

# Interview with Gail Burrill

Gail Burrill became president of the National Council of Teachers of Mathematics (NCTM) in 1996. She earned her bachelor's degree from Marquette University and her master's degree from Loyola University in Chicago, both in mathematics. She has been a high school mathematics teacher in suburban Milwaukee for over twenty-five years, teaching everything from prealgebra through calculus. She has a special interest in statistics and worked on several quantitative literacy programs through the American Statistical Association, the most recent a curriculum project called "Data Driven Mathematics". At present she is a researcher at the University of Wisconsin Center for Education Research, where she has been working on a middle school mathematics curriculum, "Math in Context". She was named a Presidential Awardee for Teaching Mathematics and Science in 1985 and a Wisconsin Distinguished Mathematics Educator in 1986. In 1994 she was elected a Fellow of the American Statistical Association. She has written numerous articles on statistics and mathematics education, as well as textbooks and curriculum materials.

At the Joint Mathematics Meetings in Baltimore Burrill will deliver an AMS-MAA-NCTM Joint Invited Address entitled "K-12 Mathematics Education in the Twenty-first Century". Her lecture will be at 11:10 a.m. on Thursday, January 8, 1998.

The following interview was conducted by Notices senior writer Allyn Jackson.

**Notices:** In 1989 the NCTM issued its first set of Standards, which have received wide attention and have spurred other disciplinary areas to issue standards. One of the major projects of the NCTM right now is the updating of its Standards. Can you de-

scribe the motivation behind this project?

**Burrill:** The first NCTM Standards weren't intended to be the final product. Education should evolve as we continue to learn about how kids learn and understand and as the face of math continues to shift. By the time the revised Standards come out, it will have been eleven years since the first Standards appeared. And in that span of eleven years, we've learned a lot. We know more now than we did before about ways to put curricula together that will help kids learn. Technology has changed dramatically over the past ten years, and that's had an impact on the important mathematics kids need to know. There is a need to build stronger articulation between the grade-level bands. We also recognize that anytime you do a first draft of something, you may need to strengthen it. And that's certainly the case with the NCTM Standards.

**Notices:** There is a network of groups working on different aspects of the updating of the Standards. Can you briefly describe this network?

**Burrill:** It's a multilayered effort. First, we have teams of five writers for each grade-level band: pre-kindergarten to second grade, third to fifth grade, sixth to eighth grade, and ninth to twelfth grade. Joan Ferrini-Mundy is the overall chair for the writing teams. Second, we have a commission, chaired by Mary Lindquist, that oversees the interaction between the writers and the rest of the community.



---

*"We disenfranchised three-fourths of the student population and raised a nation of people who are literally afraid of mathematics."*

---

We wanted to include the perspectives of other math-related groups, so the commission started what we call Association Review Groups (ARGs). Through the Conference Board of Mathematical Sciences we asked organizations if they would like to form an ARG, whose task would be to respond to questions on which the Standards writers would

like input. At last count we had twelve of these ARGs. The chair of the ARG for the AMS is Roger Howe.

Most of the current ARGs are related in some way to mathematics education. We intend to expand this network to include organizations like the PTA so that we hear the voice of the parents, the community, and others

with a vested interest in mathematics education. We have also enlisted support from people who have strong knowledge in particular areas such as technology or equity issues.

**Notices:** *I'd like to ask you about the notion of mathematical proof, which most mathematicians think is extremely important to get across in mathematics education. Is this going to be an important aspect of the updated Standards?*

**Burrill:** I can't articulate what the Standards writers have yet to write, but I can say that I think most of us agree that proof is a very important part of mathematics. So it would seem to me that this belief is going to be reflected in their work, particularly if the concern of the AMS is reinforced, as I think it will be, by other groups.

**Notices:** *Some people felt that emphasis on proof was lacking in the original Standards.*

**Burrill:** In the original Standards we had tables which listed on one side topics that should receive increased emphasis and on the other side topics that should receive decreased emphasis. And some people took the word "decrease" to mean "none". For example, in the Standards one of the topics we suggested should receive decreased attention was the two-column proof. That does not mean "throw it out altogether"! In traditional high school geometry students were spending most of the year proving, in many cases, trivial statements and putting them into this two-column proof format with little understanding of what they were proving. The notion in the Standards was not to abandon proof but to switch some of the empha-

sis from always having proofs in the two-column format to helping kids understand what proof is really about.

One of the things I hope to see is an effort, starting in elementary school, to lay the foundations for sound reasoning so students begin to learn to make mathematical judgments based on evidence and begin to have an understanding of what proof really is. We didn't lay these foundations very well before. We need a stronger emphasis on reasoning and thinking as a background before kids begin to formalize their reasoning processes.

