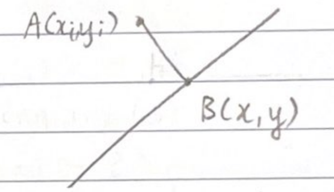
CS-E4850 COMPUTER VISION

Exercise Round 5

Exercise 1:

① Line $l: ax + by - d = 0 \quad (a^2+b^2=1)$
Point $A(x_i, y_i)$, $B(x, y)$ is a point on l s.t $AB \perp l$:



$l: ax + by - d = 0$

We have
$$\left.\begin{aligned} (l)&: y = -\frac{a}{b}x + d \\ l \perp (AB)&: y_i = mx_i + n \end{aligned}\right\}$$
$$\Rightarrow m\left(-\frac{a}{b}\right) = 1 \Rightarrow m = \frac{b}{a}$$

AB goes through $A(x_i, y_i)$ & $B(x, y)$
$$\Rightarrow \frac{y - y_i}{x - x_i} = \frac{b}{a} \Rightarrow y - y_i = \frac{b}{a}(x - x_i) \Rightarrow ay - ay_i - bx + bx_i = 0$$

Therefore, we have:
$$\begin{cases} ax + by - d = 0 \\ -bx + ay - ay_i + bx_i = 0 \end{cases} \quad (*)$$

Solving for $(*)$, we have:
$$\begin{cases} x = \dfrac{b^2x_i - aby_i + ad}{a^2+b^2} \\[2ex] y = \dfrac{a^2y_i - abx_i + bd}{a^2+b^2} \end{cases}$$

$$\Rightarrow |AB| = \sqrt{(x-x_i)^2 + (y-y_i)^2} = \sqrt{\left(\frac{b^2x_i - aby_i + ad}{a^2+b^2} - x_i\right)^2 + \left(\frac{a^2y_i - abx_i + bd}{a^2+b^2} - y_i\right)^2}$$

$$= \sqrt{\left(\frac{b^2x_i - aby_i + ad - a^2x_i - b^2x_i}{a^2+b^2}\right)^2 + \left(\frac{a^2y_i - abx_i + bd - y_ia^2 - b^2y_i}{a^2+b^2}\right)^2}$$

$$= \frac{[-a(ax_i + by_i - d)]^2 + [-b(ax_i + by_i - d)]^2}{(a^2+b^2)}$$

$$= \frac{|ax_i + by_i - d|}{} = |ax_i + by_i - d|$$

(since $a^2 + b^2 = 1$)

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(2)

$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

+) Partial derivative :

$$\frac{\partial E}{\partial d} = \sum_{i=1}^{n} -2(ax_i + by_i - d)$$

+) $\frac{\partial E}{\partial d} = 0 \Rightarrow \sum_{i=1}^{n} -2(ax_i + by_i - d) = 0$

$$\Rightarrow a.\sum_{i=1}^{n} x_i + b.\sum_{i=1}^{n} y_i - nd = 0$$

$$\Rightarrow d = \frac{a}{n}\sum_{i=1}^{n} x_i + \frac{b}{n}\sum_{i=1}^{n} y_i$$

$$= a\bar{x} + b\bar{y} \quad (\bar{x}, \bar{y} = \text{mean of all } x, y \text{, respectively})$$

(3)

$$E = \sum_{i=1}^{n} (ax_i + bx_i - d)^2 = \sum_{i=1}^{n} (ax_i + by_i - a\bar{x} - b\bar{y})^2$$

$$= \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}))^2$$

$$= \left\| \begin{bmatrix} x_1 - \bar{x} & y_i - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2$$

$$= (U(a,b)^T)^T (U(a,b)^T)$$

$$= (a,b) U^T U (a,b)^T$$