1987 Cole Prize in Number Theory Awarded in San Antonio

The Frank Nelson Cole Prize in Number Theory is awarded every five years for a notable research memoir in number theory that has appeared during the previous five years. This prize, as well as the Frank Nelson Cole Prize in Algebra, was founded in honor of Professor Frank Nelson Cole on the occasion of his retirement as Secretary of the American Mathematical Society after twenty-five years and as Editor-in-Chief of the Bulletin for twenty-one years. The original fund was donated by Professor Cole from moneys presented to him on his retirement. It has been augmented by contributions from members of the Society, including a gift made in 1929 by Charles A. Cole, Professor Cole's son, which more than doubled the size of the fund. In recent years, the Cole Prizes have been augmented by awards from the Leroy P. Steele Fund and currently amount to $4,000.

The Twenty-Second Cole Prize was awarded jointly to DORIAN M. GOLDFELD, of Columbia University, and to BENEDICT H. GROSS, of Harvard University, and DON B. ZAGIER, of the University of Maryland and the Max Planck Institute. The prize was awarded at the Society's ninety-third Annual Meeting in San Antonio. The Cole Prize was awarded by the Council of the American Mathematical Society, acting through a selection committee consisting of Paul T. Bateman (Chairman), Enrico Bombieri, and Bernard M. Dwork.

The text below includes the Committee's citation, the recipients' responses on presentation of the award, and a brief biographical sketch of the recipients. Professor Zagier was unable to attend the Annual Meeting to receive the prize in person. He did, however, send a written response to the award.

Citation

The 1987 Frank Nelson Cole Prize in Number Theory is awarded jointly to DORIAN M. GOLDFELD and to BENEDICT H. GROSS and DON B. ZAGIER for giving an effective bound for the discriminants of all imaginary quadratic fields with a given class number—thereby solving a problem that goes back to Gauss. This effective bound was developed in the following two papers:

Benedict H. Gross and Don B. Zagier, *Heegner points and derivatives of L-Series*, Inventiones Mathematicae, volume **84** (1986), pages 225-320; and

Dorian M. Goldfeld, *Gauss's class number problem for imaginary quadratic fields*, Bulletin of the American Mathematical Society, volume **13** (1985), pages 23-37

the latter of which summarizes the results of Goldfeld's earlier paper, *The class number of quadratic fields and the conjectures of Birch and Swinnerton-Dyer*, Annali della Scuola Normale Superiore di Pisa, Serie IV, volume **3** (1976), pages 623-663.

Dorian M. Goldfeld

Response

It is a great honor to receive the Cole Prize, and I should like to take this opportunity to thank the American Mathematical Society and the Cole Prize Committee for considering me.

In 1976, I developed an analytic method for solving the long-standing conjecture of Gauss on class numbers of imaginary quadratic fields; to construct an effective algorithm for finding all imaginary quadratic fields with a given class number. The first case of this problem was solved by K. Heegner in 1952 who showed that there are exactly nine imaginary quadratic fields which are unique factorization domains (class number one). His proof, however, had a gap which was later rectified by H. Stark. Subsequently, both H. Stark and A. Baker gave different proofs of this result, and they also solved the class number two problem. The higher class numbers, however, seemed inaccessible by their methods.

In my approach to this problem, I demonstrated that the Gauss conjecture would follow if one could exhibit the existence of a single L-function, $L(s)$, associated to an automorphic form of arithmetic type on $GL(2)$ for which the same L-function over an imaginary quadratic field has at least a fourth order zero at the center of its critical strip. It can be shown that this requirement will be satisfied for suitable $L(s)$ having a triple zero at the center of its critical strip.

In their brilliant and beautiful work, Gross and Zagier proved a special case of the Birch-Swinnerton-Dyer conjecture on the special values of derivatives of L-series associated to elliptic

curves. They were then able to construct an L-function with a triple zero at the center of its critical strip. Combined with my work, this solved the Gauss conjecture.

Let me also take this opportunity to thank B. Gross and D. B. Zagier for their contribution to this problem. It is a delight to share the Cole Prize with them.

