HUST

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HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

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Applied Algorithm Lab

Strongly connected component

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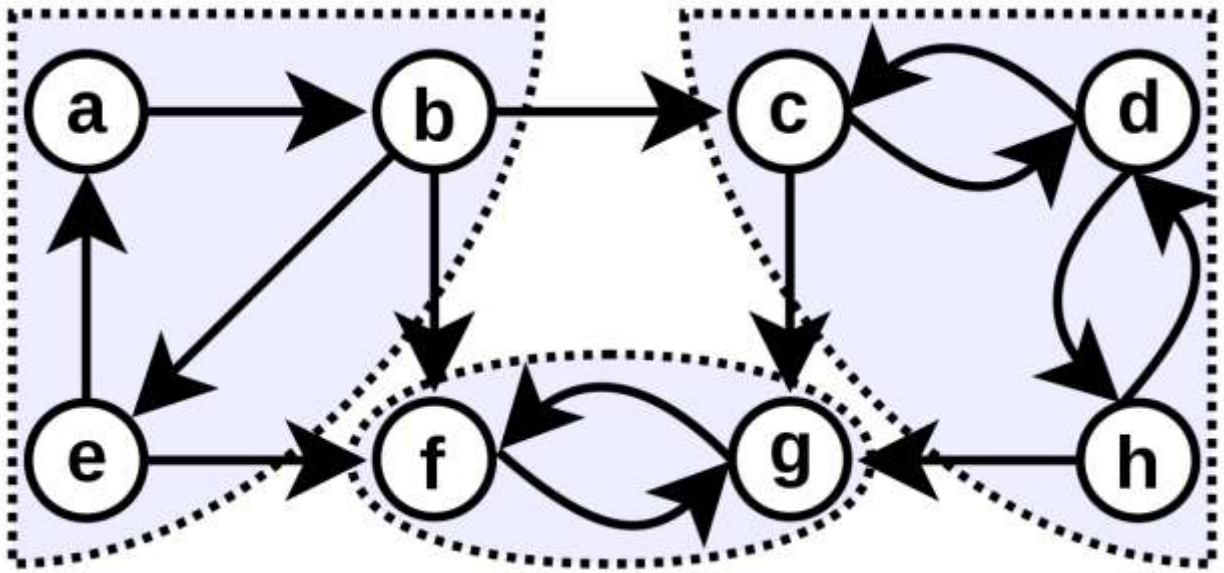
Strongly connected component

- Count #strongly connected components in a directed graph
  - Definition: A subset with maximal number of vertices that between 2 arbitrary vertices, there exists a path from a vertex to the other and vice versa.

  Thành phần liên thông mạnh là một tập con tối đa các đỉnh sao cho giữa 2 đỉnh bất kỳ luôn có đường đi từ đỉnh này đến đỉnh kia và ngược lại.
- **Input:** Edge list
  - Line 1 contains N, M
  - M lines follow, containing a pair of 2 integers a, b which is an undirected edge from a to b.
- **Output:**
  - Number of strongly connected components.

Strongly Connected Components

- BFS and DFS can find all connected component in undirected graph.
- However, in directed graph, finding all strongly connected components is not trivial.
- Can we use DFS tree to find all strongly connected components ?



Strongly connected component

- Idea to solve: DFS on residual graph. Algorithm:
  - Run DFS on G -> compute the finishing time f(v) of each node v of G
  - Build residual graph G^T of G: reverse direction of all edges
  - Run DFS on G^T:
    - the nodes are considered in a decreasing order of finishing time f:
    - Each run DFS(u) will visit all nodes of the strongly connected component containing u
  - Number of connected components on G = number of times call DFS on residual graph of G

Strongly connected component - Implementation

```cpp
// Đọc thêm tại https://cp-algorithms.com/graph/strongly-connected-components.html
#include <iostream>
#include <algorithm>
#include <vector>
using namespace std;
const int maxN = 1e5 + 5;
int n, m;
vector<int> adj[maxN], adj_trans[maxN];
vector<int> toposort;
bool visited[maxN];
int numSCC = 0;
// Tạo mảng toposort, trong đó các nút hiện đang được xếp tăng dần theo t_out
// (xem định nghĩa trong link bên trên)
void dfs1(int node){
    if (visited[node])
        return;
    visited[node] = true;
    for (auto u: adj[node])
        dfs1(u);
    toposort.push_back(node);
}
```

Strongly connected component - Implementation

```
23  // Tìm thành phần liên thông mạnh ứng với node
24  void dfs2(int node){
25      if (visited[node])
26          return;
27      visited[node] = true;
28      for (auto u: adj_trans[node])
29          dfs2(u);
30  }
```

Strongly connected component - Implementation

```
32  int main(){
33      cin >> n >> m;
34      for (int i = 0; i < m; i++){
35          int a, b;
36          cin >> a >> b;
37          adj[a].push_back(b);
38          adj_trans[b].push_back(a);
39      }
40      for (int i = 1; i <= n; i++)
41          if (!visited[i])
42              dfs1(i);
43      for (int i = 1; i <= n; i++)
44          visited[i] = false;
45      reverse(toposort.begin(), toposort.end());
46      for (int i = 0; i < n; i++){
47          if (!visited[toposort[i]]){
48              dfs2(toposort[i]);
49              numSCC++;
50          }
51      }
52      cout << numSCC << endl;
53      return 0;
54  }
```

THANK YOU !