

# Slot 11. BTVN

## Ex1

a)  $f(x, y) = cxy$ , for  $x = 1, 2, 3; y = 1, 2, 3$

$$1 = \sum_{x=1}^3 \sum_{y=1}^3 cxy = c \sum_{x=1}^3 x \sum_{y=1}^3 y = c((1+2+3)(1+2+3)) = c(6 \cdot 6) = 36c$$

$$\text{Therefore, } 36c = 1 \implies c = \frac{1}{36}$$

b)  $f(x, y) = c|x - y|$ , for  $x = -2, 0, 2; y = -2, 3$

$$1 = \sum_x \sum_y c|x - y| = c \sum_x \sum_y |x - y| = c(|(-2) - (-2)| + |0 - (-2)| + |2 - (-2)| + |(-2) - 3| + |0 - 3| + |2 - 3|) = c(0 + 2 + 4 + 5 + 3 + 1) = 15c$$

$$\text{Therefore, } 15c = 1 \implies c = \frac{1}{15}$$

## Ex2

a)

$$\begin{aligned} P(0 \leq X \leq \frac{1}{2} \text{ and } \frac{1}{4} \leq Y \leq \frac{1}{2}) &= \int_0^{\frac{1}{2}} \int_{\frac{1}{4}}^{\frac{1}{2}} f(x, y) dy dx \\ &= \int_0^{\frac{1}{2}} \int_{\frac{1}{4}}^{\frac{1}{2}} 4xy dy dx \\ &= \int_0^{\frac{1}{2}} 2x \left( \int_{\frac{1}{4}}^{\frac{1}{2}} 2y dy \right) dx \\ &= \int_0^{\frac{1}{2}} 2x \left( y^2 \right) \Big|_{\frac{1}{4}}^{\frac{1}{2}} dx \\ &= \left( \left( \frac{1}{2} \right)^2 - \left( \frac{1}{4} \right)^2 \right) \int_0^{\frac{1}{2}} 2x dx \\ &= \left( \frac{1}{4} - \frac{1}{16} \right) \left( x^2 \right) \Big|_0^{\frac{1}{2}} \\ &= \frac{3}{16} \cdot \left( \frac{1}{2} \right)^2 \end{aligned}$$

$$= \frac{3}{64}$$

b)

$$\begin{aligned}
 P(X < Y) &= P(0 < X < 1, X < Y < 1) \\
 &= \int_0^1 \int_x^1 f(x, y) dy dx \\
 &= \int_0^1 \int_x^1 4xy \, dy dx \\
 &= \int_0^1 2x \left( \int_x^1 2y \, dy \right) dx \\
 &= \int_0^1 2x(y^2) \Big|_x^1 dx \\
 &= \int_0^1 2x(1 - x^2) dx \\
 &= \int_0^1 (2x - 2x^3) dx \\
 &= \left( x^2 - \frac{x^4}{2} \right) \Big|_0^1 \\
 &= 1^2 - \frac{1^4}{2} \\
 &= \frac{1}{2}
 \end{aligned}$$