CM8: Inertial Accelerations

The distant, rapidly rotating planet Cyclo is spherical with mass M, radius R and a constant angular velocity $\underline{\omega}$ pointing out through the north pole of the planet. Accelerations, $\underline{\ddot{r}}$ measured in this rotating frame are related to those determined in an inertial frame, $\underline{a}_S$, via the equation

$$\underline{\ddot{r}} = \underline{a}_S - \underline{\omega} \times (\underline{\omega} \times \underline{r}) - 2\underline{\omega} \times \underline{\dot{r}}\,,$$

where $\underline{r}$ represents the position measured in the rotating frame.

1. Given that the effective gravity at the equator is zero, find an expression for the size of the angular velocity, ω.

2. What are the sizes of the Coriolis and centrifugal accelerations at the north pole for a particle moving with speed v along the surface, and in which directions do they point?

   A competition was held at the north pole, involving one archer from the settlement at each pole firing a single arrow at a circular target with diameter d a distance l away. The south pole competitor took aim as if at the south pole. For the rest of this question, ignore gravity and assume that $d/l \ll 1$ and that the arrows travel at a constant speed.

3. Show that the north pole archer aimed an angle $\omega l/v$ (viewed from above) to the left of the target in order to hit its centre.

4. By what distance and in which direction did the south pole archer's arrow miss the target centre?

   The following year at the north pole, the south pole competitor cunningly substituted a target with diameter fd at a distance fl, where $0 < f < 1$. The north pole competitor, having only one eye like all Cyclopsians, could not perceive this change so did not alter their aim.

5. Assuming that the southerner continued to aim as if at the south pole, albeit using their knowledge of the smaller target size and distance, find expressions for the distance by which each competitor misses the target centre and use these to infer the range of f for which the southern Cyclopsian's arrow lands nearer to the target centre.