1 Kinematic Model of a Cart with N Omnidirectional Wheels

In order to describe the movements of the cart, it is necessary to model its behavior in some way. The simplest model of the movement of a cart in space is a kinematic one. This model describes movements exclusively through the dependence of coordinates on time. That is, in the kinematic model, the movement of the body is considered, but the reasons that create it are not considered.

Consider the movement of a cart with N omnidirectional wheels ($N > 3$) on a smooth two-dimensional surface without taking into account the acting forces, and the plane of the cart wheels is vertical and stationary relative to the cart platform. Within the model, omnidirectional wheels can slide in any direction with a negligible friction force. Let a global coordinate system associated with the surface $\{o, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}$ and a local, inertial relative to the global one, rigidly connected with the cart $\{c, \boldsymbol{x_l}, \boldsymbol{y_l}, \boldsymbol{z_l}\}$, with the plane $\boldsymbol{x_l y_l}$ parallel to the plane $\boldsymbol{xy}$. Without loss of generality, we put the origin of local coordinates at the point of the center of mass of the cart. The position of the cart is determined by the coordinate vector (x, y, φ) where x, y are the coordinates, and φ is the angle between the axis $\boldsymbol{ox}$ and $\boldsymbol{cx_l}$. The speed of the cart is determined by the vector $(\dot{x}, \dot{y}, \omega)$ where $\omega = \dot{\varphi}$ is the angular velocity of the cart.

We denote by $\{c_i, \boldsymbol{x_{w,i}}, \boldsymbol{y_{w,i}}, \boldsymbol{z_{w,i}}\}$ - the local coordinate system i th wheel, shown in the figure 1.1, where c_i is the axis of rotation, $\boldsymbol{x_{w,i}}$ is the axis directed from c_i towards the point of tangency with the surface, $\boldsymbol{y_{w,i}}$ is the axis parallel to the rolling surface, directed to the right, $\boldsymbol{z_{w,i}} = \boldsymbol{x_{w,i}} \times \boldsymbol{y_{w,i}}$.

For example, the figure 1.2 schematically shows a kinematic model of a cart with three omnidirectional wheels.

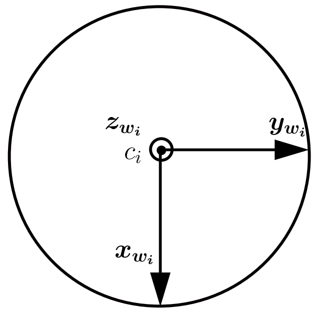

Figure 1.1 – coordinate axes of the i th wheel.

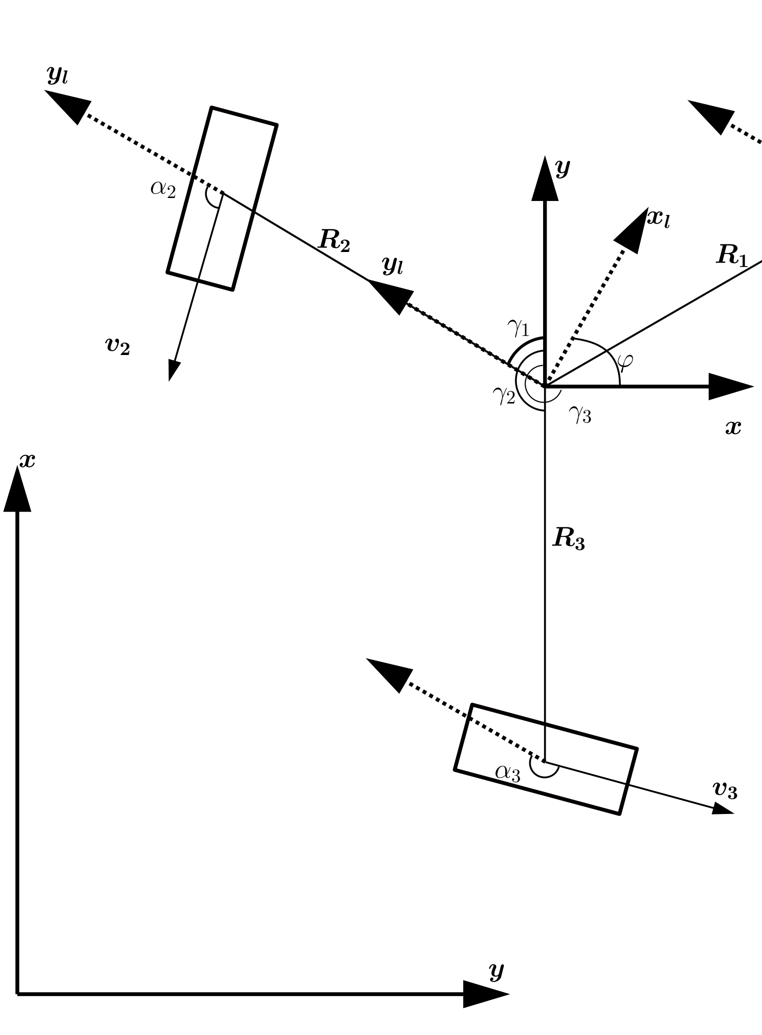

Figure 1.2 – kinematic model of a cart with three omnidirectional wheels.

Consider the velocity vector of the i th wheel, $\boldsymbol{v_i}$, $i = \overline{1, N}$. It is directed tangent to the disk i of the wheel $\boldsymbol{y_{w_i}}$. We decompose it into translational and rotational components:

$$\boldsymbol{v_i} = \boldsymbol{v_{i,tr}} + \boldsymbol{v_{i,rot}} \tag{1.1}$$

Denote by α_i the angle between $\boldsymbol{y_{w,i}}$ and the axis $\boldsymbol{y_l}$. The vectors $\boldsymbol{v_i}$ and $\boldsymbol{y_{w,i}}$ are aligned, and therefore $\boldsymbol{v_i}$ is $\boldsymbol{y_l}$ angle α_i, as shown in the figure 1.2.

