Robust On-Policy Sampling for Data-Efficient Policy Evaluation

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Abstract

Reinforcement learning (RL) algorithms are often categorized as either on-policy or off-policy depending on whether they use data from a target policy of interest or from a different behavior policy. In this paper, we study a subtle distinction between on-policy *data* and on-policy *sampling* in the context of the RL sub-problem of policy evaluation. We observe that on-policy sampling may fail to match the expected distribution of on-policy data after observing only a finite number of trajectories and this failure hinders data-efficient policy evaluation. Towards improved data-efficiency, we show how non-i.i.d., off-policy sampling can produce data that more closely matches the expected on-policy data distribution and consequently increases the accuracy of the Monte Carlo estimator for policy evaluation. We introduce a method called *Robust On-policy Sampling* and demonstrate theoretically and empirically that it produces data that converges faster to the expected on-policy distribution compared to on-policy sampling. Empirically, we show that this faster convergence leads to lower mean squared error policy value estimates.

1 Introduction

Reinforcement learning (RL) algorithms are often categorized using the dichotomy of on-policy versus off-policy. On-policy algorithms learn about a particular target policy using data collected by behaving according to the target policy. Off-policy algorithms use data collected by behaving according to a different *behavior* policy. We study a subtle distinction between on-policy *data* versus on-policy *sampling* as a step towards more data-efficient RL algorithms. To better understand this distinction, consider a simple example. In this example, a certain target policy repeatedly visits a state in which it takes action A with probability 0.2 and action B with probability 0.8. Under on-policy sampling, after five visits to this state, we might actually observe action A 2 times and action B 3 times instead of the expected 1 and 4 times. Alternatively, we could collect data off-policy by deterministically tracking the expected target policy action proportions resulting in observing action A once and action B four times. Though the latter case uses off-policy sampling, it produces data that is arguably more on-policy than the data produced by on-policy sampling.

In this paper, we study the distinction between on-policy sampling and on-policy data in the context of the RL sub-problem of policy evaluation [Zinkevich et al., 2006]. In policy evaluation, we are given an *evaluation policy* and asked to estimate the expected return that would be accrued when running the evaluation policy on a task of interest. This problem is important for high confidence deployment of RL-trained policies. In RL applications, such as robotics, data-efficient policy evaluation is of the upmost importance – we desire the most accurate estimate with minimal collected data. While much research has gone into how to most efficiently use a set of already collected data, i.e., the off-policy policy evaluation problem, an implicit assumption in the RL community is that on-policy data is preferred to off-policy data when available. When data can be collected on-policy, we can use the Monte Carlo estimator which computes a mean return estimate using trajectories sampled i.i.d. by running the evaluation policy. In the limit, with infinite trajectories, the empirical proportion of

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each trajectory will converge to its true probability under the evaluation policy and the estimate will converge to the true expected return. However, for any *finite* sample-size, the empirical proportion of each trajectory will likely fail to match the true probability and the estimate will have error. Such *sampling error* is an inevitable feature of i.i.d. sampling. The probability of each new trajectory is unaffected by the trajectories occurring in the past and thus the only way to ensure the empirical distribution matches the true probability is to sample a large enough data set. That is, it is only in the limit that on-policy sampling produces exactly on-policy data.

The observations made so far raise the question: "can non-i.i.d., off-policy trajectory sampling cause the empirical distribution of trajectories to converge to the expected on-policy distribution faster?" We answer this question affirmatively by introducing a method that adapts the data collecting behavior policy to consider what data has already been collected when selecting future actions. We call this method *Robust On-policy Sampling* (ROS) since it converges faster to the expected on-policy trajectory distribution compared to standard on-policy sampling. We present policy evaluation experiments in finite and continuous-valued state- and action-space domains showing 1) ROS reduces sampling error in finite datasets and 2) consequently lowers the MSE of policy value estimates compared to i.i.d. on-policy sampling. We complement our empirical study with theoretical results showing that the empirical distribution of data collected by ROS converges to the on-policy distribution at a faster rate than on-policy sampling.

Our paper contributes to the field of RL on two fronts. On one front, we introduce a practical method for data collection and demonstrate empirically that it leads to more accurate policy evaluation compared to on-policy sampling. Simultaneously, our work examines nuance in the on-policy versus off-policy dichotomy. A better understanding of this nuance opens up the possibility of designing new data collection procedures to improve the data efficiency of any RL algorithm that relies upon on-policy data.

2 Related Work

Data collection is a fundamental part of the RL problem. The most widely studied data collection problem is the question of how an agent should explore its environment to learn an optimal policy [Ostrovski et al., 2017, Tang et al., 2017]. In contrast to these approaches, our work focuses on the question of how an agent should collect data to *evaluate* a fixed policy. When given a choice of how to collect data for policy evaluation, on-policy data collection is generally preferable to off-policy data collection [Sutton and Barto, 1998]. Notable exceptions are adaptive importance sampling (AIS) methods [Hanna et al., 2017, Oosterhuis and de Rijke, 2020, Ciosek and Whiteson, 2017, Bouchard et al., 2016, Frank et al., 2008] and quasi-Monte Carlo methods [Arnold et al., 2022]. Both these AIS methods and the Quasi-Monte Carlo method of Arnold et al. [2022] lower variance in estimates computed with future samples while our method lowers the total error in the estimate computed from both past and future samples.

