# ECON 4003 Econometrics I

## Empirical Exercise 6

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# Picture the Scenario

- **Objective:** Investigate the impact of the **marginal tax rate** paid by high-income earners on the level of **inequality**.
- **Data:** Time-series data on five different countries
  - We use a subset of their data for Australia can be found in the file `inequality_aus.dta`.
- **Key variables:**
  - `share`: the percentage income share of the top 1% of incomes.
  - `tax`: the median marginal tax rate (as a percentage) paid on wages by the top 1% of income earners.
  - `year`: $1 = 1921, 2 = 1922, ..., 80 = 2000$.
  - `gwth`: the percentage growth rate

# Question (a)

- Estimate the equation: $share = \beta_0 + \beta_1 tax + u$
- Interpret your estimate for $\beta_1$.
  Would you interpret this as a causal relationship?
  Is the OLS estimator $\widehat{\beta}_1$ unbiased?

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  Would you interpret this as a causal relationship?
  Is the OLS estimator $\widehat{\beta}_1$ unbiased?

*Answer.*

$$\underset{(se)}{\widehat{share}} = \underset{(0.2358)}{12.0207} - \underset{(0.005336)}{0.1044} \cdot tax$$

$$R^2 = 0.6668 \quad SER = 1.346$$

# Question (b)

- It is generally recognized that inequality was high prior to the great depression, then declined during the depression and World War II, increasing again toward the end of the sample period. To capture this effect, estimate the following model with a linear trend:

$$share = \beta_0 + \beta_1 tax + \beta_2 year + \epsilon$$

# Question (b)

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$$\underset{(se)}{\widehat{share}} = \underset{(0.2261)}{12.0370} - \underset{(0.01627)}{0.07467} \cdot tax - \underset{(0.01456)}{0.02776} \cdot year \quad R^2 = 0.69 \quad SER = 1.31$$

  - Controlling for time trend, we find that an increase in the marginal tax rate for top earners by one percentage is associated with a decrease in the income share of the top earners by 0.07467 percentage on average. The p-value is 0.000; therefore, the estimate is statistically significant at 1% significance level.
  - Holding marginal tax rate constant, the income share of the top earners decreases by 0.02776 percentage per year on average.
    The p-value is 0.060; therefore, the estimate is statistically significant at 10% significance level.

# Question (b)(cont.)

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*Answer.*

$$\underset{(se)}{\widehat{share}} = \underset{(0.2358)}{\overset{\hat{\beta}_0}{12.0207}} - \underset{(0.005336)}{\overset{\hat{\beta}_1}{0.1044}} \cdot tax$$

$$R^2 = 0.6668 \quad SER = 1.346$$

$$\underset{(se)}{\widehat{share}} = \underset{(0.2261)}{\overset{\hat{\beta}_0^*}{12.0370}} - \underset{(0.01627)}{\overset{\hat{\beta}_1^*}{0.07467}} \cdot tax - \underset{(0.01456)}{\overset{\hat{\beta}_2^*}{0.02776}} \cdot year$$

$$R^2 = 0.6902 \quad SER = 1.306$$

$$\Rightarrow \hat{\beta}_1 < \hat{\beta}_1^*$$

# Question (b)(cont.)

- Can the change in this estimate, or lack of it, be explained by the correlation between `tax` and `year`?

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The **omitted variable bias** formula:

$$plim\ \widehat{\beta}_1 = \beta_1 + \beta_2 \frac{\text{Cov}(X, Z)}{Var(X)}$$

where the regressor $X = tax$ and the omitted variable $Z = year$.

- ☐ $\widehat{\beta}_2 = -0.02776$ and statistically significant $\Rightarrow \beta_2$ is likely to be negative.
- ☐ Correlation between `tax` and `year` is positive and high:

$$\text{Corr}(tax, year) = 0.8352$$

Hence, $plim\ \widehat{\beta}_1 < \beta_1$, then $\widehat{\beta}_1$ is inconsistent. The estimated coefficient for `tax` is biased downwards in (a).

# Question (c)

- The top marginal tax rate in 2000 was 64%.
  Test the hypothesis that, in the year **2000**, the expected income share of the top 1% would have been **5%** if the marginal tax rate had been **64%** at that time at 5% level of significance.

- Do your result change if the error term is homoskedastic?

