

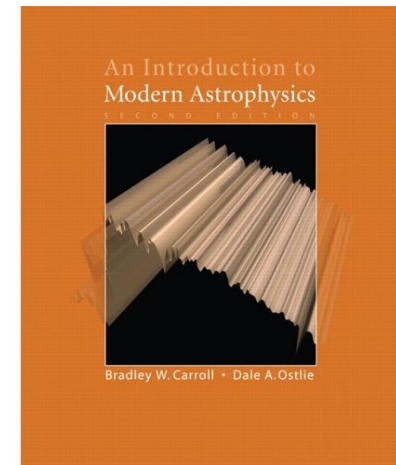
# Lecture 2:

# Observed properties –

## Spectral properties, excitation, ionisation

Professor David Alexander  
Ogden Centre West 119

Chapter 8 (some of Chapter 5 and 7) of Carroll and Ostlie

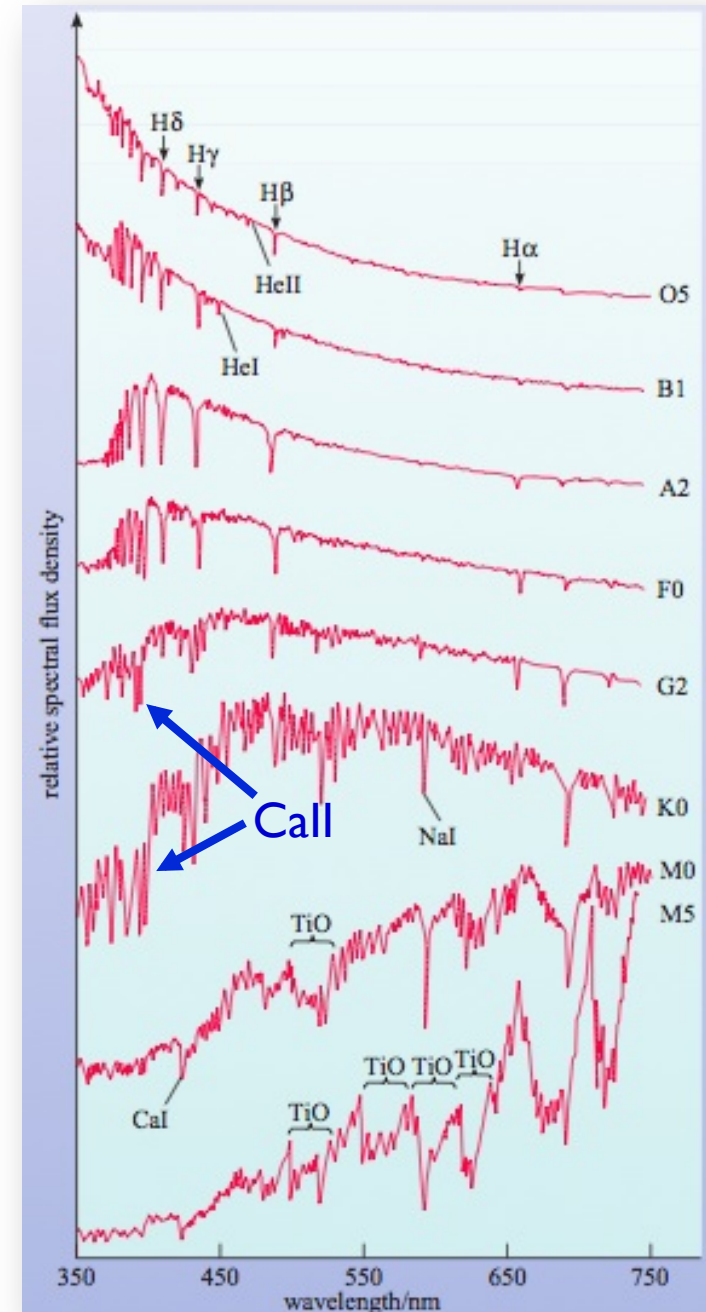


# Overall aim: to understand the spectra of stars

**In lecture 1 we understood that stars emit as a black body – the different overall (continuum) shapes of the spectra is due to the temperature of the gas**

