## Slot 11. BTVN

## Ex1

a) 
$$f(x,y)=cxy$$
, for  $x=1,2,3;y=1,2,3$  
$$1=\sum_{x=1}^3\sum_{y=1}^3cxy=c\sum_{x=1}^3x\sum_{y=1}^3y=c((1+2+3)(1+2+3))=c(6\cdot 6)=36c$$

Therefore,  $36c=1 \implies c=\frac{1}{36}$ 

b) 
$$f(x,y)=c|x-y|$$
, for  $x=-2,0,2;y=-2,3$  
$$1=\sum_x\sum_yc|x-y|=c\sum_x\sum_y|x-y|=c(|(-2)-(-2)|+|0-(-2)|+|2-(-2)|+|(-2)-3|+|0-3|+|2-3|)=c(0+2+4+5+3+1)=15c$$

Therefore,  $15c=1 \implies c=\frac{1}{15}$ 

## Ex2

a)

$$egin{aligned} P(0 \leq X \leq rac{1}{2} ext{ and } rac{1}{4} \leq Y \leq rac{1}{2}) &= \int_0^{rac{1}{2}} \int_{rac{1}{4}}^{rac{1}{2}} f(x,y) dy dx \ &= \int_0^{rac{1}{2}} \int_{rac{1}{4}}^{rac{1}{2}} 4xy \ dy dx \ &= \int_0^{rac{1}{2}} 2x igg( \int_{rac{1}{4}}^{rac{1}{2}} 2y \ dy igg) dx \ &= \int_0^{rac{1}{2}} 2x igg( y^2 igg) igg|_{rac{1}{4}}^{rac{1}{2}} dx \ &= igg( igg( rac{1}{2} igg)^2 - igg( rac{1}{4} igg)^2 igg) \int_0^{rac{1}{2}} 2x \ dx \ &= \left( rac{1}{4} - rac{1}{16} igg) igg( x^2 igg) igg|_0^{rac{1}{2}} \ &= rac{3}{16} \cdot igg( rac{1}{2} igg)^2 \end{aligned}$$

$$=\frac{3}{64}$$

b)

$$egin{aligned} P(X < Y) &= P(0 < X < 1, X < Y < 1) \ &= \int_0^1 \int_x^1 f(x,y) dy dx \ &= \int_0^1 \int_x^1 4xy \ dy dx \ &= \int_0^1 2x igg( \int_x^1 2y \ dy igg) dx \ &= \int_0^1 2x (y^2) igg|_x^1 dx \ &= \int_0^1 2x (1-x^2) dx \ &= \int_0^1 (2x-2x^3) dx \ &= igg( x^2 - rac{x^4}{2} igg) igg|_0^1 \ &= 1^2 - rac{1^4}{2} \ &= rac{1}{2} \end{aligned}$$