Review Chapter 1 Probability

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1.2 Sample space

Experiments = Situations in which the outcomes occur randomly.

The **sample space** Ω corresponding to an experiment is the set of all possible outcomes. An element of Ω is denoted by ω.

An **event** is a subsets of Ω

The **union** of two events, A and B, is the event C that either A occurs or B occurs or both occur: $C = A \cup B$

The **intersection** of two events, $C = A \cap B$, is the event that both A and B occur.

The **complement** of an event, A^c, is the event that A does not occur and thus consists of all those elements in the sample space that are not in A.

The **empty set**, $\emptyset$, is the set with no elements; it is the event with no outcomes.

A and C are said to be **disjoint**, if A and C have no outcomes in common $A \cup C = \emptyset$

Commutative Laws:

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

Associative Laws:

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive Laws:

- $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
- $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

1.3 Probability measures

1. $P(\Omega) = 1$
2. If $A \supset \Omega$, then $P(A) \geq 0$.
3. If A1 and A2 are disjoint, then $P(A1 \cup A2) = P(A1) + P(A2)$
4. $P(A^c) = 1 - P(A)$ and $P(\emptyset) = 0$.
5. If $A \subset B$, then $P(A) \leq P(B)$.
6. *Addition Law* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

1.4 Computing Probabilities: Counting methods

The elements of Ω all have <u>*equal probability*</u>; so if there are N elements in Ω, each of them has probability 1/N. If A can occur in any of n mutually exclusive ways, then P(A) = n/N

$P(A) = \frac{number\ of\ ways\ A\ can\ occur}{total\ number\ of\ outcomes}$

- **Multiplication principle**: If one experiment has m outcomes and another experiment has n outcomes, then there are mn possible outcomes for the two experiments.
- **Extended multiplication principle**: If there are p experiments, and the first has n1 possible outcomes, the second n2, . . . , and the pth np possible outcomes, then there are a total of $n1n2...np$ possible outcomes for the p experiments.
- A **permutation** is an ordered arrangement of objects.
- For a set of size n and a sample of size r , there are n^r different ordered samples $withreplacement$ and $n(n-1)(n-2)...(n-r+1) = n!$ different ordered samples $withoutreplacement$.
- A **combination** is unordered sample
- The number of unordered samples of r objects selected from n objects without replacement is $\binom{n}{r}$
- The numbers $\binom{n}{r}$, called the **binomial coefficients**, occur in the expansion

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

- In particular, $2^n = \sum_{k=0}^{n} \binom{n}{k}$
- Suppose that n items are in a lot and a sample of size r is taken. The lot contains k defective items. What is the probability that the sample contains exactly m defectives?

$P(A) = \frac{\binom{k}{m}\binom{n-k}{r-m}}{\binom{n}{r}}$

- The number of ways that n objects can be grouped into r classes with n_i in the i^{th} class, $i = 1, ..., r$, and $\sum_{i=1}^{r} n_i = n$

$\binom{n}{n_1 n_2 ... n_r} = \frac{n!}{n_1! n_2! ... n_r!} = multinomial\ coefficients$

Important examples:

- Simpson's Paradox
- Birthday problems
- Capture/recapture method

1.5 Conditional Probability

Let A and B be two events with $P(B) \neq 0$. The conditional probability of A given B is defined to be

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

- MULTIPLICATION LAW: Let A and B be events and assume $P(B) \neq 0$. Then $P(A \cap B) = P(A|B)P(B)$
- LAW OF TOTAL PROBABILITY: Let $B1, B2, ..., Bn$ be such that $\bigcup_{i=1}^{n} B_i = \Omega$ and $Bi \cap Bj = \emptyset$ for $i \neq j$, with $P(B_i) > 0$ for all i . Then, for any event A,

$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$

BAYES' RULE: Let A and $B1, .., Bn$ be events where the Bi are disjoint,$\bigcup_{i=1}^{n} B_i = \Omega$ and $P(B_i) > 0$ for all i. Then

$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{n} P(A|B_i)P(Bi)}$

1.6 Independence

A and B are said to be independent events if $P(A \cap B) = P(A)P(B)$.

a collection of events, $A_1, A_2, ..., A_n$, to be **mutually independent** if for any subcollection, $Ai1, ..., Aim$,

$P(A_{i_1} \cap ... \cap A_{i_m}) = P(A_{i_1})...P(A_{i_m})$

Special examples - Matching DNA Fragments

1.7 Remarks

One might ask what is meant by the statement “The probability that this coin will land heads up is 1/2 .” Two commonly advocated views are the **frequentist approach** and the **Bayesian approach**.

- According to the frequentist approach, the statement means that if the experiment were repeated many times, the long-run average number of heads would tend to 1/2 .
- According to the Bayesian approach, the statement is a quantification of the speaker’s uncertainty about the outcome of the experiment and thus is a personal or subjective notion;