Leon Simon Receives 1994 Bôcher Memorial Prize

The Bôcher Memorial Prize is awarded every five years for a notable research memoir in analysis which has appeared in the previous five years. The prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer (1896) and tenth president (1909–1910) and one of the founding editors of *Transactions of the AMS*. The recipient must be a member of the Society, or the memoir for which the award is given must be published in a recognized North American journal. The prize carries a cash award of $4000.

The seventeenth award was made at the Society's 100th Annual Meeting in Cincinnati, Ohio, on January 13, 1994. The 1994 recipient is LEON SIMON of Stanford University.

The prize was awarded by the AMS Council acting on the recommendation of the Committee to Select the 1994 Recipient of the Bôcher Prize. The committee members are Luis A. Caffarelli, Richard B. Melrose, and Richard M. Schoen (chair).

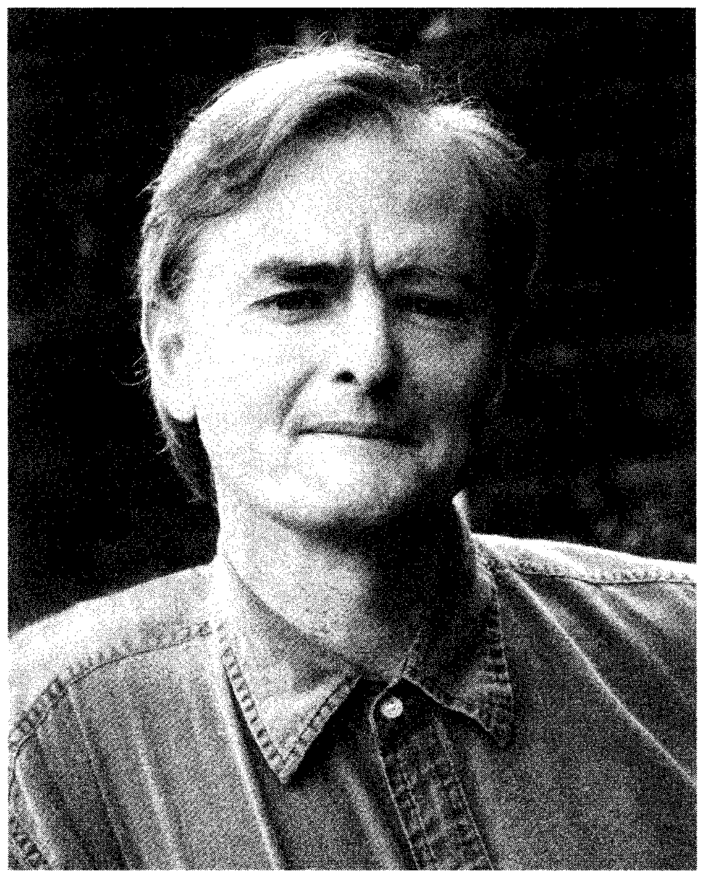
The text below includes the committee's citation, Simon's response on presentation of the award, and a brief biographical sketch.

Citation

The 1994 Bôcher Prize is awarded to Leon Simon for his profound contributions toward understanding the structure of singular sets for solutions of variational problems.

Powerful methods were developed in the 1960s to establish the partial regularity of minima and critical points of the Plateau problem and later extended to other variational problems such as the harmonic mapping problem. These results left open basic questions about the structure of the set of singularities exhibited by the solutions of such variational problems.

In a series of papers over the past ten years, Simon has developed methods for analyzing this structure. This development began with his 1983 paper on asymptotics near isolated singularities, entitled "Asymptotics for a class of nonlinear evolution equations, with applications to geometric problems", *Annals of Mathematics* **118** (1983), pp. 525–572. The first stage of his work on general singular sets is principally described in "Cylindrical tangent cones and the singular set of minimal submanifolds", *Journal of Differential Geometry* **38** (1993), pp. 585–652, and the remaining work appears in his paper "Rectifiability of the singular set of energy minimizing maps" (Preprint, Stanford University, 1993). This latter paper establishes rectifiability for the singular sets of energy minimizing maps into an arbitrary compact real analytic target manifold.



