Oregon State University

ECE 462 Final Project: 8-QAM Simulation

Jared Gaskin and Nhu Duong

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1. Problem Description

The goal of this project was to evaluate the performance of a particular 8-QAM digital communication system over an AWGN channel with power spectral density $S_W(f) = \frac{N_o}{2}$, $\forall f$. This system was to be evaluated via simulation in Matlab, and results were to be presented in terms of plots of SER (Symbol Error Rate) and BER (Bit Error Rate) versus SNR (Signal to Noise Ratio)

The communication system to be evaluated was given in terms of its signal constellation diagram, which is shown below:

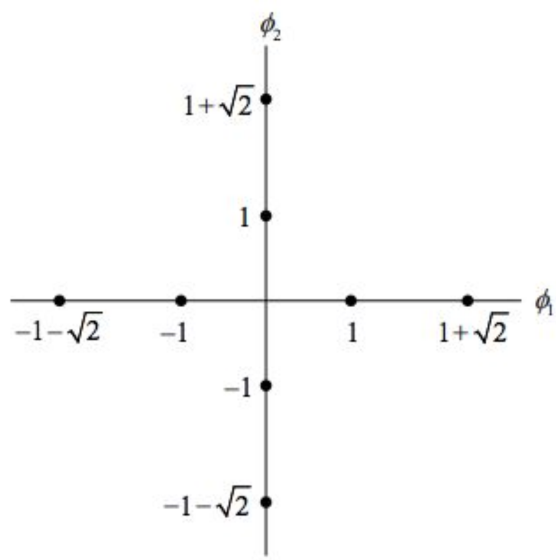


Figure 1: Given constellation diagram

2. Assumptions

A few assumptions are made in this project, including the fact that all the symbols occur with equal probability at the transmitter. Additionally, it is assumed that the symbols in the constellation are Grey encoded. This Grey encoding of the symbols is as follows:

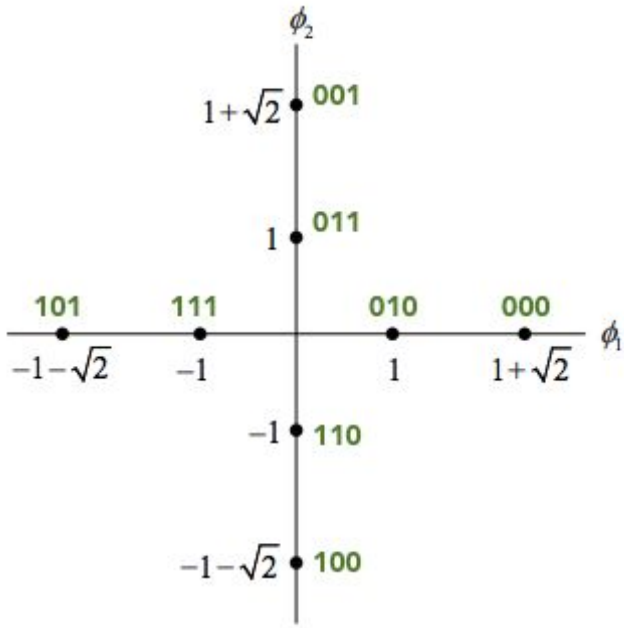


Figure 2: Constellation diagram with Grey encoding

3. Analysis

<u>Derivation of the average symbol energy in the system:</u>

First, deriving the average symbol energy: (this will be used later as well)

$$E_{s,\,av} = 4[\tfrac{1}{8}(d_1{}^2)] + 4[\tfrac{1}{8}(d_2{}^2)]$$

With d_1 = distance between 4 inner-most points and the origin = 1
d_2 = distance between 4 outer-most points and the origin = $1 + \sqrt{2}$

Substituting these values, the average symbol energy is given by : $E_{s,\,av} =$ 3.4142 J

Calculation of a theoretical upper bound on SER:

Before attempting to simulate the given communication system, a theoretical SER for the system was calculated so that it could be compared to the simulation results. The calculation of this SER is as follows:

First consider that the unconditional probability of a symbol error is given by:

$$SER = \sum_{i=1}^{8} P\{symbol\ error \cap m_i\} = \sum_{i=1}^{8} P\{ symbol\ error \mid m_i \}\ P\ \{m_i\}$$

Now, noting that all symbols are equally probable a priori, and thus that $P\{m_i\} = 1/8$ for all i, this can be re-written as:

$$SER = \frac{1}{8} \sum_{i=1}^{8} P\{ symbol\ error \mid m_i \}$$

We can find this probability that a symbol error will occur given symbol m_i is sent for each symbol by considering the notion that an error will occur if an observation lands in another symbol's decision region due to noise. Thus, an error occurs if the observation vector, X, lies in the union of all seven of the other symbols' decision regions:

$$P\{symbol\ error \mid m_i\} = P\{ X \notin R_i \mid m_i \} = P\ \{ X \in \bigcup_{j=1}^{7} R_j^{\,i} \mid m_i \}$$

Now, we will bound this quantity by considering the fact that the sum of the probabilities of X lying in each of the other symbols' decision regions will be greater than or equal to the probability that X lies in the union of these regions

$$P\{symbol\ error \mid m_i\} \leq \sum_{j=1}^{7} P\{X \in R_j^{\,i} \mid m_i \}$$

Now, in order to simplify this calculation, and because the symbols in the constellation have been gray encoded, we will consider the probability that X lies in any other points' decision region to be equal to the probability of a pair-wise bit error:

$$P\{ pairwise\ bit\ error\ (choosing\ symbol\ j\ instead\ of\ symbol\ i) \} = Q\left(\frac{d_{ij}}{\sqrt{2N_0}} \right)$$

$$P\{X \in R_j^{\,i} \mid m_i \} = Q\left(\frac{d_{ij}}{\sqrt{2N_0}} \right)$$

Where d_{ij} is the distance between signal points i and j

Re-writing the SER expression, we now have:

$$SER \leq \frac{1}{8} \sum_{i=1}^{8} \sum_{j=1}^{7} Q\left(\frac{d_{ij}}{\sqrt{2N_0}} \right)$$

To simplify the calculation of this upper bound on the SER, only the nearest neighbor points will be accounted for in this second sum (we will only consider the possibility that each point is confused with one of its nearest neighbors). This is justifiable for high SNR, but may produce an incorrect estimate at lower SNR values.