Consider the translational velocity vector of the cart $\boldsymbol{v_{tr}} = [\dot{x}, \dot{y}, 0]$. In order for the trolley to have a speed of $\boldsymbol{v_{tr}}$, the value of the speed vector of the i th wheel should be equal to the projection of $\boldsymbol{v_{tr}}$ in the direction $\boldsymbol{y_{w,i}}$. The vector of translational speed of the i - th wheel with the $\boldsymbol{y}$ axis makes the angle $\varphi + \alpha_i$, which is illustrated in the figure 1.3. Then $\boldsymbol{v_{tr,i}}$ can be represented as

$$\boldsymbol{v_{tr,i}} = [-sin(\varphi + \alpha_i)\dot{x} + cos(\varphi + \alpha_i)\dot{y}]\boldsymbol{y_{w,i}} \tag{1.2}$$

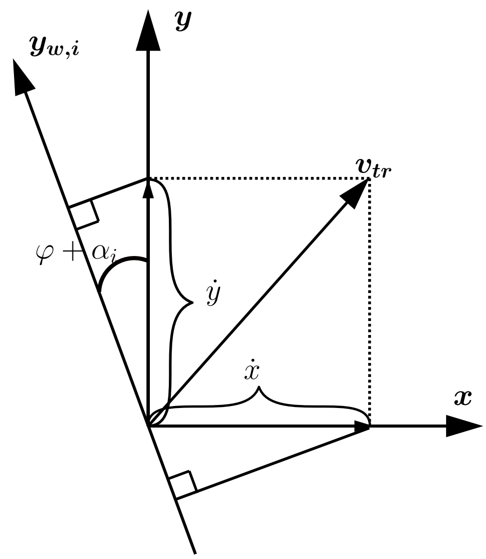

Figure 1.3 – illustration of projecting the translational velocity vector in the direction of movement of the i - th wheel

Let $\boldsymbol{R_i}$ be the radius of the vector of the axis of the i th wheel from the axis of rotation (center of mass point) are long R_i. In order to make the trolley rotate around the axis of rotation $C\boldsymbol{z_l}$ with the angular velocity ω, it is necessary to set the velocity vector of each wheel perpendicular to $\boldsymbol{R_i}$. The length of the vector should be $R_i\omega$. Thus, the rotational motion velocities of i - th are given by the following equation

$$\boldsymbol{v_{rot,i}} = \boldsymbol{R_i} \times \omega \boldsymbol{z} \tag{1.3}$$

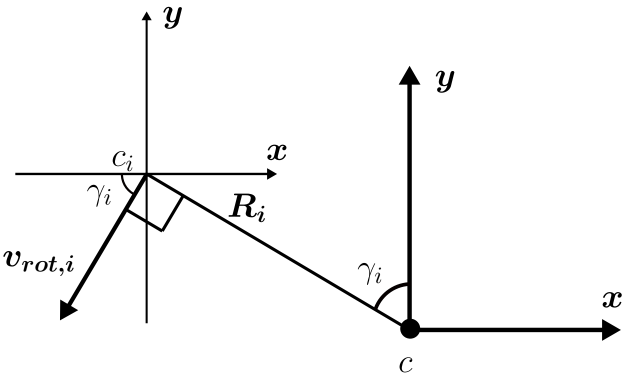

Figure 1.4 – projection of the rotational velocity vector of the i - th wheel on the global coordinate system

Let γ_i be the angle between the axis $\boldsymbol{y}$ and $\boldsymbol{R_i}$. Then first projecting $\boldsymbol{v_{rot,i}}$ onto $\boldsymbol{xy}$, as shown in the figure 1.4, we get

$$\boldsymbol{v_{rot,i}} = R_i\omega[-cos(\gamma_i)\boldsymbol{x} - sin(\gamma_i)\boldsymbol{y}] \tag{1.4}$$

Projecting $\boldsymbol{v_{rot,i}}$ in the direction $\boldsymbol{y_{w,i}}$, similar to the equation 1.2, we obtain

$$\boldsymbol{v_{rot,i}} = R_i\omega[sin(\varphi + \alpha_i)cos(\gamma_i) - cos(\varphi + \alpha_i)sin(\gamma_i)]\boldsymbol{y_{w,i}}$$

$$\boldsymbol{v_{rot,i}} = R_i\omega sin(\varphi + \alpha_i - \gamma_i)\boldsymbol{y_{w,i}} \tag{1.5}$$

It is easy to see that the angle $\theta_i = \gamma_i - \varphi = const$ is the angle between $\boldsymbol{R_i}$ and $\boldsymbol{y_l}$, therefore the coefficient the angular velocity vector is independent of φ in the general case. Obviously $\alpha_i - \theta_i$ is the angle between $\boldsymbol{y_{w,i}}$ and $\boldsymbol{R_i}$. If the tangent of the disk i of the wheel $y_{w,i}$ is perpendicular to R_i, then $\alpha_i - \theta_i = \frac{\pi}{2}$, $\boldsymbol{v_{rot,i}} = R_i\omega$.

