Tosio Kato (1917–1999)

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Tosio Kato was born August 25, 1917, in Kanuma City, Tochigi-ken, Japan. His early training was in physics. He obtained a B.S. in 1941 and the degree of Doctor of Science in 1951, both at the University of Tokyo. Between these events he published papers on a variety of subjects, including pair creation by gamma rays, motion of an object in a fluid, and results on spectral theory of operators arising in quantum mechanics. His dissertation was entitled "On the convergence of the perturbation method".

Kato was appointed assistant professor of physics at the University of Tokyo in 1951 and was promoted to professor of physics in 1958. During this time he visited the University of California at Berkeley in 1954–55, New York University in 1955, the National Bureau of Standards in 1955–56, and Berkeley and the California Institute of Technology in 1957–58. He was appointed professor of mathematics at Berkeley in 1962 and taught there until his retirement in 1988. He supervised twenty-one Ph.D. students at Berkeley and three at the University of Tokyo.

Kato published over 160 papers and 6 monographs, including his famous book *Perturbation Theory for Linear Operators* [K66b]. Recognition for his important work included the Norbert Wiener Prize in Applied Mathematics, awarded in 1980 by the AMS and the Society for Industrial and Applied Mathematics. He was particularly well known for his work on Schrödinger equations of nonrelativistic quantum mechanics and his work on the Navier-Stokes and Euler equations of classical fluid mechanics. His activity in the latter area remained at a high level well past retirement and continued until his death on October 2, 1999.

—*Michael Taylor*

Barry Simon

Tosio Kato founded the modern theory of Schrödinger operators and dominated the field for its first twenty-five years. A *Schrödinger operator* is one of the form

(1)
$$-\Delta + V$$

acting on $L^2(\mathbb{R}^n)$, V being a real-valued function. The term "Schrödinger operator" is used also for some closely related operators that include the effect of magnetic fields or of particles with differing masses. These are the basic Hamiltonians of nonrelativistic quantum mechanics, and the theory is essentially the study of the mathematical aspects of nonrelativistic quantum mechanics.

While I met Kato at conferences and corresponded with him several times, we never had extensive personal interactions. But I have studied and admired many of his seminal papers and will focus on his scientific work in this area.

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Atomic Hamiltonians

Kato's most celebrated result is undoubtedly his proof, published in 1951 [K51a], of the essential self-adjointness of atomic Hamiltonians:

(2)
$$-\sum_{i=1}^{n}(2\mu_i)^{-1}\Delta_i - \sum_{i=1}^{n} Z|x_i|^{-1} + \sum_{i<j}|x_i - x_j|^{-1}.$$

In a case of scientific serendipity, J. von Neumann concluded his basic work on the theory of unbounded self-adjoint operators just as quantum theory was being invented, and he had realized by 1928 that the critical question was to define the Hamiltonian as a self-adjoint operator. Kato proved that the operator given by equation (2), defined initially on smooth functions of compact support, has a unique self-adjoint extension (and he was even able to describe that extension).

I have often wondered why it took so long for this fundamental question to be answered. As Kato remarks in his Wiener Prize acceptance [K80], the proof is "rather easy." In modern terminology it is a combination of a Sobolev estimate and the theorem of Kato and F. Rellich on stability of self-adjointness under certain kinds of perturbations.

Rellich knew the Kato-Rellich theorem by the mid-1930s (Kato, unaware of Rellich's work, redis-covered it as part of the proof of his theorem), and Sobolev inequalities were also discovered by then (although Kato did have to understand using them on only a subset of the variables and integrating out the remaining variables). I would have expected Rellich or K. O. Friedrichs to have found the result by the late 1930s.

One factor could have been von Neumann's attitude. V. Bargmann told me of a conversation he had with von Neumann in 1948 in which von Neumann asserted that the multiparticle result was an impossibly hard problem and even the case of hydrogen was a difficult open problem (even though the hydrogen case can be solved by sepa-ration of variables and the use of H. Weyl's 1912 theory, which von Neumann certainly knew!). Perhaps this is a case like the existence of the Haar integral, in which von Neumann's opinion stopped work by the establishment, leaving the important discovery to the isolated researcher unaware of von Neumann's opinion.