In this way, the second sum will account for the three nearest neighbors for the four inner-most points, and a single nearest neighbor for the outer four points in the constellation. For each of these nearest neighbors, the intra-symbol distance can be found to be $\sqrt{2}$ by inspection. Substituting this value for d_{ij} yields the following:

$$P\{X \in R_j^{\ i} \mid m_i\} = Q\left(\frac{\sqrt{2}}{\sqrt{2No}} \right) = Q\left(\frac{1}{\sqrt{N_0}} \right)$$

Now for the four inner-most points:

$$P\{symbol\ error \mid m_i\} \leq \sum_{j=1}^{3} Q\left(\frac{1}{\sqrt{N_0}} \right) = 3Q\left(\frac{1}{\sqrt{N_0}} \right)$$

And for the four outer-most points:

$$P\{symbol\ error \mid m_i\} \leq \sum_{j=1}^{1} Q\left(\frac{1}{\sqrt{N_0}} \right) = Q\left(\frac{1}{\sqrt{N_0}} \right)$$

Finally, returning to our SER equation, we can write:

$$SER \leq \frac{1}{8}\left(4\left[3Q\left(\frac{1}{\sqrt{N_0}} \right)\right] + 4\left[Q\left(\frac{1}{\sqrt{N_0}} \right)\right] \right) = 2Q\left(\frac{1}{\sqrt{N_0}} \right)$$

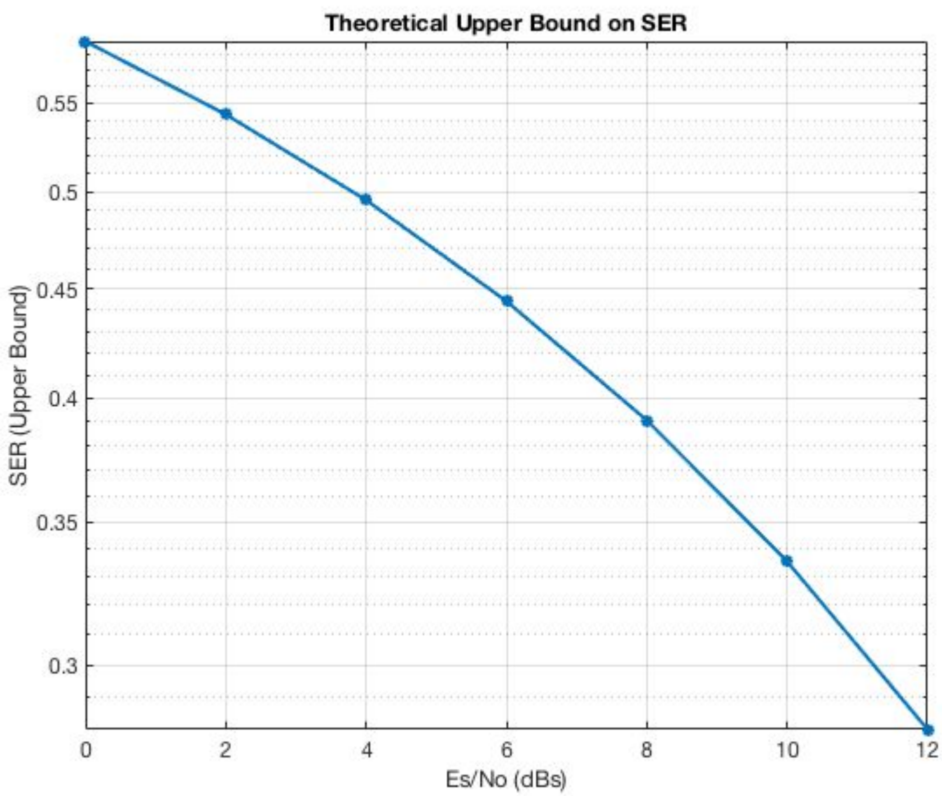
Thus,

$$SER \leq 2Q\left(\frac{1}{\sqrt{N_0}} \right)$$

Now, given a particular value of Es/No in dBs, the value of No can be determined through the following equation (as we know the average symbol energy, Es).

$$E_s/N_0\ (dBs) = 20\ log_{10}(\ E_s/N_0\ ) = 20\ log_{10}(\ 3.41/N_0\ )$$

This value of No can then be plugged into the SER bound that was just derived, and then the results can be plotted for different Es/No values in dB. This plot can then help in analyzing the results of the simulation, and has been produced for Es/No from 0dB to 12dB:



4. Simulation setup description

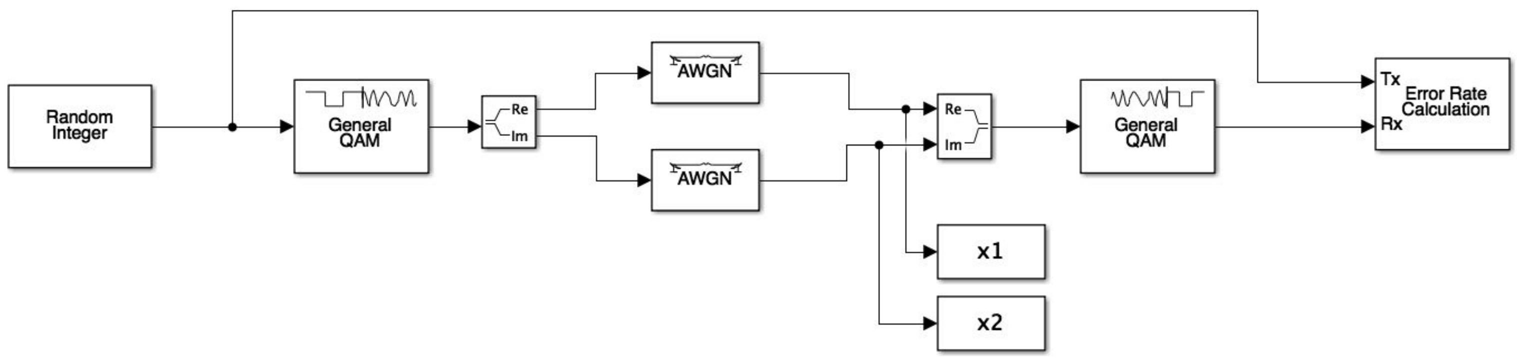


Figure 3: Simulink block diagram

<u>Explanation of Simulink Blocks</u>

The above simulink blocks simulate the given communication system in the following way:

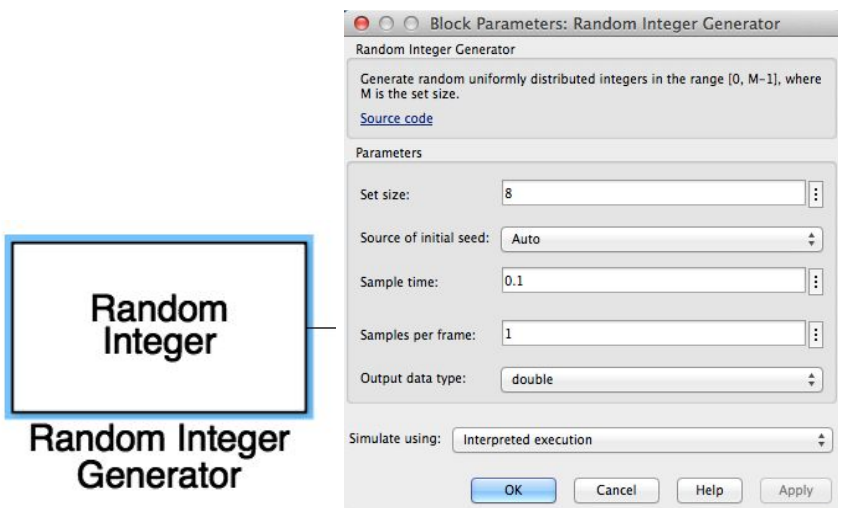


Figure 4: Parameters for the "Random Integer" block

The “Random Integer” block first produces a random integer (representing the symbol to be sent) uniformly between 0 and 7 (inclusive). This means that each of the symbols is equally likely to be transmitted. This integer is then used in the “General QAM Modulator Baseband” block to select one of the eight symbols in the given QAM constellation.

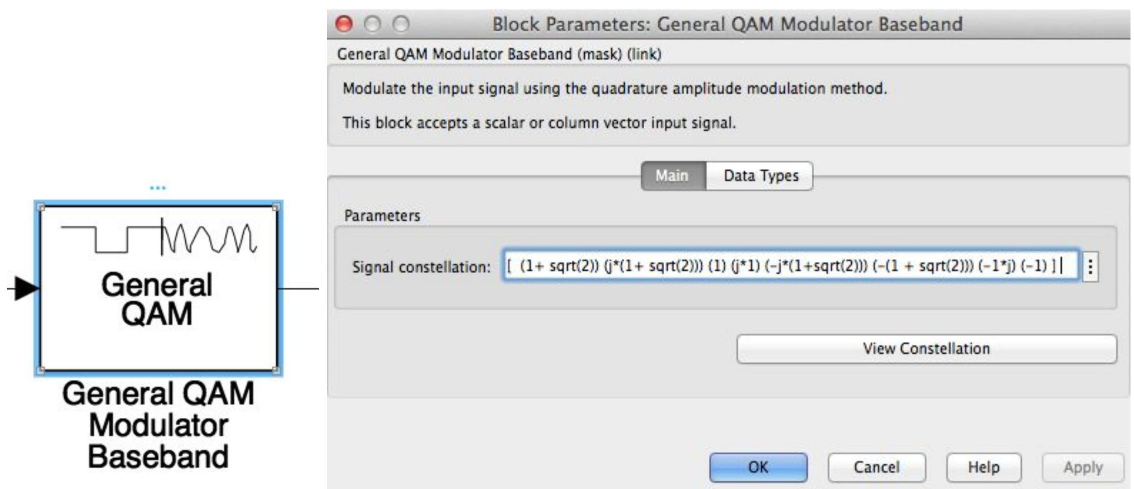


Figure 5: Parameters for the “General QAM Modulator Baseband” block

The grey encoding mentioned earlier is used to map which integer corresponds to which symbol in the constellation (by converting the integer into its corresponding three bit value). This grey encoding must be set up in the “General QAM Modulator Baseband” block by listing the signal points in increasing order from point zero (000) to point seven (111).

