Martin D. Kruskal Receives National Medal of Science

Martin D. Kruskal of Rutgers University was awarded the National Medal of Science, the nation's highest honor for achievement in science. He received the medal "for influence as a leader in nonlinear science for more than two decades". President Clinton presented the awards to eight scientists at a ceremony at the White House in late September.

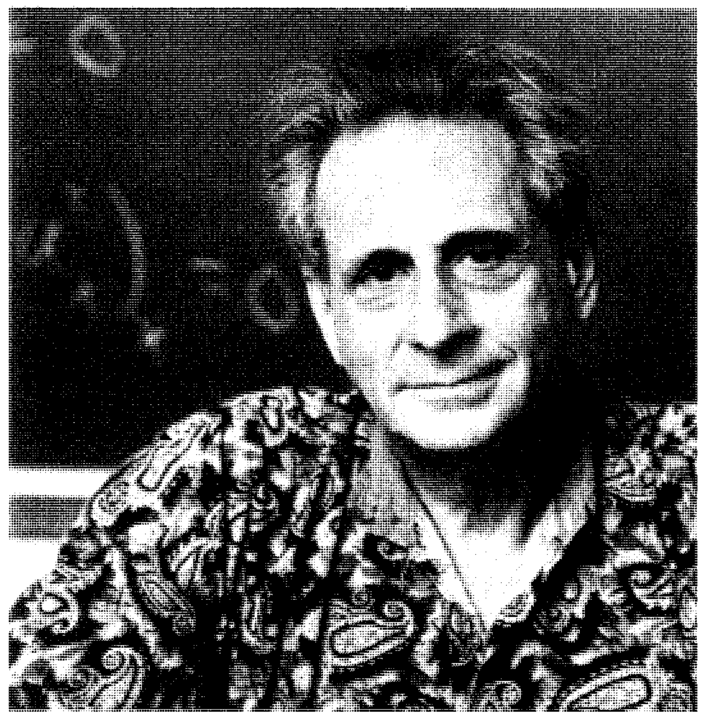
Commentary on Kruskal's Work

The following piece about Professor Kruskal's work was prepared, at the request of the *Notices*, by Mark J. Ablowitz of the University of Colorado in Boulder, John Greene of General Atomics in San Diego, and Harvey Segur of the University of Colorado in Boulder. These three authors also prepared the biographical sketch that follows this commentary.

Martin D. Kruskal has made profound contributions in pure and applied mathematics. His research is remarkably diverse and includes seminal discoveries in the mathematics of plasma physics, relativity, asymptotic analysis and perturbation theory, surreal numbers, and differential equations. In his extensive studies of differential equations he has forever changed our understanding of nonlinear partial differential equations by the discovery of solitons and the analysis of their governing equations.

The study of theoretical plasma physics dominated Martin Kruskal's early work. In this research (see for example references [1–4] below) he helped develop some of the basic principles of plasma physics, and found new exact solutions to the governing equations and novel mathematical methods to analyze the stability of plasma waves. In the 1950s there was a great deal of discussion regarding the nature of plasma oscillations. The linearized evolution equations possess a continuous spectrum, which prevents the use of standard normal-mode analysis. This raises many subtle questions about the nature of irreversibility in time-reversible systems, causality, and the physical meaning of singular eigenfunctions. The notion that there are nonlinear solutions of the governing equations that are more transparent than the linearized solutions is an important result in this context. The publication containing this revelation is known as the "BGK" (Bernstein-Greene-Kruskal) paper [1]. In a further important development arising from the study of plasma dynamics, Kruskal and Oberman introduced the "KO" principle for stability. The use of the invariants of the system presages much, if not most, of what is often referred to as the Arnol'd energy method.

In 1960 Kruskal published a paper [5] in which he concretely showed that in suitable coordinates, often termed "Kruskal coordinates" by relativists, apparent singularities in certain solutions (e.g., the Schwarzschild solution) of the equations of general relativity are in fact not singular away from the origin. These coordinates allow analysis in the neighborhood of a black hole to be carried out effectively.



Martin D. Kruskal (Photo courtesy of Rutgers University)

A recurrent theme in Kruskal's research has been the use and systematic development of asymptotic analysis and perturbation theory. He spelled out his ideas in his 1963 paper "Asymptotology" [6] in which he developed a set of principles that apply to problems in definite limiting cases.

