Bôcher, Osgood, and the Ascendance of American Mathematics at Harvard

Steve Batterson

The year 1888 is notable to members of the American Mathematical Society (AMS) for the founding of their organization under the name The New York Mathematical Society. In this same year Maxime Bôcher received his A.B. in mathematics from Harvard. The university also awarded Bôcher a fellowship that enabled him to travel to Göttingen for graduate study. At the time, knowledgeable American mathematics students with means went to Germany to pursue a Ph.D. Opportunities for course work and thesis direction in the United States were vastly inferior. The country's only significant mathematical scholars were the nonacademically employed George William Hill, the part-time professor Simon Newcomb, and the reclusive scientist J. Willard Gibbs. Over the 1890–1894 interval just two American universities would confer more than two mathematics Ph.D.'s [1], and neither of these programs was on a favorable trajectory. Johns Hopkins was in a decline that had begun with the recent departure of J. J. Sylvester. Clark University, after a promising first three years, underwent devastating turmoil and lost many of its best staff [2], [3].

Yet by 1913 the American mathematical brand was appreciated in Europe. E. H. Moore, Maxime Bôcher, W. F. Osgood, Leonard Dickson, and G. D. Birkhoff were internationally respected mathematicians. Graduate programs at Chicago, Harvard, and Princeton offered European-level training that had been unavailable in the United States when Bôcher completed his undergraduate studies a quarter century earlier. The advancement on American campuses began in the early 1890s.

Most significant was the opening of the University of Chicago in 1892. Under the leadership of E. H. Moore, Chicago recruited European emigrés to implement a high-level mathematics curriculum [2]. A steady stream of talented American students thrived in the scholarly environment. Moore's Ph.D. students Dickson (1896), Oswald Veblen (1903), and Birkhoff (1907) would each go on to deliver plenary addresses to the International Congress of Mathematicians.

Compare the Chicago ascendance with contemporaneous developments at Harvard [4], [5], [2]. After obtaining their Ph.D.'s in Germany, Osgood and Bôcher became Harvard instructors in 1890 and 1891 respectively. None of their departmental colleagues were engaged in mathematical research. Together Bôcher and Osgood steadily changed the culture, publishing their scholarly work and invigorating the graduate program. Birkhoff joined the Harvard faculty in 1912 and then discovered his famous proof of Poincaré's Geometric Theorem. With Bôcher, Osgood, and Birkhoff, Harvard was the strongest department in the United States. Given the 1890 state of American mathematics, the rise of Harvard was remarkable, even if overshadowed by the more rapid advances at Chicago. This article traces these developments, focusing on the vital roles of Bôcher, Osgood, and the Harvard traveling fellowships.

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Mathematics at Harvard Prior to 1880

In 1636 Harvard became the first college to be established in the North American British colonies. To fulfill its mission of providing the educational essentials to prospective Puritan ministers, the curriculum featured Latin, Greek, Hebrew, rhetoric, and philosophy. The small presence of mathematics was restricted to some arithmetic and geometry in the final year. During Harvard's first century mathematics was often taught by minimally trained instructors who held the title of tutor [6], [7].

The year 1727 marked the endowment of the Hollis Professorship of Mathematics and Natural Philosophy. The first holder was Isaac Greenwood. Greenwood, being knowledgeable in Newton, substantially elevated the Harvard faculty's level of scientific competence. Unfortunately his tenure ended prematurely when he was dismissed over repeated incidents of drunkenness. Greenwood's successor was his former student John Winthrop.

Serving from 1738 to 1779, Winthrop covered the broad span of mathematics and the physical sciences. Harvard historian Samuel Morison characterized Winthrop as "the first important scientist or productive scholar in the teaching staff of Harvard College" [6, page 92]. Winthrop took the then-novel initiative of setting up an experimental physics laboratory. His lectures included the topic of electricity. Winthrop's astronomical observations of the solar system earned him membership in the Royal Society.

When the nineteenth century began, no American professors were doing mathematical research. At both Harvard and Yale, scholarship in the subject meant the production of textbooks. By this time mathematics was front loaded into the Harvard curriculum. Tutors handled arithmetic and geometry in the freshman year. Subsequent topics included algebra, logarithms, trigonometry, surveying, and spherical geometry.

In 1806 the Hollis chair was offered to Nathaniel Bowditch, the author of an important handbook on navigation. Possessing only a rudimentary formal education, the self-taught Bowditch was an interesting choice. Following a maritime career, he had entered the insurance business, all the time studying mathematics on his own. Bowditch turned down the professorship but became an influential member of the Harvard Corporation, which governed the university. Meanwhile, he took on the ambitious project of translating and elucidating Laplace's multivolume work on celestial mechanics. Its successful completion was arguably the most impressive American mathematical accomplishment up to that time.

In place of Bowditch, the Hollis chair was filled by John Farrar. Farrar was a charismatic lecturer. His contribution to American mathematics was to translate French textbooks and introduce the superior continental mathematics to students in the United States. In 1824 Harvard juniors began studying Farrar's adaptation of Bezout's calculus [7].

Maxime Bôcher.

W. F. Osgood.

The following year a brilliant sixteen-year-old freshman enrolled at Harvard. Benjamin Peirce had already received mathematical training from Nathaniel Bowditch, whose son, Ingersoll, was Peirce's classmate at the Salem Grammar School. Peirce supplemented his Harvard studies by assisting Bowditch with the Laplace translation. In addition, Peirce was an avid reader of *The Mathematical Diary*, solving problems posed in this early American journal [8].

Peirce completed his A.B. in 1829. Despite his ample mathematical gifts, Peirce's opportunities for further study were severely limited; Ph.D. programs did not then exist in the United States. Over the prior decade several Harvard students had returned to campus from advanced work at Göttingen and other European institutions [6]. Their presence offered evidence and testimony to the benefits of study abroad. Yet Peirce remained in Massachusetts to teach at a prep school. His biographer speculates that recent family financial reversals forced Peirce to forgo European study and begin earning an income [8, page 52].

