Hermann Weyl in Zurich 1950–1955

Beno Eckmann

Hermann Weyl (1885–1955), a German mathematician of the Göttingen school, spent much of his mathematical career at the Swiss Federal Institute of Technology (Eidgenössische Technische Hochschule, ETH) in Zurich, and later at the Institute for Advanced Study in Princeton. Excellent expositions of his well known work in representation theory, harmonic analysis, foundations, and theoretical physics are in the literature, including ones by Weyl himself. In what follows, I give some personal recollections of Hermann Weyl.

Zurich 1913–1930

Hermann Weyl and Zurich: All biographies include the well-known fact that he was professor of mathematics at the ETH in Zurich from 1913 to 1930. The majority of his world-famous papers were written during that period, as well as his books which became best-sellers like *Space, Time, Matter* and *The Group Theoretic Method in Quantum Physics.* People from all over the world came to Zurich to see him and to discuss important problems. Zurich and the ETH were one of the centers of mathematical research.

In 1930 Weyl left Zurich to succeed David Hilbert in Göttingen. But in 1933, when the Nazis came to power in Germany, he accepted an invitation from the Institute for Advanced Study in Princeton. We should realize that, while not Jewish, Weyl was a strong opponent of the Nazi spirit and his (first) wife from Göttingen was Jewish.

He left Europe quite reluctantly; the spirit, the language, the history, everything seemed quite strange. But Princeton turned out to be a very good place for him to work and to maintain his international contacts. He made various trips, including visits to the mathematicians of the ETH in Zurich where he found a small but very interested group.

1950–1955

What is less known is the importance of his activity during his time in Zurich in 1950–55. In 1950, approaching the age of 65, he decided to retire from the Institute and move back to Zurich. The official retirement year seems to be 1951. But from April 1950 on until his death in 1955 he spent most of his time in Zurich—interrupted by shorter stays in Princeton and, of course, lecturing trips. He was remarried in 1950 to Ellen Bär, the widow of a physics professor at Zurich University who had died at a rather young age. The families Bär and Weyl had known each other well during the earlier time in Zurich. Hermann Weyl's first wife had died in Princeton in 1948. In 1947 when we, my wife Doris and I, were at the Institute she was already ill and in hospital.

Ellen Weyl-Bär (also Jewish) was a very cultivated person, active in the arts as a gifted sculptor and, in earlier times, a violinist. She knew personally many artists, painters, sculptors, musicians, and writers. She created for Weyl a most fascinating atmosphere of culture and science. From his writings we know how deeply he was interested in all cultural activities.

During 1950–55 Hermann Weyl, whenever he was in Zurich, maintained close contacts with the mathematicians of the department. He had no official status but simply came to the main building of the ETH to discuss the many aspects of mathematics he was interested in. He attended

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lectures and student seminars and, of course, gave, himself, beautiful colloquium talks. It was a very happy time for me. Since we knew each other well from the Princeton time he would often come to my office and ask questions. My answers would eventually always result in some new things I could learn from him and from his magnificent classical mathematical background. His questions centered around the de Rham theorem on differential forms on a compact manifold and generalizations; or around fixed point theorems, vector fields, Hodge and Kähler structures, and many similar things.

ICM Amsterdam 1954

A particularly interesting period was spring and early summer 1954. This was the year of the International Congress of Mathematicians in Amsterdam. Weyl, as president of the Fields Medal committee, had to report on the work of the two Fields Medal winners, Kunihiko Kodaira and Jean-Pierre Serre. He clearly was well aware of the merits of their prize-winning work, harmonic analysis on manifolds (Kodaira) and homotopy groups of spheres (Serre). He felt, however, that he should know more about the "modern" techniques in these fields. He wanted to learn for example, in connection with Kodaira's work, the details of the proof of the de Rham theorem, and of the Hodge-de Rham decomposition of the vector space of differential forms on a compact Riemannian manifold. Or, concerning Serre, the concept and importance of homotopy groups and their relation to homology, or the Hopf algebra structure of the cohomology of loop spaces, or the technique of spectral sequences, etc. So there were for me the interesting and clearly most gratifying tasks of teaching topics from a "modern" viewpoint. What an unusually gifted student!

