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ON INEQUALITY-SENSITIVE AND ADDITIVE ACHIEVEMENT MEASURES BASED ON ORDINAL DATA

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This paper, following earlier work on the measurement of inequality when only ordinal information is available, proposes an axiomatic derivation of a new class of inequality-sensitive and additive achievement measures. Use is then made of these indices to study health achievement in Europe, using information on self-assessed health in 30 countries, based on the European Health Interview Survey (wave 2).

JEL Codes: I3, I14, I31

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1. INTRODUCTION

In a summary of the contribution to Inequality Economics of the late Tony Atkinson, Brandolini (2017) emphasizes the fact that Atkinson's (1970) paper founded the modern theory of inequality measurement.[1] Brandolini considers that Atkinson's paper derives three very important results. The first one is that when two Lorenz curves do not cross, one can rank the two corresponding income distributions, as far as inequality is concerned, by agreeing on only a few properties of the social welfare function and accepting the so-called Pigou-Dalton principle of transfers. Since Lorenz curves may cross, we have here only a partial ordering. This leads Atkinson (1970) to derive a second result according to which any social welfare function corresponds to an inequality index, and conversely. Atkinson's third

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[1]As stressed by Brandolini *et al*. (2017), Atkinson "would warn us that similar ideas had been independently advanced by Kolm (1969) of which he had become aware only when his own article had been accepted for publication."

result is also remarkable: he derives a class of inequality indices, the so-called family of Atkinson indices, which depend on a parameter measuring the degree of inequality aversion. Moreover, Atkinson defines what he calls the "equally distributed equivalent income."[2] The latter is the level of income that, if obtained by every individual in the income distribution, would provide society with the same level of welfare as the one presently enjoyed with an unequal distribution of incomes. As stressed by Bourguignon (2017), Atkinson later on extends his analysis to the study of economic mobility (Atkinson, 1981), to that of multidimensional inequality (Atkinson and Bourguignon, 1982), and to the comparison of poverty in two income distributions (Atkinson, 1987). Finally, Atkinson (2003) provides a formal link between a social welfare and a counting approach to multidimensional poverty measurement.

The most popular application of such a counting approach is certainly that proposed by Alkire and Foster (2011) when they introduce their multi-dimensional poverty index (MPI),[3] based on a so-called "double cutoff." The counting approach has also been adopted by the European Union which now publishes a material deprivation measure (MD), defined as "the proportion of people living in households who cannot afford at least three (standard MD) or four (severe MD) items out of a list of nine items" (Guio and Marlier, 2017).

While both the MPI and the MD are derived from a set of binary indicators, there are also cases when the information available on the deprivation of an individual (or household), or on its achievement (the other side of the picture), does not consist of dichotomous but of ordinal variables. This is the kind of information provided, for example, by surveys on the happiness or the health of individuals. One may then wonder whether in such a case it is possible to compute an index that would be parallel to Atkinson's "equally distributed equivalent level of income." Such a measure should evidently be a function of distributions of individual achievements in general, and be a function of both the average level of achievement in society as well as of the degree of inequality of the distribution of achievements in particular. However, is it possible to measure the "location" and the "dispersion" of a distribution of ordinal variables?

Allison and Foster (2004) discuss this issue and come up with two important conclusions. The first one is that the mean cannot serve as reference since it is sensitive to the re-scaling of the ordinal categories, hence their decision to use the median as reference. The second conclusion is that traditional inequality indices, such as the Gini or Atkinson index, are inappropriate measures, when the variables are of an ordinal nature. While Allison and Foster's (2004) emphasis is on the *ranking* of the spread of distributions of ordinal variables, Abul Naga and Yalcin (2008) derive axiomatically cardinal measures of inequality for the case of ordered data, such as those provided by surveys on self-assessed health status (SAH). Apouey (2007) also derives measures of the dispersion of ordinal data.

[2]It is called "equal equivalent income" by Kolm (1969).

[3]Alkire and Foster (2011) give the theoretical basis for such an index. The Global Multidimensional Poverty Index (MPI) was, however, introduced in 2010 by the Oxford Poverty & Human Development Initiative (OPHI) and the United Nations Development Programme (UNDP). It now replaces the Human Poverty Index that UNDP used to publish.

Like Allison and Foster (2004), she suggests that, in order to measure the dispersion of ordinal data, the median rather than the mean individual should serve as a reference point. Lazar and Silber (2013), borrowing ideas from the literature on the measurement of occupational or residential segregation, extend the approach of Abul Naga and Yalcin. They suggest that the indices of ordinal segregation, proposed by Reardon (2009), are also relevant for the measurement of health inequality. Moreover, using a set of desirable axioms for a measure of health inequality when only ordinal variables are available, Lv *et al*. (2015) develop axiomatically a new class of inequality indices. They then give an empirical illustration based on SAH data from the 2007 wave of the China Household Income Project Survey (CHIPS). They also compare their results with those obtained when using the indices proposed by Apouey (2007), Abul Naga and Yalcin (2008), Reardon (2009), and Lazar and Silber (2013). Finally, Cowell and Flachaire (2017) take a different look at the inequality of ordinal variables. Their focus is on the concept of status within a distribution and status can be downward- or upward-looking, depending on the context of the analysis. They characterize a family of indices, related to the generalized entropy and Atkinson classes and conditional on a sensitivity parameter and a reference point. Note that this reference point for categorical data is neither the mean nor the median, but either the maximum or minimum possible value of status.

While all these papers introduce measures of the dispersion of ordinal variables, to the best of our knowledge, no study has yet proposed, in the case of ordinal variables, a measure of welfare that depends on the distribution of such variables and is yet sensitive to the dispersion of the distribution. The purpose of the present paper is precisely to derive axiomatically a family of inequality-sensitive achievement measures. Note that, in the context of cardinal variables, such measures were proposed (see Wagstaff, 2002) for a bivariate environment involving income and health. In the case of health however, what is measured is inequalities in health by income or some other measure of socio-economic status. Therefore, the concentration rather than the Gini index is used.