**In lecture 2 we will understand the dips (absorption features) seen in the stellar spectra**

Note that the excitation and ionisation concepts covered in this lecture will be used again when understanding stellar structure



# Aims of lecture

Key concept: formation of spectral lines

Aims:

- Understand the basis of stellar classification
- Understand the processes of excitation and ionisation
- Know and be able to use:

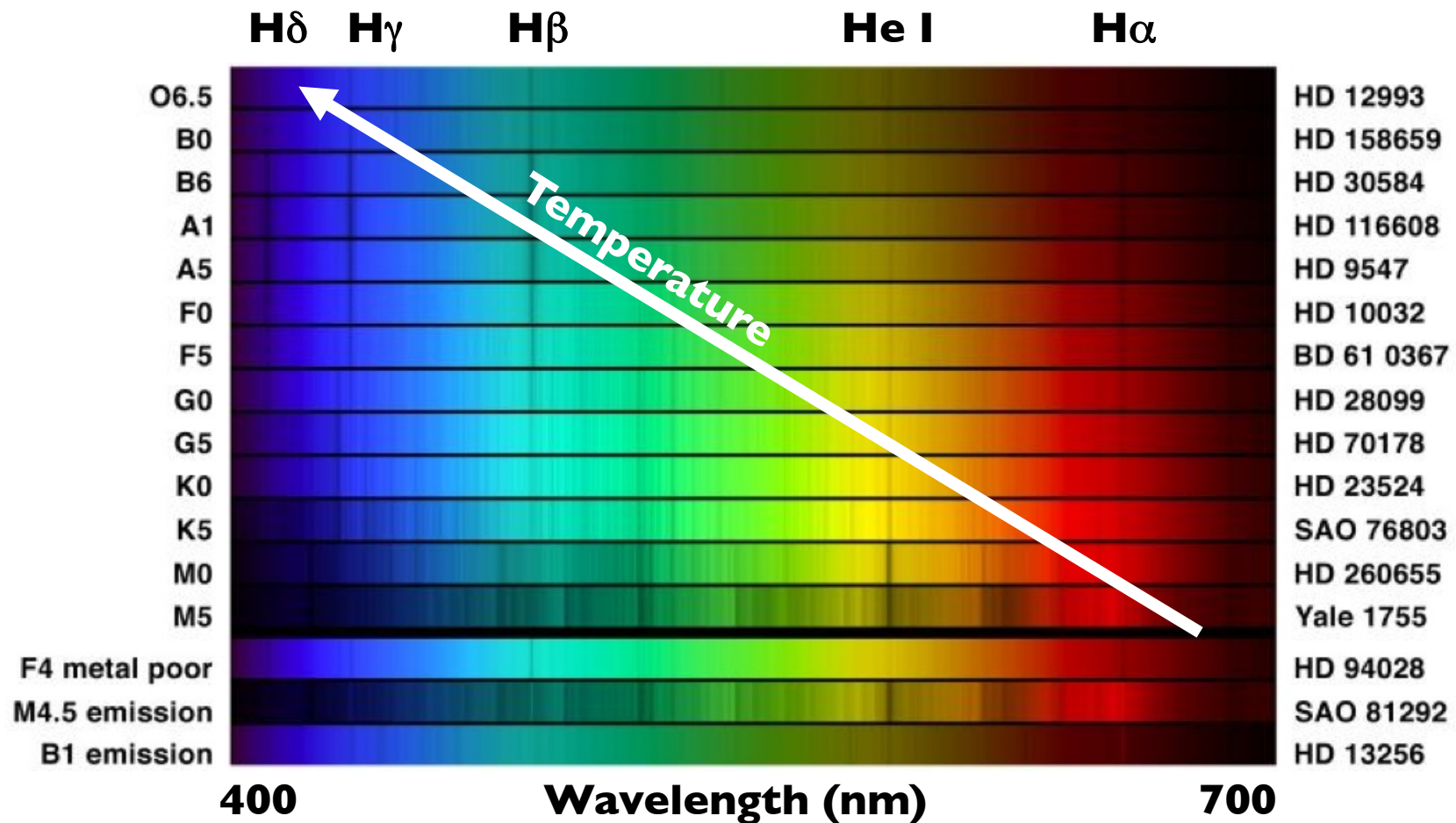
$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