The lecture in Amsterdam was a brilliant account not only of the work of the medalists but also of the role of algebraic topology throughout many fields of mathematics. Weyl himself knew very well that in earlier times, almost until the beginning of World War II, algebraic topology had not been fully recognized by the mathematical community—and now it appeared in a glorious way in the work of the two winners, in quite different contexts.

A Student Course in Zurich

An extraordinary story, typical of Hermann Weyl, happened in 1950 soon after his return to Zurich from Princeton. I was a young professor at ETH, nominated in 1948. In the summer of 1950 there was the first post-war International Congress of Mathematicians, at Harvard University. I was invited to deliver a lecture in the topology session. There were no funds available for the very high travel and living costs; so it was arranged by the American Mathematical Society that I give a short course at a midwestern university. This meant that I had to leave Zurich three weeks before the end of the semester and had to find replacements for my ETH lecture courses. This was not easy for one of them, my course on number theory. When Weyl, during one of his frequent visits to our department, heard about this he immediately offered to do it in place of an assistant. I hesitated, but he said "No red tape, don't inform the administration. Just give me your notes." We proceeded in that way. He entered the classroom, gave his name, and said that he would replace me for the last three weeks of the semester. The students were happy about his lecturing. To what extent they realized what a world-famous substitute I had is not clear. When I returned after the Congress, the dean and rector and president were rather angry at me—not having known about that activity of Weyl at the ETH. Our arrangement had of course been illegal but could not be changed. What seemed important was Weyl's feeling to still be part of the mathematics life at the ETH Zurich.

70th Birthday

In the fall of 1955 the ETH and the mathematics-physics group organized a wonderful 70th birthday celebration for Hermann Weyl. Let me just mention the remarkable talk by Wolfgang Pauli on the overall importance of Weyl's work, and the dinner party at one of the Zurich guildhouses. One month later Weyl brought to a neighorhood post office all the letters he had written to thank people for the congratulations. They were handwritten on a small white card which had, at the top, a Goethe verse as imprint: "Willst Du ins Unendliche schreiten, geh nur im Endlichen nach allen Seiten (you want to go to infinity, just go in the finite domain in all directions)." These simple words from (complex) projective geometry were an adequate description of Weyl's lifetime work and thinking. On the way back from the post office he was struck down by a heart attack. He fell to the street and died on the spot.

A volume *Selecta Hermann Weyl* was published, with the help of the ETH, the Institute for Advanced Study, and Birkhäuser Verlag Basel, in honor of the 70th birthday. Weyl himself had selected, from the huge number of his publications, those papers which should be included—and those which should not! He was enthusiastic about the project and had very strong opinions concerning his own work. We, that is Heinz Hopf, Michel Plancherel, and myself, had long discussions with him about that choice but could not always follow his arguments. Of course we accepted his decisions. The volume appeared early in 1956. It was sad that Weyl could not see it and hold the final copy in his hands.

2006 Gauss Prize

Kiyoshi Itô

On August 22, 2006, the first Carl Friedrich Gauss Prize was awarded at the opening ceremonies of the International Congress of Mathematicians (ICM) in Madrid, Spain. The prizewinner is Kiyoshi Itô. The prize honors his achievements in stochastic analysis, a field of mathematics based essentially on his groundbreaking work.

The Work of Kiyoshi Itô

The following announcement of the Gauss Prize is adapted from an IMU news release.

Carl Friedrich Gauss (1777–1855), known as "princeps mathematicorum" ("prince of mathematicians"), united within his person two sides of mathematics. He not only achieved great progress in number theory, called the "queen of mathematics" because it was glorious as well as far from any real-world applications, a statement that was valid until a few decades ago. He also created what is today called the "least-squares fit", a method that is applied every time you have to deal with real-world problems, in particular measuring inaccuracies.