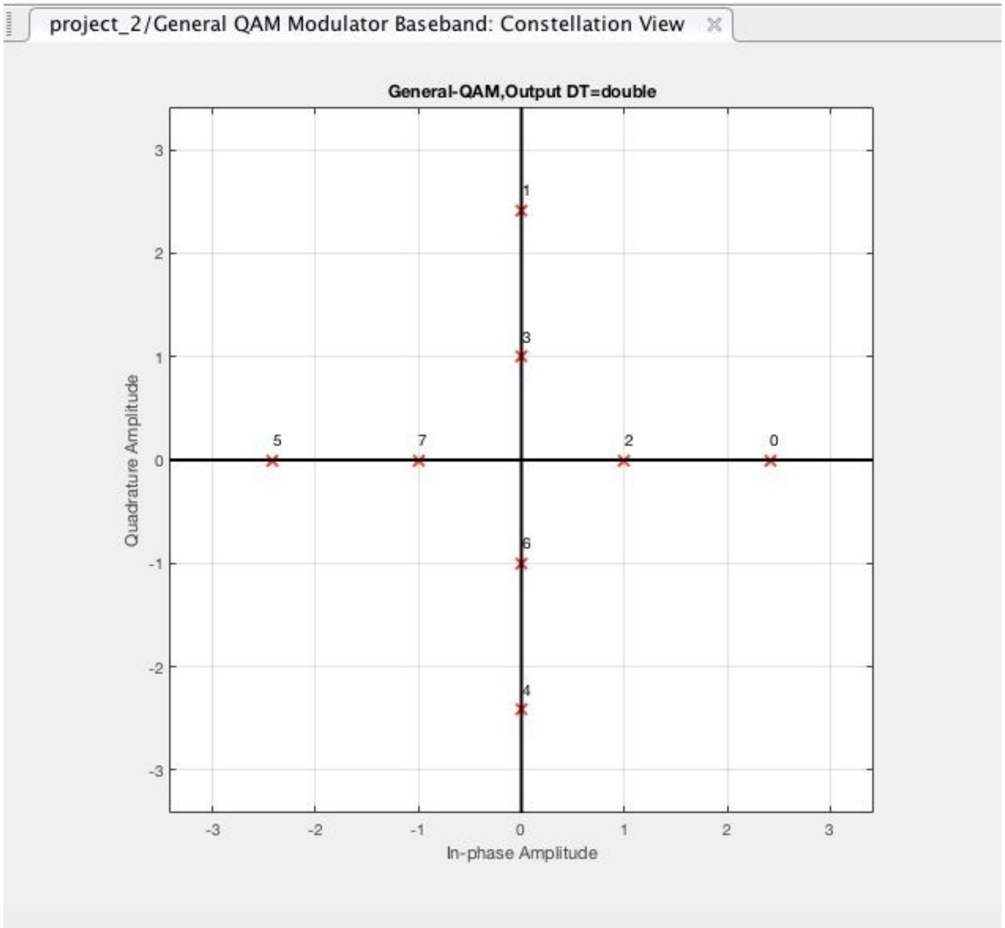


Figure 6: Signal Constellation in "General QAM Modulator Baseband" block

The "General QAM Modulator Baseband" block, after selecting the symbol, produces a complex number representing the baseband quadrature and in-phase components of the symbol in the constellation. This complex number is then split into its real and imaginary components by the next block.

Then, we add White Gaussian Noise to each of the real and imaginary components. This White Gaussian Noise is zero mean and has variance $\sigma^2 = N_o$ because this is a baseband system. This noise is added using the "AWGN Channel" block, which has a parameter that allows the user to explicitly specify the desired Es/No value in dBs or Eb/No value in dBs. These parameters are what are set to run the simulation at different SNR values. (Note: it is important to set the 'seed' parameters on these two AWGN blocks to different values)

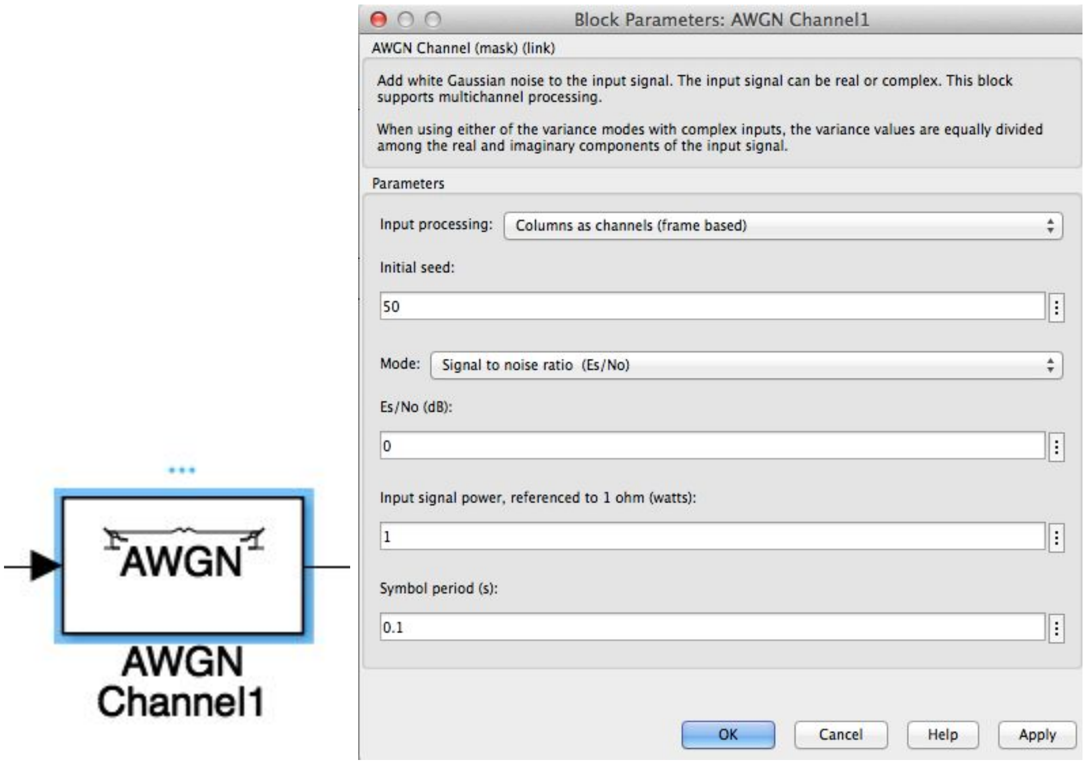


Figure 7: Parameters for the "AWGN Channel" blocks

After adding the noise, we have what will be observed at the receiver, and send these two observations to the Matlab workspace (this is the purpose of the "X1" and "X2" blocks).