Boltzmann equation:  
excitation states

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left( \frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} e^{-\chi_i/kT}$$

Saha equation:  
ionisation states

# Stellar spectral types: classification

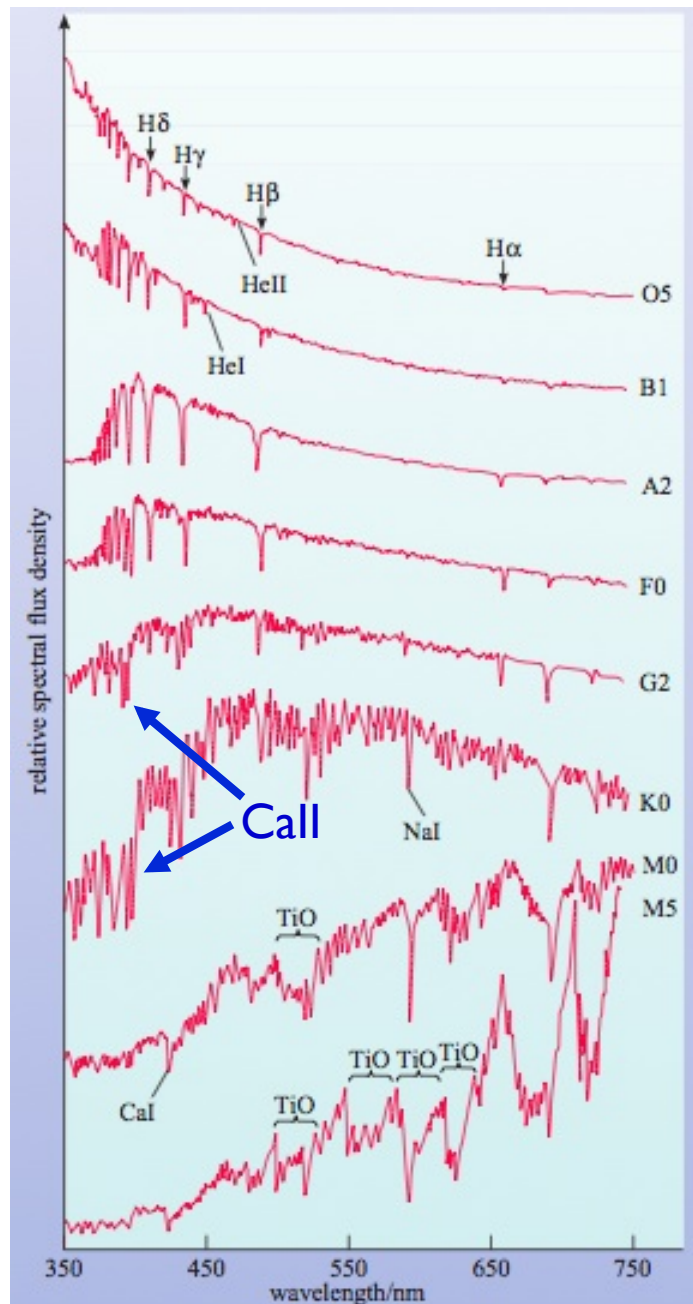


Letters in spectral classification indicate strength of Hydrogen features (A: strongest; O: weakest). Cannon redefined the sequence into temperature order - stellar spectral classification OBAFGKM with numerical sub types

A good mnemonic to help you remember this odd system:  
“Oh Be A Fine Girl/Guy Kiss Me!”

