Algorithms for Intelligent Robots

Tutorial 2 solution

Question 1:

The kinematic equations can be obtained by using the following formula:

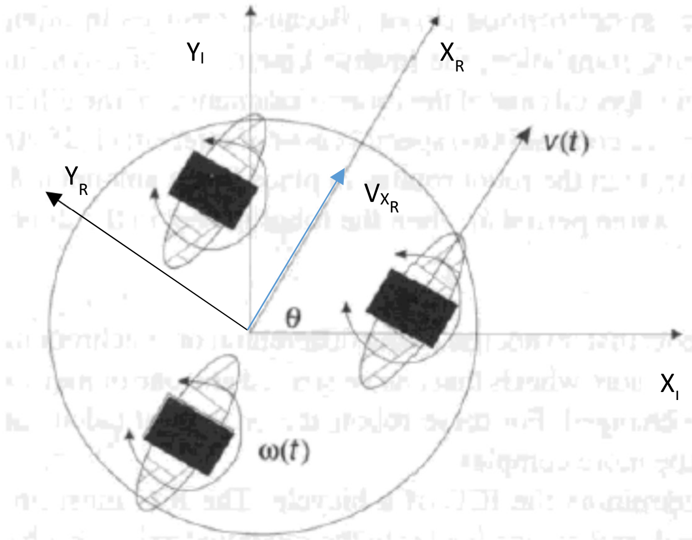
$$\dot{\xi}_I = Q\dot{\xi}_R \qquad (1)$$

From the figure below, we can see that:

$$\dot{\xi}_R = \begin{bmatrix} Vx_R \\ Vy_R \\ w_R \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ w \end{bmatrix} \qquad (2)$$

Substituting (2) into (1) we get:

$$\dot{x} = V \cos\theta$$
$$\dot{y} = V \sin\theta$$
$$\dot{\theta} = \omega$$



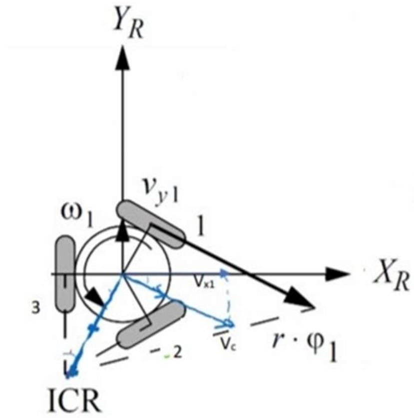
Question 2:

a) Method 1

In this method, we will find the velocities contributed by each wheel and then sum them to find the overall kinematic equations.

- Wheel 1:

When wheel 1 spins, wheel 2 and 3 will slip (rolling along the rollers of the wheel) resulting in the ICR as shown in the figure below



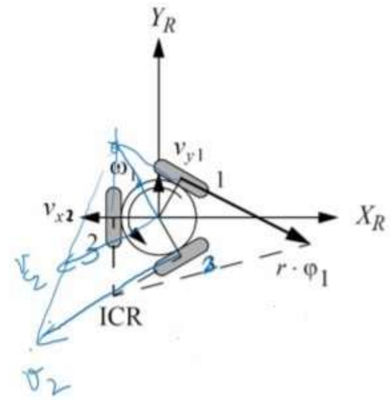
$v_{c1} = \frac{2}{3} v_1 = \frac{2}{3} r\varphi_1$

$v_{x1} = v_{c1}.\cos(30^{\circ}) = \frac{\sqrt{3}}{2}.\frac{2}{3}.r\varphi_1 = \frac{1}{\sqrt{3}} r\varphi_1$

$v_{y1} = -v_{c1}.\sin(30^{\circ}) = \frac{-1}{2}.\frac{2}{3}.r\varphi_1 = \frac{-1}{3} r\varphi_1$

$\omega_1 = \frac{-v_{c1}}{2l} = \frac{-2}{3}\frac{r\varphi_1}{2l} = \frac{-r\varphi_1}{3l}$

- Wheel 2:



$v_{c2} = \frac{2}{3} v_2 = \frac{2}{3} r\varphi_2$

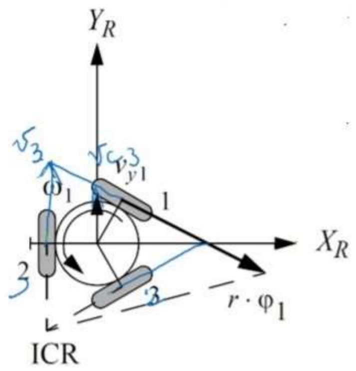
$v_{x2} = -v_{c2}.\cos(30^{\circ}) = \frac{-\sqrt{3}}{2}.\frac{2}{3}.v_2 = \frac{1}{\sqrt{3}} v_2$

$v_{y2} = -v_{c2}.\sin(30^{\circ}) = \frac{-1}{2}.\frac{2}{3}.v_2 = \frac{-1}{3} v_2$

$\omega_2 = \frac{-v_{c2}}{2l} = \frac{-2}{3}\frac{v_2}{2l} = \frac{-v_2}{3l}$

- Wheel 3:

$$v_{c3} = \frac{2}{3}v_3$$
$$v_{x3} = 0$$
$$v_{y3} = \frac{2}{3}v_3$$
$$\omega_2 = \frac{-v_3}{3l}$$



