

# Introductory Statistics for Economics

## ECON1013: TUTORIAL 3

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# Intro

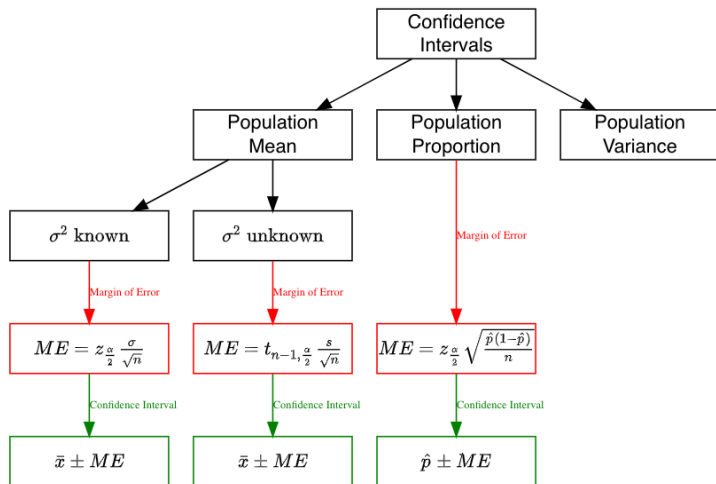
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- ◇ ECON1013 TU07
  - ◇ Tuesday 2-3 pm
  - ◇ 4 sessions (28-Jan, 11-Feb, 25-Feb, 11-March)
  
- ◇ ECON1013-TU08
  - ◇ Tuesday 3-4 pm
  - ◇ 4 sessions (28-Jan, 11-Feb, 25-Feb, 11-March)

## Record Attendance

## BRIEF REVIEW

# BRIEF REVIEW

## Warning: package 'knitr' was built under R version 4.2.3



# Exercise 1

## Redfield & Wilton Strategies

- ◇ Population Sampled: Eligible Voters in Scotland.
  - ◇ Sample Size: 1,054
  - ◇ 51% of survey respondents now say they would vote “No”.
- 
- (a) Compute the margin of error (ME) of the proportion saying “No” at the 95% confidence level.
  - (b) Compute the confidence interval at the 95% level.
  - (c) Suppose I read from somewhere that the ME of that same survey is 2.5%. Is this ME at a higher or at a lower confidence level?

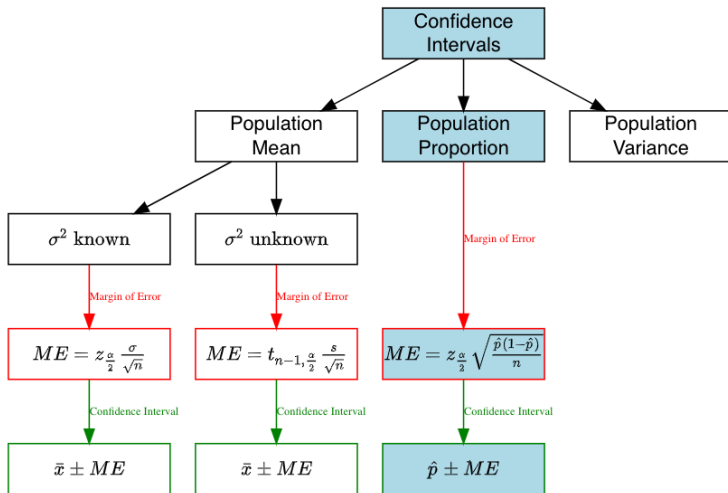
# Exercise 1

## Ipsos

- ◇ Population Sampled: Eligible Voters in Scotland.
- ◇ Sample Size: 1,004
- ◇ 54% of voters back “Yes” → The fraction for “No” in the Ipsos poll is 0.46.

- (d) Compute the margin of error (ME) of the proportion saying “No” at the 95% confidence level.
- (e) What is your view on the differences between the two polls?

# Where we currently are





(a) Compute the margin of error (ME) of the proportion saying “No” at the 95% confidence level.

The formula for the margin of error for a population proportion  $\hat{p}$  is given by

$$ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

- ◇  $z_{\frac{\alpha}{2}}$ : the value which cuts off  $(\frac{\alpha}{2}) \cdot 100\%$  of the probability mass in the right tail for a variable following a standard normal distribution.
  - ◇ Because the confidence level is  $1 - \alpha = 0.95$ , we have that  $\alpha = 0.05$  and  $\frac{\alpha}{2} = 0.025$ .
- ◇ Compute  $z_{0.025}$ 
  - ◇ Looking at a statistical table:  $z_{0.025} = 1.96$ , OR
  - ◇ Using Excel function: `=NORM.INV(0.975,0,1)`

(a) Compute the margin of error (ME) of the proportion saying “No” at the 95% confidence level.

