Introductory Statistics for Economics

ECON1013: TUTORIAL 4

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Intro

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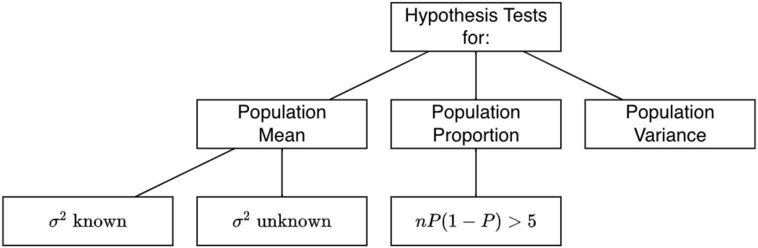
  - Email: Duong.Trinh@glasgow.ac.uk
- ECON1013-TU04
  - Monday 12-1 pm
  - 4 sessions (22-Jan, 5-Feb, 19-Feb, 4-March)
- ECON1013-TU05
  - Tuesday 12-1 pm
  - 4 sessions (23-Jan, 6-Feb, 20-Feb, 5-March)
- ECON1013-TU07
  - Tuesday 2-3 pm
  - 4 sessions (23-Jan, 6-Feb, 20-Feb, 5-March)

Your Attendance & Tutorial Feedback

BRIEF REVIEW

Hypothesis Testing



Exercise 1

Using measures developed by psychologists, we have a test score measuring a child's ability to engage in teamwork. Among childen in a given age group, the most typical test score is 78. ($\mu_0 = 78$)

This year, some children took extra sports classes in primary school. We have data for a sample of the children who took the extra classes:

- $n = 53$
- In this sample, the average test score is 82.3. ($\bar{x} = 82.3$)

Suppose that the standard deviation of this population is known to be 13. ($\sigma = 13$)

Exercise 1

Questions

a) Can it be claimed at a 1% significance level that the mean test score among students who took extra sports differs from the typical value of the age group?

b) Compute the p-value associated with the observed sample mean. Explain how it should be interpreted.

c) Can the claim from part a) be made at the 10% significance level?

(1a) Can it be claimed at a 1% significance level that the mean test score among students who took extra sports differ from the typical value of the age group?

Hypothesis Testing procedure includes 5 steps:

- Null hypothesis H_0
- Alternative hypothesis H_1
- Test statistic
- Decision rule
- Conclusion

(1a) - State Hypotheses

- Null hypothesis H_0:
  H_0: The mean test score among students who took extra sports *is equal to* the typical value of the age group
- Alternative hypothesis H_1:
  H_1: The mean test score among students who took extra sports *differs from* the typical value of the age group.

- Test statistic
- Decision rule
- Conclusion

(1a) - State Hypotheses

Denote μ the true population mean of test score among students who took extra sports.

- Null hypothesis H_0:
  H_0: The mean test score among students who took extra sports *is equal to* the typical value of the age group.
  H_0: $\mu = 78$

- Alternative hypothesis H_1:
  H_1: The mean test score among students who took extra sports *differs from* the typical value of the age group.
  H_1: $\mu \neq 78$

- Test statistic
- Decision rule
- Conclusion

(1a) - State Hypotheses

Denote μ the true population mean of test score among students who took extra sports.

- Null hypothesis H_0: $\mu = 78$
- Alternative hypothesis H_1: $\mu \neq 78$
  $\Rightarrow$ This is a two-tailed test.

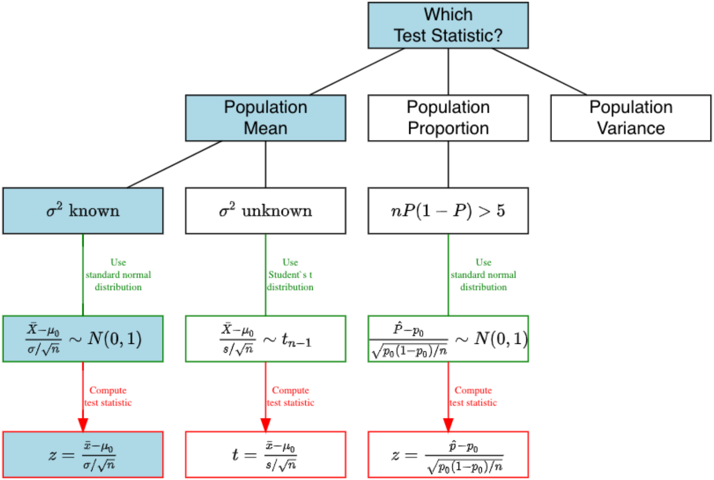
- Test statistic
- Decision rule
- Conclusion

(1a) - Compute Test statistic

Denote μ the true population mean of test score among students who took extra sports.

