BÔCHER AND STEELE PRIZES AWARDED

The Bôcher Memorial Prize, which is awarded at five-year intervals, is supported by the Bôcher Memorial Fund established in 1920 with gifts in memory of Maxime Bôcher (1867-1918), who served the Society as its tenth President (1909, 1910). The Bôcher Prize is now supplemented by a prize from the Leroy P. Steele Fund. The thirteenth award of the Bôcher Prize is to Alberto P. Calderón, Louis Block Professor of Mathematics, University of Chicago, "for his fundamental work on the theory of singular integrals and partial differential equations, and in particular for his recent paper 'Cauchy integrals on Lipschitz curves and related operators,' Proceedings of the National Academy of Sciences, USA, volume 74, number 4, pages 1324-1327, April 1977."

Leroy Powell Steele, a graduate of Harvard College (B.A., 1923), died January 7, 1968 and bequeathed the bulk of his estate to the American Mathematical Society to be used for the award from time to time of prizes in honor of George David Birkhoff, William Fogg Osgood and William Caspar Graustein. The fourteenth and fifteenth Steele Prizes are awarded to Salomon Bochner, Edgar Odell Lovett Professor of Mathematics, Rice University, and to Hans Lewy, Professor Emeritus, University of California, Berkeley.

Professor Bochner's award is for the "cumulative impact of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph. D. students." The fields cited are probability theory, Fourier analysis, several complex variables, and differential geometry. The students cited are Calabi, Cheeger, Furstenberg, Gunning, Helgason, and Hunt.

Professor Lewy's award is "for a paper...which has proved to be of seminal or lasting importance in its field, or a model of important research." Three papers in two fields of research were cited:

"On the local character of the solutions of an atypical linear differential equation in three variables and a related theorem for regular functions of two complex variables," Annals of Mathematics (2), volume 64 (1956), pages 514-522. MR 18, 473.

"An example of a smooth linear partial differential equation without solution," Annals of Mathematics (2), volume 66 (1957), pages 155-158. MR 19, 551.

"On hulls of holomorphy," Communications on Pure and Applied Mathematics, volume 13 (1960), pages 587-591. MR 27, 340.

The three prizes, currently totaling fifteen hundred dollars each, were presented at the prize session held at the Annual Meeting of the Society in Biloxi, Mississippi, January 25, 1979, on which occasion the recipients were given the opportunity to speak in response to the awards. Professor Bochner spoke informally, acknowledging the award of his Steele Prize. Professors Calderón and Lewy spoke about their scientific work. The texts of their remarks are reproduced below.

The prizes were awarded by the Council of the American Mathematical Society, based on recommendations made by committees appointed for this purpose. The committee for the Bôcher Prize for 1979 consisted of Paul J. Cohen, Donald S. Ornstein (chairman), and Walter Rudin. During 1978, the Committee on Steele Prizes consisted of Edward B. Curtis, Irving Kaplansky, H. Blaine Lawson, Hans Samelson, Stephen S. Shatz, Joseph R. Shoenfield, Frank Spitzer, Joseph L. Taylor, Raymond O. Wells, Jr., and Hans F. Weinberger (chairman).

Alberto P. Calderón

It certainly is a great honor to be awarded the Bôcher Prize. I wish to express my deep gratitude for having been chosen to be its recipient for 1979. In responding on these occasions it has been customary to describe briefly at least some of the work for which the prize is awarded. I will do this, confining myself to discussing only the most recent work mentioned in the citation.

I will begin by considering a problem related to the Cauchy integral on Lipschitz curves, which may appear at first sight to have little or no connection with singular integrals and partial differential equations, but which has suggested the most effective method to deal with them so far. The problem is that of showing that singular integral operators (or pseudo-differential operators of nonpositive order)—sufficiently general to permit their use in treating linear differential operators with nonsmooth coefficients—still have the properties that make them an effective tool. More specifically, the question is this: Do singular integral operators that can be used in dealing with differential operators with Lipschitzian coefficients have the property that they can be composed modulo smoothing operators by simply multiplying their symbols? The simplest case (to which, fortunately, the general case can be reduced without much difficulty) is the following: Let H denote the Hilbert transforms of functions on the real line and let A denote operator multiplication by the bounded Lipschitzian function a(x); is it then true that HA - AH is a smoothing operator in L^2? Equivalently, does the principal value integral

$$(1) \qquad \text{p.v.} \int_{-\infty}^{+\infty} \frac{a(x) - a(y)}{(x-y)^2} f(y)\,dy$$

with a(x) Lipschitzian define a bounded operator in L^2? This question was answered affirmatively nearly fifteen years ago by using complex variable techniques and the equivalence of the L^1-norms of the Lusin area function and the Hardy-Littlewood maximal function associated with a function that is analytic in a half-plane; this equivalence was also obtained at that time.