In one of our experiments, we consider a setting where we already have some data (collected off-policy) and must decide how to collect additional data for policy evaluation. This problem has been previously studied in the bandit literature [Tucker and Joachims, 2022] or when there are only a finite number of policies that could be ran [Konyushkova et al., 2021]. These prior works also show that on-policy data collection is a sub-optimal choice. They differ from (and are complementary to) our work in that they still use i.i.d. sampling for data collection whereas we show how non-independent sampling can be used to produce data that more closely matches a desired distribution.

The method we introduce in this paper is motivated by the idea of decreasing sampling error in all collected data. Previous work has considered how sampling error can be reduced *after* data collection by re-weighting the obtained samples. For example, Hanna et al. [2021] show how importance sampling with an estimated behavior policy can lower sampling error and lead to more accurate policy evaluation. Similar methods have also been studied for policy evaluation in multi-armed bandits [Li et al., 2015, Narita et al., 2019] and temporal-difference learning [Pavse et al., 2020]. These prior works assume data is available a priori and ignore the question of how to collect it when unavailable.

Finally, the idea of adapting the *sampling distribution*, (i.e., behavior policy) has analogs outside of policy evaluation in Markov decision processes. O'Hagan [1987] identifies flaws with i.i.d. sampling for Monte Carlo estimation that motivate taking past samples into account. Rasmussen and Ghahramani [2003] use Gaussian processes to represent uncertainty in an expectation to be evaluated and use this uncertainty to guide future sample generation. We are unaware of any adaptations of these ideas for policy evaluation in RL.

3 Preliminaries

In this section, we introduce notation, formalize the policy evaluation problem, and introduce the Monte Carlo estimator for policy evaluation.

3.1 Notation

We assume the environment is a finite-horizon, episodic *Markov decision process* (MDP) with state set $\mathcal{S}$, action set $\mathcal{A}$, transition function, $P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1]$, reward function $R : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$, discount factor γ, maximum horizon l, and initial state distribution d_0 [Puterman, 2014]. We assume that $\mathcal{S}$ and $\mathcal{A}$ are finite though our empirical analysis considers both settings. We assume that the transition and reward functions are unknown. A policy, $\pi : \mathcal{S} \times \mathcal{A} \to [0, 1]$, is a function mapping states and actions to probabilities. We use $\pi(a|s) := \pi(s, a)$ to denote the conditional probability of action a given state s and $P(s'|s, a) := P(s, a, s')$ to denote the conditional probability of state s' given state s and action a.

Let $h := (s_0, a_0, r_0, s_1, \ldots, s_{l-1}, a_{l-1}, r_{l-1})$ be a *trajectory* and $g(h) := \sum_{t=0}^{l-1} \gamma^t r_t$ be the *discounted return* of h. Any policy induces a distribution over trajectories, $\Pr(h|\pi)$. We define the *value* of a policy, $v(\pi)$, as the expected discounted return when sampling a trajectory by following policy π: $v(\pi) := \mathbf{E}[g(H)|H \sim \pi] = \sum_h \Pr(h|\pi)g(h)$ where H is a random variable representing a trajectory and $H \sim \pi$ denotes sampling H by running π in the given environment.

3.2 Policy Evaluation

In the policy evaluation problem, we are given an *evaluation policy*, π_e, for which we would like to estimate $v(\pi_e)$. Conceptually, algorithms for policy evaluation involve two steps: collecting data (or receiving previously collected data) and computing an estimate from that data. We assume that data is collected by running a policy which we call the *behavior policy*. If the behavior policy is the same as the evaluation policy data collection is *on-policy*; otherwise it is *off-policy*. Whether on-policy or off-policy, we assume the data collection process produces a set of trajectories, $D := \{H_i\}_{i=1}^n$ and write $D \sim \pi$ to denote collecting these trajectories by running policy π. The final value estimate is then computed by a policy evaluation method (PE) that maps the set of trajectories to a scalar-valued estimate of $v(\pi_e)$. Following earlier work in policy evaluation (e.g., [Thomas and Brunskill, 2016, Jiang and Li, 2016]), we set our goal to be policy evaluation with low *mean squared error* (MSE):

$$\mathrm{MSE}\Big[\mathrm{PE}\Big] := \mathbf{E}\bigg[\Big(\mathrm{PE}(D) - v(\pi_e)\Big)^2 \;\bigg|\; D \sim \pi_b\bigg], \tag{1}$$

where π_b is the behavior policy that is ran to collect D.