Peirce taught at the prep school for just two years. Then a mathematics tutorship opened up for him when Farrar's health began to fail. The 1831 Harvard appointment of Peirce was the beginning of an historic tenure for American science. Within months he submitted an original theorem for publication in *The Mathematical Diary*. It was known that if $2^{n+1} - 1$ is prime, then $(2^{n+1} - 1)2^n$ is perfect. Peirce proved that if a perfect number M does not have the above form, then M must have at least four distinct prime factors [9]. Later, a posthumously published paper of Euler showed that any even perfect number has the form stipulated above. Thus Peirce's result established that any odd perfect number has at least four distinct prime factors.

It is unclear whether the Harvard administration had any appreciation for this worthy demonstration of mathematical scholarship. Shortly afterward, however, President Josiah Quincy steered Peirce in a more traditional direction, the writing of textbooks. Peirce sought a clarification of priorities. He asked whether the Harvard Corporation wanted him to "undertake a task that must engross so much time and is so elementary in its nature and so unworthy of one that aspires to anything higher in science" [8, page 69]. Advised that it did, Peirce would publish seven textbooks over the next ten years and no further papers in number theory.

Benjamin Peirce, ca. 1859.

Harvard apparently was satisfied with Peirce's performance. As Farrar's health continued to deteriorate, Peirce took on increasing responsibility. In 1833 Peirce was promoted to professor of mathematics and natural philosophy. Nine years later he became the first Perkins Professor of Astronomy and Mathematics.

Astronomy was the discipline of Peirce's first international notoriety. In 1846 the planet Neptune was discovered by an innovative technique. Neptune was spotted after its location was predicted from inferences about perturbations to the orbit of Uranus. The mathematical calculations had been performed independently by John Couch Adams of England and Urbain Le Verrier of France. A great deal of fascination accompanied the identification of a planet by means other than direct observation. Peirce closely followed these events and did his own calculations. He found aspects of Neptune's orbit that called into question Le Verrier's original prediction. When Peirce characterized the planet's discovery as a "happy accident", it did not sit well with Le Verrier [8].

In the ensuing dispute, Peirce was up against more than an eminent astronomer. Their countries represented the scientifically undeveloped and elite respectively. That Peirce held his own gave standing to both the scholar and his country. The latter was important to him.

About this time Peirce became part of a small fraternity of scientists, known as the Lazzaroni, whose objective was to elevate American science while enjoying each other's company. The core of the group also included Smithsonian visionary Joseph Henry, Harvard professor of zoology and geology Louis Agassiz, and Coast Survey Superintendent Alexander Dallas Bache. Their individual scholarly prominence placed the Lazzaroni in a position to promote national science initiatives. Out of their efforts came the creation of the American Association for the Advancement of Science and the National Academy of Sciences.

Peirce himself continued to do research in astronomy and mathematics. His astronomical work gained him admission to the Royal Society. However, Peirce's magnum opus, *Linear Associative Algebra*, was in mathematics. Along the way he held key positions with two important government scientific agencies, The Coast Survey and The Nautical Almanac.

Peirce remained at Harvard until his death in 1880. As a teacher Peirce was generally depicted as incomprehensible to ordinary students. Two of the more complimentary assessments were made in reflections from former pupils who each rose to the presidency of the university. They portray Peirce as an inspirational, if opaque, lecturer. A perhaps more balanced view was given by a member of the next generation of the Harvard mathematics faculty, Julian Coolidge: "His great mathematical talent and originality of thought, combined with a total inability to put anything clearly, produced among his contemporaries a feeling of awe that amounted almost to dread" [4]. Through his government work and his half century at a leading university, Peirce exerted an influence on the most promising younger American mathematicians, including his son C. S. Peirce, Simon Newcomb, George William Hill, and William Story [8]. In terms of both accomplishment and impact, Peirce was the outstanding American mathematician of his time.

Graduate education was one area where Peirce missed an opportunity to advance his country. In 1860 Yale became the first American university to offer a Ph.D. Harvard was slow to embrace the higher degree, reluctantly establishing its graduate school in 1872 [6]. The first Harvard Ph.D. was awarded to William Byerly in mathematics the following year. He was the only student to earn a Ph.D. under Peirce's direction.

Mathematics at Harvard and Elsewhere in the 1880s

Peirce's death in 1880 left a void in mathematical research at Harvard. Surviving him in the department were his son James Mills Peirce and former Ph.D. student William Byerly. In 1881 a distant relative, mathematical physicist Benjamin O. Peirce, joined the faculty. As undergraduates all three had taken courses from the elder Benjamin Peirce. Each was an effective teacher and wrote textbooks for Harvard students [5]. B. O. Peirce published experimental physics papers both earlier and later in his career. The state of mathematical scholarship at Harvard in the 1880s, however, had reverted back to that at the beginning of the century. No one was proving new theorems.

Meanwhile other Harvard departments were flourishing. A new age began in 1869 with the installation of Charles Eliot as president. Eliot had a vision for Harvard as a modern university. Moreover, he possessed the skills to implement his plans. By the midpoint of his forty-year tenure, research was issuing from virtually every Harvard department except mathematics [6, page 378].

Beyond Harvard the only post-Peirce American scholars utilizing substantial mathematics were J. Willard Gibbs, George William Hill, and Simon Newcomb. Gibbs was professor of mathematical physics at Yale. His groundbreaking theoretical work in chemistry and physics was hailed in Europe by James Clerk Maxwell and Wilhelm Ostwald. Appreciation for Gibbs's ideas in his own country was limited by a lack of scientific understanding. The temperamentally withdrawn Gibbs rarely left New Haven, working in quiet isolation and seeing few students.

Hill and Newcomb were acclaimed for their research in celestial mechanics. Both held positions at the Nautical Almanac Office. Pure mathematical research was then absent from United States campuses, with the following exception. The Johns Hopkins University opened in 1876 under a twofold mission of research and graduate education. With no Americans suited to lead such a mathematics program, J. J. Sylvester was imported from England [2]. On Peirce's recommendation, Harvard tutor William Story was chosen to be second in command. Story had completed a Ph.D. in Leipzig following his undergraduate work at Harvard.