Thus, the velocity vector of the i th wheel can be written as:

$$\boldsymbol{v_i} = [-sin(\varphi + \alpha_i)\dot{x} + cos(\varphi + \alpha_i)\dot{y} + R_i\omega sin(\alpha_i - \theta_i)]\boldsymbol{y_{w,i}} \tag{1.6}$$

Let ω_i be the desired angular velocity of the i th wheel. We correlate the velocity vector of the i - th wheel with its angular velocity

$$\boldsymbol{v_i} = \boldsymbol{r_i} \times \omega_i \boldsymbol{z_{w,i}} \tag{1.7}$$

Where $\boldsymbol{r_i}$ is a vector having the coordinates $[-r_i, 0, 0]$ in the coordinate system of the i -th wheel of the cart, r_i is the radius of i wheel of the cart, ω_i is the angular velocity of this wheel. Computing the vector product in 1.7, we get

$$\boldsymbol{v_i} = \boldsymbol{r_i} \times \omega_i \boldsymbol{z_{w,i}} = \omega_i r_i \boldsymbol{y_{w,i}} \tag{1.8}$$

Thus, expressing from 1.8 w_i, substituting $\boldsymbol{v_i}$ from 1.6, we get:

$$\omega_i = \frac{1}{r_i}(-sin(\varphi + \alpha_i)\dot{x} + cos(\varphi + \alpha_i)\dot{y} + R_i \omega sin(\alpha_i - \theta_i)) \tag{1.9}$$

or in matrix form

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ ... \\ \omega_N \end{bmatrix} = \begin{bmatrix} -\frac{1}{r_1}sin(\varphi + \alpha_1) & \frac{1}{r_1}cos(\varphi + \alpha_1) & \frac{1}{r_1}R_1 sin(\alpha_1 - \theta_1) \\ -\frac{1}{r_2}sin(\varphi + \alpha_2) & \frac{1}{r_2}cos(\varphi + \alpha_2) & \frac{1}{r_2}R_2 sin(\alpha_2 - \theta_2) \\ ... & ... & ... \\ -\frac{1}{r_N}sin(\varphi + \alpha_N) & \frac{1}{r_N}cos(\varphi + \alpha_N) & \frac{1}{r_N}R_N sin(\alpha_N - \theta_N) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \omega \end{bmatrix} \tag{1.10}$$

Thus, knowing the coordinates of the path path (x, y, φ), we can obtain the magnitude of the angular velocity of each wheel. To do this, calculate $(\dot{x}, \dot{y}, \omega)$ and substitute in the formula 1.10.

2 Dynamic Model of a Cart with N Omnidirectional Wheels

The dynamic model, in contrast to the kinematic, considers the motion of solids taking into account the forces that bring this body into motion.

In order to set the trolley in motion, it is necessary to transmit T torque to its wheels. By increasing the speed of the trolley, we overcome its resting resistance and give it acceleration.

Consider the movement of a trolley with N omnidirectional wheels ($N > 3$) on a nonsmooth two-dimensional surface, taking into account the acting forces. In the description of the model, the same notation is used as in the kinematic model exceptMoreover, instead of velocity vectors, we consider the force vectors $\boldsymbol{f_i}$. The force $\boldsymbol{f_i}$ is applied to the upper point of the i th wheel in the rolling direction. For example, the figure 2.1 schematically shows a dynamic model of a cart with three omnidirectional wheels. Consider the resultant of all the forces acting on the cart $\boldsymbol{F}$.

$$\boldsymbol{F} = F_x\boldsymbol{x} + F_y\boldsymbol{y} + M_t\boldsymbol{z} \tag{2.1}$$

Force as a measure of the impact on the body characterizes the translational motion of the model, while the moment of forces $M_t\boldsymbol{z}$ characterizes the rotational motion. The resultant of all forces can be decomposed into translational and rotational components, similar to the kinematic model, as follows:

$$F = \begin{bmatrix} F_X \\ F_Y \\ M_t \end{bmatrix} = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & J_r \end{bmatrix} \cdot \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \dot{\omega} \end{bmatrix} = B \cdot \ddot{u} \tag{2.2}$$

Where M is the mass of the trolley, J_r is its moment of inertia.