The present paper focuses however on a univariate, rather than a bivariate, approach to the measurement of achievement. It attempts to extend previously mentioned work on the cardinal measurement of health inequality and achievement in the population when only ordinal variables are available. We illustrate our results using health information on 30 countries from the European Health Interview Survey (EHIS), wave 2. Using the ordinal SAH variable, we first compute our achievement index for each country. We find that achievement is particularly low in Croatia, Lithuania, and Portugal, and very high in Austria, Ireland, and Malta. Using a latent variable approach, we then create a cardinal health score and measure achievement for this score (using a standard achievement index for cardinal variables). Interestingly, the ranking of countries in terms of achievement in this cardinal approach is sometimes different from the ranking we obtain with our ordinal method, highlighting the originality of our approach.

The paper is organized as follows. Section 2 presents axiomatic derivations of some new classes of measures of the level of achievement in a population when the achievement variable is ordinal. Section 3 is devoted to the empirical illustration while concluding comments are given in Section 4.

2. Axiomatic Framework

Let $N=\{1,\dots,n\}$ be the set of individuals in the society with $n\geq 2$. The set of achievements is denoted by $\mathbb{K}=\{1,\dots,K\}$ with $K\geq 2$, a lower number indicating a better achievement. An achievement vector, $s=\left(s_1,\dots,s_i,\dots,s_n\right)\in\mathbb{K}^n$, represents the achievement of each individual in the society with $s_i\in\mathbb{K}$ being individual i's achievement.

A social achievement index is defined as a mapping $h:\mathbb{K}^n\rightarrow[0,1]$ so that, for each achievement vector $s=\left(s_1,\dots,s_i,\dots,s_n\right)\in\mathbb{K}^n$, $h(s)$ reflects the overall achievement level of the society: for any $s,t\in\mathbb{K}^n$, $h(s)\geq h(t)$ is interpreted as implying that the overall achievement level of the individuals in the society under s is at least as great as the overall achievement level of the individuals in the society under t, and, $h(s)>h(t)$ is interpreted as implying that the overall achievement level of the individuals in the society under s is greater than the overall achievement level of the individuals in the society under t.

For any $i\in N$ and any achievement vector $s=\left(s_1,\dots,s_i,\dots,s_n\right)\in\mathbb{K}^n$, we shall sometimes write s as $s\equiv\left(s_i;s_{-i}\right)$ where s_i is the achievement of individual i and s_{-i} is the achievement vector consisting of the other individuals; in particular, $\left(s_i;k\right)$ denotes the achievement vector in which s_i is individual i's achievement, and the achievement of every other individual is k.

For each $s=\left(s_1,\dots,s_i,\dots,s_n\right)\in\mathbb{K}^n$ and every $k\in\mathbb{K}$, let $p_k(s)=\#\left\{i\in N:s_i=k\right\}$, and $p(s)=\left(p_1(s),\dots,p_K(s)\right)$. Therefore, $p(s)$ is the frequency distribution of the achievement vector s.

2.1. *Basic Axioms for a Social Achievement Index and their Implications*

In this subsection, we first introduce several basic axioms for a social achievement index h and then examine their consequences.

Normalization

For any $s=\left(s_1,\dots,s_i,\dots,s_n\right)\in\mathbb{K}^n$, if [$s_i=1$ for all $i\in N$] then $h(s)=1$, and if [$s_i=K$ for all $i\in N$] then $h(s)=0$.

Independence

For all $\left(s_i;s_{-i}\right),\left(t_i;s_{-i}\right),\left(s_i;t_{-i}\right),\left(t_i;t_{-i}\right)\in\mathbb{K}^n$,
$h\left(s_i;s_{-i}\right)-h\left(t_i;s_{-i}\right)=h\left(s_i;t_{-i}\right)-h\left(t_i;t_{-i}\right)$.

Weak Pareto Principle

For all $k,k'\in s,t\in\mathbb{K}^n$, if $k<k'$, then $h(k,\dots,k)>h\left(k',\dots,k'\right)$.

Anonymity

For any $s=\left(s_1,\dots,s_i,\dots,s_n\right),t=\left(t_1,\dots,t_i,\dots,t_n\right)\in\mathbb{K}^n$, if, for some permutation π of N, [$s_i=t_{\pi(i)}$ for all $i\in N$], then $h(s)=h(t)$.

Normalization simply requires that, when each individual's achievement is at the lowest level K, the social achievement index is 0, and when each individual's achievement is at the highest level 1, the social achievement index is 1. This is a reasonable property in the context of measuring social achievement based on individuals' achievement. It may be noted that this property can be dispensed, without affecting our subsequent result in any significant way.

Independence stipulates that, starting with a given achievement vector, if the achievement of one individual changes without affecting the achievement of any other individual, the resulting change in the social achievement index is independent of the initial achievements of those other individuals. Independence seems a highly plausible property. Variants of Independence were proposed in different contexts by Chakraborty *et al.* (2008), and by Dhongde *et al.* (2016). It may be noted that, in the literature on measuring achievements and inequality, various independence-type properties have been introduced. In our context, Independence implies that the resulting social achievement index can be interpreted as a cardinal measure of social achievement.

Weak Pareto Principle is another commonly used property in welfare economics and social choice theory. It requires that an achievement vector in which every individual has the same achievement level k represents a greater social achievement level than another achievement vector in which every individual has the same achievement level k' whenever k is a better achievement level than k'.

Anonymity requires the social achievement index to be symmetric with respect to individuals' achievements. It essentially stipulates that, in the measurement of social achievement from individual achievements, the names of individuals have no significance. Again, in many contexts of measuring social achievements and inequality, the property of Anonymity is invoked.

With the help of the axioms introduced above, we are ready to examine the consequences of imposing them on a social achievement index h, and our findings are summarized in the following proposition.

Proposition 1. A social achievement index h satisfies Normalization, Independence, Weak Pareto Principle and Anonymity if and only if there exists a function $\varphi:\mathbb{K} \to [0,1]$ such that,

(i) for all $s = \left(s_1, \dots, s_i, \dots, s_n\right) \in \mathbb{K}^n$, $h(s) = \sum_{i=1}^{n} \varphi\left(s_i\right)$,
(ii) $\varphi(K) = 0$, and $\varphi(1) = 1/n$,
(iii) for all $k, k' \in \mathbb{K}$, $k \langle k' \Rightarrow \varphi(k) \rangle \varphi\left(k'\right)$.

Proof
See the Appendix.

