Transpose

Transpose:

- Turning rows of a matrix into columns, to create a new matrix.
- If matrix A is $n \times m$, then A^T is $m \times n$.
- $(A^T)^T = A$
- ex If A is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A^T is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

ex: $A = [5, 7]$

$A^T = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

ex:

$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$

A^T is 4×3

A is 3×4

From matrix A:

Row 1 becomes Col 1 in A^T

" 2 " " 2 " "

" 3 " " 3 " "

So far we went over two methods to solve linear equations:

Method 1 - Back substitution:

$x + y = 4 \rightarrow x = 4 - y$

$2x + 3y = 9$

$2(4-y) + 3y = 9$ then solve for y

then plug in y value to first equation.

Method 2 - using inverse matrix:

$AB = C$

$B = A^{-1}C$

Method 3 - Gaussian Elimination to get in reduced row echelon form:

rref

Row Echelon Form (ref):

- certain criteria for a matrix.
- Getting a matrix in ref will let you get it in rref.

ref criteria:

1. All nonzero rows (rows with at least one nonzero element) are above any rows of all zeros.
2. Each leading entry of a row is in a column to the right of a leading entry above it.

leading entry = left most nonzero number in a row.

ref

ex: $\begin{bmatrix} -2 & 5 & 0 & 3 \\ 0 & 0 & 11 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 6 & 7 \\ 0 & 0 & 0 & 8 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Reduced row echelon form (rref):
- need it in this format to solve linear equations.

rref criteria:
1. matrix must be in ref.
2. The leading entry in each nonzero row is a **1.** (Called a leading 1).
3. Each Column Containing a leading 1 has zeros everywhere else in that Column.

$$\begin{bmatrix} 1 & 5 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 6 & 10 & 35 & 11 & 7 \\ 0 & 1 & 22 & -1 & 100 & 1 & 9 \end{bmatrix}$$

Gaussian Elimination:

- steps uset to convert a matrix into ref and rref. which will allow you to solve linear equations.
- You can only use a sequence of the following etementary row operations:
  1. Swapping two rows.
  2. Multiplying a row by a nonzero number.
  3. Adding a multiple of one row to another row.

ex: Solve the following using Gaussian Elimination to get in rref:

$$\begin{cases} x+5y=7 \\ -2x-7y=-5 \end{cases}$$

$x+5y=7$

$-7y-2x=-5$

$$\cancel{\begin{bmatrix} 1 & 5 & 7 \\ -7 & -2 & -5 \end{bmatrix}}$$

Solution:

$$\begin{bmatrix} 1 & 5 & 7 \\ -2 & -7 & -5 \end{bmatrix}$$

neet this to be zero

$2 \cdot R_1 + R_2 = R_2$:

in ref

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{bmatrix}$$

neet this to be a 1

$\frac{1}{3} \cdot R_2 = R_2$

needs to be a zero

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{bmatrix}$$

$-5 \cdot R_2 + R_1 = R_1$:

$$\begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{bmatrix}$$ ← in rref

tells us that $\boxed{\begin{matrix} x = -8 \\ y = 3 \end{matrix}}$