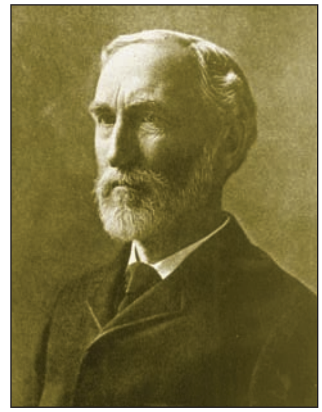
The graduate program at Johns Hopkins offered mathematical opportunities not previously available in America. Sylvester produced quality research and inspired students to follow his lead. Story taught courses in geometry. Together they began *The American Journal of Mathematics*, the first significant mathematics periodical based in the United States.

Late in 1883 Sylvester returned to England to assume Oxford's Savilian chair. Succeeding him at Hopkins was Simon Newcomb. Primarily an astronomer, Newcomb was serving as the superintendent of the Nautical Almanac Office in Washington. Newcomb continued in this position, commuting to Hopkins two days a week to conduct classes in astronomy. It was not enough to make up for the loss of Sylvester [2]. Once again, there was no United States university providing mathematical training approaching what was available in Europe.

The next notable American educational event occurred in 1889 with the founding of Clark University. Story left Hopkins to lead the new mathematics department [3]. Joining him was the German emigré Oskar Bolza, who had recently obtained his Ph.D. under Felix Klein at Göttingen. The following year another Klein student, the American Henry White, provided an additional boost to the teaching staff. Unfortunately the Clark venture was undercapitalized and a victim of competing visions. By its third year the university was roiled by acrimony among the founder, president, and faculty. Bolza, White, and several colleagues moved on to other opportunities.

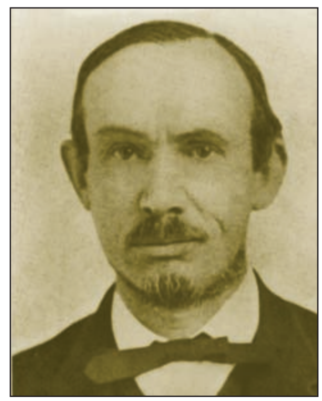
Study Abroad and the Harvard Fellowships

American students in the mid-1880s needed to look across the Atlantic Ocean for advanced mathematical training. Several found their way to Germany into the classroom of Felix Klein. These fortunate placements were hardly random. The students came from Princeton, Wesleyan, and especially Harvard, where there was knowledge of the opportunities abroad.



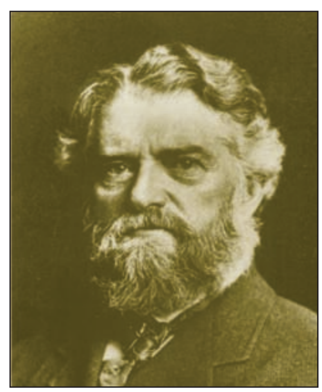
J. Willard Gibbs

In prior years Harvard graduates had occasionally sailed to Europe for graduate study. Benjamin Gould arrived in 1845 after taking several courses as an undergraduate from Benjamin Peirce. Gould studied under Carl Friederich Gauss and earned his Ph.D. in astronomy from Göttingen. Returning to the United States, he became an influential astronomer and member of the Lazzaroni.



George William Hill

The Harvard class of 1871 included two future mathematicians who pursued different educational paths. William Byerly remained at Harvard, where the graduate program was begun a few months later. Byerly received his Ph.D. in 1873. Meanwhile, his classmate William Story was in Berlin and Leipzig continuing his study of mathematics and physics. Story returned to the United States early in 1874 without an advanced degree [3].



Simon Newcomb

Graduate study in Germany posed many challenges. Young Americans needed a variety of assets to succeed. Language facility, mathematical background, and maturity were essential prerequisites for profiting from the lectures. Moreover, no European study was feasible unless the student possessed the wherewithal to pay for the voyage and for subsistence over an extended period. In 1873 Harvard began a remarkable program that eased this burden for Story and many others.

At this time, income from a $50,000 bequest by the Boston merchant John Parker Jr. became available for Harvard graduates to continue their studies at home or abroad. In his will Parker stated, "My design is to establish a fund for the highest possible education and advancement of one or more of those minds of great intellectual power, having a special adaptation to some particular science, which occasionally arise in society, and whose possessors, whether strictly poor or not, are not blessed with pecuniary means adequate to effecting the high state of improvement and advance in science for which they seem to be destined by nature" [10]. The term *science* was interpreted broadly, as the first Parker Fellow studied modern languages in Europe. Story and a philosophy student were selected for the remaining $1,000 annual fellowships, which, with satisfactory progress, were renewable for two additional years. The arrangements meant that in any given year zero to three new fellowships opened.

Felix Klein

Frank Nelson Cole

Story needed just one year to complete his Ph.D. at Leipzig. He then returned to Harvard as a tutor prior to joining Sylvester for the opening of Johns Hopkins. The Parker Fellowships were extremely attractive to Harvard's best students. These so-called "traveling fellowships" often funded a crucial transitional period from student to faculty careers. In 1877 B. O. Peirce succeeded among eighteen applicants for the two available Parker Fellowships. Peirce obtained a physics Ph.D. at Leipzig, gained valuable postdoctoral experience in Helmholtz's Berlin laboratory, and soon thereafter was appointed to the Harvard faculty.

Frank Nelson Cole was awarded a Parker Fellowship after graduating second in the class of 1882. Cole studied at Harvard for an additional year and then went to Leipzig, where he first attended courses in physics. During the summer of 1884 Cole enrolled in a mathematics class of Felix Klein on elliptic functions. Klein, then in his mid-thirties, was one of the most highly regarded mathematicians in Europe. He had recently been offered the chair to succeed Sylvester at Johns Hopkins. Although the Baltimore negotiations had broken down, Klein remained intrigued by the prospects for science in the United States [2].

