

Memo No. \_\_\_\_\_

Date    /    /

## CS - E485D COMPUTER VISION

### Exercise Round 12

#### Exercise 1.

(Done)

#### Exercise 2.

We have:

$$\begin{cases} P = [I | 0] \\ P' = [R | t] \end{cases}$$

where  $R$  is rotation matrix

$t = [t_1, t_2, t_3]^T$  is translation vector

We have:

$$\vec{O'p'} \cdot (\vec{O'O} \times \vec{Op}) = 0$$

→ Given  $x = (x, y, 1)^T$  and  $x' = (x', y', 1)^T$  as homogeneous image coordinate vectors of  $p$  and  $p'$ :

$$\vec{O'p'} = x'$$

→ From the camera coordinates and translation between camera origins:

$$\vec{Op} = Rx, \quad \vec{O'O} = t$$

Therefore,

$$\vec{Op'} \cdot (\vec{O'O} \times \vec{Op}) = x' \cdot (t \times Rx) = 0$$

$$\Rightarrow x'^T \cdot [t]_x R \cdot x = 0$$

$$\text{We have } E = [t]_x R$$

$$\Rightarrow x'^T \cdot E \cdot x = 0$$

Exercise 3.

a) Assume  $d = 1\text{cm}$ ,  $b = 6\text{cm}$  &  $f = 1\text{cm}$ .

$$d = x - x' = \frac{b \cdot f}{Z_p}$$

$$\Rightarrow Z_p = \frac{b \cdot f}{d} = 6 \text{ (cm)}$$

b)  $d_{\min} = 1 \text{ pixel}$

pixel width =  $0.01 \text{ mm}$

For points with disparity below 1 pixel:

$$d = \frac{b \cdot f}{Z_p} \leq 0.01 \text{ (mm)} = 0.001 \text{ cm}$$

$$\Rightarrow Z_p \geq \frac{b \cdot f}{0.001} = \frac{6}{0.001} = 6000 \text{ cm}$$

c)  $P_l = [I \ 0]$ , where  $I$  is identity matrix

$$P_r = [I \ t]$$

$$t = [-6, 0, 0]^T$$

$$Q(3, 0, 3)$$

We have:

$$E = [t] \times R, \text{ where } R = I \text{ & } t = [-6, 0, 0]^T$$

$$\Rightarrow E = [t] \times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix}$$

$$x = P_l \cdot (3, 0, 3, 1)^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

$$\Rightarrow E \cdot x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}$$