- Null hypothesis H_0: $\mu = 78$
- Alternative hypothesis H_1: $\mu \neq 78$
- Test statistic?
- Decision rule
- Conclusion

GUIDE


Which Test Statistic?
Population Mean
Population Proportion
Population Variance
σ^2 known
σ^2 unknown
$nP(1-P) > 5$
Use standard normal distribution
Use Student`s t distribution
Use standard normal distribution
$\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$
$\frac{\bar{X}-\mu_0}{s/\sqrt{n}} \sim t_{n-1}$
$\frac{\hat{P}-p_0}{\sqrt{p_0(1-p_0)/n}} \sim N(0,1)$
Compute test statistic
Compute test statistic
Compute test statistic
$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$
$t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$
$z = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$

(1a) - Compute Test statistic

There is a random sample of students drawn from the population who did extra sports, and the sample is relatively large ($n = 53 > 25$). This means that the sampling distribution of the sample mean is approximately normal. Moreover, the population standard deviation is known ($\sigma = 13$). Therefore, we can use a z-test. Under the null hypothesis is true

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0, 1)$$

We compute the realisation of the test statistic in our sample

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{82.3 - 78}{13/\sqrt{53}} \approx 2.41$$

(1a) - Consider Decision Rule

Denote μ the true population mean of test score among students who took extra sports.

- Null hypothesis H_0: $\mu = 78$
- Alternative hypothesis H_1: $\mu \neq 78$
- Test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{82.3 - 78}{13/\sqrt{53}} \approx 2.41$$

- Decision rule?
- Conclusion

GUIDE

Let's answer 3 questions to find appropriate decision rule

1. Is this a two-tailed test or an one-tailed (lower-tail/upper-tail) test?
   $\Longrightarrow$ Look again H_1.
2. What is the **significance level** α?
   $\Longrightarrow$ Usually chosen to be 0.01, 0.05 or 0.10.
3. Is the decision rule based on **critical values** or **p-value**?
   $\Longrightarrow$ Distinguish...

(1a) - Consider Decision Rule

1. We are conducting a two-tailed test (as $H_1 : \mu \neq 78$).
2. The significance level $\alpha = 0.01$ (given in the question).
3. Both approaches are equivalent, we first rely on **critical values** approach.

GUIDE

Approach 1: **Critical-value Test**

Test	H_1	Reject H_0 if
Two-tailed	$\mu \neq \mu_0$	$z < -z_{\frac{\alpha}{2}}$ or $z > z_{\frac{\alpha}{2}}$
Lower-tail	$\mu < \mu_0$	$z < -z_{\alpha}$
Upper-tail	$\mu > \mu_0$	$z > z_{\alpha}$

Approach 2: **p-value Test**

Test	H_1	p-value	Reject H_0 if
Two-tailed	$\mu \neq \mu_0$	sum probabilities to the right of $\lvert z\rvert$ and to the left of $-\lvert z\rvert$	p-value $< \alpha$
Lower-tail	$\mu < \mu_0$	probability to the left of z	p-value $< \alpha$
Upper-tail	$\mu > \mu_0$	probability to the right of z	p-value $< \alpha$

*Note: p-value is probability of obtaining a test statistic more extreme ($\leq$ or $\geq$) than the observed sample value given H_0 is true.

(1a) - Decision Rule using Critical values approach

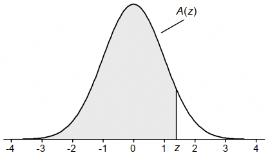
For two-tailed test, reject H_0 if

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} < -z_{\frac{\alpha}{2}} \text{ or } z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} > z_{\frac{\alpha}{2}}$$

- Compute critical value:
  $z_{\frac{\alpha}{2}} = z_{\frac{0.01}{2}} = z_{0.005} = 2.57$
  (`Excel` function: $=$ NORM.INV$(0.995, 0, 1)$).

- Compare test statistic to critical value:
  Notice that $z \approx 2.41$, which is greater than -2.57 ($-z_{0.005}$) and less than 2.57 ($z_{0.005}$)
  $\Rightarrow$ We DO NOT reject H_0 at $\alpha = 0.01$-level.