Joining Cole in Klein's course was Henry Fine of Princeton. Both Americans received thesis problems from Klein. With just American university preparation, Cole and Fine found the research to be extremely difficult. Their struggles resulted in different outcomes. A junior faculty member, Eduard Study, gave Fine considerable help on a substitute problem [2]. After one year Fine had his Ph.D. to take back to Princeton.

Cole also returned to his home university in 1885. There he continued his thesis research while proselytizing Klein's mathematics in courses at Harvard and lectures at MIT. Cole worked hard on his thesis problem but was completely isolated from anyone who might help him. Discouraged by his lack of progress, Cole wrote Klein after a year that he didn't believe "that I will finish it during my lifetime using my present method" [11]. Nevertheless, Cole had done enough to obtain his Ph.D. from Harvard.

Cole's Harvard teaching career lasted just two years. Overwork caused a breakdown that forced him to withdraw from a tutorship. He resumed his academic career elsewhere after a therapeutic year of outdoor railroad work. As a promoter, however, Cole's impact was striking. Klein, who had moved to Göttingen, suddenly experienced a surge of students from the United States, particularly the Boston area. His 1887 pupils included Harry Tyler of MIT, both Mellen Haskell and William Osgood on Harvard traveling fellowships, Henry Thompson from Princeton, and Henry White from Wesleyan. The following year another Harvard traveling fellow, Maxime Bôcher, arrived.

Osgood and Bôcher

Osgood and Bôcher would earn their Ph.D.'s in Germany and then return to Harvard. Over their careers they would establish similar impressive vitae, differing most notably with the latter's premature death in 1918 [12]. Both were born in Boston: Maxime Bôcher on August 27, 1867, and William Fogg Osgood three and one half years earlier.

Bôcher grew up in a scholarly, international household. The ancestry of his mother, Caroline, went back to the Plymouth Colony. Maxime's father, Ferdinand, was born in New York during a business trip of Maxime's grandfather from France. Ferdinand Bôcher became a Harvard French professor. He was among the early hires of President Eliot, coming from MIT, where the two had been colleagues. Ferdinand Bôcher revered Eliot, perhaps accounting in part for his son's devotion to the university and its president.

Osgood was also descended from early residents of Massachusetts. He came to Harvard as an undergraduate in 1882. Excelling in mathematics, physics, and Latin, Osgood graduated second in his class four years later. He remained at Harvard for an A.M. Then, inspired by classes from Cole, Osgood applied for a traveling fellowship to study under Klein at Göttingen.

By this time Parker Fellowship stipends had been reduced to $700, but there were four of these grants in the rotation. Another traveling fellowship, the Harris, from a smaller endowment, carried a $500 award under similar conditions. In 1887 one Parker and the Harris were vacated. The new Parker went to a law student who would later become a professor at the University of Chicago and a judge on the United States Court of Appeals. Osgood got the Harris. In his second year Osgood was upgraded to the Parker, making the Harris available again in 1888.

Bôcher had just turned sixteen when he began his undergraduate work at Harvard in 1883. It is unclear why it took him five years to complete his A.B. During the regime of President Eliot, Harvard students had substantial freedom to select elective courses. Bôcher took advantage of this opportunity in his senior year to fashion a diverse program that consisted of mathematics, Roman art, music, and an advanced course in geology [13]. He was awarded highest honors in mathematics for *A thesis on three systems of parabolic coordinates* and a second prize for the meteorological essay he entered in Harvard's long-standing Bowdoin Prize competition.

With Osgood's promotion to a Parker Fellowship, only one other Parker was available in 1888. This went to future Nobel chemistry laureate Theodore Richards for postdoctoral work in Germany. Bôcher was awarded the Harris Fellowship. He arrived in Göttingen September 1888, one year after Tyler, Osgood, and White.

The impression made by the Göttingen faculty on an American student was conveyed in a letter from Tyler to his parents following his first week of classes:

> After some 16 students are assembled, the door opens hastily, the Prof. enters, there is a slight scraping and stumping—to assure him we're glad he's no later—he deposits his tall hat and cane, and within 5 seconds of his appearance with no other preface than a hurried "meine Herren" he is in the midst of his lecture. It should not be inferred that he is a hasty instructor. Too much the contrary; he is one of the slowest men I ever heard lecture. This however later—My first impressions are that he is a large, stout dignified, fine-looking gentleman perhaps 55 years old, with full slightly gray beard and gold spectacles. In the next place he spoke with admirable distinctness, and I am agreeably surprised to find myself understanding almost every word—though it is very difficult at the same time to follow the lecture and to take notes either in English, German, or a mixture of the two…Please remember that the gentleman just introduced is Prof. Schwarz, senior professor of Mathematics in "der hiesigen Universität". He has unfortunately not the reputation, and presumably not nearly the ability of his junior Prof. Klein, and his classes are not large.
>
> Wednesday I attended the initial lecture of a course on the "higher plane curves" by Dr. Schoenflies—a Privat Docent—. It was almost beyond description, but I'll try. Dr. S. is a good-looking business like young man (30–35 perhaps) with dark hair and full beard. He talks very rapidly and somewhat indistinctly, though otherwise clear and very interesting. But he marched back and forth along the small platform, falling off 3–4 times in his apparent excitement, leaned against the desk or the black board, squinted up his eyes and wrinkled his nose, then dashed at the black board talking steadily with his back towards us…
>
> Not till Thursday did I hear and see the great Klein (so to speak) whose fame as the greatest mathematical teacher in Germany (consequently in the world) has attracted me to Göttingen. He is a tall slender man of about 40, his hair is light brown, his eyes blue, keen and alert; the strength of his face lies chiefly in his large nose and high forehead. He speaks rather quickly and with a somewhat high voice, but clearly enough, and methodically, enunciating frequently statements to be taken down verbatim. He lays much stress upon the notes taken, and has one student write up the lectures which after his own revision are put in the reading room for general reference. His subject was Potential—a subject of mathematical physics, in which I have no interest. In spite of my first disinclination, I am gradually concluding to take this course—4 lectures a week—partly for the sake of the Mathematics involved, mainly to hear the man [14].

