

The background of the entire image is a dark blue field filled with a pattern of red dots. These dots are arranged in a way that they form a large, faint, stylized circular shape in the center, with the density of the dots increasing towards the center.

HUST

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HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.



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Applied Algorithm Lab

Inversions

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Inversions

- Count the number of inversions (số cặp nghịch thế) of a permutation
- Input: a sequence of integers a_1, a_2, \dots, a_n .
- Output: The number of pairs (i, j) that $i < j$ and $a_i > a_j$
- Example

stdin	stdout
6 3 2 4 5 6 1	6

- Idea to solve #1: Brute-force from left -> right
 - At each element: count number of its right elements smaller than it -> number of inversions
 - Complexity: $O(n^2)$

- Idea to solve #2: similar to merge sort
 - Fix the code mergeSort to sort and count inversions at the same time
 - Idea of mergeSort(int left, int right):
 - divide the array into 2 parts
 - Sort the subarray (left, mid) and subarray (mid + 1, right)
 - Merge the two subarrays
 - Complexity: $O(n \log n)$

- Idea to solve #2: similar to merge sort
 - Code Merge sort:
 - Fix the code mergeSort to sort and count inversions at the same time

```
void mergeSort(int left, int right) {  
    if (right <= left) return;  
    int mid = (left + right) / 2;  
    mergeSort(left, mid);  
    mergeSort(mid + 1, right);  
  
    int i = left, j = mid + 1, k = left;  
    while (i <= mid && j <= right) {  
        if (a[i] <= a[j]) {  
            temp[k++] = a[i++];  
        } else {  
            temp[k++] = a[j++];  
        }  
    }  
    while (i <= mid) temp[k++] = a[i++];  
    while (j <= right) temp[k++] = a[j++];  
  
    for (int i = left; i <= right; i++)  
        a[i] = temp[i];  
}
```


Inversions - Implementation

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  #define N 1000001
4  const int Q = 1e9+7;
5
6  int n, a[N], temp[N], nbInversion = 0;
7
8  void input() {
9      ios_base::sync_with_stdio(0);
10     cin.tie(0); cout.tie(0);
11     cin >> n;
12     for (int i = 1; i <= n; i++) {
13         cin >> a[i];
14     }
15 }
```

Inversions - Implementation

```
17 void countAndMergeSort(int left, int right) {
18     if (left >= right) return;
19     int mid = (left + right) / 2;
20     countAndMergeSort(left, mid);
21     countAndMergeSort(mid + 1, right);
22
23     int i = left, j = mid + 1, k = left;
24     while (i <= mid && j <= right) {
25         if (a[i] <= a[j]) {
26             temp[k++] = a[i++];
27         } else {
28             temp[k++] = a[j++];
29             nbInversion += (mid - i + 1); // a[i] > a[j]
30             nbInversion %= Q; // to avoid overflow
31         }
32     }
33
34     while (i <= mid) temp[k++] = a[i++];
35     while (j <= right) temp[k++] = a[j++];
36     for (int i = left; i <= right; i++) a[i] = temp[i];
37 }
```



```
39  int main() {  
40      input();  
41      countAndMergeSort(1, n);  
42      cout << nbInversion;  
43      return 0;  
44  }
```

A large graphic on the left side of the slide. It features a dark blue background with a circular pattern of red dots of varying sizes, creating a sense of depth and movement. The word "HUST" is centered within this pattern in a white, bold, sans-serif font.

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THANK YOU !