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ORDINAL VARIABLES AND THE MEASUREMENT OF UPWARD AND DOWNWARD INTERGENERATIONAL EDUCATIONAL MOBILITY IN EUROPEAN COUNTRIES

BY BÉNÉDICTE APOUEY

Paris School of Economics-CNRS

ZOYA NISSANOV

Independent Researcher

AND

JACQUES SILBER*

Bar-Ilan University, LISER, Esch-sur-Alzette, Centro Camilo Dagum, Tuscan Interuniversity Centre, Advanced Statistics for Equitable and Sustainable Development

This article proposes a new approach to the measurement of intergenerational mobility in education. Borrowing the concept of inequality-sensitive and additive achievement measure axiomatically developed by Apouey *et al*., we derive new indices of upward, downward, and total mobility, using a "movement approach." It turns out that the Prais-Bibby and Bartholomew mobility indices are particular cases of the mobility indices we introduce. We then present an empirical illustration based on the 2016 European Social Survey. Particular attention is given to within country differences between fathers to children and mothers to children educational mobility. When comparing two countries, we also make a distinction between gross and net mobility, the latter referring to the case where country differences in the educational structure of parents and children are neutralized.

JEL Codes: I20, I24

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1. Introduction

The literature on social mobility seems to make a basic distinction between mobility as temporal independence and mobility as movement. The former approach was clearly stated by Prais (1955) who wrote that "in terms of the transition matrix, a perfectly mobile society is a society in which the probability of entering a particular social class is independent of the class of one's father; so that all the elements in each row of the matrix would be substantially equal" (p. 59). In the case of intergenerational mobility in income or education for instance, this approach, in our opinion, should be called "(in)equality in life chances" (Silber and Spadaro, 2011; Silber and Yalonetzky, 2011). As stressed out by Fields and

*Correspondence to: Jacques Silber, Bar-Ilan University, Ramat Gan, Israel (jsilber_2000@yahoo.com).

Ok (1999), it is also possible to view mobility as movement, which is a completely different notion. In this perspective, there will be no social mobility if every individual's social class is identical to that of his father, so that all elements of the transition matrix that are not on the diagonal would be nil, while all elements on the diagonal would be equal to 1. In this paper, we focus on this approach based on movement.[1] Moreover, our emphasis is on ordinal variables. Specifically, our work is based on educational categories that can be ranked and we assume that the same educational categories are available for parents and children. Needless to say, we are interested in relative mobility as we compare the educational category of a child with that of her/his father or mother.

The main novelty of our approach to mobility is that we borrow the concept of inequality-sensitive and additive achievement measure for ordered qualitative variables that was recently introduced and axiomatically derived by Apouey *et al.* (2020). We first look at upward mobility and define the extent of upward educational mobility for the educational category i of the fathers (mothers) by employing a weighted function of the proportion of children who are in category j, assuming that $j \geq i$. Our measure of upward mobility satisfies a number of principles. In particular, it takes into account equality between children (see below the Section 3.2). We similarly define the extent of downward educational mobility for the educational category i of the fathers (mothers) as a weighted function of the proportion of children who are in category j, assuming now that $j \leq i$. We then define total mobility as a weighted sum of the upward and downward mobility of the different educational categories i of the fathers (mothers). Importantly, it turns out that the widely used Prais-Bibby and Bartholomew mobility indices are particular cases of our generalized approach to mobility measurement.

We finally present an empirical illustration on educational mobility between parents and children, based on data from the European Social Survey (ESS) Round 8. The surveyed countries belong to four regions corresponding to Eastern, Northern, Southern, and Western Europe. Given the large number of countries in the survey, we chose to limit the analysis to two countries per region, for a total of eight countries.

Using these data, we first compute our upward, downward, and total mobility indices and take a close look at differences between the mobility from fathers to children and that from mothers to children. Then, for each of the four regions, we compare (upward, downward, and total) mobility in the two countries of interest, making a distinction between "gross" and "net" mobility, the latter referring to the case where we neutralize differences in the margins of the matrices that are analyzed (that is, when we are interested in mobility from fathers to children for instance, we neutralize any differences between the two countries in the educational structure of fathers and in that of children). To implement such an analysis, we combine a technique originally introduced by Deming and Stephan (1940) with a Shapley decomposition (see, Chantreuil and Trannoy, 2013 and Shorrocks, 2013). More precisely, we are able to break down the difference in gross mobility between

[1] For more detailed discussions of the concept of mobility, see Fields and Ok (1999) and Fields (2008). For a more recent but short survey of the topic, see the introduction of Chakravarty *et al.* (2017).

the two countries into the sum of differences in the "internal structure" of the two matrices (differences in "net" or "pure" mobility between countries), in the horizontal margins (difference in the educational structure of the fathers), and in the vertical margins (differences in the educational structure of the children).

The paper is organized as follows. Section 2 contains a review of the literature on intergenerational educational mobility. In Section 3, we present in more detail how we measure mobility and check the properties of the mobility indices that we introduce. Section 4 contains the empirical illustration based on the ESS. Some concluding comments are finally given in Section 5.

2. On Intergenerational Educational Mobility: A Short Review of the Literature

The study by Behrman *et al.* (2001) includes a nice introduction to the topic. It makes a clear distinction between two main approaches to this issue, one employing regressions, the other one using transition probability matrices. Erikson and Goldthorpe (2002) argue that the former approach is more popular among economists and the latter among sociologists. Stuhler and Biagi (2018) also provide an interesting survey of the methodological aspects related to the study of intergenerational mobility, beyond educational mobility.

On the one hand, in the regression approach, it is possible to use a first-order Markov model where the educational level of the children (years of schooling) depends on the parents' educational level and on a stochastic term assumed to be independent of the parents' educational level and to be independently distributed across children. If the coefficient of the parents' education is positive and greater than one, it shows that there has been an increase in schooling. Torche (2019) explains the difference that exists between such a coefficient, also called the intergenerational educational regression (IER) coefficient, and what is labeled the intergenerational educational correlation (IEC) coefficient. While the former depends on the dispersion of the education of both parents and children, the latter does not (it is equal to the multiplication of the IER by the ratio of the standard deviations of parents' and children's schooling) and is hence a number varying between -1 and $+1$. Torche (2019) stresses the fact that a decrease over time in the IER may be related only to changes in the distribution of parents' and children's schooling, in which case this decline in the IER does not say anything about the extent of "pure mobility." Note also that if the schooling of individuals (parents and children) is defined relative to the mean of its distribution, a coefficient of the parents schooling smaller than one implies regression towards the mean. It is then easy to see that this coefficient is a measure of persistence or immobility.

Some papers use rank-based measures of intergenerational mobility (e.g. Emran and Shilpi, 2018), the idea being that these measures are less affected by measurement errors and life cycle bias.

When education is measured via ordered categories, intergenerational educational mobility is analyzed via ordered logistic (or ordered probit) regressions or multinomial logistic models (e.g. Heineck and Riphahn, 2009). Such an approach assumes that the actual educational level of the children is a function of a latent variable which is a function of the parents' educational level and of other control variables.

On the other hand, when data on education are ordered categorical variables, it is also possible to measure intergenerational educational mobility via transition probability matrices. In this approach, asymmetries can be taken into account (like when the probability of moving from the bottom to the top category is different from that of moving from the top to the bottom category). A limitation of this approach is that there is no unique way of summarizing such a matrix via a scalar (on this issue, see, Dardanoni, 1993).

Early studies of the determinants of educational attainment in the US (Spady, 1967; Bowles, 1972) stress the role of the status of origin (i.e. the social class background). Hauser and Featherman (1976) note however that the impact of farm background, broken families, Southern birth, black ethnicity and Spanish origin declined over time, but this is not true of the influence of the educational level or status of fathers and of the size of families (see also Blake, 1985, on the role of the number of siblings). In a more recent study, Hilger (2015) estimates intergenerational educational mobility in the US using Census data from 1940 to 2000, while Ferrare (2016) compares educational mobility among Blacks and Whites by gender over the past century. Using the 1997 National Longitudinal Survey of Youth, Wagner (2017) concludes that, after accounting for the test score and gender, the difference in educational mobility between African Americans and Hispanics and non-Blacks and non-Hispanics is not statistically significant. Fletcher and Han (2019) look at differences in educational mobility across time (1982–2004) and across states in the US.

