ICM-94 in Zürich

Bourgain, Lions, Yoccoz, and Zelmanov Receive Fields Medals and Wigderson Receives Nevanlinna Prize

Rumors piled up like so many feet of Alpine snow, but few could be really sure who would get Fields Medals until the names were announced at the International Congress of Mathematicians (ICM) in Zürich last month. Sitting unobtrusively in the front row of the hall where the opening ceremonies were held, waiting patiently through all the honorific speeches and musical interludes, were the four medalists—Jean Bourgain, Pierre-Louis Lions, Jean-Christophe Yoccoz, and Efim Zelmanov—and the Nevanlinna Prize winner Avi Wigderson. The awardees were a composed group: the only detectable fidgeting was on the part of Wigderson's young daughter, who had a ringside seat on her father's lap throughout the proceedings.

ICM-94, held August 3–11, drew about 2,500 participants, somewhat fewer than the last couple of Congresses, perhaps due to the worldwide recession and Switzerland's prices, which tend toward Matterhorn proportions. Congressgoers were treated not to the bracing air that one tends to associate with a country boasting some of the most spectacular mountains in the world, but rather to a European heat wave that sent temperatures into the ninety degree range on several days of the Congress. The Eidegenössische Technische Hochschule (ETH) and the University of Zürich, whose adjacent buildings served as the site for most of the Congress sessions, have no air conditioning to speak of, and some of the lecture rooms sweltered. But the outdoor cafes and the sparkle of Lake Zürich, not to mention the mathematics, provided some distraction from the heat.

The Awardees' Work

This year's crop of Fields Medalists represents two or three very classical areas of mathematical research. It includes two French analysts (Lions and Yoccoz) and a Belgian analyst (Bourgain) who has worked in France; Zelmanov, a Russian, works in algebra. With more geometric work dominating the three previous rounds of Fields Medals given in Warsaw, Berkeley, and Kyoto, some speculated that this year's emphasis on analysis was an attempt to balance the different branches of mathematics. In addition, the choice of Lions as a Fields Medalist turns the spotlight on applied mathematics, an area that many say has been neglected at previous Congresses.

The medalists were mostly honored for a body of work rather than a single result, though Zelmanov is perhaps an exception in this regard. His major contribution was the solution of the Restricted Burnside Problem. There are actually three Burnside problems, the first two of which were posed by Burnside in 1902 and were solved by Novikov and Adian. These problems ask whether every finitely generated torsion group is finite or is of bounded exponent (the exponent of a group is the smallest n such that $x^n = 1$ for all x in the group). The Restricted Burnside Problem, which dates back to the 1930s, asks whether there is a bound on the order of a group with a finite number of generators and a given exponent. Zelmanov settled this question in the affirmative. A major technical tool in the solution is earlier joint work of Zelmanov and McCrimmon on quadratic Jordan algebras. Zelmanov, born in 1955, is at the University of Wisconsin at Madison.

Pictured left to right are: Fields Medalist Jean Bourgain, Nevanlinna Prize winner Avi Wigderson, Fields Medalist Jean-Christophe Yoccoz, Fields Medalist Pierre-Louis Lions, and Fields Medalist Efim Zelmanov.

Pierre-Louis Lions received the Fields Medal for contributions over the last fifteen years to several areas of partial differential equations. Lions' work is important because variations of methods he has developed have applications to broad

classes of equations. The so-called "viscosity method" has, due in large part to the work of Lions, grown into an elegant and comprehensive theory useful for many equations arising from different applications. Lions has also made important contributions to the understanding of the Boltzmann and other transport equations arising in kinetic theory and other areas of physics. Prior to Lions's work, no general theory existed for understanding these equations. He also developed the concept of "concentration compactness" for variational problems. Lions was born in 1956 and is at Université de Paris-Dauphine.

Jean-Christophe Yoccoz is one of the leading theorists in dynamical systems. The field originated in Poincaré's study of the movement of the planets in the solar system. The attraction of the planets to each other causes them to deviate from the elliptical orbits predicted by Kepler's laws, raising the question of whether or not the solar system is stable. In this generality, the question is still open today, although about fifty years ago Carl-Ludwig Siegel formulated a criterion for stability. One of Yoccoz's major achievements was to establish precise limits on when such stability theorems hold. He did this through a combination of ideas from analysis and geometry and by using techniques reminiscent of renormalization methods from mathematical physics. He also developed combinatorial methods, called "Yoccoz puzzles", for the study of fractal sets arising in complex dynamics. Yoccoz was born in 1957 and is at the Université de Paris-Sud (Orsay).

