HUST

ĐẠI HỌC BÁCH KHOA HÀ NỘI

HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.

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Applied Algorithm Lab

Bridges

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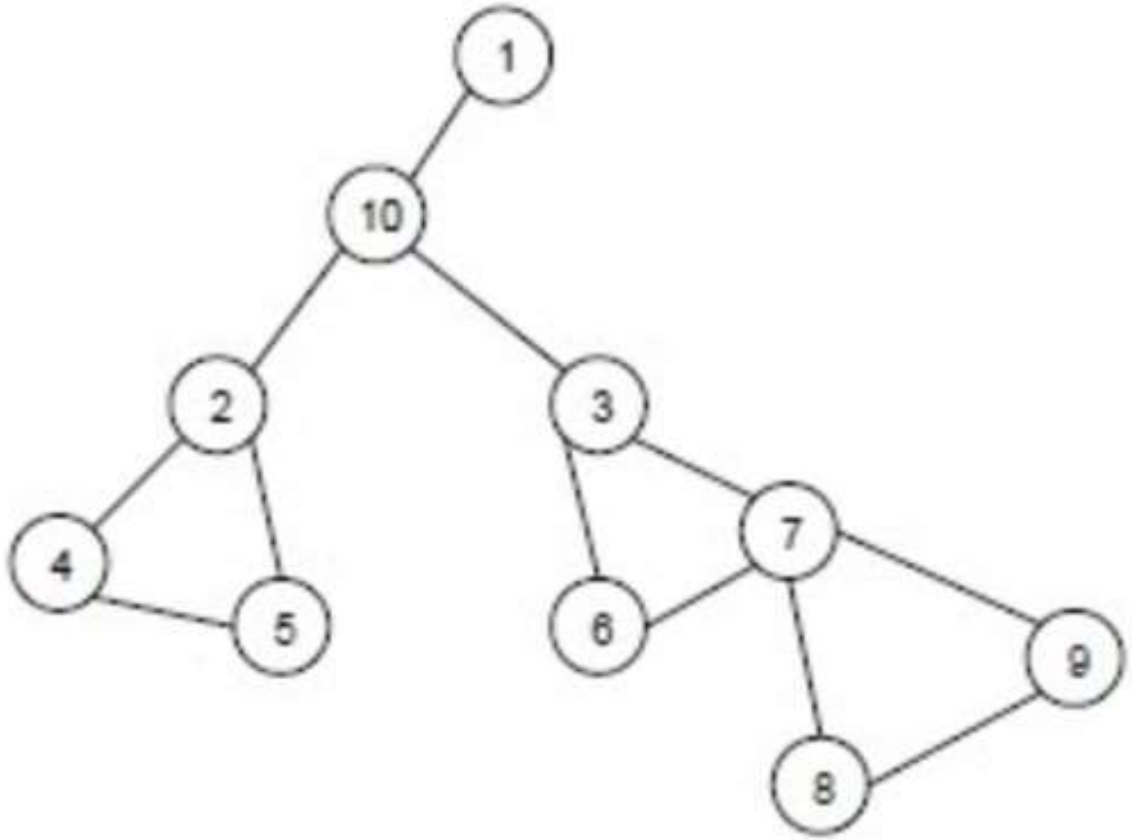
Bridges

- Find articulation points and bridges of an undirected graph.
  - Definition: Remove articulation points/bridges -> the number of connected components in the graph increase
- **Input:** Edge list
  - Line 1 contains N, M
  - M lines follow, containing a pair of 2 integers a, b which is an undirected edge from a to b.
- **Output:**
  - Number of articulation points and the number of bridges.