Many studies explore intergenerational educational mobility in other countries: see Behrman *et al.* (2001), Daude (2011), and Neidhöfer *et al.* (2018) on Latin America; Binder and Woodruff (2002) and Urbina (2018) on Mexico; Azam and Bhatt (2012), Torche (2005) on Chile; Asher *et al.* (2018) and Sinha (2018) on India; Lillard and Willis (1994) on Malaysia; Emran and Sun (2015) on rural China; Lam and Liu (2019) on Hong Kong; Niimi (2018) on Japan; Aydemir and Yazici (2019) on Turkey; Ben-Halima *et al.* (2014) on France; Azomahou and Yitbarek on Sub-Saharan African countries; Thomas (1996) on South Africa; Emran *et al.* (2019) for a comparison of rural China and rural India; Landersø and Heckman (2017) and Andrade and Thomsen (2018) for a comparison of Denmark and the US; and Neidhöfer and Stockhausen (2019) for a comparison of long run (three generations) mobility in the US, the UK, and Germany.

Of particular interest are the studies of Hertz *et al.* (2007) who estimate 50-year trends in the intergenerational persistence of educational attainments for a sample of 42 countries, Van De Werfhorst *et al.* (2017) who use survey data for 40 post-war birth cohorts in 35 advanced economies, Narayan *et al.* (2018) who describe educational mobility in 148 economies, including 111 developing economies (covering thus 96 percent of the population of the developing world), for the 1980s cohort, and Lee and Lee (2019) who measure intergenerational persistence in educational attainment by age cohort for 30 countries.

The paper by Leone (2017) on educational mobility in Brazil has the advantage of using the two approaches mentioned previously, namely mobility matrices and regressions (univariate regressions and multivariate techniques). In any case, it appears that mobility is higher for daughters than for sons.

A few studies on intergenerational educational mobility use the European Social Survey (ESS) data. The focus of the study of Schuck and Steiber (2018), who analyze waves 4 to 7 of this survey (i.e. the 2008–2014 period), is on the impact of this mobility on the subjective wellbeing of young adults. Using waves 1–7 of the ESS, Torul and Oztunali (2017) investigate the empirical evolution of intergenerational educational mobility in Europe. They show that intergenerational educational persistence is very heterogeneous both in level and in trend, across countries and country groups (i.e. Mediterranean, Post-Socialist, Nordic, and the rest of Europe). They also examine how educational inequality and intergenerational mobility interact and conclude that intergenerational educational elasticity correlates positively with educational inequality, thus confirming the hypothesis that there exists an "Educational Great Gatsby" curve in Europe (see, Durlauf and Seshadri, 2018, for more details on this curve).

3. Methodology: Measuring Upward, Downward, and Overall Intergenerational Mobility in the Case of Ordinal Variables

3.1. *Measuring Upward Intergenerational Mobility*

Assume that individuals in the generation of the parents are ranked by educational level, say, from low to high education, and that there are K educational levels, with $K \geq 3$. Suppose also that individuals belonging to the generation of the children may also be ranked using the same K categories of education.[2] It is important to state at this stage that, given that we deal with ordinal variables, we do not try to give a cardinal value to the various educational levels and we do not make any assumption concerning the distance between educational categories.

Let us now define a K by K matrix X whose typical element x_{ij} indicates how many parents with educational level i have children with educational level j. We also assume that there are at least four parents in each educational category, i.e. each row (at least two parents on the left-hand side of the diagonal and two parents on the right-hand side, the number of parents on the diagonal being zero or strictly positive).

To measure the extent of intergenerational upward mobility among parents who have educational level i ($1 \leq i \leq K$), we will therefore consider only those x_{ij} for which $j \geq i$. The inclusion of the case $i = j$ in the definition of upward mobility will be justified later on, but it should be clear that there will be no upward intergenerational mobility for those belonging to the cell x_{ii}.

In other words, we now consider only the cells of matrix X that are on, or above, the diagonal and define a new matrix V that only contains natural numbers and whose typical element[3] $v_{ij} = x_{ij}$ if $j \geq i$, $v_{ii} = \frac{1}{2}x_{ii}$ and $v_{ij} = 0$ if $j < i$.

[2]We assume that the number of educational levels is the same for parents and children.

[3]The reason for defining v_{ii} as $v_{ii} = \frac{1}{2}x_{ii}$ is that v_{ii} will also appear in the definition of downward mobility.

Let us now consider the set of parents who have educational level i. Define $v_{i.}$ as the sum of the cells on row i on matrix V, i.e. $v_{i.} = v_{ii} + \sum_{j>i}^{K} v_{ij}$. We certainly would like to assume that if $v_{i.} = v_{ii} = \frac{1}{2}x_{ii}$, upward intergenerational mobility among these parents will be nil. Conversely, there will be a maximal level of upward intergenerational mobility among these parents if $v_{i.} = v_{iK}$. We may as well assume that in the latter case, upward intergenerational mobility for this category of parents will be equal to 1.

Let us now define a matrix S whose typical element s_{ij} is defined as $s_{ij} = \left(v_{ij}/v_{i.}\right)$.

Apouey *et al.* (2020) recently derived axiomatically inequality-sensitive and additive achievement measures, for the case where data are of an ordinal nature. We can extend their approach to derive the extent h_i of upward mobility among parents with educational level i. Apouey *et al.* (2020) classify the ordinal categories by decreasing level, that is, in our case, by decreasing educational level. Since we classify here the educational categories of both parents and children by increasing educational level, we adapt the formulas that appear in Apouey *et al.* (2020) and define h_i as follows:

$$(1) \quad \text{For } i < K, h_i = \sum_{k=i}^{K} s_{ik} \frac{1-\alpha^{k-i}}{1-\alpha^{K-i}} = \sum_{k=i}^{K} \left(\frac{v_{ik}}{v_{i.}}\right)\left(\frac{1-\alpha^{k-i}}{1-\alpha^{K-i}}\right) \quad \text{if } \alpha \in (0,1)$$

It is easy to observe that if $i = 1$ and $k = 1$, $\frac{1-\alpha^{k-i}}{1-\alpha^{K-i}} = 0$, while if $i = 1$ and $k = K$, we have $\frac{1-\alpha^{k-i}}{1-\alpha^{K-i}} = 1$. For $i < K$ and if $\alpha = 1$, we can once again adapt the results given by Apouey *et al.* (2020) and derive that

$$(2) \quad h_i = \sum_{k=i}^{K} s_{ik} \frac{k-i}{K-i} = \sum_{k=i}^{K} \left(\frac{v_{ik}}{v_{i.}}\right)\left(\frac{k-i}{K-i}\right)$$

if $i = k = l$, $\frac{k-i}{K-i} = \frac{l-l}{K-l} = 0$, while if $i = l$ and $k = K$, $\frac{k-i}{K-i} = \frac{K-l}{K-l} = 1$.

When $i = j = K$, that is, when fathers (or mothers) and children have the highest level of education, there cannot be any upward mobility, by definition, and hence it will be assumed that $h_K = 0 \; \forall \alpha \in (0, 1]$.

Using (1) or (2), we observe that whatever the educational level of the parents ($\forall i$ if $i \neq K$), the measure h_i of upward mobility for these parents is equal to 0 if their children have the same educational level i as their parents and to 1 if the children reach the highest educational level K.