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974

(1a) - Conclusion

We DO NOT find evidence at this level suggesting that among children who did extra sports, the mean test score would be different from the typical test score of the age group.

(1a) Hypothesis Testing

- **Null hypothesis** H_0: $\mu = 78$
- **Alternative hypothesis** H_1: $\mu \neq 78$
- **Test statistic**

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{82.3 - 78}{13/\sqrt{53}} \approx 2.41$$

- **Decision rule** *Critical values approach

  The critical value of a two-sided z-test at the level of $\alpha = 0.01$,
  $z_{\frac{\alpha}{2}} = z_{0.005} = 2.57$.
  The test statistic computed based on our sample (z=2.41) is less extreme than the critical value.

  $\Rightarrow$ We DO NOT reject H_0 at $\alpha = 0.01$.

- **Conclusion**

  We do not find evidence at this level suggesting that among children who did extra sports, the mean test score would be different from the typical test score of the age group.

(1b) Compute the p-value associated with the observed sample mean. Explain how it should be interpreted.

To find the p-value associated with the observation, we are looking for the probability of observing a sample outcome as extreme or more extreme than what we did observe, under assumption that the null hypothesis is true.

This means that we are looking for the probability of finding outcomes which are at least 4.3 units away from the typical value 78 ($82.3 - 78 = 4.3$). In other words, we are looking for a probability of observing a sample average which is *at least* 82.3 or *at most* 73.7.

(1b) Compute the p-value associated with the observed sample mean. Explain how it should be interpreted.

$\mathbb{P}(\text{Find sample outcome as extreme or more extreme than observed} \mid H_0 \text{ true})$

$$= \mathbb{P}(\bar{X} \geq \underset{(1)}{82.3} \mid H_0 \text{ true}) + \mathbb{P}(\bar{X} \leq \underset{(2)}{73.7} \mid H_0 \text{ true})$$

$$
\begin{aligned}
(1) \quad \mathbb{P}(\bar{X} \geq 82.3 \mid H_0 \text{ true}) &= \mathbb{P}\left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \geq \frac{82.3 - \mu_0}{\sigma/\sqrt{n}} \mid H_0 \text{ true}\right) \\
&= \mathbb{P}\left(Z \geq \frac{82.3 - 78}{13/\sqrt{53}}\right) \\
&= \mathbb{P}\left(Z \geq 2.41\right)
\end{aligned}
$$

where Z follows a standard normal distribution.

(1b) Compute the p-value associated with the observed sample mean. Explain how it should be interpreted.

$\mathbb{P}(\text{Find sample outcome as extreme or more extreme than observed} \mid H_0 \text{ true})$

$=\mathbb{P}(\underbrace{\bar{X} \geq 82.3 \mid H_0 \text{ true})}_{(1)} + \underbrace{\mathbb{P}(\bar{X} \leq 73.7 \mid H_0 \text{ true})}_{(2)}$

$$(2) \quad \mathbb{P}(\bar{X} \leq 73.7 \mid H_0 \text{ true}) = \mathbb{P}\left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \leq \frac{73.7 - \mu_0}{\sigma/\sqrt{n}} \mid H_0 \text{ true}\right)$$
$$= \mathbb{P}\left(Z \leq \frac{73.7 - 78}{13/\sqrt{53}}\right)$$
$$= \mathbb{P}\left(Z \leq -2.41\right)$$

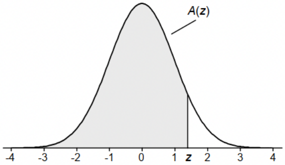
where Z follows a standard normal distribution.

(1b) Compute the p-value associated with the observed sample mean. Explain how it should be interpreted.

$$\mathbb{P}(\text{Find sample outcome as extreme or more extreme than observed} \mid H_0 \text{ true})$$
$$= \mathbb{P}(\bar{X} \geq 82.3 \mid H_0 \text{ true}) + \mathbb{P}(\bar{X} \leq 73.7 \mid H_0 \text{ true})$$
$$= \mathbb{P}(Z \geq 2.41) + \mathbb{P}(Z \leq -2.41)$$

where Z follows a standard normal distribution.