A significant feature of the traveling fellowships was their renewability. Unlike Osgood and Bôcher, who could expect three years of study abroad, Tyler was bound by a two-year leave from his faculty position at MIT. The extra year could be decisive in completing the requirements for a Ph.D. Further compounding the time limitations was Klein's course scheduling. In the fall of 1887 Klein's advanced offering was the second term of a course on hyperelliptic functions. Lacking the prerequisites Tyler, Osgood, and White took the intermediate-level potential theory.

Not until their second semester, when Klein started a sequence on Abelian functions, could the three Americans begin the sort of instruction which had drawn them to Göttingen. Two other Americans and one or two Germans were also enrolled. Osgood received an unwelcome surprise on the first day of class. He was asked by Klein to serve as the course scribe. The duties included substantial revisions and rewriting. Up to this point Osgood had not been taking careful notes. While he dreaded assuming the new responsibility, he could not say no to Klein. After two lectures Osgood was overwhelmed and prevailed upon the more clerically oriented Tyler to take over the duties.

Harry Tyler

The revision process gave Tyler more interaction with Klein than he would have otherwise had. Occasionally they discussed future plans. While Tyler had come to Göttingen with some hopes of obtaining his Ph.D., he had doubted whether the degree was possible under his two-year constraint. To make the most of his time abroad, Tyler intended to study for a semester in some other German city and another in Paris. As the first year drew to a close, Klein began encouraging Tyler to remain at Göttingen for his Ph.D.

Tyler went back and forth over where to spend his second year. At the last moment, with Klein's approval, Tyler moved to Erlangen. The particular attraction of Erlangen were its two strong mathematicians, Paul Gordan and Max Noether, and few students. Tyler's plan was to continue work on a thesis problem from Klein while receiving individual instruction from Gordan and Noether.

Both Gordan and Noether were generous with their time, offering personal attention that was not available in Göttingen. Tyler was especially drawn to Gordan, who, rather than discussing the problem from Klein, set Tyler to work on resultants. Then, as Tyler wrote Osgood, "A month or six weeks later he told me to my unbounded surprise he would accept this as a Ph.D dissertation if I chose" [15]. The plans for Paris were scrapped. Tyler spent the remainder of the second year in Erlangen, writing up his thesis and preparing for the required supplementary topics in physics and chemistry. After two years in Germany, Tyler returned to MIT with his Ph.D.

Tyler remained in touch with Osgood and White, with whom he had become close during their classes together. Back at Göttingen Klein was finishing a three-term sequence on Abelian functions. In the fall of 1889 he would begin a program in mathematical physics. Osgood sought advice from Tyler over whether to remain in Göttingen for a third year. Tyler's nine-page response, excerpted below, was both thoughtful and incisive, carefully analyzing the advantages and disadvantages of the people and venues.

> I think in the first place that it's much better for you or anyone else who has 3 years abroad not to spend the whole time in Göttingen unless for reasons of great importance…I have much admiration for Klein personally, I know of nobody who can approach him as a lecturer…He's certainly acute, fertile in resource, not only understands other people, but makes them understand him, and seems to me to have a very firm grasp of the philosophical relations and bearings of different subjects, as well as great versatility and acquaintance with literature.
>
> But quite in keeping with some of these good qualities are drawbacks that seem to me somewhat serious. So busy a man can not and will not give a student a very large share of his time and attention; so too he will not study out or interest himself especially in the painstaking elaboration of details, preferring to scatter all sorts of seed continually and let other people follow after to do the hoeing…it would seem ridiculous to claim—what he certainly would not claim for himself—that he does not sacrifice completeness of detail, and that this is not a real sacrifice…
>
> Still anyone coming here from Klein would be sure to look at mathematical things from a new standpoint and as matters are now would be practically certain of a degree of interest and attention about out of the question in Göttingen, and especially valuable when one is beginning original work. I have been and am still embarrassed by the opportunities. I might have gained a great deal from Noether had I not been so occupied with Gordan. In the present semester Noether will probably have but one student besides myself and will probably give us anything we like…The chief advantage in being here in general depends upon cultivating personal relations with Gordan and Noether. I wouldn't advise anybody to come for the lectures alone. Both men are so peculiar and so irreconcilable that the p.r. must be cultivated with some tact especially if one tries to divide his attention about equally…G. is outspoken, irascible, exasperating, violent; N. is taciturn, serious, equable, patient…

If G. is absolutely unrestrained N. is quite the contrary; but it's restraint not constraint. He may forbear from saying disagreeable things, but he doesn't go out of his way to say the other kind....

Now as to your plans, I would advise you unhesitatingly to come here if you want detailed work in pure mathematics. If you want to work especially with Gordan I wouldn't suggest any preparation unless the first volume of his book. If you had anything underway very likely it would not interest him. For Noether on the other hand I think it would be worth while to have something to propose—in Abelian Functions if you like or in any of his subjects that you know from the Annalen as well as I could tell you. I wouldn't advise you to come unless you feel sure your tastes will lie in these directions. I do not see the least reason to doubt your being able to make the Ph.D. in two semesters here, or even one if necessary [16].

Osgood followed the second branch. He went to Erlangen for his third year, bringing a problem from Klein on Abelian functions. One year later he had an Erlangen Ph.D. under Noether. For the fall of 1890 Osgood returned to Harvard with the title of instructor of mathematics.

Bôcher remained in Göttingen his entire 1888–1891 period abroad. The lectures on mathematical physics, begun by Klein in 1889, suited Bôcher nicely. In his second year Bôcher took up a substantial piece of Klein's program.