In short, for each educational category of the parents, we will have an upward mobility measure h_i that varies between 0 and 1. To derive an overall measure of upward mobility, we will use a weighted average of these measures h_i. The total number of parents who can potentially move upward is equal to $\sum_{i=1}^{K} v_{i.}$ so that the overall measure M_u of upward mobility will be defined as

$$(3) \quad M_u = \sum_{i=1}^{K} \left(\frac{v_{i.}}{\sum_{i=1}^{K} v_{i.}}\right) h_i = \sum_{i=1}^{K-1} \left(\frac{v_{i.}}{\sum_{i=1}^{K} v_{i.}}\right) h_i$$

We thus get

(4)
$$M_u = \sum_{i=1}^{K-1} \left(\frac{v_{i.}}{\sum_{i=1}^{K} v_{i.}} \right) \sum_{k=i}^{K} \left(\frac{v_{ik}}{v_{i.}} \right) \left(\frac{1-\alpha^{k-i}}{1-\alpha^{K-i}} \right)$$
$$= \sum_{i=1}^{K-1} \sum_{k=i}^{K} \left(\frac{v_{ik}}{\sum_{i=1}^{K} v_{i.}} \right) \left(\frac{1-\alpha^{k-i}}{1-\alpha^{K-i}} \right) \quad \text{if } \alpha \in (0,1)$$

(5)
$$M_u = \sum_{i=1}^{K-1} \left(\frac{v_{i.}}{\sum_{i=1}^{K} v_{i.}} \right) \sum_{k=i}^{K} \left(\frac{v_{ik}}{v_{i.}} \right) \left(\frac{k-i}{K-i} \right) = \sum_{i=1}^{K-1} \sum_{k=i}^{K} \left(\frac{v_{ik}}{\sum_{i=1}^{K} v_{i.}} \right) \left(\frac{k-i}{K-i} \right) \quad \text{if } \alpha = 1$$

3.2. *Desirable Properties of the Upward Mobility Index M_u*

We start by stating two properties that are clearly assumed to hold when looking at the definition of the upward mobility index M_u given in (3).

Focus

This property implies that for any educational level i of the parents, h_i does not depend on the value of any v_{ij} for which $j < i$.

The *Focus* axiom assumes that the index M_u is not affected by any downward mobility.

Replication Invariance

If a matrix Z is derived from matrix V in such a way that $z_{ij} = \lambda v_{ij}$ $\forall i$ and $\forall j$, with λ a strictly positive integer,[4] there will be no change in the value of the mobility index M_u.

The *Replication invariance* axioms states that "cloning" once or several times all the individuals (all the individuals are cloned the same number of times) does not affect the extent of upward mobility.

We now turn to the properties stressed in the work of Apouey *et al.* (2020) on the measurement of individual achievements when only ordinal variables are available. We adapt these properties to the case of upward mobility measurement and we state them in terms of the matrix V which was previously defined. In these properties, we focus on individuals who are on the diagonal or on the right-hand side of the diagonal (note that in matrix V, there is no individual on the left-hand side of the diagonal, by definition).

Anonymity

Assume two individuals whose parents have educational level i. We consider two different situations. In the first case, one of these individuals has educational

[4]Note that, by definition, the number of individuals after this replication is still greater than 2 on the right-hand side of the diagonal.

level j (with $j > i$) and the other has educational level k (with $k > i$). In the second case, the first individual now has educational level k, the second educational level j, and there is no change in the educational level of the other individuals. The value of our upward mobility index will be the same in the two situations.

Anonymity requires our upward mobility measures to be symmetric with respect to individuals' upward mobility. This implies that in measuring upward mobility, the names or other (than educational mobility) characteristics of individuals have no impact.

Normalization

If, for a matrix V, $v_{i.} = v_{ii} \, \forall i$, then $M_u = 0$. If $v_{i.} = v_{iK} \, \forall i$, then $M_u = 1$.

The *Normalization* axiom states that if none of the children has a higher educational level than that of her/his parents, upward mobility will be equal to zero. On the contrary, if every child (who has a higher level of education than that of her/his parents) reaches the highest level of education, then upward mobility will be equal to 1.

Independence

If a matrix V' is derived from a matrix V in such a way that $v'_{ij} = v_{ij} \, \forall i \neq h$ and $v'_{hj} = v_{hj} \, \forall j \neq k, l$, while $v'_{hk} = v_{hk} - 1$ and $v'_{hl} = v_{hl} + 1$ assuming $h < l$, the difference between the value of the upward mobility index $M_u^{V'}$ (for matrix V') and that of the upward mobility index M_u^V (for matrix V) is independent of the values of the cells v_{ij} for $i \neq h$ and of the cells v_{hj} for $j \neq k, l$.

The axiom of *Independence* stipulates that, starting with a given educational mobility matrix, if the educational achievement of a child whose parents have educational level h changes without affecting the educational achievement of any other individual, the resulting change in the social mobility matrix is independent of the initial achievements of the other individuals.

Weak Pareto Principle

If, given two matrices V and V', $v_{hj} = v_{h.}$ and $v'_{hl} = v'_{h.}$ with $l > j$, while $v_{ij} = v'_{ij}$ $\forall i \neq h, \forall j \neq l$, then $M_u^{V'} > M_u^V$.

This assumption examines the case of two mobility matrices V and V' that are identical in all respects except for the following difference. For some educational level h of the parents, it appears that children who move upward all reach the same educational level. However, the educational level they reach is higher in the case of mobility matrix V' than in that of mobility matrix V. The Weak Pareto principle then assumes that upward mobility is higher for mobility matrix V' than for mobility matrix V.

Equity Principle

This *Equity principle* requires that, other things being the same, changes in the upward mobility of two individuals whose parents have the same educational level i, from two further-apart educational categories to two "closer" educational categories, should increase the level of upward mobility. The implicit assumption here is that such a change means that inequality in the upward mobility of children, whose parents have the same educational level i, decreases. This is assumed to have a positive impact on our measure of upward mobility, because we assume that when the upward mobility varies from one individual to the other, mobility will be smaller, ceteris paribus, than when all the individuals have the same upward mobility.

Proportional Equality

Assume three mobility matrices V, V' and V'' that are identical in all respects, the only differences between these three matrices concerning the children whose parents have educational level i. In matrix V all the children whose parents have educational i have educational level k, educational level k being higher than educational level i. In matrix V' all the children whose parents have educational i have educational level $(k-1)$, educational level $(k-1)$ being still higher than educational level i. Finally, in matrix V'' all the children whose parents have educational i have educational level $(k+1)$, educational level $(k+1)$ being evidently also higher than educational level i.

Assume now three mobility matrices W, W' and W'' that are identical in all respects, the only differences between these three matrices concerning the children whose parents have educational level i. In matrix W all the children whose parents have educational i have educational level k', educational level k' being higher than educational level i. In matrix W' all the children whose parents have educational i have educational level $(k'-1)$, educational level $(k'-1)$ being still higher than educational level i. Finally, in matrix W'' all the children whose parents have educational i have educational level $(k'+1)$, educational level $(k'+1)$ being evidently also higher than educational level i.

Define now the six mobility indices M_u^V, $M_u^{V'}$, $M_u^{V''}$, M_u^W, $M_u^{W'}$, $M_u^{W''}$ that measure the extent of upward mobility for the six mobility matrices V, V', V'', W, W' and W''.

The axiom of *Proportional equality* simply assumes that

$$\left(\frac{M_u^{V''}-M_u^V}{M_u^V-M_u^{V'}}\right)=\left(\frac{M_u^{W''}-M_u^W}{M_u^W-M_u^{W'}}\right)$$

The *Proportional equality* principle assumes therefore that the ratio of the change in upward mobility described previously is independent of the initial upward mobility level.

On the basis of the six properties of *Normalization*, *Independence*, *Anonymity*, *Weak Pareto principle*, *Equity principle*, and *Proportional equality principle*, Apouey *et al.* (2020) derived axiomatically an index of achievement. Expression (3) adapts the approach of Apouey *et al.* (2020) to derive an index of upward mobility. Such an index can naturally be derived axiomatically in the same way as Apouey *et al.* (2020) derived axiomatically their index of achievement, and there is hence no need to repeat here their proofs.