3.3 Monte Carlo Policy Evaluation

Perhaps the most fundamental, model-free policy evaluation method is the *Monte-Carlo* (MC) estimator. Given a data set, D, of n trajectories, the Monte Carlo estimate is the mean return over D:

$$\mathrm{MC}(D) := \frac{1}{n}\sum_{i=1}^{n} g(H_i) = \sum_h \Pr(h|D)g(h), \tag{2}$$

where $\Pr(h|D)$ denotes the empirical probability of h, i.e. how often h appears in D.

If trajectories in D are collected i.i.d. by running π_e (i.e., *on-policy* sampling), the Monte Carlo estimator is unbiased and consistent assuming $g(h)$ is bounded [Sen and Singer, 1993]. However, this method can have high variance as on-policy sampling may require many trajectories for the empirical trajectory distribution $\Pr(h|D)$ to accurately approximate $\Pr(h|\pi_e)$. Since on-policy sampling collects each trajectory i.i.d., it relies on the law of large numbers for an accurate weighting on each possible return. We call error between $\Pr(h|D)$ and $\Pr(h|\pi_e)$ *sampling error*.

4 Data-Conditioned Monte Carlo Estimates

In this section, we motivate how an estimator that uses on-policy data can benefit from off-policy sampling. Specifically, we consider the Monte Carlo estimator and suppose that we have already collected a data set, $\mathcal{D}_1$, of trajectories. We now wish to collect an additional set of trajectories, D_2, and compute the Monte Carlo estimate with the set $\mathcal{D}_1 \cup D_2$. Note that $\mathcal{D}_1$ is a fixed set (the trajectories actually observed) while D_2 is a random variable (the trajectories yet to be observed). How should the additional trajectories be collected for minimal MSE policy evaluation with the Monte Carlo estimator? Our analysis in this section suggests that i.i.d. sampling of trajectories with π_e may be a sub-optimal choice.

In the setting described in the preceding paragraph, the Monte Carlo estimator using $\mathcal{D}_1 \cup D_2$ can be written as:

$$\mathrm{MC}(\mathcal{D}_1 \cup D_2) \coloneqq \underbrace{\frac{1}{n}\sum_{i=1}^{n_{\mathcal{D}_1}} g(h_i)}_{\text{fixed value}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n_{D_2}} g(H_i)}_{\text{random variable}}, \tag{3}$$

where $n_{\mathcal{D}_1}$ and n_{D_2} are the number of trajectories in $\mathcal{D}_1$ and D_2, respectively and $n = n_{\mathcal{D}_1} + n_{D_2}$ We refer to (3) as the *data-conditioned Monte Carlo estimator*.

Viewing the Monte Carlo estimator as a sum between a fixed quantity and a random quantity changes how we view the statistical properties of the estimator. For instance, while the Monte Carlo estimator is known to be unbiased under on-policy sampling, its data-conditioned estimate is biased as shown in the following proposition.

Proposition 1. *The data conditioned Monte Carlo estimator is biased under on-policy sampling of D_2 unless $\mathrm{MC}(\mathcal{D}_1) = v(\pi_e)$ or $\mathcal{D}_1 = \emptyset$. That is:*

$$\mathbf{E}\left[\mathrm{MC}(\mathcal{D}_1 \cup D_2) \,\middle|\, D_2 \sim \pi_e\right] \neq v(\pi_e).$$

Proof. See Appendix A. □

Remark 1. *Proposition 1 holds even if $\mathcal{D}_1$ was collected under on-policy sampling as well. When $\mathcal{D}_1$ was collected under on-policy sampling then the Monte Carlo estimator is unbiased considering all possible realizations of $\mathcal{D}_1$. However, once the trajectories in $\mathcal{D}_1$ are fixed, it no longer matters what others values they could have taken.*

Can we reduce the bias of the data-conditioned Monte Carlo estimator by collecting D_2 with a policy that is different than π_e? We conclude this section with an example showing that we can. Consider a one-step MDP with one state, s, and two actions, a_0 and a_1. The return following a_0 is 2 and the return following a_1 is 4. The evaluation policy is $\pi_e(a_0|s) = \pi_e(a_1|s) = 0.5$. Suppose that, after sampling 3 trajectories, $\mathcal{D}_1$ contains two of $\{s, a_0, 2\}$ and one occurrence of $\{s, a_1, 4\}$. Note that action a_0 is over-sampled relative to its true probability in s and a_1 is under-sampled. If we collect an additional trajectory with π_e the expected value of the Monte Carlo estimate is: $\frac{1}{4}(2+2+4+2\pi_e(a_0)+4\pi_e(a_1)) = \frac{11}{4} = 2.75$. The true value, $v(\pi_e) = 3$ and thus, *conditioned on prior data*, the Monte Carlo estimate is biased in expectation as shown in Proposition 1. If instead we choose the behavior policy such that $\pi_b(a_1) = 1$ then neither action is over- or under-sampled and the expected value of the Monte Carlo estimate is the exact true value: $\frac{1}{4}(2+2+4+4) = \frac{12}{4} = 3$.

