VNU University of Engineering and Technology

ALGORITHMS FOR INTELLIGENT ROBOTS

Lecture 5
Robot Localization with the Extended Kalman Filter

Particle filter

- Particle filter (aka Monte Carlo localisation) is a global localiser.
- It can process raw sensor measurements
- It is non-parametric – it can handle multi-modal distribution
- Both linear and nonlinear process and measurement models can be used.
- It can solve ‘kidnapping’ problem – recover from global locations
- It is very popular and ‘simple’ to implement
- It could suffer from dimensionality problems – computational cost grow exponentially with dimension of the space.

Particle filter: Basic idea

- Tracking problems can be solved by recursively applying the predict–update cycle that is common in Bayesian filtering or Kalman filter.
  1. The process model is used to compute the state that is expected at time *k* given all measurements up to time k-1.
  2. A measurement z_k at time k is used to refine the expected state estimate leading to the posterior.
- Step 1 is computing the prior using a process model, step 2 is refining the estimate using Bayes' theorem.

Particle filter: Basic idea

- The previous integrals can only be solved analytically under strong assumptions, e.g., for finite dimensional discrete state variables or linear models and Gaussian pdfs.
- Rather than restricting the models, the particle filter approximates the pdf representing the posterior by a discrete PDF such that there are minimal restrictions on the models involved.
- The optimal Bayesian solution is approximated by a sum of weighted samples:

$$p(x_{0:k} \mid z_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(x_{0:k} - x_{0:k}^i)$$

$$w_k^i \propto w_{k-1}^i \frac{p(z_k \mid x_k^i) p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid x_{k-1}^i, z_k)}$$

Particle filter: Basic idea

Mathematically these steps can be written as follows:

- The posterior distribution at the previous time step is combined with the process model that describes how the state evolves over time in the prediction step. The result is referred to as the prior state:

$$p(x_k \mid z_{1:k-1}) = \int p(x_k \mid x_{k-1}) p(x_{k-1} \mid z_{1:k-1}) dx_{k-1}$$

- The measurement z_k at time *k* is used to compute the posterior using Bayes' theorem

$$p(x_k \mid z_{1:k}) = \frac{p(z_k \mid x_k) p(x_k \mid z_{1:k-1})}{p(z_k \mid z_{1:k-1})}$$

The procedure of the particle filter for robot localization

1. Start with *N* random number of possible robot locations (N particles)
2. From each particle perform an estimation of possible measurements to the known landmarks
3. Calculate the likelihood (importance weight) of each particle based on the real measurement and the estimated measurement – according to normal distribution

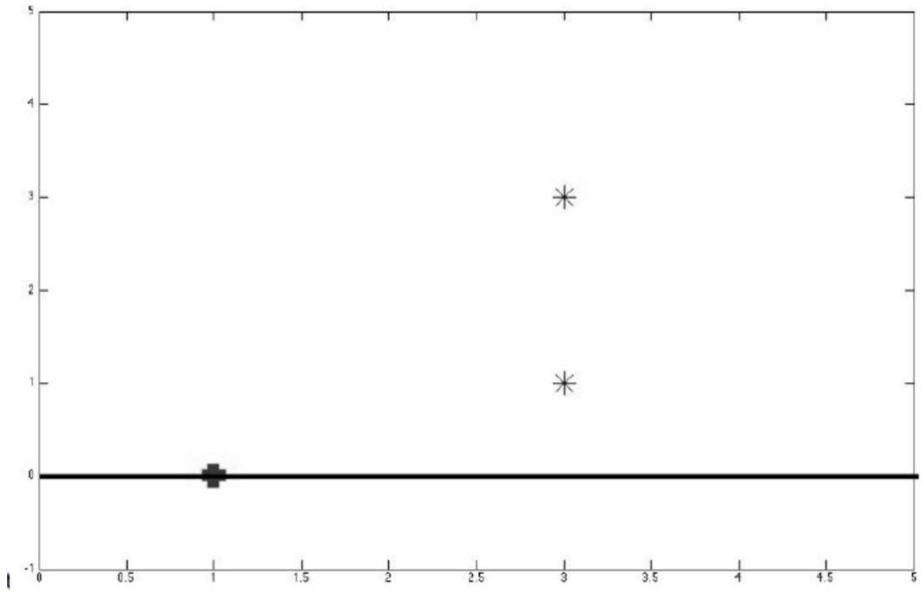
$$w_i \propto \exp\left(-\frac{1}{2}\frac{(x_i - Z_i)^2}{\sigma^2}\right)$$

4. Normalise the importance weight
5. Resample particle according to the normalised weight
6. Move each particle (robot) by the set motion command + noise
7. Go to step 2 and repeat.