Another factor surely was the Second World War. My generation and later generations are sufficiently removed from the dislocations of the war and its aftermath that we often forget its effect. In [K80] Kato remarks dryly: "During World War II, I was working in the countryside of Japan." In fact, from a conversation I had with Kato one evening at a conference, it was clear that his experiences while evacuated to the countryside and in the chaos immediately after the war were horrific. He barely escaped death several times, and he caught tuberculosis. (Charles Dolph once told me that he regarded his most important con-tribution to American mathematics was that when he learned Kato was having trouble getting a visa because of an earlier bout with tuberculosis, Dolph contacted physicist Otto Laporte, then American scientific attaché in Tokyo, to get Kato a waiver.)

Formally trained as a physicist, Kato submitted his paper to *Physical Review*, which could not figure out how and who should referee it, and that journal eventually transferred it to the *Transactions of the American Mathematical Society*. Along the way the paper was lost several times, but it finally reached von Neumann, who recommended its acceptance. The refereeing process took over three years.

Later Self-Adjointness Results

Kato returned to the issue of self-adjointness sev-eral times after his initial work, most notably once in the early 1960s and once in the early 1970s. The first of these cases was a paper with his student T. Ikebe [KI], which among other things estab-lished the proper $(-x^2)$ borderline for situations where the potential was allowed to go to minus in-finity at spatial infinity.

The second involved some work I did. The general wisdom by 1970 was that for gen-eral Schrödinger opera-tors of the form (1) on $\mathbb{R}^n$, the right local con-dition on V to assure self-adjointness is that it be in L^p with $p \geq n/2$ (at least if n is 5 or more). It was known that one could not im-prove this as far as L^p properties alone were concerned. As an off-shoot of work I had done in quantum field theory, I realized in 1972 that this was only the right property for the negative part of V, but that as far as local

Tosio Kato, February 15, 1997.

behavior was concerned, L^2 was the proper con-dition for the positive part.

I conjectured that for positive V's, a sufficient condition (it is clearly necessary) for essential self-adjointness was that V be locally L^2; because of the nature of my proof, I could get the result for locally L^2 positive potentials only under the unnatural additional condition that the L^2 norm over a ball of radius R does not grow any faster than $\exp(cR^2)$.

Within weeks of my mailing out my preprint, a letter arrived from Kato and shortly afterwards a brilliant paper [K72] that is my personal favorite among all his works. He not only settled the gen-eral case but did it by introducing a totally new idea—a distributional inequality now called Kato's inequality—that for all functions u such that u and its distributional Laplacian are both locally in-tegrable,

(3) $$\Delta|u| \geq (\operatorname{sgn} u)\Delta u.$$

He had results (later improved by H. Leinfelder and C. Simader) for magnetic fields; indeed, Kato's version of (3) with magnetic fields led others to what are now called universal diamagnetism and diamagnetic inequalities, as well as to abstract results on comparison of semigroups. Kato also took advantage of this paper to redo the situation for negative potentials, work that led to a class of functions now known as the Kato class, which turns out to be the natural class from a path-integral point of view.

Eigenvalue Perturbation Theory

Kato also returned many times to the issue of eigen-value perturbation theory. Independently, but later than Rellich, he developed the theory of regular operator perturbations (what he later called type A).

As early as 1955 he appreciated the significance of quadratic forms [K55] and developed what he later called type B perturbations. All these situations are regular ones for which the eigenvalues are analytic in the perturbation parameters. But there are many standard quantum examples to which the theory does not apply, and Kato developed methods for proving that eigenvalue perturbation series are often asymptotic. He understood the critical notion of stability used by many later authors.

While Kato's perturbation-theoretic work initially and often focused on eigenvalue perturbations, the notion of controlling things perturbatively was a major theme in much of his other work, including semigroup perturbations, the adiabatic theorem, scattering theory, and his later work on nonlinear equations. And, of course, any discussion of perturbation theory has to mention Kato's masterpiece *Perturbation Theory for Linear Operators* [K66b], used as a bible by a generation of mathematical physicists and operator theorists.

Scattering Theory

From 1956 until 1980 a major theme in Kato's work was scattering theory and the understanding of the absolutely continuous spectra of Schrödinger operators. Among his many papers in this area, I shall focus on three topics. The first, in 1957, involves finite-rank and trace-class perturbations [K57]; M. Rosenblum shares the credit for developing the initial theory. Later critical developments were made by S. T. Kuroda, M. Birman, and Kato again when he wrote an important paper on the invariance principle [K65]. One can also see the interplay between time-dependent and time-independent methods, a theme that recurred.

The slickest marriage of time-dependent and time-independent methods occurred in what has come to be called the theory of Kato smoothness [K66a]. The link is essentially the Plancherel theorem! I regard this paper, published in 1965 when Kato was forty-eight, as his most beautiful and in some ways his deepest. There have been applications to positive commutator theory (the Putnam-Kato theorem), which was a precursor to Mourre theory.

