section 4.6

RSA

$n = p \cdot q$

Encryption

$C = M^e \bmod n$

(key(n, e) ← both n and e are given p·q also given

M is the block of the original message.

(the integer representation of it)

gcd(e(p-1)(q-1)) = 1 ← You'll utilize this fact for decryption

encrypted message (integer)

Blocks of M:

Divide the original message into equally sized blocks of 2N digits, where 2N is the largest even number such that the number 2525....25 with 2N digits doesn't exceed n.

↑ This passage tells you what the block size is.

ex: if $n = 2537$

2525 < 2537

largest grouping of 25's that doesn't exceed n because 2525 is four digits, the block size is four.

ex: if $n = 713345$

$252525 < 713345$

↖ because it's six digits, block size is six

$n = 113345$

$2525 < 113345$

block size is 4

for RSA,
A to J is two digits, padded with 0 on the left.
eg. A is 00
B is 01
etc

pad last block with x's if it doesn't meet the block size.

ex:

Encrypt the message STOP using RSA with key (2537, 13).
Note that $2537 = 43 \cdot 59$. $p = 43$ and they're primes
$q = 59$

$\gcd(e, (p-1)(q-1)) = \gcd(13, 42 \cdot 58) = 1$

Solution:
Block size: $2525 < 2537$, so each block is size 4.

convert letters to numbers: STOP
18, 19, 14, 15

$m_1 = 1819$
$m_2 = 1415$

$c_1 = m_1^e \bmod n$ $c_2 = m_2^e \bmod n$

STOPS
$m_1 = 1819$
$m_2 = 1415$
$m_3 = 1823$ ← X

$C_1 = 1819^{13} \bmod 2537$ ↔ $C_2 = 1415^{13} \bmod 2537$ → ⋮ → $C_2 = 2182$

use modular exponentiation

$(13)_{10} = (1101)_2$

$\checkmark 1819^1 \bmod 2537 = 1819$

$1814^2 \bmod 2537 = 513$

$\checkmark 1819^4 \bmod 2537 = 513^2 \bmod 2537 = 1858$

$\checkmark 1819^8 \bmod 2537 = 1858^2 \bmod 2537 = 1849$

$(1819 \cdot 1858 \cdot 1849) \bmod 2537 = 2081$

2081	2182