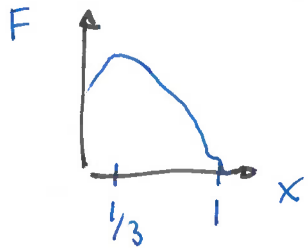
The proton and the neutron have a substructure:

$F(x, Q^2) \sim$ flat with Q^2 $\longrightarrow$ pointlike

$F_2(x, Q^2) = 2x\, F_1(x, Q^2)$ $\longrightarrow$ spin $\frac{1}{2}$



$\longrightarrow$ most likely 3

$\longrightarrow$ gluons necessary to hold them together.

Structure functions can be described them

$$F_2(x) = x \sum_c Q_c^2\, f_c(x)$$

particle distribution function

constituents

el. charge of the constituent c

charge conservation $\displaystyle\int_0^1 \sum_q Q_q\, f_q(x)\, dx = 1$

momentum conservation $\displaystyle\int_0^1 \sum_c x\, f_c(x)\, dx = 1$

Split up valence and sea quarks

$$f_q(x) = f_q^v(x) + f_q^s(x)$$

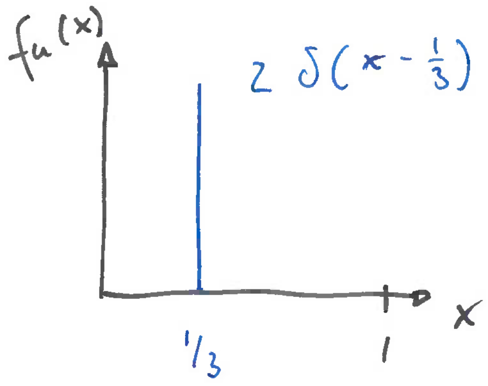
$$\Rightarrow \int_0^1 dx\, [f_u(x) - f_{\bar{u}}(x)] = +2$$

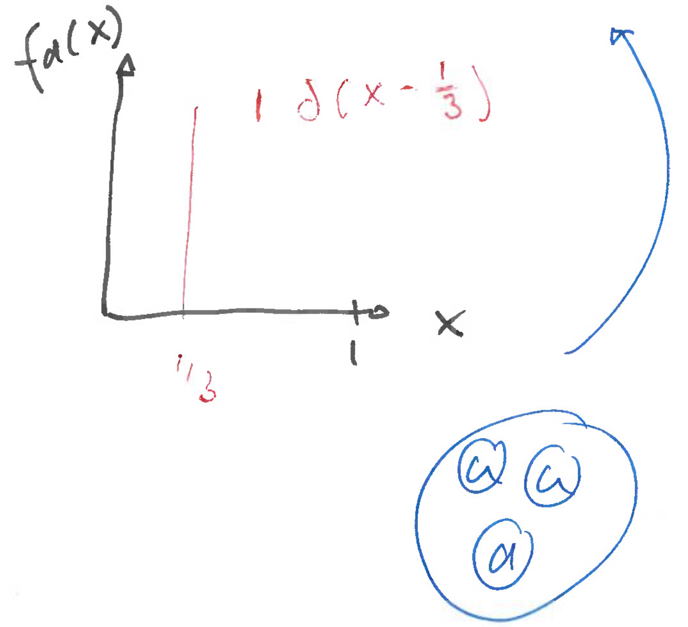
$$\int_0^1 dx\, [f_d(x) - f_{\bar{d}}(x)] = +1$$

$$\int_0^1 dx\, [\, f_s(x) - f_{\bar{s}}(x)\,] = 0$$

What would the PDFs look like?

Consider the proton with no interactions between the quarks.