This example highlights that adapting the behavior policy to consider previously collected data can lower the expected finite-sample error of policy evaluation. In the next section, we introduce an adaptive data collection method that adjusts the behavior policy based on what data has already been observed so as to lower the MSE of a Monte Carlo estimate using all observed data.

5 Robust On-Policy Data Collection

In this section, we introduce a data collection method that adapts the data collecting behavior policy online to minimize sampling error in the data used by the Monte Carlo estimator. Our method can be used starting with an empty $\mathcal{D}_1$ or can start with $\mathcal{D}_1$ already containing trajectories generated from any policy. In either case, the goal of the method is to adjust the behavior policy to reduce sampling error, i.e., divergence between $\Pr(h|\pi_e)$ and $\Pr(h|\mathcal{D}_1 \cup D_2)$.

To reduce sampling error when collecting future trajectories, we want to adjust the behavior policy to increase the probability of under-sampled trajectories, i.e., h for which $\Pr(h|\mathcal{D}_1) < \Pr(h|\pi_e)$.

Unfortunately, the trajectory distributions are unknown because the transition function, P, is also unknown. Instead, we will increase the probability of under-sampled actions. Let $\pi_D : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$ denote the *empirical policy* which gives the proportion of times that each action was taken in each state in $\mathcal{D}_1$. If $\pi_D(a|s) > \pi_e(a|s)$ then a has appeared more often in the data than it would in expectation under π_e. Thus, we should decrease the probability of a in s for future data collection. If $\pi_D(a|s) < \pi_e(a|s)$ then a is under-sampled and we should increase its probability.

When the state and action spaces are finite, π_D can be computed exactly as the maximum likelihood policy under $\mathcal{D}_1$:

$$\pi_D := \arg\max_{\pi} \mathcal{L}(\pi), \qquad \mathcal{L}(\pi) := \sum_{h \in \mathcal{D}_1} \sum_{t=0}^{l-1} \log \pi(a_t|s_t) \tag{4}$$

where the argmax is taken with respect to all policies. In larger MDPs, we require function approximation which may make π_D hard to compute and update online as new data is collected. Fortunately, with an additional assumption we can determine the direction to adjust action probabilities without explicitly computing π_D. We assume that π_e belongs to a class of differentiable, parameterized policies (e.g., neural networks) and is parameterized by vector $\boldsymbol{\theta} \subseteq \mathbb{R}^d$. We use $\boldsymbol{\theta}_e$ to represent the parameter values for π_e. As we show, the the gradient of the log-likelihood at $\boldsymbol{\theta}_e$, $\nabla_{\boldsymbol{\theta}} \mathcal{L}(\pi_{\boldsymbol{\theta}})|_{\boldsymbol{\theta}=\boldsymbol{\theta}_e}$ can be used to make sampling-error-reducing changes to the behavior policy. This gradient can be updated online each time an action is taken and only requires computation linear in the dimensionality of $\boldsymbol{\theta}$. The assumption of a differentiable, parameterized π_e is mild for many RL applications.

5.1 Robust On-policy Sampling

Our primary algorithmic contribution – **R**obust **O**n-Policy **S**ampling (ROS) – reduces sampling error by adapting the behavior policy with a single step of gradient descent on the log-likelihood at each time-step. From here on, we use $\nabla_{\boldsymbol{\theta}} \mathcal{L}$ to denote the gradient of the log-likelihood evaluated at $\boldsymbol{\theta}_e$. $\nabla_{\boldsymbol{\theta}} \mathcal{L}$ provides a direction to adjust $\boldsymbol{\theta}_e$ to *increase* the probability of actions that were over-sampled relative to their probability under π_e. Thus, $-\nabla_{\boldsymbol{\theta}} \mathcal{L}$ provides a direction to adjust $\boldsymbol{\theta}_e$ to *decrease* the probability of over-sampled actions for which $\pi_D(a|s) > \pi_e(a|s)$. ROS changes the evaluation policy parameters with a single step of gradient descent so that at each time-step, under-sampled actions have greater probability than they would have under π_e.

Pseudocode for ROS is given in Algorithm 1. ROS first computes $\nabla_{\boldsymbol{\theta}} \mathcal{L}$ with $\mathcal{D}_1$ if any initial trajectories are provided (Line 3). ROS then collects n trajectories by interacting with the given MDP (Lines 6-14). For each action selection, ROS sets the behavior policy parameters as $\boldsymbol{\theta}_e - \alpha \nabla_{\boldsymbol{\theta}} \mathcal{L}(\pi_{\boldsymbol{\theta}})|_{\boldsymbol{\theta}=\boldsymbol{\theta}_e}$ (Lines 9 and 10). It then computes $\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(A|s)|_{\boldsymbol{\theta}=\boldsymbol{\theta}_e}$ and updates $\nabla_{\boldsymbol{\theta}} \mathcal{L}(\pi_{\boldsymbol{\theta}})|_{\boldsymbol{\theta}=\boldsymbol{\theta}_e}$ (Lines 11 and 12). Finally, the chosen action is executed in the environment, a reward received, and the agent moves to the next state (Line 13). Importantly, note that per-timestep computation is linear in the number of parameters and remains constant as the size of $\mathcal{D}$ grows.