Jean Bourgain, born in 1954, has held positions at the Institut des Hautes Études Scientifiques in Bûres-sur-Yvette and at the University of Illinois at Urbana-Champaign; he will move this year to the Institute for Advanced Study in Princeton. Bourgain received the Fields Medal for outstanding contributions to several areas of analysis, including the geometry of Banach spaces, convexity in high dimensions, harmonic analysis, ergodic theory, and nonlinear evolution equations. For example, he proved well-posedness for the nonlinear Schrödinger equation and the Korteweg-de Vries equation with singular boundary conditions. Bourgain has tackled analytical problems using ideas from outside analysis in unexpected ways; his work on nonlinear partial differential equations combined concepts from harmonic analysis and number theory. Bourgain has also made important contributions to Fourier analysis, where he used ideas from geometry in his work.

Avi Wigderson, the Nevanlinna Prize winner, was born in 1956 and is at the Hebrew University in Jerusalem. Working on the mathematical foundations of computer science, he achieved important insights into the concept of zero-knowledge interactive proofs. Using this technique, a "prover" can convince a "verifier", within certain probabilistic bounds, of the truth of a statement without revealing any of the details of the proof of the statement. Interactive proofs find application in distributed systems, where the problem arises of how to insure that a network is able to perform its task even when some of the computing agents in the network are faulty. Wigderson's work established the exact maximal possible ratio of faulty to nonfaulty agents. He has also made contributions to the theory of pseudorandom generators.*

Computers and Ethics in Mathematics

Onstage throughout the opening ceremonies were Jacques-Louis Lions of the Collège de France, president of the International Mathematical Union (IMU) (and father of Medalist Lions); David Mumford of Harvard University, IMU vice-president (and recently named president for the coming four years); and Henri Carnal of the University of Berne, chair of the ICM Organizing Committee and president of the Congress. Beno Eckmann, founder of the Mathematical Research Institute at the ETH and former IMU secretary, was named honorary president of the ICM; and, after presenting a speech, he joined the others onstage and later presented the Fields Medals to the recipients. The proceedings took place in English, although most of the speakers greeted the audience in French and German. Eckmann threw in an Italian introduction as well, but conceded that he did not speak Switzerland's fourth language, Romansh.

Eckmann, like some of the other speakers, pointed to the computer as having a major influence on mathematical research. "But," he said, "we should not forget that the most important tool of a mathematician is the fellow mathematician. And that is why we are all here today—to exchange ideas, views, and results, and to listen to each other." Reformulating the aphorism, "whether mathematicians like it or not, the computer is here to stay," he declared that "whether the computer likes it or not, mathematicians are here to stay." "Mathematics is and remains an abstract intellectual enterprise, despite the fact that the natural sciences and technology bear witness to its practical usefulness." He issued a stern warning about the danger of "worldwide trends trying to completely replace rigorous reasoning and proof by computer visualization and experimentation."

By popular acclaim, the speech of Ruth Dreifuss, a Swiss federal minister whose portfolio includes science and education, was the hit of the opening ceremonies. To prepare for her speech, Dreifuss sent three questions to more than a dozen of the world's leading mathematicians, including Eckmann, Raoul Bott, René Thom, Armand Borel, Phillip Griffiths, Jürgen Moser, Friedrich Hirzebruch, Gerd Faltings, and Sir Michael Atiyah. Her first question concerned the difficulty of communicating mathematical results to the public. "In contrast to a harpist who delights others by her music, I fear the pure mathematician cannot make his art accessible to a wider public," Dreifuss said. "How can pure mathematics justify its art to the state which finances it?" The second question was, Has mathematics avoided discussion of the ethical implications of what it does? The third struck a lighter tone: If ten new professorships were to be created in Swiss universities, how many should go to mathematics?

Perhaps the most interesting answers came in response to the second question. Although Thom wrote that "mathematics

*An upcoming issue of the *Notices* will carry a longer article about the work of the Fields Medalists and the Nevanlinna Prize winner.

itself is ethically neutral" and Eckmann that such discussions are "not relevant" to mathematics, Dreifuss seemed unconvinced. "I do not think that making a distinction between abstract theory and practical application can altogether eliminate the problem," she said. "We owe much of our progress in society to mathematicians, and we have to recognize their merits, while at the same time they have to assume their responsibilities." She seemed to be closest in spirit to Bott, who responded that "the age of innocence has come to an end for us all."

