Tutorial 4

1. As part of a “Math for the Twenty-First Century" initiative, Bayview High was chosen to participate in the evaluation of a new algebra and geometry curriculum. In the recent past, Bayview’s students were considered “typical", having earned scores on standardized exams that were very consistent with national averages.

   Two years ago, a cohort of eighty-six Bayview students, all randomly selected, were assigned to a special set of classes that integrated algebra and geometry. According to test results that have just been released, those students averaged 502 on the math exam; nationwide seniors averaged 494 with a standard deviation of 124.

   (a) Can it be claimed at 5% significance level that the new curriculum had effect? Justify your answer.

   (b) Compute the p-value associated with the test statistics. How should it be interpreted?

   Short Solution

   (a) Given the sample size and the sampling scheme, the sample average is asymptotically normally distributed. $\frac{502-494}{124/\sqrt{n}} = 0.5983$ is lower than 1.96, so we don’t reject the null (even at 10% significance level).

   (b) Looking at the statistical tables

   $$P\left(Z \leq 0.5983\right) \simeq 0.7257$$

   Then the p-value is $2(1-0.7257) = 0.5486$. The large p-value should be interpreted as evidence against the rejection of the null hypothesis. We would reject the null hypothesis for unusual large $\alpha > 0.5486$.

2. Supporters claim that a new windmill can generate an average of at least 800 kilowatts of power per day. Daily power generation for the windmill is assumed to be normally distributed with a standard deviation of 120 kilowatts. A simple random sample of 100 days is taken to test this claim against the alternative hypothesis that the true mean is less than 800 kilowatts. The claim will not be rejected if the sample mean is 776 kilowatts or more and rejected otherwise.

   (a) What is the probability α of a Type I error using the decision rule if the population mean is, in fact, 800 kilowatts per day?

   (b) What is the probability β of a Type II error using this decision rule if the population mean is, in fact, 740 kilowatts per day?

   (c) Suppose that the same decision rule is used, but with a sample of 200 days rather than 100 days.

i. Would the value of α be larger than, smaller than, or the same as that found in part (a)? Explain.

ii. Would the value of β be larger than, smaller than, or the same as that found in part (b)? Explain.

Short Solution See the solution of Exercise 9.58 Textbook (see Moodle, Unit 5).