Slot 12. BTVN

a)

$P(1 < Y < 3 \mid X = 1) = P(1 < Y < 2 \mid X = 1) + P(2 < Y < 3 \mid X = 1) = 0 + P(2 < Y < 3 \mid X = 1)$

We have formula:

$f(y|x) = \frac{f(x,y)}{g(x)}$

Find $g(x) = \int_2^4 \frac{6-x-y}{8} dy = \frac{3-x}{4}$, where $0 < x < 2$

Find $f(y|x)$, where $x = 1$:

$P(2 < Y < 3|X = 1) = \int_2^3 (\frac{6-x-y}{8} : \frac{3-x}{4})dy|_{x=1} = (\frac{5}{4}y - \frac{y^2}{8})|_2^3 = \frac{5}{8}$

b) Marginal probability of X and Y

$f_X(x) = \int_2^4 \frac{6-x-y}{8} dy = \frac{-x+3}{4}$

$f_Y(y) = \int_0^2 \frac{6-x-y}{8} dx = \frac{-y+5}{4}$

c) Conditional probability distribution of Y given that $X = x$

$f(y|x) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{(6-x-y):8}{(-x+3):4} = \frac{6-x-y}{-2x+6}$

d)

$E(X) = \int_0^2 xp(x)dx = \frac{5}{6}$

$E(Y) = \int_2^4 yp(y)dy = \frac{17}{6}$

$V(Y) = \int_2^4 (y - E(Y))^2 p(y)dy = \frac{11}{36}$

$V(X) = \int_0^2 (x - E(X))^2 p(x)dx = \frac{11}{36}$

$Cov(X,Y) = E(XY) - E(X)E(Y) = \int_0^2 \int_2^4 xy * p(x,y)dxdy - \frac{5}{6} \times \frac{17}{6} = \frac{-1}{36}$

$Corr(X,Y) = Cov(X,Y)/std(X)/std(Y) = 7/3/(11/36) = -1/11$

e) $f(y|x) = \frac{6-x-y}{-2x+6} <> f_Y(y) = \frac{-y+5}{4} \implies Dependent$