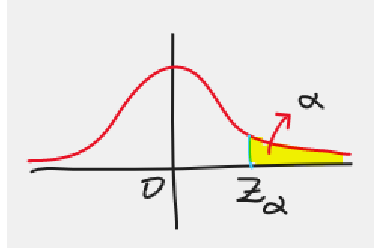
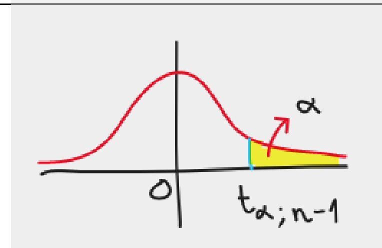
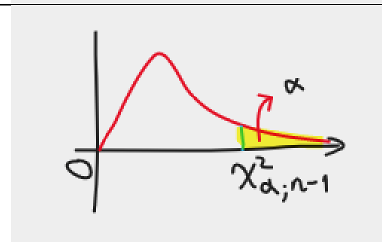
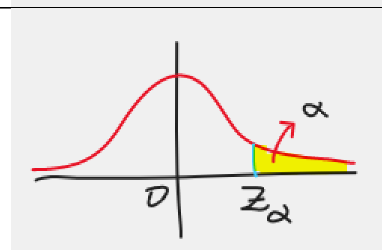
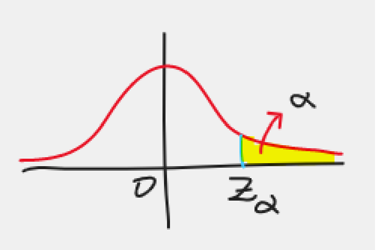
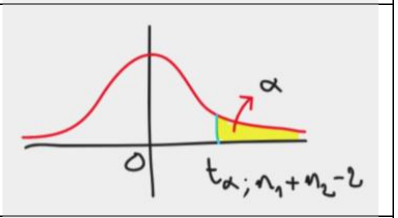
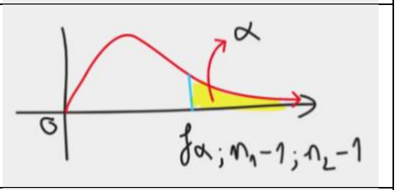
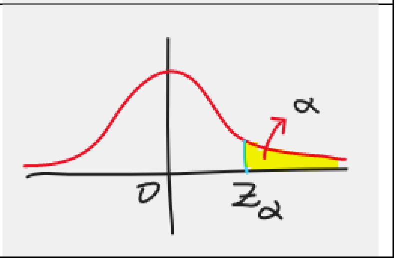
Params	Context	Dist Z-stat	Shape	Critical Value
μ	$n > 30$ normal σ known	$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0;1)$		Z_α
μ	$n > 30$ normal σ unknown	$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t(n-1)$		$t_{\alpha;n-1}$
σ^2	Normal μ unknown	$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$		$\chi^2_{\alpha;n-1}$
p	n large enough	$\frac{F-p}{\sqrt{p(1-p)/n}} \sim N(0;1)$		Z_α

Params	Context	Sampling dist	Test statistic	Shape
$\mu_1-\mu_2$	Normal $n\geq 30$ $\sigma_1^2;\sigma_2^2$ known	$\overline{X_1}-\overline{X_2}\sim N(\mu_1;\mu_2;\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2})$	$Z=\frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}\sim N(0;1)$	
$\mu_1-\mu_2$	Normal $n\geq 30$ $\sigma_1^2;\sigma_2^2$ unknown	$S_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$	$T=\frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{\sqrt{S_p^2(\frac{1}{n_1}+\frac{1}{n_2})}}\sim t(n_1+n_2-2)$	
$\frac{\sigma_1^2}{\sigma_2^2}$	Normal $\mu_1;\mu_2$ unknown	$F=\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}\sim F(n_1-1;n_2-1)$	$F=\frac{s_1^2/s_2^2}{\sigma_1^2/\sigma_2^2}\sim F(n_1-1;n_2-1)$	
p_1-p_2	Normal n is large enough	$\widehat{p_1}-\widehat{p_2}\sim N(p_1-p_2;\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2})$	$Z=\frac{\widehat{p_1}-\widehat{p_2}-(p_1-p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}}\sim N(0;1)$	

9.47.

a)