The newly created Gauss Prize honors "mathematical research that has had an impact outside mathematics—either in technology, in business, or simply in people's everyday lives". The prize is awarded jointly by the Deutsche Mathematiker-Vereinigung (DMV, German Mathematical Society) and the International Mathematical Union (IMU) and administered by the DMV. It consists of a medal and a monetary award (currently valued at 10,000 €, approximately US$13,000). The source of the prize is the surplus from the ICM held in Berlin in 1998. The prize was awarded for the first time at this year's ICM in Madrid.

Kiyoshi Itô's theoretical work has had an enormous impact and shows that the route from real-world phenomena, to abstract mathematical description, and back to real-world application, is often long and circuitous.

In Itô's case, the route begins with a look in a microscope at water containing pollen grains or dust particles that move around in an erratic way. Today, this strange dance is called Brownian motion after the Scotch botanist Robert Brown, who observed it in 1827 and gave a detailed description. The motion of the particles is the net effect of a large number of collisions between the particles and the water molecules. It was Albert Einstein (1879–1955) who formulated a mathematical model of Brownian motion in one of his three papers of 1905, each of which amounted to a revolution in science. Norbert Wiener (1894–1964), better known as the founder of cybernetics, followed in 1923 with a proof that Einstein's model was mathematically sound.

It turned out that Einstein and Wiener had found a mathematical idealization of pure chance. A Wiener process can be used to describe how dust particles move in water, how lines at a checkout counter grow or shrink, or how the price of a share of stock fluctuates. This model is about as universal as Gauss's normal distribution, which turns up every time a quantity is influenced by many independent perturbations.

This idealization, however, came at a price. Physicists normally assume that nature behaves smoothly in some sense, but a Wiener process violates this assumption in a fundamental way. The particle's path is infinitely wiggly, or nowhere differentiable in technical terms. Moreover, it is infinitely long. Just a hundred years ago, mathematicians used to turn away in horror from those "monsters", as they are intractable with the classical tools to handle curves.

One of those classical tools is the integral. Beginning in 1942, Itô reformulated the classical way of defining the integral and created a new concept called the "stochastic integral". This work included calculation rules and a solution theory for stochastic differential equations. An ordinary differential equation is the form mathematicians use to describe the motion of a particle under a known (deterministic) force; a stochastic differential equation incorporates forces that depend on, for instance, a Wiener process.

A stochastic integral in itself can never be the final solution of a problem. You cannot know a random process in advance—if you could, it wouldn't be random—so it's useless to ask where a point subject to a stochastic differential equation will be five minutes from now, or when it will cross a given line for the first time. However, Itô's method provides probabilities of the occurrence of events of that kind.

Consider a gambler's assets in a game like roulette, or the value of an investor's portfolio. The gambler and the investor have a vital interest in knowing when a certain value, which depends on chance as well as on their own actions, first crosses the zero line. Risk-averse players, in particular bankers, want to direct their own actions so as to minimize the effects of chance.

This is the idea underlying financial instruments like options and futures. A call option amounts to a bet on a future event, such as the price of a share reaching a certain value. The bank offering the option must make provisions to have the money available on the day it might lose the bet; the cost of those provisions is the price of the option that the bank charges the customer. The bank, however, is free to take these provisions, depending on the actual share price, at any time from now on up to the time the option is due. The option price can be expressed and calculated by a stochastic integral.

Today the most influential applications of Itô's theory are found in finance. At the beginning of the 1970s, the economists Fischer Black, Robert C. Merton, and Myron S. Scholes found an explicit formula to calculate the price of an option. Today the Black-Scholes formula, which contains only known data, underlies almost all financial transactions that involve options or futures; moreover it won Merton and Scholes the Nobel Prize in Economics in 1997 (Black died in 1995).

