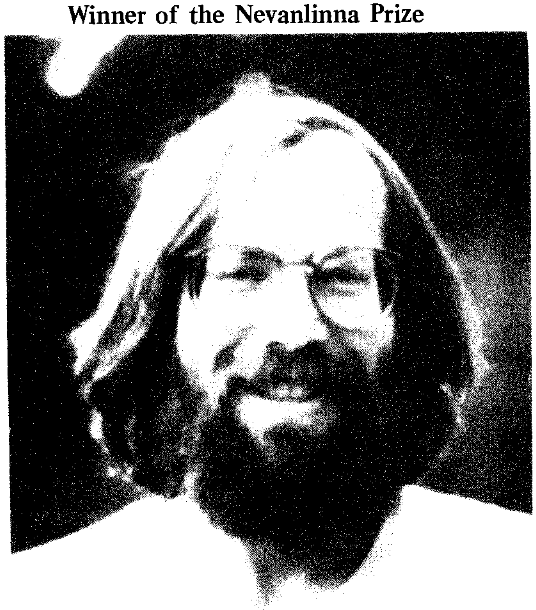
U.S. Recipients of Fields Medals

William P. Thurston

Shing-Tung Yau

Attempts to obtain a photograph of the third recipient of a Fields Medal, Alain Connes of France, for publication here were not successful. We hope to print one in the next issue of the *Notices* – Editors

Winner of the Nevanlinna Prize



Robert E. Tarjan

Fields Medals and Nevanlinna Prize

At the meeting of the General Assembly of the International Mathematical Union in Warsaw early in August, the names of recipients of the Fields Medals and the new Nevanlinna Prize in Information Science were announced.

Fields Medals are to be presented to ALAIN CONNES of the Institut des Hautes Études Scientifiques, WILLIAM P. THURSTON of Princeton University, and SHING-TUNG YAU of the Institute for Advanced Study and the University of California, San Diego. ROBERT E. TARJAN of Bell Laboratories, Murray Hill, is to be the first recipient of the Nevanlinna Prize. Present plans call for the awards to be made at the International Congress of Mathematicians, which is currently scheduled for August 1983 in Warsaw. At that time, lectures are to be presented on the work and accomplishments of each of the recipients.

ALAIN CONNES was born April 1, 1947 in Darguignan, France. In June 1973 he received a Doctorat d'État from University of Paris VI for a thesis written under the supervision of Jacques Dixmier. Connes was affiliated with the Centre National de la Recherche Scientifique from 1970 to 1974, he was at Queen's University, Ontario, Canada in 1974-1975 and held a faculty appointment at University of Paris VI from 1975 to 1977. In 1978-1979 he was a visiting member of the Institute for Advanced Study, Princeton, and since 1979 he has been a Professor at the Institut des Hautes Études Scientifiques, Bures-sur-Yvette. In 1975 he was awarded the Prix Aimeé Berthé, in 1976 the Prix Pecot-Vimont, in 1977 the Médaille d'Argent of the CNRS, in 1980 the Prix Ampère, and in 1981 the Prix de Electricité de France. In 1980 he was elected a Corresponding Member of the Academie des Sciences, in 1982 he and André Weil were the only mathematicians elected to full membership in the Academie, bringing the number of mathematicians who are full members of the Academie up to thirteen.

Among honors earned by WILLIAM P. THURSTON are the Society's Veblen Prize in Geometry in 1976 and the National Science Foundation's Waterman Award in 1979. The August 1979 issue of the *Notices* contains biographical and bibliographical information, as well as descriptions of Professor Thurston's work by H. Blaine Lawson and Dennis Sullivan (pages 293 to 296).

SHING-TUNG YAU received the Society's Veblen Prize in 1981. See the February 1981 (pages 162 to 164) issue of the *Notices* for biographical and bibliographic information, as well as the text of the citation of the Veblen Prize Committee outlining the work for which he received that award.

ROBERT ENDRE TARJAN was born April 30, 1948 in Pomona, California. He was educated at the California Institute of Technology (B.S. in Mathematics, 1969) and Stanford University (M.S. in 1971 and Ph.D. in 1972, both in Computer Science). He was assistant professor of computer science at Cornell University, 1972 to 1974, Miller Research Fellow at the University of California, Berkeley, 1973 to 1975, and assistant and associate professor of computer science at Stanford University, from 1974 to 1981. Since September 1980 he has been a member of the technical staff at Bell Laboratories, Murray Hill, and since September 1981 an adjunct professor at New York University.

Fields Medals are awarded by the International Mathematical Union on the occasion of an International Congress of Mathematicians. The awards were established in accordance with the will of Professor J. C. Fields of the University of Toronto. Professor Fields died in 1932 and the first awards were made at the Congress in Oslo in 1936.