Bridges

- Example

Input	Output
10 12 1 10 10 2 10 3 2 4 4 5 5 2 3 6 6 7 7 3 7 8 8 9 9 7	4 3 Explain: Articulation points: 10, 2, 3, 7 Bridges: (2-10), (10-3), (10-1)



Bridges

- Idea to solve: DFS
  - Store graph in adjacency list: vector<vector<int>> adj(N);

Bridges

- **DFS**
- DFS tree and Num, Low structure
- Finding bridges
- Finding articulation points

Depth First Search

- DFS is a basic graph traversing technique (to visit all nodes and edges in graph).
  - DFS can answer if there exist a path from node u to node v in graph or not, and show the path if exists.
  - DFS can also answer from u, we can goes to which nodes on graph G.
- The traversing order in DFS follow LIFO – Last In First Out mechanism, start from some beginning node u.
  - We may use backtracking recusion or stack
- Complexity: $O(|V| + |E|)$, where V is node set and E is edge set of G, since each node and each edge is visited once.

Programming

- Graph G = (V, E) represented by adjacency lists A[1…n]
- Marking array:
  - visited[u] = true, if u is visited
  - visited[u] = false, otherwise

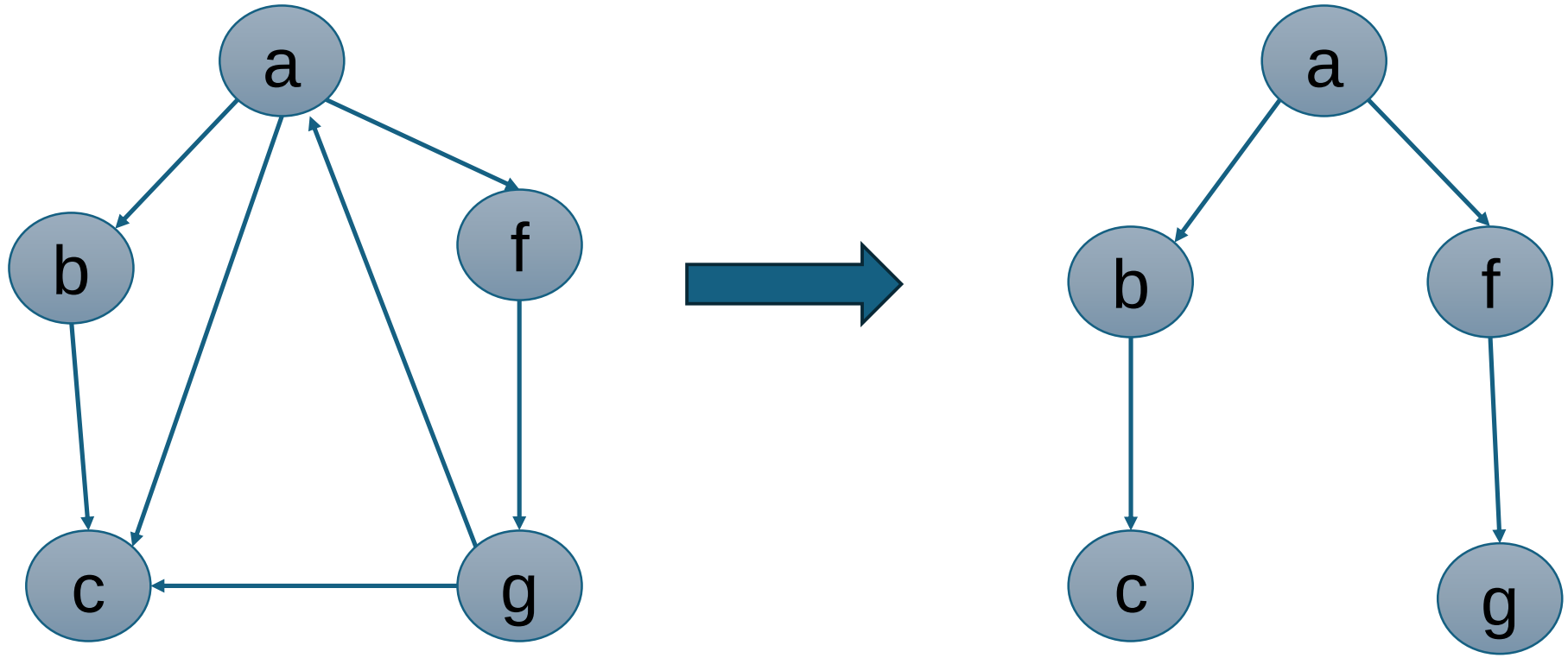
```
DFS(u) {
   visit(u); // assign visited[u] = true
   for v in A[u] do {
      if not visited[v] then {
          DFS(v);
      }
   }
}
DFS(){
  for u in V  do {   visited[u] = false;  }
  for u in V do {
     if not visited[u] then
        DFS(u);
  }
}
```