Cumulative Standardized Normal Distribution



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2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936

(1b) Compute the p-value associated with the observed sample mean. Explain how it should be interpreted.

$$
\begin{aligned}
&\mathbb{P}(\text{Find sample outcome as extreme or more extreme than observed} \mid H_0 \text{ true}) \\
&\quad = \mathbb{P}(\bar{X} \geq 82.3 \mid H_0 \text{ true}) + \mathbb{P}(\bar{X} \leq 73.7 \mid H_0 \text{ true}) \\
&\quad = \mathbb{P}\left(Z \geq 2.41\right) + \mathbb{P}\left(Z \leq -2.41\right) \\
&\quad = \mathbb{P}\left(Z \geq 2.41\right) \times 2 \\
&\quad = (1 - 0.992) \times 2 \\
&\quad = 0.008 \times 2 = 0.016
\end{aligned}
$$

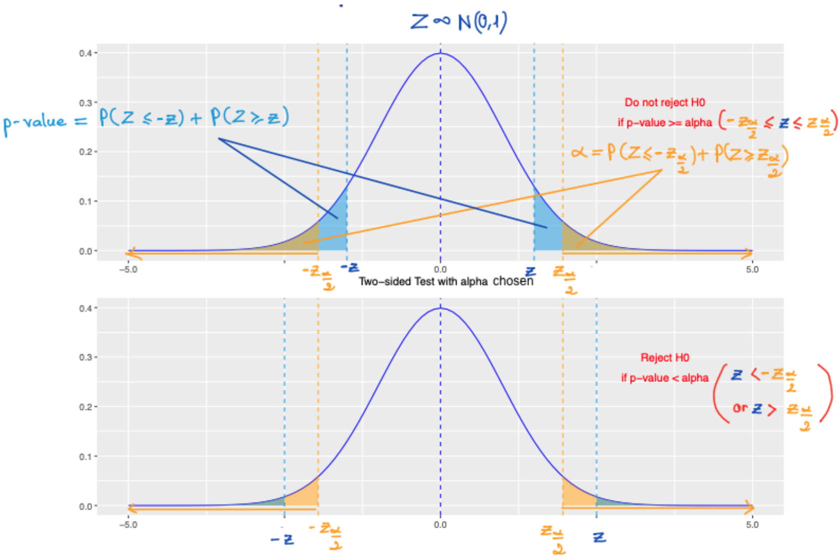
Thus, **p-value** $= 0.016$.

(1b) Interpretation

The p-value is the probability of obtaining a value of the test statistic *as extreme as or more extreme* than the actual value obtained when the null hypothesis is true.

Thus, the p-value is the *smallest* significance level at which a null hypothesis can be *rejected*, given the observed sample statistic.

A p-value larger than the significant level α should be interpreted as evidence against the rejection of the null hypothesis.



Z ∾ N(0,1)
p-value = P(Z ≤ -z) + P(Z ≥ z)
Do not reject H0
if p-value >= alpha
α = P(Z ≤ -z_{α/2}) + P(Z ≥ z_{α/2})
Two-sided Test with alpha chosen
Reject H0
if p-value < alpha
-5.0
0.0
5.0

Revisit Hypothesis Testing in (1a)

- **Null hypothesis** H_0: $\mu = 78$
- **Alternative hypothesis** H_1: $\mu \neq 78$
- **Test statistic**

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{82.3 - 78}{13/\sqrt{53}} \approx 2.41$$

- **Decision rule** *P-value approach

  The p-value associated with the test statistic computed based on our sample (z=2.41) is $\mathbb{P}(Z \geq 2.41) + \mathbb{P}(Z \leq -2.41) = 0.016$, which is larger than $\alpha = 0.01$.

  $\Rightarrow$ We DO NOT reject H_0 at $\alpha = 0.01$ (the same as Critical values approach).

- **Conclusion**

  We do not find evidence at this level suggesting that among children who did extra sports, the mean test score would be different from the typical test score of the age group.

(1c) Can the claim from (1a) be made at the 10% significance level?

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Comparing the p-value 0.016 to $\alpha = 0.1$, we see that p-value $< \alpha$ (since $0.016 < 0.1$). We can reject the null hypothesis at the 10% significance level.

Exercise 2

- It is claimed that the average number of pieces of litter found on a 100-meter interval on UK beaches is more than 300. $(\mu_0 = 300)$
- The number of items found on a 100-meter interval is known to have a standard deviation of 144. $(\sigma = 144)$
- In total, data was collected from 81 randomly selected beaches, on a 100-meter interval on each beach. $(n = 81)$
- We adopt the following test:
    - H_0: The mean number of pieces of litter on a 100-meter interval on a UK beach is at least 300.
    - H_1: The mean number of pieces of litter on a 100-meter interval on a UK beach is less than 300.
    - Decision rule: "The null hypothesis will not be rejected if the sample mean is 276 items or more, and will be rejected otherwise."