Therefore, according to Proposition 1, the implication of imposing the four properties discussed earlier on a social achievement index h is that the measure of social achievement has an additive structure: the social achievement level of an achievement vector $s = \left(s_1, \dots, s_i, \dots, s_n\right)$ is the sum of the "values", $\varphi\left(s_1\right), \dots, \varphi\left(s_i\right), \dots, \varphi\left(s_n\right)$, of individual achievements, and the "value function" $\varphi(\cdot)$ is strictly monotonic.

It may be noted that, if $K = 2$, then, Proposition 1 implies $\varphi(K = 2) = 0$ and $\varphi(1) = 1/n$ so that the social achievement index can be regarded as a "counting" measure: the social achievement level is equivalent to counting the number of individuals who have the better achievement!

2.2. *Inequality-Sensitive Axioms and the Structure of Social Achievement Indices*

In the last subsection, we derived a class of additive measures of social achievement indices. In this subsection, we consider and introduce several axioms that reflect various perspectives on distributional concerns in constructing a social achievement index.

First, as we noted earlier, if $K=2$, then, from Proposition 1, we have $\varphi(K=2)=0$ and $\varphi(1)=1/n$. Consequently, when there are only two levels of individual achievements, the structure of social achievement index is completely determined by the axioms of Normalization, Independence, Weak Pareto Principle, and Anonymity, and therefore, there is no further scope for discussing and considering distributional concerns. Thus, in the following discussions of this subsection, we assume that $K \geq 3$.

Consider the following axiom first:

Equity Principle

For all $i,j \in N$ and all $s=\left(s_1,\ldots,s_i,\ldots,s_j,\ldots,s_n\right) \in \mathbb{K}^n$, if, for some $k' \in \mathbb{K}$, $s_i < k' < s_j$ or $s_j < k' < s_i$, then, there exists $t=\left(t_1,\ldots,t_i,\ldots,t_j,t_n\right) \in \mathbb{K}^n$ with $[\forall i' \in N \backslash \{i,j\} : s_{i'} = t_{i'}]$ and $[s_i < t_i \leq t_j < s_j]$ such that $h(s) < h(t)$.

To understand the axiom of Equity Principle, suppose the society has three individuals ($n=3$) and there are four levels of individual achievements ($K=4$). Consider $i=2,j=3$, and the achievement vector $s=(1,1,4)$. Then, the axiom of Equity Principle requires the **existence** of an achievement vector $t=\left(t_1,t_2,t_3\right)$ such that $1 < t_2 \leq t_3 < 4$, $t_1=1$ and $h(s) < h(t)$. Clearly, in this example, t can be any of the following achievement vectors: $(1,2,2)$, $(1,2,3)$, and $(1,3,3)$, and consequently, the Equity Principle implies that at least one of the following holds: $h(1,1,4) < h(1,2,2)$, $h(1,1,4) < h(1,2,3)$, and $h(1,1,4) < h(1,3,3)$.

Basically, the Equity Principle requires that, other things being the same, changes in the achievements of two individuals from two further-apart levels to certain two "closer" levels should increase the level of social achievement. The idea is that, in such changes, the (local) 'inequality' of the achievement levels among the individuals in the society seems to have decreased and this should have a positive bearing on a social achievement index. The Equity Principle can be viewed as a weaker version of Hammond's (Hammond, 1976) equity principle in the social choice literature. This is because, our Equity Principle requires the existence of such a "locally inequality-reduced" achievement vector that is ranked higher than the initial achievement vector, while Hammond's equity principle insists on *any* locally inequality-reduced achievement vector being ranked higher than the initial achievement vector. It may also be noted that our Equity Principle resembles the Pigou-Dalton transfer principle in the literature on measurement of income inequality, and can be viewed as an attempt to capture the Pigou-Dalton transfer principle when levels of individual achievement are ordinal.

The imposition of the Equity Principle on a social achievement index puts more restrictions on the structure of the social achievement indices characterized in Proposition 1. Our next result summarizes those restrictions.

Proposition 2. A social achievement index h satisfies Normalization, Independence, Weak Pareto Principle, Anonymity, and Equity Principle if and only if, there exists a function $\varphi:\mathbb{K}\rightarrow[0,1]$ such that,

(i) for all $s=\left(s_1,\ldots,s_i,\ldots,s_n\right)\in\mathbb{K}^n$, $h(s)=\frac{1}{n}\sum_{k=1}^{K}p_k(s)\,\varphi(k)$,
(ii) $\frac{1}{n}=\varphi(1)>\ldots>\varphi(K)=0$,
(iii) for all $k=2,\ldots,K-1$, $2\varphi(k)>\varphi(k-1)+\varphi(k+1)$.

Proof
See the Appendix.

We note that, with the help of Equity Principle, the function φ figuring in Proposition 2 has the property (vi). Property (vi) conveys some information about and puts certain restrictions on the "curvature" or the "degree of aversion to achievement inequality" in the search of a social achievement index. Specifically, we have the following relations:

$$2\varphi(2)>\varphi(1)+\varphi(3),2\varphi(3)>\varphi(2)+\varphi(4),\ldots,2\varphi(K-1)>\varphi(K-2)+\varphi(K)$$

In a certain way, these relations resemble the strict "concavity" of the φ function when the domain of φ is an interval.

Proposition 2 is an interesting result as it puts certain "concave-type" restrictions on a φ function. And yet, it is still quite open about specific functional forms that φ can take. To narrow down the functional forms that a φ function can take, in what follows, we shall explore a class of specific social achievement indices each of which satisfies the Equity Principle. For this purpose, we consider the following axiom:

Proportional Equality

For all $s,t,u,s',t',u'\in\mathbb{K}^n$, if $s=(k,\ldots,k)$, $t=(k-1,\ldots,k-1)$, $u=(k+1,\ldots,k+1)$, $s'=\left(k',\ldots,k'\right)$, $t'=\left(k'-1,\ldots,k'-1\right)$, $u'=\left(k'+1,\ldots,k'+1\right)\in\mathbb{K}^n$, then

$$\frac{h(t)-h(s)}{h(s)-h(u)}=\frac{h\left(t'\right)-h\left(s'\right)}{h(s')-h(u')}$$

The interpretation of Proportional Equality is as follows. Assume an original achievement vector, where everyone has the same achievement level k. If now the achievement level of everyone changes one level up, to $k+1$, or if the achievement level of everyone changes one level down, to $k-1$, then the ratio of the changes in the social achievement index of two such changes is independent of the initial achievement level k: $\frac{h(k-1,\ldots,k-1)-h(k,\ldots,k)}{h(k,\ldots,k)-h(k+1,\ldots,k+1)}$ is independent of k. Proportional Equality thus requires two things: (a) the change $h(k-1,\ldots,k-1)-h(k,\ldots,k)$ is *proportional* to the change $h(k,\ldots,k)-h(k+1,\ldots,k+1)$, and (b) this proportionality is independent of k. As indicated earlier, we study social achievement indices that are cardinal. For cardinal measures of social achievement, Proportional Equality has certain appeals and plausibility. Though in the context of cardinal measures of social achievement the axiom of Proportional Equality has certain appeals, it is certainly not as appealing as the other axioms considered thus far. Nevertheless, we will see that Proportional Equality puts a further structural restriction on social achievement indices.