Potential functions for many partial differential equation problems in mathematical physics could be obtained by series methods after employing an orthogonal change of coordinates and separation of the new variables. Bôcher had dabbled with a few of these coordinate systems in his undergraduate thesis. Now he sought to develop series solutions under general cyclidic coordinate transformations. The ordinary differential equation and other issues that arose from the technique required difficult analysis. Klein arranged for a prize to be awarded for a general development of this theory. Bôcher's success earned him the prize and his Ph.D.

In 1891 Bôcher returned to Harvard. Like Osgood, who arrived one year earlier, he was an instructor of mathematics with a German bride.

Mathematics at Harvard 1890–1913

Bôcher immediately experienced the conflict between a desire to continue his research and the overwhelming teaching obligations of a beginning instructor. As with new Ph.D.'s throughout time, this led to another dilemma. Bôcher wanted to stay in touch with his advisor but was embarrassed by having no great results to report. He delayed writing for fear that Klein might attribute the lack of progress to laziness. These emotions are vividly displayed, along with the 1891 Harvard teaching load, in this New Year's letter.

Dear Professor,

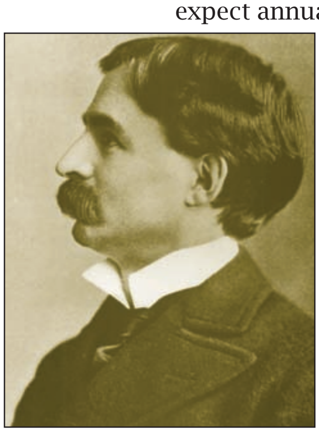
I apologize that you have not heard from me. The main reason for my silence is my work. I wanted to have something to write about, and I did not want you to think that I abandoned the research entirely. Since the end of September I have been very busy with my lectures. I have to give lectures 12 hours each week. Half of that time I devote to little foxes [Füchsen] to whom I teach elementary algebra. Of course I do not have to do much preparation for this, therefore I have more time. The students have to write assignments on a daily basis, which I have to correct. In addition I teach a second 3 hour lecture on analytical geometry. Here I discuss primarily the projective geometry of the two dimensional plane, mostly through homogeneous coordinates, etc., but partly through pure geometric methods. This lecture gives me a lot of pleasure although the audience could be better. Finally I lecture for 3 hours on Lamé's functions, the linear development of the potential theory, etc. Here I have two listeners. I would be happy with the numbers, also with the individuals but they do not find the time to work on the project as one of them gives elementary lectures at the Polytechnic in Boston [MIT] and the other has to do much work at the physics laboratory. Therefore it is almost impossible to go into details in this lecture. The penta-spherical coordinates, for example, have to be left out entirely.

Up to the Christmas break I have had practically no time for my own research. But I have managed to improve on some small points...

Over the Christmas holidays I did further research on the Bessel functions. I used the time when I did not have to do work for the university. I did the research in preparation for the definitive formula of those parts to be given for the prize for the composition that dealt with the degenerate cases, etc. I am happier with this formula and hope

> to send you the printed results of these functions in the spring.
>
> As you can see it is impossible to finish the definitive reduction of the equation of the potential theory during this winter. As far as next summer goes, I would like to visit Goettingen; this is for a number of reasons. I think you agree with me that it is better I stay here at the university and work at my own pace on the research. I will have three months; in that time I can work, without interruptions, and can complete the work if I remain in good health. After that I can come to Göttingen.
>
> The situation is such that one can do little research, in particular in the first few years of employment. You have to believe me that in the last few months I have tried to do as much work as possible [17].

Osgood, a second-year instructor, was in a similar position to Bôcher. Both were earning $1,250 on one-year contracts. If all went well, they could expect annual renewals to serve as instructors for three years and then be promoted to an assistant professorship on a five-year term. As Bôcher and Osgood were adjusting to their circumstances at Harvard, a significant development was taking place for mathematics in the United States. Staffing was under way for the opening of the University of Chicago in the fall.



E. H. Moore

Chicago's president, William Rainey Harper, was working with Rockefeller funding to establish a new university model emphasizing research and graduate education. Still, in 1892, no American mathematician possessed the credentials to lead such a venture. Harper elected to take a chance on E. H. Moore to be professor and acting head of mathematics. Moore had received his Ph.D. at Yale under Hubert Newton in 1885 [2], [18]. Newton had then lent Moore the money for a year of postdoctoral study in Berlin. Over the following six years Moore had held lower-level positions at Yale and Northwestern while publishing four papers.

Moore's first task was to recruit a junior faculty member to work with him in realizing Harper's ideals. The offer of an associate professorship went to Bôcher as he was approaching the end of his second semester at Harvard. The teaching load was to be ten hours and the salary $2,500, twice what he was making at Harvard. Bôcher discussed the offer with President Eliot, who made no commitments but indicated that the chances were good for promotion to assistant professor in two years. Bôcher declined the offer from Chicago [19].

Moore subsequently hired Oskar Bolza and Heinrich Maschke, German mathematicians who had studied with Felix Klein. The results were almost magical. Moore immediately blossomed into an important mathematician. He teamed with Bolza and Maschke to give Chicago the first American graduate program offering training comparable to what was available in Europe [2]. Over the next fifteen years Dickson, Veblen, and Birkhoff received their Ph.D.'s, going on to become the next generation of American mathematical leaders.

Progress at Harvard was more gradual but sustained over a longer period. As a preface to these developments, consider the following retrospective analysis given by Bôcher in 1912:

> When Chicago was founded, Osgood and I were just beginning as young instructors, with far slighter mathematical equipments than it is easy to imagine now. I remember, during my first year of teaching, learning what uniform convergence of series means. For several years after that we were the only persons here who in any way represented modern mathematics or research. Many students of mathematics never took our courses at all, and those who did usually gave us only a small share of their time. These conditions changed only very slowly, whereas in Chicago the department was organized from the start on a thoroughly modern scientific basis [20].