Plugging in  $z_{0.025} = 1.96$ , the sample size  $n = 1054$  and the sample proportion  $\hat{p} = 0.51$ , we find

$$ME = 1.96 \sqrt{\frac{0.51(1 - 0.51)}{1054}} = 1.96 \cdot 0.0153 \approx 0.03$$

$\Rightarrow$  The margin of error is approximately 3% at the 95% confidence level.

(b) Compute the confidence interval at the 95% level.

The confidence interval is given by

$$\hat{p} \pm ME$$

$0.51 \pm 0.03$  gives the interval  $[0.48, 0.54]$ .

(c) Suppose I read from somewhere that the ME of that same survey is 2.5%. Is this ME at a higher or at a lower confidence level?

◇ Confidence level =  $1 - \alpha$

$\alpha$  is smaller  $\Leftrightarrow$  Confidence level is higher.

◇  $ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\alpha$  is smaller  $\Leftrightarrow z_{\frac{\alpha}{2}}$  is ...  $\Leftrightarrow$  ME is ....

If we compare  $z_{\frac{\alpha}{2}}$  for different values of  $\alpha$ , we see the following:

$$z_{\frac{0.10}{2}} = 1.64,$$

$$z_{\frac{0.05}{2}} = 1.96,$$

$$z_{\frac{0.01}{2}} = 2.58,$$

We see that when  $\alpha$  is smaller,  $z_{\frac{\alpha}{2}}$  is bigger.

(c) Is this  $ME = 2.5\%$  at a higher or at a lower confidence level?

◇ Confidence level  $= 1 - \alpha$

$\alpha$  is smaller  $\Leftrightarrow$  Confidence level is higher. (1)

◇  $ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\alpha$  is smaller  $\Leftrightarrow z_{\frac{\alpha}{2}}$  is bigger  $\Leftrightarrow$  ME is larger. (2)

(c) Is this  $ME = 2.5\%$  at a higher or at a lower confidence level?

◇ Confidence level  $= 1 - \alpha$

$\alpha$  is smaller  $\Leftrightarrow$  Confidence level is higher. (1)

◇  $ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\alpha$  is smaller  $\Leftrightarrow z_{\frac{\alpha}{2}}$  is bigger  $\Leftrightarrow$  ME is larger. (2)

◇ From (1) and (2)

ME is larger  $\Leftrightarrow$  Confidence level is higher.

(c) Is this  $ME = 2.5\%$  at a higher or at a lower confidence level?

◇ Confidence level  $= 1 - \alpha$

$\alpha$  is smaller  $\Leftrightarrow$  Confidence level is higher. (1)

◇  $ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\alpha$  is smaller  $\Leftrightarrow z_{\frac{\alpha}{2}}$  is bigger  $\Leftrightarrow$  ME is larger. (2)

◇ From (1) and (2)

ME is larger  $\Leftrightarrow$  Confidence level is higher.

Since we found  $ME = 3\%$  at the 95% confidence level, when ME is 2.5% (lower), the confidence level has to be **lower**.



(d) Compute the margin of error (ME) of the proportion saying “No” at the 95% confidence level.

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◇ The margin of error

$$ME = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$\Rightarrow ME = 1.96 \sqrt{\frac{0.46(1 - 0.46)}{1004}} \approx 0.031$$

◇ The 95% confidence level is approximately  $[0.429, 0.491]$ .

(e) What is your view on the differences between the two polls?

(e) What is your view on the differences between the two polls?

- ◇ The confidence intervals are clearly different:  $[0.429, 0.491]$  is different from  $[0.48, 0.54]$ , and in this sense, the estimates do differ.
- ◇ However, at the same time, the intervals overlap (for example, 49% is contained in both), so they are not necessarily making different statements about the population parameter.
- ◇ **Remark:** There exist statistical tests to determine whether *two sample averages* are different from each other in a statistically significant way. We will learn about statistical testing procedures in Unit 5.