With the help of Proportional Equality, we obtain the following result.

Proposition 3. A social achievement index h satisfies Normalization, Independence, Weak Pareto Principle, Anonymity, Equity Principle, and Proportional Equality if and only if, there exists $\alpha \in (0,1)$ such that, for all $s = \left(s_1, \ldots, s_i, \ldots, s_n\right) \in \mathbb{K}^n$,

$$h(s) = \frac{1}{n} \sum_{k=1}^{K} p_k(s) \frac{1-\alpha^{K-k}}{1-\alpha^{K-1}}$$

Proof
See the Appendix.

The index,

$$h(s) = \frac{1}{n} \sum_{k=1}^{K} p_k(s) \frac{1-\alpha^{K-k}}{1-\alpha^{K-1}}$$

characterized in Proposition 3 satisfies Equity Principle for $0 < \alpha < 1$, and therefore reflects an "aversion" to achievement inequality. Clearly, this index is a member of the class of social achievement indices characterized in Proposition 2.

To give a glimpse into the parameter α and the nature of achievement-inequality aversion of the index, we note that, from the proof of Proposition 3 (see the Appendix),

$$\alpha = \frac{h(K-2,\ldots,K-2) - h(K-1,..,K-1)}{h(K-1,\ldots,K-1) - h(K,\ldots,K)} = \frac{h(K-2,\ldots,K-2) - h(K-1,..,K-1)}{h(K-1,\ldots,K-1)}.$$

When $\alpha < 1$, it is equivalent to having

$$h(K-2,\ldots,K-2) - h(K-1,..,K-1) < h(K-1,\ldots,K-1).$$

In other words, assume that everyone has an achievement level $(K-1)$, then the absolute change in the social achievement index when the achievement level of everyone moves one level up (to have a better achievement) is smaller than the absolute change in the social achievement index when the achievement of everyone moves one level down (to have a worse achievement). Given this and given the additive structure of the social achievement index, it illustrates, from a different perspective, that, for a given $\alpha < 1$, the index characterized in Proposition 3 exhibits an aversion to achievement inequality, and as α becomes smaller, aversion to achievement inequality becomes greater.

The class of indices characterized in Proposition 3 has some other interesting features. For example, in the case of extreme inequality of an achievement vector s, that is, when $p_1(s) = p_K(s) = 0.5$ while $p_k(s) = 0$ for any $k \neq 1, K$ we obtain $h(s) = 0.5\left[\left(\frac{1-^{K-1}}{1-^{K-1}}\right) + \left(\frac{1-^{K-K}}{1-^{K-1}}\right)\right] = 0.5\,(1+0) = 0.5$. It can be further verified that when s is symmetrically distributed and when $\alpha \to 1$, $h(s) \to 0.5$ as well.

Finally, we note the following feature of the index characterized in Proposition 3. Assume that there are J population subgroups where each subgroup $j$has size n_j and $\sum_{j=1}^{J} n_j = n$. The index $h(s)$ can then be expressed as

$$\sum_{j=1}^{J}\left(\frac{n_j}{n}\right)\sum_{k=1}^{K}\left(\frac{p_{kj}(s)}{n_j}\right)\left(\frac{1-^{K-k}}{1-^{K-1}}\right)$$

where $p_k(s) = \sum_{j=1}^{J} p_{kj}(s)$. Therefore the contribution C_j of population subgroup j to the index $h(s)$ can be written as

$$C_j = \sum_{k=1}^{K}\left(\frac{p_{kj}(s)}{n}\right)\left(\frac{1-^{K-k}}{1-^{K-1}}\right)$$

with $\sum_{j=1}^{J} C_j = h(s)$.

3. Empirical Illustration

In this section, we employ survey data on the distribution of SAH to compute the achievement index for several values of α. We show that the choice of α has some influence on the rankings of health distributions according to achievement. We also transform the ordinal SAH into a cardinal health score and compute an alternative achievement index for cardinal variables. It then appears that our achievement measures sometimes lead to conclusions that are different from those based on the existing achievement measure for cardinal variables, highlighting the usefulness of our approach.

3.1. *Data*

Our data come from the European Health Interview Survey (EHIS), which is a general population survey. It includes a wide range of information on individual demographic characteristics and on general, mental, and physical health. Two waves are available: the first wave was conducted between 2006 and 2009 and the second wave was implemented between 2013 and 2015. Detailed pieces of information about the EHIS have been published online.[4] Ideally, we would like to exploit these two waves. However, questions on chronic conditions are not comparable across waves and consequently we make use of wave 2 only. This

[4]See https://ec.europa.eu/eurostat/web/microdata/european-health-interview-survey.

wave covers 30 countries.[5] In most countries, the reference population is composed of individuals aged 15 and over.[6] The full sample contains 284,240 individuals, after excluding respondents with missing values, and the number of observations per country ranges from 3,313 (Malta) to 24,687 (Italy).

We create a series of dummies for each sex-age group, using the following age categories: 15-19 years old, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, and 75+.

Our first health variable is SAH, which is an ordinal measure that captures general health. SAH comes from the question: "How is your health in general?" with the following response categories: "very good" (first category), "good," "fair," "bad," and "very bad" (fifth category). When we compute our achievement index for SAH, we thus assume five categories ($K=5$).

