L2 Foundation of Physics 2B Optics 2019-20

O.7 Many waves: Summary

February 3, 2020

Learning outcomes:

1. To interpret **multi-path interference** in terms of a discrete phasor sum. [Optics $f2f$ Sec. 3.8]

2. To discuss the case of a large number of paths (e.g. the diffraction grating). [Optics $f2f$ Sec. 3.9]

Key equations:

Building on the double-slit example, in Lecture 6 we found that the electric field at $t = 0$ in the far-field of an aperture containing 3-slits is

$$E = \frac{E_0}{\sqrt{\mathrm{i}kz}} \mathrm{e}^{\mathrm{i}k\bar{r}} \left(\mathrm{e}^{\mathrm{i}kdx/z} + 1 + \mathrm{e}^{-\mathrm{i}kdx/z} \right) \ . \tag{1}$$

The terms inside the bracket is a **discrete phasor sum**. For N-slits the expression becomes

$$E = \frac{E_0}{\sqrt{\mathrm{i}kz}} \mathrm{e}^{\mathrm{i}k\bar{r}} \left[\mathrm{e}^{-\mathrm{i}k(N-1)dx/2z} + \mathrm{e}^{-\mathrm{i}k(N-1)dx/2z} \mathrm{e}^{\mathrm{i}kdx/z} + \cdots + \mathrm{e}^{-\mathrm{i}k(N-1)dx/2z} \right] \ , \tag{2}$$

where each successive term is multiplied by $\mathrm{e}^{\mathrm{i}kdx/z}$. As this is a geometric progression it can be summed analytically. The generic feature of the sum are:

1. Positions where all phasors are align (complete constructive interference) are known as **principal maxima**. Principal maxima occur at $x = m(\lambda/d)z$, where m is an integer, and have an intensity proportional to N^2.

2. The intensity is zero when the phasors are isotropically distributed in the complex plane. The position of the first zero is $x = [\lambda/(Nd)]z$. There are $N-1$ zeros between the principal maxima.

3. Position of partial constructive alignment are known as **subsidiary maxima**. There are $N-2$ subsidiary maxima between the principal maxima.

Outlook: In the next lecture, we shall study the case of an infinite sum: the **Fresnel diffraction integral**, [Optics $f2f$ Section 5.3] which says that the light distribution in any plane z can be written as a **sum of paraxial spherical waves** originating from the input plane at $z = 0$:

$$E^{(z)} = \frac{E_0}{\mathrm{i}\lambda z} \iint_{-\infty}^{\infty} \mathrm{f}(x', y') \mathrm{e}^{\mathrm{i}kr_\mathrm{p}} \mathrm{d}x' \mathrm{d}y' \ , \qquad \text{where} \qquad r_\mathrm{p} = z + \frac{(x-x')^2 + (y-y')^2}{2z} \ , \tag{3}$$

is the **paraxial distance** from the source point $(x', y', 0)$ to the observation point (x, y, z).