Now, the in-phase and quadrature components are re-combined, and set as the input to the "General QAM Demodulator Baseband" block. This block acts as the decision making device, and uses the maximum-likelihood criteria to determine which symbol it believes was sent. (Note: it is important that the signal constellation in this block is exactly the same as the one that was input into the "General QAM Modulator Baseband" block). It then outputs the integer that this symbol corresponds to (from 0 to 7).

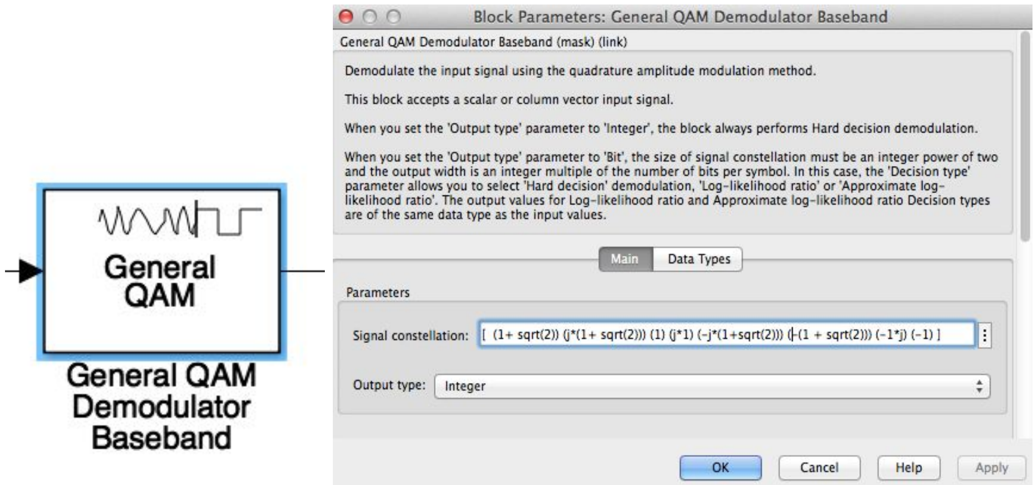


Figure 8: Parameters for the "General QAM Demodulator Baseband" block

Finally, the symbol that was sent can be compared with the symbol that was chosen by the receiver using the "Error Rate Calculation" block. This block is given inputs from the integer that represents the symbol sent and the integer that represents the symbol received. It automatically keeps track of the number of times that these two values do not match up during a simulation and calculates the symbol error rate (SER) and outputs it to the Matlab workspace.

