1993 Steele Prizes

Three Leroy P. Steele Prizes were awarded at the International Joint Mathematics Meetings in Vancouver, British Columbia.

These prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein and are endowed under the terms of a bequest from Leroy P. Steele.

Three Steele Prizes are awarded each year: one for expository mathematical writing, one for a research paper of fundamental and lasting importance, and one in recognition of cumulative influence extending over a career, including the education of doctoral students. The current award is $4,000 in each of these categories.

The recipients of the Steele Prizes for 1993 are WALTER RUDIN for the expository award, GEORGE DANIEL MOSTOW for research work of fundamental importance, and EUGENE B. DYNKIN for the career award. The prizes were presented at the AMS-CMS-MAA opening banquet on August 15, 1993, at the International Joint Mathematics Meetings in Vancouver.

The Steele Prizes are awarded by the Council of the Society acting through a selection committee whose members at the time of these selections were Eugenio Calabi, Sylvain E. Cappell, Vaughan F. R. Jones, Harry Kesten, Joseph J. Kohn, Robert P. Langlands, Paul Rabinowitz, Jane Cronin Scanlon, and Jean E. Taylor.

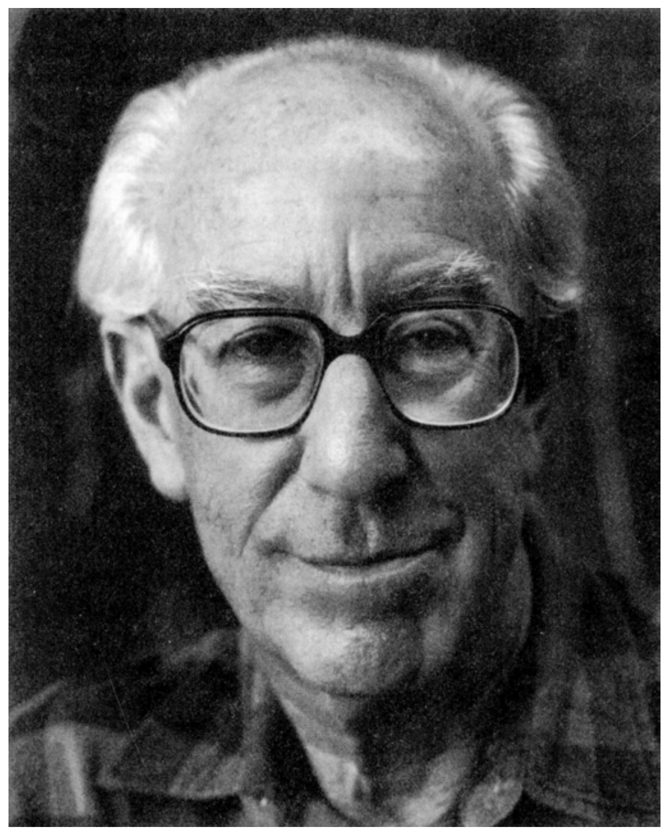
The text that follows contains the committee's citations for each award, the recipients' responses, and a brief biographical sketch of each recipient. The biographical sketches were written by the recipients or were based on information provided by them.

Expository Writing

Walter Rudin

Citation

To Walter Rudin for his books, in particular the classics *Principles of Mathematical Analysis* and *Real and Complex Analysis*. These books have served as the introduction to analysis to several generations of students both in the United States and abroad. They have set a standard for what undergraduate mathematics majors and beginning graduate students should know of analysis. The books are characterized by Rudin's personal style of elegance, clarity, and brevity. While giving full treatments of many important subjects, these books have presented their wealth of material to students in an integrated, coherent, and unified development. In writing these textbooks Rudin exhibits the same care, depth, and originality that can be found in his high-quality research monographs.



Walter Rudin

Biographical Note

Rudin was born in Vienna in 1921. After receiving his Ph.D. from Duke University in 1949, Rudin's academic career has included teaching at MIT (Moore Instructor, 1950–1952), the University of Rochester (1952–1959), and the University of Wisconsin, Madison (1959–1991) where he is currently Professor Emeritus. Rudin was a Sloan Fellow

(1956–1960). His research interests include mathematical analysis, especially abstract harmonic analysis, Fourier series, and holomorphic functions of one and several variables.