Leon Simon

Response

I am very honored to be awarded the 1994 Bôcher Prize.

In this response I would first like to offer thanks to the Bôcher Prize Committee for their kind recognition of my work, and also to the various people to whom I am most indebted, both personally and professionally: To my family, and especially to my wife Sandra; to Jim Michael for his guidance and mathematical insight during the period of my undergraduate and graduate work at the University of Adelaide; and to David Gilbarg for his constant support and for the numerous mathematical conversations we enjoyed during my time as an assistant professor at Stanford University and later. There are also many colleagues, both present and past, to thank. Of these I want to mention especially Shing-Tung Yau, Richard Schoen, and Robert Hardt.

As the citation mentions, much of my recent work addresses questions about the structure of the singular sets of the solutions of various geometric variational problems. These questions arose naturally from the work of the pioneers in the field of geometric measure theory/geometric calculus of variations, including De Giorgi, Reifenberg, Federer, Fleming, Almgren, and Allard, who are principally responsible for the initial development of the partial regularity and existence theory for minimal surfaces. An analogous partial regularity theory for energy minimizing maps between Riemannian manifolds was later established by Schoen and Uhlenbeck. The latter work, for example, established that if the domain of a bounded energy minimizing map has dimension n, then the dimension of the singular set (i.e., the set of points where the map fails to be locally smooth) is $n-3$; the dimension referred to here is Hausdorff dimension, so the result is that the singular set has Hausdorff measure zero in any dimension *larger* than $n-3$. Part of my work cited above shows that if the target manifold is real-analytic, then the singular set locally decomposes into finitely many locally $(n-3)$-rectifiable, locally compact pieces. (Here "locally" means in a neighborhood of each point of the relevant set.) There are analogous results for various classes of minimal surfaces, always in the appropriate dimension. For example, for n-dimensional mod 2 area minimizing surfaces, the singular set is shown to locally decompose into finitely many locally $(n-2)$-rectifiable, locally compact pieces. In some special cases it is possible to prove more, including even that the singular set is a union of smooth manifolds together with a compact set of lower dimension.

The methods used in the proof of these results are a mixture of geometric measure theory and PDE methods. The PDE methods involve in part ideas originating in quasilinear elliptic theory, developed by C. B. Morrey, E. De Giorgi, O. Ladyzhenskaya, N. Ural'tseva, J. Moser, and others, principally during the period from the late 1930s to the mid-1970s.

There are still many fascinating and fundamental questions remaining in the analytic side of the geometric calculus of variations, and these questions have an added dimension of interest by virtue of their close connection to important problems in geometry. For example, in recent times there has been much interest in geometric evolution problems, beginning with the work of Hamilton, Brakke, and Huisken on Ricci and mean-curvature flow; and here there are important unsolved problems related to singularities, including how singularities form and the structure of the actual set of singularities in space-time.

Certainly it seems clear that both measure-theoretic and hard analysis methods have an important role to play in the field for the foreseeable future.

Biographical Sketch

Leon Simon received his Ph.D. from the University of Adelaide, South Australia, in 1971. He held positions as assistant professor at Stanford University (1973–1975), associate professor at the University of Minnesota (1976–1977), and professor of mathematics at the University of Melbourne (1978–1980) and The Australian National University (1981–1986). Since 1986 he has been professor of mathematics at Stanford University.

Professor Simon was a Sloan Fellow (1974–1975), was elected Fellow of the Australian Academy of Sciences (1983), and received an Australian Mathematical Society Medal (1983).

He has held visiting positions in various institutions, including the Mittag-Leffler Institute (Sweden), Courant Institute of Mathematical Sciences (New York), University of Adelaide (South Australia), Institute for Advanced Study (Princeton), and Eidgenössische Technische Hochschule (Zürich).

His major research interests are geometric measure theory and partial differential equations.