

# Varadhan Receives 2007 Abel Prize



**Srinivasa S. R. Varadhan**

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2007 to SRINIVASA S. R. VARADHAN of the Courant Institute of Mathematical Sciences, New York University, “for his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations.”

The Abel Prize carries a cash award of NOK 6,000,000 (US\$875,000). Previous recipients are

Jean-Pierre Serre (2003), Michael Atiyah and I. M. Singer (2004), Peter D. Lax (2005), and Lennart Carleson (2006).

## Citation

Probability theory is the mathematical tool for analyzing situations governed by chance. The law of large numbers, discovered by Jacob Bernoulli in the eighteenth century, shows that the average outcome of a long sequence of coin tosses is usually close to the expected value. Yet the unexpected happens, and the question is: how? The theory of large deviations studies the occurrence of rare events. This subject has concrete applications to fields as diverse as physics, biology, economics, statistics, computer science, and engineering.

The law of large numbers states that the probability of a deviation beyond a given level goes to zero. However, for practical applications, it is crucial to know how fast it vanishes. For example, what capital reserves are needed to keep the probability of default of an insurance company below acceptable levels? In analyzing such actuarial “ruin problems”, Harald Cramér discovered in 1937 that standard approximations based on the Central Limit Theorem (as visualized by the bell curve) are actually misleading. He then found the first precise estimates of large deviations for a sequence of independent random variables. It took thirty years before Varadhan discovered the underlying general principles and began to demonstrate their

tremendous scope, far beyond the classical setting of independent trials.

In his landmark paper “Asymptotic probabilities and differential equations” in 1966 and his surprising solution of the polaron problem of Euclidean quantum field theory in 1969, Varadhan began to shape a general theory of large deviations that was much more than a quantitative improvement of convergence rates. It addresses a fundamental question: what is the qualitative behavior of a stochastic system if it deviates from the ergodic behavior predicted by some law of large numbers or if it arises as a small perturbation of a deterministic system? The key to the answer is a powerful variational principle that describes the unexpected behavior in terms of a new probabilistic model minimizing a suitable entropy distance to the initial probability measure. In a series of joint papers with Monroe D. Donsker exploring the hierarchy of large deviations in the context of Markov processes, Varadhan demonstrated the relevance and the power of this new approach. A striking application is their solution of a conjecture of Mark Kac concerning large time asymptotics of a tubular neighborhood of the Brownian motion path, the so-called “Wiener sausage”.

Varadhan’s theory of large deviations provides a unifying and efficient method for clarifying a rich variety of phenomena arising in complex stochastic systems, in fields as diverse as quantum field theory, statistical physics, population dynamics, econometrics and finance, and traffic engineering. It has also greatly expanded our ability to use computers to simulate and analyze the occurrence of rare events. Over the last four decades, the theory of large deviations has become a cornerstone of modern probability, both pure and applied.

Varadhan has made key contributions in several other areas of probability. In joint work with Daniel W. Stroock, he developed a martingale method for characterizing diffusion processes, such as solutions of stochastic differential equations. This new approach turned out to be an extremely powerful way of constructing new Markov processes, for example infinite-dimensional diffusions arising in population genetics. Another major theme is the analysis of hydrodynamical limits describing the macroscopic behavior of very large systems of

interacting particles. A first breakthrough came in joint work with Maozheng Guo and George C. Papanicolaou on gradient models. Varadhan went even further by showing how to handle non-gradient models, greatly extending the scope of the theory. His ideas also had a strong influence on the analysis of random walks in a random environment. His name is now attached to the method of “viewing the environment from the travelling particle”, one of the few general tools in the field.

Varadhan's work has great conceptual strength and ageless beauty. His ideas have been hugely influential and will continue to stimulate further research for a long time.

### Biographical Sketch

Srinivasa S. R. Varadhan was born January 2, 1940, in Madras (Chennai), India. He is currently professor of mathematics and Frank J. Gould Professor of Science at the Courant Institute of Mathematical Sciences, New York University.