The integral (1) is a special case of the following:

$$(2) \quad \text{p.v.} \int_{-\infty}^{+\infty} \frac{1}{x-y} F\left[\frac{a(x) - a(y)}{x-y}\right] f(y)\,dy$$

where a(x) is again Lipschitzian and F is analytic. This integral has several interesting specializations other than (1). Consider, for example,

$$(3) \ \tfrac{1}{2}f(x) - \frac{1}{2\pi}\,\text{p.v.} \int_{-\infty}^{+\infty} \frac{a(x) - a(y) - (x-y)a'(y)}{[a(x) - a(y)]^2 + (x-y)^2} f(y)\,dy$$

which gives, if they exist, the boundary values of the logarithmic potential of a double layer distributed on the graph of the function a(x), and is a simple expression in terms of integrals like (2). Integrals like this, and their generalizations to several variables, can be used effectively in the study of boundary value problems for the Laplace and other elliptic equations.

Another example is the integral of Cauchy type

$$(4) \qquad \text{p.v.} \int_{\Gamma} \frac{f(w)}{w-z}\,dw, \quad w \in \Gamma,$$

where Γ is the graph of the function a(x), which also can be expressed in terms of integrals like (2).

A natural approach to (2) would be to expand F in power series as

$$(5) \ \sum_{n=0}^{\infty} c_n\, \text{p.v.} \int_{-\infty}^{+\infty} \frac{1}{x-y} \left[\frac{a(x) - a(y)}{x-y}\right]^n f(y)\,dy;$$

thus the study of (1) can be regarded as the first step in the study of (2). Unfortunately, the complex method employed originally in the treatment of (1) fails utterly when applied to terms of (5) with $n > 1$. It was not until 1975 that the case $n = 2$ was finally resolved by R. Coifman and Y. Meyer. Shortly after that they showed that each of the terms of (5) represents a bounded operator in L^p, $1 < p < \infty$. Their method actually yielded more general results applicable to pseudo-differential operators of classical type, but, unfortunately, their estimates for the norms of the terms of (5) did not permit one to sum the series. People working in this area had been aware of the fact that the study of (2) could be reduced to that of (4), and therefore it came as a surprise when soon after it was realized that (4) could be treated by a method similar to that used in treating (1). Suppose one introduces a family of curves Γ_t, Γ_t being the graph of the function $ta(x)$, $0 \leq t \leq 1$, and considers the integral

$$(6) \qquad A_t f = \text{p.v.} \int_{-\infty}^{+\infty} \frac{f(y)}{z_t(x) - z_t(y)}\,dy,$$

where $z_t(x) = x + ita(x)$, and which for $t = 1$ is essentially the same as (4). If one assumes that a(x) is in C_0^∞, in which case A_t is well defined and bounded in L^2, and succeeds in estimating the norm of A_t in terms of $\|a'\|_\infty$ and t alone, then the existence and boundedness of (4) as an operator in L^2 can be obtained by using standard techniques. If one differentiates (6) with respect to t, one obtains the integral

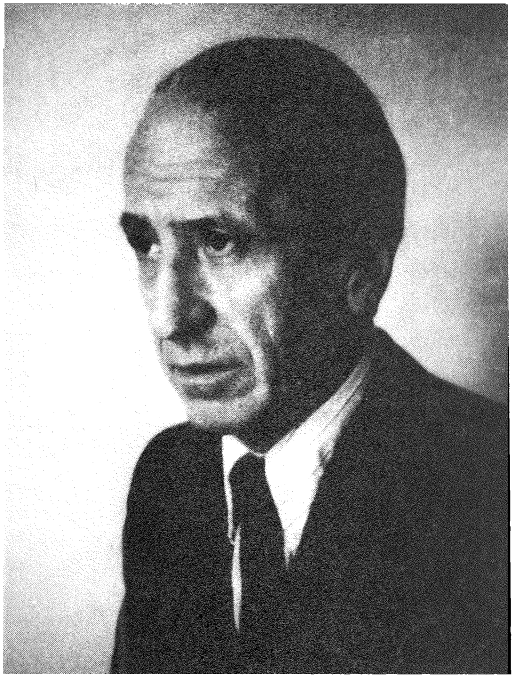
$$(7) \qquad \frac{dA_t}{dt} f = \text{p.v.} \int_{-\infty}^{+\infty} \frac{a(x) - a(y)}{[z_t(x) - z_t(y)]^2} f(y)\,dy$$