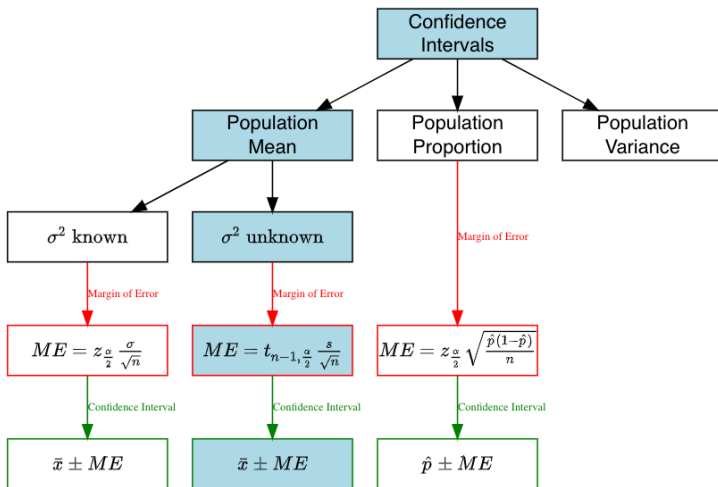
## Exercise 2

The government introduces a housing benefit of 100 pounds per month for low-income households.

After the introduction of the policy, we collect a sample of rents paid by 31 low-income households:  $(r_1, r_2, \dots, r_{31})$ . In this sample, the sample average rent is  $\bar{r} = 709$  pounds. In this sample, the standard deviation is  $s_r = 34$ .

- (a) Construct the margin of error for  $\bar{r}$ .
- (b) Construct the 95% confidence interval for  $\bar{r}$ .
- (c) We know that the average rent paid by low-income households, prior to the reform, was 700 pounds. Is this value, 700, still a likely population rent after the reform, based on the sample?

# Where we currently are



## (a) Construct the margin of error for $\bar{r}$ .

The margin of error of the sample mean for a sample of size  $n$ , when the population standard deviation is not known, is given by

$$ME = t_{n-1, \frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

- ◇  $t_{n-1, \frac{\alpha}{2}}$ : the value which cuts off  $(\frac{\alpha}{2}) \cdot 100\%$  of the probability mass in the right tail for a variable following a t-distribution with  $n - 1$  degrees of freedom.
  - ◇ We use the Student's t distribution with  $n - 1$  degrees of freedom.
  - ◇ Because  $1 - \alpha = 0.95$ , we have that  $\alpha = 0.05$  and  $\frac{\alpha}{2} = 0.025$ .
- ◇ Calculate  $t_{30, 0.025}$ :
  - ◇ Looking at a statistical table:  $t_{30, 0.025} = 2.042$ , OR
  - ◇ Using Excel function: `=T.INV(0.975, 30)`

(a) Construct the margin of error for  $\bar{r}$ .

Plugging in  $t_{30,0.025} = 2.042$ , the sample size  $n = 31$  and the sample standard deviation  $s = 34$ , we find

$$ME = 2.042 \times \frac{34}{\sqrt{31}} \approx 12.47$$



(b) Construct the 95% confidence interval for  $\bar{r}$ .

(b) Construct the 95% confidence interval for  $\bar{r}$ .

$\bar{r} = 709 \Rightarrow \bar{r} \pm 12.47$  give the confidence interval  $[696.5, 721.5]$ .

(c) We know that the average rent paid by low-income households, prior to the reform, was 700 pounds. Is this value, 700, still a likely population rent after the reform, based on the sample?

(c) We know that the average rent paid by low-income households, prior to the reform, was 700 pounds. Is this value, 700, still a likely population rent after the reform, based on the sample?

700 is contained in the 95% confidence interval. Therefore we don't have very strong evidence suggesting that 700 is not a possible value for the population mean after the reform, at the 95% confidence level.

EXCEL NOTE

# Relevant functions (I)

Launch the **Excel** online

<https://www.office.com/launch/excel?auth=2>

**NORM.INV()** To return the inverse of the normal cumulative distribution for the specified mean and standard deviation (*real number*).

```
= NORM.INV(probability,mean,standard_dev)
```

**T.INV()** To return the t-value of the Student's t-distribution as a function of the probability and the degrees of freedom (*real number*).

```
= T.INV(probability,degrees_freedom)
```

## Relevant functions (II)

Launch the **Excel** online

<https://www.office.com/launch/excel?auth=2>

**NORMSDIST()** To return the standard normal cumulative distribution (*probability*).

= NORMSDIST(z)