Compared to other health measures, SAH offers several advantages. First, it provides a summary of individual global health status, which includes both mental and physical components. Moreover, individual SAH contains relevant health information. Indeed, SAH is an independent predictor of future mortality and morbidity. The correlation between SAH and mortality remains strong even after controlling for other health variables and for socio-economic characteristics (Idler and Benyamini, 1997). However, a shortcoming of SAH is that individuals with the same "true" (but unobserved) health level may interpret the SAH question in different ways and thus provide different answers to the question. This is the so-called "reporting heterogeneity."[7] This measurement error could be related to cultural factors and individual characteristics.[8] A similar concern is raised for subjective well-being measures by Exton *et al.* (2015/04), and Gluzmann and Gasparini (2017). Anchoring vignettes may be used to test and correct for heterogeneity (Bago D'Uva *et al.*, 2008; Ravallion *et al.*, 2013). Moreover, the ordinal nature of SAH prevents the use of traditional achievement and inequality indices for cardinal variables (such as the Gini coefficient). Transforming SAH into a cardinal measure allows researchers to use traditional indices.

We also use information on a number of "objective" health symptoms from the EHIS. First, the data indicate whether the individual has a long-standing health problem (i.e. a problem which has been lasting for at least six months) and whether she has been suffering from limitations in activities because of health problems for at least the past six months. We also use a series of dichotomous variables indicating the presence of diseases and chronic conditions, in the 12 months preceding the interview. Here is the list of these diseases: asthma, chronic bronchitis, chronic obstructive pulmonary disease or emphysema, myocardial infarction (heart attack), coronary heart disease or angina pectoris, high blood pressure, stroke (cerebral

[5]Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Germany, Denmark, Estonia, Greece, Spain, Finland, France, Croatia, Hungary, Ireland, Iceland, Italy, Lithuania, Luxembourg, Latvia, Malta, the Netherlands, Norway, Poland, Portugal, Romania, Sweden, Slovenia, Slovakia, and the United Kingdom (UK).

[6]In Sweden and the UK, the reference population is composed of individuals aged 16 and over.

[7]For Lindeboom and Van Doorslaer (2004), reporting heterogeneity could be due to cut-point shift and index shift in SAH.

[8]In the SAH question, there is no reference to any comparison group, and consequently we do not believe that differences in interpretations of the question could be due to differences in reference groups between respondents.

hemorrhage, cerebral thrombosis), arthrosis (arthritis excluded), low back disorder or other chronic back defect, neck disorder or other chronic neck defect, diabetes, allergy such as rhinitis, eye inflammation, dermatitis, food allergy or other types of allergy (allergic asthma excluded), cirrhosis of the liver, urinary incontinence, problems in controlling the bladder, kidney problems, and depression. In addition, we use information on whether the individual has difficulty in seeing and walking half a kilometer on level ground without the use of any aid. Moreover, we take advantage of information on the intensity of bodily pain and on the extent to which pain interfered with normal work, during the four weeks preceding the interview. Finally, we use a series of dummies for whether the individual is underweight or has a normal weight, is overweight, or is obese.

3.2. *Construction of the Cardinal Health Score*

We construct a cardinal health score by combining information on individual characteristics (i.e. objective health symptoms, gender-age groups, and countries). Specifically, we estimate the contribution of the characteristics to a cardinal general health measure.

More precisely, we first regress SAH on characteristics. Our approach assumes that SAH is generated by a latent health variable that captures "true" health and cannot be observed directly. We assume that the latent variable is related to the characteristics in the following way:

$$h_i^* = \beta' x_i + \varepsilon_i$$

where i represents an individual, h_i^* is the latent health measure, x_i is the vector of characteristics (health symptoms, sex-age group dummies, and country fixed effects), and ε_i is the error term which is assumed to be normally distributed.

Responses to the SAH question are linked to "true" health in the following manner:

$SAH_i = 1$ (very good) if $h_i^* < \mu_1$
For $k = 2$ (good), 3 (fair), or 4 (poor), $SAH_i = k$ if $\mu_{k-1} < h_i^* < \mu_k$
$SAH_i = 5$ (very poor) if $\mu_4 < h_i^*$

We estimate the model using an ordered probit model. In that case, the thresholds (μ_1, μ_2, μ_3, and μ_4) are assumed to be the same for all individuals and do not depend on their characteristics. We thus assume homogeneous reporting. Coefficients and thresholds are then estimated. We use the estimated coefficients to linearly predict individual health. We denote this prediction h^*. Finally, we standardize the predictions to get a general health score in the $[0,1]$ interval such that a higher value captures better health. Note that we need our score to be positive to compute the Gini coefficient of health (see below). Let h^{*min} and h^{*max} denote the smallest and largest predictions. Our cardinal score h is defined as follows:

$$h = \frac{h^{*max} - h^*}{h^{*max} - h^{*min}}$$

The score equals 0 for the individual with the poorest health level and 1 for the respondent with the best health.

Similar methods to transform SAH into a cardinal score are employed in the literature. Various lists of right-hand side variables are included in regressions and different re-scaling procedures are used. For instance, in his study on the effect of lottery prizes on health, Lindahl (2005) creates a summary health score by regressing SAH on a series of 48 health symptoms, gender, and age. He then computes the linear prediction and standardizes the prediction to obtain a score with a mean of 0 and a standard deviation of 1. His score may thus take negative values. In our study, we need positive health scores to calculate the Gini index (see below).

Alternative methods to impose cardinality are available. In particular, Van Doorslaer and Jones (2003) suggest using an existing cardinal health variable – the McMaster Health Utility Index (HUI) – to scale the responses to the SAH question, by mapping HUI into SAH, for Canadian data. They first find the boundaries of SAH intervals in HUI units. They then regress these boundaries on a number of socio-demographic factors, using an interval regression, compute the linear prediction, and finally re-scale it. Their cardinal score can be interpreted as predicted HUI score. In their study of Spain, Coveney *et al.* (2016) employ this method (with Canadian thresholds), using income, region, age, and gender as explanatory variables. In addition, Christiansen *et al.* (2018) convert SAH to a cardinal score using a Swedish scale developed by Burström *et al.* (2014) for Nordic countries. The approach uses a time trade-off model based on EQ-5D health states.

However, the cardinalization of SAH is a supra-ordinal approach whose limitations are well-known (Allison and Foster, 2004; Apouey, 2007).