In conclusion, I should like to mention the still unsolved conjecture of Chowla; to construct an effective algorithm for finding all real quadratic fields (with discriminant equal to one plus a square) which have a given class number. My method would solve this problem if one could find a suitable L-function with a fourth order zero. Of course, if no condition is put on the discriminant, another famous conjecture of Gauss asserts that class number one occurs infinitely often. This problem appears quite intractible at the moment.

Dorian M. Goldfeld

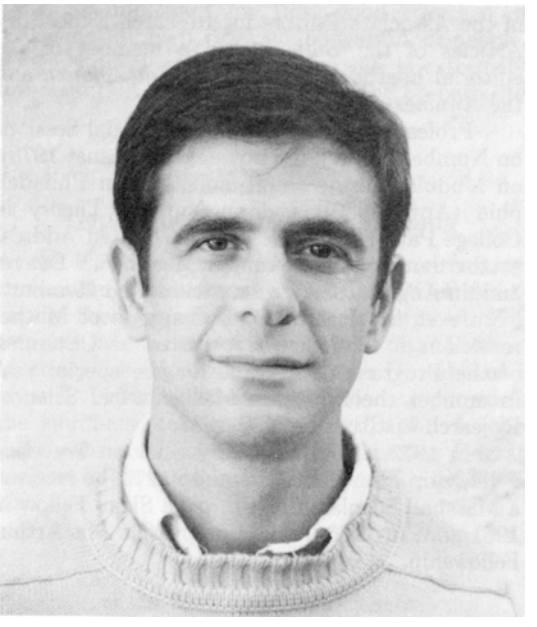
Biographical Sketch

Dorian M. Goldfeld was born January 21, 1947, in Marburg, Germany. He received both his B.S. degree (1967) and his Ph.D. (1969) from Columbia University.

Professor Goldfeld was a Miller Fellow at the University of California, Berkeley (1969–1971), a postdoctoral fellow at Hebrew University, Jerusalem (1971–1972), and a lecturer at Tel-Aviv University, Israel (1972–1973). He spent the 1973–1974 year as a member of the Institute for Advanced Study in Princeton, and the years 1974–1976 as a visiting professor at Scuola Normale Superiore, Pisa. Professor Goldfeld then became assistant professor at the Massachusetts Institute of Technology (1976–1979), advancing to associate professor (1979–1982). He was an associate professor at the University of Texas, Austin, (1983–1985) and a visiting associate professsor at Harvard University (1982–1985). Since 1985, he has been a professor at Columbia University.

Professor Goldfeld spoke at the Special Session on Elliptic Curves and Arithmetic Geometry in Providence (October 1980), on Number Theory in San Francisco (January 1981), and on Number Theory and Related Parts of Analysis in Austin (November 1981). Also, Professor Goldfeld gave an Invited Address at the Annual Meeting in Anaheim (January 1985) and an invited 45-minute address at the International Congress of Mathematicians in Berkeley (August 1986).

Professor Goldfeld was a Sloan Fellow (1977–1979) and, in 1985, received the Vaughn Prize.



Benedict H. Gross

Benedict H. Gross

Response

I would like to thank the American Mathematical Society for this honor. The Cole Prize means a great deal to me, as many of my teachers were former recipients.

The joint paper cited by the committee establishes a new type of limit formula, which connects the canonical heights of special points on modular curves to the first derivatives of L-series at $s = 1$. One application is to give examples of L-series which vanish to order 3, and this has applications to the class number problem via previous work of Dorian Goldfeld.

I was lucky both to find this limit formula and to collaborate with Don Zagier on the proof. The essential miracles underlying our computations are still not well understood; I look forward to the day when some of these secrets will be revealed.

Biographical Sketch

Benedict H. Gross was born on June 22, 1950, in South Orange, New Jersey. He received his B.A. from Harvard University in 1971, his M. Sc. from Oxford University in 1974, and his Ph.D.

from Harvard University in 1978. He began his teaching career at Princeton University in 1978 as an instructor and then as an assistant professor. In 1982, he went to Brown University where he held the positions of associate professor and then professor. Since 1985, he has been a professor of mathematics at Harvard University.

Professor Gross has been a member of the AMS Committee on Postdoctoral Fellowships (1980–1982) and of the AMS Committee on Summer Research Conferences (1982–1985). He is one of the Associate Editors for Research-Expository Articles of the *Bulletin* and a member of the editorial boards of *Compositio Mathematica* and the *Annales de l'Institut Fourier*.

