Training Problems 5 - Solutions

The below exercises are based on Chapter 2 from the *Coding the Matrix* book by Philip Klein.

For Python related problems, you can assume the following:

- from math import pi,e
- from plotting import plot
- scalar_vector_mult(alpha, v) and add2(v, w) are already implemented.

1. Write a comprehension that would plot a line of 51 points, connecting points [2, 1] and [4, 3]. Your graph should have a scale of 10 and your code should just be one line.

   Solution:

   Subtract [2,1] from [4,3]

   [4,3] – [2,1] = [2,2]

   Then implement comprehension:

   plot([add2(scalar_vector_mult(i/50, [2,2]),[2,1]) for i in range(51)], 10)

   or

   Subtract [4,3] from [2,1]

   [2,1] – [4,3] = [-2,-2]

   Then implement comprehension:

   plot([add2(scalar_vector_mult(i/50, [-2,-2]),[4,3]) for i in range(51)], 10)

2. What is 3([4, 9, 2] + [3, 3, 1]) equal to?

   Solution:

   3([4, 9, 2] + [3, 3, 1]) = 3[7,12, 3] = [21, 36, 9]

3. Provide the convex combination of the following:
   a.

$$u_1 = [25]$$
$$v_1 = [75]$$
$$\alpha = .25$$
$$\beta = .75$$

Solution:

An expression of the form $\alpha \boldsymbol{u} + \beta \boldsymbol{v}$ where $\alpha, \beta \geq 0$ and $\alpha + \beta = 1$ is called a *convex combination* of $\boldsymbol{u}$ and $\boldsymbol{v}$.

$$.25[25] + .75[75] = [6.25] + [56.25] = [62.5]$$

   b.

$$u_1 = \begin{bmatrix}16\\8\end{bmatrix}$$
$$v_1 = \begin{bmatrix}12\\15\end{bmatrix}$$
$$\alpha = .5$$
$$\beta = .5$$

Solution:

$$.5\begin{bmatrix}16\\8\end{bmatrix} + .5\begin{bmatrix}12\\15\end{bmatrix} = \begin{bmatrix}8\\4\end{bmatrix} + \begin{bmatrix}6\\7.5\end{bmatrix} = \begin{bmatrix}14\\11.5\end{bmatrix}$$

4. Provide the dot product of the following vectors:
   a. [5, 3, 2, 19, 2] and [1, 2, 3, 4, 5]

Solution:

$$5 \times 1 + 3 \times 2 + 2 \times 3 + 19 \times 4 + 2 \times 5 = 103$$

   b. [5, 2, 11] and [7, 12, 5]

Solution:

$$5 \times 7 + 2 \times 12 + 11 \times 5 = 114$$