Bridges

- DFS
- **DFS tree and Num, Low structure**
- Finding bridges
- Finding articulation points

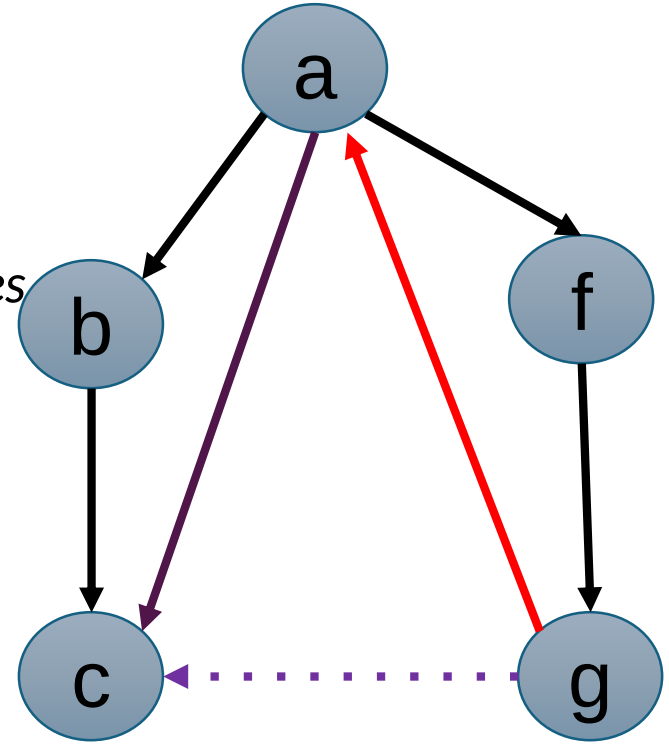
DFS tree

- Trace of the DF search will construct a tree, called DFS tree



DFS tree

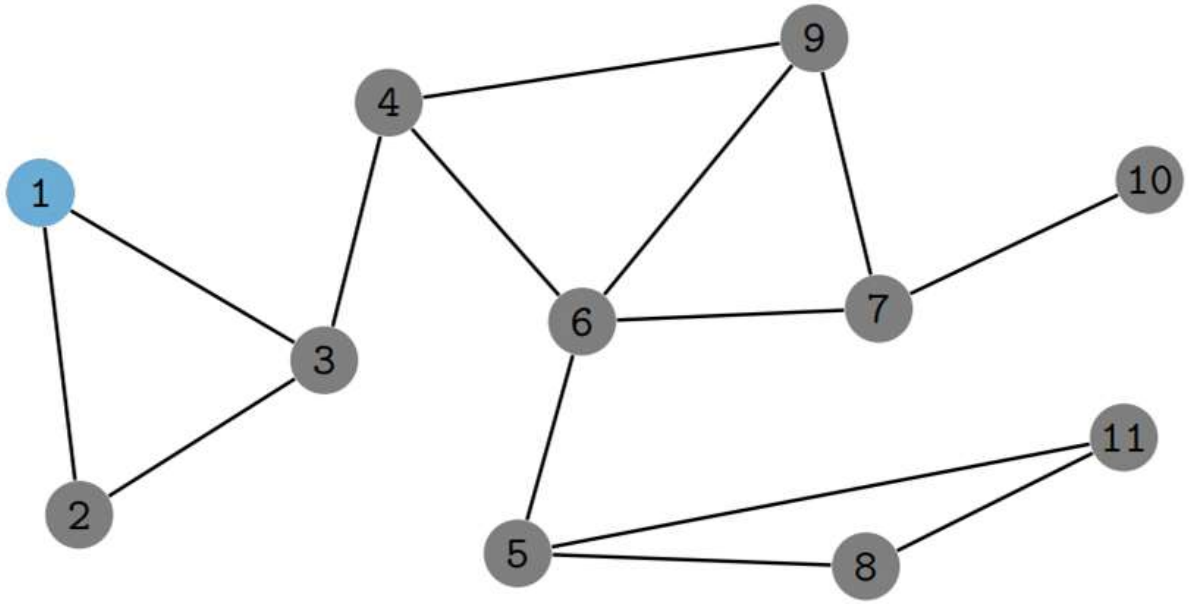
- Trace of the DF search will construct DFS tree
- Some type of edge in DFS:
  - Tree Edge: edge in DFS tree, *e.g. black edges in figure*
  - Back Edge: edge from descendants to ancestors, *e.g. red edges*
  - Forward Edge: edge from ancestors to descendants, *e.g. blue edges*
  - Crossing Edge: edge between non-relational nodes, *e.g. purple edges*



