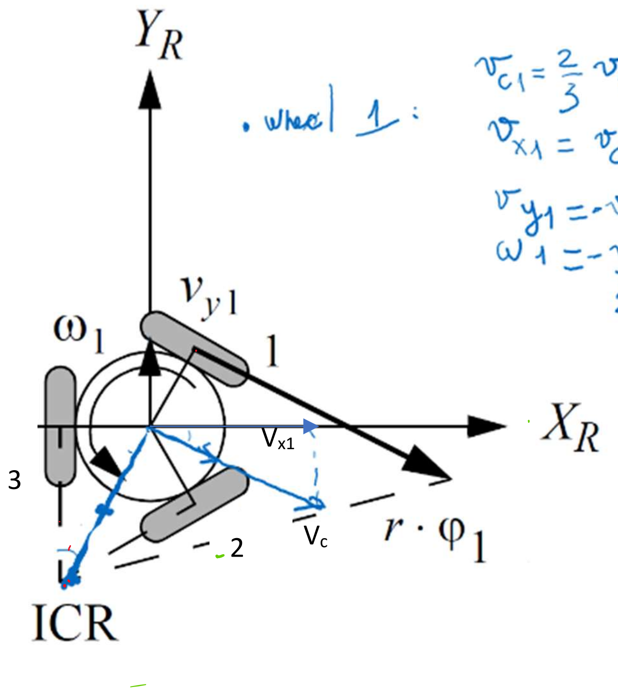
Tutorial 2

Question 2 solution:

Part a) Method 1

Swedish robot kinematic derivation:

When wheel 1 spins, wheels 2 and 3 will slip resulting in the ICR as shown in the figure below.



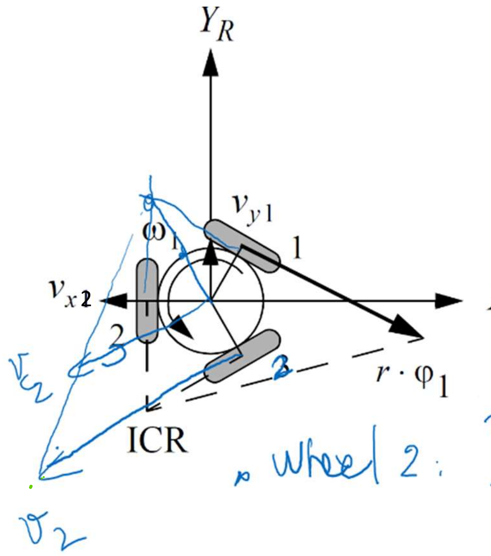
• wheel 1:

$$v_{c1} = \frac{2}{3} v_1 = \frac{2}{3} r\varphi_1 = \frac{2}{3} v_1$$

$$v_{x1} = v_{c1} \cos 30^\circ = \frac{\sqrt{3}}{2} \cdot \frac{2}{3} r\varphi_1 = \frac{1}{\sqrt{3}} r\varphi_1$$

$$v_{y1} = -v_{c1} \sin 30^\circ = -\frac{1}{2} \frac{2}{3} r\varphi_1 = -\frac{1}{3} r\varphi_1$$

$$\omega_1 = -\frac{v_{c1}}{2l} = -\frac{2}{3} r\varphi_1 / 2l = -\frac{r\varphi_1}{3l}$$



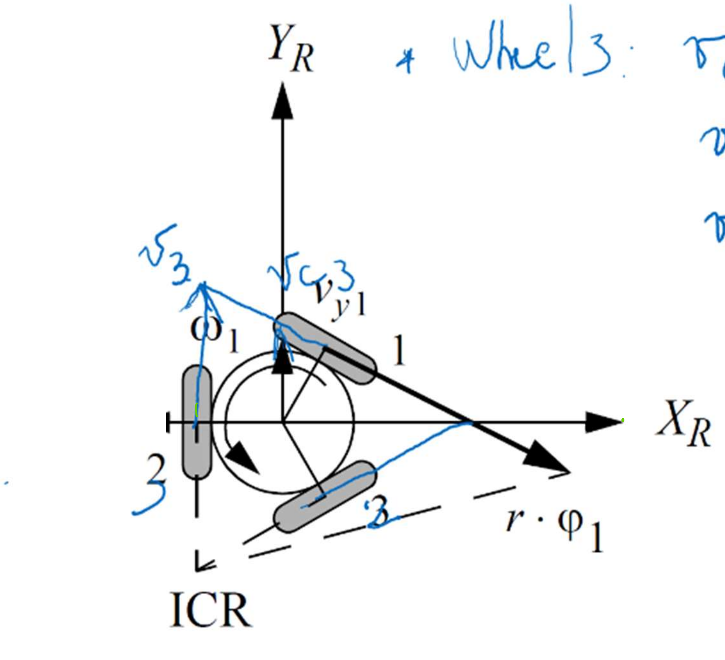
* Wheel 2:

$$v_{c_2} = \frac{2}{3} v_2$$

$$v_{x_2} = -v_{c_2} \cos 30^\circ = -\frac{\sqrt{3}}{2} \frac{2}{3} v_2 = -\frac{1}{\sqrt{3}} v_2$$