Finally, in the early 1970s, Kato together with Kuroda [KK] made important contributions to the development of the limited absorption principle in scattering theory, a subject raised to high art by S. Agmon and L. Hörmander.

Other Work

It is a tribute to Kato's depth and breadth that some of his other, "less important", papers would be considered major parts of the oeuvre of many mathematicians. Not only did Kato write the first significant mathematical analysis of the quantum adiabatic theorem [K50], but the ideas presented in it remain to this day central to most further work on the subject, including most mathematical presentations of M. Berry's phase.

At the same time as his fundamental paper on self-adjointness, he proved that the helium atom Hamiltonian in the limit of infinite nuclear mass had an infinity of bound states [K51b]. It was later realized by others that as long as one is careful about using the right coordinate system, Kato's method extends to finite nuclear masses and to arbitrary atoms (recovering a theorem of G. Zhislin proven later than Kato's work).

Kato understood the nature of the singularities of atomic wave functions ("Kato cusp conditions") and wrote about these in 1957. In a 1950 paper he found the right way of formulating G. Temple's lower bounds on eigenvalues, and in 1959 he obtained some of the earliest results on the absence of embedded positive energy eigenvalues under suitable assumptions on the decay of the potential.

Tosio Kato leaves a rich mathematical legacy to everyone working on mathematical problems associated with nonrelativistic quantum mechanics.

Gustavo Ponce

Kato was one of the most influential figures in the study of nonlinear evolution equations. He formulated a general abstract approach to the well-posedness of the initial value problem that isolated and illuminated the basic features of a broad class of problems. As Kato stated in his acceptance of the Wiener Prize [K80], "I have been fascinated by the Hille-Yosida-Phillips theory of operators semigroups, and continuously have tried to apply it to solving various evolution equations, linear and nonlinear." However, his work in this area went far beyond this general local existence theory. Kato developed the basic tools and the framework for the qualitative study of many fundamental problems in mathematical physics: nonlinear symmetric hyperbolic systems, the equations of

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motions for incompressible and compressible fluids, and nonlinear dispersive models with a wide variety of boundary conditions.

I first met Kato in the fall of 1982 at Berkeley. As a postdoc I attended his lectures in nonlinear functional analysis. After one of his classes he invited me to come to his office. On the board he had written a mathematical statement, and he asked me whether or not I believed it was true. After a few minutes of thought I admitted that I had no idea. With a worried look on his face he showed me a paper where this result was used and credited to one of his papers. He said that it was not really there, and in fact he did not know how to prove it. Some days later he told me that the result as stated was false, but happily he was able to get around this point in the paper. Perhaps without realizing it, the author had stumbled on a novel method of mathematical proof by appealing to Kato's honesty.

From this period I enjoyed a mathematical and personal relationship with Kato. I was privileged to have the opportunity to collaborate with him and to know his pure and straightforward intellectual integrity. Our communication continued until his death.

Of Kato's many contributions in this area I will only comment on a select few that seem especially important to me. Following his general framework, consider the problem

$$(4) \qquad \frac{du}{dt} = Au + f(u), \quad u(0) = \psi \in X,$$

where A is a linear operator on a function space X and f is a nonlinear function on X. Kato's notion of well-posedness in X includes: existence, uniqueness, the continuity of the map from the data to the solution, and persistence (i.e., the question whether the solution describes a continuous curve in X).

In [K83] Kato studied the initial value problem associated to the Korteweg de Vries equation, for which $A = -\partial_x^3$ and $f(u) = -u\partial_x u$ in (4). Kato established a smoothing effect, which almost contradicts the time reversibility of the equation. Roughly, he showed that if $u(0) = \psi$ is in $L^2(\mathbb{R})$, then for any $r > 0, \partial_x u(t)$ is in $L^2(-r, r)$ for almost every time $t \in \mathbb{R}$. From the time reversibility one has that if $\partial_x u(0) = \partial_x \psi$ is not in $L^2(\mathbb{R})$, then $\partial_x u(t)$ is not in $L^2(\mathbb{R})$ for all $t \in \mathbb{R}$. His proof of this result, which was new even for the associated linear case $f \equiv 0$, was extremely simple. This was quite a surprise, since the Korteweg de Vries equation is one of the most famous nonlinear evolution equations. However, this was not the first regularizing effect established by Kato.

