1980 COLE PRIZES AWARDED

In San Antonio, the 1980 Cole Prizes in Algebra were awarded to Michael Aschbacher of the California Institute of Technology for his paper *A characterization of Chevalley groups over fields of odd order*, Annals of Mathematics 106 (1977), 353—398, and to Melvin Hochster of the University of Michigan for his paper *Topics in the homological theory of commutative rings*, CBMS Regional Conference Series in Mathematics No. 24, American Mathematical Society, 1975.

The prizes were awarded by the Council of the American Mathematical Society on the recommendation of a selection committee consisting of Nathan Jacobson (chairman), Daniel G. Quillen, and Richard G. Swan.

Frank Nelson Cole served the Society for twenty-five years as Secretary and for twenty-one years as Editor-in-Chief of the *Bulletin*. In 1920 when he retired from both of these positions, a number of his friends collected a sum of money in his honor, which he in turn offered to the Society. In February 1921 the Council accepted the gift and created the "Cole Fund" which has been used for over fifty years for the award of the Cole Prizes in Algebra and in Number Theory.

The original fund has grown through earnings and additional gifts (including one made in 1929 by Professor Cole's son, Charles A. Cole, which more than doubled its size). The awards are now supplemented by money from the Steele Fund, bringing each award to $1500.

The 1980 awards were made at the Prize Session in San Antonio, held at 4:00 p.m. on Friday, January 4, 1980.

Professor Jacobson made the following comments on these awards.

It is a widely held opinion that the problem of classifying finite simple groups is close to a complete solution. This will certainly be one of the great achievements of mathematics of this century. That this is the present situation in finite group theory is due to the work of many people. Among the most fundamental contributions are those made by Michael Aschbacher. In particular his paper "A characterization of Chevalley groups over fields of odd order" lifted the subject to a new plateau and brought the classification within reach.

Melvin Hochster has made many important contributions to the theory of commutative rings. Perhaps his most important contribution to date is the proof of the existence of "big" Cohen-Macaulay modules for local rings that contain fields. This has led to the proof of a large number of conjectures in the homological theory of commutative rings that had been made by Auslander and Buchsbaum, Bass, Kaplansky, Serre, and others. The monograph "Topics in the homological theory of commutative rings" gives an account of these results.

In accepting their prizes, Professors Aschbacher and Hochster made the statements reproduced below.

Michael Aschbacher

It is certainly an honor to receive the Cole prize; to be recognized by one's peers in this way is most gratifying.

The prize committee cited the paper *A characterization of Chevalley groups over fields of odd order*. I will refer to that paper as C. I would like to say a little about the history of C and its applications. For the most part I will concentrate on the applications of C in the program to classify the finite simple groups.

The nonabelian finite simple groups known at present are the groups of Lie type, the alternating groups, and 26 simple groups which belong to no known infinite family of simple groups, and which are therefore termed *sporadic groups*. Most finite group theorists now believe there are no more infinite families of simple groups and, quite possibly, no more sporadic groups. The classification program seeks to establish this conjecture.

The classification is in terms of so called *p-local subgroups*. These are the normalizers of nontrivial p-subgroups. The first result in local group theory is Sylow's theorem. For purposes of the classification, the collection of finite groups is divided into two subcollections, according to the structure of 2-local subgroups. The groups of Lie type and odd characteristic are, for the most part, in the collection of *groups of component type*. C provides the 2-local characterization of the groups of Lie type and odd characteristic necessary to the classification of the groups of component type.

