Training Problems #1 Solutions

Section 4.1:

2. **a)** $1 \mid a$ since $a = 1 \cdot a$. **b)** $a \mid 0$ since $0 = a \cdot 0$.

6. Under the hypotheses, we have $c = as$ and $d = bt$ for some s and t. Multiplying we obtain $cd = ab(st)$, which means that $ab \mid cd$, as desired.

10.

b) Since $21 \cdot 37 = 777$, we have quotient 777 **div** $21 = 37$ and remainder 777 **mod** $21 = 0$.

c) As above, we can compute 123 **div** $19 = 6$ and 123 **mod** $19 = 9$. However, since the dividend is negative and the remainder is nonzero, the quotient is $-(6+1) = -7$ and the remainder is $19 - 9 = 10$. To check that -123 **div** $19 = -7$ and -123 **mod** $19 = 10$, we note that $-123 = (-7)(19) + 10$.

12. **a)** Because 100 **mod** $24 = 4$, the clock reads the same as 4 hours after 2:00, namely 6:00.

b) Essentially we are asked to compute $12 - 45$ **mod** $24 = -33$ **mod** $24 = -33 + 48$ **mod** $24 = 15$. The clock reads 15:00.

14. This problem is equivalent to asking for the right-hand side **mod** 19. So we just do the arithmetic and compute the remainder upon division by 19.

c) $11 - 3 = 8 \pmod{19}$

f) $11^3 + 4 \cdot 3^3 = 1439 \equiv 14 \pmod{19}$

Section 4.2:

2. To convert from decimal to binary, we successively divide by 2. We write down the remainders so obtained from right to left; that is the binary representation of the given number.

a) Since $321/2$ is 160 with a remainder of 1, the rightmost digit is 1. Then since $160/2$ is 80 with a remainder of 0, the second digit from the right is 0. We continue in this manner, obtaining successive quotients of 40, 20, 10, 5, 2, 1, and 0, and remainders of 0, 0, 0, 0, 1, 0, and 1. Putting all these remainders in order from right to left we obtain $(1\ 0100\ 0001)_2$ as the binary representation. We could, as a check, expand this binary numeral: $2^0 + 2^6 + 2^8 = 1 + 64 + 256 = 321$.

b) We could carry out the same process as in part **(a)**. Alternatively, we might notice that $1023 = 1024 - 1 = 2^{10} - 1$. Therefore the binary representation is 1 less than $(100\ 0000\ 0000)_2$, which is clearly $(11\ 1111\ 1111)_2$.

4.

c) $2 + 4 + 8 + 16 + 32 + 128 + 256 + 512 = 958$

6. We follow the procedure of Example 7.

b) $(1010\ 1010\ 1010)_2 = (101\ 010\ 101\ 010)_2 = (5252)_8$

8. Following Example 7, we simply write the binary equivalents of each digit. Since $(A)_{16} = (1010)_2$, $(B)_{16} = (1011)_2$, $(C)_{16} = (1100)_2$, $(D)_{16} = (1101)_2$, $(E)_{16} = (1110)_2$, and $(F)_{16} = (1111)_2$, we have $(BADFACED)_{16} = (10111010110111111010110011101101)_2$. Following the convention shown in Exercise 3 of grouping binary digits by fours, we can write this in a more readable form as 1011 1010 1101 1111 1010 1100 1110 1101.

12. Following Example 7, we simply write the hexadecimal equivalents of each group of four binary digits. Note that we group from the right, so the left-most group, which is just 1, becomes 0001. Thus we have $(0001\ 1000\ 0110\ 0011)_2 = (1863)_{16}$.

21. **a)** 1011 1110, 10 0001 0000 0001

22. We can just add and multiply using the grade-school algorithms (working column by column starting at the right), using the addition and multiplication tables in base three (for example, $2 + 1 = 10$ and $2 \cdot 2 = 11$). When a digit-by-digit answer is too large to fit (i.e., greater than 2), we "carry" into the next column. Note that we can check our work by converting everything to decimal numerals (the check is shown in parentheses below). A calculator or computer algebra system makes doing the conversions tolerable. For convenience, we leave off the "3" subscripts throughout.

a) $112 + 210 = 1022$ (decimal: $14 + 21 = 35$)
$112 \cdot 210 = 101{,}220$ (decimal: $14 \cdot 21 = 294$)

23.

c) 2110, 1,107,667

24. We can just add and multiply using the grade-school algorithms (working column by column starting at the right), using the addition and multiplication tables in base sixteen (for example, $7 + 8 = F$ and $7 \cdot 8 = 38$). When a digit-by-digit answer is too large to fit (i.e., greater than F), we "carry" into the next column. Note that we can check our work by converting everything to decimal numerals (the check is shown in parentheses below). A calculator or computer algebra system makes doing the conversions tolerable, specially if we use built-in functions for doing so. For convenience, we leave off the "16" subscripts throughout.

b) $20CBA + A01 = 21{,}6BB$ (decimal: $134{,}330 + 2561 = 136{,}891$)
$20CBA \cdot A01 = 14{,}815{,}0BA$ (decimal: $134{,}330 \cdot 2561 = 344{,}019{,}130$)