The 1936 recipients were Lars V. Ahlfors and Jesse Douglas. Later recipients were Atle Selberg and Laurent Schwartz in 1950; Kunihiko Kodaira and Jean-Pierre Serre in 1954; Klaus Roth and René Thom in 1958; Lars Hörmander and John Milnor in 1962; Michael Atiyah, Paul J. Cohen, Alexander Grothendieck and Stephen Smale in 1966; Alan Baker, Heisuke Hironaka, Sergei Novikov and John G. Thompson in 1970; Enrico Bombieri and David Mumford in 1974; and Pierre Deligne, Charles Fefferman, Gregori Aleksandrovitch Margulis and Daniel Quillen in 1978.

The following essays describe some of the achievements of the present recipients of these awards.

The Work of Alain Connes

Calvin C. Moore

To place Alain Connes's fundamental and pioneering contributions to operator algebras in context, recall that von Neumann and Murray in the 1930s and 1940s were led by, among other things, the spectral theory of operators on Hilbert space, and by considerations of constructing mathematical models for quantum mechanical systems, to introduce what they called rings of operators—since renamed von Neumann algebras. These are weakly closed self adjoint algebras of operators on a Hilbert space, containing the identity operator. One of the main problems has been and remains the classification of these algebras as intrinsic algebraic and topological objects. One easily reduces this to the study of factor algebras—those with one dimensional center, or equivalently those which are simple in a well defined sense.

In their original papers, von Neumann and Murray introduced a type classification: type I algebras are those which turn out to have a Wedderburn type structure theory and they constitute the "expected" examples; for instance, the only factors of this kind are $B(H)$, the algebra of all bounded operators on a Hilbert space. The algebras of types II and III seemed rather more exotic and mysterious; for example one can have families of projections in a factor of type II whose generalized "dimensions" fill out an interval. The type II (and I) algebras are the ones possessing linear functionals with the formal properties of a trace. The factors of type II are either finite, called II_1 factors (these have an everywhere defined trace) or infinite, called II_∞ factors, where the trace is only densely defined. Type III factors, lacking such a trace, seemed especially intractible.

In a first attempt to classify factors not of type I, von Neumann and Murray introduced the notion of hyperfinite algebras—now called approximately finite algebras. These are the algebras that can be approximated by finite dimensional algebras in the sense that they are the weak closure of an ascending chain of finite dimensional algebras. This class of von Neumann algebras turns out to be of exceptional importance both for reasons internal to the subject and for applications.

Connes's thesis **[1]** was already a major, stunning breakthrough in the classification problem. Building on work of Powers, Araki and Woods, and Krieger, Connes introduced his S-invariant for factors—a subset of $[0,\infty]$. This provides a subdivision of the type III algebras into subclasses of type III_λ, $0 \le \lambda \le 1$, and provided great structural insight. He further showed how to obtain these algebras for $\lambda \neq 1$ from type II algebras and their automorphisms. Takesaki, using more general crossed products, then proved this in all cases. In all this work an absolutely key tool without which one cannot get started is the Tomita-Takesaki theory of modular automorphisms. One upshot of Connes's result is that classification in general comes down to classification of type II algebras and their (outer) automorphisms. Connes further realized that the type III algebra is approximately finite iff the type II algebra from which it is built is approximately finite.

Now one of the very early results of von Neumann and Murray was that, up to isomorphism, there is one and only one approximately finite algebra of type II_1. Connes consequently undertook in **[2]** an intensive study of the outer automorphisms of this algebra and of the associated II_∞ factor of infinite matrices over it. The amazing result is that up to conjugacy there are very few such automorphisms. With these results in hand, there remained one crucial point for the classification of approximately finite factors—whether the algebra of matrices over the unique II_1 approximately finite algebra is the only II_∞ approximately finite factor. This seemingly simple problem turns out to be enormously difficult, and Connes's work in this area culminated in the affirmative resolution of this problem in **[3]**. In the process he established more, including the equivalence of several other important conditions, including injectivity, with approximate finiteness, and of course achieved the complete classification of all approximately finite factors except for those of type III_1; it is conjectured but unproved that there is just one such algebra. One elegant formulation of this classification, coming from a combination of this work with that of Krieger, is that the infinite approximately finite factors are in one-to-one correspondence with ergodic flows up to conjugacy (the flow of weights of the factor), and also in turn with ergodic transformations up to orbit equivalence. This classification is one of great simplicity and elegance, and one that had hardly seemed possible a decade earlier.