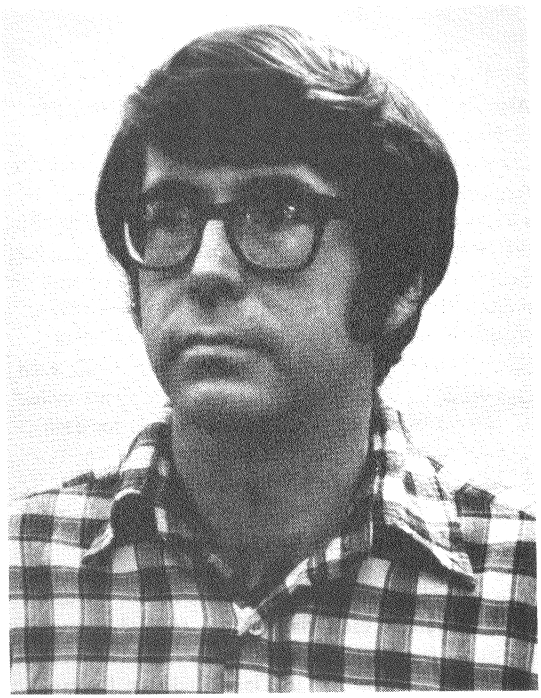
During the sixties, Daniel Gorenstein and John Walter became aware of a local property which seems to be shared by all finite groups. This observation was later formalized as the B-conjecture. In 1973, I showed that, if the B-conjecture holds, then each group G of component type possesses an element of order 2 whose centralizer has a certain structure. In the interesting situations, there is a normal subgroup A of the centralizer which is the perfect central extension of a simple group, and such that the centralizer K of A is *tightly embedded*: that is K is of even order, but distinct conjugates of K intersect in subgroups of odd order. A is called a *standard subgroup*. It can be shown that, with known exceptions, the Sylow 2-groups of K are cyclic or quaternion. In the first case, a large body of work done in the sixties, and initiated by Brauer, suggested that the group G could be determined from A. For fixed A, this is the *standard form problem for A*.

In his Colloquium Lectures at the annual meeting of the American Mathematical Society in 1974, John Thompson suggested a program to establish the B-conjecture. One step in this program involved the classification of the finite groups with a tightly embedded subgroup whose Sylow 2-subgroups are quaternion. An extension of this result constitutes the main theorem of C. A more precise statement of the theorem would probably not increase the understanding of those who do not work in the area of finite groups; those who do probably know the theorem already. One observation may be helpful to Lie theorists: if G is of Lie type, the tightly embedded subgroup in question is generated by a long root group and its negative.

While Thompson's program motivated my work on C, that program was never implemented. Instead, an alternate program gained favor. However, this second approach was motivated in turn by C, and C remained a major step in the second program. It appears that the latter program will be successful, since, at this writing, only one obstacle remains to the verification of the B-conjecture: the solution of the standard form problem for the 4-dimensional unitary group over the field of order 3. I believe it to be a relatively small obstacle.

Thus the first application of C is in the verification of the B-conjecture. The second application is in the solution of the standard form problems for the groups of Lie type and odd characteristic. A method of John Walter reduces most cases to C. The third major application is an immediate corollary of C: if the centralizer of a standard subgroup has quaternion Sylow 2-subgroups, then G is a group of Lie type.

Incidentally, only a handful of standard form problems remain to be completed. If and when these problems are finished, the groups of component type will be classified.



Michael Aschbacher

These are the principal applications of C in the classification program. C is also useful in investigating the groups of Lie type. For example C has been used by Kantor and Cooperstein to determine the subgroups of such groups containing a long root element.

BIOGRAPHICAL SKETCH

Michael Aschbacher is professor of mathematics at the California Institute of Technology. He was born on April 8, 1944, in Little Rock, Arkansas. He received a B.S. from the California Institute of Technology in 1966 and a Ph.D. from the University of Wisconsin in 1969. He held an Alfred P. Sloan Foundation Fellowship from 1973 to 1975.

Professor Aschbacher served on the AMS Committee for the 1979 Summer Institute on Finite Group Theory. He spoke at the Special Session on Finite Groups at the annual AMS meeting in Dallas (January 1973) and gave invited addresses at the Summer Meeting in Toronto (August 1976), the British Mathematical Colloquium at Aberystwyth in 1976, and the 1978 International Congress of Mathematicians in Helsinki, Finland. His major fields of interest are finite simple groups and combinatorial geometries.

