Robert W. Thomason (1952–1995)

Charles A. Weibel

Like many of his colleagues, Bob Thomason hated to waste energy on trivial matters like fashion. He made the decision early in life to dress only in black clothing, thus simplifying that portion of his life. With his pointed goatee, he looked like a beat poet to outsiders, but mathematicians knew him as one of the greatest talents of his generation. Few have had the simultaneous grasp of topology, algebraic geometry, and K-theory that Thomason did.

Bob had diabetes and always had to strictly control what he ate. This made going to restaurants with Bob an awkward affair, because he would not eat something until he was sure it had no nutritional content. Late in October 1995, just before his forty-third birthday, he went into diabetic shock and died in his apartment in Paris. We are all saddened by his passing.

Here is an overview of Thomason's career. For simplicity I have focused upon what I think are his three major results. A retrospective article, describing some of his mathematical contributions in more detail, will appear in a future issue of the *Bulletin of the AMS*.

Robert Wayne Thomason was born in Tulsa, Oklahoma, on November 5, 1952. Attracted to Michigan State University by a flexible undergraduate Honors Mathematics program, he spent two years there (1971–73). During his second year at MSU he published his first paper [6], in point-set topology. He then spent 1973–77 as a graduate student in the Princeton University mathematics department, writing his Ph.D. dissertation [7] under the direction of John Moore.

Charles A. Weibel is professor of mathematics at Rutgers University. His e-mail address is `weibel@math.rutgers.edu`.

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His thesis [7] describes and analyzes a simple but fundamental construction in category theory: the "canonical cofibered category" associated to any diagram D of (small) categories. Since the geometric realization of a small category is a topological space, we obtain a corresponding diagram $|D|$ of topological spaces. The main result in his thesis is that the geometric realization of the canonical cofibered category of D is the homotopy colimit of the diagram $|D|$ of spaces. Because of the elegance and thoroughness of his analysis, this construction has become a basic tool used routinely by topologists.

As he was graduating in June 1977, Thomason discovered the first of his major results: a proof that all infinite loop space machines produce equivalent output. In order to straighten out the technical details of his insight, he immediately enlisted the aid of J. Peter May. In a collaboration May recalls as "delightful interaction", they reduced Bob's argument to a characterization of infinite loop space machines by just one axiom: the "group completion" axiom; see [4].

A variation on this theme occurs in a 1979 paper by Thomason, showing that all one-fold delooping machines also produce equivalent output. Recently in [10], Thomason showed that every infinite loop space, and every -1-connected spectrum, arises from an infinite loop space machine applied to a symmetric monoidal category.

Thomason then went to M.I.T. as a Moore Instructor (1977–79). During this time he developed the ideas in his thesis into a series of papers studing the homotopy theory of categories, especially symmetric monoidal categories. In one paper he proved the reassuring result that the abstract homotopy theory of categories does not depend upon a passage to geometric realizations, because fibrations, cofibrations, etc., of categories exist as part of a "closed model structure". In two detailed papers he constructed mapping cones, mapping cylinders, and other homotopy colimits within the category of small symmetric monoidal categories and showed that infinite loop space machines send these constructions to the appropriate homotopy colimits of spectra. Thomason's homotopy colimit constructions have since been central to the work of several people.

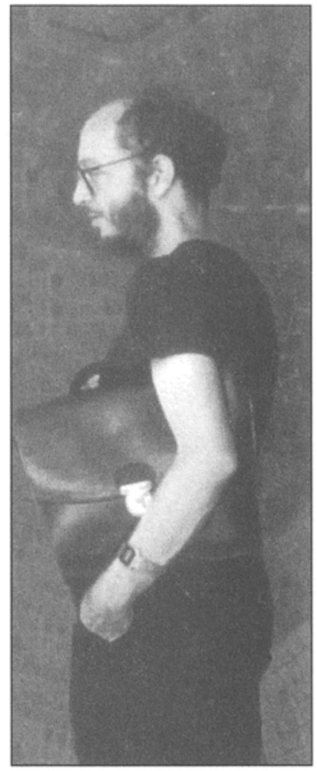
In 1979 Thomason went to the University of Chicago to begin a three-year appointment as a Dickson Assistant Professor. There he developed the notion of cohomological descent for spectra, parallel to the notion of hypercohomology in homological algebra; it has since become a basic notion in algebraic K-theory. In a paper humorously entitled "*Beware the phony multiplication on Quillen's* $\mathcal{A}^{-1}\mathcal{A}$" [8], he exposed a subtle but lethal flaw in a putative construction for the ring structure on the K-groups of commutative rings. He also began a four-year effort to settle the Quillen-Lichtenbaum conjectures, which connect algebraic K-theory to étale cohomology. After the proof of an early partial result collapsed in 1980, Thomason began to feel uncomfortable about the skepticism expressed by others. Perceiving this as persecution, he resigned from his position at Chicago in June 1980.