3.3. *Measuring Downward Intergenerational Mobility in the Case of Ordinal Variables*

Assume, as before, that individuals in the generation of the parents are ranked by educational level, say, from low to high education, and that there are K educational levels. Suppose also, as previously, that individuals belonging to the generation of the children may be also ranked by educational level and that there are again K levels[5] of education. The K by K matrix X is defined as previously.

To measure the extent of intergenerational downward mobility among parents that have educational level i $(1 \leq i \leq K)$, we will therefore consider only those x_{ij} for which $i \geq j$. The inclusion of the case $i = j$, in the definition of downward mobility, will be justified later on, but it should be clear that there will be no downward intergenerational mobility for those belonging to the cell x_{ii}.

In other words, we now consider only the cells of matrix X that are on or below the diagonal and we define a new matrix W whose typical element[6] $w_{ij} = x_{ij}$ if $i \geq j$, $w_{ii} = \frac{1}{2}x_{ii}$, and $w_{ij} = 0$ if $i < j$

Let us now consider the set of parents who have educational level i. Define $w_{i.}$ as $w_{i.} = w_{ii} + \sum_{j=1}^{i-1} w_{ij}$. We certainly would like to assume that if $w_{i.} = x_{ii}$, downward intergenerational mobility will be nil. Conversely, there is a maximal level of downward intergenerational mobility among these parents if $w_{i.} = w_{i1}$. We may as well assume that in the latter case, downward intergenerational mobility will be equal to 1.

Let us now define a matrix Γ whose typical element γ_{ij} is defined as $\gamma_{ij} = \left(w_{ij}/w_{i.}\right)$.

We can now define the extent l_i of downward mobility among parents with educational level i as

$$\text{(6)} \qquad \text{For } i > 1, l_i = \sum_{j=1}^{i} \gamma_{ij}\, c_i = \sum_{j=1}^{i} \left(\frac{w_{ij}}{w_{i.}}\right)\left(\frac{1-\alpha^{i-j}}{1-\alpha^{i-1}}\right) \text{ if } \alpha \in (0,1)$$

We observe that for $j = i$, we have $\frac{1-\alpha^{i-i}}{1-\alpha^{i-1}} = 0$, while for $j = 1$, we have $\frac{1-\alpha^{i-1}}{1-\alpha^{i-1}} = 1$.

For $i > 1$, but if $\alpha = 1$, we can once again adapt the results given by Apouey *et al.* (2020) and write that in such a case l_i will be expressed as

$$\text{(7)} \qquad l_i = \sum_{j=1}^{i} \gamma_{ij}\frac{i-j}{i-1} = \sum_{j=1}^{i}\left(\frac{w_{ij}}{w_{i.}}\right)\left(\frac{i-j}{i-1}\right)$$

It is easy to verify that if $j = i$, we have $\frac{i-j}{i-1} = 0$, while if $j = 1$, we have $\frac{i-j}{i-1} = 1$.

For $i = 1$, it is clear that there is no possibility of downward mobility and, in such a case, we will simply assume that $l_1 = 0$.

[5]Here also we assume that the number of educational levels is the same for "parents" and "children."

[6]The reason for writing $\frac{1}{2}x_{ii}$ in the definition of w_{ii} is that w_{ii} (or rather v_{ii}) also appeared in the definition of upward mobility, as we saw previously. Writing $\frac{1}{2}x_{ii}$ will also be useful in our definition of total mobility, as will be shown later on.

We therefore observe that whatever the educational level of the parents ($\forall i$), the measure l_i of downward mobility is equal to 0 if their children have the same educational level i and to 1 if the children have the lowest educational level 1.

As a consequence, for each educational category of the parents, our downward mobility measure l_i varies between 0 and 1. To derive an overall measure of downward mobility, we use a weighted average of these measures l_i. The total number of parents who can potentially move downwards is equal to $\sum_{i=1}^{K} w_{i.}$ so that the overall measure M_d of downward mobility will be defined as

$$M_d = \sum_{i=1}^{K} \left(\frac{w_{i.}}{\sum_{i=1}^{K} w_{i.}} \right) l_i = \sum_{i=2}^{K} \left(\frac{w_{i.}}{\sum_{i=1}^{K} w_{i.}} \right) l_i \tag{8}$$

$$\begin{aligned} M_d &= \sum_{i=2}^{K} \left(\frac{w_{i.}}{\sum_{i=1}^{K} w_{i.}} \right) \sum_{j=1}^{i} \left(\frac{w_{ij}}{w_{i.}} \right) \left(\frac{1-\alpha^{i-j}}{1-\alpha^{i-1}} \right) \\ &= \sum_{i=2}^{K} \sum_{j=1}^{i} \left(\frac{w_{ij}}{\sum_{i=1}^{K} w_{i.}} \right) \left(\frac{1-\alpha^{i-j}}{1-\alpha^{i-1}} \right) \quad \text{if } \alpha \in (0,1) \end{aligned} \tag{9}$$

$$\begin{aligned} M_d &= \sum_{i=2}^{K} \left(\frac{w_{i.}}{\sum_{i=1}^{K} w_{i.}} \right) \sum_{j=1}^{i} \left(\frac{w_{ij}}{w_{i.}} \right) \left(\frac{i-j}{i-1} \right) \\ &= \sum_{i=2}^{K} \sum_{j=1}^{i} \left(\frac{w_{ij}}{\sum_{i=1}^{K} w_{i.}} \right) \left(\frac{i-j}{i-1} \right) \quad \text{if } \alpha = 1 \end{aligned} \tag{10}$$

The list of properties that were mentioned above for the upward mobility index can easily be adapted to downward mobility and hence are not repeated here.

3.4. *Defining a Total or Overall Mobility Index*

To derive an overall mobility index, we cannot just sum the upward and downward mobility indices M_u and M_d, since the number of parents for whom there is a possibility of upward mobility is likely to be different from the number of parents for whom there is a possibility of downward mobility. We therefore have to use a weighted average of these two mobility indices, the weights being respectively equal to the shares of the parents for whom there is a possibility of upward and downward mobility. In other words, the "total mobility index" M_T may well be expressed as

$$M_T = \left(\frac{\sum_{i=1}^{K} v_{i.}}{\sum_{i=1}^{K} \left(v_{i.} + w_{i.} \right)} \right) M_u + \left(\frac{\sum_{i=1}^{K} w_{i.}}{\sum_{i=1}^{K} \left(v_{i.} + w_{i.} \right)} \right) M_d \tag{11}$$

Combining (3), (8), and (11), we obtain

$$M_T = \left(\frac{\sum_{i=1}^{K} v_{i.}}{\sum_{i=1}^{K} \left(v_{i.} + w_{i.} \right)} \right) \sum_{i=1}^{K} \left(\frac{v_{i.}}{\sum_{i=1}^{K} v_{i.}} \right) h_i \tag{12}$$

$$+\left(\frac{\sum_{i=1}^{K} w_{i.}}{\sum_{i=1}^{K}\left(v_{i.}+w_{i.}\right)}\right)\sum_{i=1}^{K}\left(\frac{w_{i.}}{\sum_{i=1}^{K} w_{i.}}\right)l_i$$

$$\leftrightarrow M_T=\left(\frac{1}{\sum_{i=1}^{K}\left(v_{i.}+w_{i.}\right)}\right)\left\{\sum_{i=1}^{K}\left[\left(v_{i.}h_i\right)+\left(w_{i.}l_i\right)\right]\right\}$$