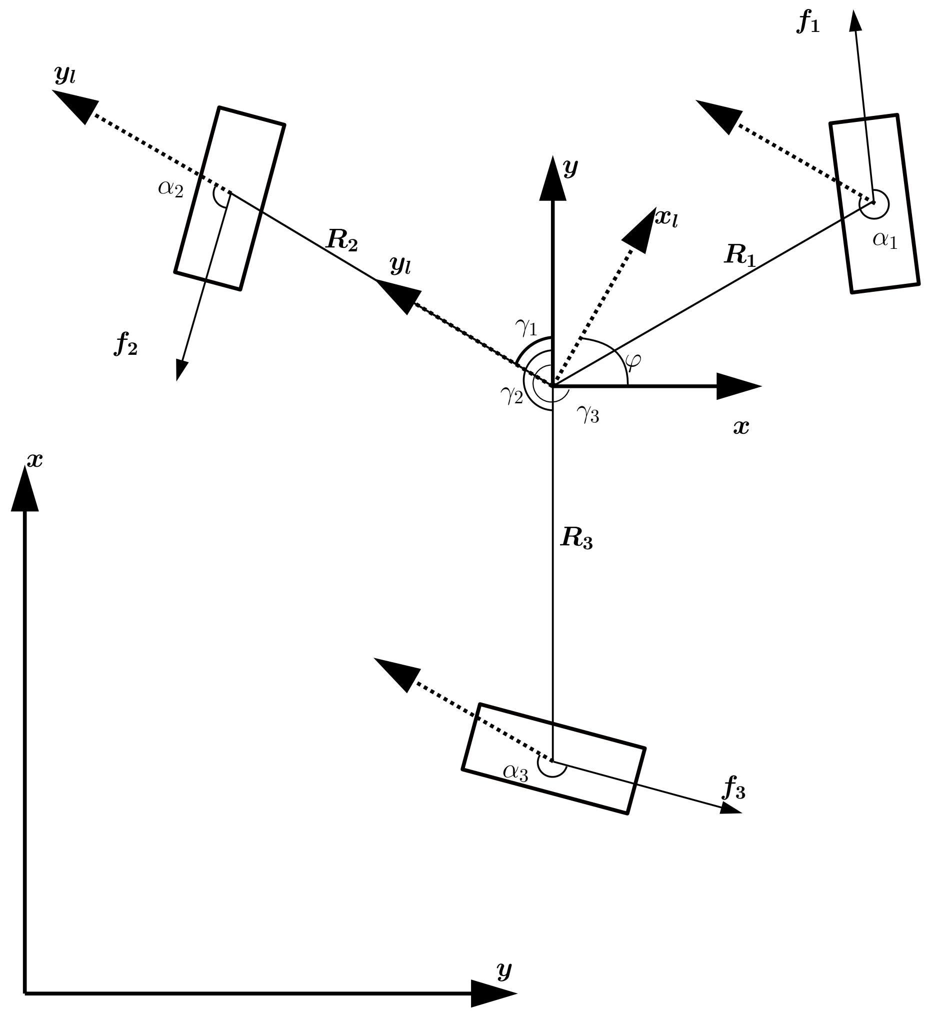

Figure 2.1 – dynamic model of a cart with three omnidirectional wheels

The moment of inertia of the bogie can be considered approximately equal to the moment of inertia of a homogeneous cylinder of radius $R = max_i(R_i)$

$$J_r = \frac{1}{2}MR^2 \tag{2.3}$$

In addition, the vector $\boldsymbol{F}$ can be represented as:

$$\boldsymbol{F} = \sum_{i=1}^{N} \boldsymbol{f_i} \tag{2.4}$$

where $\boldsymbol{f_i}$ is the force vector applied to the top point of the i th wheel. It has a direction of $\boldsymbol{y_{w,i}}$, a value of f_i and makes an angle $\varphi + \alpha_i$ with the axis $\boldsymbol{y}$.

Then, expanding $\boldsymbol{F}$ into the components and projecting the vectors $\boldsymbol{f_i}$ onto the coordinate axes, we get:

$$\boldsymbol{F_X} = -\boldsymbol{x}\sum_{i=1}^{N} f_i sin(\varphi + \alpha_i) \tag{2.5}$$

$$\boldsymbol{F_Y} = \boldsymbol{y}\sum_{i=1}^{N} f_i cos(\varphi + \alpha_i) \tag{2.6}$$

$$\boldsymbol{M_t} = \sum_{i=1}^{N} \boldsymbol{f_i} \times \boldsymbol{R_i} = \boldsymbol{z}\sum_{i=1}^{N} f_i R_i sin(\alpha_i - \theta_i) \tag{2.7}$$

Then the values of the components of the force vector have the following relationship:

$$\begin{bmatrix} F_X \\ F_Y \\ M_t \end{bmatrix} = \begin{bmatrix} -sin(\varphi + \alpha_1) & -sin(\varphi + \alpha_2) & ... & -sin(\varphi + \alpha_N) \\ cos(\varphi + \alpha_1) & cos(\varphi + \alpha_2) & ... & cos(\varphi + \alpha_N) \\ R_1 sin(\alpha_1 - \theta_1) & R_2 sin(\alpha_2 - \theta_2) & ... & R_N sin(\alpha_N - \theta_N) \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ ... \\ f_n \end{bmatrix} = Af \tag{2.8}$$

We get the equation

$$F = af \tag{2.9}$$

Solving the equation 2.9 with respect to f, we obtain

$$f = A^{-1} \cdot F = A^{-1} \cdot B \cdot \ddot{u} \tag{2.10}$$

Knowing that the torque of the wheel is equal to the force applied at its top point on its radius and denoting

$$\overline{r} = \begin{bmatrix} r_1 \\ r_2 \\ ... \\ r_N \end{bmatrix} \tag{2.11}$$

We obtain the dependence of the coordinates of the path (x, y, φ) on the torques of each wheel:

$$\begin{bmatrix} T_1 \\ T_2 \\ \cdots \\ T_N \end{bmatrix} = \overline{r} f^T = \overline{r}(A^{-1} \cdot B \cdot \ddot{u})^T \tag{2.12}$$

In turn, the magnitude of the moment of forces of each wheel can be expressed through its angular acceleration:

$$T_i = J_{w,i}\dot{\omega}_i \tag{2.13}$$

Where $J_{w,i}$ is the moment of inertia of the i th wheel, $\dot{\omega}_i$ is the angular acceleration of the i th wheel.

We take into account the rolling friction force of the model on a certain surface, assuming a uniform distribution of weight on each wheel. It is known that the rolling friction force is opposite in direction to the force that drives the wheel and for the i -th wheel can be calculated by the formula:

$$f_{t_i} = \frac{Mg\eta}{Nr_i} \tag{2.14}$$

Where Mg is the support reaction force equal to the gravity of the cart, η is the rolling friction coefficient, which depends on the surface characteristics. $\eta = 0$ in the case when friction between the wheel and the surface does not occur and $\eta = \infty$ when the friction of the surface is insurmountably strong. The rolling friction force is directed in the opposite direction from the direction of wheel movement. Therefore, in order to compensate for it, it is necessary to increase the force applied at the upper point of the wheel by the value of the friction force.

$$\tilde{f}_i = f_i + f_{t_i} \tag{2.15}$$

We denote

$$\tilde{f} = f + \begin{bmatrix} f_{t_1} \\ f_{t_2} \\ \dots \\ f_{t_N} \end{bmatrix} = f + \frac{Mg\eta}{N} \cdot \begin{bmatrix} \frac{1}{r_1} \\ \frac{1}{r_2} \\ \dots \\ \frac{1}{r_N} \end{bmatrix} \tag{2.16}$$

Then replacing in the formula 2.12 f with $\tilde{f}$, getm the dependence of the moment of forces of each wheel on the coordinates of the path, taking into account the friction force:

$$\begin{bmatrix} T_1 \\ T_2 \\ \dots \\ T_N \end{bmatrix} = \overline{r}(A^{-1} \cdot B \cdot \ddot{u} + \frac{Mg\eta}{N} \cdot \begin{bmatrix} \frac{1}{r_1} \\ \frac{1}{r_2} \\ \dots \\ \frac{1}{r_N} \end{bmatrix})^T \tag{2.17}$$

Thus, we obtain a dynamic model of the trolley, taking into account its mass and rolling friction force. Knowing the coordinates of the path (x, y, φ) and calculating the acceleration of each coordinate $(\ddot{x}, \ddot{y}, \dot{\omega})$, from the equation 2.17 we can get the moments of forces of each wheel corresponding to a given path.