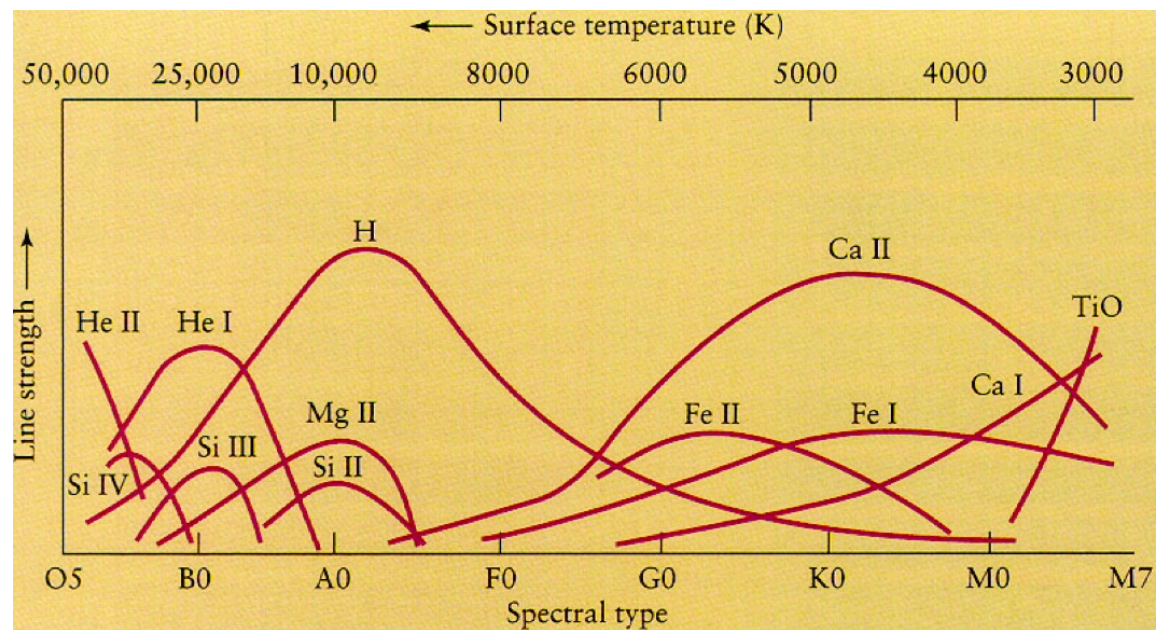


# Stellar spectral types: line strengths



Are the differences in the line strengths due to different elemental abundances? Or is the primary driver of the observed features something else?

Predictions for same abundances but different temps

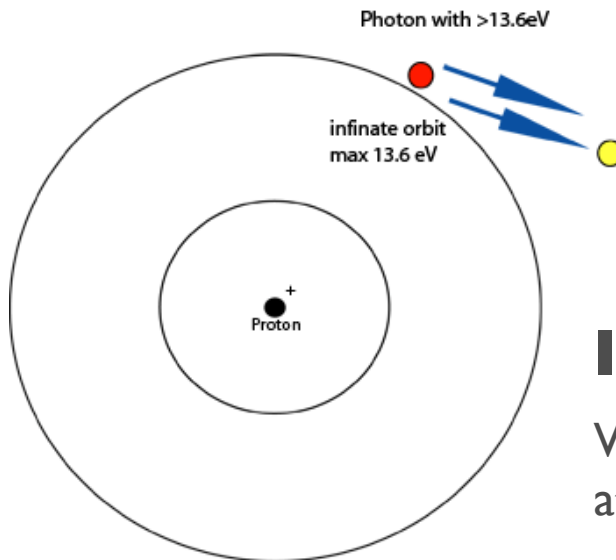
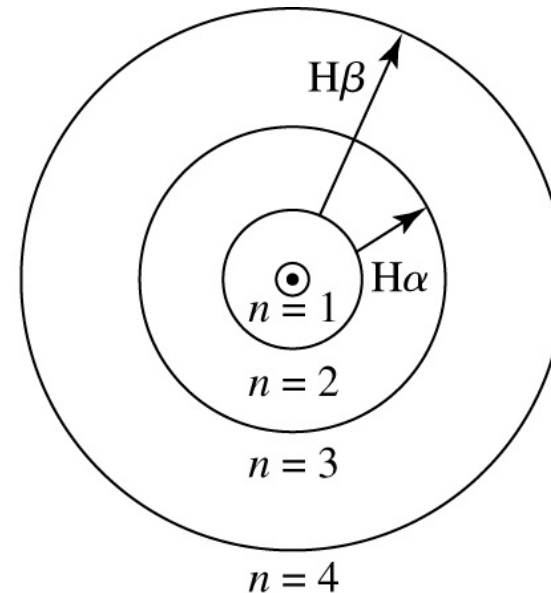