The solution $u(t) = e^{it\Delta}\psi$ of the linear Schrödinger equation, namely, (4) with $A = -i\Delta$ and $f \equiv 0$ and data $\psi \in L^2(\mathbb{R}^n)$, describes a continuous curve in $L^2(\mathbb{R}^n)$. In [K66a] Kato showed that for almost every time $t \in \mathbb{R}$, $e^{it\Delta}\psi$ takes values in a small set (of first category). This smoothing effect

anticipated by more than ten years a celebrated inequality due to R. Strichartz.

Later, in [KY], in collaboration with K. Yajima, Kato quantified this effect by showing that for any fixed $q \in L^n(\mathbb{R}^n)$,

$$(5) \qquad \int_{-\infty}^{\infty} \|q\,e^{it\Delta}\psi\|_{L^2}^2 dt \le c\|\psi\|_{L^2}^2.$$

Moreover, Kato showed that the estimate in (5) still holds with the operator

$$(1 + |x|^2)^{-1/2}(1 - \Delta)^{1/4}$$

instead of multiplication by q.

This smoothing effect or gain of derivatives is fundamentally related to the dispersive character of the equation. In particular, it does not hold for hyperbolic equations.

The results in [K83] together with questions raised there led to the great activity in the last decade on the problem of the optimality of the initial function space X to guarantee the local well-posedness for various nonlinear evolution equations.

In [KF] Kato and his student H. Fujita consider the Navier-Stokes system, namely, (4) with $A = \nu\Delta$, $f = (f_1, \ldots, f_n)$, and $f_j(u) = P((u \cdot \nabla)u^j)$. Here x is in $\mathbb{R}^n$, $u = (u^1, \ldots, u^n)$, ν is > 0, and P denotes the projection onto divergence-free vector fields. They introduced an argument (weighted-in-time norms) that in the 3-dimensional case allowed them to obtain the local well-posedness in the critical Sobolev space $\dot{H}^{1/2}(\mathbb{R}^3)$, the dot indicating that the space is invariant under the map carrying $u(x, t)$ to $\lambda u(\lambda x, \lambda t)$. In particular, the criticality of the space guarantees the existence of global solutions for small data. The existence of global classical solutions with large data has remained as an open problem since J. Leray's work in 1934. In 1984 Kato extended his results to the space $L^3(\mathbb{R}^3)$. The further extensions and applications of the techniques introduced by Kato in these works have generated a long list of interesting results.

One of my favorite of Kato's papers is [K86]. In the 2-dimensional case the global well-posedness for the Navier-Stokes and Euler equations are due to Leray and W. Wolibner respectively. In [K86] Kato gave a unified and extremely simple proof of these global results. His proof is based on a logarithmic type of inequality for the boundedness in $L^\infty(\mathbb{R}^2)$ of singular integral operators. As often in Kato's works, [K86] was a fountain of ideas. The extension of this logarithmic inequality to the 3-dimensional case led to a joint result of Kato with J. T. Beale and A. Majda. Questions raised there initiated my collaboration with Kato and yielded the results in a joint paper of mine with Kato in 1988; this concerned sharp energy estimates involving fractional derivatives and the extension of his abstract approach to the $L^{s,p}(\mathbb{R}^n)$-well-posedness of the Navier-Stokes and Euler systems.
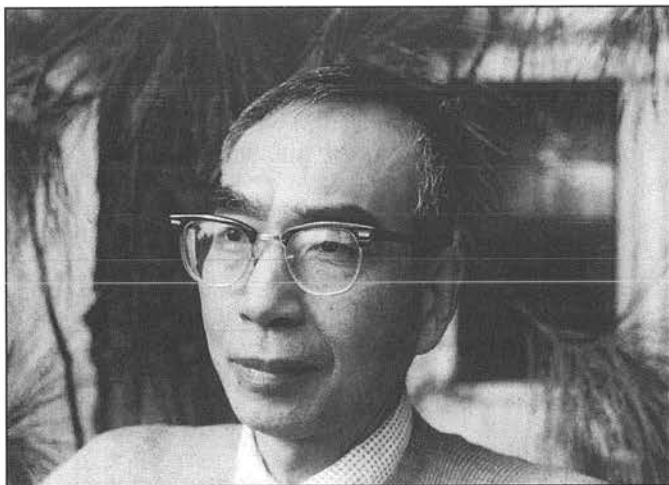
Tosio Kato (from family photo album).