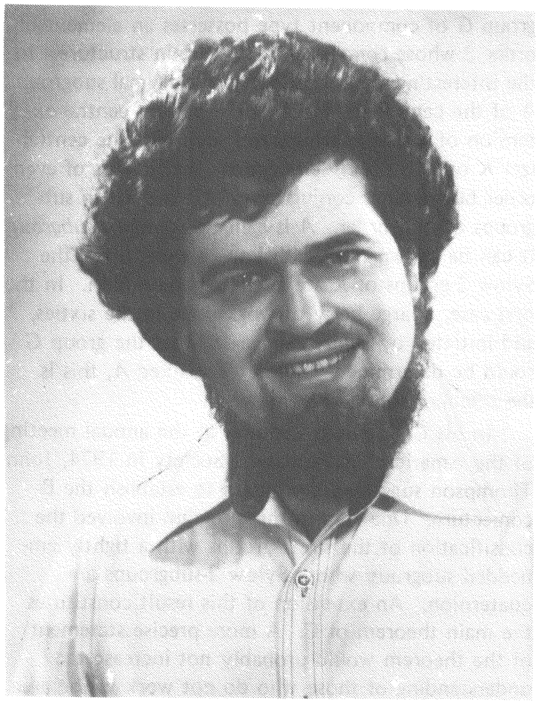
Melvin Hochster

I am very honored to receive a Cole Prize in Algebra. The work cited deals with the properties of Noetherian local rings ("local" means having a unique maximal ideal): rings of germs of appropriate functions at a point of an algebraic or analytic variety are good examples. The best local rings, like the ring of convergent power series $R = C\{z_1, \ldots, z_n\}$ (which is the local ring at a smooth point of an n-dimensional complex analytic variety), contain a sequence of elements $x_1, \ldots, x_n$ in the maximal ideal, of length n equal to the dimension of R, such that $R/\Sigma_{i=1}^{n} x_iR$ has dimension 0 (the x_i are called a "system of parameters") and such that for each i, $0 \leqslant i \leqslant n-1$, x_{i+1} is not a zerodivisor on $R/\Sigma_{j=1}^{i} x_jR$. (In the example above, we may choose $x_i = z_i^{m_i}$, $1 \leqslant i \leqslant n$.) When the local ring R is associated with a bad singularity there is usually no such sequence, but one can at least ask whether there exists a system of parameters $x_1, \ldots, x_n$ and an R-module M such that $\Sigma_{i=1}^{n} x_iM \neq M$ and x_{i+1} is not a zerodivisor on $M/\Sigma_{j=1}^{i} x_jM$, $0 \leqslant i \leqslant n-1$. The main result is that this is indeed true for every local ring which contains a field. Note that M need not be finitely generated. M is called a "big Cohen-Macaulay module."

The existence of these modules can then be used to give rather easy proofs for a substantial number of homological conjectures whose relationship to each other becomes clarified in the process. These questions were posed by M. Auslander, H. Bass and others, but especially Auslander, whose fundamental contributions provided most of the inspiration for work in this area. C. Peskine and L. Szpiro made the first great breakthrough on the homological conjectures and, while they did not study big Cohen-Macaulay modules, they made innovative use of several techniques (the Frobenius endomorphism, Artin approximation) which were essential to the work discussed here.

To show the existence of big Cohen-Macaulay modules one translates the question into one concerning the impossibility of solving certain systems of polynomial equations. Using a beautiful and deep theorem of M. Artin (algebraic approximation—but applied to the "ring structure" as well as to the equations) one can reduce to the algebro-geometric case. From there, even if one started out over a field of characteristic 0, it is possible to reduce to the case of characteristic $p > 0$. One then applies the Frobenius endomorphism to the equations to get a contradiction.

It is still not known, in dimension three or more, whether well-behaved (e.g. algebro-geometric, or analytic, or complete) local rings possess a finitely generated Cohen-Macaulay module, nor whether arbitrary local rings possess a big Cohen-Macaulay module.



Melvin Hochster

BIOGRAPHICAL SKETCH

Melvin Hochster was born in Brooklyn, New York on August 2, 1943. He attended Stuyvesant High School, received his A.B. from Harvard College in 1964, and his Ph.D. from Princeton University in 1967. From 1967 until 1973 he was assistant and then associate professor at the University of Minnesota. From 1973 until 1977 he was professor of mathematics at Purdue University. Since 1976 he has been at the University of Michigan, first as visiting professor and then as professor. He has been a visiting professor at the Mathematics Institute of Aarhus University in Denmark.

Professor Hochster has given invited addresses at the annual AMS meeting in San Antonio (January 1976) and at the International Congress of Mathematicians in Helsinki, Finland, in August 1978. He has twice been the principal lecturer at NSF-CBMS Regional Conferences: at the University of Nebraska in June 1974 and at George Mason University in August 1979. He has served as a member of the AMS Committee to Select Hour Speakers for Western Sectional Meetings (1977—1979; chairman 1979). His main research interests are commutative rings and algebraic geometry.