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# CS-E4850 COMPUTER VISION

## Exercise Round 8

### Exercise 1:

a) Run the notebook
b) Run the notebook with different input ('obama')
c)
Most of the features are not tracked very long in part b) probably because the camera rotates and the movement is quite fast. These rotations and fast movements may compromise the key assumptions of optical flow: small motion & spatial coherence.

d) In order to avoid gradually losing the features, one solution is to avoid rapid movements in short time periods so that the model can have enough time to learn new features. In addition to that, trying to minimize the learning time or increasing the processing speed so that it can keep up with the fast movements. Some adjustments to the method of handling outliers for better tracking are also helpful.

### Exercise 2:

+ The paper's equation (10):

$$\Delta p = H^{-1} \sum_x \left[\nabla I \frac{\partial W}{\partial p}\right]^T \left[T(x) - I(W(x;p))\right]$$

In this equation,

$$\Delta p = \begin{bmatrix} u \\ v \end{bmatrix} ; \quad W(x;p) = \begin{bmatrix} x + u \\ y + v \end{bmatrix} ; \quad \frac{\partial W}{\partial p} = \begin{bmatrix} \frac{\partial W_x}{\partial u} & \frac{\partial W_x}{\partial v} \\ \frac{\partial W_y}{\partial u} & \frac{\partial W_y}{\partial v} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore,

$$H = \sum_x \left[\nabla I \frac{\partial W}{\partial p}\right]^T \left[\nabla I \frac{\partial W}{\partial p}\right] = \sum_x \left(\frac{\partial W}{\partial p}\right)^T \nabla I^T \nabla I \frac{\partial W}{\partial p}$$

$$= \sum_x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} \partial I/\partial x \\ \partial I/\partial y \end{bmatrix} \begin{bmatrix} \partial I/\partial x & \partial I/\partial y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix}$$

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Substituting these calculated values into equation (10):

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix}^{-1} \sum_x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_x \\ I_y \end{bmatrix} \left[ T(\mathbf{x}) - I(W(\mathbf{x};p)) \right] \quad \circledast$$

$$\Leftrightarrow \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \sum_x \begin{bmatrix} I_x \\ I_y \end{bmatrix} \left[ T(\mathbf{x}) - I(W(\mathbf{x};p)) \right]$$

Note that the paper's notation of $T(\mathbf{x})$ means $I(x,y,t-1)$

$I(W(x;p))$ means $I(x,y,t)$

from the slides $\rightarrow$ $T(x) - I(W(x;p)) = -I_t$

Hence, equation $\circledast$ can be rewritten as:

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \sum_x \begin{bmatrix} I_x \\ I_y \end{bmatrix} \left[ -I_t \right] = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$