5.2 ROS Convergence

This section develops our theoretical understanding of ROS. First, we show that if the MDP state-space has a DAG structure then ROS converges to the expected state visitation frequencies under π_e. Second, we show, for a fixed state, that $\pi_D(\cdot|s)$ converges to $\pi_e(\cdot|s)$ faster under ROS compared to on-policy sampling. To prove these results, we make the following assumptions:

Assumption 1. *The discrete state-space of the* MDP *has a directed acyclic graph (*DAG*) structure. Specifically, states in $\mathcal{S}$ can be partitioned into l disjoint sets $\mathcal{S}_t$ indexed by episode step. The transition function is such that $P(s'|s, a) > 0$ implies that $s \in \mathcal{S}_t$ and $s' \in \mathcal{S}_{t+1}$.*

Assumption 2. ROS *uses a step-size of $\alpha = \infty$ and the behavior policy is parameterized as a softmax function, i.e., $\pi_{\boldsymbol{\theta}}(a|s) \propto e^{\theta_{s,a}}$, where for each state, s, and action, a, we have a parameter $\theta_{s,a}$. As we formally show in Appendix B, this assumption implies that* ROS *always takes the most under-sampled action in each state.*

Theorem 1. *Let $d^t_\pi(s)$ be the probability of visiting state s at episode time t while following policy π. Let $d^t_n(s)$ be the empirical frequency of visitations to state s at episode time t after observing n trajectories. Then we have that, under Assumptions 1, 2, and* ROS *action selection, $d^t_n(s)$ converges to $d^t_\pi(s)$ with probability 1 for all $s \in \mathcal{S}$ and $0 < t < l$:*

$$\lim_{n \to \infty} d^t_n(s) = d^t_\pi(s), \forall s \in \mathcal{S}, 0 \leq t < l.$$

Algorithm 1 Robust On-policy Sampling.

Input: Evaluation policy π_e with parameters $\boldsymbol{\theta}_e$, step size α, previously collected trajectories to be used for policy evaluation, $\mathcal{D}_1$ (possibly empty), number of trajectories to collect, n.
Output: Data set of trajectories.
$\nabla_{\boldsymbol{\theta}} \mathcal{L} \leftarrow \sum_{(s,a) \in \mathcal{D}_1} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a|s)|_{\boldsymbol{\theta}=\boldsymbol{\theta}_e}$
$k \leftarrow$ number of state-action tuples in $\mathcal{D}_1$
$\mathcal{D} \leftarrow \mathcal{D}_1$
for $0 \leq i < n$ **do**
    $s_0 \sim d_0$
    for $0 \leq t < l$ **do**
        $\boldsymbol{\theta}_b \leftarrow \boldsymbol{\theta}_e - \alpha \nabla_{\boldsymbol{\theta}} \mathcal{L}$
        $a_t \leftarrow A \sim \pi_{\boldsymbol{\theta}_b}(\cdot|s_t)$
        $\nabla_{\boldsymbol{\theta}} \mathcal{L} \leftarrow \frac{k}{k+1} \nabla_{\boldsymbol{\theta}} \mathcal{L} + \frac{1}{k+1} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t|s_t)|_{\boldsymbol{\theta}=\boldsymbol{\theta}_e}$
        $k \leftarrow k+1$
        $s_{t+1} \sim P(\cdot|s_t, a_t)$, $r_t \leftarrow r(s_t, a_t)$
    end for
    $\mathcal{D} \leftarrow \mathcal{D} \cup \{(s_0, a_0, r_0, ..., s_{l-1}, a_{l-1}, r_{l-1})\}$
end for
Return $\mathcal{D}$

Theorem 2. *Let s be a particular state that is visited m times during data collection and assume that $|\mathcal{A}| \geq 2$. Under Assumption 2, we have that $D_{\texttt{KL}}(\pi_D(\cdot|s)||\pi(\cdot|s)) = O_p(\frac{1}{m^2})$ under* ROS *sampling while $D_{\texttt{KL}}(\pi_D(\cdot|s)||\pi(\cdot|s)) = O_p(\frac{1}{m})$ under on-policy sampling where O_p denotes stochastic boundedness.*

Due to space constraints, we defer all proofs to Appendix B. Note that the DAG assumption is not overly restrictive as any finite-horizon MDP can be made a DAG by including the current episode step as part of the state. In addition to the DAG structure used by Theorem 1, both Theorems 1 and 2 require a finite number of states and actions. In the next section, we complement our theoretical results with an empirical study that considers non-DAG MDPs and continuous state and action spaces.

