Introductory Statistics for Economics

ECON1013: LAB 2

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Picture the Scenario

- **Objective:**
  - illustrate how the statistic $\bar{X}_n$ varies with repeated samples
  - describe what might happen if we repeat the entire sampling process and compute the sample average again and again.
  - illustrate the CLT and the LLN
- **Implementation:**
  - We use a *pseudo-random number generator*, already built in R, to simulate the samples. Simulations allow the user to perform experiments and answer questions in a rapid manner.

PRELIMINARIES

The normal distribution

For each distribution, R provides functions that allow users calculate value of: quantile, probability and generate random variable of the distribution.

- To calculate the *probability* of $P(X \leq 0.05)$ where $X \sim N(0, 1)$:

```
pnorm(q = 0.05, mean = 0, sd = 1)
```

- To calculate the *probability* of $P(X \leq 1)$ where $X \sim N(4, 3^2)$:

```
pnorm(q = 1, mean = 4, sd = 3)
```

The normal distribution

For each distribution, R provides functions that allow users calculate value of: quantile, probability and generate random variable of the distribution.

- To calculate the *quantile* at $p = 0.05$ of $N(0, 1)$:

```
qnorm(p = 0.05,mean = 0,sd = 1)
```

- To calculate the *quantile* at $p = 0.05$ of $N(4, 3^2)$:

```
qnorm(p = 0.05,mean = 4,sd = 3)
```

The normal distribution

For each distribution, R provides functions that allow users calculate value of: quantile, probability and generate random variable of the distribution.

- To calculate value of *density function* of $N(0, 1)$ at 0.4:

```r
dnorm(x = 0.4, mean = 0, sd = 1)
```

- To calculate value of *density function* of $N(4, 3^2)$ at 0.4:

```r
dnorm(x = 0.4, mean = 4, sd = 3)
```

The normal distribution

For each distribution, R provides functions that allow users calculate value of: quantile, probability and generate random variable of the distribution.

- To generate randomly 1000 numbers from $N(0, 1)$:

```
R <- rnorm(n = 1000, mean = 0, sd = 1)
head(R)
length(R)
summary(R)
```

Loops (for) and conditional expressions (if)

- Create the vector GENDER where the entries are either `male` or `female`

```
GENDER = c('male', 'male', 'female', 'male', 'female')
```

- Define the random variable X taking value 1 if the individual is a male, and 0 if a female.

```
X=numeric(0)
for (i in 1:length(GENDER)) {
  if (GENDER[i] == 'male') {
    X[i] = 1
  } else {
    X[i]=0
  }
}
```

Exercise 1: The Central Limit Theorem

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- Simulate 10,000 samples of size n from an exponential distribution with mean $\mu = 10$. Consider different sample sizes, e.g. $n = 10, 30, 500$.
- For each sample, compute the standardized sample average:

$$\tilde{X} = \frac{\bar{X} - \mathbb{E}(\bar{X})}{\sigma_{\bar{x}}}$$

- Compute the empirical quantiles of the sample distribution of $\tilde{X}$. Compare the results with the theoretical quantiles of the standard normal random variable Z.
- Compute quantile z satisfying $P(Z < z) = 0.975$. Calculate the percentage of values of $\tilde{X}$ greater than z.

Comment on your results.

Exercise 1: The Central Limit Theorem

Specify the parameters of the experiment

```
# "n" is the sample size. Try n = 10, 30, 500
n = 10
# "R" is the number of replications
R = 10000
# Parameter exponential
mu = 10 # mean
```

To reproduce results, setting a seed in R means to initialize a pseudorandom number generator

```
set.seed(12) # take an (arbitrary) integer
```

Exercise 1: The Central Limit Theorem

Simulate R samples of size n from an exponential distribution with mean $\mu = 10$ and compute the standardized sample average for each sample:

$$Y = \frac{\bar{X} - \mathbb{E}(\bar{X})}{\sigma_{\bar{x}}}$$

```
# Create vector Y (pre-allocation)
Y = numeric(0)
# Loop through R replications
for (i in 1:R) {
  X = rexp(n,1/mu) # The sample will change for every "i"
  # Compute standardized sample average
  Y[i] = (mean(X)-mu)/(mu/sqrt(n))
}
```

Exercise 1: The Central Limit Theorem

To assess the how well the sampling distribution of X is approximated by the standard normal we compare the quantile of of X and the quantile for a standard normal distribution.

