Problem 1

Given the dataset in DailyPrices.csv, for the stocks SPY, AAPL, and EQIX

A. Calculate the Arithmetic Returns. Remove the mean, such that each series has 0 mean. Present the last 5 rows and the total standard deviation.

Arithmetic Returns (Last 5 Rows):

	SPY	AAPL	EQIX
499	-0.011492	-0.014678	-0.006966
500	-0.012377	-0.014699	-0.008064
501	-0.004603	-0.008493	0.006512
502	-0.003422	-0.027671	0.000497
503	0.011538	-0.003445	0.015745

Arithmetic Returns Standard Deviation:

SPY	0.008077
AAPL	0.013483
EQIX	0.015361

B. Calculate the Log Returns. Remove the mean, such that each series has 0 mean. Present the last 5 rows and the total standard deviation.

Log Returns (Last 5 Rows):

	SPY	AAPL	EQIX
499	-0.011515	-0.014675	-0.006867
500	-0.012410	-0.014696	-0.007972
501	-0.004577	-0.008427	0.006602
502	-0.003392	-0.027930	0.000613
503	0.011494	-0.003356	0.015725

Log Returns Standard Deviation:

SPY 0.008078

AAPL 0.013446

EQIX 0.015270

Problem 2

Given the dataset in DailyPrices.csv, you have a portfolio of

- 100 shares of SPY
- 200 shares of AAPL
- 150 shares of EQIX

A. Calculate the current value of the portfolio given today is 1/3/2025

Portfolio Value on 1/3/2025: $251862.50

B. Calculate the VaR and ES of each stock and the entire portfolio at the 5% alpha level assuming arithmetic returns and 0 mean return, for the following methods:

a. Normally distributed with exponentially weighted covariance with lambda=0.97

Portfolio:

VaR (Delta-Normal VaR (5%)): 3886.02

ES (Delta-Normal ES (5%)): 4873.23

Individual Stocks:

VaR (5%) for SPY: $832.16

ES (5%) for SPY: $1043.57

VaR (5%) for AAPL: $952.06

ES (5%) for AAPL: $1193.92

VaR (5%) for EQIX: $2953.92

ES (5%) for EQIX: $3704.34

b. T distribution using a Gaussian Copula

Portfolio:

VaR (T-distribution with Gaussian Copula, 5%): $4460.94

ES (T-distribution with Gaussian Copula, 5%): $6102.83

Individual Stocks:

VaR (5%) for SPY: $ 765.8586813582463

ES (5%) for SPY: $ 1025.4728006790565

VaR (5%) for AAPL: $ 1032.3262229517113

ES (5%) for AAPL: $ 1459.9167861312446

VaR (5%) for EQIX: $ 3436.298646009848

ES (5%) for EQIX: $ 4884.366944391886

c. Historic simulation using the full history.

Portfolio:

VaR (Historical, 5%): $4575.03

ES (Historical, 5%): $6059.39

Individual Stocks:

VaR (5%) for SPY: $ 872.4038627047056

ES (5%) for SPY: $ 1080.1042035242676

VaR (5%) for AAPL: $ 1067.1149556453588

ES (5%) for AAPL: $ 1437.7852719672164

VaR (5%) for EQIX: $ 3635.0770911278046

ES (5%) for EQIX: $ 4714.893996144099

C. Discuss the differences between the methods.

The three Value at Risk (VaR) and Expected Shortfall (ES) estimation methods produce different results due to their underlying assumptions and methodologies. The Delta-Normal method, which assumes normally distributed returns and uses an exponentially weighted covariance matrix with $\lambda = 0.97$, resulted in a VaR of $3,886.02 and an ES of $4,873.23. This is the lowest among the three methods, which suggests that assuming normality underestimates tail risk. In contrast, the T-distribution with Gaussian Copula method, which captures non-normal dependencies between assets, resulted in a VaR of $4,460.94 and an ES of $6,102.83. This indicates a higher risk estimate due to the ability of the t-copula to model fat tails and extreme co-movements. Finally, the Historical Simulation method, which directly uses past return data without distributional assumptions, produced the highest VaR of $4,575.03 and an ES of $6,059.39. This suggests that historical data contains more extreme losses than those predicted by the normal assumption. The results show that the normality assumption tends to underestimate potential losses, while the historical and t-copula methods provide more conservative risk estimates. Among them, the historical method gives the highest VaR, suggesting that past extreme events had significant impacts on risk estimation, while the t-copula method also captures fat tails effectively.

Problem 3

You have a European Call option with the following parameters

- Time to maturity: 3 months (0.25 years)
- Call Price: $3.00
- Stock Price: $31
- Strike Price: $30
- Risk Free Rate: 10%
- No dividends are paid.

A. Calculate the implied volatility

Implied Volatility: 0.3351 (33.51%)

B. Calculate the Delta, Vega, and Theta. Using this information, by approximately how much would the price of the option change is the implied volatility increased by 1%. Prove it.

Delta: 0.6659296527386923

Vega: 0.05640705439230115

Theta (annually): -5.544561508358889

Actual price change: $0. 056498427517343686

```
new_volatility = implied_volatility + 0.01  # Adding 1 percentage point (0.01 
in decimal)
d1_new, d2_new = calculate_d1_d2(S0, K, T, r, new_volatility)
new_price = S0 * norm.cdf(d1_new) - K * np.exp(-r * T) * norm.cdf(d2_new)
print(f"Option price with volatility +1%: ${new_price}")

# Calculate actual price change
actual_change = new_price - current_price
print(f"Actual price change: ${actual_change}")
```

C. Calculate the price of the put using Generalized Black Scholes Merton. Does Put-Call Parity Hold?

Put price: 1.2593

LHS (C + K e^(-rT)): 32.2593

RHS (P + S0): 32.2593

Put-Call Parity holds, because the difference between the left and right sides is exactly 0.