which clearly resembles (1). Now this similarity is not merely superficial. In fact, the complex variable techniques used in treating (1) combined with more recent results on weighted norm inequalities between the area function of Lusin and the Hardy-Littlewood maximal function due to B. Muckenhoupt, R. P. Gundy and R. L. Wheeden, as well as the fact that the function mapping the upper half-plane conformally on the portion of the plane above Γ_t has a derivative whose boundary values have modulus in the class A_2 of Muckenhoupt with constant depending only on $\|a'\|_\infty$, yield an estimate of the norm of (7) in terms of the norm of (6), namely

$$\left\|\frac{dA_t}{dt}\right\| \leq c(1 + \|A_t\|)^2$$

where c depends only on $\|a'\|_\infty$. For $t = 0$, the operator A_t is the ordinary Hilbert transform. Thus, integrating this differential inequality one obtains an estimate for $\|A_t\|$ depending only on $\|a'\|_\infty$, and from this follows the existence of (4) as an operator in L^p and an estimate for its norm, provided that $\|a'\|_\infty$ does not exceed a certain positive constant α. Having established this, one obtains the continuity in L^p of operators of the form (2) and generalizations of these to several variables.

With these results it is possible, for example, to treat in detail boundary value problems for the Laplace equation in C^1 domains and Lipschitzian domains with small local oscillation of the plane tangent to the boundary, as E. Fabes, M. Jodeit and N. M. Rivière have done. Other applications worth mentioning are the existence almost everywhere of (4), with Γ merely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rectifiable and f integrable with respect to arc length, and a proof by D. E. Marshall of an old conjecture of Denjoy concerning the analytic capacity of subsets of rectifiable curves.

Notwithstanding all this work, there remain important problems in this area which seem to be beyond the scope of the methods sketched here. For example, is it true that (2) represents a bounded operator in some of the L^p provided that $a(x)$ is Lipschitzian and the quotient

$$\frac{a(x) - a(y)}{x - y}$$

remains in a compact subset of the domain of analyticity of F? In particular, does (4) represent a bounded operator in L^2 without restriction on the size of $\|a'\|_\infty$?

An affirmative answer to these questions would have important consequences in the theory of boundary value problems for elliptic equations in general Lipschitzian domains.

For a bibliography on this subject see A. P. Calderón, "Commutators, singular integrals on Lipschitz curves and applications," Proceedings of the International Congress of Mathematicians, Helsinki, 1978 (to appear).

BIOGRAPHICAL SKETCH

Alberto P. Calderón was born September 14, 1920, in Mendoza, Argentina. He holds a Civil Engineer degree from the University of Buenos Aires (1947) and a Ph.D. from the University of Chicago (1950). He was a visiting associate professor at Ohio State University (1950-1953), a member of the Institute for Advanced Study (1953-1955), and associate professor of mathematics at the Massachusetts Institute of Technology (1955-1959). He has been professor of mathematics at the University of Chicago since 1959; in 1968 he was appointed to the Louis Block Professorship.

Professor Calderón was a member-at-large of the Council of the AMS (1965-1967). AMS Committees on which he has served include the Transactions and Memoirs Editorial Committee (Associate Editor, 1959-1964); the Organizing Committee for the Symposium on Singular Integrals (Chairman, April 1966); the Nominating Committee for the 1968 Election; the Colloquium Editorial Committee (1971-1976); the Organizing Committees for the 1971 Summer Research Institute, and the July 1978 Summer Institute on Harmonic Analysis in Euclidean Spaces and Related Topics.

Professor Calderón gave an invited address at the summer meeting of the Society in University Park, Pennsylvania (August 1957) and has spoken at symposia on Partial Differential Equations (Berkeley, April 1960) and on Singular Integrals (Chicago, April 1966), and at the International Congress of Mathematicians in 1966 (Moscow) and 1978 (Helsinki). He delivered the Colloquium Lectures on the topic "Singular Integrals" at the summer meeting of the Society in Ithaca (August 1965).

Professor Calderón was elected to membership in the National Academy of Sciences in 1968. His interests include Fourier series, harmonic analysis, ergodic theory, functional analysis, singular integrals, and partial differential equations. He has been a member of the Society since 1949.