Exercise 1: The Central Limit Theorem

```
# Empirical quantiles
quantile(Y)
```

```
##         0%        25%        50%        75%       100%
## -2.6105220 -0.7338397 -0.1024090  0.5983728  5.6335527
```

```
# Population quantiles of a standard normal distribution
qnorm(p = 0.25, mean = 0, sd = 1)
```

```
## [1] -0.6744898
```

```
qnorm(p = 0.5, mean = 0, sd = 1)
```

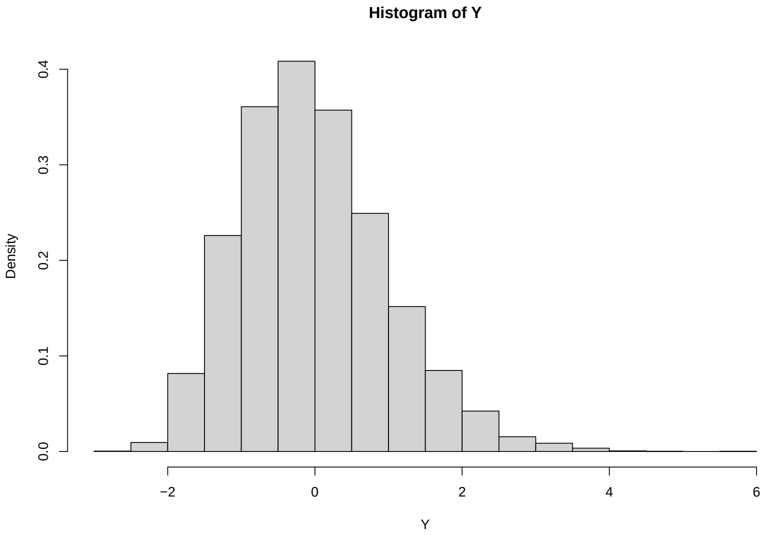
```
## [1] 0
```

```
qnorm(p = 0.75, mean = 0, sd = 1)
```

```
## [1] 0.6744898
```

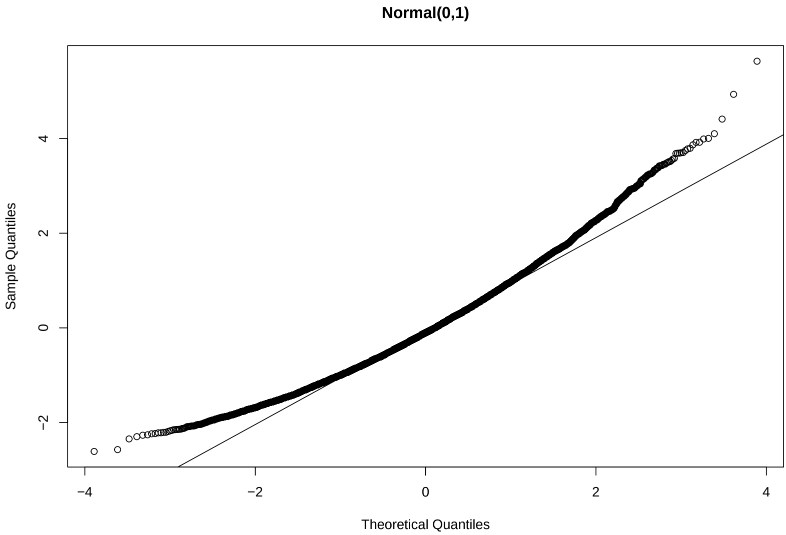
Exercise 1: The Central Limit Theorem

```
hist(Y, prob = T)
```



Exercise 1: The Central Limit Theorem

```
qqnorm(Y,main='Normal(0,1)');qqline(Y) #normal plots
```



Exercise 1: The Central Limit Theorem

```
# Compute the quantile z satisfying P(Z<z) = 0.975
z = qnorm(p = 0.975, mean = 0, sd = 1)
z
```

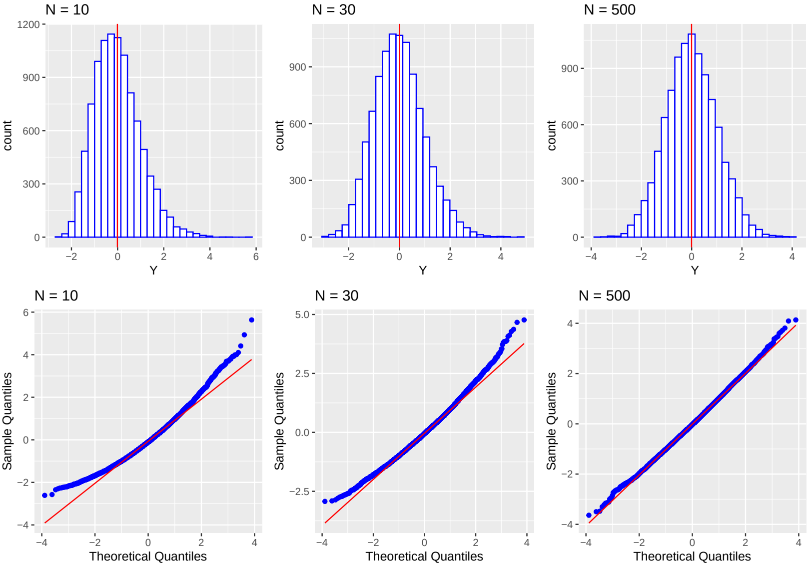
```
## [1] 1.959964
```

```
# The percentage of values of Y greater than z
sum(Y>z)/R*100
```

```
## [1] 3.79
```

If the sample is well approximated by a standard normal distribution, the realization of Y should be greater than z : $(1 - 0.975 * R) = 250$ times, or 2.5% of the times.

