Condensed Matter Physics: Weekly Problem 4

These problems are to be formatively self-assessed by you, the student. Students taking part in the peer-marking pilot scheme will also be required to mark one of their peer's weekly problems. A mark scheme, out of 10, will be provided with each solution to aid your assessment before your timetabled weekly workshop.

Summary: In this problem, we will use the classical Drude model of electrons in metals to predict the electrical and thermal properties of lithium and compare the results obtained with experimental data.

a. The mean atomic mass of lithium is 6.94 u and its density is 0.534×10^3 kg m^{-3}. Li has a single valence electron in its outer shell. What is the density n of valence electrons per unit volume? **[1 mark]**

b. Use the conductivity data in the table below (from Ashcroft and Mermin Table 1.2) and the Drude model to calculate the average relaxation time and the mean free path for electrons in Li for each temperature given in the data.

$\boldsymbol{T}$ (K)	$\boldsymbol{\sigma \times 10^8}$ ($\boldsymbol{\Omega^{-1}\mathrm{m}^{-1}}$)
77	0.962
273	0.117
373	0.081

[4 marks]

c. Consider your results:

- **i.** Assuming Li crystallises in a primitive cubic structure, how does your result for the mean free path compare with the spacing between atoms? What possible mechanisms for the scattering of the electrons does this suggest? **[1 mark]**
- **ii.** If the mean free path is independent of temperature, how would you expect the electrical conductivity to vary with temperature? Is your expectation confirmed by the data? **[1 mark]**

d. Estimate the thermal conductivity of Li at 273 K using the Wiedemann-Franz law. How accurate do you expect your estimate to be? Why should this ratio be a better test of the Drude model than either the electrical or thermal conductivities separately? **[3 marks]**

Condensed Matter Physics: Weekly Problem 4 - Solutions

When completing your assessment please enter the numerical marks for each question. Please also give information on any parts which you found difficult, as this will allow me to go over any common issues in the workshops. The workshops also provide the opportunity to individually talk to myself, and other staff members about any issues you faced when solving the problem.

a. Li has mass 6.94 u = $6.94 \times 1.67 \times 10^{-27}$ kg = 1.16×10^{-26} kg.

Density = 0.534×10^{3} kg m^{-3}.

$$\text{Atomic Density } n = \frac{0.534 \times 10^3}{1.16 \times 10^{-26}} = 4.60 \times 10^{28}\,\text{m}^{-3}$$

The electron density is the same as there is one electron per atom. (Li is in group 1 in the periodic table – single electron in outer shell.) **[1 mark]**

b. The Drude model predicts the electrical conductivity to be:

$$\sigma = \frac{ne^2\tau}{m_e}$$

This gives a mean time between collisions of:

$$\tau = \frac{m_e\sigma}{ne^2}$$ **[1 mark]**

The velocity is obtained from the classical equipartition theorem (the Drude model is entirely classical):

$$\frac{1}{2}mv^2 = \frac{3}{2}k_BT \Rightarrow v = \left(\frac{3k_BT}{m}\right)^{1/2}$$ **[1 mark]**

Using these relationships gives the mean free path of τv.

Putting all this together gives the values below:

T (K)	$\sigma \times 10^8$ $(\Omega^{-1}\text{m}^{-1})$	$\tau \times 10^{-15}$ (s)	$v \times 10^5$ (m s^{-1})	$\tau v \times 10^{-10}$ (m)
77	0.962	73.74	0.5915	43.62
273	0.117	8.968	1.114	9.989
373	0.081	6.209	1.302	8.083

[2 marks for correct values]

c. i. The spacing between the atoms is calculated from the density. We assume a primitive cubic structure. The volume occupied by one atom is:

$$\frac{1.16 \times 10^{-26}}{0.534 \times 10^{3}} = 2.17 \times 10^{-29}\mathrm{m}^3$$

Spacing is then 0.279 nm.

The mean free path is similar to this ranging from 15 atomic spacing's at 77 K to 3 atomic spacing's at 373 K. This suggests that collisions occurring with atoms is a plausible explanation in the Drude model. **[1 mark]**

ii. If the mean free path is independent of temperature (because the atom spacing changes only a few % going from 77 K to 300 K) then the conductivity should be more or less independent of temperature. This is not supported by the data, the mean free path changes by an order of magnitude. This suggests that scattering from atoms is not a good explanation. **[1 mark]**

d. The Wiedemann Franz law predicts that:

$$\frac{\kappa}{\sigma} = \frac{3}{2}\left(\frac{k_\mathrm{B}}{e}\right)^2 T \Rightarrow \frac{\kappa}{\sigma T} = 1.11 \times 10^{-8}\ \mathrm{W\Omega K^{-2}}$$

[1 mark]

From this we have: $\kappa = 1.11 \times 10^{-8} \times 0.117 \times 10^{8} \times 273 = 35.45\ \mathrm{Wm^{-1}K^{-1}}$.

This ratio is half the experimental value given in Ashcroft and Mermin Table 1.6 – see Handout Lecture 8. **[1 mark]**

The Wiedemann Franz law does not involve the mean collision time τ.

The uncertainty in the mean collision time τ cancels out for in the ratio of the electrical to thermal conductivity. This means that the Wiedemann Franz Law is a better test of the Drude model than using either the thermal or electrical conductivity alone. **[1 mark]**