When $\alpha \in (0, 1)$, by combining (1), (6), and (12), we thus get

$$M_T=\left(\frac{1}{\sum_{i=1}^{K}\left(v_{i.}+w_{i.}\right)}\right)\left\{\sum_{i=1}^{K-1} v_{i.}\left[\sum_{k=i}^{K}\frac{v_{ik}}{v_{i.}}\left(\frac{1-\alpha^{k-i}}{1-\alpha^{K-1}}\right)\right]\right. \tag{13}$$

$$\left.+\sum_{i=2}^{K} w_{i.}\left[\sum_{j=1}^{i}\frac{w_{ij}}{w_{i.}}\left(\frac{1-\alpha^{i-j}}{1-\alpha^{i-1}}\right)\right]\right\}$$

$$\leftrightarrow M_T=\left(\frac{1}{\sum_{i=1}^{K}\left(v_{i.}+w_{i.}\right)}\right)\left\{\sum_{i=1}^{K-1}\left[\sum_{k=i}^{K} v_{ik}\left(\frac{1-\alpha^{k-i}}{1-\alpha^{K-i}}\right)\right]\right.$$

$$\left.+\sum_{i=2}^{K}\left[\sum_{j=1}^{i} w_{ij}\left(\frac{1-\alpha^{i-j}}{1-\alpha^{i-1}}\right)\right]\right\}$$

When $\alpha = 1$, by combining (2), (7), and (12), we get

$$M_T=\left(\frac{1}{\sum_{i=1}^{K}\left(v_{i.}+w_{i.}\right)}\right)\left\{\sum_{i=1}^{K-1} v_{i.}\left[\sum_{k=i}^{K}\frac{v_{ik}}{v_{i.}}\left(\frac{k-i}{K-i}\right)\right]\right. \tag{14}$$

$$\left.+\sum_{i=2}^{K} w_{i.}\left[\sum_{j=1}^{i}\frac{w_{ij}}{w_{i.}}\left(\frac{i-j}{i-1}\right)\right]\right\}$$

$$\leftrightarrow M_T=\frac{1}{\sum_{i=1}^{K}\left(v_{i.}+w_{i.}\right)}\left[\sum_{i=1}^{K-1}\sum_{k=i}^{K} v_{ik}\left(\frac{k-i}{K-i}\right)+\sum_{i=2}^{K}\sum_{j=1}^{i} w_{ij}\left(\frac{i-j}{i-1}\right)\right]$$

3.5. *Alternative Existing Intergenerational Mobility Indices*

Let A be a matrix whose typical element a_{ij} refers to the number of parents belonging to educational category i whose children belong to educational category j. Prais (1955) and Bibby (1975, 1980) discuss the following social mobility measures.

The first one is defined as

$$g=1-\left[\left(\frac{1}{n}\right) Trace\,(A)\right] \tag{15}$$

where *Trace* (A) is equal to the sum of the elements on the main diagonal (i.e. the diagonal that goes from the upper left to the lower right of matrix A) and n is the total number of parents.

Using our previous notations, expression (15) may be also written as

$$g = 1 - \left[\left(\frac{1}{\sum_{i=1}^{K} \sum_{j=1}^{K} x_{ij}} \right) \left(\sum_{i=1}^{K} x_{ii} \right) \right] \tag{16}$$

$$\leftrightarrow g = \frac{\sum_{i=1}^{K} \sum_{j=1}^{K} x_{ij} - \sum_{i=1}^{K} x_{ii}}{\sum_{i=1}^{K} \sum_{j=1}^{K} x_{ij}} = \frac{\sum_{i=1}^{K} \sum_{j=1, j \neq i}^{K} x_{ij}}{\sum_{i=1}^{K} \sum_{j=1}^{K} x_{ij}} \tag{17}$$

Note however that if we assume in (13) that $\alpha \to 0$, it is easy to show that we end up with

$$M_T = \frac{\sum_{i=1}^{K} \sum_{j \neq i}^{K} x_{ij}}{\sum_{i=1}^{K} \sum_{j=1}^{K} x_{ij}} \tag{18}$$

The Prais and Bibby index turns out to be a particular case of our total mobility index M_T, the one where the parameter α tends to 0.

Another popular measure of mobility, also mentioned by Bibby (1975, 1980), is the Bartholomew (1967) index B. Using our previous notations, the formulation of this index in absolute terms (see, Bibby, 1975) is

$$B = \sum_{i=1}^{K} \sum_{j=1}^{K} \left(\frac{x_{i.}}{x_{..}} \right) \left(\frac{x_{ij}}{x_{i.}} \right) |i - j| = \sum_{i=1}^{K} \sum_{j=1}^{K} \left(\frac{x_{ij}}{x_{..}} \right) |i - j| \tag{19}$$

where $x_{..} = \sum_{i=1}^{K} \sum_{j=1}^{K} x_{ij}$ and $x_{i.} = \sum_{j=1}^{K} x_{ij}$.

There exists also a definition of a standardized Bartholomew index $B_{\text{standardized}}$ (see, Chakravarty *et al.*, 2017) where

$$B_{\text{standardized}} = \sum_{i=1}^{K} \sum_{j=1}^{K} \left(\frac{x_{i.}}{x_{..}} \right) \left(\frac{x_{ij}}{x_{i.}} \right) \left(\frac{|i - j|}{K} \right) = \sum_{i=1}^{K} \sum_{j=1}^{K} \left(\frac{x_{ij}}{x_{..}} \right) \left(\frac{|i - j|}{K} \right) \tag{20}$$

However, since $|i - j|$ can never be greater than $(K - 1)$, it might be better to define the standardized index as

$$B_{\text{standardized}} = \sum_{i=1}^{K} \sum_{j=1}^{K} \left(\frac{x_{i.}}{x_{..}} \right) \left(\frac{x_{ij}}{x_{i.}} \right) \left(\frac{|i - j|}{K - 1} \right) = \sum_{i=1}^{K} \sum_{j=1}^{K} \left(\frac{x_{ij}}{x_{..}} \right) \left(\frac{|i - j|}{K - 1} \right) \tag{21}$$

Note however that while for upward mobility, $|i{-}j|$ can never be higher than $K{-}i$, for downward mobility, $|i{-}j|$ can never be higher than $i{-}1$. We therefore suggest using a slightly different standardized definition of the Bartholomew index and express it as

$$B'_{\text{standardized}} = \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \left(\frac{x_{ij}}{x_{..}} \right) \left(\frac{j - i}{K - i} \right) + \sum_{i=2}^{K} \sum_{j=1}^{i} \left(\frac{x_{ij}}{x_{..}} \right) \left(\frac{i - j}{i - 1} \right) \tag{22}$$

It is however easy to observe that expression (22) is actually equivalent to expression (14). Our definition $B'_{\text{standardized}}$ of the standardized Bartholomew index turns out to be identical to our total mobility index M_T when $\alpha = 1$.

Our total mobility index M_T is hence in a way a generalization of both the Prais and Bibby index and of the Bartholomew index.

3.6. *Interpreting the Parameter α*

The mobility index introduced in this paper is an extension of the achievement index proposed by Apouey *et al.* (2020) for the case of ordinal variables. It has therefore the advantage, ceteris paribus, of taking into consideration both the "average achievement" of children of parents with a given level of education, as well as the inequality of the achievement of these children. The Bartholomew index on the contrary is not sensitive to this inequality.

4. Methodology: Breaking Down the Difference Between the Intergenerational Mobility of Two Countries

If we make a simple comparison of the values taken in two countries by the upward, downward, and total mobility indices defined previously (M_u, M_d and M_T), we will obtain a difference in what should be considered as "gross mobility," because such a gap will ignore differences between the two countries in both the horizontal and vertical margins, that is in the educational structure of fathers and in that of the children. If we manage to neutralize such differences, we will be able to compare the "net mobility" (upward, downward, or total) in the two countries. Such a distinction between differences in "gross" and "net" mobility has been made in the mobility literature based on regression analysis. There is thus a difference between the notion of intergenerational educational regression (IER) coefficient and that of intergenerational educational correlation (IEC) coefficient, since the former depends on the dispersion of the education of both parents and children while the latter does not (see, Torche, 2019, for more details).

