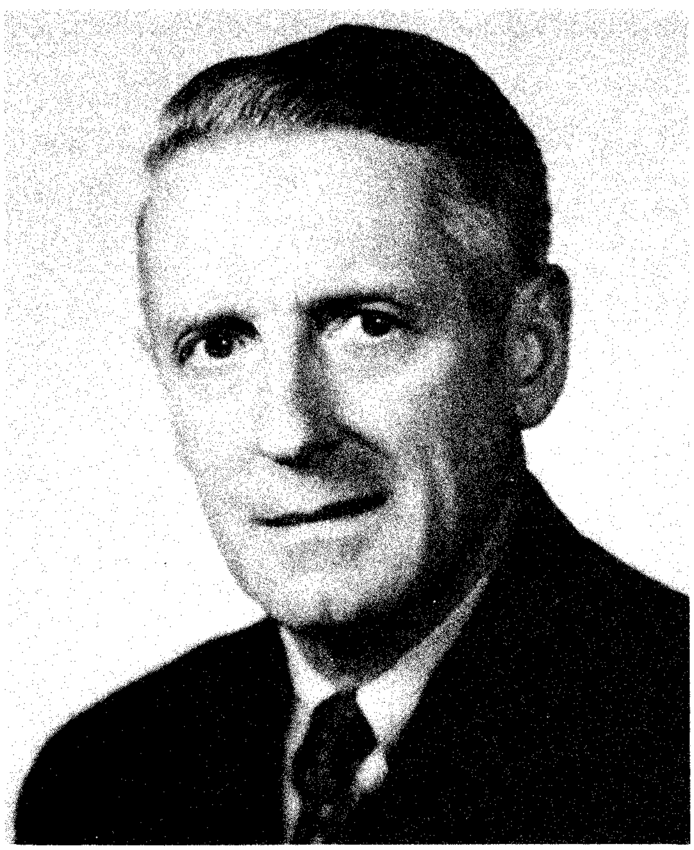
Deane Montgomery
1909–1992

Deane Montgomery was born on September 2, 1909, in Weaver, Minnesota. He received a B.A. from Hamline University in 1929, a M.S. in 1930 and a Ph.D. in 1933 from the University of Iowa.

After having held various fellowships at Harvard University, Princeton University, and the Institute for Advanced Study, he went to Smith College, where he was successively assistant professor (1935–1938), associate professor (1938–1941), and professor (1941–1946). During that period, he was also a Guggenheim fellow at the Institute and a visiting associate professor at Princeton University. After two years at Yale University as an associate professor (1946–1948), he came to the Institute, where he was a permanent member from 1948 to 1951 and a professor from 1951 to 1980, at which time he became emeritus.



His thesis adviser had been E. W. Chittenden, and he had a solid background in real analysis and point set topology. His initial research interests focused on the latter, to which he devoted his first four papers. In the tradition of L. E. J. Brouwer and "Polish topology", they already show considerable technical strength and expertise. As soon as he came to Harvard and Princeton, he broadened his interests, first to algebraic or geometric topology (initially on his own and in a private study group including N. Steenrod and Garrett Birkhoff), and then gradually to transformation groups, which became his major interest for the rest of his career.

His first papers in that area, many written in collaboration with Leo Zippin, were in part in the spirit of earlier work of Brouwer and Kerejarkto, aiming at characterizing groups of familiar euclidean motions such as translations or rotations by topological conditions. They were motivated by questions on the foundations of geometry and, foremost, by Hilbert's fifth problem. In the broad sense, the latter asks, given a locally euclidean topological group acting effectively (i.e., no element $\neq 1$ acts trivially) on an analytic manifold, whether coordinates can be introduced to make the group and the operation analytic (the answer is no). In its narrow sense, it asks whether a locally euclidean topological group is, after a suitable change of coordinates, a(n analytic) Lie group. Variants of the first problem and the second one became points of major interest in the next fifteen years or so, but not of sole interest, though.

Among Deane's contributions to the first question, let me mention the following results pertaining to a (separable metric) locally compact group G acting effectively on a manifold M: (i) If G is compact, M analytic, and each transformation is analytic, then G is a Lie group (1945); (ii) (with S. Bochner, 1946). If M is C^2 and every transformation is C^2, and no element $\neq 1$ leaves pointwise fixed a nonempty open subset, then G is a Lie group; (iii) (with S. Bochner, 1947). If M is a compact complex analytic manifold and G the group of automorphisms of M, then G is a complex Lie group acting holomorphically. On the fifth problem proper, after a series of papers with L. Zippin, Deane gave a positive solution in dimension three (1948). Then came shortly afterwards the decisive results proved jointly with L. Zippin: The existence of a closed subgroup isomorphic to $\mathbb{R}$ in a locally compact, noncompact, connected, separable metric group of strictly positive finite dimension (1951) (also established by A. Gleason) and then the reduction to groups without small subgroups (1952). Since A. Gleason had just proved that such a group is a Lie group, that gave a positive answer to Hilbert's fifth problem. In fact, the whole investigation had been carried out for separable metric finite-dimensional locally compact groups and it was shown more generally that such a group is a "generalized Lie group", i.e., possesses an open subgroup that is a projective limit of Lie groups, hence is a Lie group if it is locally connected. The assumption of finite dimensionality was soon removed by H. Yamabe, who was Deane's assistant at the time.

