Tutorial 3

1. In the mid-term elections of 2006, the political winds were shifting. One of the key races for control of the Senate was in Virginia, where challenger Jim Webb and incumbent George Allen were in a very tight race. Just a week before the election, the Associated Press reported on a CNN poll based on telephone interviews of 597 registered voters who identified themselves as likely to vote. Webb was the choice of 299 of those surveyed. The article went on to state, "Because Webb's edge is equal to the margin error of plus or minus 4 percentage point, it means that he can be considered slightly ahead"[1].

   (a) The article stated that the margin error was 4%. Which is the significance level?

   (b) Compute the confidence interval. Would you agree with that statement that Webb is slightly ahead?

   (c) What is the interpretation of the margin error?

   (d) Would you consider consecutive polls as samples from the same population? Justify your answer.

   Solution

   (a) $0.04 = c_{\alpha/2}\sqrt{\frac{0.25}{597}}$. Then $c_{\alpha/2} = 1.9547$. $P(Z > 1.9547) \simeq 0.975$.

   (b) $\hat{p} = 299/597$, hence $\hat{p} \pm 0.04$ gives the interval $(0.4608, 0.5408)$. No statistical reason to say Webb is slightly ahead, interval symmetric around parity ($\hat{p} \simeq 0.5$).

   (c) The ME refers to the sample variability, that is it reflects the extent to which $\hat{p}$ varies if repeated sample of the same size are drawn from the same population.

   (d) No, the opinions can change between one poll and the other (different elections) because the opinion of the voting population can be affected the scandals, reputation of the candidate, position of the candidates on important events, etc.

2. A random sample of size 2, Y_1 and Y_2 is drawn from the the pdf

$$f(y;\theta) = 2y\theta^2, \qquad 0 < y < \frac{1}{\theta}$$

   (a) What must c be equal if the statistic

$$c(Y_1 + 2Y_2)$$

   to be an unbiased estimator of $1/\theta$?

   (b) Find an alternative unbiased estimator for $1/\theta$.

   Solution

[1] Adapted from Lars and Morris (2014), *Introduction to Mathematical Statistics and Its Applications*.

(a) Note that

$$\mathbb{E}(Y) = 2\theta^2 \int_0^{1/\theta} y^2 dy = \frac{2\theta^2}{3} \left[y^3\right]_0^{1/\theta} = \frac{2}{3\theta}$$

By the linearity of the expectation

$$\mathbb{E}\left[c(Y_1 + 2Y_2)\right] = c\left(\frac{2}{3\theta} + \frac{4}{3\theta}\right) = \frac{2c}{\theta}$$

Hence, $c = 1/2$.

(b) Recalling that $\mathbb{E}(Y) = 2/(3\theta)$, and unbiased estimator is

$$\frac{3}{4}\left(Y_1 + Y_2\right)$$