Phys. Soc. **9** (1964), 386.

2. (with J. F. Plante) *Anosov flows and the fundamental group*, Topology **11** (1972), 147–150. MR **45** #4455.

3. *Foliations of* 3*-manifolds which are circle bundles*, Thesis, Univ. of California, Berkeley, 1972.

4. *Noncobordant foliations of* S^3, Bull. Amer. Math. Soc. **78** (1972), 511–514. MR **45** #7741.

5. (with H. Rosenberg) *Some remarks on foliations*, Dynamical Systems (Proc. Sympos., Univ. of Bahia, Salvador, 1971, M. M. Peixoto, editor), Academic Press, New York, 1973, pp. 463–478. MR **49** #3974.

6. *Foliations and groups of diffeomorphisms*, Bull. Amer. Math. Soc. **80** (1974), 304–307. MR **49** #4027.

7. *On the construction and classification of foliations*, Proc. Internat. Congress of Mathematicians (Vancouver, 1974), Canadian Math. Congr., Montreal, Que., 1975, Vol. I, pp. 547–549. MR **54** #7.

8. *A generalization of the Reeb stability theorem*, Topology **13** (1974), 347–352. MR **50** #8558.

9. *The theory of foliations of codimension greater than one*, Comment. Math. Helv. **49** (1974), 214–231. MR **51** #6846.

10. (with M. W. Hirsch), *Foliated bundles, invariant measures and flat manifolds*, Ann. of Math. (2) **101** (1975), 369–390. MR **51** #6842.

11. (with H. E. Winkelnkemper) *On the existence of contact forms*, Proc. Amer. Math. Soc. **52** (1975), 345–347. MR **51** #11561.

12. *A local construction of foliations for three-manifolds*, Differential Geometry (Proc. Sympos. Pure Math., Vol. 27, Part 1, Stanford Univ., Stanford, Calif., 1973), Amer. Math. Soc., Providence, R. I., 1975, pp. 315–319. MR **52** #1725.

13. *The theory of foliations of codimension greater than one*, ibid., p. 321. MR **51** #11540.

14. *Existence of codimension-one foliations*, Ann. of Math. (2) **104** (1976), 249–268. MR **54** #13934.

15. *Some simple examples of symplectic manifolds*, Proc. Amer. Math. Soc. **55** (1976), 467–468. MR **53** #6578.

16. (with D. M. Kan) *Every connected space has the homology of a* $K(\pi, 1)$, Topology **15** (1976), 253–258. MR **54** #1210.

17. (with J. Plante), *Polynomial growth in holonomy groups of foliations*, Comment. Math. Helv. **51** (1976), 567–584. MR **55** #9117.

18. (with F. J. Almgren, Jr.) *Examples of unknotted curves which bound only surfaces of high genus within their convex hulls*, Ann. of Math. (2) **105** (1977), 527–538. MR **56** #1209.

19. *Structure of the group of volume preserving diffeomorphisms* (preprint).

20. *On the geometry and dynamics of diffeomorphisms of surfaces*. I (preprint).

21. *A norm for the homology of* 3*-manifolds* (preprint).

22. (with A. Hatcher) *A presentation for the mapping class group of a closed orientable surface* (preprint).

23. *The geometry and topology of* 3*-manifolds* (book, in preprint form).