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## Extended Master theorem : Not the official name, but that's how we can refer to it in this class.

- More portable (can be applied to more recurrences)
- Just more difficult to memorize

$\log^p n = (\log n)^p$

★ Definition :

$$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$$

Cases :

1. If $a > b^k$, then $T(n) = \Theta(n^{\log_b a})$
2. If $a > b^k$,
   a. If $p > -1$, then $T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$
   b. If $p = -1$, then $T(n) = \Theta(n^{\log_b a} \log\log n)$
   c. If $p < -1$, then $T(n) = \Theta(n^{\log_b a})$
3. If $a < b^k$, then
   a. If $p \geq 0$, then $T(n) = \Theta(n^k \log^p n)$
   b. If $p < 0$, then $T(n) = O(n^k)$ ↳ Big O

Note: the following requirements:

$a \geq 1$
$b > 1$
$k \geq 0$
$p$ is a real number

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4.5.1 : Use the Master theorem to give tight asymptotic bounds for the following recurrences.

ⓐ $T(n) = 2T\left(\frac{n}{4}\right) + 1$

$a = 2$
$b = 4$ $\quad b^k = 4^0 = 1$
$k = 0$ $\quad$ Is $a > b^k$ ?
$p = 0$ $\quad$ Is $2 > 1$ ? yes

$T(n) = \Theta(n^{\log_4 2})$
$= \Theta(n^{.5})$
$= \Theta(\sqrt{n})$

$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$

Important

$$\boxed{\begin{array}{l} x^0 = 1 \\ x^1 = x \\ \log^0 n = 1 \\ \log^1 n = \log n \end{array}}$$

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$T(n) = T\left(\frac{n}{2}\right) + n^2$

$a = 1$ $\quad b^k = 2^2 = 4$
$b = 2$ $\quad$ ① Is $a > b^k$ ?
$k = 2$ $\quad$ Is $1 > 4$ ? No
$p = 0$

② Is $a = b^k$ ?
$1 = 4$ ? No

3a: $p \geq 0$ ? yes
then $T(n) = \Theta(n^k \log^p n)$
$T(n) = \Theta(n^2)$

③ Is $a < b^k$ ?

$1 < 4$ ? yes

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$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

$$\frac{n}{\log n} = n \log^{-1} n$$

$a = 2$

$b = 2$

$k = 1$

$p = -1$

$b^k = 2^1 = 2$

Since $a = b^k$ $(2 = 2)$,

we go with case 2.

Use case 2b because $p = -1$, thus

$$T(n) = \Theta(n^{\log_b a} \log\log n)$$
$$= \Theta(n^{\log_2 2} \log\log n)$$
$$= \Theta(n \log\log n)$$

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End of Day