Exercise 1: The Central Limit Theorem



The Central Limit Theorem - Reflection

The distribution of the sample average $\bar{X}_n$ will be approximately normal as long as the sample size n is large enough.

Exercise 2: The Law of Large Numbers

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- Compute the sample averages

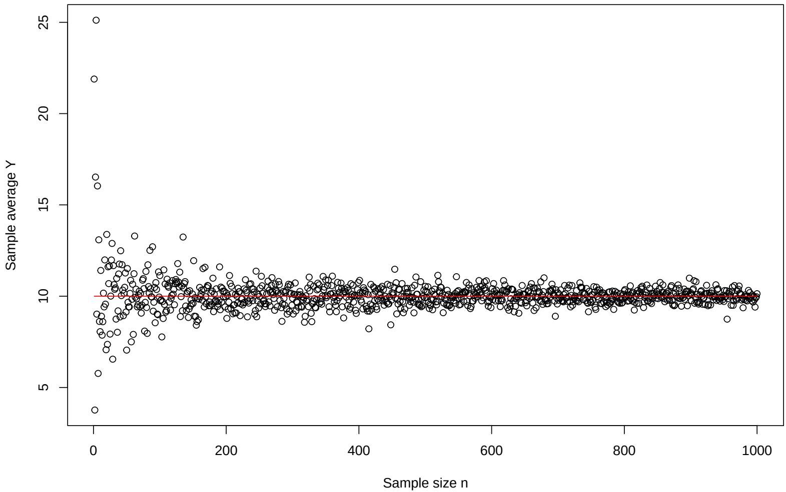
$$\bar{x}_n = \frac{\sum_{i=1}^{n} x_i}{n}$$

  for n draws from the exponential distribution considered in Exercise 1, for $n = 1, 2, \ldots, 1000$.
- Plot the sample averages. What do you conclude about the variability of the sample average?

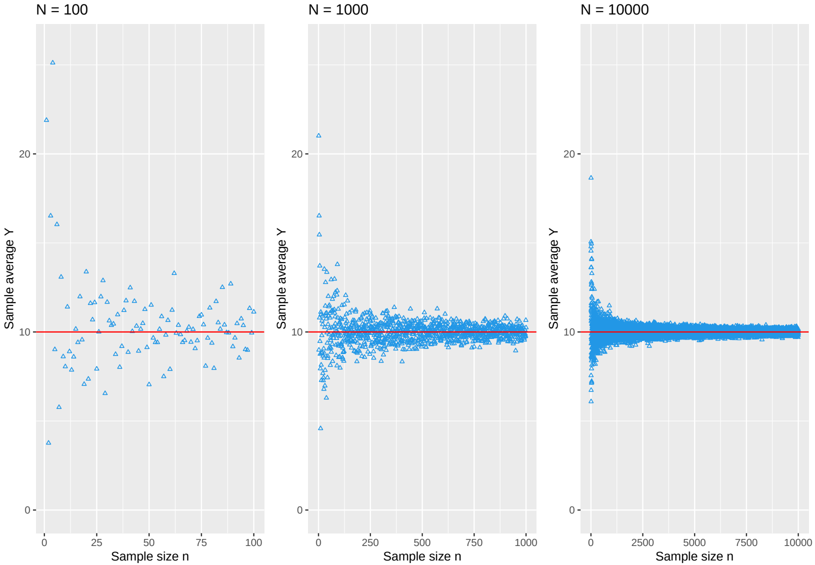
Exercise 2: The Law of Large Numbers

```r
set.seed(12) # to reproduce results
N = 1000 # maximum sample size
mu = 10 # parameter exponential
Y = numeric(0) # pre-allocation
for (i in 1:N) {
  X = rexp(i,1/mu) # 'i' draws from Exp(1/mu)
  Y[i] = mean(X) # sample average
}
plot(Y, xlab = "Sample size n", ylab = "Sample average Y")
lines(rep(mu,N), col = 'red')
```

Exercise 2: The Law of Large Numbers



Exercise 2: The Law of Large Numbers



The Law of Large Numbers - Reflection

Given a random sample of size n from a population, the sample average $\bar{X}_n$ will approach the population mean μ as the sample size n becomes large.