- Example

A robot is moving on a straight line along x direction. The robot is moving from x=1m, at a velocity of 0.1 m/s, with a variance of 2.5×10^{-3} (m^2/s^2).
The robot is equipped with a range sensor that can measure landmarks within a 5 m range.

The sensor can get multiple range readings at each scan.
The measurement variance is 10^{-5} m^2.
Assuming there is one landmark located at x_1=3m and y_1=3m from the start, while there is a 2_{nd} landmark at x_2=3m and y_2=1m appearing at t=5s.



If the robot can be anywhere in the range of x=[0, 10], Implement a Particle Filter that estimates the robot location in space.

6.1 Dead Reckoning

- Most devices for measuring position and distance are relative measurement tools.
- By counting the number of rotations executed by a vehicles drive wheels, for example, and using knowledge of the wheel's size and the vehicle's kinematics, an estimate of the rate of position change can be obtained.
- Computing absolute coordinates thus involves the integration of such local differential quantities; for example, changes in position, orientation, or velocity.
- When someone walks in a silent, odor-free environment with his or her eyes closed, this is the form of position estimation to which that person is reduced.
- In the context of biological systems, this observation of internal parameters (for example, how many steps are taken) is referred to as *proprioception*.

- Based on an idealized error-free velocity vector d**x**/dt, the robot's position **x** can be calculated as where the motion takes place over a time interval t_o through t_f.

$$x = \int_{t_0}^{t_f} \frac{dx}{dt} dt$$

- More generally, from higher-order derivatives (such as acceleration) we can integrate repeatedly to recover position. However, that errors in the sensing or integration process are manifested as higher-order polynomials of the time interval over which we are integrating.

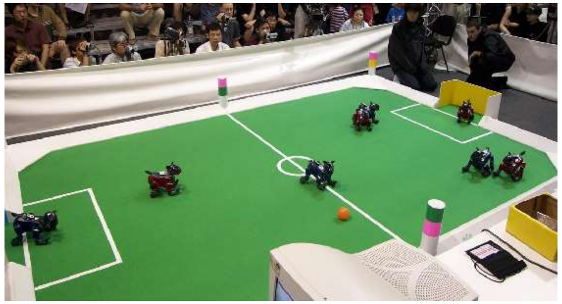
$$x = \sum \delta_i$$

- For discrete motions where positional change is expressed by a difference vector d_i, we can compute the absolute position.

This localization method can have acceptable accuracy over sufficiently small steps given a suitable terrain and drive mechanism.

6.2 Simple Landmark Measurement

- The fundamental approaches to position estimation are based on the solution of geometric or trigonometric problems involving constraints on the positions of landmarks in the environment.
- In principle, the problem is related to pose estimation of a landmark with respect to a fixed sensor.
- Important variations of the problem arise when the landmarks are unlabeled instead of labeled (that is, their individual identities are unknown), when the landmarks are difficult to detect, or when the measurements are inaccurate.
- Essentially all methods for pose estimation can be described in the context of landmark-based methods.
- The primary factors governing the use of landmarks are as follows:
  - Over what region can the landmarks be detected?
  - What is the functional relationship between landmark measurements and position?
  - How are errors manifested?



- Additional factors that characterize a particular position estimation system include:
  - Are the landmarks passive or active
  - What is the sensing modality
  - What are the geometric properties of the landmarks
  - How easy is it to detect, identify, or measure a landmark
- A key issue in practice is whether the landmarks to be used are *Artificial* or *natural*.
  - Artificial landmarks used for the purposes of robot localization are typically much easier to detect and can be uniquely labelled.