3 Kinematic Model of a Omnidirectional Wheel

For a non-supported wheel that rotates on a surface without slipping, the instantaneous speed of the lowest point of contact is zero [Link to source]. In practice, contacting bodies, due to physical limitations, are always in contact with many points called the contact patch. As an illustration, you can bring a caterpillar trolley moving on the surface without slipping. The caterpillar's contact spot is large enough to notice that at a certain moment it does not move relative to the surface. At a fairly low speed of the cart, you can even notice that some sub-area of the track's contact spot does not move relative to the surface for a certain period of time.

A feature of roller-bearing wheels is that the instantaneous point of contact has a non-zero velocity relative to the rolling surface, since the rollers mounted on the wheel under the gravity of the trolley rotate about its axis.

The simplest non-holonomic model of a roller-bearing wheel is a flat disk, for which the speed of the point of contact with the bearing surface is directed along a straight line, making a constant angle with the plane of the wheel.

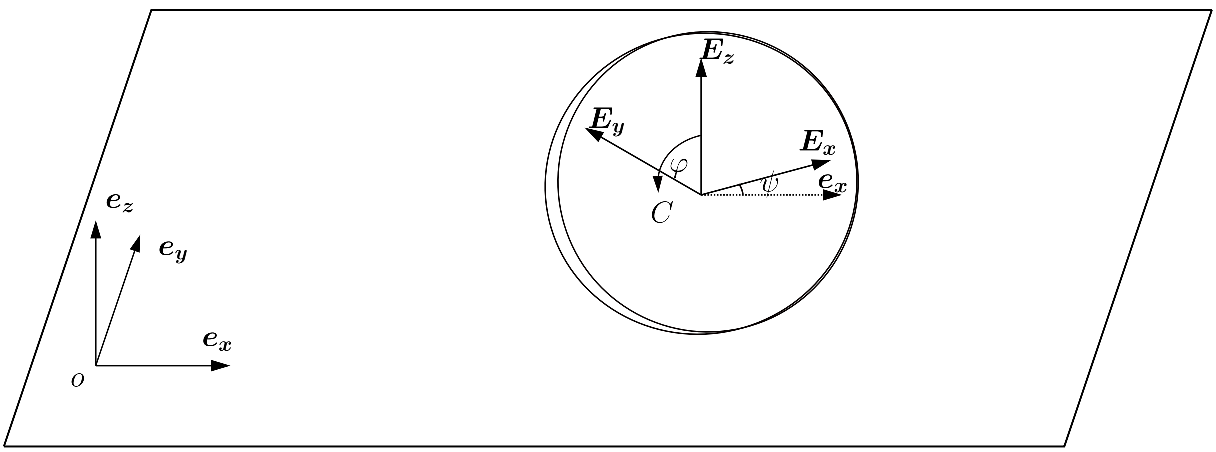

Figure 3.1 – roller model: side view

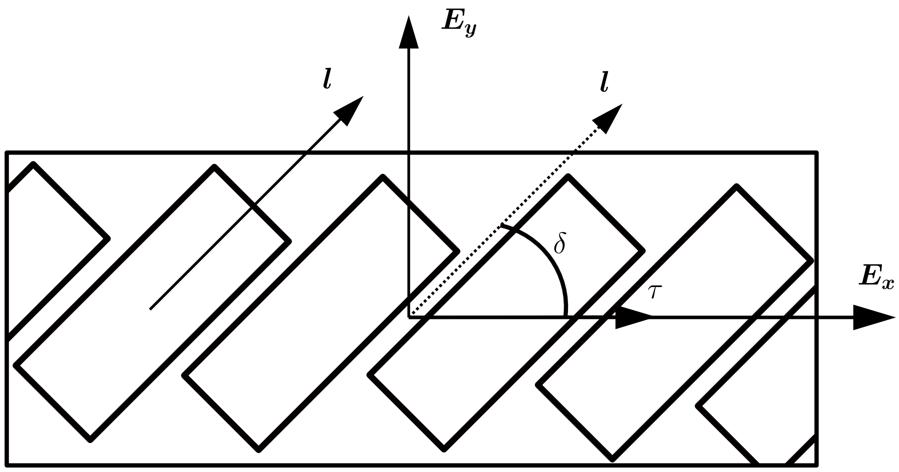

Figure 3.2 – roller model: top view

As a model of a roller-bearing wheel, we consider a disk of radius R centered at the point C, and the plane of the wheels of the cart is vertical and stationary relative to the platform of the cart. Let $\boldsymbol{l}$ be the unit vector of the axis of rotation of the roller, τ be the tangent to the disk plane, $\delta = \widehat{\boldsymbol{l}\tau}$. Then the coupling equation is written as follows:

$$(\boldsymbol{v}_M, \boldsymbol{l}) = 0 \tag{3.1}$$

where M is the point of tangency, $\boldsymbol{v}_M$ is the speed of the point of tangency. The vector $\boldsymbol{v}_M$ is defined as follows:

$$\boldsymbol{v}_M = \boldsymbol{v}_C + \boldsymbol{\omega} \times \boldsymbol{CM} \tag{3.2}$$

