Homework 03

Submission Notices:

- *Conduct your homework by filling answers into the placeholders in this file (in Microsoft Word format). Questions are shown in black color, instructions/hints are shown in italics and blue color, and your content should use any color that is different from those.*
- *After completing your homework, prepare the file for submission by exporting the Word file (filled with answers) to a PDF file, whose filename follows the following format,*
  <StudentID-1>_<StudentID-2>_HW01.pdf (Student IDs are sorted in ascending order)
  *E.g., **2112001_2112002_HW02.pdf***
  and then submit the file to Moodle directly WITHOUT any kinds of compression (.zip, .rar, .tar, etc.).
- *Note that you will get zero credit for any careless mistake, including, but not limited to, the following things.*
  1. *Wrong file/filename format, e.g., not a pdf file, use "-" instead of "_" for separators, etc.*
  2. *Disorder format of problems and answers*
  3. *Conducted not in English*
  4. *Cheating, i.e., copying other students' works or letting other students copy your work.*

Problem 1. (3pts) For each English sentence below, write the FOL sentence that best expresses its intended meaning, using the following given predicates.

Child(x) for "x is a child" Cupcake(x) for "x is a cupcake"

Loves(x, y) for "x loves y" Eats(x, y) for "x eats y"

1. Every child loves every cupcake.
2. For every child, there is a cupcake that the child loves.
3. There is a child who loves every cupcake.
4. Some child loves some cupcake.
5. There is a cupcake that every child loves.
6. For every cupcake, there is a child who loves that cupcake.
7. Every child eats exactly one cupcake.
8. Some child eats at least two cupcakes.
9. Every child eats at most one cupcake.

Please be aware of the precedence of logical connectives if you do not use brackets.

Please fill your answers in the table below

No.	Original statements and FOL sentences
1.	Every child loves every cupcake.
	$\forall x.\ \forall y.\ \ [Child(x) \land Cupcake(y)] \rightarrow Loves(x, y)$
2.	For every child, there is a cupcake that the child loves.
	$\forall x.\ \ Child(x) \rightarrow [\exists y.\ \ Cupcake(y) \land Loves(x, y)]$

3.	There is a child who loves every cupcake
	$\exists x.\ \ Child(x) \land [\forall y.\ \ Cupcake(y) \rightarrow Loves(x, y)]$
4.	Some child loves some cupcake.
	$\exists x.\ \exists y.\ \ Child(x) \land Cupcake(y) \land Loves(x, y)$
5.	There is a cupcake that every child loves.
	$\exists y.\ \ Cupcake(y) \land [\forall x.\ \ Child(x) \rightarrow Loves(x, y)]$
6.	For every cupcake, there is a child who loves that cupcake.
	$\forall y.\ \ Cupcake(y) \rightarrow [\exists x.\ \ Child(x) \land Loves(x, y)]$
7.	Every child eats exactly one cupcake.
	$\forall x.\ \ Child(x)$ $\rightarrow [\exists y.\ \ Cupcake(y) \land Eats(x, y) \land (\forall z.\ \ Cupcake(z) \land Eats(x, z) \rightarrow z = y)]$
8.	Some child eats at least two cupcakes.
	$\exists x.\ \ Child(x)$ $\land [\exists y.\ \exists z.\ \ Cupcake(y) \land Eats(x, y) \land Cupcake\ (z) \land Eats(x, z) \land \neg(z = y)]$
9.	Every child eats at most one cupcake.
	$\forall x.\ \ Child(x)$ $\rightarrow [\forall y.\ \forall z.\ \ (Cupcake(y) \land Eats(x, y) \land Cupcake(z) \land Eats(x, z)) \rightarrow y = z]$

Problem 2. (3pts) Consider the following statements

"(1) Either taxes are increased or if expenditures rise then the debt ceiling is raised. (2) If taxes are increased, then the cost of collecting taxes rises. (3)If a rise in expenditures implies that the government borrows more money, then if the debt ceiling is raised, then interest rates increase. (4) If taxes are not increased and the cost of collecting taxes does not increase then if the debt ceiling is raised, then the government borrows more money. (5)The cost of collecting taxes does not increase. (6) Either interest rates do not increase, or the government does not borrow more money. Prove by resolution that *(7)* Either the debt ceiling is not raised, or expenditures do not rise."

Let T	be	taxes are increased	C	be	cost of collecting taxes rises
E	be	expenditures rise	G	be	government borrows more money
D	be	debt ceiling is raised	I	be	interest rates increase

Note that you do not need to handle the exclusiveness of the grammar structure "either…or", just consider them as simple "or".