Response

One of my first duties as a Moore Instructor was to teach an Advanced Calculus course in which a certain list of topics was to be covered. When I said, "But there is no good book which contains all of this," Ted Martin, who was department chairman and also a consulting editor for McGraw-Hill, answered, "Why don't you write one?" So I did. *Principles of Mathematical Analysis* was published in 1953 and, to my amazement, is still going strong forty years later. I very much wanted to present a beautiful and important part of mathematics in a straightforward, logical, well-organized way, with complete and concise proofs. Constructing the book gave me real pleasure. That it turned out to be so successful is, of course, a welcome additional bonus.

Real and Complex Analysis was harder to write. I started it because the traditional Real Variables and Complex Variables courses were usually so structured that neither of these areas even acknowledged the existence of the other. The resulting mathematical version of apartheid gave students quite a misleading impression of what really goes on in analysis. That the book was so well received by so many mathematicians made it all worthwhile.

I thank the AMS for honoring my work with this Steele Prize.

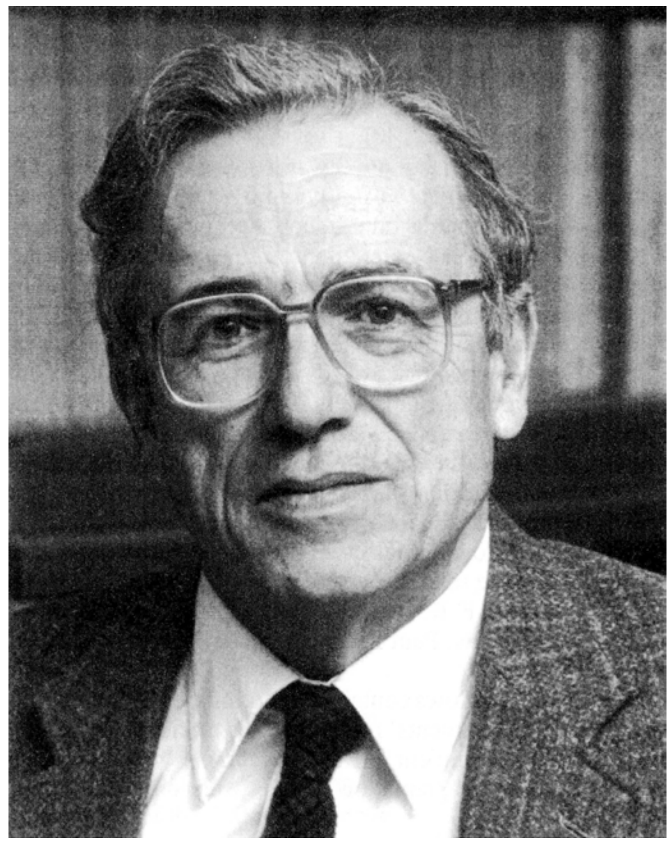
Fundamental Paper

George Daniel Mostow

Citation

To George Daniel Mostow for his monograph on strong rigidity. The Strong Rigidity Theorems of Mostow proven in his *Annals of Math. Studies*, vol. 78 (1973) are central and landmark achievements in modern mathematics. Previously there had been the local rigidity results of Selberg, Calabi-Vesentini, and Weil. These assert that if $\Gamma \leq G$ is a lattice (that is, a finite co-volume discrete subgroup) of a semisimple Lie group G which is (essentially) not $SL_2(R)$, then all local deformations of Γ arise from conjugations by elements of G; that is, there are no deformations.

Mostow's *Annals Study* (and his paper in I.H.E.S. Publications, 1967) changed the subject by proving the first global results. First, in the I.H.E.S. paper he showed that if G is the isometry group of hyperbolic n-space $n \geq 3$, Γ_1 and Γ_2 lattices in G and $\Theta\colon \Gamma_1 \to \Gamma_2$ is a group isomorphism of Γ_1 to Γ_2, then they are conjugate in G. In this paper Mostow introduces the crucial idea of extending equivariant maps (which are defined via Θ) to the boundary of hyperbolic space. He then uses and develops creatively the theory of quasiconformal maps and ergodic theory to construct the conjugation. In his *Annals of Math. Study* Mostow proves the strong rigidity theorem for all semisimple groups G (not $SL_2(\mathbf{R})$). The other cases of real rank 1 are dealt with by techniques similar to the hyperbolic space case but are by no means easy. Indeed, his treatment of the complex, quaternionic, and Cayley hyperbolic spaces is an excellent source for the study of these spaces. The higher rank groups are also dealt with by extending maps to the boundary but also making extensive use of the rich combinatorial structure of the boundary (e.g., Tits buildings) to construct the conjugation. Besides the greatness of these results, these papers are beautifully written. The reader not only learns the proofs but gains many other insights along the way. A number of fundamental concepts which were central in other new directions (pseudo-isometry, quasiconformal mapping over division algebras, etc.) were introduced in this paper.



