

4.6

Chinese Remainder Theorem ($[2, 3], [3, 5], [2, 7]$)

23

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 2 \pmod{7}$$

4.6 - Classical cryptography

• Caesar Cipher:

• Replace each letter by an element of \mathbb{Z}_{26}

• Can be represented by the function f that assigns the non-negative integer p , $p \leq 25$, the integer $f(p)$ in the set

$\{0, 1, \dots, 25\}$ with

$$f(p) = (p+3) \pmod{26}$$

original message's letter (integer)
↑
encrypted letter (integer)

Shift cipher
Encryption:

A B C D E F G H I J K
0 1 2 3 4 5 6 7 8 9 10

L M N O P Q R S T U V
11 12 13 14 15 16 17 18 19 20 21

W X Y Z
22 23 24 25

an integer from 0 to 25

ex: What is the secret message from the message

"MEET YOU IN THE PARK" using the
Caesar cipher

solution:

• Because it's a Caesar cipher, shift key is 3.

• Step 1: Convert letters to numbers.

• Step 2: Shift by 3 and mod by 26.

• Step 3: Convert back to letters

15, 7, 7, 22, 1, 17, 23, 11, 16, 22, 0, 7, 18, 3, 20, 13

→ PHHW BRX LA WKH SPUN

Shift cipher decryption:

$$f^{-1}(p) = (p - k) \bmod 26$$

ex: ENCRYPT the plaintext message "STOP" using
shift cipher with shift key $K=11$

Solution:

Step 1: STOP

18, 19, 14, 15

$\downarrow (p+11) \bmod 26$

Step 2: 2, 4, 25, 0

Step 3: DEZA ← final answer