Solutions:

The system model is:

$$x_1(t) = x_1(\tau) + (t-\tau)x_2(\tau) + \frac{a}{2}(t-\tau)^2$$

$$x_2(t) = x_2(\tau) + a(t-\tau)$$

This can be written in matrix form as:

$$\begin{bmatrix} \hat{x}_1(k+1|k) \\ \hat{x}_2(k+1|k) \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \mathbf{B}a$$

Where:

$$\mathbf{A} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \; and \; \mathbf{B} = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$$

Therefore, the system model uncertainty can be estimated as:

$$P(k+1|k) = \mathbf{A}P(k)\mathbf{A}^T + \mathbf{Q}$$

It can also be seen that the measurement model can be written as:

$$\hat{z}_1(k+1) = \mathbf{H} \begin{bmatrix} \hat{x}_1(k+1|k) \\ \hat{x}_2(k+1|k) \end{bmatrix}$$

where:

$$\mathbf{H} = [1 \quad 0]$$

Kalman gain is:

$$K(k+1) = P(k+1|k)\mathbf{H}^T[\mathbf{H}P(k+1|k)\mathbf{H}^T + \mathbf{R}(k+1)]^{-1}$$

To perform the update of state estimation, the following is carried out:
First find the innovation:

$$r(k+1) = z(k+1) - \hat{z}_1(k+1)$$

Then the state estimate is updated as:

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \hat{x}_1(k+1|k) \\ \hat{x}_2(k+1|k) \end{bmatrix} + K(k+1)r(k+1)$$

The state uncertainty is updated by:

$$P(k+1) = [I - K(k+1)\mathbf{H}]P(k+1|k)$$

Repeat the whole calculation from k = 1 to k = 30.