I last saw Kato in the spring of 1999. He was emptying his house at Berkeley and asked me to help him get rid of some publications and books. It was a hard and sad time for him. However, Kato was willing to talk mathematics. His enthusiasm for it was intact. He was full of ideas and questions.

S. T. Kuroda

In this article I concentrate on personal reminiscences about the old days when Tosio Kato was still in Japan.

My recollection of Kato goes back to my younger days when I attended his course on mathematical physics at the Department of Physics, University of Tokyo. It was 1953–54. The course covered, thoroughly but efficiently, most of the standard material from the theory of functions through partial differential equations. The style of his lecture never gave an impression that he went quickly, but at the end of each class I was surprised by how much he had covered within one session. The course could vividly live today, and a plan of editing materials from old notes has been slowly going on. I deeply regret that it could not be completed while he was alive.

Two years later I was a graduate student under the supervision of Kato. It was several years after the publication of his first major work on self-adjointness of Hamiltonians of Schrödinger type and his comprehensive analysis of perturbations of the discrete spectrum. His work on self-adjointness is now more famous as a turning point in mathematical physics or "functional analysis of quantum mechanics" (this is the title of an essay he wrote in Japanese). Nevertheless, I recall that I put more energy into reading his big paper on perturbation theory [K51c]. He gave me a thick reprint, which was precious in those days before photocopiers. This

paper perhaps led to my later fondness for abstract operator theory. Also, I learned much from his lecture notes from Berkeley [K55]. The simple directness of these notes is unforgettable.

Immediately after his return from his first visit to the United States in 1954–55 (University of California at Berkeley and New York University), he focused his attention on perturbations of the continuous spectrum and the associated scattering theory. As he completed his papers on scattering theory for perturbations of finite rank and then of trace class, he passed the manuscripts to us to read and check. I still keep a carbon copy of these papers. He then suggested that I investigate the case of relative trace class, with a view to applications to Schrödinger operators. With some of my effort and ideas, this resulted in my doctoral thesis.

At that time the group led by Kato was a small one in the physics department. Kôsaku Yosida had just moved to Tokyo (to the mathematics department) from Osaka and was building up a functional analysis group. It was apparent even to the eyes of a beginning student that two giants of functional analysis in Japan had very close academic ties and respect for each other. So, these were days when Kato initiated a tradition of mathematical physics in Japan.

As far as I recall, the last three visits of Kato to Japan were in 1989 (on the occasion of a belated celebration of his seventieth birthday), 1991, and 1992. During these visits he actively participated in conferences and symposia and gave lectures on varied subjects in operator theory, evolution equations, and nonlinear partial differential equations. Recognizing persistent influences of his ideas in mathematical physics and partial differential equations, we in Japan were looking forward to another occasion. We mourn that the chance has gone forever.

Arne Jensen

Tosio Kato passed away on October 2, 1999. His death came as a shock to me and was wholly unexpected.

I spent two and a half years in Berkeley as a visitor, from October 1976 to March 1979. I came to Berkeley on a Danish grant, after studying at the University of Aarhus. During my last year there S. T. Kuroda was a visitor and gave a course on scattering theory. So it was natural for me to go to Berkeley to learn more from Tosio Kato. I learned a great deal from him. As preprints came in, I was asked to read through those he thought would be of interest to me and to give my opinions. This often resulted in lengthy discussions.

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Some of these discussions resulted in two joint papers. I still recall vividly his attention to detail. Every proof had to be discussed and distilled to its essence. Often we would discuss a mathematical problem, and he would think for a while and then give a reply which could not be improved on. In several cases I found the next morning in my mailbox a note complementing the discussion.

The discussions took place both in his office in Berkeley and during walks in the surrounding area. We often took walks in Tilden Park or Strawberry Canyon. It was during these walks that I learned of his interest in botany. He could identify a large number of plants and trees and knew the Latin names as well. He had great respect for the classification system introduced in the eighteenth century by the Swedish botanist Carl von Linné. Thus many years later, in 1993, when he visited the Mittag-Leffler Institute, a visit to Uppsala and in particular to Linné's garden was a must. At home in Berkeley he tried to grow many kinds of plants, with varying success. In particular, the drought in 1977 was hard on his plants.

Of his work I particularly like the paper [K66a]. It has been highly influential, including on work in the 1980s on the many-body problem. After 1980 Tosio Kato concentrated his work on nonlinear partial differential equations.