DFS tree: Num and Low structure

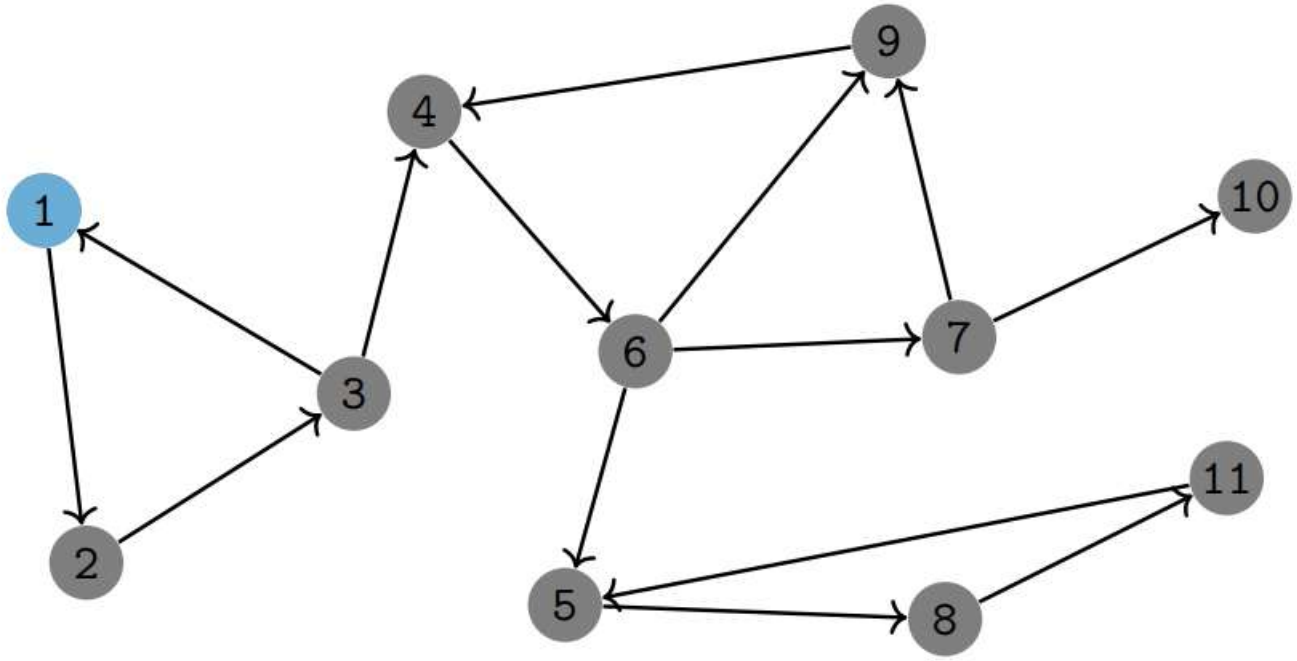
- Defining 2 arrays Num and Low for each node of DFS tree.
- $Num[u]$: visiting order of u in DFS traversal
- $Low[u]$: the minimum value of:
  - $Num[v]$ if (v,u) is back edge
  - $Low[v]$:

    ~minimum num[v] where v and descendents can visit



i	1	2	3	4	5	6	7	8	9	10	11
Num[i]	1	2	3	4	6	5	9	7	10	11	8
Low[i]	1	1	1	4	6	4	4	6	4	11	6

Example



i	1	2	3	4	5	6	7	8	9	10	11
Num[i]	1	2	3	4	6	5	9	7	10	11	8
Low[i]	1	1	1	4	6	4	4	6	4	11	6

DFS and Num, Low programming by recursion

- p[v]: father of v in DFS
- Num[u] = 0: node u is not visited
- Num[u] > 0: node u is visited and Num[u] is the order

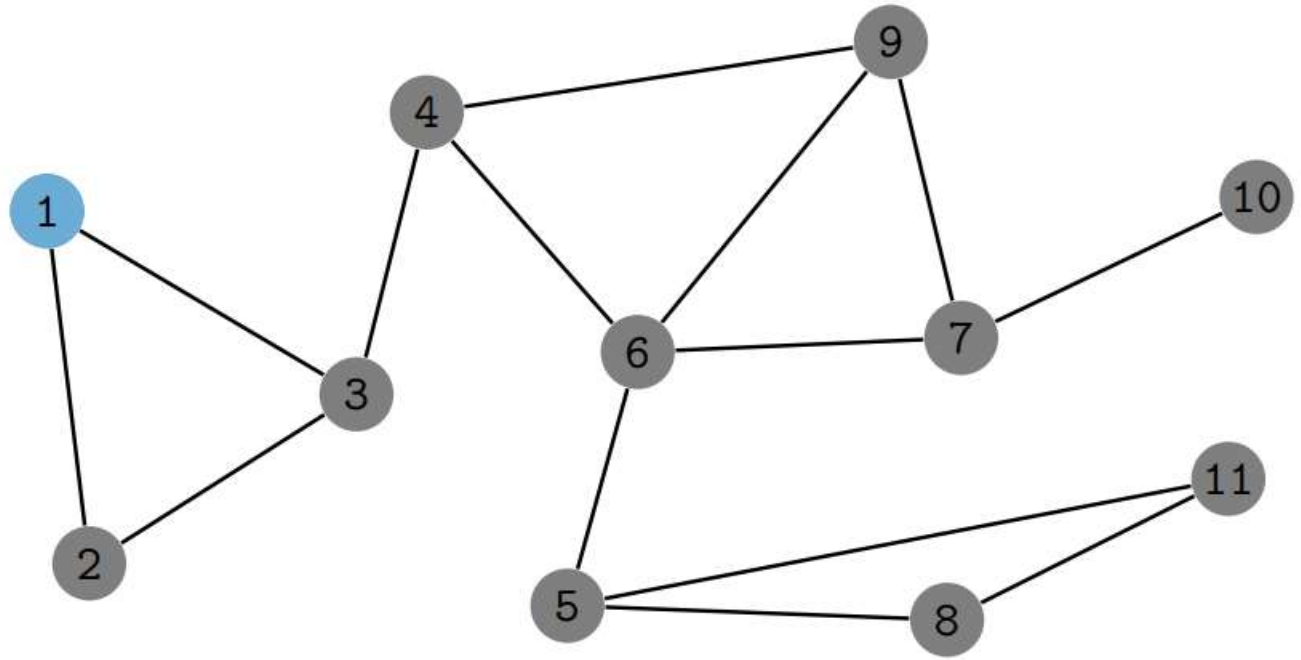
```
DFS(u) {
   T += 1; Num[u] = T; Low[u] = T;
   for v in A[u] do {
      if v = p[u] continue;
      if Num[v] > 0 then { // v was visited
         Low[u] = min(Low[u], Num[v]);
      } else {
         p[v] = u;
         DFS(v);
         Low[u] = min(Low[u], Low[v]);
      }
   }
}
```