This was the climax of a major effort and, as I remember it, some people were mildly curious to see where Deane would turn, now that this big problem had been solved. But he did not have to look around at all. Apart from writing with L. Zippin a systematic exposition of the work on the fifth problem (1955), he just went back full time to what was really his main interest (and is already the subject matter of the last chapter of that book): Lie groups (especially compact Lie groups) of transformations on manifolds, so that, in the context of his whole work, the contributions of the fifth problem appear almost as a digression, albeit a most important one.

Even during that hot pursuit, Lie transformation groups were very much on his mind, and he brought a number of interesting contributions, in particular in joint works with L. Zippin and with H. Samelson. In fact, two papers with H. Samelson on compact Lie groups transitive on spheres or tori (1943) have a special place in my memory: When I was an assistant in Zürich, H. Hopf once gave me copies of them, and I could generalize and sharpen some of their results. This led to my first single author paper, which I submitted for publication in the *Proceedings of the AMS* to Deane, then an editor; my first contact with him.

The general problem in transformation groups is, roughly, to relate the structures of the group G, the manifold M operated upon, the orbits, fixed points, and the quotient space. At the time, there was one body of special, but deep, work, that of P. A. Smith on homeomorphisms of prime power order of homology spheres or acyclic spaces. Very little was known otherwise, and Deane was a prime mover in the development of a general theory, which he pushed in many directions. He and various collaborators proved a number of foundational results, as well as more special ones, which often opened up fruitful directions for others. A survey of these contributions and of work they led to is given by F. Raymond and R. Schultz in the Proceedings of a Conference honoring Deane on his 75th birthday (*Contemporary Mathematics*, vol. 36 (1983)), and I shall not try to duplicate it. It ranges from basic results such as the existence of a slice (with C. T. Yang, 1957), a powerful tool to study a group action near an orbit, the existence of a principal type of orbits (with C. T. Yang, 1958), to more special ones, such as actions on euclidean space or spheres with orbits of small codimension or the existence of smooth actions of SO_3 on euclidean space without fixed points (with P. E. Conner, 1962). In a first phase, the emphasis was on continuity, i.e., on topological properties, but Deane kept up with the great advances of differential topology and soon veered more and more to differentiable actions, adapting techniques and points of view of differential topology. This led to his last major effort, a long series of joint papers with C. T. Yang on free or semi-free (i.e., free outside the fixed point set) actions of the circle group on homotopy 7-spheres, which produced notably many interesting examples of homotopy complex projective 3-spaces (1966–1973).

During his tenure as a professor at the Institute, Deane was at the center of activity in topology (algebraic, geometric, differential), one of the highlights in the life of the School, first by his seminar, a perennial feature and a meeting ground for topologists in the Princeton community, but also in more informal ways. He frequently organized seminars in his office, usually with some younger members with whom he would go through some recent developments. He was always seeking out and encouraging young mathematicians. He and his wife Kay would regularly and very warmly receive the visiting members at their home. Maybe remembering his own beginnings in an out of the way place, he had a special interest, and talent, in finding out people with considerable potential among some applicants from rather isolated places about whom not much information was available.

His concern for the Institute went far beyond his immediate scientific interests and was all encompassing. He had a very high view of the role the Institute should play and served this ideal with unwavering and thoroughly unselfish loyalty. In day to day contacts, he was very kind, informal, full of understanding, always ready to help, and struck one as a very mild person, but deceptively so for anyone who, in his eyes, would threaten the Institute's standards, and who would then soon see rising an iron-willed and formidable opponent. His care for the highest standards at the Institute, later gratefully acknowledged in citations by the Trustees, was not always universally understood or shared at the time, so that he and like-minded colleagues had to weather some rather stormy moments, during which he was totally unshakable.

His abiding interest in the welfare of mathematics also led him to accept a number of official positions. In particular, he was Vice President (1952–1953), elected Trustee (1955–1961) and President (1960–1963, includes terms as President-Elect and Ex-President) of the AMS, where he also served on a number of committees, and President of the International Mathematical Union (1974–1978).

Honors, too, came his way: Honorary Doctor of Science from Hamline University (1954), Yeshiva University (1961), the University of Illinois (1977), and the University of Michigan (1986), as well as a Doctor of Laws degree from Tulane University (1967); election to the National Academy of Sciences in 1955, to the American Academy of Arts and Sciences, and the American Philosophical Society in 1958; and receipt of the Steele Prize of the AMS in 1988.

Deane was an early riser and it was a rare event for anyone to be at the Institute before him. Being very gregarious, he talked to practically everybody working in any capacity at the Institute, which won him the respect and affection of members and staff alike and gave him an exhaustive knowledge of the

Institute. Through O. Veblen, to whom he had been very close during the latter's late years, it reached to the very beginnings of the Institute so that he was a walking encyclopedia on all aspects of the Institute's history and operations.

In 1988, he and Kay moved to Chapel Hill, NC to be close to their daughter and granddaughters. That prospect did not fully compensate for the severance of the ties with an institution which had meant and still meant so much to him, and it was altogether a rather sad occasion, the sadness of which was hardly mitigated by promises to keep in touch. Being myself a fairly early riser, I often started my day by knocking at his door, sure to find him, to have a chat, mostly about mathematics, mathematicians, and Institute affairs. That I have not been able to do so after his departure has left for me a void which could not be filled.

Deane died in his sleep in Chapel Hill, on March 15, 1992. He is survived by his wife, his daughter Mary Heck, and two granddaughters.

Armand Borel
Institute for Advanced Study