W10_Empirical Exercise 6

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★ Hypothesis testing

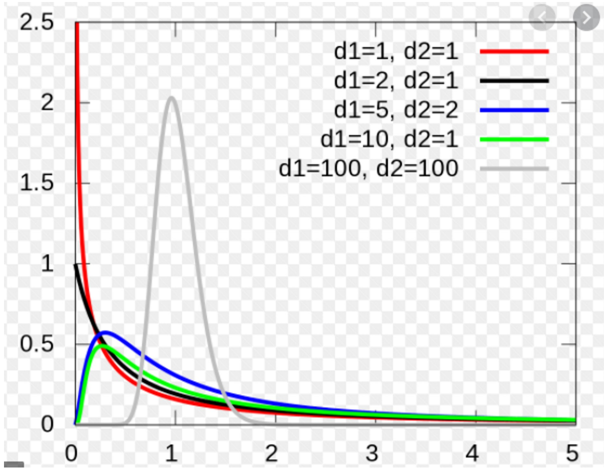
n: number of observations
k: number of regressors (independent varibles) under the unrestricted model
k+1: number of parameters under the unrestricted model (= number of estimated coefficients)
q: number of restrictions (number of linear hypothese with **equal** sign)

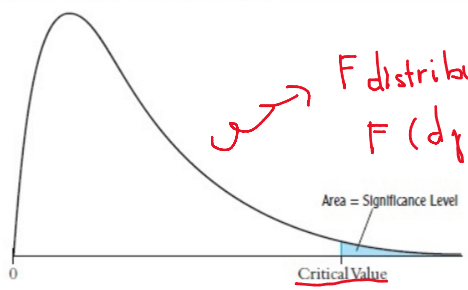
Regression Model	$share = \beta_0 + \beta_1 tax + \beta_2 year + \epsilon$	$share = \beta_0 + \beta_1 tax + \beta_2 year + \epsilon$	$share = \beta_0 + \beta_1 tax + \beta_2 year + \beta_3 gwth + e$
Test hypothesis	(c) In the year **2000**, the expected income share of the top 1% would have been **5%** if the marginal tax rate had been **64%** Test the hypothesis at 5% significance level	(d) A marginal tax rate of **64%** would lead to the same **5%** share for the top income earners in both **1925** and **2000** Test the hypothesis at 10% significance level	(f) Test the overall significance of the model at 1% significance level
1. Null hypothesis H0	$H_0: \beta_0 + 64\beta_1 + 80\beta_2 = 5$ $q=1$	$H_0: \beta_0 + 64\beta_1 + 5\beta_2 = 5$ and $\beta_0 + 64\beta_1 + 80\beta_2 = 5$ $q=2$	$H_0: \beta_1 = 0$ and $\beta_2 = 0$ and $\beta_3 = 0$
2. Alternative hypothesis H1	$H_1: \beta_0 + 64\beta_1 + 80\beta_2 \neq 5$	H_1 : at least one equality in H_0 does not hold	H_1 At least one of β_j is non-zero $j = 1, 2, 3$

3. Test statistic
F-statistic
	`. quiet reg share tax year, robust`
	`. test (_cons + 64*tax + 80*year = 5)`
	`( 1) 64*tax + 80*year + _cons = 5`
	`F( 1, 77) = 0.02`
	`Prob > F = 0.8891`
	`. display r(F)`
	`.01957989`
	Heteroskedasticity-robust *F*-statistic = 0.0196
		`. quiet reg share tax year, robust`
		`. test (_cons + 64*tax + 5*year = 5) (_cons + 64*tax + 80*year = 5)`
		`( 1) 64*tax + 5*year + _cons = 5`
		`( 2) 64*tax + 80*year + _cons = 5`
		`F( 2, 77) = 2.83`
		`Prob > F = 0.0651`
		`. display r(F)`
		`2.8313648`
		Heteroskedasticity-robust *F*-statistic = 2.8314
			`. quiet reg share tax year gwth, robust`
			`. test (tax = 0) (year = 0) (gwth = 0)`
			`( 1) tax = 0`
			`( 2) year = 0`
			`( 3) gwth = 0`
			`F( 3, 76) = 127.07`
			`Prob > F = 0.0000`
			Heteroskedasticity-robust *F*-statistic = 127.07

4. Rejection region
F-statistic $> F_c$
$\Rightarrow$ Reject H_0
or
Pvalue $< \alpha$
$\Rightarrow$ Reject H_0
	*Calculate Critical Value or p-value
	+, alpha = 5%
	+, Degree of freedom:
	df1 = q = 1
	df2 = n - k - 1 = 80 - 2 - 1 = 77
	=> Critical value F(1,77) at 5% level of significance
	`. display invFtail(1,77,0.05)`
	`3.9650941`
	*Compare F-statistic and Critical value
	Or Compare p-value and level of significance
	F-statistic = 0.0196 < 3.9651 = Critical Value
	Or p-value = 0.8891 > 0.05 = level of significance
	=> **Do not reject H0** at 5% significance level
		*Calculate Critical Value or p-value
		+, alpha = 10%
		+, Degree of freedom:
		df1 = q = 2
		df2 = n - k - 1 = 80 - 2 - 1 = 77
		=> Critical value F(2,77) at 10% level of significance
		`. display invFtail(2,77,0.1)`
		`2.3728344`
		*Compare F-statistic and Critical value
		Or Compare p-value and level of significance
		F-statistic = 2.8314 > 2.3728 = Critical Value
		Or p-value = 0.0651 < 0.1 = level of significance
		=> **Reject H0** at 10% significance level
			*Calculate Critical Value or p-value
			+, alpha = 1%
			+, Degree of freedom:
			df1 = q = 3
			df2 = n - k - 1 = 80 - 3 - 1 = 76
			=> Critical value F(3,76) at 10% level of significance
			`. display invFtail(3,76,0.01)`
			`4.0502821`
			*Compare F-statistic and Critical value
			Or Compare p-value and level of significance
			F-statistic = 127.07 > 4.0503 = Critical Value
			Or p-value = 0.0000 < 0.01 = level of significance
			=> **Reject H0** at 1% significance level

5. Conclusion	Data do not contradict conjecture about income share in 2000 for a marginal tax rate of 64%	A marginal tax rate of 64% does not lead to the same 5% share for the top income earners in both 1925 and 2000.	At least one of the regressors has a statistically significant relationship with share.





F distribution

$F(d_{f_1}, d_{f_2})$

α

Relationship Between t-Test and F-Test

- What happens if we have a null hypothesis which has only one restriction?
  - Example: $H_0 : \beta_1 = 2$ vs $H_1 : \beta_1 \neq 2$
- For a two-sided test with a single restriction (q=1), either a t-test or an F-test can be used
  - Two-sided t-tests are equivalent to F-tests when there is a single hypothesis H_0
- When $q = 1$, $F = t^2$
- This result holds for both homoskedastic and heteroskedastic errors