Bridges

- DFS
- DFS tree and Num, Low structure
- **Finding bridges**
- Finding articulation points

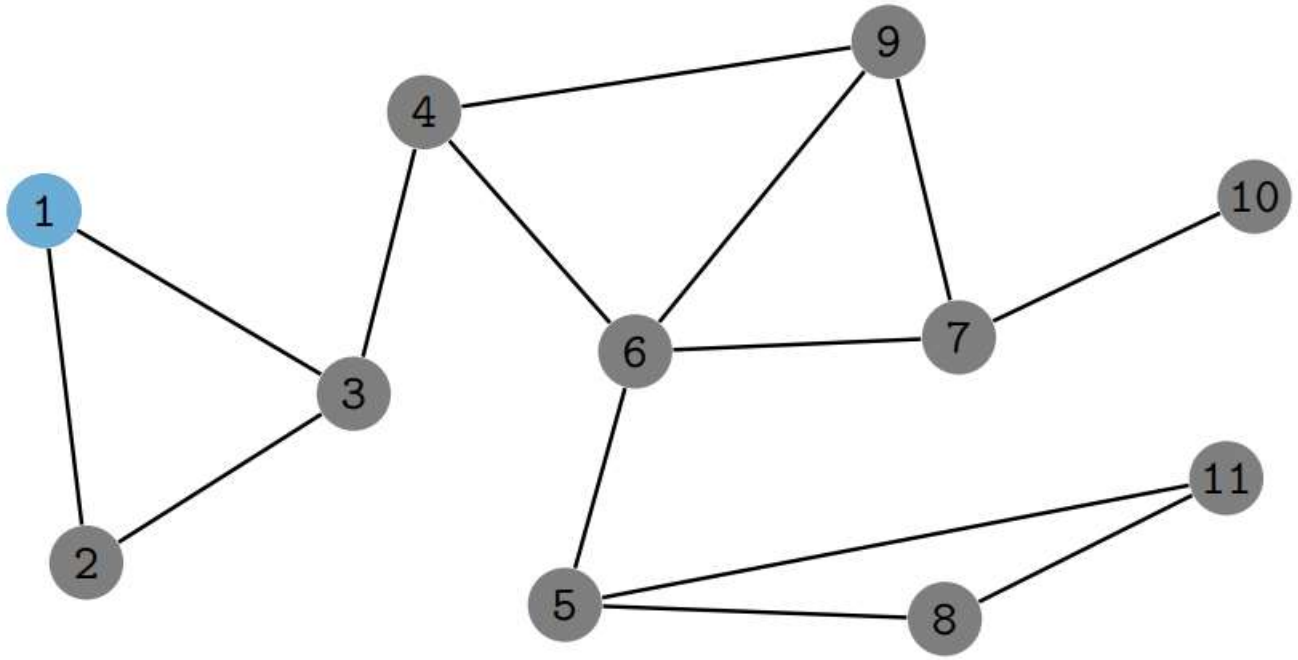
Finding bridges

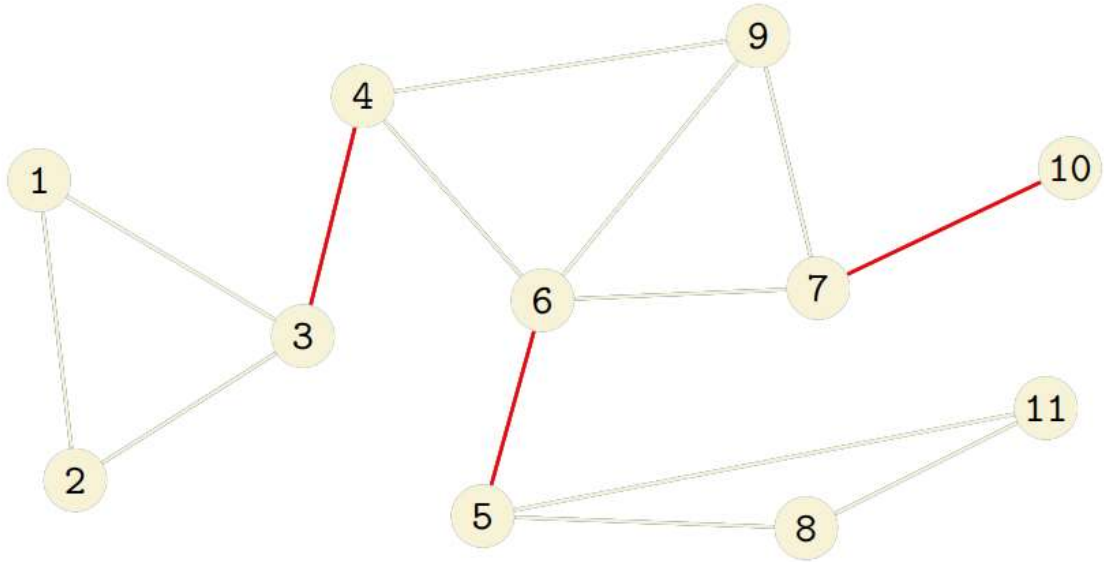
- **Definition**: Bridge is an edge where if we remove it, the number of connected components in the graph increases.
- **Note**: A forwarde edge (u, v) is an edge if and only if $Low[v] > Num[u]$



Finding bridges

- **Note**: A forwarde edge (u, v) is an edge if and only if $Low[v] > Num[u]$





i	1	2	3	4	5	6	7	8	9	10	11
Num[i]	1	2	3	4	6	5	9	7	10	11	8
Low[i]	1	1	1	4	6	4	4	6	4	11	6

Programming

- p[v]: father of v in DFS
- Num[u] = 0: node u is not visited
- Num[u] > 0: node u is visited and Num[u] is the order
- update low[u]
- check bridge (u,v)

```
DFS(u) {
   T += 1; Num[u] = T; Low[u] = T;
   for v in A[u] do {
      if v = p[u] continue;
      if Num[v] > 0 then { // v was visited
         Low[u] = min(Low[u], Num[v]);
      } else {
         p[v] = u;
         DFS(v);
         Low[u] = min(Low[u], Low[v]);
         if Low[v] > Num[u] then (u,v) is a bridge;
      }
   }
}
```

Programming - Implementation

```cpp
#include <bits/stdc++.h>
using namespace std;

const int N = 1e5 + 2;

int timeDfs = 0, bridge = 0, cntJoint = 0;
vector<vector<int>> adj(N);  // Danh sách kề của đồ thị