6 Empirical Study

We next conduct an empirical study of ROS in policy evaluation problems. Our primary goal is to answer the following questions:

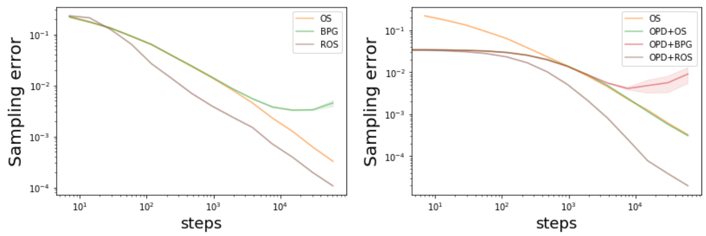
1. Does ROS reduce sampling error compared to on-policy sampling?
2. Does ROS lower policy evaluation MSE when starting with and without off-policy data?

We conduct policy evaluation experiments in four domains covering discrete and continuous state and action spaces: a multi-armed bandit problem [Sutton and Barto, 1998], Gridworld [Thomas and Brunskill, 2016], CartPole, and Continuous CartPole [Brockman et al., 2016]. Since these domains are widely used, we defer their descriptions to Appendix C. Our primary baseline for comparison is on-policy sampling (OS) of i.i.d. trajectories with the Monte Carlo estimator used to compute the final policy value estimate. We also compare to BPG which finds a minimum variance behavior policy for the ordinary importance sampling policy value estimator [Hanna et al., 2017].

We obtain the evaluation policy, π_e, in each domain by improving an initial random policy with REINFORCE [Williams, 1992] and stopping once the policy has improved but is still far from convergence. We obtain the true value $v(\pi_e)$ and the average episode steps $\overline{T}$ in each domain with 10^6 Monte Carlo roll-outs. Additional details are found in Appendix C.1. We measure sampling error with the KL-divergence (KL) between π_e and a parametric maximum likelihood estimate of π_D from the observe data. The complete definition of the sampling error metric is given in Appendix D.

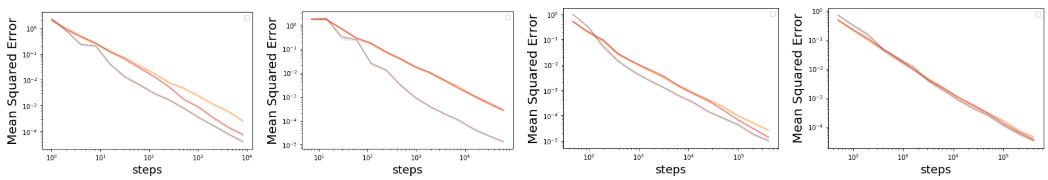
6.1 Policy Evaluation without Initial Data

We first run experiments in a **without initial data** setting in which all policy evaluation data is collected from scratch. For each domain, we run each data collection method to collect a total of



(a) Without initial data (b) With initial data

Figure 1: Sampling error (KL) curves of data collection in the GridWorld domain. Each strategy is followed to collect data with $2^{13}\overline{T}$ steps, and all results are averaged over 200 trials with shading indicating one standard error intervals. Figures 1(a) and 1(b) show the sampling error curves of data collection **without** and **with initial data**, respectively. Axes in these figures are log-scaled.



(a) Bandit (b) GridWorld (c) CartPole (d) CartPoleContinuous

Figure 2: Mean squared error (MSE) of policy evaluation in the **without initial data** setting. Policy evaluation is conducted on the data collected from each strategy, and these curves show the MSE of the estimates (lower is better). The vertical axis gives MSE and the horizontal axis is the amount of environment steps taken (both are log-scaled). Shading indicates one standard error.

$2^{13}\overline{T}$ environment steps (approximately 2^{13} trajectories) and compute metrics every $2^1, 2^2, ..., 2^{13}$ trajectories. The hyper-parameter settings for these experiments are presented in Appendix E.

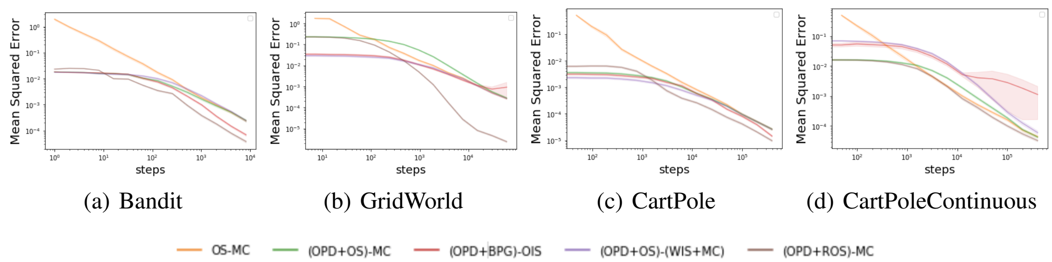
We first verify that ROS reduces sampling error (as measured with $D_{\text{KL}}(\pi_D||\pi_e)$) compared to on-policy sampling. Due to space constraints, we only show this result for the GridWorld domain (Figure 1(a)); results for other domains are qualitatively the same and can be found in Appendix D.1. Figure 1(a) shows that with ROS sampling error decreases faster than OS. Unsurprisingly, BPG increases sampling error as it is an off-policy method which adapts the behavior policy away from π_e. These results answer our first empirical question and support the central hypothesis of this work that non-i.i.d. off-policy sampling can cause the empirical distribution of data to converge to the expected on-policy distribution faster.