George Daniel Mostow

Improvements of Mostow's methods have led to the proof of many other results (super-rigidity for lattices and for cocycles for group actions on measure spaces, strong rigidity for discrete subgroups more general than lattices, etc.). Moreover, the strong rigidity theorems are some of the deepest and most important results used regularly by topologists working with hyperbolic 3-manifolds for they assert that the fundamental group of such a (say, compact) manifold determines the hyperbolic structure uniquely. Sullivan gave important extensions of Mostow's rigidity and techniques to Kleinian groups in $SL_2(C)$. The most striking development beyond (and partly from) Mostow's papers is that of Margulis.

He proved "super-rigidity" in rank ≥ 2. In particular he showed that all lattices in higher rank groups come from arithmetic constructions.

A geometric development emerging from Mostow's papers was achieved by Siu (1980). He gave a new proof of Mostow's strong rigidity for certain hermitian symmetric spaces using harmonic maps. In this way the rigidity could be stated and proved in the geometric context of Kähler manifolds (with certain negative curvature assumptions). Recently the subject took a big step forward when Corlette, using Siu's methods but applied to quaternionic hyperbolic space, proved archimedian superrigidity for these real rank 1 lattices. This was an unexpected result. It was followed by Gromov and Schoen's recent proof of arithmeticity for such lattices.

It is clear that the subject initiated by Mostow is still very much alive and exciting. To quote M. Gromov (see his paper "Asymptotic invariants of infinite groups", p. 11), "The hyperbolic geometry took a new turn in 1968 when G.D. Mostow discovered his amazing asymptotic proof of the rigidity of lattices in $0(n, 1)$." These papers of Mostow merit this award both in their own right as foundational achievements and as the fountainhead of this great stream of mathematics.

Biographical Note

Mostow was born in Boston in 1923 and received his Ph.D. from Harvard in 1948. He began his academic career as an instructor of mathematics at Princeton (1947–1948) and was a member at the Institute for Advanced Study (1947–1949). He was an assistant professor at Syracuse University (1949–1952), then moved to Johns Hopkins (1952–1961), where he advanced to professor. Mostow is currently at Yale University, where he has served as department chair (1971–1974), the James E. English Professor (1963–1983), and Henry Ford II Professor of Mathematics (1983–).

During his 44-year membership in the Society Mostow has served admirably in many capacities, most notably as president (1987–1989). He was a Guggenheim Fellow (1958) and is a member of both the National Academy of Sciences and the American Academy of Arts and Sciences. He served on the Executive Committee of the International Mathematical Union (1983–1987), the Board of Trustees of the Institute for Advanced Study (1982–1992), and the Scientific Advisory Council of the Mathematical Sciences Research Institute (1988–1991).

Response

The honor that the American Mathematical Society bestows upon me is the most meaningful professional reward to which I aspire. I am deeply grateful.

The work that has been cited brought me great pleasure as it evolved. And on this occasion it is appropriate to share some of that experience with you and to add some historical remarks. My point of view emerged from trying to understand the deformation results of Selberg, Calabi-Vesentini, and Weil as a geometric phenomenon. Thereby, I was led to look at the infinitely far regions of the symmetric spaces associated with the relevant groups—that is, at their boundaries. In the *Proceedings of the 1965 Boulder Summer Institute on Algebraic Groups and Discrete Subgroups*, I published an intermediate result: if a lattice isomorphism induces a smooth mapping of the corresponding boundaries, then the lattice isomorphism extends to a group isomorphism. Curiously, the proof was highly algebraic, involving case-by-case checking of root diagrams.

This result thus posed the challenge: how to prove that the mapping of a symmetric space induced by a lattice isomorphism extends to a smooth mapping at the boundary? I was stuck at that point until one day I inquired of my next door office neighbor, Tsuneo Tamagawa, if he had ever encountered such an extension phenomenon. "Why, yes," replied Tamagawa, "I had a friend in Japan, A. Mori, who studied such a phenomenon in the plane for mappings that he called quasiconformal." That was the first time I heard the word "quasiconformal".

After absorbing the literature on quasiconformal mappings, I found that the 1962 paper of Fred Gehring on quasiconformal mappings in three space allowed me to prove the existence of the extension for real hyperbolic n-space. But still lacking was the reason why the boundary mapping had to be smooth. The reason for that hit me one day at the corner of Whalley Avenue and Fitch Street in New Haven as I waited in my car for the traffic light to turn green.