Joint Hypothesis

# Question (c)(cont.)

- **Null hypothesis $H_0$:**
  - $H_0$ : In the year **2000**, the expected income share of the top 1% would have been **5%** if the marginal tax rate had been **64%**

    $H_0 : \beta_0 + 64\beta_1 + 80\beta_2 = 5$

    *(single restriction involving multiple coefficients)*

- **Alternative hypothesis $H_1$:**
  - $H_1$ : In the year **2000**, the expected income share of the top 1% would have been different from **5%** if the marginal tax rate had been **64%**

    $H_1 : \beta_0 + 64\beta_1 + 80\beta_2 \neq 5$

# Question (c)(cont.)

- **Test statistic** - Case 1. Heteroskedasticity-robust standard error:

```
. quiet reg share tax year, robust

. test (_cons + 64*tax + 80*year=5)

 ( 1)  64*tax + 80*year + _cons = 5

       F(  1,    77) =    0.02
            Prob > F =    0.8891
```

- **Decision Rule**
  - At $\alpha = 5\%$ and degrees of freedom $df_1 = q = 1$, $df_2 = n - k - 1 = 80 - 3 = 77$, the critical value $F_{1,77} = 3.9651$
  - Heteroskedasticity-robust $F$-statistic $= 0.0196 < 3.9651$ and $p$-value $= 0.8891 > 0.05 \Rightarrow$ We do not reject $H_0$ at 5% significance level.
- **Conclusion**

  Data do not contradict conjecture about income share in 2000 for a marginal tax rate of 64%.

# Question (c)(cont.)

- **Test statistic** - Case 2. Homokedasticity-only standard error:

```
. quiet reg share tax year

. test (_cons + 64*tax + 80*year=5)

 ( 1)  64*tax + 80*year + _cons = 5

       F(  1,    77) =    0.02
            Prob > F =    0.8965
```

- **Decision Rule**
  - ☐ At $\alpha = 5\%$ and degrees of freedom $df_1 = q = 1$, $df_2 = n - k - 1 = 80 - 3 = 77$, the critical value $F_{1,77} = 3.9651$
  - ☐ Homokedasticity-only $F$-statistic $= 0.0170 < 3.9651$ and $p$-value $= 0.8965 > 0.05 \Rightarrow$ We still do not reject $H_0$ at 5% significance level.
- **Conclusion**

  Our conclusion does not change.

# Question (d)

- Test jointly the hypothesis in (c) and that a marginal tax rate of **64%** in **1925** would have led to an expected income share of **5%** for the top 1% of income earners at 10% level of significance.

# Question (d)(cont.)

- **Null hypothesis $H_0$:**
  - $H_0$ : A marginal tax rate of **64%** would lead to the same **5%** share for the top income earners in both **1925** and **2000**

    $H_0 : \beta_0 + 64\beta_1 + 5\beta_2 = 5$ and $\beta_0 + 64\beta_1 + 80\beta_2 = 5$

- **Alternative hypothesis $H_1$:**
  - $H_1$ : at least one equality in $H_0$ does not hold

# Question (d)(cont.)

- **Test statistic**

```
. quiet reg share tax year, robust

. test (_cons + 64*tax + 80*year=5) (_cons + 64*tax + 5*year=5)

 ( 1)  64*tax + 80*year + _cons = 5
 ( 2)  64*tax + 5*year + _cons = 5

       F(  2,    77) =    2.83
            Prob > F =    0.0651
```

- **Decision Rule**
  - ☐ At $\alpha = 10\%$ and degrees of freedom $df_1 = q = 2$, $df_2 = n - k - 1 = 80 - 3 = 77$, the critical value $F_{2,77} = 2.3728$
  - ☐ Heteroskedasticity-robust $F$-statistic $= 2.8314 > 2.3728$ and $p$-value $= 0.0651 < 0.10 \Rightarrow$ We reject $H_0$ at 10% significance level.
- **Conclusion**

  We conclude that a marginal tax rate of 64% does not lead to the same 5% share for the top income earners in both 1925 and 2000.

# Question (e)

- Add the growth rate (`gwth`) to the equation in (b) and re-estimate. Interpret the estimated coefficient for `tax`.

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  *Answer.*

$$\underset{(se)}{\widehat{share}} = \underset{(0.2187)}{11.9728} - \underset{(0.01620)}{0.07548} \cdot tax - \underset{(0.01451)}{0.02928} \cdot year + \underset{(0.04199)}{0.08776} \cdot gwth$$

# Question (e)(cont.)

- Has adding the variable `gwth` led to substantial changes to your estimated coefficient for `tax`?

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# Question (e)(cont.)

- Can the changes, or lack of them, be explained by the correlations between `gwth` and the other variables in the equation?
- Why is `gwth` still included in the regression?

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- Can the changes, or lack of them, be explained by the correlations between `gwth` and the other variables in the equation?
- Why is `gwth` still included in the regression?

*Answer.* The lack of changes can be explained by the low correlations between `gwth` and the other regressors in the equation:

$$\text{Corr}(tax, gwth) = 0.1849 \quad \text{and} \quad \text{Corr}(year, gwth) = 0.1991$$

# Question (f)

- Test the overall significance of the following model at 1% significance level:

$$share = \beta_0 + \beta_1 tax + \beta_2 year + \beta_3 gwth + e$$

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$$share = \beta_0 + \beta_1 tax + \beta_2 year + \beta_3 gwth + e$$

*Answer.*

To test the overall significance of the model:

$H_0 : \beta_1 = 0$ and $\beta_2 = 0$ and $\beta_3 = 0$

vs. $H_1$ At least one of $\beta_j$ is non-zero $\quad j = 1, 2, 3$

Heteroskedasticity-robust $F$-statistic is 127.07 and $p$-value is 0.0000.
At $\alpha = 0.01$, the critical value $F_{3,76}$ is 4.0503.