3.3. *Methodology*

Using information on the distribution of SAH ($K=5$), we first compute the index from Proposition 3 for $\alpha=0.1$, 0.5, 0.9, and 0.99, as well as the index from Lemma 1 in the Appendix for $\alpha=1$. Second, employing the cardinal health score, we compute an alternative achievement index, namely the "equally distributed equivalent level of health." Let $\bar{h}$ denote the average cardinal health score and G the Gini coefficient of the cardinal score. This index is expressed as:

$$ED=\bar{h}\times(1-G)$$

The index equals 0 when inequality attains its maximum value, i.e. when $G=1$. The index increases when the average health level improves and/or when inequality goes down in the population.

This index is inspired by the work of Atkinson (1970). This author introduces the notion of the "equally distributed equivalent level of income" that captures "the level of income per head which if equally distributed would give the same level of social welfare […]" as the observed income distribution (Atkinson, 1970, p. 250). Atkinson (1970, p. 250) then defines a new measure of inequality as "1 minus the ratio of the equally distributed equivalent level of income to the mean of the actual distribution." He then identifies the functional form for his inequality index, assuming that the index should be relative. He finally derives what is known as Atkinson's inequality index. Silber (1983) applies Atkinson's approach to the measurement of development. Following Hicks and Streeten (1979), Silber (1983) considers that the duration of life is a good indicator of development and computes the "equivalent length of life" in a number of countries across the world.

While Silber (1983) measures inequality in life durations via the Atkinson index, we measure it via the Gini index.[9] Our approach is thus similar to what Apouey and Silber (2016) do in their study of health attainment and performance in Asia. We report 95% confidence intervals for the index, based on 500 bootstrap replications.

4. Results

Table 1 reports the achievement measure for SAH for three values of α (=0.1, 0.5, and 0.9). Interestingly, for all three indices, we find that achievement is especially low in Croatia, Lithuania, and Portugal, and noticeably high in Austria, Ireland, and Malta. Romania and Slovenia exhibit average achievement levels.

The ranking of countries in terms of achievement depends on the choice of α. This is especially clear when we compare rankings for $\alpha = 0.1$ and $\alpha = 0.9$. For instance, for $\alpha = 0.1$, achievement in Greece is significantly smaller than in Finland, while for $\alpha = 0.9$, the opposite is true. Similarly, for $\alpha = 0.1$, achievement in Iceland is significantly smaller than in Germany, but for $\alpha = 0.9$, the situation is reversed. To better illustrate this point, we represent the three indices in Figure 1. Countries are ranked by achievement level when $\alpha = 0.9$, which explains why the curve at the bottom of the figure has a positive slope. If the ranking of countries remained the same for different values of α, then the curves for $\alpha = 0.1$ and $\alpha = 0.5$ at the top of Figure 1 should also be upward sloping. This is not always the case, which means that the ranking of the countries depends on the value of α that is selected.

In Table 2, we show the achievement index for SAH for high values of α ($\alpha = 0.99$ and $\alpha = 1$). For $\alpha = 0.99$, we use the formula from Proposition 3, whereas for $\alpha = 1$, we employ the index shown in Lemma 1 in the Appendix. As expected, the values of the indices for $\alpha = 0.99$ and $\alpha = 1$ are very similar. Moreover, both indices provide the same ranking of countries. Finally, like for other values of α, Croatia, Lithuania, and Portugal exhibit low levels of achievement, whereas Austria, Ireland, and Malta have high levels of achievement.

Table 3 reports the equally distributed equivalent levels of health for the cardinal health score. Note that alternative cardinalization methods would lead to different rankings of countries, highlighting the limitations of scaling (Allison and Foster, 2004). The ranking of countries according to the achievement indices for ordinal and cardinal variables are sometimes different. To illustrate this point, we represent in Figure 2 achievement indices for SAH and for the health score. In the top sub-figure, countries are ranked according to the achievement index for

[9]However, Silber (1988) uses the Gini index.

TABLE 1
Achievement Index for SAH

Country	$\alpha = 0.1$				$\alpha = 0.5$				$\alpha = 0.9$			
	Index	95% Confidence Interval		Rank (from low to high)	Index	95% Confidence Interval		Rank (from low to high)	Index	95% Confidence Interval		Rank (from low to high)
Austria	0.989	0.988	0.99	28	0.919	0.917	0.921	28	0.809	0.807	0.812	28
Belgium	0.982	0.98	0.985	19	0.899	0.895	0.902	23	0.771	0.766	0.776	23
Bulgaria	0.956	0.951	0.96	5	0.847	0.842	0.853	9	0.703	0.697	0.71	10
Cyprus	0.984	0.981	0.987	23	0.912	0.908	0.917	27	0.814	0.807	0.82	29
Czech Republic	0.973	0.97	0.976	14	0.853	0.849	0.857	10	0.695	0.69	0.7	8
Germany	0.989	0.988	0.99	27	0.897	0.895	0.898	22	0.752	0.75	0.755	18
Denmark	0.986	0.984	0.989	26	0.895	0.892	0.899	21	0.755	0.749	0.76	19
Estonia	0.966	0.963	0.97	9	0.842	0.837	0.846	7	0.679	0.673	0.685	5
Greece	0.968	0.965	0.972	11	0.878	0.874	0.882	15	0.755	0.75	0.76	20
Spain	0.967	0.965	0.969	10	0.866	0.864	0.868	11	0.722	0.719	0.725	11
Finland	0.985	0.983	0.988	24	0.881	0.878	0.885	16	0.728	0.722	0.733	12
France	0.982	0.98	0.983	18	0.885	0.882	0.887	18	0.747	0.743	0.75	16
Croatia	0.945	0.939	0.951	2	0.822	0.815	0.828	4	0.673	0.665	0.68	4
Hungary	0.955	0.95	0.96	4	0.846	0.84	0.851	8	0.697	0.691	0.703	9
Ireland	0.991	0.989	0.993	29	0.929	0.926	0.932	30	0.828	0.823	0.832	30
Iceland	0.975	0.971	0.98	15	0.894	0.888	0.899	20	0.776	0.768	0.783	24
Italy	0.973	0.971	0.975	12	0.875	0.873	0.877	12	0.735	0.732	0.737	14
Lithuania	0.947	0.942	0.952	3	0.804	0.798	0.809	2	0.626	0.62	0.633	1
Luxembourg	0.984	0.981	0.987	21	0.887	0.882	0.892	19	0.747	0.74	0.754	17
Latvia	0.96	0.957	0.964	8	0.818	0.813	0.822	3	0.635	0.63	0.639	3
Malta	0.993	0.991	0.995	30	0.92	0.917	0.923	29	0.799	0.794	0.805	27
Netherlands	0.986	0.984	0.988	25	0.901	0.898	0.904	24	0.768	0.763	0.772	22
Norway	0.982	0.98	0.985	20	0.905	0.902	0.908	26	0.788	0.783	0.793	26
Poland	0.957	0.955	0.959	7	0.836	0.834	0.839	5	0.681	0.678	0.684	6
Portugal	0.945	0.942	0.948	1	0.803	0.8	0.806	1	0.628	0.624	0.631	2
Romania	0.976	0.974	0.977	16	0.877	0.874	0.879	14	0.741	0.738	0.744	15
Sweden	0.984	0.981	0.986	22	0.903	0.899	0.906	25	0.776	0.771	0.782	25
Slovenia	0.978	0.975	0.981	17	0.876	0.873	0.88	13	0.731	0.726	0.736	13
Slovakia	0.956	0.952	0.961	6	0.841	0.836	0.846	6	0.693	0.687	0.7	7
UK	0.973	0.971	0.975	13	0.885	0.882	0.887	17	0.76	0.757	0.764	21

