Introductory Statistics for Economics

ECON1013: TUTORIAL 3

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Intro

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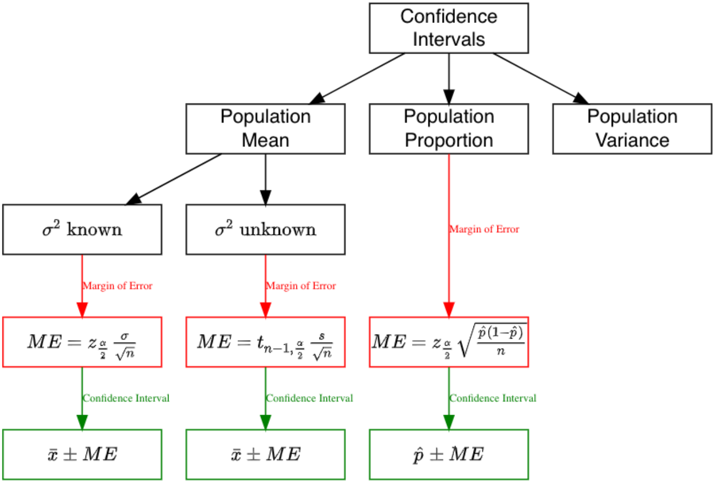
  - Email: Duong.Trinh@glasgow.ac.uk
- ECON1013-TU04
  - Monday 12-1 pm
  - 4 sessions (22-Jan, 5-Feb, 19-Feb, 4-March)
- ECON1013-TU05
  - Tuesday 12-1 pm
  - 4 sessions (23-Jan, 6-Feb, 20-Feb, 5-March)
- ECON1013-TU07
  - Tuesday 2-3 pm
  - 4 sessions (23-Jan, 6-Feb, 20-Feb, 5-March)

Record Attendance

BRIEF REVIEW

BRIEF REVIEW



Exercise 1

Redfield & Wilton Strategies

- Population Sampled: Eligible Voters in Scotland.
- Sample Size: $1,054$
- 51% of survey respondents now say they would vote "`No`".

(a) Compute the margin of error (ME) of the proportion saying "`No`" at the 95% confidence level.

(b) Compute the confidence interval at the 95% level.

(c) Suppose I read from somewhere that the ME of that same survey is 2.5%. Is this ME at a higher or at a lower confidence level?

Exercise 1

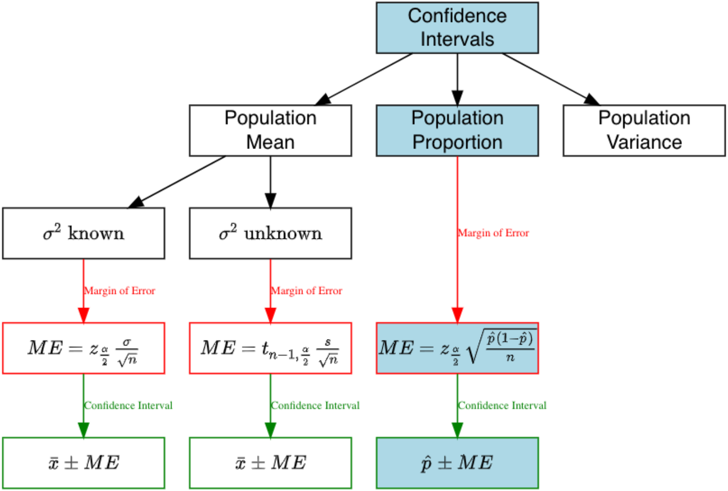
Ipsos

- Population Sampled: Eligible Voters in Scotland.
- Sample Size: $1,004$
- 54% of voters back “Yes” $\rightarrow$ The fraction for “No” in the Ipsos poll is 0.46.

(d) Compute the margin of error (ME) of the proportion saying “No” at the 95% confidence level.

(e) What is your view on the differences between the two polls?

Where we currently are



(a) Compute the margin of error (ME) of the proportion saying "`No`" at the 95% confidence level.