An important application of Kruskal's systematic use of asymptotic analysis was an approach to the understanding of what we would now call the chaotic nature of magnetic lines of force in a toroidal configuration. Since magnetic lines are a characteristic of virtually any model of plasma physics, solutions of these equations cannot be understood without some conception of the magnetic geometry. In the pre-computer days of the 1950s he formulated the problem as one of understanding the unavoidable inaccuracy of asymptotic series for the approximate locations of the magnetic lines. In our computer age there are alternative ways to study chaos, but the value of such asymptotic analyses remains in providing analytical results that can be used to verify computational results.

In the course of his work on plasma physics, Kruskal illuminated the importance of asymptotic approximations as opposed to ad hoc approximations, and a number of significant results emerged, e.g., the analysis of the adiabatic invariance of the magnetic moment of a charged particle gyrating in a magnetic field. An adiabatic invariant is a formal constant in the asymptotic theory, but usually not in the exact problem. Kruskal, in various papers (see [7] for an historical perspective), represented the adiabatic invariant by an appropriate asymptotic series, which when truncated at any order, undergoes a change of smaller order than the last retained term. This shows that the actual change is small beyond all orders of the expansion.

Generalizations of the asymptotic studies in plasma physics led Kruskal to the understanding of solutions of Hamiltonian equations that display, in their time dependence, multiple scales that depend on the perturbation amplitude. In the paper on Hamiltonian systems with nearly periodic solutions [7] Kruskal systematically constructs new variables and recursively obtains better and better approximations to nearly periodic solutions.

The question of analyzing asymptotic solutions "beyond all orders" has been a strong motivation for much of his recent work on asymptotic expansions. Kruskal and Segur [8] analyzed a nonlinear ordinary differential equation arising in crystal growth and developed a method showing that small terms in the equations (e.g., surface tension) have a physically significant, though exponentially small, effect on the solution. This effect, which is beyond all orders, showed that many of the models of crystal growth were nonphysical. The method developed in their paper has since been used by numerous researchers in a variety of problems.

The issue of analyzing, in general, asymptotic series with terms beyond all orders has also been one of the motivations behind Kruskal's analysis and development of surreal numbers. Surreal numbers are an extension of the real number system that comprise both infinitely large (e.g. the ordinal numbers) and infinitesimal numbers. They were originally conceived and developed by John Conway over twenty years ago. Kruskal has given a direct and constructive way to generate the surreal numbers and has moved towards being able to use surreal numbers as fundamental entities in calculus. Anyone who discusses surreal numbers with Kruskal comes away appreciating his firm belief that they are beautiful, fundamental, and natural.

Perturbation theory also led Kruskal from the anharmonic lattice system of Fermi-Pasta-Ulam to the Korteweg-deVries (KdV) equation,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0,$$

which arose first in 1895 in the context of long water waves of small amplitude. The KdV equation was known to possess special solitary wave solutions, namely steady travelling localized waves (e.g. a moving heap of water of invariant form). In what can only be characterized as a monumental contribution, Zabusky and Kruskal in 1965 [9] recognized that these solitary waves are extremely special. Upon collision, asymptotically in time, the solitary waves regain their amplitudes and speeds and only suffer a phase shift. Zabusky and Kruskal termed these particle-like waves "solitons", a term that is recognized throughout science today. This work was based upon numerical simulation of the KdV equation and was an outstanding early success of scientific computation.

Following the discovery of solitons was a series of far-reaching contributions that culminated in the "inverse scattering method" for solving the KdV equation [10, 11] as an initial-value problem on the line with rapidly decreasing initial data. If the KdV equation were the only such equation with solitons and solvable by inverse scattering, the contribution would have been important but limited in scope. However, the KdV equation is the prototype of a class of "integrable" infinite-dimensional Hamiltonian nonlinear equations that have many features in common, including solitons, infinitely many conserved quantities and related symmetries, Miura and Backlund transformations, special structure in the complex plane such as reductions to ordinary differential equations of Painlevé type, and linearizations by Lax pairs. Such integrable equations have applications in many areas of mathematics and physics, for example: group theory, numerical analysis, fluid dynamics, nonlinear optics, plasma physics, relativity, quantum field theory, and differential, algebraic, and symplectic geometry. The wide-ranging importance and usefulness of the KdV work is perhaps best appreciated by realizing that there have been an enormous number of papers and many textbooks written on various aspects of this subject. In short, this work created a field of study.