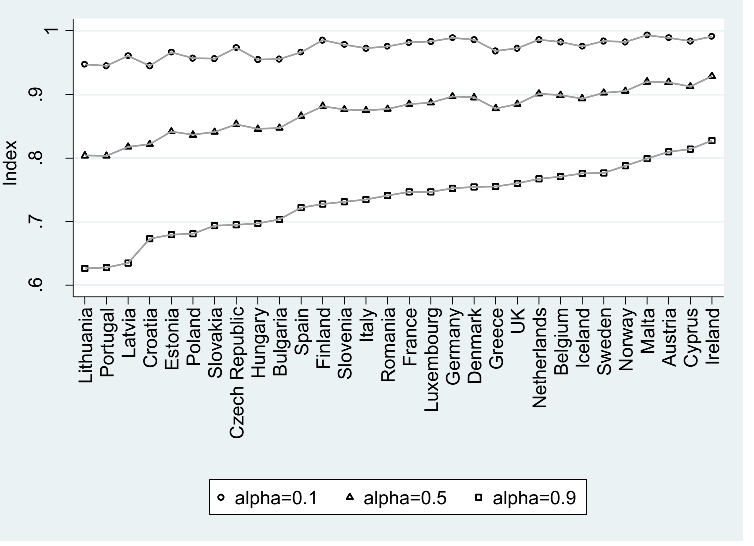

Figure 1. Achievement Index for SAH
Notes: Countries are ranked according to the achievement index for SAH for $\alpha = 0.9$.
[Colour figure can be viewed at wileyonlinelibrary.com]

SAH for $\alpha = 0.1$, whereas in the bottom sub-figure, they are ranked according to the index for $\alpha = 0.9$. In both sub-figures, the curve at the bottom captures the index for the cardinal score and the three curves at the top represent the measures for SAH. We find that the curve for SAH for $\alpha = 0.9$ and for the cardinal score lead to a rather similar ranking of countries. However, this ranking is different from that generated by the index for SAH for $\alpha = 0.1$ and $\alpha = 0.5$, highlighting the usefulness of our ordinal approach.

5. Concluding Comments

This paper, following earlier work on the measurement of inequality, when only ordinal data are available, presents axiomatic derivations of some new classes of inequality-sensitive achievement measures. The proposed measures have several features: (i) they are cardinal, (ii) when they satisfy Equity Principle, they are inequality sensitive, and (iii) they take specific functional forms when Proportional Equality is satisfied.

As noted in Section 2, the Equity Principle can be considered as a weaker version of Hammond's equity principle in our context. Hammond's equity principle was primarily used for characterizing maximin and/or leximin type rankings of achievement vectors or profiles of individual utilities (see, for example, Cowell, 2000; Bossert and Weymark, 2004) for various uses of Hammond's equity principle in social choice theory and in the measurement of inequality). It is therefore

TABLE 2
Achievement for the Ordinal SAH Variable for High Values of α

Country	$\alpha=0.99$				$\alpha=1$			
	Index	95% Confidence Interval		Rank (from low to high achievement)	Index	95% Confidence Interval		Rank (from low to high achievement)
Austria	0.786	0.783	0.789	28	0.783	0.78	0.787	28
Belgium	0.743	0.738	0.749	23	0.74	0.735	0.746	23
Bulgaria	0.674	0.667	0.681	10	0.671	0.664	0.677	10
Cyprus	0.793	0.786	0.8	29	0.791	0.784	0.798	29
Czech Republic	0.663	0.658	0.668	7	0.66	0.654	0.665	7
Germany	0.721	0.718	0.724	18	0.718	0.715	0.721	18
Denmark	0.725	0.719	0.73	19	0.721	0.716	0.727	19
Estonia	0.646	0.64	0.652	5	0.643	0.637	0.648	5
Greece	0.73	0.724	0.735	20	0.727	0.722	0.733	20
Spain	0.692	0.689	0.695	11	0.689	0.686	0.691	11
Finland	0.695	0.69	0.701	12	0.692	0.686	0.698	12
France	0.718	0.714	0.722	17	0.715	0.711	0.719	17
Croatia	0.644	0.636	0.651	4	0.64	0.633	0.648	4
Hungary	0.667	0.66	0.673	9	0.663	0.657	0.67	9
Ireland	0.806	0.801	0.811	30	0.803	0.798	0.809	30
Iceland	0.751	0.743	0.759	25	0.748	0.741	0.756	25
Italy	0.705	0.702	0.708	14	0.702	0.699	0.705	14
Lithuania	0.592	0.585	0.598	1	0.588	0.582	0.594	1
Luxembourg	0.718	0.71	0.725	16	0.714	0.707	0.722	16
Latvia	0.598	0.593	0.603	3	0.594	0.589	0.599	3
Malta	0.773	0.767	0.779	27	0.77	0.764	0.776	27
Netherlands	0.739	0.734	0.743	22	0.736	0.731	0.74	22
Norway	0.763	0.758	0.768	26	0.76	0.755	0.765	26
Poland	0.649	0.646	0.652	6	0.646	0.643	0.649	6
Portugal	0.593	0.589	0.596	2	0.589	0.586	0.593	2
Romania	0.713	0.709	0.717	15	0.71	0.706	0.714	15
Sweden	0.75	0.744	0.755	24	0.747	0.741	0.752	24
Slovenia	0.701	0.696	0.707	13	0.698	0.692	0.703	13
Slovakia	0.663	0.657	0.67	8	0.66	0.654	0.667	8
UK	0.735	0.731	0.738	21	0.732	0.728	0.735	21