Bôcher was hoping to expand his thesis into a book. During the 1892 spring break he got to work in earnest on the project, maintaining his momentum through the remainder of the semester. By the middle of the summer he was able to report substantial progress to Klein. Over the next two years Bôcher obtained deep new results that went beyond his thesis. The book was written in German and published in Leipzig. Klein was sufficiently impressed to provide the preface and to upgrade his Bôcher correspondence salutation from Doctor to Colleague. A byproduct of the publication, as it circulated among European mathematicians, was to demonstrate that strong scholarship existed in the United States.

Advancing through the ranks on schedule, Bôcher and Osgood became assistant professors after three years. They did their part to modernize the Harvard graduate offerings (for details see [5]) but continued to share teaching duties with Byerly and the Peirces. Bôcher's first Ph.D. student, James Glover, came from Michigan in 1892, where he had studied with Cole.

The stories of two students who were awarded graduate fellowships in 1894 illustrates the challenges faced by the Harvard mathematics department in establishing its doctoral program. Leonard Dickson rescinded his acceptance when an offer arrived from Chicago. Charles Bouton came to Harvard, obtained his A.M., and then went to Leipzig as a Parker Fellow to study with Sophus Lie. Harvard was attracting notice but was not yet a destination school.

Nevertheless, Bôcher and Osgood were gaining stature in the nascent American mathematical community. In 1896 the AMS decided to experiment with colloquium lectures. Bôcher and James Pierpont were each selected to deliver a series of talks following the summer meeting in Buffalo. The thirteen attendees adjudged the experiment to be a success, and colloquium lectures became part of the program every two or three years. Osgood was chosen for the second offering. Then, as today, designation as a colloquium speaker was regarded as a prestigious mathematical recognition.

One factor in Bôcher's selection must have been that he was a lucid lecturer. The topic he chose for his colloquium series was *Linear differential equations and their applications*. Motivated by the inadequate treatment of existence and uniqueness theorems in contemporary texts, Bôcher gave a comprehensive theoretical development for second-order equations. He began with the case when the coefficients are analytic and then weakened the hypothesis to merely continuous. A careful uniqueness argument accompanied the presentation. With the foundation established he then went into applications and dependence of the solutions on parameters, issues that arose in his own work on potential theory.

Bôcher was a superb analyst with a broad command of mathematics. Quite a bit of his research involved aspects of linear differential equations. Representative of the work was a 1900 paper treating regular singular points in substantial generality. He considered points a where the coefficient functions have an isolated discontinuity that satisfies a weaker condition than becoming analytic when multiplied by $(x - a)$ to the appropriate power. For example, Bôcher only required that the coefficient of the linear term in a second-order equation be expressed in the form $\frac{c}{x-a} + p(x)$, where $|p|$ has an improper integral that converges on a neighborhood of a. Without analyticity of p the standard Frobenius Method is not applicable. Bôcher obtained solutions around a by using the method of successive approximations to develop a series with terms consisting of a power function times a continuous function.

The regular singular points article appeared in the first issue of the *Transactions of the AMS*. Bôcher was one of several younger AMS members who had provided the impetus for the creation of the *Transactions*. For the emerging American mathematical community the new journal was a source of pride as well as a vehicle for demonstrating its bona fides. Americans submitted their strongest work. The most definitive statement was made by Osgood with a seminal result in the third issue of the initial 1900 volume. He gave the first rigorous proof of the Riemann Mapping Theorem for arbitrary simply connected regions in the plane. In eliminating restrictions on the boundary, an American achieved the crowning position on a provenance that featured some of the greatest mathematicians of the nineteenth century.

Meanwhile, around the turn of the century, the Harvard mathematics department was bolstered by the hiring of Bouton, Julian Coolidge, and Edward Huntington (see [5]). Each was an alumnus who received his Ph.D. in Europe. More and better students were getting their doctorates at Harvard, mostly under Bôcher. Yet the mathematics faculty continued to encourage their best students to go to Europe for thesis work. Unlike at other universities, the traveling fellowships opened study-abroad opportunities to students of all financial means. One side effect of this marvelous resource was that the list of Harvard Ph.D.'s was less impressive than it otherwise would have been. E. R. Hedrick began graduate study in 1897 and then two years later, like Bouton, was awarded a Parker Fellowship. Hedrick studied with David Hilbert for his Ph.D. at Göttingen. He then returned to the United States, where he became a leading figure in the AMS and at UCLA.

That Harvard was closing the gap with Chicago can be seen from their competition in the graduate recruitment of G. D. Birkhoff. Birkhoff had entered Chicago in 1902 as an advanced undergraduate. He quickly came under the influence of E. H. Moore, who recognized a student of considerable potential. Surprisingly, Birkhoff transferred to Harvard in 1903. It is unclear why Birkhoff left Chicago after only sampling its scholarly resources, especially with Moore anxious to supervise him in research. The choice of Harvard is easily understood from the high esteem in which Moore held Bôcher and Osgood, but why did Birkhoff leave Chicago prematurely? Some notion of the reason possibly may be inferred from a summer letter by Moore offering Birkhoff advice on preparing for Cambridge. The first item was "*to take much enough exercise this summer to come back to work in perfect trim in the autumn*" [21] (emphasis included). After making some mathematical suggestions on a book and problem, Moore closed with the admonishment: "*Don't* forget *no. 1*: the rich red blood I want you to have for next year."

In his two years at Harvard, Birkhoff took courses from Bôcher and Osgood while obtaining A.B. and A.M. degrees. Early in 1905 Birkhoff contemplated whether to remain at Harvard or return to Chicago for his Ph.D. Both institutions offered graduate fellowships. Moore advised Birkhoff (and his father) that "except for two considerations" he should come to Chicago, where "we have a more catholic attitude towards mathematics in general than they have at Harvard" [22]. One of the exceptions was if he were already engaged in an important research project that could best be prosecuted at Harvard. The second was to receive a guarantee that he would be awarded a traveling fellowship after remaining at Harvard for another year or two. Bôcher, who was also impressed by Birkhoff, tended to be less assertive with students, leaving it to them to choose their own path. Birkhoff decided to return to Chicago and work with Moore for his Ph.D.