(1pt) Translate the English sentences into propositional logic sentences

No.	***Sentences***	**No.**	***Sentences***	**No.**	***Sentences***
1	$T \lor (E \rightarrow D)$	4	$(\neg T \land \neg C) \rightarrow (D \rightarrow G)$	7	$\neg D \lor \neg E$

2	$T \rightarrow C$	5	$\neg C$		
3	$(E \rightarrow G) \rightarrow (D \rightarrow I)$	6	$\neg I \vee \neg G$		

(1pt) Convert propositional logic sentences into CNF

No.	***Sentences***	***Notes the sentence's sources***
1	$T \vee \neg E \vee D$	Premise
2	$\neg T \vee C$	Premise
3	$E \vee \neg D \vee I$	Premise
4	$\neg G \vee \neg D \vee I$	Premise
5	$T \vee C \vee \neg D \vee G$	Premise
6	$\neg C$	Premise
7	$\neg I \vee \neg G$	Premise
8	D	Negated goal
9	E	Negated goal

(1pt) Prove by resolution that either the debt ceiling is not raised, or expenditures do not rise.

No.	***Sentences***	***Notes the sentence's sources***
10	$\neg G \vee I$	4 and 8
11	$\neg G$	7 and 10
12	$T \vee \neg D \vee G$	5 and 6
13	$\neg T$	2 and 6
14	$\neg D \vee G$	12 and 13
15	$\neg D$	11 and 14
16		

Conclusion: The debt ceiling is not raised, or expenditures do not rise.

Problem 3. (2.5pts) Consider a vocabulary with the following symbols:

Dog(x) *predicate "x is a dog"* — Cat(x) *predicate "x is a cat"*

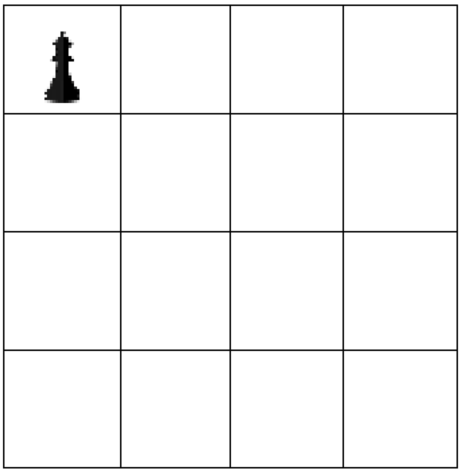
Cute(x) *predicate "x is cute"* — Scary(x) *predicate "x is scary"*

Owns(x, y) *predicate "x owns y"* — Bites*(x,y) predicate "x bites y"*

Please fill your answers in the table below

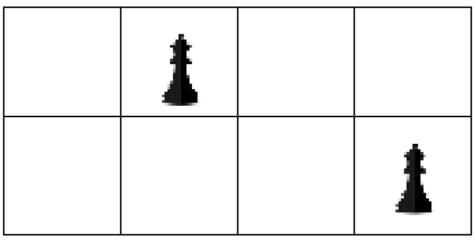
No.	Original statements and FOL sentences
1.	$\exists x\ (Owns(Joe, x) \land Dog(x) \land Cute(x))$
	Joe has a cute dog.
2.	$\forall x\ ((Owns(Joe, x) \land Dog(x)) \Rightarrow Cute(x))$
	All of Joe's dogs are cute.
3.	$\neg(\exists x\ (Owns(Joe, x) \land Dog(x))) \Rightarrow Scary(Joe)$
	Unless Joe owns a dog, he is scary.
4.	$\forall x\ (Dog(x)) \land(\exists y\ Dog(y) \land (\exists z\ Cat(z)) \land(Bites(x,y) \lor Bites(x,z)) \Rightarrow \neg\ Cute(x)$.
	A dog which bites a cat or another dog is not cute
5.	$\exists x\ (Dog(x) \land\neg\ (Scary(x) \land Cute(x)))$
	Not all dogs are both scary and cute.

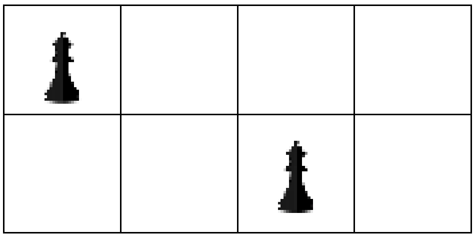
Problem 4. (1.5 pts) Given a chess board with 4 rows and 4 columns (4x4) as below.



a) (0.5pt) Place 4 queens on the board so that they are not able to attack each other.

Please fill your answer in the given table below





b) (1.0pt) Assign a Boolean variable to each cell of the board as below (*1, 2, 3, etc. are variable names*)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

If a variable is True, then there is a queen on the corresponding cell and vice versa.

Use CNF clauses to describe constraints to place a queen on cell no. 1 (*use -1 to denote NOT 1*).

Hint: if there is a queen on cell no. 1, then there mustn't any queens on cell no. 2, 3, etc.

Please fill your answer in the table below (add more rows as needed)

1.	If there is a queen at [1], there must not exist ones at [2], [3], and [4] 1 → -2 ^ -3 ^ -4	Convert into CNFs
2.	If there is a queen at [1], there must not exist ones at [6], [11], and [16] 1 → -6 ^ -11 ^ -16	Convert into CNFs
3.	If there is a queen at [1], there must not exist ones at [5], [9], and [13] 1 → -5 ^ -9 ^ -13	Convert into CNFs