Finally, I would like to mention an intriguing problem that he left for posterity to solve. It is sometimes referred to as Kato's square root problem. Briefly, the problem can be stated as follows: Let $L = -\text{div}(A \, \text{grad})$, where $A(x)$ is a matrix with complex-valued bounded entries. The operator L is the maximally accretive operator on $L^2(\mathbb{R}^n)$ defined via the quadratic form on the Sobolev space $H^1(\mathbb{R}^n)$. Then one can define the square root of L, and the problem is whether its domain equals $H^1(\mathbb{R}^n)$. The answer is not known in general today. In a 1998 article P. Auscher and P. Tchamitchian gave a survey and wrote about some recent results concerning the problem.

Heinz Cordes

I miss my friend Tosio Kato. After twenty-five years of work side by side at UC Berkeley and another eleven years together after his retirement, we had grown close.

I first heard of Kato in 1952 in Göttingen. My "Doktorvater" Franz Rellich showed me the work of a young Japanese scholar who had improved some very nontrivial estimates in the thesis of Rellich's student E. Heinz. An example, in terms of matrices: for self-adjoint positive A, B, if a matrix

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Q satisfies $\|Qx\| \le \|Bx\|$ and $\|Q^*x\| \le \|Ax\|$ for all x, then

$$|(Qx, y)| \le \|B^\nu x\| \, \|A^{1-\nu} y\|$$
$$\text{for all } x, y, \text{ and } 0 \le \nu \le 1.$$

There is an analogue for unbounded closed operators, but the inequality is already interesting for matrices.

Kato and I first met at CalTech in 1957, and it was in 1962 that he came permanently to Berkeley. Our early contacts at Berkeley were mainly in the PDE seminar, organized jointly by C. Morrey, M. Protter, H. Lewy, Kato, and me. We worked independently, and never wrote a joint paper, although our backgrounds were quite close. Kato had been raised as a physicist in the glorious age after quantum mechanics was created, when J. von Neumann and F. Riesz introduced the spectral theory of unbounded operators as a generalization of Hilbert's spectral theory, to fit the needs of a mathematically rigorous quantum theory. In the 1940s Rellich had laid out perturbation theory for the spectral resolution of unbounded self-adjoint operators, and I was a member of Rellich's group.

Rellich had an "analytic" approach to perturbation theory, mainly working with power series. It may have been B. Sz. Nagy who introduced us to resolvent methods, which proved so powerful. Kato entered this area, and perturbation theory became "his field" after Rellich passed away and Heinz turned to nonlinear problems. Kato's book *Perturbation Theory of Linear Operators* appeared in 1966. Not only did it extend many techniques from Hilbert spaces to Banach spaces, but it also discussed semigroups and their perturbations. It was carefully and patiently laid out and easy to digest and proved to be the standard reference in the field.

In Berkeley in the early 1960s there were several strong currents in analysis. Morrey was building on the work of J. Nash on estimates for divergence-form second-order elliptic operators. I made some progress on the nondivergence case (a problem finally settled in the late 1970s by N. Krylov and M. Safonov). There was also strong interest in Bourbaki-style functional analysis, with J. Kelley. In the wake of the movement crowned by the Atiyah-Singer index theorem, I turned my interests to C^*-algebras of pseudodifferential operators. Kato never veered from classical analysis into this terrain.

Nevertheless, Kato's work and mine did wind up influencing each other. One example is his paper [K58] on nullity and deficiency of operators between Banach spaces. As another example, in 1972 P. Chernoff gave his spectacularly short proof of my result on essential self-adjointness of powers of the Laplace operator on $C_0^\infty(M)$, when M is a complete Riemannian manifold. Kato gave a further improvement, extending Chernoff's argument to

Kato and wife Mizue (about 1984).

treat some non-semibounded operators.

Our most intensive contact may have been in matters of L^2 boundedness of pseudodifferential operators. His encouragement and kind words helped me in my work on compactness of commutators and boundedness of pseudodifferential operators, published in 1975. In turn, he extended my version to a proof of the Calderon-Vaillancourt theorem for symbols of type (ρ, ρ), for $\rho < 1$ (in the Hörmander category), while I had looked at $\rho = 0$. Subsequently, in an improvement of a result of Schulenberger and Wilcox, I made use of his ideas from a 1976 paper. We also had a joint Ph.D. student (G. Childs) who studied L^2 boundedness under a weaker (Hölder-type) condition on the symbol.

In the late 1960s our personal acquaintance grew, and our families grew closer together. We spent time together in Hamburg in 1968. In 1972 the Katos visited us in the Sierra Nevada. We made a habit of arranging joint picnics on Sundays, at places such as Mount Diablo or a Sonoma winery, frequently accompanied by another mathematician visiting the department.