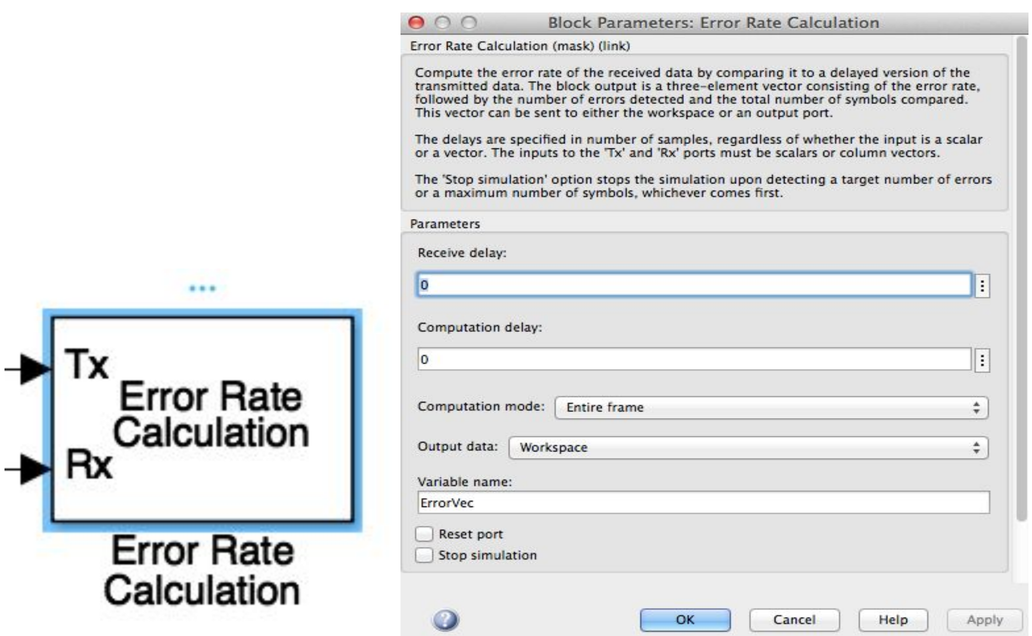
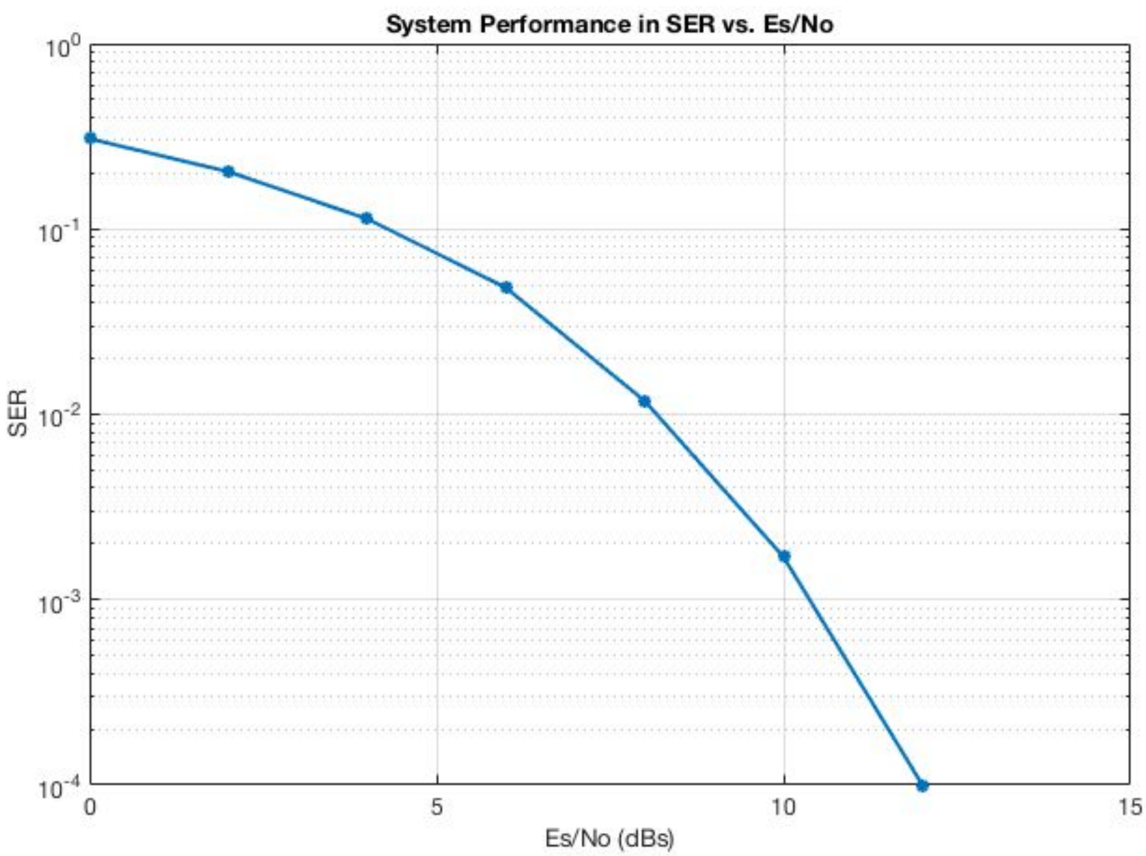
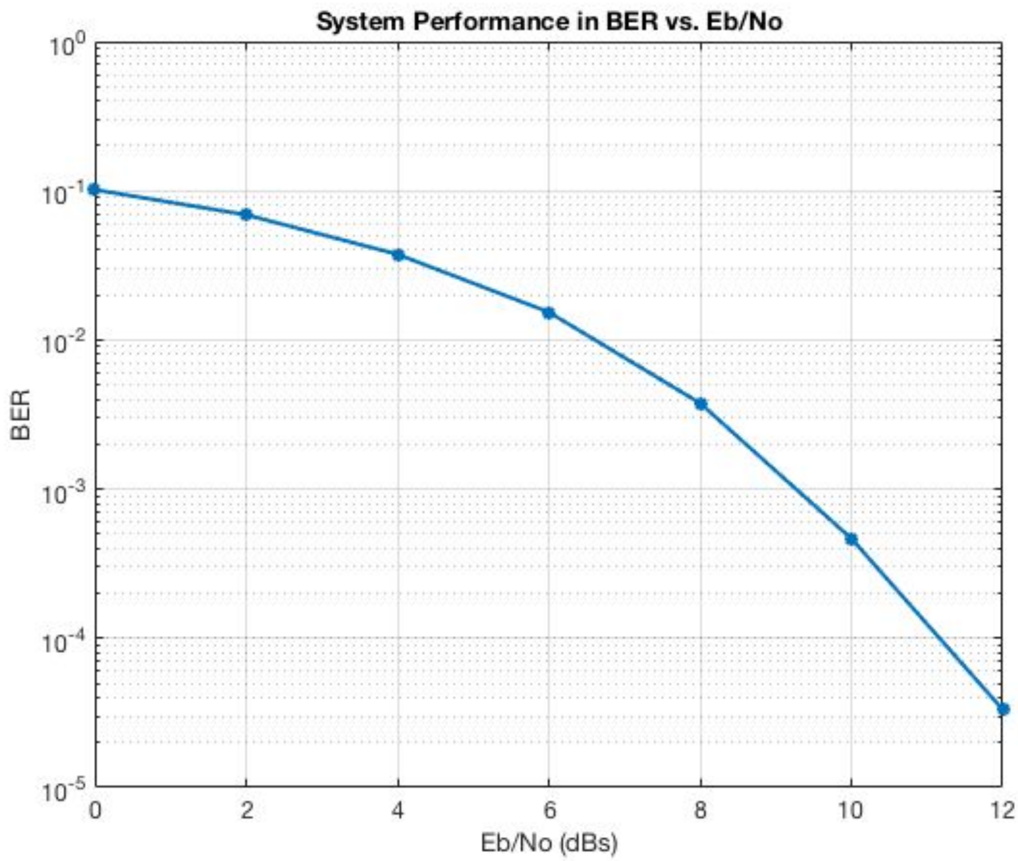