Since completing this work, Connes has gone on to the very fruitful study of the connections between operator algebras, foliations, and index theorems. Associated with a compact foliated manifold together with a transverse measure, there is a natural von Neumann algebra. Connes shows that the kernel and cokernel of a differential operator that is tangential to the leaves of the foliation, and that is tangentially elliptic in the obvious sense, can be viewed as projections in this von Neumann algebra. If one further assumes that the transverse measure is invariant, this produces a trace on this algebra, making it type II (or type I in degenerate cases). In particular there now is a numerical index, and Connes obtains in **[4]** a beautifully simple formula for this index in terms of topological data from the symbol, in complete analogy with the Atiyah-Singer theorem. Connes has recently announced in **[5]** a far more general and powerful version of the theorem, freed from assumptions about invariant transverse measures, and formulated, as it should be, in terms of the C^* algebra associated to the foliation and its K-theory.

Taken altogether Connes's work in the last decade on operator algebras and its applications has transformed the subject and opened up entire new areas of research. In this short space we have discussed only some of the highlights of his many contributions.

[1] *Une classification des facteurs de type* III, Annales Scientifiques de l'École Normale Supérieure (4) **6** (1973), 133–152.

[2] *Outer conjugacy classes of automorphisms of factors*, Annales Scientifiques de l'École Normale Supérieure (4) **8** (1975), 383–419.

[3] *Classification of injective factors, cases* II_1, II_∞, III_λ, $\lambda \neq 1$, Annals of Mathematics (2) **104** (1976), 73–115.

[4] *Sur la théorie non-commutative de l'integration.* Lecture Notes in Mathematics **725** (1979), 19–143 (Springer-Verlag).

[5] *Théorème de l'indice pour les feuilletages* (with G. Skandalis), Comptes Rendus des Séances de l'Académie des Sciences. Série I. Mathématique. (Paris) **292** (18 May 1981), 871–876.

The Work of William P. Thurston

William Browder and W.-c. Hsiang

For half a century, the study of the topology of 2-dimensional manifolds has rested heavily on the *geometric* structures that can be introduced, e.g. as complex manifolds or Riemannian manifolds of constant curvature. Thurston's audacious idea is a 3-dimensional extension.

He considers eight basic kinds of geometries which might be introduced, based on certain homogenous spaces of Lie groups. The most interesting and useful of these is the Lobachevskian geometry, i.e. a space of constant negative curvature.

> **Conjecture of Thurston.** The interior of any compact 3-dimensional manifold is the union of submanifolds, each of which carries a geometric structure of one of these eight types.

Such a manifold Thurston calls "geometric".

Thurston has proved this conjecture for many wide classes of 3-manifolds such as Haken manifolds (which includes knot complements).

The existence of this geometric structure makes available a whole range of new techniques to study 3-manifolds. For example, for hyperbolic manifolds (constant negative curvature) the Mostow rigidity theorem says the isomorphism type of the fundamental group determines the manifold up to isometry. One may use this type of result to study the group of homeomorphisms of a geometric 3-manifold, by reducing to problems about isometries. He shows that on a Haken manifold M^3 satisfying an additional condition called "homotopically atoroidal," $\pi_o(\mathrm{Diff}\, M^3)$ (the set of isotopy classes of diffeomorphisms of S^3) is finite, and there is a splitting of groups $\pi_o(\mathrm{Diff}\, M^3) \to \mathrm{Diff}(M^3)$. A well-defined notion of volume may be used to play for 3-manifolds the role played by the Euler characteristic in 2-manifold theory.

Perhaps the most spectacular achievement of this program is the positive solution of the Smith conjecture: *Any periodic homeomorphism of S^3 fixing a simple closed curve is conjugate to a rotation.* This result uses Thurston's geometric method plus the equivariant loop theorem of Meeks and Yau [8].

The Thurston method in 3-manifolds is closely related to the theory of Kleinian groups and Teichmüller spaces, in which Thurston has introduced novel methods from the theory of foliations of considerable interest to specialists in this area.

Thurston's earlier work in foliation theory, for which he was awarded the 1976 Veblen Prize, included a spectrum of results ranging from new constructions realizing uncountably many values of the Godbillon-Vey invariant, extending by new geometrical techniques the Haefliger foliation theory to closed manifolds, calculating homology of classifying spaces, etc. One dramatic consequence: Any closed manifold of Euler characteristic 0 admits a codimension 1 foliation. Reeb had produced such a foliation for S^3 thirty years ago, while other odd dimensional spheres were given such foliations in the 1970s through work of Lawson, Durfee and Tamura.

[1] W. Thurston, *Three dimensional manifolds, Kleinian groups and hyperbolic geometry*, Bulletin of the AMS (New Series) **6** (1982), 357–381.

[2] A. Fathi, F. Laudenbach, V. Poenaru, et al, *Travaux de Thurston sur les surfaces*, Astérisque, no. 66, 67, Société Mathématique de France, 1979.