In 1996 Kato's wife Mizue became ill. After a while they moved into a retirement home in Oakland, and things stabilized to the point that he could start to work again.

His plans then aimed at a 3-dimensional extension of his work on the 2-dimensional Euler equations. He almost got me to the point of working with him then.

His death came suddenly. On a Saturday at midnight I was called by a nurse—he had passed away of heart failure.

Michael Taylor

I first met Tosio Kato in the summer of 1968, near the end of my first year as a graduate student at Berkeley. Berkeley was hosting a summer symposium on global analysis, and the place was abuzz with activity. Over lunch on Telegraph Avenue,

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R. Anderson, a mathematical companion with a recent Ph.D. from Princeton, told me that the problem of studying fractional powers of elliptic differential operators was hot. Looking into it quickly led me to seek out faculty advice, and Kato was willing to provide it. He told me I should learn interpolation theory. He put me onto the recent works of J.-L. Lions and E. Magenes. Their books, in French, on boundary problems were not out yet, and of their papers some were in French and some in Italian. But a \$2 paperback on Italian for beginners from Moe's Bookstore helped make them accessible.

I took Kato's PDE (partial differential equations) course in the fall of 1968. It followed S. Agmon's book on elliptic boundary problems, which emphasizes the realization of elliptic operators as closed, unbounded (sometimes self-adjoint) operators via the study of quadratic forms on a Hilbert space. This approach evolved from fundamental work of K. O. Friedrichs, and it was also close to Kato's heart, playing a role in his work on Schrödinger operators. At the end of the course Kato invited the students to lunch at the Golden Bear. There were only about four students in the course: Frank Massey and I and maybe two others. In those days at Berkeley a course on topological vector spaces might get twenty students. Students would fill a large room to attend S. S. Chern's course on index theory, but the study of PDE itself was not popular.

Fortunately for the handful of us who needed it, the department ran a three-quarter sequence of courses on PDE. The second and third quarters were taught that year by I. Kupka and H. O. Cordes. They both treated the theory of pseudodifferential operators. This theory, evolving from classical studies of layer potentials, had gained panache from its role in the Atiyah-Singer index theorem. L. Hörmander was in the midst of producing spectacular results in the area, and I was seduced by the microlocal side of analysis.

To be sure, Kato's work had influences on microlocal analysis, both direct and indirect. I first mention an indirect influence. Around 1970 there arose definitive solutions to some classes of mixed initial-boundary problems for hyperbolic systems. Important progress was made by H. Kreiss, who constructed "symmetrizers" for such boundary problems satisfying an analogue of what in the elliptic case were termed Lopatinsky conditions; these symmetrizers were pseudodifferential operators. At this time, Kato's student Massey conducted a study of mixed problems for a class of symmetric hyperbolic systems, while J. Rauch, working with P. Lax, produced important refinements of Kreiss's results. Then Massey and Rauch got together and produced sharp regularity results for solutions to certain symmetric-hyperbolic mixed problems.

More direct was Kato's role in a lovely proof of the Calderón-Vaillancourt theorem. This result is an estimate on pseudodifferential operators with symbols of Hörmander type (ρ, δ), in the borderline cases $\rho = \delta \in [0, 1)$. Proven by A. P. Calderón and R. Vaillancourt in 1971, it quickly had a spectacular application in work of R. Beals and C. Fefferman, in the case $\rho = \delta = 1/2$. Such operators are also useful in the study of hypoelliptic operators with double characteristics, such as arise in treatments of the $\bar{\partial}$-Neumann problem. On the other hand, estimates on operators of type $(0, 0)$ have been important in work on semiclassical asymptotics for Schrödinger operators. In 1975 Cordes produced a new proof of the theorem, in the case $\rho = \delta = 0$. One particularly interesting feature of the proof was that it required estimates on relatively few derivatives of the symbol. This paper was followed by Kato's paper [K76] extending Cordes's method to treat the general case $\rho = \delta \in [0, 1)$. Kato's paper contains a general lemma that makes clear what the basic mechanism is behind this approach to the proof. In representation-theoretic language one would say it is the fact that the representation of the Heisenberg group defined by $U(p, q)f(x) = e^{iq \cdot x}f(x + p)$ is square integrable, a feature also emphasized by R. Howe in work published a few years later.