The formula for the margin of error for a population proportion $\hat{p}$ is given by

$$ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- $z_{\frac{\alpha}{2}}$: the value which cuts off $\left(\frac{\alpha}{2}\right) \cdot 100\%$ of the probability mass in the right tail for a variable following a standard normal distribution.
  - Because the confidence level is $1 - \alpha = 0.95$, we have that $\alpha = 0.05$ and $\frac{\alpha}{2} = 0.025$.
- Compute $z_{0.025}$
  - Looking at a statistical table: $z_{0.025} = 1.96$, OR
  - Using `Excel` function: $= \text{NORM.INV}(0.975, 0, 1)$

(a) Compute the margin of error (ME) of the proportion saying "`No`" at the 95% confidence level.

Plugging in $z_{0.025} = 1.96$, the sample size $n = 1054$ and the sample proportion $\hat{p} = 0.51$, we find

$$ME = 1.96\sqrt{\frac{0.51(1-0.51)}{1054}} = 1.96 \cdot 0.0153 \approx 0.03$$

$\Rightarrow$ The margin of error is approximately 3% at the 95% confidence level.

(b) Compute the confidence interval at the 95% level.

The confidence interval is given by

$$\hat{p} \pm ME$$

0.51 ± 0.03 gives the interval $[0.48, 0.54]$.

(c) Suppose I read from somewhere that the ME of that same survey is 2.5%. Is this ME at a higher or at a lower confidence level?

- Confidence level $= 1 - \alpha$

  α is smaller $\Leftrightarrow$ Confidence level is higher.

- $ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

  α is smaller $\Leftrightarrow$ $z_{\frac{\alpha}{2}}$ is ... $\Leftrightarrow$ ME is ....

If we compare $z_{\frac{\alpha}{2}}$ for different values of α, we see the following:

$z_{\frac{0.10}{2}} = 1.64$,

$z_{\frac{0.05}{2}} = 1.96$,

$z_{\frac{0.01}{2}} = 2.58$,

We see that when α is smaller, $z_{\frac{\alpha}{2}}$ is bigger.

(c) Is this $ME = 2.5\%$ at a higher or at a lower confidence level?

- Confidence level $= 1 - \alpha$

$$\alpha \text{ is smaller} \Leftrightarrow \text{Confidence level is higher.} \tag{1}$$

- $ME = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$\alpha \text{ is smaller} \Leftrightarrow z_{\frac{\alpha}{2}} \text{ is bigger} \Leftrightarrow \text{ME is larger.} \tag{2}$$

(c) Is this $ME = 2.5\%$ at a higher or at a lower confidence level?

- Confidence level $= 1 - \alpha$

  α is smaller $\Leftrightarrow$ Confidence level is higher. (1)

- $ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

  α is smaller $\Leftrightarrow$ $z_{\frac{\alpha}{2}}$ is bigger $\Leftrightarrow$ ME is larger. (2)

- From (1) and (2)

  ME is larger $\Leftrightarrow$ Confidence level is higher.

(c) Is this $ME = 2.5\%$ at a higher or at a lower confidence level?

- Confidence level $= 1 - \alpha$

  α is smaller $\Leftrightarrow$ Confidence level is higher. (1)

- $ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

  α is smaller $\Leftrightarrow$ $z_{\frac{\alpha}{2}}$ is bigger $\Leftrightarrow$ ME is larger. (2)

- From (1) and (2)

  ME is larger $\Leftrightarrow$ Confidence level is higher.

Since we found ME $= 3\%$ at the 95% confidence level, when ME is 2.5% (lower), the confidence level has to be **lower**.

(d) Compute the margin of error (ME) of the proportion saying “`No`” at the 95% confidence level.

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- The margin of error

$$ME = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\Rightarrow ME = 1.96\sqrt{\frac{0.46(1-0.46)}{1004}} \approx 0.031$$

- The 95% confidence level is approximately $[0.429, 0.491]$.

(e) What is your view on the differences between the two polls?

(e) What is your view on the differences between the two polls?

- The confidence intervals are clearly different: $[0.429, 0.491]$ is different from $[0.48, 0.54]$, and in this sense, the estimates do differ.
- However, at the same time, the intervals overlap (for example, 49% is contained in both), so they are not necessarily making different statements about the population parameter.
- **Remark**: There exist statistical tests to determine whether *two sample averages* are different from each other in a statistically significant way. We will learn about statistical testing procedures in Unit 5.