It is however also possible to make such a distinction when working with transition matrices. The idea is to apply to the study of intergenerational mobility a technique originally proposed by Deming and Stephan (1940) and applied by Karmel and MacLachlan (1988) to the analysis of occupational segregation by gender. Deutsch *et al.* (2009) later generalized the approach of Karmel and MacLachlan by combining the technique proposed by Deming and Stephan (1940) with the concept of Shapley decomposition (for more details on the latter, see, Sastre and Trannoy, 2002, Chantreuil and Trannoy, 2013, Shorrocks, 2013, and Appendix A). This technique allows one, when comparing mobility in two countries (whether upward, downward, or total mobility) to make a distinction between a difference in mobility due to differences in the margins of the matrices compared and a difference in what is called "the internal structure" of the matrices, the latter referring to the degree of independence between the lines (educational structure of the parents in our case) and the columns (education structure of the children). Differences in the margins are evidently the consequence of differences between the two countries compared in the educational structure (composition) of the parents (fathers or mothers) as well as that of the children. Appendix A explains in more details how to derive the components of the breakdown of the difference in the mobility of two countries.

Naturally, we can also implement such a decomposition when comparing, for a given country, the gross mobility of a matrix focusing on the intergenerational mobility from fathers to children, and the gross mobility of a matrix giving data on the intergenerational mobility from mothers to children. In such a case, the impact of the margins will be only that of the educational structure of the parents (fathers or mothers).

To get a better idea of how such a decomposition is obtained, assume two countries A and B, whose data on intergenerational mobility are given respectively by two matrices l and m, whose typical element are l_{ij} and m_{ij}, the subscript i referring to the educational level of the parents (father or mother) and the subscript j to that of the children. Let also adopt the following definitions:

$$l_{i.} = \sum_j l_{\mathrm{ij}}; \quad m_{i.} = \sum_j m_{\mathrm{ij}}; \quad l_{.j} = \sum_i l_{\mathrm{ij}}; \quad m_{.j} = \sum_i m_{\mathrm{ij}}.$$

Let in stage one multiply all the elements m_{ij} of the matrix m by the ratios $\left(l_{i.}/m_{i.}\right)$ and call p the matrix you obtain. It should be clear that the matrices l and p have the same horizontal margins.

Multiply now all the elements p_{ij} of the matrix p by the ratios $\left(l_{.j}/m_{.j}\right)$ and call q the matrix you obtain. If you repeat this procedure several times, you will quite quickly end up with a matrix z that will have the same horizontal as well as vertical margins as those of the matrix l. The difference between the gross mobility of the matrix z and that of the matrix l will then be only the consequence of differences in what we called previously the internal structure of the matrices. Moreover, since we derived the matrix z by starting from the matrix m we can also conclude that the difference between the gross mobility of matrix z and that of matrix m is solely due to differences in their margins. In Appendix A, we explain how to decompose this difference in the margins into a component due to differences in the horizontal margins and one related to differences in the vertical margins.

Naturally, we could have started from matrix l and multiply all the elements l_{ij} of such a matrix by the ratios $\left(m_{i.}/l_{i.}\right)$ and implement the procedure that has just been described to end up with a matrix y that would have the same margins as matrix l. The so-called Shapley decomposition procedure simply takes into account these two possibilities (starting from matrix m or from matrix l) as explained in Appendix A.

It should also be clear that such a decomposition may be applied to any gross mobility index, whether it refers to upward, downward or overall mobility, not only to the new mobility indices introduced in this paper (M_u, M_d and M_T) but also to alternative mobility indices such as, for example, the Bartholomew mobility index.

5. Empirical Illustration: Intergenerational Mobility in European Countries

5.1. *Data*

This section presents an empirical illustration that focuses on educational mobility between parents and children. We employ data from the European Social

Survey (ESS) Round 8 (2016). This survey collects information on attitudes, beliefs, and behavior in the 23 participating countries. In each country, samples are representative of individuals aged 15+. The survey includes a core module and two rotating modules. Information on education comes from the core module, which indicates the highest educational level reached by the respondent and her/his father and mother. We distinguish between the following levels of education: pre-primary, primary, lower secondary, upper secondary, post-secondary but non-tertiary, bachelor, master, and doctoral degree.

In our tables, we organize results using the United Nations Geoscheme created by the United Nations Statistics Division. This approach distinguishes between Eastern, Northern, Southern, and Western Europe and Western Asia. Note that there are many definitions of Central Europe; in this paper, we refer to the area that includes Austria, Czechia, Hungary and Poland, but excludes Germany.

5.2. *Empirical Results*

We compute the upward, downward, and total mobility indices introduced in this paper. In Table 1, we compute our three mobility indices (i.e. upward, downward, and total mobility indices), using information on either the father's or the mother's education, for the case where the parameter α is equal to 0.5. For total mobility, we also show the results when $\alpha \to 0$, namely $\alpha = 0.001$, since, as was shown previously, this case gives us the Prais-Bibby index. In addition, we show the value of the total mobility index when $\alpha \to 1$, namely $\alpha = 0.999$, since, as mentioned before, this case corresponds to what we define as the standardized Bartholomew index. Finally, we compute the Bartholomew (1967) index, using its traditional definition. The table shows that the country rankings and the fathers-mothers comparisons are often rather similar, but not perfectly identical.

Total mobility is quite high in the countries of Northern Europe, especially in Finland, Ireland, and Sweden, in which both mobility from fathers to children and from mothers to children are high. In Northern Europe, upward and downward mobility is generally high or very high, compared to other countries. In Israel, upward mobility is very high.

Total mobility is rather low in France, and very low in Switzerland and in Central Europe (Austria, Hungary, Poland). These results are driven by a low or very low level of upward mobility (Switzerland, Austria, Hungary, Poland) combined with a very small level of downward mobility (Austria, Hungary, Poland).

Finally, we notice that in Italy and Portugal, downward mobility is very low.

Importantly, while Table 1 shows a high correlation between our index and the Bartholomew index, this is not always the case. In Appendix B, we provide two illustrations where the two indices produce opposite orderings of two mobility matrices. This change is due to the fact that the Bartholomew index takes into account the weighted number of rank changes $\left(\sum_{j=1}^{K}\left(\frac{x_{1j}}{x_{1.}}\right)|i-j|\right)$, while our index takes into account not only the number of rank changes, but also the degree of equality of the mobility taking place. This means that if a researcher considers that inequality in

TABLE 1

Upward, Downward, and Total Educational Mobility in Europe

	FA M_u ($\alpha = 0.5$)	FA M_d ($\alpha = 0.5$)	FA M_T ($\alpha = 0.5$)	MO M_u ($\alpha = 0.5$)	MO M_d ($\alpha = 0.5$)	MO M_T ($\alpha = 0.5$)	FA Prais-Bibby M_T with $\alpha \to 0$	MO Prais-Bibby M_T with $\alpha \to 0$	FA Bartholomew Standardized M_T with $\alpha \to 1$	MO Bartholomew Standardized M_T with $\alpha \to 1$	FA Bartholomew (Traditional Definition)	MO Bartholomew (Traditional Definition)
Eastern Europe												
Czechia	0.338	0.353	0.346	0.387	0.327	0.360	0.506	0.533	0.203	0.209	0.106	0.109
Hungary	0.412	0.191	0.338	0.472	0.181	0.397	0.514	0.611	0.163	0.186	0.098	0.116
Poland	0.553	0.247	0.478	0.557	0.284	0.485	0.671	0.67	0.266	0.273	0.170	0.173
Northern Europe												
Estonia	0.617	0.429	0.558	0.600	0.448	0.549	0.754	0.748	0.313	0.308	0.193	0.187
Finland	0.699	0.470	0.639	0.697	0.473	0.639	0.803	0.803	0.369	0.373	0.248	0.248
Iceland	0.633	0.498	0.579	0.653	0.486	0.606	0.77	0.787	0.349	0.364	0.204	0.230
Ireland	0.679	0.329	0.618	0.673	0.311	0.608	0.811	0.796	0.339	0.340	0.227	0.220
Lithuania	0.652	0.43	0.597	0.640	0.458	0.586	0.787	0.772	0.341	0.336	0.216	0.211
Norway	0.624	0.463	0.568	0.656	0.481	0.605	0.742	0.779	0.344	0.371	0.203	0.222
Sweden	0.706	0.473	0.645	0.702	0.463	0.649	0.817	0.823	0.366	0.371	0.246	0.246
UK	0.643	0.349	0.561	0.656	0.318	0.574	0.694	0.713	0.341	0.351	0.228	0.234
Southern Europe												
Italy	0.540	0.261	0.485	0.563	0.236	0.507	0.744	0.77	0.224	0.231	0.154	0.162
Portugal	0.612	0.231	0.541	0.623	0.240	0.563	0.739	0.786	0.295	0.296	0.210	0.216
Spain	0.637	0.349	0.578	0.659	0.325	0.604	0.779	0.804	0.305	0.320	0.221	0.237
Western Europe												
Austria	0.394	0.254	0.340	0.474	0.146	0.39	0.507	0.601	0.177	0.192	0.106	0.118
France	0.617	0.323	0.537	0.657	0.297	0.579	0.69	0.735	0.292	0.318	0.197	0.212
Germany	0.518	0.427	0.478	0.604	0.367	0.533	0.629	0.702	0.289	0.324	0.16	0.188
Netherlands	0.594	0.418	0.542	0.632	0.392	0.580	0.726	0.771	0.322	0.346	0.205	0.228
Switzerland	0.535	0.393	0.480	0.596	0.291	0.521	0.646	0.707	0.279	0.297	0.161	0.179
Western Asia												
Israel	0.681	0.429	0.614	0.690	0.417	0.624	0.782	0.785	0.347	0.351	0.229	0.236