Ultimately, this paper focuses on reducing sampling error for lower MSE policy evaluation. Figure 2 shows that ROS lowers MSE compared to both OS and BPG across all domains.[1] Overall, these results address our second empirical question and support the claim that reducing sampling error decreases the MSE of the Monte Carlo estimator for policy evaluation.

6.2 Policy Evaluation with Initial Data

Our next set of experiments considers a **with initial data** setting in which a set of 100 trajectories are already available and we wish to use these trajectories in our policy value estimate. These trajectories are collected via i.i.d. *off-policy* sampling with a behavior policy that is slightly different than π_e. This setting is intended to represent a setting where π_e has just been updated from an older policy but we would still like to use off-policy data already collected from the older policy and combine it with new data. In addition to the off-policy data (OPD), we collect an additional $2^{13}\overline{T}$ steps of environment interaction with each method.

[1]Numeric values for the final MSE of each method can be found in Appendix F. We also report median and interquartile ranges of the error of each method in Appendix G.



(a) Bandit (b) GridWorld (c) CartPole (d) CartPoleContinuous

Figure 3: Mean squared error (MSE) of policy evaluation in the **with initial data** setting. Policy evaluation is conducted on the data collected from each strategy and a small set of initial data collected off-policy. Axes and confidence intervals are the same as in Figure 2.

For ROS, we use the OPD to initialize $\nabla_{\boldsymbol{\theta}}\mathcal{L}$. We expect to see that ROS will collect data to combine with the OPD such that the aggregate data set looks as if it had been collected with π_e to begin with. We compare ROS to the following baseline methods. **(OPD + OS)-MC** collects additional data with OS and uses the Monte Carlo estimator with the total data-set. **(OPD + OS)-(WIS + MC)** uses weighted importance sampling (WIS) to compute an estimate from the OPD combines the WIS estimate with a Monte Carlo estimate using on-policy data. **(OPD + BPG)-OIS** collects additional data with BPG and uses ordinary importance sampling as the estimator with all data. Finally, **(OS - MC)** replaces the 100 initial trajectories with trajectories from OS, then collects the remaining data with OS and uses the Monte Carlo estimator.

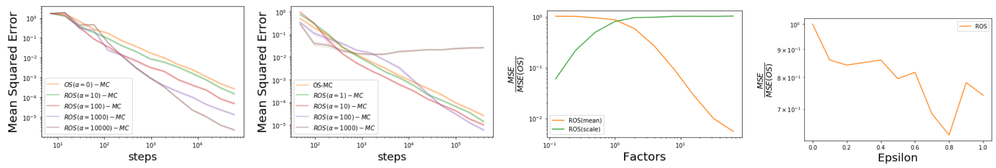
Figure 1(b) shows that sampling error decreases for all methods (except BPG) as the additional data is collected. Sampling error decreases fastest for ROS. For policy evaluation, we show MSE for varying amounts of data in Figure 3. These figures show that ROS can collect additional data that reduces sampling error in the aggregate data set and produce lower MSE estimates compared to other data collection methods. It is worth noting that in Figure 3(d), it is hard for OS to correct the bias brought by using OPD, while ROS is able to correct the empirical off-policy distribution to an empirical on-policy distribution, and thus make estimation without any off-policy corrections. Numerical results for the final MSE are in Appendix F.

6.3 Sensitivity Study

Finally, we evaluate the sensitivity of our results to hyper-parameter, environment, and policy settings. ROS involves a single hyper-parameter, step size α, which controls how much ROS updates the behavior policy away from π_e. We show MSE curves for ROS with different step size α on GridWorld and CartPole in Figures 4 ($\alpha = 0$ corresponds to OS). Figure 4(a) shows that, in GridWorld, ROS with any tested step-size produces lower MSE policy evaluation than OS for any data set size. As it collects more data, ROS with larger α enables lower MSE because the norm of $\nabla_{\boldsymbol{\theta}}\mathcal{L}$ decreases as sampling error decreases, and thus a larger α is required to make significant updates. A larger α value is also in line with our theoretical results which prescribe $\alpha = \infty$. However, in CartPole, (Figure 4(b)), ROS with the largest tested α (1000) diverges and the second largest ($\alpha = 100$) requires many steps before it improves upon OS. Thus, in domains with continuous state-spaces, more conservative α values may be preferred.