1. The parameter of interest is the true mean speed, μ.
2. $H_0: \mu = 100$
3. $H_1: \mu < 100$
4. $z_0 = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
5. Reject H_0 if $z_0 < -z_\alpha$ where $\alpha = 0.05$ and $-z_\alpha = -1.65$
6. $\bar{x} = 102.2, \sigma = 4$

$$z_0 = \frac{102.2 - 100}{\frac{4}{sqrt(8)}} = 1.56$$

7. Because 1.56 > -1.56 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean speed is less than 100 at $\alpha = 0.05$

b) $z_0 = 1.56$, then $P - value = \Phi(z_0) \approx 0.94$

c) $\beta = 1 - \Phi\left(-z_{0.05} - \frac{(95-100)\sqrt{8}}{4}\right) = 0.0294$

Power = $1 - \beta = 0.9706$

9.75.

a) $H_0 : \mu = 98.2$ and $H_1 : \mu \neq 98.2$
b) Test statistic:

$$z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{98.285 - 98.2}{0.625/\sqrt{52}} = 0.98$$

We know that, $\alpha = 0.05 \rightarrow z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

If $-z_\alpha \leq z_0 \leq z_\alpha$, then we should fail to reject H_0. Therefore: $0.98 = z_0 < z_\alpha = 1.96$ and $H_0 : \mu = 98.2$ is true

The P-value for the test:

$$\begin{aligned} P(z_\alpha > z_0) &= 2[1 - \Phi(|z_0|)] \\ &= 2[1 - \Phi(0.98)] \\ &= 2[1 - 0.836457] \end{aligned}$$

$$P(z_a > z_0) = 2[1 - \Phi(|z_0|) = 2[1 - \Phi(0.98)] = 2[1 - 0.836457] = 0.327$$

c) 95% two-sided CI on the mean:

$$98.285 - 1.96\frac{0.625}{\sqrt{52}} \leq \mu \leq 98.285 + 1.96\frac{0.625}{\sqrt{52}}$$

$$98.285 - \frac{1.96(0.625)}{\sqrt{52}} \leq \mu \leq 98.285 + \frac{1.96(0.625)}{\sqrt{52}}$$

$$98.11 \leq \mu \leq 98.45$$

We fail to rejected $H0$ because $98.2 \in (98.11, 98.45)$

9.91.

a) A one-sided test because the alternative hypothesis is $p < 0.6$
b) The test is the normal approximation ($np > 5$ and $n(1-p) > 5$
c) Sample: $\hat{p} = \frac{X}{N} = 0.574$
Z-value: $z_0 = \frac{x-np}{\sqrt{np(1-p)}} = \frac{287-500(0.6)}{\sqrt{500(0.6)(0.4)}} = -1.1867$
P-value: $\mathrm{P-value} = \Phi(-1.1867) = 0.117$

95% upper confident interval:

$$p \le \hat{p} + z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$p \le 0.574 + 1.65\sqrt{\frac{0.574(0.426)}{500}}$$

$p \le 0.6105$
d) P-value for two-side test is: $\mathrm{P-value} = 2[1 - \Phi(|-1.1867|)] = 0.234$

10.66.

1) The parameters of interest are the std σ_1, σ_2
2) $H_0: \sigma_1^2 = \sigma_2^2$
3) $H_1: \sigma_1^2 > \sigma_2^2$
4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 > f_{0.01;19;7} = 6.18$

6) $f_0 = \frac{4.5}{2.3} = 1.9565$
7) Conclusion: 1.9565 < 6.16 → fail to reject the null hypothesis.
95% CI:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{0.99;n_2-1;n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2}$$

$$1.9565\left(\frac{1}{3.77}\right) \leq \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.519 \leq \frac{\sigma_1^2}{\sigma_2^2}$$

→ There is no significant difference in the variances.

10.67.

a)

1) The parameters of interest are the standard deviations σ_1, σ_2
2) $H_0: \sigma_1^2 = \sigma_2^2$
3) $H_1: \sigma_1^2 \neq \sigma_2^2$
4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{0.975;14;14} = 0.33$ and $f_0 > f_{0.025;14;14} = 3$ for $\alpha = 0.05$
6) $f_0 = \frac{2.3}{1.9} = 1.2105$
7) Conclusion: Because 0.33 < 1.2105 < 3, we fail to reject the null hypothesis.
95% CI:

$$1.2105(0.33) \leq \frac{\sigma_1^2}{\sigma_2^2} \leq 1.2105(3)$$

$$0.399465 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq 3.6315$$

→ There is no significant difference in the variances.

b) We get $\beta = 0.35 \rightarrow 1 - \beta = 0.65$

c) We know that $\beta = 0.05$ and the σ_2 is half of $\sigma_1$than $\lambda = 2$. That sample size is: $n_1 = n_2 = n = 31$