vector<int> low(N);          // low[u] là số hiệu nhỏ nhất của các đỉnh có thể đến được từ u
                             // bằng cả cây DFS và một số cạnh ngoài cây DFS
vector<int> num(N);          // num[u] là số hiệu của đỉnh u trong quá trình duyệt DFS
vector<bool> joint(N);       // joint[u] là true nếu u là điểm phân cực
```

Ý tưởng cài đặt bằng đệ quy - Implementation

```
13  void dfs(int u, int pre) {
14      int child = 0;  // Số lượng con trực tiếp của đỉnh u trong cây DFS
15      num[u] = low[u] = ++timeDfs;
16      for (auto v : adj[u]) {
17          if (v == pre) continue;
18          if (!num[v]) {  // Nếu đỉnh v chưa được duyệt, thực hiện DFS trên nó
19              dfs(v, u);
20              low[u] = min(low[u], low[v]);
21              if (low[v] == num[v]) bridge++;  // Cập nhật số cầu
22              child++;
23              if (u == pre) {  // Nếu u là đỉnh gốc của cây DFS
24                  if (child > 1) joint[u] = true;  // Nếu có nhiều hơn một con, u là điểm phân cực
25              } else {
26                  if (low[v] >= num[u]) joint[u] = true;  // Nếu không, xét điều kiện để u là điểm phân cực
27              }
28          } else {
29              low[u] = min(low[u], num[v]);  // Nếu v đã được duyệt, cập nhật low[u]
30          }
31      }
32  }
```