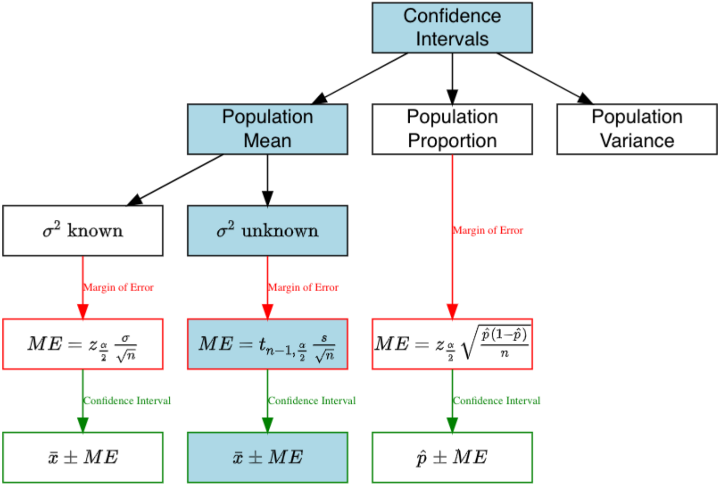
Exercise 2

The government introduces a housing benefit of 100 pounds per month for low-income households.

After the introduction of the policy, we collect a sample of rents paid by 31 low-income households: $(r_1, r_2, \ldots, r_{31})$. In this sample, the sample average rent is $\bar{r} = 709$ pounds. In this sample, the standard deviation is $s_r = 34$.

(a) Construct the margin of error for $\bar{r}$.
(b) Construct the 95% confidence interval for $\bar{r}$.
(c) We know that the average rent paid by low-income households, prior to the reform, was 700 pounds. Is this value, 700, still a likely population rent after the reform, based on the sample?

Where we currently are



(a) Construct the margin of error for $\bar{r}$.

The margin of error of the sample mean for a sample of size n, when the population standard deviation is not known, is given by

$$ME = t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

- $t_{n-1,\frac{\alpha}{2}}$: the value which cuts off $(\frac{\alpha}{2}) \cdot 100\%$ of the probability mass in the right tail for a variable following a t-distribution with $n-1$ degrees of freedom.
  - We use the Student's t distribution with $n-1$ degrees of freedom.
  - Because $1-\alpha = 0.95$, we have that $\alpha = 0.05$ and $\frac{\alpha}{2} = 0.025$.
- Calculate $t_{30,0.025}$:
  - Looking at a statistical table: $t_{30,0.025} = 2.042$, OR
  - Using `Excel` function: $= \text{T.INV}(0.975, 30)$

(a) Construct the margin of error for $\bar{r}$.

Plugging in $t_{30,0.025} = 2.042$, the sample size $n = 31$ and the sample standard deviation $s = 34$, we find

$$ME = 2.042 \times \frac{34}{\sqrt{31}} \approx 12.47$$

(b) Construct the 95% confidence interval for $\bar{r}$.

(b) Construct the 95% confidence interval for $\bar{r}$.

$\bar{r} = 709 \Rightarrow \bar{r} \pm 12.47$ give the confidence interval $[696.5, 721.5]$.

(c) We know that the average rent paid by low-income households, prior to the reform, was 700 pounds. Is this value, 700, still a likely population rent after the reform, based on the sample?

(c) We know that the average rent paid by low-income households, prior to the reform, was 700 pounds. Is this value, 700, still a likely population rent after the reform, based on the sample?

700 is contained in the 95% confidence interval. Therefore we don't have very strong evidence suggesting that 700 is not a possible value for the population mean after the reform, at the 95% confidence level.

EXCEL NOTE

Relevant functions (I)

Launch the **Excel** online

https://www.office.com/launch/excel?auth=2

NORM.INV() To return the inverse of the normal cumulative distribution for the specified mean and standard deviation (*real number*).

```
= NORM.INV(probability,mean,standard_dev)
```

T.INV() To return the t-value of the Student's t-distribution as a function of the probability and the degrees of freedom (*real number*).

```
= T.INV(probability,degrees_freedom)
```

Relevant functions (II)

Launch the **Excel** online

https://www.office.com/launch/excel?auth=2

NORMSDIST() To return the standard normal cumulative distribution (*probability*).

```
= NORMSDIST(z)
```