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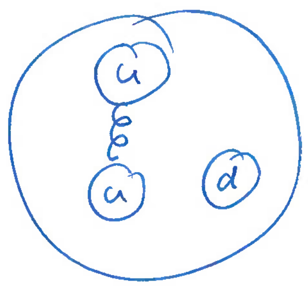
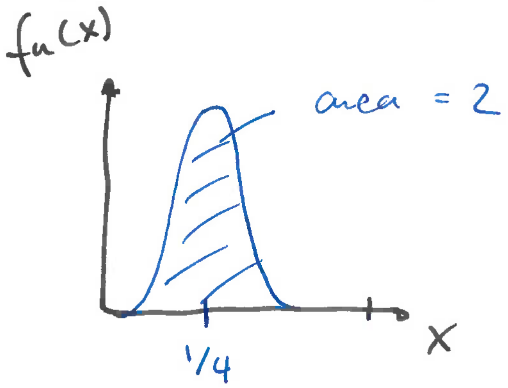
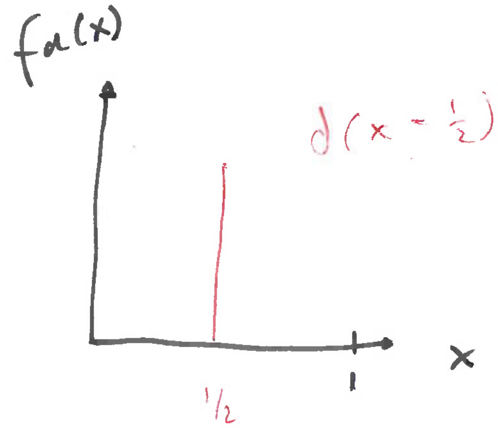




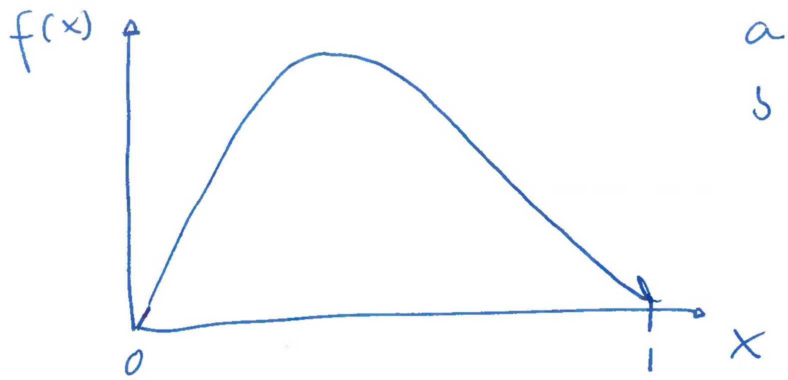


How to construct PDFs for valence quarks?

Ansatz: $f(x) = x^a (1-x)^b \quad , \; a, b > 0$

$\lim_{x \to 0} f(x) \to 0$ because it is a valence quark

$\lim_{x \to 1} f(x) \to 0$ because it would carry all the momentum.



$a = 1$

$b = 5$

Quark structure of the nucleons

Now

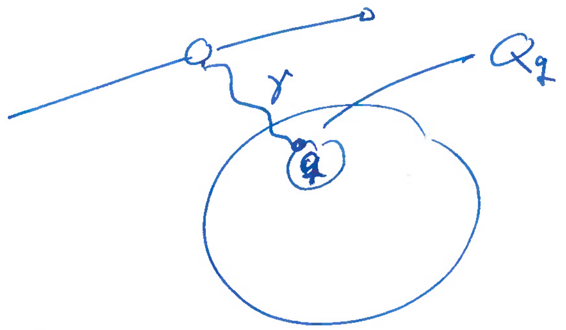
$$f_q(x) \longrightarrow q(x)$$

$$f_{\bar{q}}(x) \longrightarrow \bar{q}(x)$$

For electron - proton scattering

$$F_2(x) = x \sum_q \sum_{t=s,v} Q_q^2 \left(q_t(x) + \bar{q}_t(x) \right)$$

Structure functions are process-dependent.