For the next two years Thomason held an irregular appointment at M.I.T., and then spent a year as a Member at the Institute for Advanced Study. During this period, he finished his opus [9] on the Quillen-Lichtenbaum Conjecture, which contains his second major result. Roughly, it states that the groups K_n can be calculated in terms of étale cohomology for large n, using a formula due to Dwyer and Friedlander. This result established the first half of the Quillen-Lichtenbaum Conjecture. The final part of the conjecture, which pins down the values of n, is currently the focus of intensive work in Motivic Cohomology.

As part of his four-year effort culminating with [9] Thomason wrote five other papers, including the four-author paper [1] with Dwyer, Friedlander, and Snaith. Two nice applications of his descent machinery appear in other papers from that era: a proof of $\mathbb{Q}_\ell$-adic cohomological purity and a rigidity theorem for K-theory with Gillet (their proof was contemporaneous with Gabber's).

In 1983 Thomason joined the mathematics faculty of Johns Hopkins University in Baltimore, where he stayed for six years. During this time he supervised two Ph.D. dissertations, by Masana Harada [3] and Dongyuan Yao [12]. During 1983–86 he wrote a series of papers about equivariant algebraic K-theory.

Starting in 1985, he mounted a sustained three-year attack upon the problems left over from Grothendieck's opus [5], especially an analysis of how the K-theory of a scheme depends upon its derived category of vector bundles and how to describe the effect of localization upon K-theory. His successful solution of this problem in 1988 [11] forms his third major result.



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The story of this three-year attack reveals much about Thomason's methods. The first step in this program, which he discovered in 1985, was a "cofinality theorem" for Waldhausen K-theory. Today his cofinality theorem is viewed as one of the fundamental results in K-theory. The next step was taken during the calendar year 1987, which he spent at Rutgers University as part of a Sloan Fellowship (1985–87). That year he put everything into place except for one step: extending perfect complexes from an open subscheme to the entire scheme. On January 22, 1988, he had a dream in which his recently deceased friend Thomas Trobaugh told him how to solve the final step: use "the direct limit characterization of perfect complexes." Awakening with a start, he worked out the argument for the missing step. In gratitude he listed his friend as a coauthor of the resulting paper [11].

In recognition of the importance of his work in [9] and [11], Thomason was chosen to give an address at the Kyoto International Congress of Mathematicians in 1990.

In October 1989 Thomason made what turned out to be his final career move, to Paris. He accepted a position in the C.N.R.S., attached to Max Karoubi's laboratory URA 212 at the University of Paris VII. While there, he helped Karoubi, Kahn, and Kassel run the monthly Paris K-theory seminar and wrote six more papers. He remained in this position in Paris until his untimely death last October.

References

[1] W. DWYER, E. FRIEDLANDER, V. SNAITH, and R. W. THOMASON, *Algebraic K-theory eventually surjects onto topological K-theory*, Invent. Math. **66** (1982), 481–491. MR **84i**:14015

[2] H. GILLET and R. W. THOMASON, *The K-theory of strict hensel local rings and a theorem of Suslin*, J. Pure Appl. Algebra **34** (1984), 241–254. MR **86e**:18014

[3] M. HARADA, *A proof of the Riemann-Roch theorem*, Ph.D. thesis, Johns Hopkins University, Baltimore, 1987.

[4] J. P. MAY and R.W. THOMASON, *The uniqueness of infinite loop space machines*, Topology **17** (1978), 205–224. MR **80g**:55015

[5] P. BERTHELOT, A. GROTHENDIECK, and L. ILLUSIE, *Théorie des Intersection et Théorème de Riemann-Roch (SGA6)*, Lecture Notes in Math., vol. 225, Springer-Verlag, 1971. **MR 50**:7133

[6] R.W. THOMASON, *A note on spaces with normal product with some compact space*, Proc. Amer. Math. Soc. **44** (1974), 509–510. MR **51**:4165

[7] ——, *Homotopy colimits in the category of small categories*, Math. Proc. Cambridge Philos. Soc. **85** (1979), 91–109. MR **80b**:18015

[8] ——, *Beware the phony multiplication on Quillen's* $\mathcal{A}^{-1}\mathcal{A}$, Proc. Amer. Math. Soc. **80** (1980), 569–573. MR **81k**:18010

[9] ——, *Algebraic K-theory and étale cohomology*, Ann. Sci. École Norm. Sup. (4) **18** (1985), 437–552; erratum **22** (1989), 675–677. MR **87k**:14016, MR **91j**:14013

[10] ——, *Symmetric monoidal categories model all connnective spectra*, Theory Appl. Categories **1** (1995), 78–118; (electronic journal http://www.tac.mta.ca/tac/).

[11] R. W. THOMASON and T. TROBAUGH, *Higher algebraic K-theory of schemes and of derived categories*, The Grothendieck Festschrift III, Progress in Math., vol. 88, Birkhäuser, 1990, pp. 247–435. MR **92f**:19001

[12] D. YAO, *Higher algebraic K-theory of admissible abelian categories and localization theorems*, Ph.D. thesis, Johns Hopkins University, Baltimore, 1990.