Notes: FA refers to mobility from fathers to children, and MO to mobility from mothers to children.

mobility is something that should be taken into account, he/she should prefer our index to the Bartholomew index, as explained earlier.

In Tables 2–4, we take a close look at differences between the mobility from fathers to children and that from mothers to children, the comparison being made on the basis of bootstrap 5 percent to 95 percent confidence intervals. Let us assume, for instance, that, in a given country, I_{MO} and I_{FA} are the (expected) values of index I (upward, downward, or total mobility index) for mobility from mothers to children and for mobility from fathers to children, while $I_{MO,5\%}, I_{MO,95\%}, I_{FA,5\%}$, and $I_{FA,95\%}$ are the values of these indices corresponding to the 5 percent–95 percent interval. Assume, for example, that $I_{FA} < I_{MO,5\%}$ and that $I_{MO} > I_{FA,95\%}$. We can

TABLE 2

Total Educational Mobility from Fathers to Children versus Educational Mobility from Mothers to Children, for Two Countries for Each of the Following Areas: Eastern, Northern, Southern, and Western Europe (With Bootstrap 5%–95% Confidence Intervals). The case where $\alpha = 0.001$ (M_T is then almost identical to the Prais-Bibby index)

	MO Actual Value of M_T	MO Lower Bound (5%) of M_T	MO Upper Bound (95%) of M_T	FA Actual Value of M_T	FA Lower Bound (5%) of M_T	FA Upper Bound (95%) of M_T
Czechia	0.533	0.532	0.535	0.506	0.504	0.507
Poland	0.670	0.668	0.672	0.671	0.669	0.673
Estonia	0.748	0.731	0.767	0.754	0.738	0.772
Sweden	0.823	0.807	0.839	0.817	0.815	0.818
Italy	0.770	0.768	0.771	0.744	0.743	0.745
Spain	0.804	0.802	0.805	0.779	0.777	0.780
Austria	0.601	0.599	0.602	0.507	0.505	0.509
France	0.735	0.733	0.737	0.690	0.688	0.692

Notes: MO refers to educational mobility from mothers to children, and FA to educational mobility from fathers to children.

TABLE 3

Total Educational Mobility from Fathers to Children versus Educational Mobility from Mothers to Children, for Two Countries for Each of the Following Areas: Eastern, Northern, Southern, and Western Europe (With Bootstrap 5%–95% Confidence Intervals). The case where $\alpha = 0.5$

	MO Actual Value of M_T	MO Lower Bound (5%) of M_T	MO Upper Bound (95%) of M_T	FA Actual Value of M_T	FA Lower Bound (5%) of M_T	FA Upper Bound (95%) of M_T
Czechia	0.360	0.359	0.361	0.346	0.333	0.359
Poland	0.485	0.483	0.486	0.478	0.476	0.479
Estonia	0.549	0.535	0.564	0.558	0.544	0.572
Sweden	0.649	0.633	0.665	0.645	0.630	0.662
Italy	0.507	0.506	0.508	0.485	0.484	0.486
Spain	0.604	0.603	0.605	0.578	0.565	0.592
Austria	0.390	0.389	0.391	0.340	0.338	0.341
France	0.579	0.577	0.580	0.537	0.536	0.539

Notes: MO refers to educational mobility from mothers to children, and FA to educational mobility from fathers to children.

TABLE 4

Total Educational Mobility from Fathers to Children versus Educational Mobility from Mothers to Children, for Two Countries for Each of the Following Areas: Eastern, Northern, Southern, and Western Europe (With Bootstrap 5%–95% Confidence Intervals). The case where $\alpha = 0.999$ (M_T is then almost identical to our standardized Bartholomew index $B'_{\text{STANDARDIZED}}$)

	MO Actual Value of M_T	MO Lower Bound (5%) of M_T	MO Upper Bound (95%) of M_T	FA Actual Value of M_T	FA Lower Bound (5%) of M_T	FA Upper Bound (95%) of M_T
Czechia	0.209	0.208	0.210	0.203	0.202	0.204
Poland	0.273	0.272	0.275	0.266	0.265	0.267
Estonia	0.308	0.298	0.319	0.313	0.303	0.324
Sweden	0.371	0.358	0.383	0.366	0.353	0.378
Italy	0.231	0.230	0.232	0.224	0.223	0.225
Spain	0.320	0.319	0.321	0.305	0.304	0.306
Austria	0.192	0.191	0.193	0.177	0.176	0.178
France	0.318	0.317	0.319	0.292	0.291	0.293

Notes: MO refers to educational mobility from mothers to children, and FA to educational mobility from fathers to children.

then conclude that the value of the index I_{MO} describing the mobility from mothers to children is significantly greater than that of the index I_{FA} describing the mobility from fathers to children.

Such an analysis is conducted separately for Eastern, Northern, Southern, and Western Europe, two countries being selected in each of these four areas. The countries selected were those with the highest and lowest values of the total mobility from fathers to children, when the parameter α is equal to 0.5. These countries are Czechia and Poland for Eastern Europe, Estonia and Sweden for Northern Europe, Italy and Spain for Southern Europe, and Austria and France for Western Europe. We concentrate our attention on total mobility and it turns out that when $\alpha = 0.001$ (the case of the Prais-Bibby index, Table 2), total mobility is significantly higher from mothers than from fathers to children in Czechia, Italy, Spain, Austria, and France. For Poland, Estonia, and Sweden, there is no significant difference between the total mobility from mothers or fathers to children. When $\alpha = 0.5$ (Table 3), we again observe that in Czechia, Italy, Spain, Austria, and France, total mobility from mothers to children is significantly higher than that from fathers to children. But this time this is also true for Poland. The same conclusions are derived when $\alpha = 0.999$ (Table 4) which is the case of our standardized Bartholomew index.

It appears therefore that for five to eight of the countries selected, total mobility from mothers to children is higher than that from fathers to children, whatever the value of the parameter α.

