(*) Prove $\sum_{i=1}^{n} 2 X_i \overline{X}_n = 2n \overline{X}_n^2$

<u>Solution:</u>

$$\sum_{i=1}^{n} 2 X_i \overline{X}_n \overset{1}{=} \sum_{i=1}^{n} 2 \overline{X}_n X_i$$

$$\overset{2}{=} 2\overline{X}_n X_1 + 2\overline{X}_n X_2 + \dots + 2\overline{X}_n X_n$$

$$\overset{3}{=} 2\overline{X}_n (X_1 + X_2 + \dots + X_n)$$

$$\overset{4}{=} 2\overline{X}_n \sum_{i=1}^{n} X_i$$

$$\overset{5}{=} 2\overline{X}_n \cdot n\overline{X}_n$$

$$\overset{6}{=} 2n\overline{X}_n^2$$

Notice that: 5 because $\sum_{i=1}^{n} X_i = n\overline{X}_n$

or $\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$

by definition

2 and 3 are implicit steps in solution 3a.

Here $\overline{X}_n$ is a constant and we apply distributive property. /

⊛ You wonder:

$$\underbrace{\sum_{i=1}^{n} 2\bar{X}_n X_i}_{LHS} = \underbrace{2n\bar{X}_n \sum_{i=1}^{n} X_i}_{RHS} \quad ?$$

$$RHS = 2n\bar{X}_n (X_1 + X_2 + \dots + X_n)$$

$$= 2n\bar{X}_n X_1 + 2n\bar{X}_n X_2 + \dots + 2n\bar{X}_n X_n$$

$$\neq 2\bar{X}_n X_1 + 2\bar{X}_n X_2 + \dots + 2\bar{X}_n X_n = LHS$$

$\longrightarrow$ Incorrect.