$$F_2^{e,p}(x) = x \sum_{q=u,d,s} \sum_{t=v,s} Q_q^2 \left(q_t(x) + \bar{q}_t(x) \right)$$

drop x-argument

$$= x \left[\frac{1}{9} \left(d_v^p + d_s^p + \bar{d}_s^p \right) + \frac{4}{9} \left(u_v^p + u_s^p + \bar{u}_s^p \right) + \frac{1}{3} \left(s_s^p + \bar{s}_s^p \right) \right]$$

$$F_2^{e,n}(x) = x \sum_q \sum_t Q_q^2 \left(q_t(x) + \bar{q}_t(x) \right)$$

$$= x \left[\frac{1}{9} \left(d_v^n + d_s^n + \bar{d}_s^n \right) \right.$$
$$+ \frac{4}{9} \left(u_v^n + u_s^n + \bar{u}_s^n \right)$$
$$\left. + \frac{1}{9} \left(s_s^n + \bar{s}_s^n \right) \right]$$

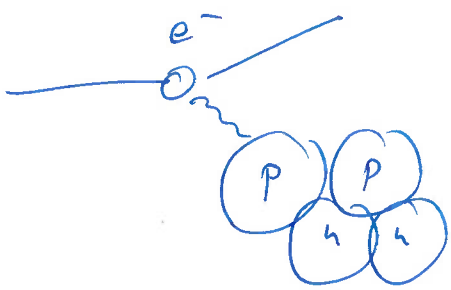
Symmetry

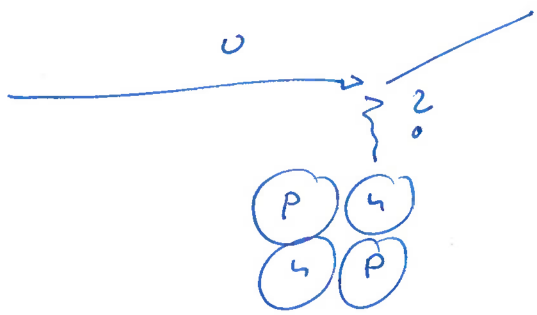
$$\begin{array}{ccc} p & \to & n \\ u & \to & d \\ d & \to & u \\ s & \to & s \end{array}$$

$$F_2^{e,n}(x) = x \left[\frac{1}{9} \left(u_v^p + u_s^p + \bar{u}_s^p \right) \right.$$
$$\left. + \frac{4}{9} \left(d_v^p + d_s^p + \bar{d}_s^p \right) + \frac{1}{9} \left(s_s^p + \bar{s}_s^p \right) \right]$$

For scattering off a nucleus with ~~even~~ equal numbers of p and n:

$$F_2^{e,N} = \frac{F_2^{e,p} + F_2^{e,n}}{2} = x \left[\frac{5}{18} \left(d_v^p + d_s^p + \bar{d}_s^p \right) \right.$$
$$+ \frac{5}{18} \left(u_v^p + u_s^p + \bar{u}_s^p \right)$$
$$\left. + \frac{1}{9} \left(s_s^p + \bar{s}_s^p \right) \right]$$





For neutrino-nucleus scattering

$$F_2^{\nu,N} = x \sum_q \sum_t \underbrace{(q_t(x) + \bar{q}_t(x))}_{\text{no el. charge}}$$

Therefore, if we ignore the strange quarks

$$\frac{F_2^{e,N}(x)}{F_2^{\nu,N}(x)} = \frac{5}{18}$$

Momentum distribution

The PDFs give the probability of finding a parton with a momentum fraction x of the proton.

The momentum carried by quarks and antiquarks of type q is then

$$P_q = \sum_t \int_0^1 dx \, (q_t(x) + \bar{q}_t(x)) \, x \underbrace{P}_{\text{total proton momentum}}$$

$$\Rightarrow \frac{\sum_q P_q}{P} = \sum_q \sum_t \int_0^1 dx \, (q_t(x) + \bar{q}_t(x)) \, x$$

$$= \int_0^1 dx \, F_2^{\nu,N}(x) = \frac{18}{5} \int_0^1 dx \, F_2^{e,N}(x) \approx 0.5$$

experimentally $\int_0^1 dx \, F_2^{e,N}(x) \sim 0.15$

$\rightarrow$ exp evidence for gluons.