- ➢ Their optimal placement is an interesting issue.
  - Naturally occurring landmarks’ stable and robust detection can be a major issue.
- ➢ Finally, positional constraint provided by observations of a landmark depends on the sensor and the geometry of the landmark itself.
  - Planar landmarks, for example, may provide only one-dimensional constraints on the robot’s pose (distance along the normal to the landmark).
  - Sensors – range or bearing

6.2.1 Landmark Classes

- Landmarks can be active or passive, natural or artificial.
- In principle, landmarks can be defined in terms of any sensing modality.
  - Video-based sensing. In its simplest form, it can provide bearing and perhaps range to visually defined landmarks.
  - Laser transmission accompanied by video sensing deserves special mention.
  - Active radio beacons form a class of very well-established position estimation landmark. (GPS)
  - Sonar has been considered for positioning despite its drawbacks in terms of beam dispersion, specular reflection, and background noise.
    It has been shown to work particularly well with large, simple geometric structures that are sometimes referred to as *geometric beacons*.

6.2.2 Triangulation

- *Triangulation* refers to the solution of constraint equations relating the pose of an observer to the positions of a set of landmarks.
- Although landmarks and robots exist in a three-dimensional world, the limited accuracy associated with height information often results in a two-dimensional problem in practice; elevation information is sometimes used to validate the results.
  - Thus, more commonly the task is posed as a two-dimensional (or three-dimensional) problem with two- or three-dimensional landmarks.
- Depending on the combinations of sides (S) and angles (A) given, the triangulation problem is described as “side-side-side” (SSS), “side-angle-side” (SAS), and so forth.
  - In practice, a given sensing technology often returns either an angular measurement (bearing) or a distance measurement (range), and the landmark positions are typically known.

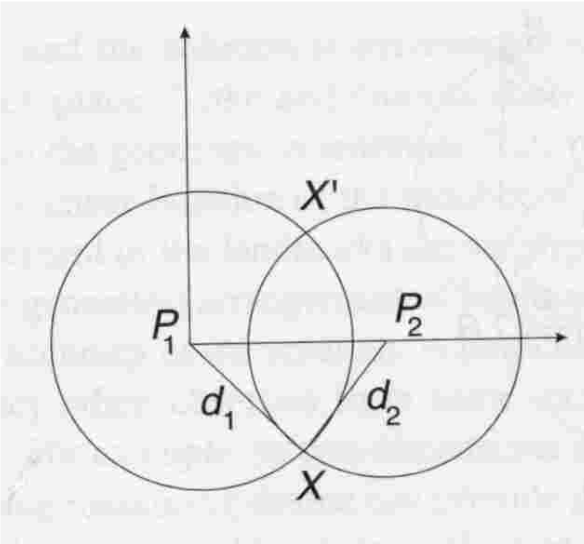
This can be formulated as: $$X = F(m_1, m_2, \cdots, m_n)$$

where the vector **X** expresses the pose variables to be estimated and **M** = m_i, ..., m_n is the vector of measurements to be used. In the specific case of estimating the position of an oriented robot in the plane, this becomes

$$\begin{aligned} x &= F_1(m_1, m_2, \cdots, m_n) \\ y &= F_2(m_1, m_2, \cdots, m_n) \\ \theta &= F_3(m_1, m_2, \cdots, m_n) \end{aligned}$$

- CASE 1: the distance to a landmark is available, a single measurement constrains the robot's position to the arc of a circle:

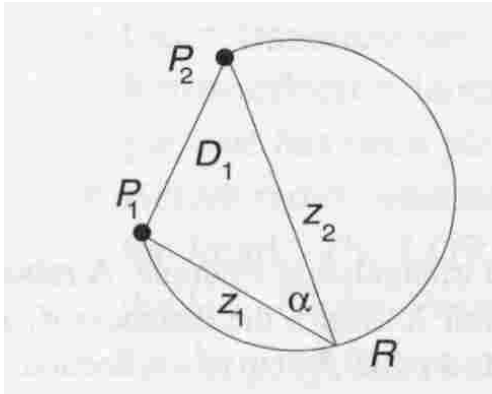
The following figure illustrates the simplest triangulation case. Without loss of generality we can assume that P_1 is at the origin and that P_2 is at $(a, 0)$. Then:



$$x = (a^2 + d_1^2 - d_2^2)/(2a)$$
$$y = \pm(d_1^2 - x^2)^{1/2}$$

In a typical application, beacons are located on walls, and thus the spurious solution can be identified because it corresponds to the robot's being located on the wrong side of (inside) the wall.