**Notices:** *In the 1980s mathematics education reform was introduced in England. In 1995 the London Mathematical Society, together with two other organizations, produced a report about the reform called "Tackling the Mathematics Problem". The report was critical of students' declining technical facility and lack of understanding of proof. This seems similar to some of the criticisms in the U.S. about mathematics education reform. Do you see parallels between the two? Will what happened in England help the updating of the NCTM Standards?*

**Burrill:** I don't have in-depth knowledge about either the reform or the report, but I worry about this kind of thing. For instance, in California people have been saying the students' performance had been terrible because the state has adopted new standards. Someone asked how they knew that the new standards were actually being implemented and had contributed to the decline of the student performance, and it turns out that nobody knew whether or not the reforms were actually being used in classrooms. They didn't seem to have any real evidence. In many cases, it was just one person who heard something, or somebody had read a statement and misinterpreted it in one way or another. It's very hard to gather this kind of evidence. The closest evidence we have in the U.S. is TIMSS [Third International Mathematics and Science Study], and it says that many of our teachers know about mathematics education reform, but when it comes to implementing it, they haven't moved beyond the isolated techniques to focusing on higher-level thinking and reasoning, which is the real message in the Standards. We need to get better evidence before we can actually make statements about the influence of the reform.

There never was a golden era of mathematics education. We did produce—and I was part of it actually—some wonderful students right after Sputnik. But we concentrated all of our energies on a very, very few students. We disenfranchised three-fourths of the student population and raised a nation of people who are literally afraid of mathematics. They don't want to help their children with mathematics because it's too hard; they never "got it". We can't afford to do things that way anymore. Everybody needs to be mathematically literate in

order to function in the kind of world in which they're going to be living.

**Notices:** *The idea of "mathematics for all" is a major theme of the Standards. Within that framework, can you insure that the mathematically talented also get what they need?*

**Burrill:** Mathematically gifted children are part of the "all". I think it's important that we continue to find ways to nurture and to challenge those kids. It's not enough to just let them sprout by themselves. They need guidance and help and opportunities. It certainly needs to be a priority.

**Notices:** *Is that going to be part of the updated Standards?*

**Burrill:** Of course. I can't tell you how it will happen for sure, because the way the NCTM set up the Standards revision was to pick very good people and put them in charge.

**Notices:** *Do you think kids' mathematical abilities have declined in the time you've been teaching?*

**Burrill:** No, I do not. When I started teaching, in my school there were two precalculus classes and two second-year algebra classes. The school enrollment is about the same now as when I started there. When I left, there were two calculus classes, four precalculus classes, and six second-year algebra classes. Over that time we at least doubled the enrollment in our upper-level math classes. And the kids that I had were doing more high-powered thinking and reasoning about mathematics. They were asking me questions that I had to go home and look up or discuss with my colleagues to a much greater degree than they were when I first started teaching.

**Notices:** *I'd like to ask you about teacher background and qualifications. Many people worry that teachers don't have enough background to teach in the traditional way or in the reform way. Do you think that these worries are justified?*

**Burrill:** I think these worries are very much justified. It's a matter of very much concern to me that we're placing demands on teachers for which we have not prepared them. We have many teachers who are not certified in mathematics. About 37 percent of mathematics teachers in our high schools do not have minors in mathematics or mathematics education, and about 89 percent of teachers in grades 5-8 do not. And that means that these teachers have a minimal background in mathematics. In many schools in many states, to teach K-8 mathematics all you need to have is one or two survey mathematics courses. We're asking these teachers to teach things like discrete math and statistics and algebra and geometry, to go beyond the arithmetic level, but we haven't given them the background and content knowledge that they need in order to make sense of what they're supposed to teach. And that's real scary.

Right now, many places are experiencing teacher shortages. For example, Milwaukee is desperately

in need of teachers. I know of at least two suburban areas where they called all over looking for people to take very good teaching jobs and found no one. So they have retired community people who are holding down the fort. The need for teachers—especially highly qualified teachers who know their mathematics and how to teach it—is critical.

Taken together, this means that we need to have a real emphasis on long-term, coordinated professional development as well as strong pre-service programs that will prepare teachers to teach the math kids need to know.

**Notices:** *This year President Clinton proposed the development of a national 8th-grade mathematics test. Congress didn't fund it, so the idea may be dead. Do you think it would be a good idea?*

**Burrill:** I think it presents a really wonderful opportunity, but it also has potentially some serious disadvantages. It could establish a common platform for important mathematics students should know by grade 8, set high expectations for kids throughout the United States, and provide feedback that parents and teachers could find useful. We need, however, to put in place the support mechanisms and resources that teachers and schools would need in order to actually change what they're doing and help bring their students to the level where they have the background to do well on the test. If we could get the attention and energy of the United States focused on high standards and good performance in mathematics, think how wonderful it would be.