The idea was to exploit the ergodic action of the group of homotheties in the space of all lattices in this group. Thereby one could show for the case of real hyperbolic space that the boundary map was not only smooth but even Moebius. Over the ensuing few years, in combination with other ideas, a corresponding result was obtained for every group except $SL_2(\mathbf{R})$, in which the rigidity phenomenon does not occur.

The pleasurable excitement of working that out was its own reward. The subsequent contributions to rigidity of Prasad, Margulis, Sullivan, Zimmer, Siu, Mok, Pansu, Corlette, Gromov-Schoen added to the pleasure. The topological results of Thurston and Farrell-Jones added even more. Your award today makes my cup run over.

Career Award

Eugene B. Dynkin

Citation

Eugene B. Dynkin has made major contributions to the theory of Lie algebras and to probability theory. Dynkin's most famous contribution to the theory of Lie algebras was his use of the "Coxeter-Dynkin diagrams" to describe and classify the Cartan matrices of semisimple Lie algebras. This work was done while Dynkin was still a student at Moscow University.

Even though Dynkin has directed many Ph.D. theses in Lie group/algebra theory, most of his career has been devoted to probability theory. Dynkin has laid much of the foundations of the general theory of Markov processes as we know it today. His books (*Foundations of the Theory of Markov Processes*, Moscow 1959, translation published by Pergamon 1960; and *Markov Processes*, vols. I and II, Moscow, 1963, translation published by Springer-Verlag, 1963) have had a tremendous influence. He formulated and proved the strong Markov property (in 1956, together with his student Yushkevitch; this was done almost at the same time and independently of Hunt's introduction of the strong Markov property). Dynkin proved the measurability of certain hitting times (again almost at the same time as Hunt and independently of him). He developed the semigroup theory of Markov processes and characterized Markov processes by the generator of their semigroup. He also showed the usefulness of what is now known as "Dynkin's formula". This formula, which expresses expectations of functionals of the Markov process as an integral involving its generator, has become a standard and indispensable tool which is still used all the time. Dynkin further studied such topics as excessive functions, Martin boundary, additive functionals, entrance and exit laws, random time change, control theory, and mathematical economics.

Eugene B. Dynkin

Around 1980 Dynkin interpreted and vastly generalized an identity which had first come up in the context of quantum field theory. In his hands it became a remarkable relation between occupation times of a Markov process and a related Gaussian random field. This identity has led to many deep studies, by Dynkin himself as well as a host of others, of the properties of local times of Markov processes as well as to the detailed study of multiple points or self-intersections of Brownian motion.

In the last few years Dynkin has obtained exciting results in the theory of "super processes". This is a class of measure-valued Markov processes, which in many cases can be constructed as a suitable scaled limit of branching processes. These processes can be used to give probabilistic solutions to certain nonlinear PDE's in a way which is analogous to the classical solution of the Dirichlet problem by means of Brownian motion. Dynkin has used this to relate analytic properties of solutions of such PDE's (e.g., removability of singularities) to probabilistic properties of super processes.

Even though Dynkin has dealt with quite concrete probability problems, one of his strengths is his ability to build general theories and an apparatus to answer broad questions (e.g., characterize a certain natural class of additive functionals of a Markov process, or find all super processes with a branching property).

The list of Dynkin's Ph.D. students in Moscow is a "Who's Who" in Russian probability theory. In Moscow he has been extremely active in a special high school for gifted students in mathematics. He has also had many Ph.D. students since he immigrated to the U.S. He has been invited several times to speak at the International Congress of Mathematicians, and he is a member of the National Academy of Sciences.

This career award is in recognition of Dynkin's foundational contributions to two areas of mathematics over a long period and his production of outstanding research students in both countries to whose mathematical life he contributed so richly.

Biographical Note

Dynkin was born in Leningrad on May 11, 1924. After receiving his Ph.D. from Moscow University in 1948 he ultimately became professor at the same institution and held that position until 1968. He was a senior research fellow at the Academy of Sciences, Moscow (1968–1976). Since 1977 he has been a professor of mathematics at Cornell University.

He received the Prize of the Moscow Mathematical Society (1951) and also served on the council and as vice-president of that organization. He is a member of the National Academy of Sciences; the American Academy of Arts and Sciences; the Institute of Mathematical Statistics; and the Bernoulli Society for Mathematics, Statistics, and Probability. His research interests include Lie theory and probability theory, stochastic processes, optimal control, probabilistic models of economic growth and equilibrium, and sufficient statistics.

Response

I feel deeply honored to have been awarded a Steele Prize by the American Mathematical Society.