Salomon Bochner

Salomon Bochner was born in Cracow in Austria-Hungary on August 20, 1899. He received a Ph.D. from the University of Berlin, and was lecturer at the University of Munich from 1927-1932. In 1933 he joined Princeton University, starting out as an associate, and becoming Henry Burchard Fine Professor in 1959. He retired from Princeton in 1968 and has since then been Edgar Odell Lovett Professor at Rice University. Professor Bochner was an International Education Board Fellow at Copenhagen, Oxford and Cambridge Universities (1925-1927). He has been consultant to the Los Alamos Project at Princeton University (1951-1952) and to the National Science Foundation and Air Research and Development Command (1952-1962). He was also visiting professor at the University of California in 1953, and a member (part time) at the Institute for Advanced Study (1945-1948).

Professor Bochner was vice president of

the American Mathematical Society (1957, 1958). He has served on many Society committees, including the Committee to Select Hour Speakers for Eastern Sectional Meetings (1941-1942); the Committee on the Conference in Analysis of the International Congress of Mathematicians (1950); the Committee to Select Gibbs Lecturers for 1952 and 1953; the Organizing Committee for Summer Institutes (1952-1955); the Project Committee for the Second Summer Mathematical Institute (1954); the Colloquium Editorial Committee (chairman, 1955-1960); the Committee to Award the Bôcher Prize in 1958 (chairman); the Committee on Expository Books (1958-1959); the Committee to Consider By-laws Concerning Election and Term of the Executive Director (1960); the Selection Committee for Expository Articles in the Bulletin (chairman, 1962-1966); and the Committee to Nominate Members of the Executive Committee for 1962. He was a delegate to the International Congress of Mathematicians in 1958.

Professor Bochner has given addresses at the Conference on Theory of Integration, Chicago, 1941, and at the International Congress of Mathematicians in 1950. He gave invited addresses at New York (October 1942) and at Kingston, Ontario (August-September 1953). He delivered the Colloquium Lectures at the 1956 Summer Meeting of the Society in Seattle.

He has been a Consulting Editor for the McGraw-Hill Encyclopedia of Science and Technology since its inception, and he was an Editor of the Dictionary of the History of Ideas (Scribner, 1973).

Professor Bochner is a member of the National Academy of Sciences. His research interests include probability theory, harmonic analysis, complex manifolds, complex variables, complex and almost periodic functions, Fourier analysis. He has been a member of the Society since 1934.

Salomon Bochner

Hans Lewy

The Cauchy-Kovalevski theorem (C.-K.) is the fundamental theorem in the study of partial differential equations. It concerns analytic equations and their local-analytic solutions. In an attempt to see more clearly how dropping this double analyticity hypothesis would affect the local existence of solutions, I undertook the work for which you kindly awarded a Steele prize.

The simplest case to consider, of course, is the linear case

(E) $\sum_{j=1}^{n} A_j(x)\frac{\partial u}{\partial x} = f(x), \quad x \in \mathbb{R}^n;$

$A_j(x)$ and $f(x)$ complex-valued C^∞ functions. Here the simplest case is $n = 3$, $f = 0$, since the case $n = 2$ leads to the well-studied elliptic system occurring in the quasi-conformal mapping of the plane. Even if the $A_j(x)$, $j = 1,2,3$, are real and real-analytic, the solution u need not be analytic, as is well known. In the general case of truly complex-valued, real-analytic $A_j(x)$, one can construct by C.-K. two independent solutions z_1, z_2. They define a patch of a 3-dimensional surface S of $\mathbb{C}^2$, and every third local-analytic solution $u(x)$ then turns out to be the trace on S of a holomorphic function of z_1, z_2 with (E) serving as the tangential linear combination of the Cauchy-Riemann equations with respect to z_1, z_2. If we drop the hypothesis of the analyticity of $u(x)$, however, it could be expected—and it was proved—that, in general, u can still be extended as a holomorphic function of z_1, z_2, but only on one side of S, in an open set of $\mathbb{C}^2$ depending on the domain of existence of $u(x)$ but not on the particular solution u.

Next, let the $A_j(x)$ be no longer analytic; here C.-K. no longer yields the solutions z_1, z_2, but one obtains the same result by postulating their existence. Will there always be such z_1, z_2, however? In the search for an answer, I tried a continuity method in which the $A_j = A_j(x,t)$ are made to depend upon a real parameter t, and which reduces, for $t = 0$, to the simplest possible case, namely linear coefficients. This in turn leads to the corresponding inhomogeneous (E) with $f(x)$ no longer analytic but still C^∞. I was surprised to notice that the existence of even a single solution to this equation imposes a severe restriction on $f(x)$; in fact, one can choose $f(x) \in C^\infty$ such that no solution u exists in any open set of $\mathbb{R}^3$.