Note: the Roman numerals indicates how ionised – II means singly ionised, III doubly ionised etc

# We can understand the spectral properties when we consider:

## Excitation

In what orbitals are the electrons most likely to be found?



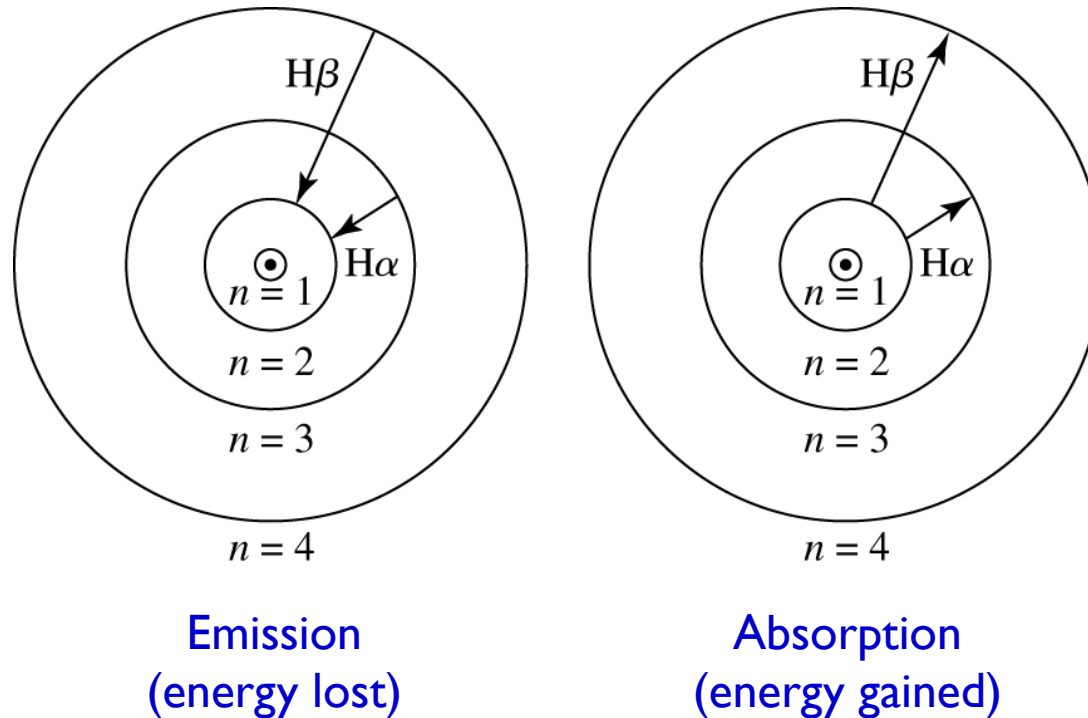
## Ionisation

What are the relative numbers of atoms in various stages of ionisation?