These results refer however to "gross mobility," because they ignore differences that may exist between the horizontal margins of the matrices analyzed, that is, differences between the educational structures of fathers versus that of mothers. If rather than comparing for a given country the mobility from mothers to children and that from fathers to children, we compare two countries and look, say, at the mobility from fathers to children in these two countries, it is likely that there will be differences between the two countries in both the horizontal and vertical margins, that is in the educational structure of fathers and in that of the children. If we manage to neutralize such differences, we will be able to compare the "net mobility" (upward, downward, or total) in the two countries. As mentioned previously such a distinction between differences in "gross" and "net" mobility has been made in the mobility literature based on regression analysis. But it is also possible to make this distinction when working with transition matrices. As indicated in Section 4 and explained in detail in Appendix A, the idea is to apply to the study of intergenerational mobility a technique originally proposed by Deming and Stephan (1940) and used by Karmel and MacLachlan (1988) to explore occupational segregation by gender. Deutsch *et al.* (2009) later generalized the approach of Karmel and MacLachlan. This technique allows, when comparing mobility in two countries (whether upward, downward, or total mobility) to make a distinction between difference in mobility due to differences in the margins of the matrices compared and a difference in what is called "the internal structure" of the matrices, such a gap being really the consequence of a difference in "pure" mobility, that is, net of differences in the margins. Differences in the margins are evidently the consequence of differences between the two counties compared in the educational structure (composition) of the parents (fathers or mothers) as well as of the children.

The results of such an investigation are presented in Table 5 for our total mobility index M_T and for the traditional total mobility index B_T. We compare the four pairs of countries. We compute for each comparison of countries the relative contribution of differences in the margins and in the "internal structure" when using the indices M_T and B_T as well as the relative impact of differences in the horizontal and vertical margins.

The comparison of Czechia and Poland shows that the relative contributions of differences in the margins is the same, whether we use M_T or B_T, but the index M_T shows a somewhat higher impact of differences in the horizontal margins (53 percent versus 47 percent).

When Estonia and Sweden are compared, we see that while the index M_T attributes 100 percent of the difference in total mobility to differences in the margins, the index B_T indicates that only 80 percent is related to differences in the margins. Moreover, we observe that more importance is given by the index M_T to differences in the horizontal margins.

Regarding the comparison of Italy and Spain, the index M_T indicates again a greater impact of the role of differences in the margins. The impact of differences in the horizontal margins is much higher (97 percent versus 56 percent) than in the previous columns.

Finally, when Austria and France are compared, we observe that there is not much difference between the results based on the index M_T and those derived from the index B_T when we look at the relative impact of the margins. We see some differences when we look at each margin separately though.

In Appendix C, we present additional results. In Appendix C-1, using the indices M_T (with $\alpha = 0.5$) and B_T of total mobility from fathers to children, we decompose the difference between two countries in these two indices and give the 5 percent to 95 percent bootstrap confidence intervals. This decomposition is similar to that presented in Table 5 but in Appendix C-1 we give the actual and not the percentage decomposition.

TABLE 5

Percentage Decomposition of the Difference in Mobility Between Two Countries (Indices M_T and B_T)

	Czechia versus Poland Mobility Index M_T	Czechia versus Poland Mobility Index B_T	Estonia versus Sweden Mobility Index M_T	Estonia versus Sweden Mobility Index B_T	Italy versus Spain Mobility Index M_T	Italy versus Spain Mobility Index B_T	Austria versus France Mobility Index M_T	Austria versus France Mobility Index B_T
Total mobility of first country	0.346	0.120	0.217	0.216	0.485	0.173	0.34	0.119
Total mobility of second country	0.478	0.192	0.277	0.276	0.578	0.248	0.537	0.221
Difference in total mobility	0.132	0.072	0.060	0.059	0.093	0.076	0.198	0.102
Percentage contribution of "internal structure"	30.8	30.3	0.0	19.8	3.6	13.6	11.3	14.4
Percentage contribution of margins	69.1	69.8	100.0	80.2	96.5	86.4	88.9	85.8
Percentage contribution of horizontal margin	53.3	46.9	88.5	72.9	96.9	56.1	69.2	59.8
Percentage contribution of vertical margin	15.9	22.9	11.5	7.2	−0.4	30.3	19.3	26.0

In Appendix C-2, we decompose, for the pairs of countries mentioned previously, the difference in our indices M_u and M_d of upward and downward mobility, for the case where the parameter α is equal to 0.5.

In Appendix C-3, we present a similar decomposition for the traditional Bartholomew indices of upward and downward mobility.

Finally, in Appendix C-4, we compare for Germany total mobility from fathers to children with total mobility from mothers to children, using our index M_T and assuming that the parameter α is equal to 0.5.

6. Conclusion

Income and social inequalities have recently widened in a number of European countries, and citizens are concerned about the transmission of advantage and disadvantage within families (European Foundation for the Improvement of Living and Working Conditions, 2016). In this context, our paper started by proposing a new approach to the measurement of intergenerational mobility in education. Using a "movement approach" to the measurement of mobility, it borrowed the concept of inequality-sensitive and additive achievement measure recently introduced by Apouey *et al.* (2020). Upward educational mobility for educational category i of the fathers (or mothers) was defined as a weighted function of the proportion of their children who belong to category j, assuming that $j \geq i$. Similarly downward educational mobility for educational category i of the fathers (mothers) was expressed as a weighted function of the proportion of their children who belong to category j, assuming now that $j \leq i$. The overall upward and downward mobility levels were then defined as weighted averages of the upward and downward mobility observed for the different educational levels of the fathers (or mothers). Finally, total mobility was also defined as a weighted sum of the of the overall upward and downward mobility levels. We also showed that, given the weights we selected for the computation of this total mobility, the Prais-Bibby index turns out to be identical to our total mobility index when the parameter α is close to 0, while a standardized Bartholomew index is identical to our mobility index when α is close to 1.

In the empirical section, we first looked at upward, downward, and total mobility in 20 countries, and noted that mobility, whether upward, downward, or total, is particularly high in most countries of Northern Europe and in Israel.

We then compared the mobility from fathers to children with that from mothers to children, focusing our attention on two countries in each European area. To check whether the difference between these two types of mobility is statistically significant, we built 5 percent to 95 percent bootstrap confidence intervals, for mobility indices.

Finally, we drew the attention of the reader to the fact that in many cases differences in mobility, whether upward, downward, or total mobility, may be the consequence of differences in the margins of the matrices analyzed, that is, of differences in the educational structure of the parents (fathers or mothers) and of the children. We applied a technique sometimes used in occupational segregation analysis, to break down the overall difference in the mobility of two countries (whether upward, downward, or total) in components related to differences in the margins and differences in "the internal structure" of the matrices compared (which corresponds to what could be called "pure or net mobility"). In the illustrations we gave, most of the difference in mobility between the two countries we compared was related to differences in the margins. It should be clear that making a distinction between "total" and "net" or mobility is of outmost importance from a policy point of view since it allows policy makers to find out what the sources of differences in mobility (or its opposite, persistence) are.

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Supporting Information

Additional supporting information may be found in the online version of this article at the publisher's web site:

Appendix A: Combining the Deming and Stephan algorithm with the Shapley decomposition

Appendix B: Illustrations showing the difference between the index introduced in the paper and the Bartholomew mobility index

Appendix C-1: Decomposition of the difference between two countries in the indices M_T (assuming that $\alpha = 0.5$) and B_T of the total mobility from fathers to children (with 5%-95% bootstrap confidence intervals)

Table C.1.1: Total mobility index M_T

Table C.1.2: Total mobility index B_T

Appendix C-2: Decomposition of the difference between two countries in the indices M_u and M_d of mobility from fathers to children (with 5%–95% bootstrap confidence intervals)

Table C.2.1: Upward mobility index M_u, assuming that $\alpha = 0.5$

Table C.2.2: Downward mobility index M_d, assuming that $\alpha = 0.5$

Appendix C-3: Decomposition of the difference between two countries in the Bartholomew indices B_u, B_d, and B_T of mobility from fathers to children (with 5%–95% bootstrap confidence intervals)

Table C.3.1: Upward mobility index B_u

Table C.3.2: Downward mobility index B_d

Table C.3.3: Total mobility index B_T

Appendix C-4: Decomposition of the difference between the total mobility from mothers to children and from fathers to children, in Germany, using the index M_T, assuming that $\alpha = 0.5$ (with 5%–95% bootstrap confidence intervals).