G. D. Birkhoff

Birkhoff had situated himself well. Over the first decade of the twentieth century, Bôcher, Osgood, and Moore were the foremost pure mathematicians in the United States. During this period each was inducted into the National Academy of Sciences and served a two-year term as president of the AMS. The Chicago graduate program peaked about the time of Birkhoff's Ph.D. in 1907. Maschke died the following year, and then Bolza returned to Germany. The homegrown Dickson and Gilbert Bliss were able replacements on the Chicago faculty, but Harvard began to turn out superior students.

Griffith Evans

The first outstanding mathematician to complete a Harvard Ph.D. was Griffith Evans. Bôcher supervised his 1910 thesis on integral equations. Evans then received a traveling fellowship to do postdoctoral work with Vito Volterra in Rome. Returning to the United States, Evans led the build-up of the mathematics departments at Rice and Berkeley.

Dunham Jackson

Although Evans remained at Harvard for the entirety of his undergraduate and graduate education, other gifted students still took their Ph.D.'s in Europe. Dunham Jackson entered Harvard one year after Evans. Jackson obtained a Harvard A.M. in 1909 and then went to Göttingen on a traveling fellowship. Bôcher was instrumental in connecting Jackson with Edmund Landau, under whom Jackson wrote an important thesis in approximation theory.

Jackson completed his Ph.D. in 1911 and returned to Harvard as an instructor. His position was a new line that came about as a result of curricular changes that increased mathematics enrollments at Harvard. Jackson was not the first choice, but he was one of the three nominations put forward by the department. The others were Max Mason of Wisconsin and G. D. Birkhoff.

Bôcher had had his eye on Birkhoff since he left Harvard for Chicago. The two had maintained a correspondence on various mathematical matters. Their communication continued as Birkhoff held junior-level appointments at Wisconsin and Princeton. As a journal editor Bôcher came to rely on Birkhoff's taste and judgment. Meanwhile Birkhoff's theorems attracted offers from a number of institutions, including Princeton, which had begun its own mathematical ascendance a few years earlier. At the end of 1910 Harvard offered Birkhoff an assistant professorship at a salary of \$2,500. Princeton countered with a promotion to full professor at \$3,500. The Harvard terms called for two 5-year contracts as an assistant professor, the salary for the second at \$3,000. This was the standard procedure at Harvard, where Bôcher and Osgood had each served as assistant professors for ten years.

Birkhoff attempted to leverage better terms from Harvard through a less than sympathetic Bôcher [23]. Harvard's only concession was a shortening of the first assistant professor term from five to three years, meaning that his Harvard salary in eight years would be \$500 less than what was immediately available at Princeton. After Birkhoff declined, Jackson was then hired to fill the new position at Harvard.

Over the following year Birkhoff came to regret his decision. He wrote Bôcher hinting at a desire for a renewed offer. Bôcher replied that another position might become available but that Birkhoff would have to guarantee his unconditional acceptance in advance of further efforts on his behalf [24]. Birkhoff promised to accept the assistant professorship of the previous offer, with only the modification of reducing the first contract from three years to two. The deal was completed and Birkhoff came to Harvard in 1912.

The addition of Birkhoff was the most significant development for Harvard mathematics since the hiring of Osgood and Bôcher just over two decades earlier. During the intervening period both Harvard and American mathematics had made impressive advances. While the German and French schools were still superior, American scholarship, especially at Harvard, was becoming appreciated in Europe. Both Osgood and Bôcher were invited to deliver plenary addresses to the 1912 International Congress of Mathematicians in England. By the end of 1912 Birkhoff had proved Poincaré's Geometric Theorem, the proof of which appeared in the

January 1913 issue of the *Transactions*. According to Richard Courant, Birkhoff's result was the first piece of American mathematics to be admired by the Göttingen community [25].

The French mathematician Émile Borel invited Bôcher to serve as an exchange professor at the University of Paris for 1913–14. Osgood was recognized throughout Europe. His 1897 work on term-by-term integration for series of continuous functions was influential in Henri Lebesgue's development of integration theory [26]. Osgood's book *Funktionentheorie* was the leading primer on the subject both at home and abroad. In 1913 the Norwegian algebraist Ludwig Sylow expressed his high regard for the work of two Americans, Osgood and Leonard Dickson [27].

Birkhoff exerted an immediate impact on the Harvard graduate program. Marston Morse entered Harvard in 1914 and wrote his doctoral dissertation under Birkhoff. Birkhoff's presence was especially timely as Bôcher's health began to fail. The passing of the baton symbolically occurred through Joseph Walsh, who began his thesis with Bôcher and after his death in 1918 finished with Birkhoff. Over his first fifteen years at Harvard, Birkhoff supervised twenty-six Ph.D.'s, including that of Marshall Stone. During this period Birkhoff became regarded as the leading mathematician in the United States. Osgood, who was never active in thesis direction, remained an important presence at Harvard. Unfortunately his distinguished career was marred by a personal matter late in life. Osgood was ostracized by his colleagues and forced to retire in 1933 as a result of his relationship with the former wife of Marston Morse [28] and [29]. Morse had joined the department in 1926.

The 1913 Harvard mathematics faculty with Bôcher, Osgood, Birkhoff, and Jackson was the strongest that had ever been assembled in the United States. While the lopsided concentration in analysis has been noted, its effects were mitigated by several factors. Both Bôcher and Birkhoff were especially broad in their knowledge of mathematics. Moreover, Coolidge and Huntington added coverage to other areas. Finally, at a time when as many as four research mathematicians could be found on only a few university faculties, mathematical diversity was a different consideration than in more modern times.

A remarkable transformation occurred in the Harvard mathematics department from 1890 to 1913. Together, Bôcher and Osgood successfully installed research as the primary mission. Jackson would leave for Minnesota in 1919, but Birkhoff was firmly entrenched as the department's anchor. Harvard was on course to be a world mathematical power of the twentieth century.

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