10-6.

a)

1) The parameter of interest is the difference in mean burning rate, $\mu_1 - \mu_2$
2) $H_0: \mu_1 - \mu_2 = 0\ or\ \mu_1 = \mu_2$
3) $H_1: \mu_1 - \mu_2 \neq 0\ or\ \mu_1 \neq \mu_2$
4) The test statistic is

$$z_0 = \frac{\overline{x_1} - \overline{x_1} - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_{\frac{\alpha}{2}} = -1.96\ or\ z_0 > z_{\frac{\alpha}{2}} = 1.96$ for $\alpha = 0.05$
6)

$$z_0 = \frac{18 - 24}{\sqrt{\frac{3^2}{20} + \frac{3^2}{20}}} = -6.325$$

7) Conclusion: Because $-6.325 < -1.96$ reject the null hypothesis and conclude the mean burning rates differ significantly at $\alpha = 0.05$

$$P - value = 2\big(1 - \Phi(6.325)\big) = 2(1 - 1) = 0$$

b) CI for $\mu_1 - \mu_2$

$$(18-24) - 1.96\sqrt{\frac{3^2}{20}+\frac{3^2}{20}} \le \mu_1 - \mu_2 \le (18-24) + 1.96\sqrt{\frac{3^2}{20}+\frac{3^2}{20}}$$
$$-7.859 \le \mu_1 - \mu_2 \le -4.1406$$

We are 95% confident that the mean burning rate for solid fuel propellant 2 exceeds that of propellant 1 by between 4.14 and 7.86 cm/s.

c)

$$\beta = \Phi\left(1.96 - \frac{2.5}{\sqrt{\frac{3^2}{20}+\frac{3^2}{20}}}\right) - \Phi\left(-1.96 - \frac{2.5}{\sqrt{\frac{3^2}{20}+\frac{3^2}{20}}}\right)$$
$$= \Phi(-0.675) - \Phi(-4.595) = 0.25 - 0 = 0.25$$

→ The power of the test in part a) is $1 - \beta = 0.75$

d) Assume the sample sizes are to be equal, use $\alpha = 0.05, \beta = 1 - power = 0.1, \Delta = 14$

$$n = \frac{\left(z_{\frac{\alpha}{2}} + z_\beta\right)^2 (\sigma_1^2 + \sigma_2^2)}{\delta^2} = \frac{(1.96 + 1.28)^2(3^2 + 3^2)}{14^2}$$

So the sample size is $n = 1$

10-82.

a) The test is two-sided.

b) $\widehat{p_p} = \frac{x_1+x_2}{n_1+n_2} = \frac{114}{540} \approx 0.2111$

Value of test statistic

$$z = \frac{\widehat{p_1} - \widehat{p_2}}{\sqrt{\widehat{p_p} - (1 - \widehat{p_p})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{0.216 - 0.206897}{\sqrt{0.2111(1 - 0.2111)}\sqrt{\frac{1}{250} + \frac{1}{290}}} \approx 0.26$$

$$\mathrm{P-value} = P(Z < -0.26\ or\ Z > \ 0.26) = 2P(Z < -0.26) = 2(0.397432) = 0.794864$$

c) P > 0.05 →Fail to reject H_0

There is not sufficient evidence to reject the null hypothesis.

d) For confidence level $1 - \alpha = 0.9$, we have $z_{\alpha/2} = z_{0.05} = 1.64$

$$\widehat{p_1} - \widehat{p_2} - z_{\frac{\alpha}{2}}\sqrt{\frac{\widehat{p_1}(1 - \widehat{p_1})}{n_1} + \frac{\widehat{p_2}(1 - \widehat{p_2})}{n_2}}$$

$$= (0.216 - 0.206897) - 1.64\sqrt{\frac{0.216(1 - 0.216)}{250} + \frac{0.206897(1 - 0.206897)}{290}} \approx -0.0487$$

$$\widehat{p_1} - \widehat{p_2} + z_{\frac{\alpha}{2}} \sqrt{\frac{\widehat{p_1}(1 - \widehat{p_1})}{n_1} + \frac{\widehat{p_2}(1 - \widehat{p_2})}{n_2}}$$

$$= (0.216 - 0.206897) + 1.64 \sqrt{\frac{0.216(1 - 0.216)}{250} + \frac{0.206897(1 - 0.206897)}{290}} \approx 0.0669$$

$$\Rightarrow -0.0487 < p_1 - p_2 < 0.0669$$