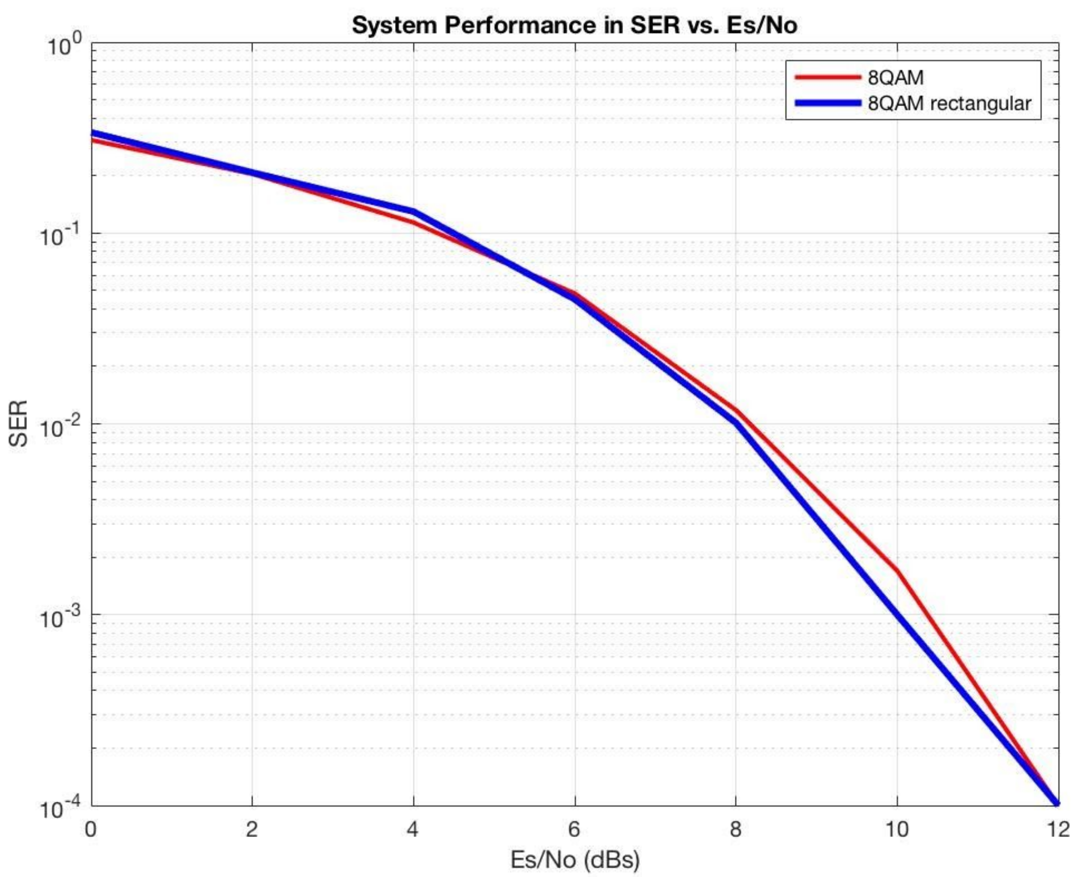
Figure 9: Parameters for the "Error Rate Calculation" block

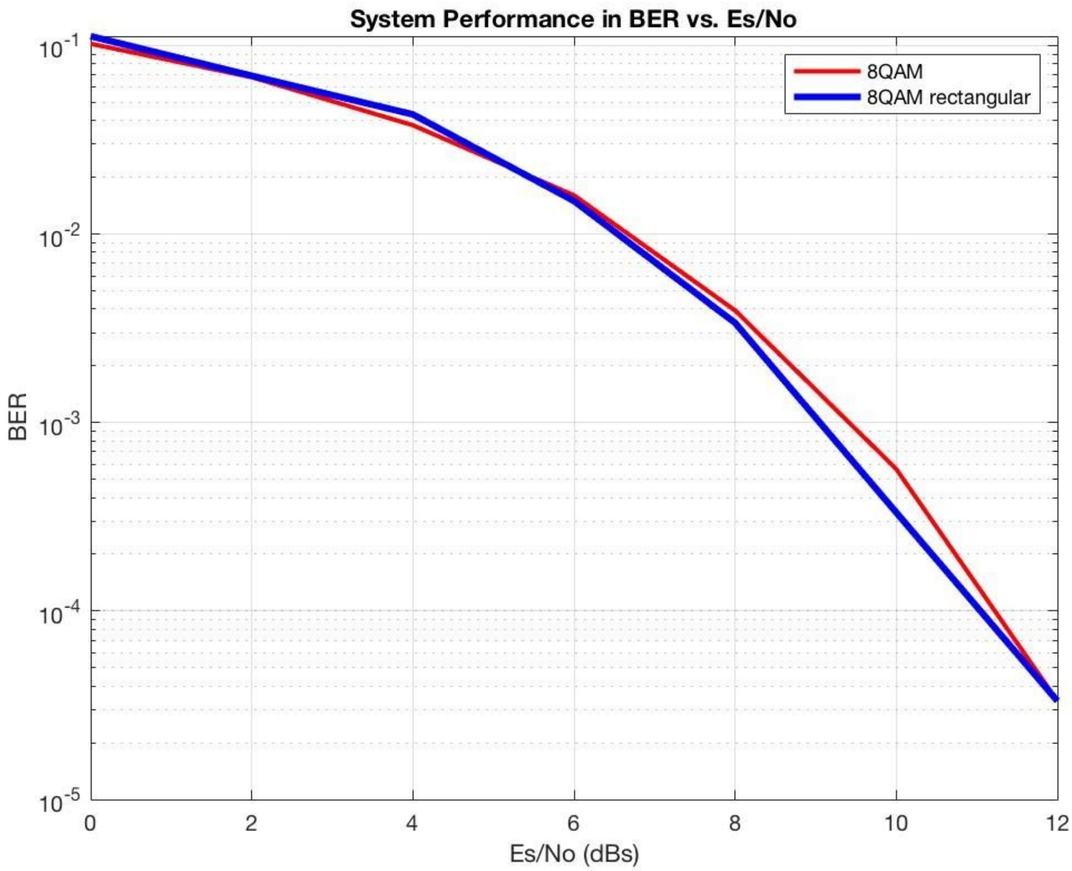
To run the simulation, it is important that all parameters in the blocks referring to “sample time” or similar time related fields be set to the same value. We chose 0.1 for this value as it is the default for most blocks. This means that for every time period = 1 that the simulation is run, 10 symbols will be sent. To obtain our results, we ran the simulation with different Es/No SNR values set in the AWGN blocks, and ensured that at least 10 symbol errors occurred at each SNR. We then plotted the resulting SER vs the SNR used to obtain it. To get BER results, we set the Eb/No field in the AWGN blocks and divided the resulting SER by three (as there are 3 bits per symbol).