In response to her third question, most replied, rather modestly, that only four or five of the ten new professorships should go to mathematicians. "In fact," said Dreifuss, "in Switzerland today only one chair out of twenty is for mathematics." A few other respondents were more ambitious. Griffiths said that they should all go to mathematics. Faltings was similarly generous in his response, Dreifuss noted: "Nine chairs for mathematics, but, as he likes music, he leaves the tenth chair to the harpist."

Making History

ICM-94 made history in the number of women plenary speakers. The number was not large, but there were as many at this Congress as in all previous ICMs together, namely, two: Marina Ratner of Stanford University and the University of California at Berkeley, and Ingrid Daubechies of AT&T Bell Laboratories and Princeton University. After Ratner boycotted ICM-86 because of the lack of women speakers, and after her work was described in lectures by other people at ICM-90, her appearance as a plenary speaker held special significance. She described her ground-breaking work that solved some conjectures of Raghunathan and of Margulis, who received a Fields Medal in 1978, and the applications of this work to ergodic theory and to number theory.

Plenary speaker Clifford H. Taubes of Harvard University presented an accessible lecture about "anti-self dual geometry". Under this heading he discussed three topics in the geometry of four-dimensional smooth manifolds: the theory of conformal structures with anti-self-dual Weyl tensor, the theory of anti-self-dual Yang-Mills connections, and symplectic geometry. Of these, the first is a young subject, and Taubes described some significant theorems he has proved in it. According to Taubes, the second subject is a mature one, due to the work of Donaldson and others that was reported on at previous ICMs. Symplectic geometry is not yet part of (anti-)self-dual geometry, but, because a symplectic form on a four-manifold is self-dual, Taubes speculated that it would one day be incorporated. Throughout his lecture, given at the beginning of the Congress, Taubes made references to lectures that others would later deliver in the geometry and topology sessions.

Drawing a crowd despite its being held after the regular sessions had finished was a special lecture by Olga Ladyzhenskaya of the St. Petersburg Branch of the Steklov Institute of the Russian Academy of Sciences. The lecture was sponsored by the Association for Women in Mathematics, European Women in Mathematics, and the Committee on Women of the Canadian Mathematical Society. This was an unusual opportunity to hear the seventy-two-year-old Ladyzhenskaya, who since the 1950s has been a leader in the theory of partial differential equations, discuss some of her recent work. The lecture was centered on two principles which have oriented her work in this area: first, that uniqueness implies existence, and second, that the solvability of a problem follows from boundedness of all possible solutions of the problem (or a family of problems connected with it). The quality of her lecture led some to wonder afterward why she had never been invited to give an ICM plenary lecture. Following the lecture was a panel discussion about women in mathematics.

Throughout the Congress, everyone was wondering what Andrew Wiles of Princeton University would say in his talk, which was scheduled to be the very last plenary lecture. Wiles, who has been working for the past year to fill a gap in his proof of Fermat's Last Theorem, has made a tremendous advance toward proving the Taniyama-Shimura Conjecture, prompting some to speculate before the Congress that he might receive a Fields Medal. The large room used for plenary lectures was jammed for Wiles's lecture, and a few people had to sit in the aisle. There were no earth-shattering announcements about Fermat, just an excellent expository presentation of his results. At the end of his lecture, Wiles remarked that eight years ago, he had been thinking of working on something else, an extension of his work on the Iwasawa Conjecture. "I thought that problem would not be too taxing, but I thought it lacked enough interest, and I should try something new and more challenging," he remarked. "Six years later I found that I had reduced the Taniyama-Shimura Conjecture to precisely the problem I thought had insufficient interest." Whether or not he was correct in his assessment of the difficulty of that problem, only time will tell.

Next ICM in Berlin

The Closing Ceremonies were held immediately following Wiles's lecture. There it was announced that, at its meeting prior to ICM-94, the IMU General Assembly voted its approval of Berlin as the site for ICM-98. It was also announced that Friedrich Hirzebruch, director of the Max-Planck Institute for Mathematics in Bonn, will chair the Organizing Committee for ICM-98. It is unusual to have back-to-back ICMs in sites so close geographically and linguistically. But for Berlin, which lost out on hosting the Olympics, having an ICM was perhaps the next best thing.

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