Professor Gross spoke at the Special Sessions on Number Theory in Providence (August 1978), on Modular Forms and L-Functions in Philadelphia (April 1980), and on Number Theory in College Park (1982). He gave an Invited Address at the January 1983 Annual Meeting in Denver and, in August 1986, he gave an invited 45-minute lecture at the International Congress of Mathematicians in Berkeley. He served as Chairman of the Program Committee for the special year in number theory at the Mathematical Sciences Research Institute in 1986–1987.

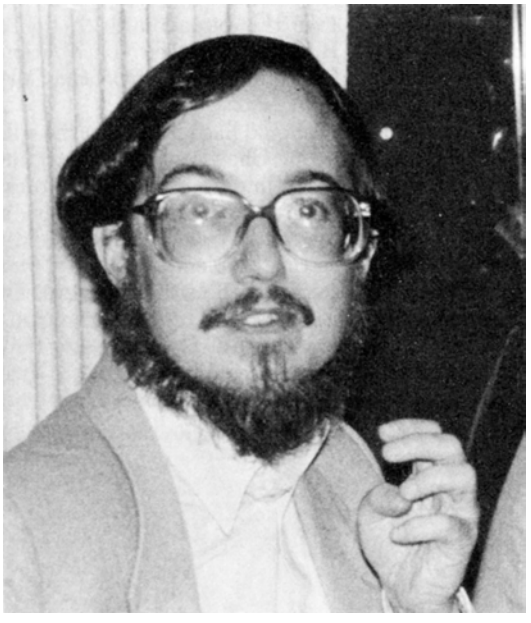
In 1972, Gross received a Sheldon Traveling Fellowship from Harvard and, in 1973, he received a Marshall Scholarship. He was a Sloan Fellow in 1981 and, in 1986, he was awarded a MacArthur Fellowship.

Don B. Zagier

Response

I feel greatly honored to be one of the recipients of the Cole Prize, which has gone in the past to such illustrious number theorists.

Working on the paper which has been cited by the Cole Prize Committee was a tremendously satisfying mathematical experience, for several reasons. First of all, the perhaps most important recent development in number theory has been the realization of the phenomenal interdependence between purely arithmetic questions and the theory of automorphic forms, epitomized by the Langlands program but visible at a more naive level in the conjectures and theorems relating modular curves to elliptic curves and associated with the names of Taniyama, Shimura, Weil, Birch, and Swinnerton-Dyer. It is, however, very rare to find a situation where a diophantine object and a related modular one are both understood so well that they can be calculated, and it was very exciting to be able to work out one such case. Secondly, the actual progress of my collaboration with Dick Gross had a lot of drama, since we were both feeling in the dark and each step of one of us cast just enough light for the other to take one step in turn. And finally, I love to calculate, and this paper let me indulge as never before. The application to Gauss's class number formula via the beautiful prior work of Goldfeld was, of course, a wonderful and undeserved bonus.



Don B. Zagier

Biographical Sketch

Don B. Zagier, an American citizen, was born on June 29, 1951, in Heidelberg, Germany. He finished high school at the age of 13, attended an English public school (Winchester) for one year, and then attended the Massachusetts Institute of Technology, where he received Bachelor's degrees in mathematics and physics two years later. He studied topology under Michael Atiyah (Oxford) and Friedrich Hirzebruch (Bonn), obtaining his D.Phil. degree from Oxford in 1972.

Professor Zagier has been attached to the faculty of the Universitat Bonn since 1971, a professor at the University of Maryland since 1979, and a scientific member of the Max-Planck-Institut für Mathematik (Bonn) since 1984. He has also held visiting appointments at the Eidgenössische Technische Hochschule Zurich (1972–1973), the Institut des Hautes Etudes Scientifiques (1973–1974), Universiteit Leiden (1976), and Harvard University (1977). He has lectured in India, China, Japan and, extensively, in America and Eastern and Western Europe. He was awarded the Carus Prize of the Leopoldina Academy in Germany in 1983 and a special research grant of the French Ministry of Research and Technology in 1985.

Professor Zagier is a member of the editorial boards of *Compositio Mathematica* and the *Journal of Number Theory*. His research interests include number theory and the theory of modular forms.