Where $\boldsymbol{v}_C$ - speed of the center of the wheel, $\boldsymbol{\omega}$ - vector angular speed of the wheel. Let $\{O, \boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z\}$ be the global inertial coordinate system, $\{C, \boldsymbol{E}_x, \boldsymbol{E}_y, \boldsymbol{E}_z\}$ - a coordinate system that is rigidly connected to the disk. The plane $\{C, \boldsymbol{E}_x, \boldsymbol{E}_y\}$ is parallel to the plane of the surface. Let φ be the angle of rotation of the disk around the perpendicular plane of the disk axis passing through the point C counterclockwise, $\psi = \widehat{\tau \boldsymbol{e}_x}$. The designations are illustrated in the figures 3.1 and 3.2. Then the vectors $\boldsymbol{\omega}$ and $\boldsymbol{CM}$ can be represented as:

$$\boldsymbol{\omega} = \dot{\varphi}\boldsymbol{E}_y + \dot{\psi}\boldsymbol{E}_z \tag{3.3}$$

$$\boldsymbol{CM} = -R\boldsymbol{E_z} \tag{3.4}$$

From here it is fair

$$\boldsymbol{\omega} \times \boldsymbol{CM} = -R\dot{\varphi}\boldsymbol{E_x} \tag{3.5}$$

Denoting by x_C y_C the coordinates of the point C in global coordinates, projecting the velocity vector of the point C onto the local coordinate system, as illustrated in the figure 3.3, we can write

$$\boldsymbol{v_C} = (\dot{x_C}cos\psi + \dot{y_C}sin\psi)\boldsymbol{E_x} + (-\dot{x_C}sin\psi + \dot{y_C}cos\psi)\boldsymbol{E_y} \tag{3.6}$$

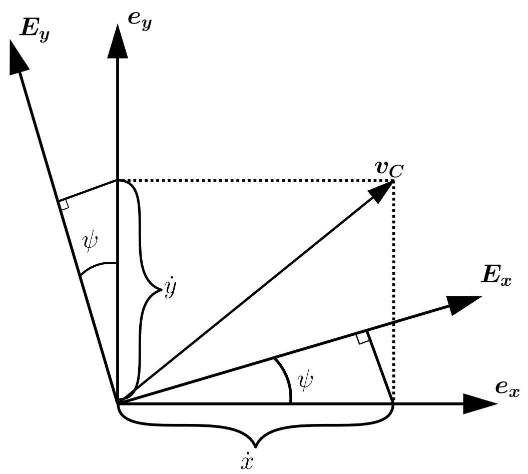

Figure 3.3 – illustration of designing a velocity vector on a local coordinate system

Finally, projecting the unit vector l onto the local coordinate system

$$\boldsymbol{l} = cos\delta\boldsymbol{E_x} + sin\delta\boldsymbol{E_y} \tag{3.7}$$

We can write the coupling equation in the form

$$(\boldsymbol{v_m}, \boldsymbol{l}) = (\boldsymbol{v_c} + \boldsymbol{\omega} \times \boldsymbol{CM}, \boldsymbol{l}) =$$
$$((\dot{x_C}cos\psi + \dot{y_C}sin\psi - R\dot{\varphi})\boldsymbol{E_x} + (-\dot{x_C}sin\psi + \dot{y_C}cos\psi)\boldsymbol{E_y}, cos\delta\boldsymbol{E_x} + sin\delta\boldsymbol{E_y}) =$$
$$(\dot{x_C}cos\psi + \dot{y_C}sin\psi - R\dot{\varphi})cos\delta + (-\dot{x_C}sin\psi + \dot{y_C}cos\psi)sin\delta = 0$$

$$\dot{x_C} cos(\psi + \delta) + \dot{y_C} sin(\psi + \delta) = R\dot{\varphi} cos\delta \quad (3.8)$$

The equation 3.8 connects the three coordinates (x_C, y_C, ψ) with the angular velocity of the wheel φ. In the work [link to source], the dynamics of a trolley with N roller-bearing wheels was studied and a controllability criterion was formulated: if $N \geq 3$, at least one pair of vectors $\boldsymbol{l_i}, \boldsymbol{l_j}$ $i, j = \overline{1, N}$, which correspond to the axes of the rollers , is not parallel and the contact points of the wheels do not lie on one straight line, then for any trajectory it is always possible to find such control moments (that is, functions $\varphi_i(t)$) that the cart, controlled by these moments, moves along a given path .

4 Kinematic Model of a Cart with Four Omnidirectional Wheels

Consider the kinematic model of a cart on four roller-bearing wheels, illustrated in the figure 4.1. Let C be the point of rotation of the cart, $\{O, \boldsymbol{e_x}, \boldsymbol{e_y}, \boldsymbol{e_z}\}$ is the global inertial coordinate system, $\{C, \boldsymbol{E_x}, \boldsymbol{E_y}, \boldsymbol{E_z}\}$ is the coordinate system rigidly connected to the cart. The plane $\{C, \boldsymbol{E_x}, \boldsymbol{E_y}\}$ is parallel to the plane of the surface. Let r_i be the radius of the i th wheel, δ_i be the angle between the tangent to the plane of the wheel $\boldsymbol{p_i}$ and the roller axis $\boldsymbol{l_i}$, ψ is the angle between the axis $\boldsymbol{e_x}$ and $\boldsymbol{E_x}$, α_i - the angle between the axis $\boldsymbol{E_x}$ and the radius with the vector $\boldsymbol{R_i}$ are R_i long, directed from the point C to the point of contact with the surface of the i th wheel C_i , $i = \overline{1, 4}$.

