Workshop 8: Inertial Forces

1. A stone is dropped from a helicopter hovering a height h over a point on the equator. h is small in comparison with the radius of the Earth, R, so that the acceleration due to gravity can be taken as a constant, g. Ignore air resistance.

   (a) The operator relating time derivatives of a vector viewed in an inertial frame, S, to those measured in a frame B, rotating with angular velocity $\underline{\omega}$ (for example that associated with the rotation of the Earth), is

   $$\left[\frac{d}{dt}\right]_{\text{in } B} = \left(\left[\frac{d}{dt}\right]_{\text{in } S} - \underline{\omega}\times\right).$$

   Use this to show that the force measured in the rotating frame satisfies

   $$m\ddot{\underline{r}} = \underline{F} - m\underline{\omega} \times (\underline{\omega} \times \underline{r}) - 2m\underline{\omega} \times \dot{\underline{r}} - m\dot{\underline{\omega}} \times \underline{r},$$

   where $\dot{\underline{r}}$ and $\ddot{\underline{r}}$ represent the velocity and acceleration measured in frame B, and $\underline{F}$ is the force in the inertial frame. (Note that $\underline{r}$ is the instantaneous position measured relative to an origin on the axis of rotation, and does not depend upon the frame of reference.)

   In which directions do the three inertial forces point for the falling stone?

   The rate at which the Earth's rotation is decreasing is very small, so you should now assume $\dot{\omega} = 0$.

   (b) Ignoring inertial forces, find expressions for the speed and height of the stone as a function of time, and the time taken before the stone hits the ground, t_0. What condition must be satisfied for the inertial forces to be unimportant in determining t_0?

   (c) Using the Coriolis force and the approximation that it always acts in the same direction, how far, in terms of t_0, does the stone land from the point beneath the helicopter, and in which direction?

   (d) Solve the same problem by using the conservation of angular momentum, and compare your answer with that from (c).

2. What angular velocity would be required for your effective weight at the equator to be half that at the north pole? (Take the earth's radius to be 6378 km). How many minutes would a full rotation of the earth take?