- CASE 2: a visual sensor (bearing only).



(a)

The loci of points that satisfy the bearing difference is given by:

$$D_1^2 = z_1^2 + z_2^2 - 2|z_1 z_2|\cos\alpha$$

where Z_1 and Z_2 are the distances from the robot's current position to landmarks P_1 and P_2, respectively.

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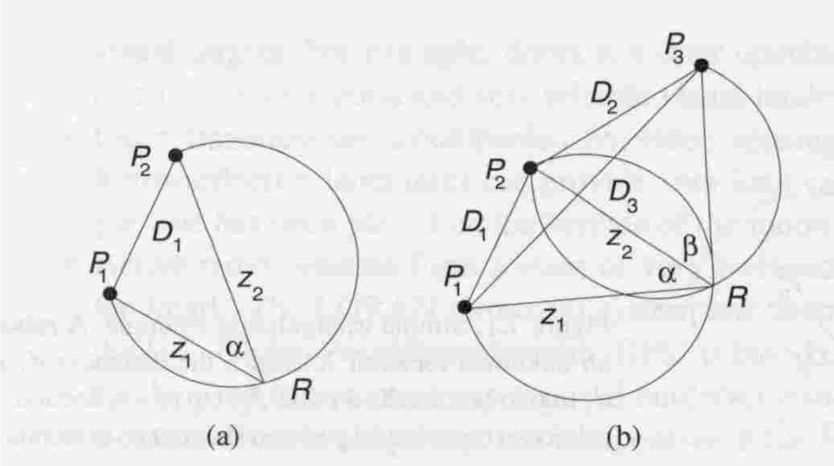
- The visibility of a third landmark gives rise to three nonlinear constraints on Z_1, Z_2 and Z_3:



$$D_1^2 = z_1^2 + z_2^2 - 2|z_1 z_2|\cos\alpha$$
$$D_2^2 = z_2^2 + z_3^2 - 2|z_2 z_3|\cos\beta$$
$$D_3^2 = z_1^2 + z_3^2 - 2|z_1 z_3|\cos(\alpha+\beta)$$

- Knowledge of Z_1, Z_2, and Z_3 leads to the robot's position and orientation. Due to noise and measuring errors, more landmarks are needed.
- The use of very distant landmarks is to be avoided if closer ones are available.

Global Positioning System (GPS) (1)

- Developed for military use
- In 1995 it became accessible for commercial applications
- 24 satellites (including three spares) orbiting the earth every 12 hours at a height of 20.190 km (since 2008: 32 satellites)
- Four satellites are located in each of six planes inclined 55 degrees with respect to the plane of the earth's equators
- Location of any GPS receiver is determined through a time of flight measurement (satellites send orbital location (*ephemeris*) plus time; the receiver computes its location through **trilateration** and **time correction**)

  - Technical challenges:
    - Time synchronization between the individual satellites and the GPS receiver
    - Real time update of the exact location of the satellites
    - Precise measurement of the time of flight
    - Interferences with other signals

