Q5.11

Find the Hessian matrix of the function

a) $f(x,y) = x^2 + 2y^2 + 2xy - 4x + 6y + 1$

$f_x(x,y) = \frac{\partial}{\partial x}(x^2 + 2y^2 + 2xy - 4x + 6y + 1) = 2x + 2y - 4$

$f_y(x,y) = \frac{\partial}{\partial y}(x^2 + 2y^2 + 2xy - 4x + 6y + 1) = 4y + 2x + 6$

Then compute second partial derivatives for every f_x, f_y

$f_{xx}(x,y) = \frac{\partial}{\partial x}(2x + 2y - 4) = 2$

$f_{yx}(x,y) = \frac{\partial}{\partial y}(2x + 2y - 4) = 2$

$f_{xy}(x,y) = \frac{\partial}{\partial x}(4y + 2x + 6) = 2$

$f_{yy}(x,y) = \frac{\partial}{\partial y}(4y + 2x + 6) = 4$

The Hessian matrix is: $H_f = \begin{bmatrix} f_{xx}(x,y) & f_{yx}(x,y) \\ f_{xy}(x,y) & f_{yy}(x,y) \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}$

b) $f(x,y) = e^{x+2y^2}$ at $(0,0)$

$f_x(x,y) = \frac{\partial}{\partial x}(e^{x+2y^2}) = e^{x+2y^2}$

$f_y(x,y) = \frac{\partial}{\partial y}(e^{x+2y^2}) = 4y \cdot e^{x+2y^2}$

Then compute second partial derivatives for every f_x, f_y

$f_{xx}(x,y) = \frac{\partial}{\partial x}(e^{x+2y^2}) = e^{x+2y^2}$

$f_{yx}(x,y) = \frac{\partial}{\partial y}(e^{x+2y^2}) = 4y \cdot e^{x+2y^2}$

$f_{xy}(x,y) = \frac{\partial}{\partial x}(4y \cdot e^{x+2y^2}) = 4y \cdot e^{x+2y^2}$

$f_{yy}(x,y) = \frac{\partial}{\partial y}(4y \cdot e^{x+2y^2}) = 4 \cdot e^{x+2y^2} + 16y^2 \cdot e^{x+2y^2}$

Then Hessian matrix is:

$$H_f = \begin{bmatrix} f_{xx}(x,y) & f_{yx}(x,y) \\ f_{xy}(x,y) & f_{yy}(x,y) \end{bmatrix} = \begin{bmatrix} e^{x+2y^2} & 4y \cdot e^{x+2y^2} \\ 4y \cdot e^{x+2y^2} & 4 \cdot e^{x+2y^2} + 16y^2 \cdot e^{x+2y^2} \end{bmatrix}$$

$$H_{f(0,0)} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$