[3] W. Thurston, *The geometry and topology of three-manifolds*, Princeton University Press (in preparation).

[4] W. Thurston, *The theory of foliations of codimension greater than one*, Commentarii Mathematici Helvetica **49** (1974), 214–231.

[5] W. Thurston, *existence of codimension-one foliations*, Annals of Mathematics (2) **104** (1976), 249–268.

[6] W. Thurston, *Foliations and groups of diffeomorphisms*, Bulletin of the AMS **80** (1974), 304–307.

[7] W. Thurston, *Noncobordant foliations of S^3*, Bulletin of the AMS **78** (1972), 511–514.

[8] W. H. Meeks III and S.-T. Yau, *Topology of three-dimensional manifolds and the embedding problems in minimal surface theory*, Annals of Mathematics (2) **112** (1980), 441–484.

The Work of Shing-Tung Yau

Louis Nirenberg

S.-T. Yau has done extremely deep and powerful work in differential geometry and partial differential equations. He is an analyst's geometer (or geometer's analyst) with enormous technical power and insight. He has cracked problems on which progress had been stopped for years. A few of his achievements:

1. **The Calabi Conjecture.** This comes from algebraic geometry and involves proving the existence of a Kähler metric, on a compact Kähler manifold, having a prescribed volume form. The

analytic problem is that of proving the existence of a solution of a highly nonlinear (complex Monge-Ampère) elliptic equation. Yau's solution is classical in spirit, via *a priori* estimates. His derivation of the estimates is a *tour de force* and the applications in algebraic geometry are beautiful.

2. **Positive Mass Conjecture**, from general relativity theory. This involves global Riemannian geometry and nonlinear elliptic partial differential equations. In joint work with R. Schoen, Yau settled this problem. The solution involves construction of global minimal surfaces and a study of their stability and behaviour near infinity. The work is very technical and highly ingenious.

3. **Real and complex Monge-Ampère equations.** In joint work with S. Y. Cheng, Yau gave a complete proof of the higher dimensional Minkowski problem (based partly on work of A. V. Pogorelov). They also constructed Einstein manifolds with given Ricci curvature in pseudoconvex domains in $\mathbf{C}^n$. Great technical power, and estimates, are involved here.

4. In a series of papers, some with P. Li, Yau obtained deep estimates on the first eigenvalue, as well as others, for the Laplace operator on a compact manifold (or manifold with boundary) under various hypotheses on the Ricci curvature—but in terms of little geometric information about the manifold. The arguments used are highly varied and most ingenious.

5. Using minimal surfaces Y. T. Siu and Yau gave a beautiful proof of the Frankel conjecture that a complete simply connected Kähler manifold with positive holomorphic bisectional curvature is biholomorphic to complex projective space. This was proved earlier by S. Mori with the aid of algebraic geometry.

6. With W. H. Meeks III, Yau used topological methods of 3-manifolds to settle some old problems in the classical theory of minimal surfaces. Conversely, they used minimal surface theory to obtain results in 3-dimensional topology: Dehn's lemma and equivariant versions of the loop and sphere theorems.

A remarkable aspect of some of Yau's work is his use of minimal surfaces in the way that, previously, people had used geodesics. This usually involves extremely difficult technical problems—topological as well as analytic. In addition to great technical power and depth, his work shows remarkable courage.

The Work of Robert Tarjan

Jacob T. Schwartz

Robert Tarjan is a leading designer of combinatorial, and especially graph-theoretic, algorithms. His work is distinguished for uniting combinatorial insight and ingenious, economical data structures to produce combinatorial procedures of remarkable elegance and efficiency. Among these is the striking method (developed jointly with Hopcroft) for testing a graph for planarity (and constructing a planar imbedding if one exists) in time linearly proportional to the number of edges in the graph, for finding node "dominators" in rooted directed graphs, and for analyzing the loop structure of certain classes of directed graphs which play an important role in the global structural analysis of computer programs. He and his collaborators have also applied high efficiency graph-theoretic techniques to problems important in numerical analysis, including determination of optimal elimination orders for inversion of sparse matrices, and of advantageous dissection orders for planar graphs.

Tarjan has also contributed new data structures useful in high-efficiency algorithm design, for example combinatorial tree structures which can be used to keep sorted lists in order while elements are efficiently inserted into and deleted from them, and has repeatedly used such structures to obtain striking and unsuspected improvements in the efficiency of algorithms designed by other combinatorialists. He has also supplied refined analyses of the performance of many such algorithms. Finally, he is the author of several very valuable studies of the fundamental question of the extent to which computation time rises as the memory available for carrying out a computation is progressively constricted. These techniques make use of a combinatorial "pebbling" technique of which Tarjan and his collaborators and students are principal developers.