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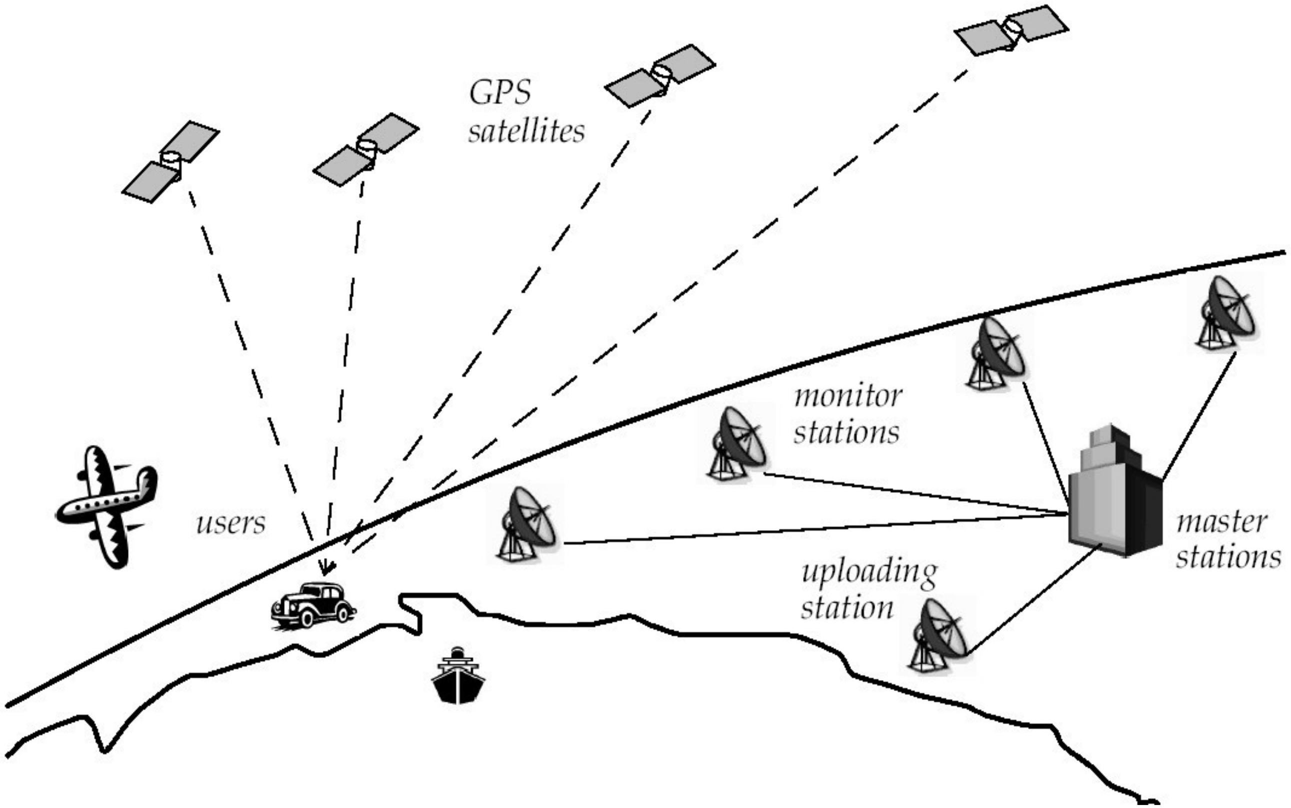
Global Positioning System (GPS) (2)



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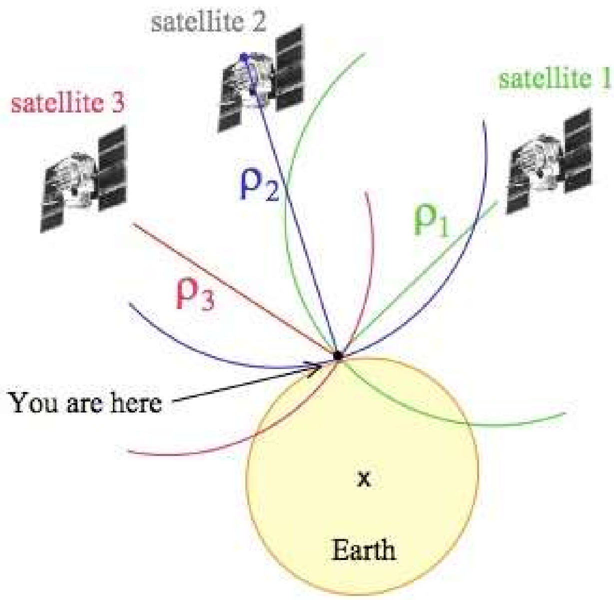
GPS positioning

- Simple positioning principle
- Satelites send signals, receivers received them with delay



$$\rho = (t_r - t_e) \times speed\ of\ \ light$$

$$\rho = \sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2}$$

If we know at least three distance measurements, we can solve for position on earth.

In practice four are used the time difference between the GPS receiver's clock and the synchronized clocks of the satellites is unknown.

Higher accuracy GPS

- DGPS (Differential GPS) uses a second static receiver at know exact position. This way errors can be correct and resolution improved (~1m accuracy)
- Take into account phase of carrier signal. There are two carriers at 19cm and 24 cm. (1cm accuracy) (both DGPS + phase less than 1cm)

DGPS Reference Station