5. Simulation results




System Performance in BER vs. Eb/No
BER
Eb/No (dBs)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
2
4
6
8
10
12


System Performance in SER vs. Es/No
8QAM
8QAM rectangular
SER
Es/No (dBs)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
2
4
6
8
10
12



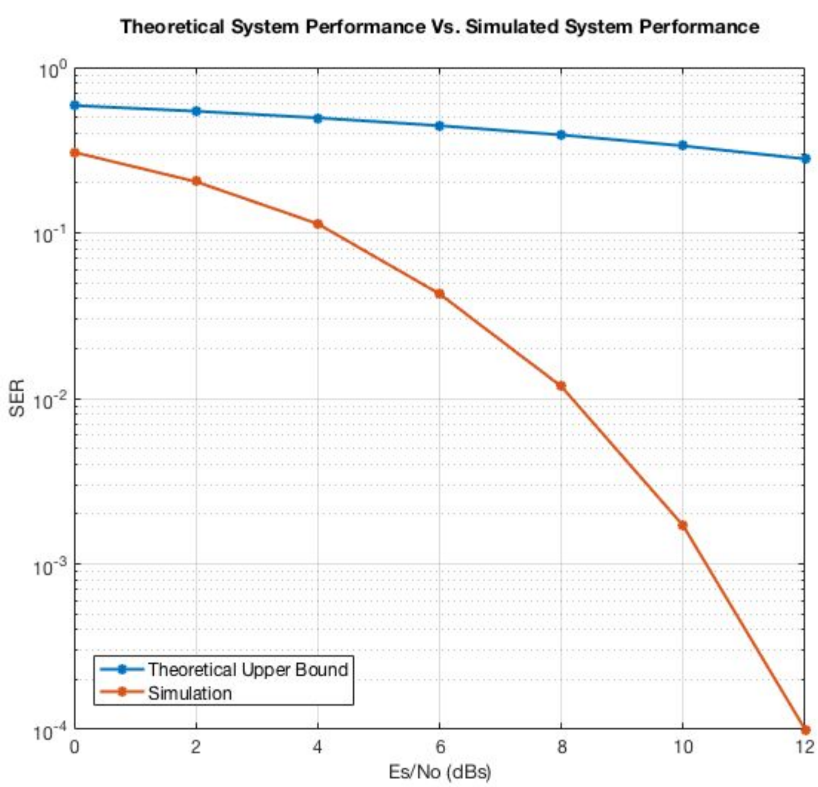
SNR(Es/No)	SER	SER (rectangular)
0dB	0.3070	0.3367
2dB	0.2039	0.2075
4dB	0.1135	0.1295
6dB	0.0482	0.04496
8dB	0.0118	0.01013
10dB	0.0017	0.001
12dB	0.0001	0.0001

6. Discussion of results.

From the graphical results, we can say that the system performance is fairly good, as the system has $BER \simeq 10^{-3}$ around 9dBs. For comparison, the "gold standard" of digital communication, BPSK, achieves $BER \simeq 10^{-3}$ at about 7dBs. Thus, this system must increase its power output by about 50% in order to achieve the same symbol error rate as binary PAM (as there is an increase of about 2dBs).

The given QAM with the constellation has an interesting placement of its signal points (namely, on the axes), so we decided to run a similar simulation with traditional rectangle QAM to compare the performance. As the simulation shows, the rectangular 8-QAM performs a little bit better but not by very much. The simulation results exceeded our expectations. At the beginning, we discussed and expected the given 8-QAM would show lower performance due to the points lying on the axes. However, it seems that the signal constellation points lying on the axes have little to no effect on system performance in this case.

Comparing the performance of the simulated system to the original calculated (theoretical) upper bound on SER, we can see that the calculated upper bound is a true upper bound and perhaps that a tighter upper bound could be achieved. Both the performance of the simulated system and the theoretical upper bound are plotted below:



7. Appendix

```
%**********************************************************************
****
clear;
clc;
SNR = [0 2 4 6 8 10 12];

%SER from simulation given 8-QAM with SNR (Es/No) from 0:2:12 dB
SER = [0.3070 0.2039 0.1135 0.0482 0.0118 0.0017 0.0001];

%SER from simulation of rectangular 8-QAM  with SNR (Es/No) from
0:2:12dB
SER_rec = [0.3367 0.2075 0.1295 0.04496 0.01013 0.001 0.0001];


figure(1)
s = semilogy(SNR,SER,'r',SNR,SER_rec,'b')
s(1).LineWidth = 2;
s(2).LineWidth = 3;
title('System Performance in SER vs. Es/No');
legend('8QAM', '8QAM rectangular');
ylabel('SER');
xlabel('Es/No (dBs)');
grid on;

figure(2)
s2 = semilogy(SNR,SER/3,'r',SNR,SER_rec/3,'b')
s2(1).LineWidth = 2;
s2(2).LineWidth = 3;
title('System Performance in BER vs. Es/No');
legend('8QAM', '8QAM rectangular');
ylabel('BER');
xlabel('Es/No (dBs)');
grid on;


%**********************************************************************
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```