Exercise 2

Questions

a) What is the level of significance α (probability of a type I error) of this test?

b) Suppose that the true population mean is actually $\mu^* = 290$. What is the probability of a type-II error (β) of this test in this case?

Bonus: Try part b) with different sample sizes.

(2a) What is the level of significance α (probability of a type I error) of this test?

- **Null hypothesis** H_0: $\mu \geq 300$
- **Alternative hypothesis** H_1: $\mu < 300$
- **Test statistic**

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 300}{144/\sqrt{81}}$$

- **Decision rule**
  This is a *lower-tail* test, we reject H_0 if $z < -z_\alpha$. Here, the critical value $-z_\alpha$ satisfies $\mathbb{P}(Z < -z_\alpha) = \alpha$ where Z follows a standard normal distribution.
  $\Rightarrow$ We reject H_0 if:

$$\frac{\bar{x} - 300}{144/\sqrt{81}} < -z_\alpha \Leftrightarrow \bar{x} < -z_\alpha \cdot 144/\sqrt{81} + 300 \quad (3)$$

GUIDE

Approach 1: **Critical-value Test**

Test	H_1	Reject H_0 if
Two-tailed	$\mu \neq \mu_0$	$z < -z_{\frac{\alpha}{2}}$ or $z > z_{\frac{\alpha}{2}}$
Lower-tail	$\mu < \mu_0$	$z < -z_\alpha$
Upper-tail	$\mu > \mu_0$	$z > z_\alpha$

Approach 2: **p-value Test**

Test	H_1	p-value	Reject H_0 if
Two-tailed	$\mu \neq \mu_0$	sum probabilities to the right of $\lvert z\rvert$ and to the left of $-\lvert z\rvert$	p-value $< \alpha$
Lower-tail	$\mu < \mu_0$	probability to the left of z	p-value $< \alpha$
Upper-tail	$\mu > \mu_0$	probability to the right of z	p-value $< \alpha$

*Note: p-value is probability of obtaining a test statistic more extreme ($\leq$ or $\geq$) than the observed sample value given H_0 is true.

(2a) What is the level of significance α (probability of a type I error) of this test?

- On the other hand, the decision rule tells us that H_0 is rejected if $\bar{x} < 276$. Therefore, the righ-hand side quantity in (3) has to be equal to 276. That means

$$\begin{aligned} -z_\alpha \cdot 144/\sqrt{81} + 300 &= 276 \\ z_\alpha &= \frac{300 - 276}{144/\sqrt{81}} \\ &= 1.5 \end{aligned}$$

- Thus, we are looking for α such that the critical value of a upper-tail z-test is 1.5. Based on statistical table/Excel function `NORMSDIST()`, we will obtain $\alpha = 1 - 0.9332 = 0.0668$.

(2b) Suppose $\mu^* = 290$.
What is the probability of a type-II error (β) of this test?

- β gives the probability of a type-II error, meaning the probability of "maintaining a null hypothesis which is false".
- Note that if $\mu = 290$, then the null hypothesis is false! In this case we would like to reject the null hypothesis.
- We are thus looking for

$$\beta = \mathbb{P}(\text{maintain } H_0 \mid \mu = 290)$$

(2b) Suppose $\mu^* = 290$. What is the probability of a type-II error (β) of this test?

- Using the given decision rule,

$$\begin{aligned}\mathbb{P}(\text{maintain } H_0 \mid \mu = 290) &= \mathbb{P}(\bar{X} > 276 \mid \mu = 290) \\ &= \mathbb{P}\left(\frac{\bar{X} - 290}{144/\sqrt{81}} > \frac{276 - 290}{144/\sqrt{81}} \mid \mu = 290\right) \\ &= \mathbb{P}(Z > -0.875) \\ &= 1 - \mathbb{P}(Z < -0.875) \\ &\approx 0.809\end{aligned}$$

- In other words, if the true population mean would have been 290, then based on our hypothesis test we would be falsely maintaining the H_0 more than 80% of the time.

(2b) Suppose $\mu^* = 290$.
What is the probability of a type-II error (β) of this test?

Remark: $1 - \beta$ is called the **power** of a test. The power of the test depends on the true value of the population parameter. The power of this test, if $\mu = 290$, is around 19%, which is quite low. The power of this test increases if *n* grows.