Differential equations and their solutions has also been a subject to which Kruskal returns over and over again. In a lovely example of concrete analysis, Clarkson and Kruskal [12] developed a direct method of obtaining special solutions of the Boussinesq equation of fluid dynamics. It turns out that their method yields a class of solutions that can not be found by classical similarity analysis. As such it has regenerated great interest in such similarity methods.

Even after discussing this list of important papers, we note that Kruskal's scientific influence has been augmented by significant unpublished work and the lasting influence on those people lucky enough to have worked with him. Their research often bears his imprint. In particular, they can attest to his dogged insistence on logical thinking, his desire for clear understanding of the basic concepts, and his emphasis on the usefulness of concrete examples.

Biographical Sketch

Martin Kruskal was born on September 28, 1925, in New York City. No doubt his childhood was stimulating; his brothers William (University of Chicago), and Joseph (AT&T Bell Laboratories) are both noted mathematicians. After completing high school at the Fieldston School in Riverdale, NY, he went to the University of Chicago and obtained a BS in mathematics. Fortunately, the Kruskal family lived in New Rochelle, NY, and one of their neighbors was Richard Courant, who was in the process of building an internationally acclaimed academic institute at New York University, now known as the Courant Institute of Mathematical Sciences. Kruskal decided to do his graduate work at this new institute and wrote his Ph.D. thesis on minimal surfaces with Courant as his advisor.

In 1951 Kruskal moved to Princeton and became one of the earliest employees of what is now called the Princeton Plasma Physics Laboratory. At that time it was called Project Matterhorn, and it was among the first places in the world to start research on the possibility of producing useful energy from controlled thermonuclear fusion. Plasma physics was in its infancy, and much of the work was security-classified. In 1956 he was promoted to Associate Head of the Theoretical Division of Project Matterhorn, a position he held until 1964. In 1959 he became a lecturer in astronomy at Princeton University and in 1961 was promoted to Professor of Astrophysical Sciences.

In 1959–1960 Kruskal was awarded a Senior Fellowship of the National Science Foundation, which he used at the Max Planck Institute for Physics and Astrophysics in Munich, Germany. In the winter of 1965–1966 he was the first long-term thermonuclear fusion exchange visitor to the then-USSR. In 1979 he led the US delegation for a binational Academy of Sciences workshop on Soliton Theory in Kiev, USSR, and from there went on to a five-month visit to the University of Nagoya and the Nagoya Plasma Institute. In recent years he has been invited to universities and research institutes around the world, including several trips to India and Australia.

From 1968–1988 Kruskal was director of the Program in Applied Mathematics at Princeton University, and in 1979 he was appointed Professor of Mathematics at Princeton. In 1989 he retired from Princeton, where he is now Professor Emeritus, and took the newly created position of the David Hilbert Professor of Mathematics at Rutgers University.

In 1980 Martin Kruskal was elected to the National Academy of Sciences and in 1983 to the American Academy of Arts and Sciences. He has numerous honors and prizes to his credit, including the AMS Gibbs lectureship in 1979, the Dannie Heineman Prize in Mathematical Physics in 1983, the Potts Gold Medal of the Franklin Institute in 1986, the National Academy of Sciences Award in Applied Mathematics and Numerical Analysis in 1989, and now the 1993 National Medal of Science.

Martin Kruskal has been a long-time member of the American Mathematical Society. He is also a member of the Society for Industrial and Applied Mathematics and a former two-term member of its Board of Trustees, a member of the Mathematical Association of America, and a Fellow of the American Physical Society. He has also served on numerous governmental and external university committees, including the Mathematical Sciences Education Board of the National Research Council.

Martin Kruskal has written more than sixty published papers on a wide range of topics, some of which were described above. He has had a number of Ph.D. students, including Stephen Orszag, currently in the Program in Computational and Applied Mathematics at Princeton; Alfred Ramani at the Ecole Polytechnique in France; Nalini Joshi at the University of New South Wales in Australia; and Ji-Shan Hu at Hong Kong University of Science and Technology.

Finally we mention two of Martin Kruskal's deep interests apart from mathematics: limericks and origami. For years, as director of the Program in Applied Mathematics at Princeton, he wrote an appropriate original limerick to appear on the announcement of the applied mathematics colloquium for each speaker. His interest in origami was motivated by his wife Laura, a world renowned creator and teacher of origami, and his mother, the late Lillian Oppenheimer, who founded the Origami Center of America. Lucky individuals who attend meetings with the Kruskals often receive their own original models!

Martin and Laura Kruskal have three children and six grandchildren, including a set of triplets.

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