I came to the United States from the Soviet Union in 1977. Since then I have taught at Cornell University, a great center in probability theory. I found here kind and friendly colleagues; gorgeous scenery of forests, lakes, and waterfalls; and a few bright graduate students with whom I have started a seminar of the Moscow type. The most exciting was a new feeling of freedom and independence of big and little bosses—something which I never enjoyed in my previous life.

I had a great advantage to spend my formative years in an extremely stimulating atmosphere of the Moscow mathematical school; to be a pupil of such great mathematicians as A. N. Kolmogorov, I. M. Gelfand, I. F. Petrovskii; and later to enjoy interaction with brilliant young mathematicians who started their research at my seminars, among them are (in chronological order) R. L. Dobrushin, F. I. Karpelevich, A. V. Skorokhod, F. A. Berezin, A. A. Yushkevich, I. V. Girsanov, Ya. G. Sinai, A. L. Onishchik, R. Z. Khasminskii, V. A. Volkonskii, M. I. Freidlin, M. G. Shur, A. A. Kirillov, A. D. Wentzell, E. B. Vinberg, V. N. Tutubalin, N. V. Krylov, S. A. Molchanov, M. B. Malyutov, G. A. Margulis, S. E. Kuznetsov, I. V. Evstigneev, M. I. Taksar. On the other hand, the life was hard under the oppression of a totalitarian regime. I was eleven when my family was exiled from Leningrad to Kasakhstan and I was thirteen when my father, one of millions of Stalin's victims, disappeared in the Gulag. It was almost a miracle that I was admitted (at the age of sixteen) to Moscow University. Every step in my professional career was difficult because the fate of my father, in combination with my Jewish origin, made me permanently undesirable for the party authorities at the university. Only special efforts by A. N. Kolmogorov, who put, more than once, his influence at stake, made it possible for me to progress through the graduate school to a teaching position at Moscow University.

My poor vision made me unfit for the draft, so during the war I continued my undergraduate study. The motherly attitude of S. A. Yanovskaya (a professor at Moscow University) softened the hardship of the most difficult years and allowed me to concentrate on mathematics. (Her support and encouragement are gratefully remembered by a number of other mathematicians and logicians.) I worked at Gelfand's seminar on Lie groups and at Kolmogorov's seminar on Markov chains. Both were important for my development as a research mathematician.

Gelfand requested that I review the H. Weyl-Van der Waerden papers on semisimple Lie groups. I found them very difficult to read, and I tried to find my own ways. It came to my mind that there is a natural way to select a set of generators for a semisimple Lie algebra by using simple roots (i.e., roots which cannot be represented as a sum of two positive roots). Since the angle between any two simple roots can be equal only to $\frac{\pi}{2}$, $\frac{2\pi}{3}$, $\frac{3\pi}{4}$, $\frac{5\pi}{6}$, a system of simple roots can be represented by a simple diagram. An article was submitted to *Matematicheskii Sbornik* in October 1944. Only a few years later, when recent literature from the West reached Moscow, I discovered that similar diagrams have been used by Coxeter for describing crystallographic groups.

Lie algebras remained my main field for about ten years. I used simple roots and the corresponding diagrams to investigate automorphisms and semisimple subalgebras of Lie algebras. After coming to the West I learned that these results have been used by a number of physicists to deal with elementary particles. (I was flattered when Yuval Ne'eman told me that his work on this subject was based on my dissertation, which he had read in one of the London libraries.)

A few times I was lucky to find a new approach which simplified an important theory. One of them is related to the celebrated Campbell-Hausdorff theorem claiming that the formal series $\log(e^X e^Y)$ can be expressed in terms of commutators. In 1947 I found a simple explicit expression: it is sufficient to replace all multiplications by commutators and then to divide each monomial by its degree. My debut in probability theory was made about the same time as my debut in algebra. In 1945 I solved a problem on Markov chains posed by Kolmogorov. In 1948 I became an assistant professor at Kolmogorov's Probability Chair, and I continued to work on probability and statistics parallel to algebra (the results on exponential families and sufficient statistics are probably the best known). Beginning in the middle of the 1950s, I switched almost completely to work on stochastic calculus, especially on Markov processes. Some detail about this part of my research can be found in the citation.

Contacts with my friends and colleagues in the Soviet Union were severed when I moved to the United States. Now, after the end of the Cold War, they flourish again. We started a joint project with S. E. Kuznetsov and A. V. Skorokhod on the structure of branching measure-valued processes. During my recent visits to Moscow we had two very emotional reunions with a large group of scientists who were my pupils in the 1960s in a special Moscow high school for mathematically gifted students.