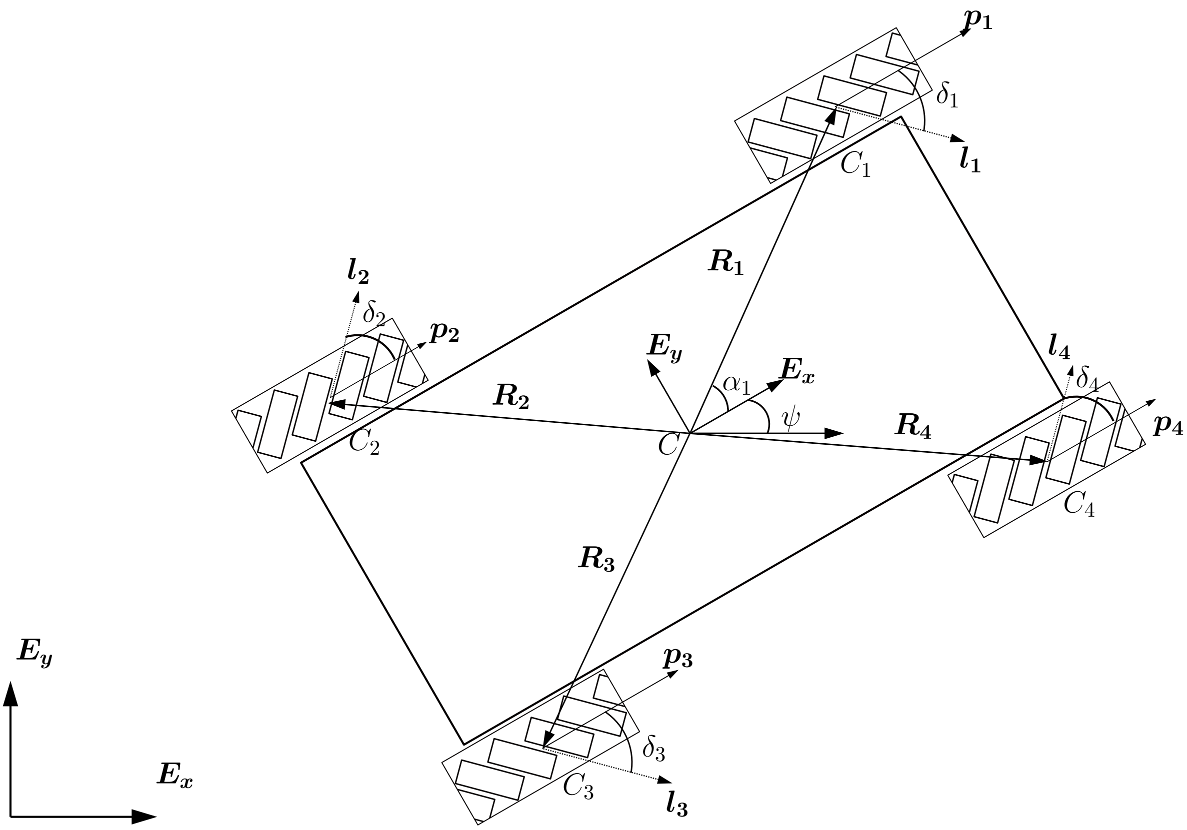

Figure 4.1 – cart model on four roller wheels

Consider the velocity vector of the center of the i th wheel $\boldsymbol{v_{c,i}}$. We decompose it into translational and rotational components:

$$\boldsymbol{v_{c,i}} = \boldsymbol{v_{tr,i}} + \boldsymbol{v_{rot,i}} \tag{4.1}$$

The translational velocity vector $\boldsymbol{v_{tr,i}}$ has the coordinates $[\dot{x}, \dot{y}, 0]$. The equation of communication with the angular velocity of the i - th wheel φ_i, based on the equation 3.8 has the form:

$$\dot{x}cos(\psi + \delta) + \dot{y}sin(\psi + \delta) = r_i\dot{\varphi}_i cos\delta_i \tag{4.2}$$

Where $[\dot{x}, \dot{y}, 0]$ are the coordinates of the translational velocity vector of the cart.

In order to make the cart rotate around the axis of rotation $C\boldsymbol{E_z}$, it is necessary to set the velocity vector of each wheel perpendicular to the radius of the vector $\boldsymbol{R_i}$. The length of the vector should be $R_i, \dot{\psi}$. The arrangement of the vectors is illustrated in the figure 4.2.