But Itô's theory is sufficiently abstract to serve completely different needs. It applies also to the size of a population of living organisms, to the frequency of a certain allele within the gene pool of a population, or to even more complex biological quantities. Chance is not completely blind in these cases: The average fluctuation of a population size is not a constant but proportional to the size, and the frequency of two alleles that occupy about half of the population tends to change more rapidly than if one of them is close to extinction. For such applications the concept of a Wiener process had to be generalized, a task that would have been almost impossible without Itô's theoretical framework. Biologists can now assess the probability with which a gene will dominate the whole population or a species will survive.

These generalizations, in turn, came in handy for the economists. In 1985 John C. Cox, Stephen A. Ross, and Jonathan E. Ingersoll found a mathematical model for the time evolution of interest rates that has now become standard. Stephen L. Heston generalized the Black-Scholes model in 1993 so as to bring it closer to reality.

On the other hand, it took mathematicians themselves quite a while to appreciate the importance of Itô's results. This is partly due to Japan's isolation during World War II. Itô lectured on his achievements during a stay at the Institute for Advanced Study in Princeton in 1954–1956.

Moreover, there was a competing theory available to describe the effects of pure chance on a more global level. If you want to know how a drop of ink disperses in water, you can either try to follow the Brownian motion of single ink particles, or you can consider both ink and water as a continuum and formulate their motion in terms of a partial differential equation—a diffusion equation in this case. As both methods describe the same physical phenomenon, they should lead to the same results, so there should be some connection between them. It took some time to bring this connection to light, but it has already been put to good use. The Black-Scholes formula contains the solution of a diffusion equation.

Today, there is no doubt that stochastic analysis is a rich, important, and fruitful branch of mathematics with a formidable impact on technology, business, and everyday life.

Biographical Sketch

Kiyoshi Itô was born on September 7, 1915, in Japan. He graduated from the Imperial University of Tokyo in 1938. He served in the Statistics Bureau of the Cabinet Secretariat of Japan from 1939 until 1943, when he became an assistant professor in the Faculty of Science at Nagoya Imperial University. In 1945 he received his D.Sc. from the Imperial University of Tokyo. He became a professor at Kyoto University in 1952 and professor emeritus in 1979. He then joined the faculty of Gakushin University and retired from there in 1985. He held visiting positions at the Institute for Advanced Study (1954–56), Aarhus University (1966–69), and Cornell University (1969–75). He served as director of the Research Institute for Mathematical Sciences at Kyoto University from 1976 to 1979.

Itô has received some of Japan's highest honors, including the Asahi Prize (1978), the Imperial Prize and the Japan Academy Prize (1978), and the Fujiwara Prize (1987). He also received the Wolf Prize in 1987 and the Kyoto Prize in 1998. He is an associate foreign member of the Académie des Sciences of France, a member of the Japan Academy, and a foreign member of the U.S. National Academy of Sciences. He has received honorary degrees from the Eidgenössisches Technische Hochschule in Zurich and the University of Warwick.

—*Allyn Jackson*

Mathematics People

PECASE Awards Announced

Fifty-six young researchers were chosen to receive the 2005 Presidential Early Career Awards for Scientists and Engineers (PECASE). This award is the highest honor bestowed by the U.S. government on outstanding young scientists, mathematicians, and engineers who are in the early stages of establishing their independent research careers. JONATHAN C. MATTINGLY of Duke University was selected for his work in developing mathematical tools that include the effects of randomness in studying models of complex systems.

The recipients were selected from nominations made by eight participating federal agencies. Each awardee receives a five-year grant ranging from US$400,000 to nearly US$1 million to further his or her research and educational efforts.

—From an NSF announcement

MAA Writing Awards Presented

The Mathematical Association of America (MAA) presented several awards for excellence in expository writing at its Summer Mathfest in Knoxville, Tennessee, in August 2006.