$$v_{y_2} = -v_{c_2} \sin 30^\circ = -\frac{1}{3} v_2$$

$$\omega_2 = -\frac{v_{c_2}}{2l} = -\frac{v_2}{3l}$$



* Wheel 3:

$$v_{c_3} = \frac{2}{3} v_3$$

$$v_{x_3} = 0$$

$$v_{y_3} = \frac{2}{3} v_3$$

$$\omega_3 = -\frac{v_3}{3l}$$

$$\dot{\xi}_R = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} v_1 & -\frac{1}{\sqrt{3}} v_2 & 0 \\ -\frac{1}{\sqrt{3}} v_1 & -\frac{1}{\sqrt{3}} v_2 & \frac{2}{3} v_3 \\ -\frac{v_1}{3l} & -\frac{v_2}{3l} & -\frac{v_3}{3l} \end{bmatrix}$$

$$\hookrightarrow \dot{\xi}_I = Q\,\dot{\xi}_R$$

Part a) Method 2

The solution based on wheel constraints:

From the equations of the Swedish wheels it can be seen that the pure rolling constraints are:

$$[\sin(\alpha+\beta+\gamma) \quad -\cos(\alpha+\beta+\gamma) \quad -l cos(\beta+\gamma)]R\dot{\xi} - r\dot{\phi}\cos(\gamma) = 0 \qquad (1)$$

For the three wheels, it can be seen that $\alpha_1 = 60°, \alpha_2 = -60°, and\ \alpha_3 = 180°$, and $\beta = 0,\ \gamma = 0$ for all three wheels. By substituting these values into (1), the following equations can be obtained:

$$\begin{bmatrix} \sin(60°) & -\cos(60°) & -l \\ \sin(-60°) & -\cos(-60°) & -l \\ \sin(180°) & -\cos(180°) & -l \end{bmatrix} R\dot{\xi}_I = \begin{bmatrix} r\dot{\phi}_1 \\ r\dot{\phi}_2 \\ r\dot{\phi}_3 \end{bmatrix} \qquad (2)$$

where the rotation matrix ***R*** can be expressed as:

$$R = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From (2), it can be calculated that:

$$\dot{\xi}_I = R^{-1} \begin{bmatrix} \sin(60°) & -\cos(60°) & -l \\ \sin(-60°) & -\cos(-60°) & -l \\ \sin(180°) & -\cos(180°) & -l \end{bmatrix}^{-1} \begin{bmatrix} r\dot{\phi}_1 \\ r\dot{\phi}_2 \\ r\dot{\phi}_3 \end{bmatrix} \qquad (3)$$

Part b)

When $R = I_{3\times3}$

It can be calculated from (2):

$$\begin{bmatrix} r\dot{\phi}_1 \\ r\dot{\phi}_2 \\ r\dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} \sin(60°) & -\cos(60°) & -l \\ \sin(-60°) & -\cos(-60°) & -l \\ \sin(180°) & -\cos(180°) & -l \end{bmatrix} \begin{bmatrix} V_{XR} \\ V_{YR} \\ \dot{\theta}_R \end{bmatrix} \qquad (4)$$

By substituting desired values of V_{XR} and V_{YR} into (4), wheel velocities can be calculated.

• As the local frame is chosen to be ~~coinci~~ in parallel with the global frame : $\cancel{\theta} = 0 \rightarrow R = I_{3\times3}$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -0.5 & -2 \\ -\sqrt{3}/2 & -0.5 & -2 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} v_{xR} \\ v_{yR} \\ \dot{\theta}_R \end{bmatrix}$$

b) $v_{xR} = 1\ m/s$, $v_{yR} = 0$, $\dot{\theta}_R = 0$

$$\rightarrow \begin{cases} v_1 = \frac{\sqrt{3}}{2} v_{xR} = \frac{\sqrt{3}}{2} \\ v_2 = -\frac{\sqrt{3}}{2} v_{xR} = -\frac{\sqrt{3}}{2} \\ v_3 = 0 = 0 \end{cases} \rightarrow \begin{cases} \omega_1 = \frac{v_1}{R} = \sqrt{3} \\ \omega_2 = -\sqrt{3} \\ \omega_3 = 0 \end{cases}$$

c) $v_{yR} = 1\ m/s$, $v_{xR} = 0$, $\dot{\theta}_R = 0$

$$\begin{cases} v_1 = -0.5 \\ v_2 = -0.5 \\ v_3 = 1 \end{cases} \Rightarrow \begin{cases} \omega_1 = -1 \\ \omega_2 = -1 \\ \omega_3 = 2 \end{cases}$$

d) $v_{xR} = 1$, $v_{yR} = 1$, $\dot{\theta}_R = 0$

$$\begin{cases} v_1 = \frac{\sqrt{3}}{2} - 0.5 \\ v_2 = -\frac{\sqrt{3}}{2} - 0.5 \\ v_3 = 1 \end{cases} \rightarrow \begin{cases} \omega_1 = \sqrt{3} - 1 = 0.73 \\ \omega_2 = -\sqrt{3} - 1 = -2.73 \\ \omega_3 = 2. \end{cases}$$

e) Circle