Name: Tran Thanh Duong (SE160185)

Q4.3

a. $\begin{vmatrix} x-2 & -1 \\ -3 & x \end{vmatrix} = (x-2)x - (-1)(-3) = x^2 - 2x - 3$

b. $\begin{vmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{vmatrix} = (-3) \times 5 \times 1 + 2 \times 6 \times 2 + 1 \times 4 \times (-3) - 1 \times 5 \times 2 - (-3) \times 6 \times (-3) - 2 \times 4 \times 1 = (-15) + 24 - 12 - 10 - 54 - 8 = -75$

c. $\begin{vmatrix} 2 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{vmatrix} = (-1)^{3+3} \times 4 \times \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = 4 \times (2 \times 2 - (-1) \times 0) = 4 \times 4 = 16$

d. $\begin{vmatrix} x & y & 1 \\ -1 & -2 & 1 \\ 1 & 5 & 1 \end{vmatrix} = x \times (-2) \times 1 + y \times 1 \times 1 + 1 \times (-1) \times 5 - 1 \times (-2) \times 1 - x \times 1 \times 5 - y \times (-1) \times 1 = -2x + y - 5 + 2 - 5x + y = -7x + 2y - 3$

e. $\begin{vmatrix} m & -1 & 0 \\ 1 & 2 & 1 \\ 2 & m & -3 \end{vmatrix} = (-1)^{1+1} \times m \times \begin{vmatrix} 2 & 1 \\ m & -3 \end{vmatrix} + (-1)^{1+2} \times (-1) \times \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = m \times (-6 - m) + (-3 - 2) = -m^2 - 6m - 5$

Q4.8a. $A = \begin{pmatrix} -3 & 5 \\ 10 & 2 \end{pmatrix}$

Find eigenvalues:

$$\det(A - \lambda I) = 0$$

$$\Leftrightarrow \begin{vmatrix} -3 - \lambda & 5 \\ 10 & 2 - \lambda \end{vmatrix} = 0$$

$$\Leftrightarrow (-3 - \lambda)(2 - \lambda) - 5 \times 10 = 0$$

$$\Leftrightarrow \lambda^2 + \lambda - 56 = 0$$

$$\Leftrightarrow \begin{bmatrix} \lambda = -8 \\ \lambda = 7 \end{bmatrix}$$

Find eigenvectors:

$$(A - \lambda I)x = 0, x \in R^2 - \{0\}$$

Suppose $x = \begin{pmatrix} a \\ b \end{pmatrix}$

With $\lambda = -8$:

$$(A + 8I)x = 0$$

$$\Leftrightarrow \begin{pmatrix} -3 + 8 & 5 \\ 10 & 2 + 8 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 5 & 5 \\ 10 & 10 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 5a + 5b \\ 10a + 10b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -b \\ b \end{pmatrix} = b\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\Rightarrow x = b\begin{pmatrix} -1 \\ 1 \end{pmatrix}, b \neq 0$$

With $\lambda = 7$:

$$(A - 7I)x = 0$$

$$\Leftrightarrow \begin{pmatrix} -3 - 7 & 5 \\ 10 & 2 - 7 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} -10 & 5 \\ 10 & -5 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} -10a + 5b \\ 10a - 5b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ 2a \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a \\ b \end{pmatrix} = a\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow x = a\begin{pmatrix} 1 \\ 2 \end{pmatrix}, a \neq 0$$