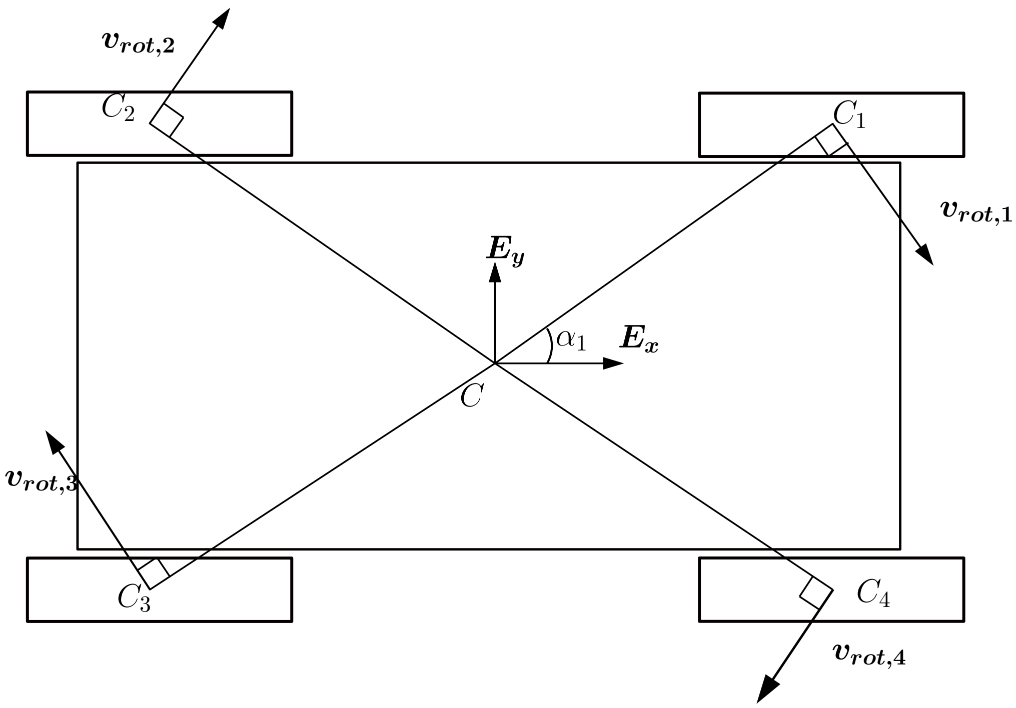

Figure 4.2 – illustration of the rotational movement of the trolley on four roller-bearing wheels

Consider the rotational velocity vector of the i th wheel $\boldsymbol{v_{rot,i}}$. Projecting it on the axis $\boldsymbol{E_x}$ $\boldsymbol{E_y}$, as shown in the figure 4.3, we get:

$$\boldsymbol{v_{rot,i}} = R_i\dot{\psi}sin(\alpha_i)\boldsymbol{E_x} - R_i\dot{\psi}cos(\alpha_i)\boldsymbol{E_y} \tag{4.3}$$

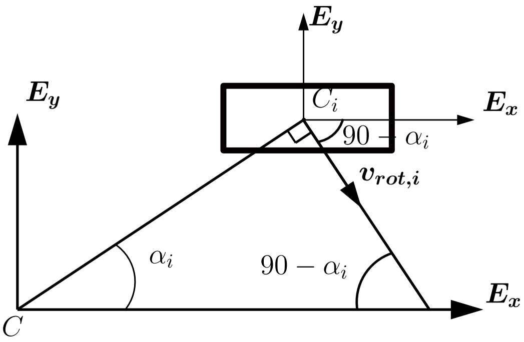

Figure 4.3 – projection of the rotational velocity vector on the local coordinate system

Projecting the vector $\boldsymbol{v_{rot,i}}$ on the $\boldsymbol{e_x}$ $\boldsymbol{e_y}$ axis, as shown in 4.4 we get

$$\boldsymbol{v_{rot,i}} = R_i\dot{\psi}sin\alpha_i(cos\psi\boldsymbol{e_x} + sin\psi\boldsymbol{e_y}) - R_i\dot{\psi}cos\alpha_i(-sin\psi\boldsymbol{e_x} + cos\psi\boldsymbol{e_y})$$

$$\boldsymbol{v_{rot,i}} = R_i\dot{\psi}sin(\alpha_i + \psi)\boldsymbol{e_x} - R_i\dot{\psi}cos(\alpha_i + \psi)\boldsymbol{e_y} \tag{4.4}$$

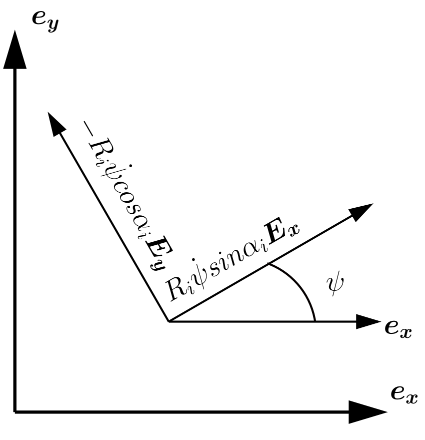

Figure 4.4 – projection of the rotational velocity vector onto the global coordinate system

Then substituting 4.4 in the communication equation 3.8, we obtain

$$R_i\dot{\psi}sin(\alpha_i+\psi)cos(\psi+\delta_i)-R_i\dot{\psi}cos(\alpha_i+\psi)sin(\psi+\delta_i)=r_i\dot{\varphi}_icos\delta_i$$

$$R_i\dot{\psi}sin(\alpha_i-\delta_i)=r_i\dot{\varphi}cos\delta_i \tag{4.5}$$

Then the coupling equation for $\boldsymbol{v_{c,i}}$, proceeding from 4.1, will take the form

$$\dot{x}cos(\psi+\delta_i)+\dot{y}sin(\psi+\delta_i)+R_i\dot{\psi}sin(\alpha_i-\delta_i)=r_i\dot{\varphi}_icos\delta_i \tag{4.6}$$

Expressing $\dot{\varphi}_i$, we teach

$$\dot{\varphi}_i=\frac{\dot{x}cos(\psi+\delta_i)+\dot{y}sin(\psi+\delta_i)+R_i\dot{\psi}sin(\alpha_i-\delta_i)}{r_icos\delta_i} \tag{4.7}$$

With purely translational ($\dot{\psi}_i=0$) or purely rotational ($\dot{x}=\dot{y}=0$) movements, the connection 4.7 is holonomic, that is, the angular acceleration of each wheel does not depend on previous movements of the cart. Otherwise, to calculate 4.7 it is necessary to take into account the current angle of rotation as follows:

$$\psi=\int\limits_{t_0}^{t_{cur}}\psi(t)dt \tag{4.8}$$

Where t_0, t_{cur} are the initial and current time points, respectively.

Thus, the dependence of the angular velocity on the coordinates of the path is found. In order to find the angular velocity of the i th wheel, $i=\overline{1,4}$ corresponding to the given path (x,y,ψ), it is necessary to calculate $(\dot{x},\dot{y},\dot{\psi})$ and substitute in the equation 4.7.