Our final set of experiments considers how the stochasticity of a domain and the particulars of π_e affect the relative improvement that ROS offers. In this sub-section, we study these settings in the Bandit domain for its simplicty; similar experimental results in GridWorld can be found in Appendix H. We choose $\alpha = 1000$ for the following experiments.

To study domain stochasticity, we first create variants of the Bandit environment by multiplying either the mean or scale of the reward distribution of each action by a varying factor. In each experimental trial, we use ROS to collect $1000\overline{T}$ steps for the Monte Carlo estimator and compute the relative MSE compared to the Monte Carlo estimator using OS with the same number of steps. Figure 4(c) shows that as the factor on the mean increases, ROS provides a greater reduction in MSE as even small amounts of sampling error translate into large MSE when the reward means are large. On the other hand, as the scale factors increase, the MSE is dominated by reward noise and the relative benefit of reducing sampling error disappears.



(a) GridWorld (b) CartPole (c) Environment Noise (d) Policy Noise

Figure 4: MSE of ROS with different step-size, α (4(a) and 4(b)). Relative improvement of ROS in Bandit compared to OS with different stochasticity in the environment (4(c)) and policy (4(d)). Results in these figures are averaged over 500 trials.

We also evaluate the relative improvement of ROS when π_e has different amounts of stochasticity. For π_e, we use ϵ-greedy policies which select the optimal action in a state with probability $1 - \epsilon$ and otherwise select an action uniformly at random. Relative improvement in MSE is shown in Figure 4(d). For all ϵ, ROS improves upon the MSE of OS. The improvement is generally larger for more stochastic π_e when sampling error in action selection will be highest.

7 Discussion and Future Work

This work has shown that off-policy non-i.i.d. sampling can produce data sets that more closely approximate the on-policy data distribution than on-policy i.i.d. sampling. We considered the problem of policy evaluation and showed that more closely approximating the on-policy data distribution leads to more data efficient policy evaluation across several domains. As far as we know, ROS is the first data collection method for policy evaluation that uses off-policy sampling to produce more closely on-policy data than the data produced by on-policy sampling.

While ROS is a first step towards off-policy algorithms that produce data matching a target distribution, we highlight a few limitations of the algorithm and our study. In our view, the main limitations of the ROS algorithm are the need to set a step-size parameter (in contrast to parameter-free on-policy sampling) and the need to update $\nabla_{\boldsymbol{\theta}} \mathcal{L}$ at each action step. For the former, future work should investigate robust methods for setting the step-size, particularly in settings where $\pi_{\boldsymbol{\theta}}$ generalizes across the state-space. For the latter limitation, a future study could consider only updating $\nabla_{\boldsymbol{\theta}} \mathcal{L}$ at the end of each episode instead of after each action choice (assuming more computation can be done between episodes). In terms of our study, for this paper we chose to study many different facets of ROS on a suite of simpler domains; a future study should assess the scalability of ROS with more complex function approximators. Finally, our theoretical results were conducted in the tabular setting; an important open question is at what rate ROS converges when $\pi_{\boldsymbol{\theta}}$ uses a function approximator that must generalize across states. Beyond these minor technical limitations, our paper addresses fundamental research questions in RL and thus we do not see obvious negative societal impacts that are unique to this work in comparison to other work in reinforcement learning and policy evaluation.

While we evaluated ROS for policy evaluation, the long term importance of this work may be in exploring the distinction between on-policy sampling and on-policy data. On-policy RL algorithms require on-policy data but our work suggests that adaptive off-policy sampling can produce on-policy data more efficiently than on-policy sampling. In the future, we wish to study these insights for on-policy policy improvement algorithms (e.g., policy gradient methods [Williams, 1992, Schulman et al., 2017]) and to extend our convergence results to non-tabular settings that require generalization across states.

8 Conclusion

In this paper we have introduced a novel data collection method for policy evaluation in reinforcement learning environments. Our method – Robust On-policy Sampling (ROS) – considers previously collected data when selecting actions to reduce sampling error in the entire collected data set. We show both in theory and in practice that data from ROS converges faster to the on-policy data distribution compared to on-policy sampling. Empirically, we find that faster convergence to the on-policy data distributions lowers the MSE of policy evaluation.

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Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] See Section 7.

(c) Did you discuss any potential negative societal impacts of your work? [Yes] See Section 7.
(d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]

2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes] See Section 5.2 and Appendix B.
   (b) Did you include complete proofs of all theoretical results? [Yes] See Appendices A and B.
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] Included in supplementary material.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] See Section 6 and Appendices C.1 and E.
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes]
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No] Experiments use RL domains and algorithms that can be ran on a typical personal computer. Minimal compute resources required to reproduce any experiment in the paper.
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