We will see that these are a function of temperature for a given atomic state

# Basic Bohr model of the atom: electron orbits

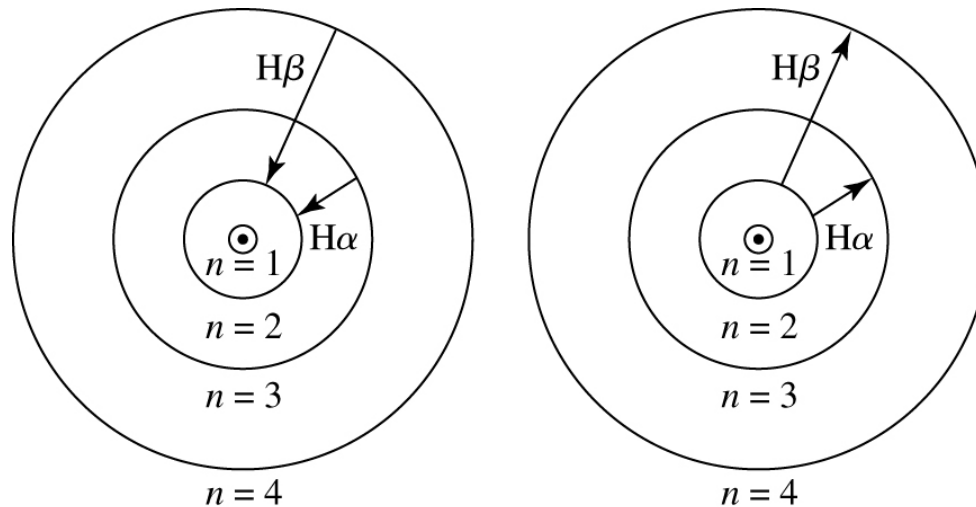
Electrons can only orbit atomic nuclei at specific, allowed, energies (shells or orbitals)



An electron will move to a higher orbital when a photon corresponding to the specific energy difference between the orbitals is absorbed

After  $\sim 10^{-8}$  s the electron will de excite (drop to a lower orbital) causing the emission of a photon (corresponding to the energy difference between the orbitals)

# Basic Bohr model of the atom: excitation energies



Emission  
(energy lost)

Absorption  
(energy gained)

Hydrogen excitation energies

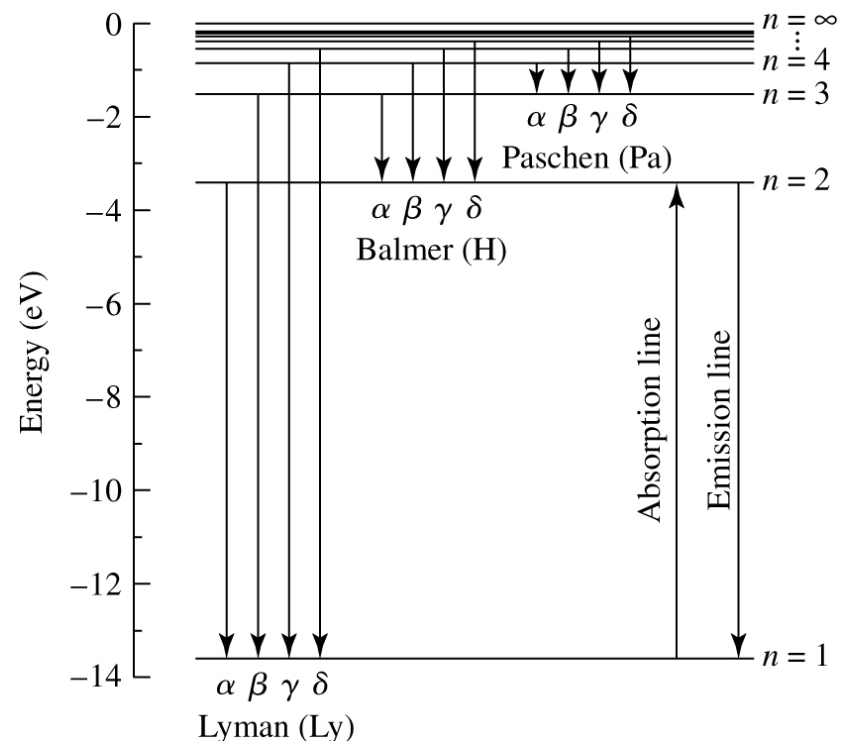
$$E = -13.6 \left( \frac{1}{n_{high}^2} - \frac{1}{n_{low}^2} \right) \text{ eV}$$

$$E = h\nu \text{ or } E = hc/\lambda$$

The Balmer lines from Hydrogen (from  $n=2$  orbital) are prominent at optical wavelengths (thus seen in optical spectra)

