Algorithms for Intelligent Robots

Tutorial 4: Robot localization with Kalman filtering

Question 1

Consider a robot that moves in a single direction with a constant velocity v = 1 m/s. Due to the imperfectness of the controller and actuator, the actual velocity of the robot is subjected to noise that has the Gaussian distribution with zero mean and the variance of 0.2 m^2/s^2.

The robot is equipped with a sensor that can measure its position. However, this sensor is also subjected to Gaussian noise with zero mean and the variance of 0.1 m^2/s^2. With the sampling rate of 1 Hz, the sensor readings are given in the table below:

Time (s)	1	2	3	4	5
Value (m)	1.2	2	3.3	4.1	5.5

a) Estimate the position $\boldsymbol{x}$ and its variance $\boldsymbol{P}$ of the robot using the Kalman filter at time 1s and 2s given that the initial position of the robot is $\boldsymbol{x_0} = 0$ with the variance $\boldsymbol{P_0} = 0.1$.
b) Write a MATLAB script to carry out the calculation for all 5 time steps. Compare the result with part a).

Question 2

Assume that a vehicle is driving on a straight rail line with a constant acceleration (Figure 1). If only the position of the vehicle can be measured using an instrument, design a Kalman filter to estimate the positions and velocities of the vehicle based on this measurement.

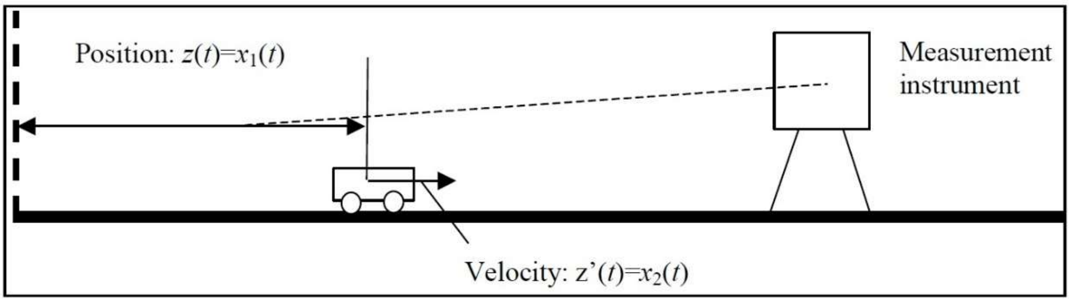

Figure 1: Measurements of a moving vehicle for application of Kalman filter

The discrete-time state-space equations of the vehicle can be expressed as:

$$x_1(t) = x_1(\tau) + (t-\tau)x_2(\tau) + \frac{a}{2}(t-\tau)^2$$

$$x_2(t) = x_2(\tau) + a(t-\tau)$$

where $x_1(t)$ is the position of the vehicle at time t, $x_2(t)$ is the velocity of the vehicle at time t, $t = kT$, $\tau = (k-1)T$, and $a = 0.2\ m/s^2$ is the constant acceleration of the vehicle. It is assumed that the sampling time T of the observation is 0.5 second and the initial position of the vehicle is $x_1(0) = 10\ m$.

a) Derive the equations for calculating the positions and velocities of the vehicle using the Kalman filter based on the measured position values.

b) The measurements of the first 30 positions of the vehicle are shown in Table 1 (a soft copy of this data, called "measurement.mat", is placed on the Course website). These measurements have been affected by independent random disturbances. Write a program in MATLAB to calculate the estimated positions and velocities of the vehicle using the Kalman filter developed in 1. It is also assumed that the initial values of the state vector (position and velocity) and the initial errors are:

$$\hat{x}(0) = \begin{bmatrix} 9.0 \\ 0 \end{bmatrix} \qquad v(k) = 0, \qquad R(k)=1, \qquad \text{and} \qquad P(0) = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$$

Table 1: The position measurements of the vehicle

k	Position Measurement (m)	k	Position Measurement (m)	k	Position Measurement (m)
1	10.229	**11**	12.586	**21**	19.135
2	10.125	**12**	13.449	**22**	20.816
3	10.229	**13**	14.082	**23**	21.793
4	11.265	**14**	14.886	**24**	23.585
5	9.261	**15**	14.868	**25**	24.570
6	10.795	**16**	16.284	**26**	26.066
7	11.045	**17**	16.514	**27**	27.042
8	11.556	**18**	16.510	**28**	28.152
9	11.310	**19**	18.025	**29**	29.555
10	12.469	**20**	18.773	**30**	31.170

Compare the estimated results from Kalman filter with the measured data, the modelled data and actual data. Comment on the effects the above initial conditions have on the accuracy of the results.