**Problem 1**

Given the dataset in DailyPrices.csv, for the stocks SPY, AAPL, and EQIX

A. Calculate the Arithmetic Returns. Remove the mean, such that each series has 0 mean. Present the last 5 rows and the total standard deviation.

Arithmetic Returns (Last 5 Rows):

SPY AAPL EQIX

499 -0.011492 -0.014678 -0.006966

500 -0.012377 -0.014699 -0.008064

501 -0.004603 -0.008493 0.006512

502 -0.003422 -0.027671 0.000497

503 0.011538 -0.003445 0.015745

Arithmetic Returns Standard Deviation:

SPY 0.008077

AAPL 0.013483

EQIX 0.015361

B. Calculate the Log Returns. Remove the mean, such that each series has 0 mean. Present the last 5 rows and the total standard deviation.

Log Returns (Last 5 Rows):

SPY AAPL EQIX

499 -0.011515 -0.014675 -0.006867

500 -0.012410 -0.014696 -0.007972

501 -0.004577 -0.008427 0.006602

502 -0.003392 -0.027930 0.000613

503 0.011494 -0.003356 0.015725

Log Returns Standard Deviation:

SPY 0.008078

AAPL 0.013446

EQIX 0.015270

**Problem 2**

Given the dataset in DailyPrices.csv, you have a portfolio of

● 100 shares of SPY

● 200 shares of AAPL

● 150 shares of EQIX

A. Calculate the current value of the portfolio given today is 1/3/2025

Portfolio Value on 1/3/2025: $251862.50

B. Calculate the VaR and ES of each stock and the entire portfolio at the 5% alpha level assuming arithmetic returns and 0 mean return, for the following methods:

a. Normally distributed with exponentially weighted covariance with lambda=0.97

VaR (Delta-Normal VaR (5%)): 6.5764

ES (Delta-Normal ES (5%)): 8.2471

b. T distribution using a Gaussian Copula

VaR (T-distribution with Gaussian Copula): -6.6107

ES (T-distribution with Gaussian Copula): -9.3932

c. Historic simulation using the full history.

VaR (Historic): -7.3931

ES (Historic): -9.3871

C. Discuss the differences between the methods.

Version 1(结果正确)

The three methods—Delta-Normal VaR, T-distribution with Gaussian Copula, and Historical Simulation—yield different VaR and ES values due to their underlying assumptions and approaches. Delta-Normal VaR assumes normally distributed returns and uses an exponentially weighted covariance matrix, normally resulting in lower VaR and ES values, as it underestimates tail risk. However, in our cases, the value of the standard deviation of the return rate is indeed lower, indicating less volatility of returns. In this case, the Delta-Normal approach may overestimate risk, as the normal distribution is less sensitive to extreme events in the case of low volatility. The T-distribution with Gaussian Copula captures fat tails in returns and models dependencies using a copula, normally leading to higher risk estimates. However, in our case, the overestimation effect of the Delta-Normal approach on risk in low volatility scenarios may outweigh the ability of the T-distribution approach to capture tail risk. Moreover, because of the high weighting of SPY in the portfolio and the near-normal distribution of SPY's returns, the Delta-Normal approach may dominate the overall portfolio risk estimate. Historical Simulation, which relies entirely on historical data without distributional assumptions, produces the most conservative risk measures (VaR: -7.3931, ES: -9.3871), as it fully reflects past extreme events. While Delta-Normal VaR is computationally efficient, it fails to account for non-normality. The T-distribution method normally improves tail risk estimation but is more complex. Historical Simulation is straightforward and data-driven but may lack predictive power if historical data is unrepresentative of future risks. The choice of method depends on the trade-off between computational simplicity, accuracy in capturing tail risk, and reliance on historical data.

Version 2 （结果错误，需要替换最终结果）

The three methods—Delta-Normal VaR, T-distribution with Gaussian Copula, and Historical Simulation—yield different VaR and ES values due to their underlying assumptions and approaches. Delta-Normal VaR assumes normally distributed returns and uses an exponentially weighted covariance matrix, resulting in lower VaR and ES values (6.5764 and 8.2471, respectively), as it underestimates tail risk. The T-distribution with Gaussian Copula captures fat tails in returns and models dependencies using a copula, leading to higher risk estimates (VaR: -6.6107, ES: -9.3932). Historical Simulation, which relies entirely on historical data without distributional assumptions, produces the most conservative risk measures (VaR: -7.3931, ES: -9.3871), as it fully reflects past extreme events. While Delta-Normal VaR is computationally efficient, it fails to account for non-normality. The T-distribution method improves tail risk estimation but is more complex. Historical Simulation is straightforward and data-driven but may lack predictive power if historical data is unrepresentative of future risks. The choice of method depends on the trade-off between computational simplicity, accuracy in capturing tail risk, and reliance on historical data.

**Problem 3**

You have a European Call option with the following parameters

● Time to maturity: 3 months (0.25 years)

● Call Price: $3.00

● Stock Price: $31

● Strike Price: $30

● Risk Free Rate: 10%

●No dividends are paid.

A. Calculate the implied volatility

0.355137080428929

ANS33.51%

B. Calculate the Delta, Vega, and Theta. Using this information, by approximately how much would the price of the option change is the implied volatility increased by 1%. Prove it.

delta\_value: 0.6606490184972353

vega\_value: 6.160772344619695

theta\_value: -5.767612201319409

price\_change: 0.06160772344619695

C. Calculate the price of the put using Generalized Black Scholes Merton. Does Put-Call Parity Hold?

Put price: 1.2593

LHS (C + K e^(-rT)): 32.2593

RHS (P + S0): 32.2593

Put-Call Parity holds.

D. Given a portfolio of

a. 1 call

b. 1 put

c. 1 share of stock

Assuming the stock’s return is normally distributed with an annual volatility of 25%, the expected annual return of the stock is 0%, there are 255 trading days in a year, and the implied volatility is constant. Calculate VaR and ES for a 20 trading day holding period, at alpha=5%

using:

d. Delta Normal Approximation

e. Monte Carlo Simulation

Hint: Don’t forget to include the option value decay in your calculations

Delta-Normal Approximation:

VaR (5%): 0.5231

ES (5%): 0.8660

Monte Carlo Simulation:

VaR (5%): 3.9391

ES (5%): 4.5715

E. Discuss the differences between the 2 methods. Hint: graph the portfolio value vs the stock value and compare the assumptions between the 2 methods.

A graph with a green line and blue line

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The graph clearly illustrates why the Delta-Normal Approximation and Monte Carlo Simulation produce significantly different results for VaR (5%) and ES (5%). The Delta-Normal method assumes a linear relationship between portfolio value and stock price, represented by a straight line, which underestimates the impact of large stock price movements and ignores nonlinear effects like the Gamma of options. This leads to lower risk measures (VaR: 0.5231, ES: 0.8660) because it fails to capture tail risk and extreme scenarios. In contrast, the Monte Carlo Simulation captures the nonlinear relationship, shown by the curved line, accurately modeling the impact of options and tail risk. This results in higher risk measures (VaR: 3.9391, ES: 4.5715), as the method reflects the increased losses in extreme scenarios due to the portfolio's nonlinear behavior. The scattered Monte Carlo points further demonstrate the distribution of portfolio values, highlighting the method's ability to model complex, real-world risks. In summary, the Delta-Normal method is suitable for linear portfolios but underestimates risk for portfolios with options, while Monte Carlo provides a more accurate and realistic assessment of risk for complex portfolios.