Matrix Multiplication

multiplying Matrices, for example $A \times B$, assuming A and B are matrices, to create a new matrix.

- Not commutative:

  $A \times B \neq B \times A$

- Must ensure matrices can be multiplied.
- The column count of first matrix must be the same as the row count of the second, otherwise, can't multiply
- use dot product of each row of the first matrix, with each column of the second matrix
- If you can multiply the two matrices, the resulting matrix has the following dimension:

  row count of first matrix $\times$ column count of second matrix ex:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Solution:

You can't multiply it because the column count in A doesn't equal the row count in B. $2 \neq 3$

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ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Solution:

we can multiply this.

A is 2×2

B is 2×3

$A \times B$ is 2×3

$$A \times B = \begin{bmatrix} 1\times1+2\times4 & 1\times2+2\times5 & 1\times3+2\times6 \\ 3\times1+4\times4 & 3\times2+4\times5 & 3\times3+4\times6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \end{bmatrix}$$

Determinants:

- Allows you to see if a matrix is invertible
- The determinant is scalar, not a matrix
- If zeros populate entire row(s) and/or column(s), then the determinant is zero.

If

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then

$$\det(A) = ad - bc$$

- only square matrices have a determinant.

$$\begin{bmatrix} + & - & + & \cdots \\ - & + & - & \cdots \\ + & - & + & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

ex:

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\begin{aligned} \det(A) &= 3 \cdot 5 - 4 \cdot 2 \\ &= 15 - 8 \\ &= 7 \end{aligned}$$

ex:

$$A = \begin{bmatrix} 20 & 5 & 0 & 0 \\ 6 & 2 & 0 & 4 \\ 2 & 3 & 2 & 0 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

Solution:

$$\det(A) = 20 \cdot \det\left(\begin{bmatrix} 2 & 0 & 4 \\ 3 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}\right) - 5 \cdot \det\left(\begin{bmatrix} 6 & 0 & 4 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}\right)$$

$$+ 0 \cdot \det\left(\begin{bmatrix} 6 & 2 & 4 \\ 2 & 3 & 0 \\ 2 & 2 & 2 \end{bmatrix}\right) - 0 \cdot \det\left(\begin{bmatrix} 6 & 2 & 0 \\ 2 & 3 & 2 \\ 2 & 2 & 2 \end{bmatrix}\right)$$

$$= 20 \cdot \left(2 \cdot \det\left(\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}\right) - 0 \cdot \det\left(\begin{bmatrix} 3 & 0 \\ 2 & 2 \end{bmatrix}\right) + 4 \cdot \det\left(\begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}\right)\right)$$

$$- 5 \cdot \ldots$$

Matrix Inverse:

- Allows you to solve linear equations
- only square matrices can have an inverse.
- If determinant of the square matrix is zero, then inverse is undefined.
- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Classic method to solve linear equations, example:

$x + y = 6$

$3x - 4y = 11$

Solution:

$x = 6 - y$

$3(6-y) - 4y = 11$

$18 - 3y - 4y = 11$

$18 - 7y = 11$

$-7 = 11 - 18$

$-7y = -7$

$y = 1$

$x = 6 - 1 = 5$

using matrix inverse:

If A, B, and C are matrices:

$$AB = C$$

$$B = A^{-1}C$$

$$x + y = 6$$

$$3x - 4y = 11$$

<u>Solution:</u>

using matrix inverse:

$$AB = C$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 3 & -4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} 6 \\ 11 \end{bmatrix}}_{C}$$

$$B = A^{-1}C$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 11 \end{bmatrix} = \begin{bmatrix} 4/7 & 1/7 \\ 3/7 & -1/7 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \end{bmatrix} = \begin{bmatrix} (4/7)(6) + (1/7)(11) \\ (3/7)(6) + (-1/7)(11) \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -4 & -1 \\ -3 & 1 \end{bmatrix} = \frac{1}{1 \cdot (-4) - 1 \cdot 3} \begin{bmatrix} -4 & -1 \\ -3 & 1 \end{bmatrix}$$

$$= -\frac{1}{7} \begin{bmatrix} -4 & -1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4/7 & 1/7 \\ 3/7 & -1/7 \end{bmatrix}$$