$\Rightarrow$ We reject $H_0$ at 1% significance level and conclude at least one of the regressors has a statistically significant relationship with `share`.

# Testing of Joint Hypotheses

Procedure includes 5 steps:

- Null hypothesis $H_0$
- Alternative hypothesis $H_1$
- Test statistic
- Decision rule
- Conclusion

# Testing of Joint Hypotheses

- Null hypothesis $H_0$: imposes a restriction on two or more coefficients

$$\text{e.g.} \quad H_0 : \beta_1 = 0 \text{ and } \beta_2 = 0$$

- Alternative hypothesis $H_1$
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Procedure includes 5 steps:

- Null hypothesis $H_0$: imposes a restriction on two or more coefficients

$$\text{e.g.} \quad H_0 : \beta_1 = 0 \text{ and } \beta_2 = 0$$

- Alternative hypothesis $H_1$:

$$\text{e.g.} \quad H_0 : \beta_1 \neq 0 \text{ or } \beta_2 \neq 0 \text{ or both are non-zero}$$

- Test statistic
- Decision rule
- Conclusion

# Testing of Joint Hypotheses

- Null hypothesis $H_0$
- Alternative hypothesis $H_1$
- Test statistic:

$$\textbf{F-statistic} = \frac{(SSR_R - SSR_U)/q}{SSR_U/(n-k-1)} = \frac{\left(R_U^2 - R_R^2\right)/q}{\left(1 - R_U^2\right)/(n-k-1)}$$

  Where:
  - n: number of observations
  - k: number of regressors (independent variables) under the unrestricted model
  - k+1: number of parameters under the unrestricted model (= number of estimated coefficients)
  - q: number of restrictions (number of linear hypotheses with **equal** sign)
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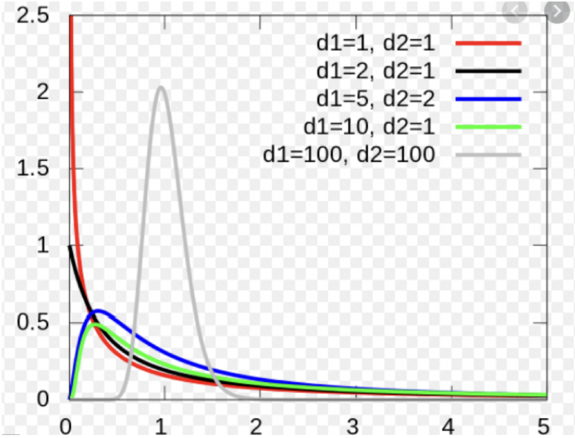
- Test statistic:

$$\textbf{F-statistic} = \frac{(SSR_R - SSR_U)/q}{SSR_U/(n-k-1)} = \frac{\left(R_U^2 - R_R^2\right)/q}{\left(1 - R_U^2\right)/(n-k-1)}$$

  Follows a $F_{q,n-k-1}$ distribution with degrees of freedom $df_1 = q$ and $df_2 = n-k-1$.
- Decision rule
- Conclusion

# F-distribution



# Testing of Joint Hypotheses

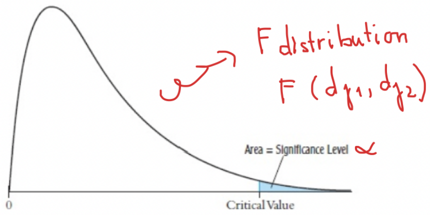
- Null hypothesis $H_0$
- Alternative hypothesis $H_1$
- Test statistic
- Decision rule:
  - What is the **significance level** $\alpha$?

    $\Longrightarrow$ Usually chosen to be 0.01, 0.05 or 0.10.
  - Is the decision rule based on **critical values** or **p-value**?

    $\Longrightarrow$ Distinguish...
- Conclusion

# Decision Rule

- Approach 1: **Critical-value Test**
  Reject $H_0$ if F-statistic $>$ Critical value $F_\alpha$
- Approach 2: **p-value Test**
  Reject $H_0$ if p-value $\leq \alpha$



# Testing of Joint Hypotheses

Procedure includes 5 steps:

- Null hypothesis $H_0$
- Alternative hypothesis $H_1$
- Test statistic
- Decision rule
- Conclusion:
  - Do you *reject* or or *fail to reject* the null hypothesis at the significance level $\alpha$?
  - AVOID saying that you "*accept*" the null hypothesis, which can be very misleading

Back