I then tried to generalize the homogeneous $n = 3$ case described earlier to $n = 4$. Given are the three independent solutions z_1, z_2, z_3 of

(E), which becomes the tangential linear combination of the Cauchy-Riemann equations on a 4-dimensional surface of $\mathbb{C}^3$ (codimension 2), parametrized by $x_1, \ldots, x_4$. Does there exist an open set of $\mathbb{C}^3$ into which all solutions u(x) of (E), existing in the same domain of $\mathbb{R}^4$, can be continued as holomorphic functions of z_1, z_2, z_3? All I could do at that point was to construct an example which confirmed that conjecture.

The papers alluded to above go back some twenty years. Fortunately, these and related problems have found an echo in the minds of some powerful analysts, who have immensely enlarged the scope of these problems and who have provided answers which have gone beyond my most daring hopes and which are basic to the theory of partial differential equations.

BIOGRAPHICAL SKETCH

Hans Lewy was born in Breslau, Germany, on October 20, 1904. He received a Ph.D. from the University of Göttingen in 1926. He was Privat-docent at the University of Göttingen from 1927-1933, and associate at Brown University from 1933-1935. He was lecturer at the University of California, Berkeley, 1936 to 1937, and advanced from assistant professor to professor there between 1937 and 1972. He has been professor emeritus at Berkeley since 1972. Professor Lewy has held Rockefeller Foundation fellowships at the University of Rome (1929-1930) and the University of Paris (1930-1931).

Professor Lewy was a member of the AMS Subcommittee on Preparation for Research of the War Preparedness Committee in 1941 and 1942. He was an AMS representative to the Division of Physical Sciences (1964-1967) and to the Division of Mathematics of the National Research Council (1965-1967). He has been on the Editorial Boards of the Indiana University Mathematical Journal and of the Annali della Scuola Normale di Pisa. He accepted invitations to address the Society at the California Institute of Technology (November 1945), and at the Annual Meeting in Berkeley (December 1954). He also presented an invited address at the International Congress of Mathematicians in 1950.

Professor Lewy is a member of the Mathematics Section of the National Academy of Sciences (U.S.A.) and of the Accademia dei Lincei (Rome). His major research interests include calculus of variations, partial differential equations, and hydrodynamics. He has been a member of the Society since 1934.

Organizers and Topics of Special Sessions

Names of the organizers of special sessions to be held at meetings of the Society are listed below, along with the topic of the session. Papers will be considered for inclusion in special sessions, if their abstracts are submitted to the Providence office by the deadlines given below. These deadlines are three weeks earlier than those for abstracts for regular sessions of ten-minute contributed papers. The most recent abstract form has a space for indicating that the abstract is for a special session. If you do not have a copy of this form, be sure your abstract is clearly marked "For consideration for special session (title of special session)." Papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

765th Meeting	New York, New York, April 1979	Deadline: Expired
Donald Dawson and Harry Kesten	Probability theory inspired by applications	
Harold M. Hastings	Homotopy theory	
Heisuke Hironaka and George R. Kempf	Algebraic geometry	
Gangaram S. Ladde	Mathematical modelling	
Gerard J. Lallement	Algebraic and topological semigroups	
Louis F. McAuley	Monotone and open mappings	
766th Meeting	**Iowa City, Iowa, April 1979**	**Deadline: Expired**
Daniel D. Anderson	Commutative ring theory	
Kent R. Fuller	Noncommutative ring theory	
William H. Jaco	Three-dimensional manifold theory	
James P. Kuelbs and Walter V. Philipp	Probability on Banach spaces	
Richard P. McGehee	Celestial mechanics	
Paul S. Muhly	Operator theory	
John C. Polking	Several complex variables	
767th Meeting	**Vancouver, Canada, June 1979**	**Deadline: April 3**
Priscilla E. Greenwood	Probability	
Stanley S. Page	Representations and ring theory	
Lon M. Rosen	Mathematical physics	
	Boulder, Colorado, March 1980	**Deadline: To be announced**
Lawrence W. Baggett and Arlan B. Ramsay	Nonabelian harmonic analysis	
Alan Day and Walter F. Taylor	Lattice theory and general algebra	
Karl E. Gustafson	Topics in mathematical physics	