Condensed Matter Physics: Workshop 3

Summary: The material in this workshop will explore thermal properties of crystals, and how these can be modelled and described by phonons. In particular we will examine the implications of using the Einstein approximation for phonon behaviour and use it to predict the thermal properties of crystals. Material for this workshop is covered in lectures 6 and 7.

a. In small groups discuss:

i. What is a phonon?

ii. What assumptions are made in deriving phonon dispersion relations?

iii. Why are there two branches in the phonon dispersion curve for a two atoms basis and what do these represent?

iv. How do phonons contribute to the heat capacity of a solid?

b. Describe the Einstein approximation as used to determine the temperature dependence of the heat capacity of a crystalline solid.

c. Using this approximation show that the internal energy U of the vibrating atoms of a two-dimensional solid is given by:

$$U(T) = \frac{2Nk_{\mathrm{B}}\theta_{\mathrm{E}}}{\exp(\theta_{\mathrm{E}} / T) - 1}$$

where N is the number of atoms in the solid and θ_{E} is the Einstein temperature.

(Think about the number of degrees of freedom a 2-D solid has).

d. Use this expression to obtain an equation for the heat capacity of the solid at constant volume. Show that at very low temperatures:

$$C_V = 2nR\left(\frac{\theta_{\mathrm{E}}}{T}\right)^2 \exp(-\theta_{\mathrm{E}} / T)$$

e. A single layer of silicon atoms with a total mass of 8.4×10^{-11} kg behaves as a 2-D solid with an Einstein temperature of 400 K. Using this information calculate the energy required to raise its temperature from 1 K to 50 K. The atomic mass of Si is 28.1 u.

f. Is your calculated value likely to be larger or smaller than the experimental value? Why?