By adding up the velocities from all three wheels, we get

$$\dot{\xi}_R = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}}v_1 & \frac{-1}{\sqrt{3}}v_2 & 0 \\ \frac{-1}{3}v_1 & \frac{-1}{3}v_2 & \frac{2}{3}v_3 \\ \frac{-v_1}{3l} & \frac{-v_2}{3l} & \frac{-v_3}{3l} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{-1}{3l} & \frac{-1}{3l} & \frac{-1}{3l} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (3)$$

From here, we can find the final kinematic equations for the robot in the global frame by using equation (1) as follows:

$$\dot{\xi}_I = Q\dot{\xi}_R = Q \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{-1}{3l} & \frac{-1}{3l} & \frac{-1}{3l} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (3b)$$

a) Method 2: Using wheel constraints

From the equation of the Swedish wheels it can be seen the pure rolling constraints are:

$$[\sin(\alpha+\beta+\gamma) \quad -\cos(\alpha+\beta+\gamma) \quad -l\cos(\beta+\gamma)]R\dot{\xi}_I - r\dot{\varphi}\cos(\gamma) = 0 \quad (4)$$

For the three wheels, it can be seen that $\alpha_1 = 60^\circ, \alpha_2 = -60^\circ, \alpha_3 = 180^\circ$ and $\beta = 0, \gamma = 0$ for all three wheels. By substituting these values into (4), the following equations can be obtained:

$$\begin{bmatrix} \sin(60^\circ) & -\cos(60^\circ) & -l \\ \sin(-60^\circ) & -\cos(-60^\circ) & -l \\ \sin(180^\circ) & -\cos(180^\circ) & -l \end{bmatrix} R\dot{\xi}_I = \begin{bmatrix} r\dot{\varphi}_1 \\ r\dot{\varphi}_2 \\ r\dot{\varphi}_3 \end{bmatrix} \quad (5)$$

Where the rotation matrix R can be expressed as:

$$R = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From (5), it can be calculated that:

$$\dot{\xi}_I = R^{-1}\begin{bmatrix} \sin(60^\circ) & -\cos(60^\circ) & -l \\ \sin(-60^\circ) & -\cos(-60^\circ) & -l \\ \sin(180^\circ) & -\cos(180^\circ) & -l \end{bmatrix}^{-1} \begin{bmatrix} r\dot{\varphi}_1 \\ r\dot{\varphi}_2 \\ r\dot{\varphi}_3 \end{bmatrix} \quad (6)$$

b) Since the robot frame and the global frame are aligned, we have θ=0 resulting in $R = Q = I_{3x3}$.

With method 1, taking the inverse of equation (3b) we get:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{-1}{3l} & \frac{-1}{3l} & \frac{-1}{3l} \end{bmatrix}^{-1} \begin{bmatrix} V_{XR} \\ V_{yR} \\ \dot{\theta}_R \end{bmatrix} \quad (7a)$$

With method 2, It can be calculated from (5):

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} r\dot{\varphi}_1 \\ r\dot{\varphi}_2 \\ r\dot{\varphi}_3 \end{bmatrix} = \begin{bmatrix} \sin(60^\circ) & -\cos(60^\circ) & -l \\ \sin(-60^\circ) & -\cos(-60^\circ) & -l \\ \sin(180^\circ) & -\cos(180^\circ) & -l \end{bmatrix} \begin{bmatrix} V_{XR} \\ V_{yR} \\ \dot{\theta}_R \end{bmatrix} \quad (7b)$$

Noting that equations (7a) and (7b) should give the same result. Therefore, by substituting desired values of V_{XR} and V_{YR} into (7a) or (7b), wheel velocities can be calculated.

- $V_{XR} = 1$ m/s, $V_{YR} = 0$, $\dot{\theta}_R = 0$

$$\begin{cases} v_1 = \frac{\sqrt{3}}{2} V_{XR} = \frac{\sqrt{3}}{2} \\ v_2 = \frac{-\sqrt{3}}{2} V_{XR} = \frac{-\sqrt{3}}{2} \\ v_3 = 0 \end{cases} \Rightarrow \begin{cases} \omega_1 = \frac{v_1}{R} = \sqrt{3} \\ \omega_2 = -\sqrt{3} \\ \omega_3 = 0 \end{cases}$$

c, $V_{XR} = 0$ m/s, $V_{YR} = 1$, $\dot{\theta}_R = 0$

$$\begin{cases} v_1 = -0.5 \\ v_2 = -0.5 \\ v_3 = 1 \end{cases} \Rightarrow \begin{cases} \omega_1 = -1 \\ \omega_2 = -1 \\ \omega_3 = 2 \end{cases}$$

d, $V_{XR} = 1$ m/s, $V_{YR} = 1$, $\dot{\theta}_R = 0$

$$\begin{cases} v_1 = \frac{\sqrt{3}}{2} - 0.5 \\ v_2 = \frac{-\sqrt{3}}{2} - 0.5 \\ v_3 = 1 \end{cases} \Rightarrow \begin{cases} \omega_1 = 0.73 \\ \omega_2 = -2.73 \\ \omega_3 = 2 \end{cases}$$

e, Circle – Students need to find the answer.