The Trevor Evans Award is given to authors of expository articles that are accessible to undergraduates and that were published in *Math Horizons*. This prize carries a cash award of US$250. The 2006 prizes were awarded to RONALD BARNES and LINDA BECERRA of the University of Houston for their joint article "The evolution of mathematical certainty", *Math Horizons*, September 2005, and to STUART BOERSMA of Central Washington University for "A mathematician's look at Foucault's Pendulum", *Math Horizons*, February 2005.

The George Pólya Award is given for articles published in *The College Mathematics Journal* and has a cash prize of US$500. The awardees for 2006 are EZRA BROWN of Virginia Tech for "Phoebe floats!", *College Mathematics Journal*, March 2005, and JAMES SANDEFUR of Georgetown University for "A geometric series from tennis", *College Mathematics Journal*, May 2005.

The Carl B. Allendoerfer Award is given for articles published in *Mathematics Magazine* and carries a cash award of US$500. The 2006 awardees are ROBB T. KOETHER and JOHN K. OSOINACH JR. of Hampden-Sydney College for their joint article "Outwitting the lying oracle", *Mathematics Magazine*, vol. 78, no. 2, 2005, and JEFF SUZUKI of Brooklyn College for "The lost calculus (1637–1670): Tangency and optimization without limits", *Mathematics Magazine*, vol. 78, no. 5, 2005.

The Lester R. Ford Award honors articles published in *The American Mathematical Monthly* and carries a cash prize of US$500. The 2006 awardees are: IBTESAM BAJUNAID of King Saud University, JOEL M. COHEN of the University of Maryland, FLAVIA COLONNA of George Mason University, and DAVID SINGMAN of George Mason University for their joint article "Function series, Catalan numbers, and random walks on trees", *Monthly*, November 2005; VIKTOR BLÅSJÖ of Marlboro College for "The evolution of the isoperimetric problem", *Monthly*, June–July 2005; EDWARD B. BURGER of Williams College for "A tail of two palindromes", *Monthly*, April 2005; KARL DILCHER of Dalhousie University and KENNETH B. STOLARSKY of the University of Illinois, Urbana-Champaign, for their joint article "A Pascal-type triangle characterizing twin primes", *Monthly*, October 2005; and WILLIAM DUNHAM of Muhlenberg College for "Touring the calculus gallery", *Monthly*, January 2005.

A new prize, the Annie and John Selden Prize for Research in Undergraduate Mathematics Education, honors a researcher who has established a significant record of published research in undergraduate mathematics. The 2006 prize was awarded to CHRIS RASMUSSEN of San Diego State University.

The Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member honors a beginning college or university teacher whose teaching has been extraordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have influence beyond his or her own classroom. The 2006 awardees are GERIKAI (KAI) CAMPBELL of Swarthmore College, CHRISTOPHER N. SWANSON of Ashland University, and LESLEY WARD of Harvey Mudd College.

—From an MAA announcement

NSF Postdoctoral Fellowships Awarded

The Mathematical Sciences Postdoctoral Research Fellowship program of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) awards fellowships each year for postdoctoral research in pure mathematics, applied mathematics and operations research, and statistics. Following are the names of the fellowship recipients for 2006, together with their Ph.D. institutions (in parentheses) and the institutions at which they will use their fellowships.