TABLE 3
Achievement Index for the Cardinal Health Score

Country	Index (using symptoms, sex-age groups and countries)	95% Confidence Interval		Rank (from low to high achievement)
Austria	0.705	0.702	0.708	28
Belgium	0.664	0.659	0.668	23
Bulgaria	0.585	0.579	0.59	10
Cyprus	0.71	0.704	0.716	29
Czech Republic	0.577	0.572	0.581	8
Germany	0.639	0.636	0.641	19
Denmark	0.643	0.638	0.648	20
Estonia	0.567	0.563	0.572	6
Greece	0.637	0.632	0.641	18
Spain	0.61	0.607	0.612	11
Finland	0.618	0.613	0.622	13
France	0.633	0.63	0.636	16
Croatia	0.553	0.547	0.559	4
Hungary	0.578	0.572	0.583	9
Ireland	0.729	0.724	0.733	30
Iceland	0.666	0.66	0.673	24
Italy	0.619	0.616	0.622	14
Lithuania	0.503	0.498	0.508	1
Luxembourg	0.633	0.627	0.64	17
Latvia	0.509	0.504	0.514	3
Malta	0.693	0.687	0.699	27
Netherlands	0.657	0.653	0.661	22
Norway	0.689	0.685	0.693	26
Poland	0.562	0.559	0.565	5
Portugal	0.508	0.505	0.511	2
Romania	0.629	0.626	0.632	15
Sweden	0.668	0.664	0.673	25
Slovenia	0.615	0.61	0.62	12
Slovakia	0.569	0.563	0.575	7
UK	0.651	0.648	0.654	21

interesting to note that a weaker version of Hammond's equity principle, our Equity Principle, is being proposed and used for additive measures of social achievement.

Note also that the idea behind the Equity Principle is similar to that underlying the Pigou-Dalton transfer principle in the context of ordinal individual achievements. Coupled with the intuition behind Hammond's equity principle, measures that satisfy the Equity Principle exhibit an aversion to achievement inequality and are thus inequality sensitive.

In the context of cardinal measures of social achievement, Proportional Equality seems a reasonable requirement on a social achievement index. As our result (Proposition 3) shows, Proportional Equality gives rise to a family of parametrized measures. The parameter captures the degree of aversion to achievement inequality, of the society.

The paper also provides an empirical application using SAH for individuals in 30 European countries. We obtain these assessments from the European Health Interview Survey (EHIS), wave 2. We compute for each country our achievement

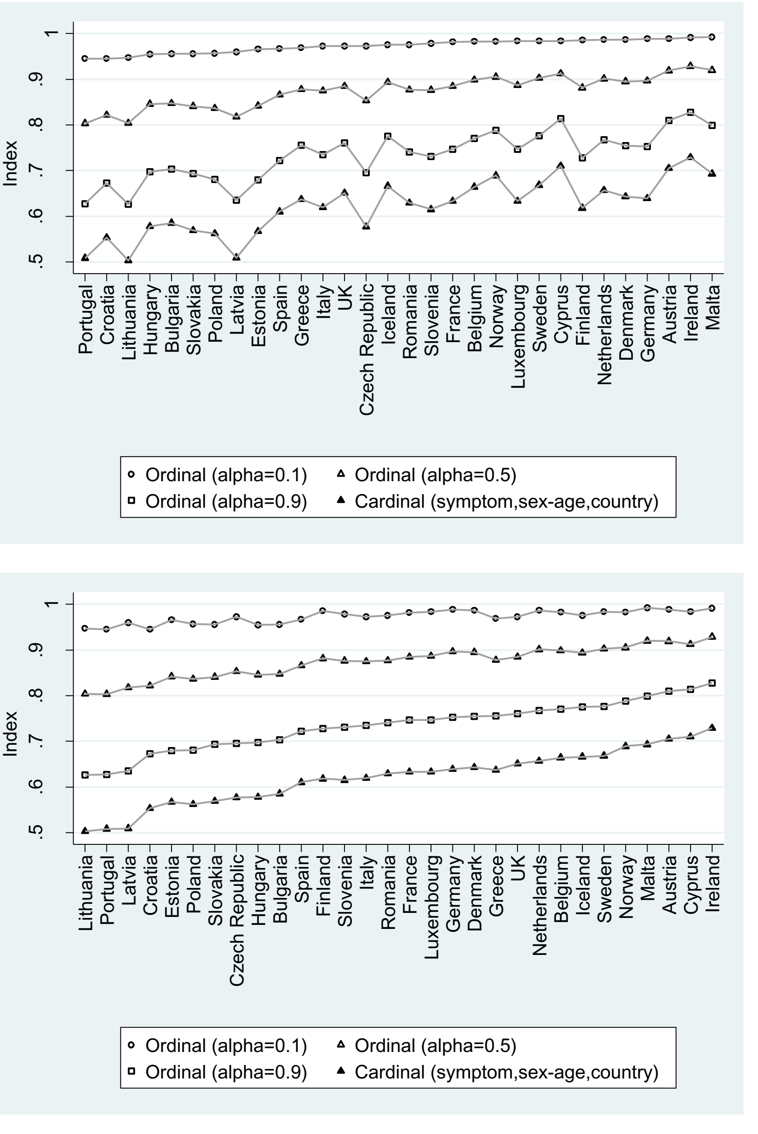

Figure 2. Achievement Indices for SAH and the Health Score
Notes: In the top subfigure, countries are ranked according to the achievement index for SAH for $\alpha = 0.1$, whereas in the bottom subfigure, they are ranked according to the index for SAH for $\alpha = 0.9$. [Colour figure can be viewed at wileyonlinelibrary.com]

index and rank the countries by level of achievement. Then, using a latent variable approach, we estimate a cardinal health score and derive an alternative measure of health achievement based on this score and ranked again the countries. The results indicate that the rankings are sometimes different. We plan in future work to determine the reasons for such a difference.

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