Bridges

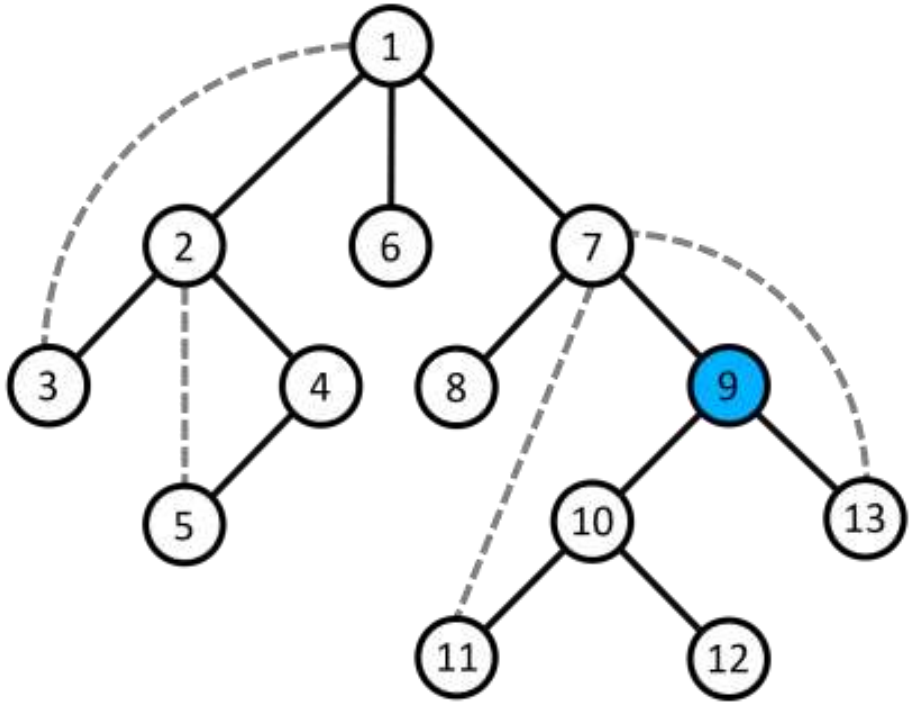
- DFS
- DFS tree and Num, Low structure
- Finding bridges
- **Finding articulation points**

Finding articulation points

- **Definition**: In undirected graph, an articulation point is a node where if we remove it and its adjacent edge, the number of connected components in graph increase.
- **Note**: Node u is an articulation point iff:
  - Node u is the root of DFS tree and

$$Low[v] \geq Num[u]$$

    where v is a direct child of u in DFS tree
  - Or u is the root of DFS tree having at least 2 direct children



Finding articulation points - Implementation

```
35  int main() {
36      ios_base::sync_with_stdio(0);
37      cin.tie(0);
38      cout.tie(0);
39      int n, m;
40      cin >> n >> m;
41      int x, y;
42      for (int i = 1; i <= m; i++) {
43          cin >> x >> y;
44          adj[x].push_back(y);
45          adj[y].push_back(x);
46      }
47      for (int i = 1; i <= n; i++) {
48          if (!num[i])
49              dfs(i, i);
50      }
51      for (int i = 1; i <= n; i++) {
52          cntJoint += joint[i];  // Đếm số điểm phân cực
53      }
54      cout << cntJoint << " " << bridge;  // In kết quả
55      return 0;
56  }
```

THANK YOU !