DANIEL ABRAMS (Cornell University), Massachusetts Institute of Technology; NATHAN L. ALBIN (University of Utah), University of Maryland, College Park; PEDRO ALBIN (Stanford University), Massachusetts Institute of Technology; DREW D. ARMSTRONG (Cornell University), University of Minnesota, Minneapolis; JAYADEV S. ATHREYA (University of Chicago), Yale University; MICHAEL F. BARAD (University of California, Davis), Stanford University; MARGARET A. BECK (Boston University), University of Surrey; OREN BEN-BASSAT (University of Pennsylvania), Columbia University; NICHOLAS K. ERIKSSON (University of California, Berkeley), Stanford University; TALIA FERNOS (University of Illinois, Chicago), University of California, Los Angeles; JULIA E. GRIGSBY (University of California, Berkeley), Columbia University; MICHAEL N. GURSKI (University of Chicago), Yale University; LAWRENCE D. GUTH (Massachusetts Institute of Technology), Stanford University; AARON D. HOFFMAN (Brown University), Boston University; PAUL M. JENKINS (University of Wisconsin, Madison), University of California, Los Angeles; JESSE JOHNSON (University of California, Davis), Yale University; RICHARD P. KENT (University of Texas, Austin), Brown University; ROBERT LIPSHITZ (Stanford University), Columbia University; SARAH K. MASON (University of Pennsylvania), University of California, Berkeley; KELLY L. MCKINNIE (University of Texas, Austin), Emory University; KARIN H. MELNICK (University of Chicago), Yale University; ROBERT W. NEEL (Harvard University), Columbia University; JAMES H. NOLEN (University of Texas, Austin), Stanford University; WILLIAM R. OTT (University of Maryland, College Park), Courant Institute of Mathematical Sciences, New York University; JESSE D. PETERSON (University of California, Los Angeles), University of California, Berkeley; ARJUN RAJ (Courant Institute of Mathematical Sciences, New York University), Massachusetts Institute of Technology; BENJAMIN I. SCHMIDT (University of Michigan, Ann Arbor), University of Chicago; SHAWN C. SHADDEN (California Institute of Technology), Stanford University; ROBERT M. STRAIN (Brown University), Harvard University; CHRIS M. WENDL (New York University), Massachusetts Institute of Technology; BENJAMIN D. WIELAND (University of Chicago), Brown University.

—NSF announcement

Zoller Awarded 2006 Dirac Medal

PETER ZOLLER of the University of Innsbruck and the Austrian Academy of Sciences has been awarded the Dirac Medal for 2006 by the Abdus Salam International Centre for Theoretical Physics (ICTP). Zoller was honored for his "innovative and prolific accomplishments in atomic physics, including his seminal work in proposing methods to use trapped ions for quantum computing and describing how to realize the Bose-Hubbard model and associated phase transitions in ultracold gases."

The ICTP awarded its first Dirac Medal in 1985. Given in honor of P. A. M. Dirac, the medal is awarded annually on Dirac's birthday, August 8, to an individual or individuals who have made significant contributions to theoretical physics and mathematics. The medalists also receive a prize of US$5,000. An international committee of distinguished scientists selects the winners from a list of nominated candidates. The Dirac Medal is not awarded to Nobel laureates, Fields Medalists, or Wolf Foundation Prize winners.

—From an ICTP announcement

European Mathematical Society Article Competition

The European Mathematical Society (EMS), through its Committee for Raising Public Awareness of Mathematics (RPA), has announced the winners of its article competition for 2005. Articles appearing in newspapers or other general-interest magazines in the authors' home countries were eligible.

F. THOMAS BRUSS of Université Libre de Bruxelles, Belgium, was awarded first prize for his article "Die Kunst der richtigen Entscheidung", published in the magazine *Spektrum der Wissenschaft* (*Scientific American*, German edition), June 2005, and in *Pour La Science* (*Scientific American*, French edition), September 2005.

TOM M. APOSTOL of the California Institute of Technology was awarded second prize for his article "A visual approach to calculus problems", published in the magazine *Engineering and Science*, California Institute of Technology, 2000; the article is available at the website `http://pr.Caltech.edu/periodicals/EandS/archives/LXIII_3.html`.

HANSJÖRG GEIGES of the Mathematisches Institut, Universität Köln, Germany, received third prize for his article "Christiaan Huygens and contact geometry", published in the Dutch magazine *Nieuw Archief voor Wiskunde* in 2005. The article is available at `http://www.mi.uni-koeln.de/~geiges/naw05.pdf`.

Editor's Note:

Because of incorrect information received by the *Notices*, an out-of-date announcement about this competition appeared in the September 2006 issue. We regret the error.

—From International Mathematical Union email newsletter