Level 2 Stars, Workshop 3

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1. Hydrostatic Equilibrium

(a) State the equation of hydrostatic support. Why do we believe most stars are in hydrostatic equilibrium?

(b) Starting from the hydrostatic equilibrium equation show that the pressure at the centre of the star of uniform density is given by:

$$P_c = \frac{3GM^2}{8\pi R^4}$$

where M is the mass of the star and R its radius.

(c) Calculate the pressure at the centre of the Sun. Explain whether you expect your calculation to provide an accurate answer to the true pressure.

(d) What are the main two forms of pressure that prevent a star from collapsing? Assuming a central-core temperature of 10^7 K, a central-core mass of 10^{30} kg, a central-core radius of $2\text{x}10^8$ m, and a pure ionised Hydrogen gas, estimate the contributions from the different forms of pressure to the overall pressure at the centre of the star, stating any further assumptions made. Which of these two forms of pressure dominates in the core of the Sun?

(e) How would your answer to part d change if the central-core temperature was 2 orders of magnitude higher?

2. Gravitational Collapse

(a) Derive an expression for the energy (E) available to radiate away from the gravitational collapse of a spherically symmetrical star of constant density to show that $E \sim \frac{3}{10}\frac{GM^2}{R}$, where M is the mass of the star and R is the radius of the star. Assume that the star is in virial equilibrium. Recall that the gravitational potential of a point mass is $dU_{g,i} = -\frac{GM_r dm_i}{r}$.

(b) Calculate the Kelvin-Helmholtz time for a star of mass $5\text{x}10^{30}$ kg, radius of $2\text{x}10^9$ m, and surface temperature of 10,000 K.

NOTE: constants listed on the next page

[$m_H = 1.67 \times 10^{-27}$ kg; $e_c = 1.60 \times 10^{-19}$ C; $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}; $k = 1.38 \times 10^{-23}$ J K^{-1}; $M_\odot = 1.99 \times 10^{30}$ kg; $R_\odot = 6.96 \times 10^8$m; $\sigma = 5.67 \times 10^{-8}$ W K^{-4} m^{-2}; $a = 7.57 \times 10^{-16}$ J m^{-3} K^{-4}; $G = 6.67$ x 10^{-11} m^3 kg^{-1} s^{-2}]