Kato made other contributions to microlocal analysis in the course of various investigations on nonlinear evolution equations. His 1984 paper with T. Beale and A. Majda on the Euler equations had to deal with a situation where things would be easy if pseudodifferential operators, which act nicely on L^p for $1 < p < \infty$, did so on L^∞. But they do not, and the day is saved by borrowing the logarithm of a slightly stronger norm. Work of Kato and G. Ponce in 1988 on the Navier-Stokes equations produced the Kato-Ponce estimate, a commutator estimate that can be viewed as a microlocalized Moser estimate. This result can be analyzed from the point of view of paradifferential operators, introduced by J.-M. Bony as a tool for nonlinear analysis. This is a connection I found particularly intriguing, and Kato and I corresponded about related issues in paradifferential operator calculus as recently as 1996.

The last time I saw Kato was in the summer of 1993, at a conference at Stanford on scattering theory. We had a conversation that opened my eyes about a very classical topic in analysis. For completely different reasons, we had both needed to obtain very accurate numerical approximations to $\Gamma(1/3)$. I related that I had proceeded by taking an integer k large enough that a standard integral for $\Gamma(k + 1/3)$ was amenable to accurate numerical integration. Kato replied that he picked a similar value of k and plugged in Stirling's asymptotic formula, carried out to sufficiently high order. In this way he got a terrific approximation using a tiny fraction of the computer time required by the other method. And his result was accompanied by provable error bounds. That there is more to Stirling's formula than an application of the Laplace asymptotic method is a piece of lore that, thanks to Kato, I will not forget.

References

[K50] Kato, T., On the adiabatic theorem of quantum mechanics, *J. Phys. Soc. Japan* **5** (1950), 435–439.

[K51a] _____ , Fundamental properties of Hamiltonian operators of Schrödinger type, *Trans. Amer. Math. Soc.* **70** (1951), 195–211.

[K51b] _____ , On the existence of solutions of the helium wave equation, *Trans. Amer. Math. Soc.* **70** (1951), 212–218.

[K51c] _____ , On the convergence of the perturbation method, *J. Fac. Sci. Univ. Tokyo* **6** (1951), 145–226.

[K55] _____ , Quadratic forms in Hilbert spaces and asymptotic perturbation series, Technical Report No. 7, University of California, 1955.

[K57] _____ , On perturbations of self-adjoint operators, *J. Math. Soc. Japan* **9** (1957), 239–249; Perturbation of continuous spectra by trace class operators, *Proc. Japan Acad.* **33** (1957), 260–264.

[K58] _____ , Perturbation theory for nullity, deficiency and other quantities of linear operators, *J. d'Analyse Math.* **6** (1958), 261–322.

[K65] _____ , Wave operators and unitary equivalence, *Pacific J. Math.* **15** (1965), 171–180.

[K66a] _____ , Wave operators and similarity for some nonselfadjoint operators, *Math. Ann.* **162** (1966), 258–279.

[K66b] _____ , *Perturbation Theory for Linear Operators*, Springer-Verlag, New York, 1966.

[K72] _____ , Schrödinger operators with singular potentials, *Israel J. Math.* **13** (1972), 135–148.

[K80] _____ , Remarks on receiving the 1980 Wiener Prize, *Notices Amer. Math. Soc.* **27** (1980), 528–529.

[K83] _____ , On the Cauchy problem for the (generalized) Korteweg-de Vries equation, *Stud. Appl. Math.*, Adv. Math. Suppl. Stud., vol. 8, Academic Press, New York-London, 1983, pp. 93–128.

[K86] _____ , Remark on the Euler and Navier-Stokes equations in R^2 (preprint Jan. 1984), *Nonlinear Functional Analysis and Its Applications*, Proc. Sympos. Pure Math., vol. 45, Part 2, Amer. Math. Soc., Providence, RI, 1986, pp. 1–7.

[KF] T. Kato and H. Fujita, On the nonstationary Navier-Stokes system, *Rend. Sem. Mat. Univ. Padova* **32** (1962), 243–260.

[KI] T. Kato and T. Ikebe, Uniqueness of the self-adjoint extension of singular elliptic differential operators, *Arch. Rational Mech. Anal.* **9** (1962), 77–92.

[KK] T. Kato and S. T. Kuroda, Theory of simple scattering and eigenfunction expansions, *Functional Analysis and Related Fields*, Springer-Verlag, 1970, pp. 99–131; The abstract theory of scattering, *Rocky Mountain J. Math.* **1** (1971), 127–171.

[KY] T. Kato and K. Yajima, Some examples of smooth operators and the associated smoothing effect, *Rev. Math. Phys.* **1** (1989), 481–496.