There's the potential danger, however, that if kids don't do well and we don't have interventions in place to help them and their teachers, it could turn into another round of bashing teachers and bashing schools where students have low performance. We already know some districts do poorly. We don't need another test to give us the same information. So there could be a lot of blaming, or we could begin to rank districts and schools in ways that I don't think this test was ever intended to set the stage for. So, while on one hand there are lots of really good and exciting things that can happen from it,

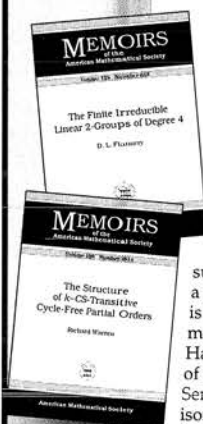
---

*"If we could get the attention and energy of the United States focused on high standards and good performance in mathematics, think how wonderful it would be."*

---



# Algebra and Algebraic Geometry



## The Finite Irreducible Linear 2-Groups of Degree 4

D. L. Flannery, *University of Canberra, ACT, Australia*

This memoir contains a complete classification of the finite irreducible 2-subgroups of  $GL(4, \mathbb{C})$ . Specifically, the author provides a parametrized list of representatives for the conjugacy classes of such groups, where each representative is defined by a generating set of monomial matrices. The problem is treated by a variety of techniques, including elementary character theory, a method for describing Hasse diagrams of submodule lattices, and calculation of 2-cohomology by means of the Lyndon-Hochschild-Serre spectral sequence. Related questions concerning isomorphism between the listed groups, and Schur indices of their defining characters, are also considered.

### Features:

- A complete classification of a class of  $p$ -groups
- A first step towards extending presently available databases for use in proposed "soluble quotient algorithms"
- Groups presented explicitly; may be used to test conjectures or to serve generally as a resource in group-theoretic computations

*Memoirs of the American Mathematical Society*, Volume 129, Number 613; 1997; 77 pages; Softcover; ISBN 0-8218-0625-4; List \$36; Individual member \$22; Order code MEMO/129/613NA

## The Structure of $k$ -CS-Transitive Cycle-Free Partial Orders

Richard Warren, *University of Leeds, England*

The class of cycle-free partial orders (CFPOs) is defined, and the CFPOs fulfilling a natural transitivity assumption, called  $k$ -connected set transitivity ( $k$ -CS-transitivity), are analyzed in some detail. Classification in many of the interesting cases is given. This work generalizes Droste's classification of the countable  $k$ -transitive trees ( $k \geq 2$ ). In a CFPO, the structure can branch downwards as well as upwards, and can do so repeatedly (though it never returns to the starting point by a cycle). Mostly it is assumed that  $k \geq 3$  and that all maximal chains are finite. The main classification splits into the sporadic and skeletal cases. The former is complete in all cardinalities. The latter is performed only in the countable case. The classification is considerably more complicated than for trees, and skeletal CFPOs exhibit rich, elaborate and rather surprising behavior.

### Features:

- Lucid exposition of an important generalization of Droste's work
- Extended introduction clearly explaining the scope of the memoir
- Visually attractive topic with copious illustrations
- Self-contained material, requiring few prerequisites

*Memoirs of the American Mathematical Society*, Volume 129, Number 614; 1997; 166 pages; Softcover; ISBN 0-8218-0622-X; List \$45; Individual member \$27; Order code MEMO/129/614NA

All prices subject to change. Charges for delivery are \$3.00 per order. For optional air delivery outside of the continental U. S., please include \$6.50 per item. Prepayment required. Order from: American Mathematical Society, P. O. Box 5904, Boston, MA 02206-5904, USA. For credit card orders, fax (401) 455-4046 or call toll free 800-321-4AMS (4267) in the U. S. and Canada, (401) 455-4000 worldwide. Or place your order through the AMS bookstore at <http://www.ams.org/bookstore/>. Residents of Canada, please include 7% GST.



on the other hand there's the potential for much misuse.

**Notices:** Starting in 1968, the government funded a huge study called Project Follow-Through. It cost a billion dollars and ran almost thirty years. The purpose was to examine how different teaching methods or philosophies affected student performance. What they found was that the traditional, "direct instruction" method was the most effective. Are you familiar with this study?

**Burrill:** I have never heard of it. I believe in designing lessons using appropriate strategies to enable kids to learn. And that means sometimes I need to stand up and lecture, sometimes my kids need to explore and investigate, sometimes they need to work by themselves. Some problems are good for teamwork where each person does a different part, some problems are good for teamwork where students brainstorm about best ways to solve the problem, and some problems you just need to do by yourself. So to say that any one method is the right method I think is not correct. One thing that research has found and reform has emphasized is that everybody comes to understand and know in different ways. If a student has a misconception about, say, something in probability and they come to my class and I teach them all the right things but I never confront their misconceptions, the thing that they don't get about probability, then a year later—and, again, research backs this up—they will abandon all the right ways that I taught them, go back to their misconceptions, and make decisions based on those misconceptions. If we don't build on the thinking and the reasoning that kids bring to mathematical situations, we're never going to be able to move them beyond their own misunderstandings.

**Notices:** What you are describing requires a lot of discernment and perception on the part of the teacher not only to understand the kids but also to understand the mathematics enough to see what the misconception is and figure out a way to move the kids to the right idea.

**Burrill:** Right. And it's much easier for me as a teacher to have one way to do it—the way I learned or the way the book showed me. But if that's not the way that the kid understands it, or if the kid brings a different understanding to what they think I'm saying, I have to be ready to shift. I have to know enough mathematics in order to give me the freedom to make that shift. And one of the scary things is that when you listen to kids, they ask you questions that take you down different paths, and you as the teacher need to know if those paths lead to the same end in mathematics, and you have to decide whether they are mathematically beneficial. You need to know a lot of mathematics to make those choices to help kids learn what they are supposed to learn.