D. Given a portfolio of

a. 1 call

b. 1 put

c. 1 share of stock

Assuming the stock's return is normally distributed with an annual volatility of 25%, the expected annual return of the stock is 0%, there are 255 trading days in a year, and the implied volatility is constant. Calculate VaR and ES for a 20 trading day holding period, at alpha=5%

using:

d. Delta Normal Approximation

e. Monte Carlo Simulation

Hint: Don't forget to include the option value decay in your calculations

Delta-Normal Approximation:

VaR (5%): 5.3951

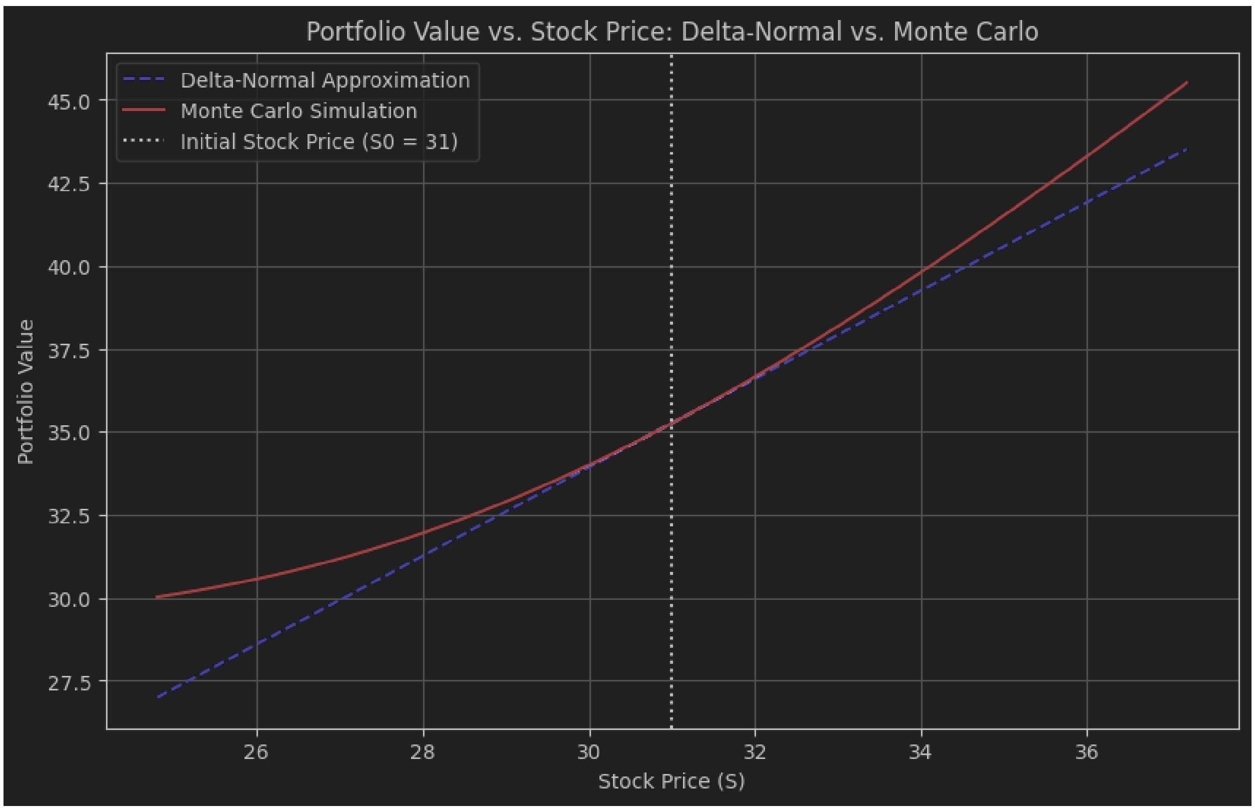
ES (5%): 6.6030

Monte Carlo Simulation:

VaR (5%): 3.9273

ES (5%): 4.2456

E. Discuss the differences between the 2 methods. Hint: graph the portfolio value vs the stock value and compare the assumptions between the 2 methods.



The Delta-Normal Approximation and Monte Carlo Simulation provide different risk estimates due to their underlying assumptions. The Delta-Normal method assumes a linear relationship between portfolio value and stock price, making it less accurate for portfolios with options, whose values exhibit convexity due to time decay and gamma effects. This explains why it produced a higher VaR (5.3951) and ES (6.6030) than Monte Carlo, as it assumes a constant delta and normally distributed returns, failing to capture option price dynamics. In contrast, Monte Carlo directly simulates stock price movements, incorporating option non-linearity and decay effects, leading to a more flexible and potentially more accurate risk estimation. The graph confirms this, as the Monte Carlo simulation (red line) follows a curved trajectory, deviating from the linear Delta-Normal estimate (blue dashed line). The lower Monte Carlo VaR (3.9273) and ES (4.2456) suggest that it better captures the full distribution of potential losses. Therefore, for portfolios containing options, Monte Carlo provides a more realistic risk assessment, while Delta-Normal can be misleading due to its reliance on normality and linearization.