Varadhan received his B.Sc. honors degree in 1959 and his M.A. the following year, both from Madras University. In 1963 he received his Ph.D. from the Indian Statistical Institute, Calcutta, with the distinguished Indian statistician C. R. Rao as his thesis advisor. It is reported that during his thesis defense Varadhan noticed a visitor in the room whom he did not know and who asked many penetrating questions. After the exam he discovered that it was the famous Russian mathematician and probabilist A. N. Kolmogorov. Apparently Rao arranged the date of the exam knowing that Kolmogorov would be visiting India then in order to show off his star student, and Kolmogorov was not disappointed.

Srinivasa Varadhan began his academic career at the Courant Institute of Mathematical Sciences as a postdoctoral fellow (1963–66), strongly recommended by Monroe Donsker. Here he met Daniel Stroock, who became a close colleague and co-author.

In an article in the *Notices of the American Mathematical Society* Stroock recalls these early years: “Varadhan, whom everyone calls Raghu, came to these shores from his native India in the fall of 1963. He arrived by plane at Idlewild Airport and proceeded to Manhattan by bus. His destination was that famous institution with the modest name, the Courant Institute of Mathematical Sciences, where he had been given a postdoctoral fellowship.” Varadhan was assigned to one of the many windowless offices in the Courant building, which used to be a hat factory. Yet despite the somewhat humble surroundings, as Stroock puts it, “from these offices flowed a remarkably large fraction of the postwar mathematics of which America is justly proud.”

Srinivasa Varadhan has stayed loyal to Courant, where he served as assistant professor (1966–68),

associate professor (1968–72), and became a full professor in 1972. When he and Stroock were awarded the AMS Steele Prize in 1996, Varadhan did not fail to mention that “The Courant Institute provided us with an ideal intellectual environment, active encouragement and the support of our senior colleagues, particularly Louis Nirenberg and Monroe Donsker.”

Varadhan must have lived up to the high expectations he was met with as a postdoctoral fellow. In 1965 Louis Nirenberg wrote to Monroe Donsker recommending Varadhan for a faculty appointment at Courant: “I think very highly indeed of Varadhan and predict a great future for him. He is very young, and I think in many ways he might be the best appointment as assistant professor in probability we could make.” Fifteen years later Srinivasa Varadhan was appointed director of Courant (1980–84), following Peter Lax.

In a letter of recommendation addressed to the president of New York University, Lax wrote, “We feel that now, when the Courant Institute is full of renewed vigor and is facing the future with confidence, is the time to pass on the leadership to a new generation.” Thus, Srinivasa S. R. Varadhan followed Peter Lax both as director of Courant and now also as an Abel Laureate. Varadhan came back to serve a second period as director of Courant (1992–94).

Varadhan has held visiting positions at Stanford University (1976–77), the Mittag-Leffler Institute (1972), and the Institute for Advanced Study (1991–92). Varadhan was an Alfred P. Sloan Fellow (1970–72) and a Guggenheim Fellow (1984–85). His awards and honors include the AMS Birkhoff Prize (1994), the Margaret and Herman Sokol Award of the Faculty of Arts and Sciences of New York University (1995), and the AMS Steele Prize (1996). He also has two honorary degrees from Université Pierre et Marie Curie in Paris (2003) and from the Indian Statistical Institute in Kolkata, India (2004).

Varadhan was an invited speaker (1978) and a plenary speaker (1994) at the International Congress of Mathematicians (ICM) in 1978 and 1994. He was elected a member of the American Academy of Arts and Sciences (1988), the Third World Academy of Sciences (1988), and the National Academy of Sciences (1995). He was elected a Fellow of the Institute of Mathematical Statistics (1991), the Royal Society (1998), and the Indian Academy of Sciences (2004).

Srinivasa Varadhan is married to Vasundra Varadhan, who is a professor at New York University. They have one son, Ashok. Their eldest son, Gopal, was one of the victims of the 9/11 terrorist attacks on the World Trade Center Twin Towers.

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