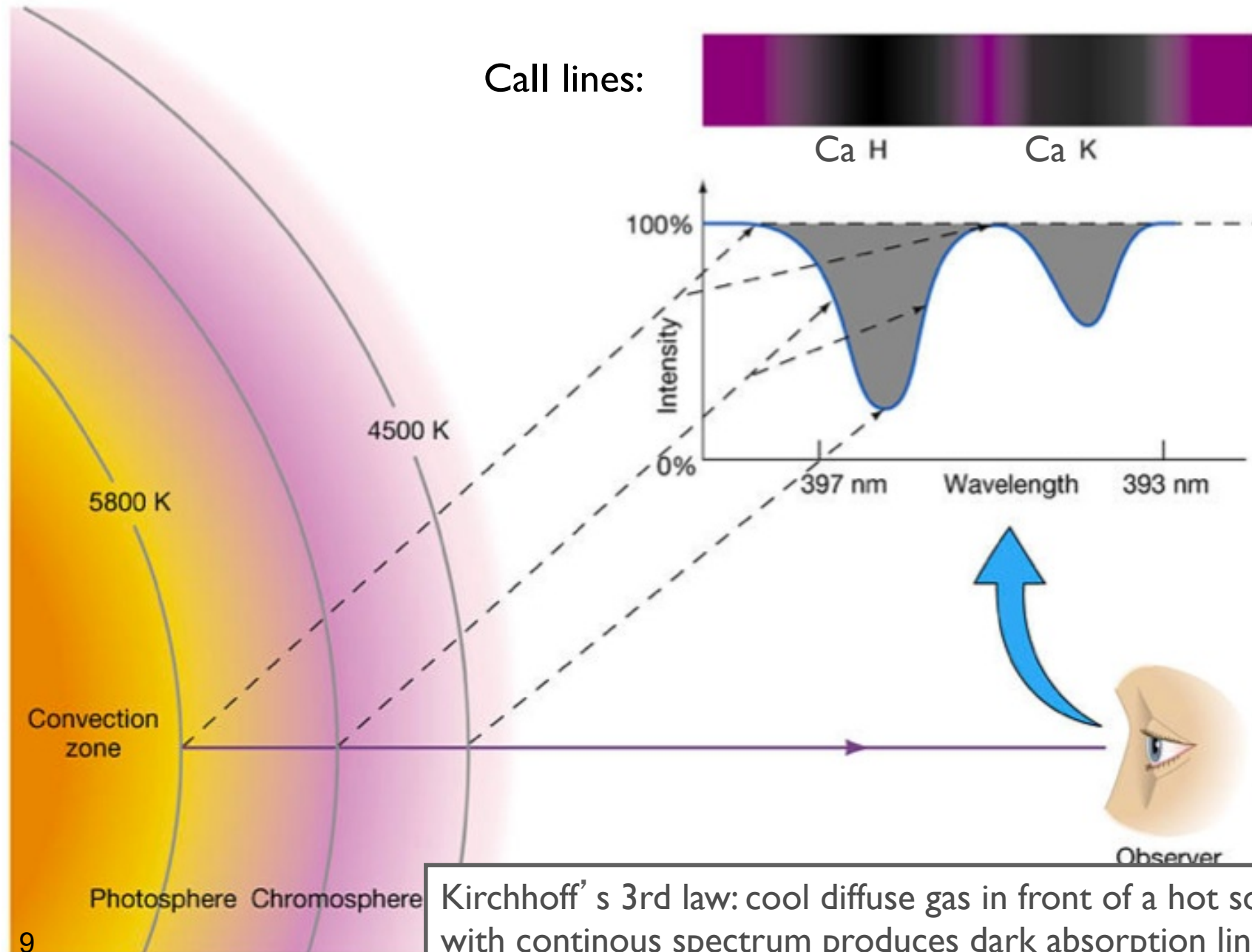
Hydrogen excitation

Orbitals





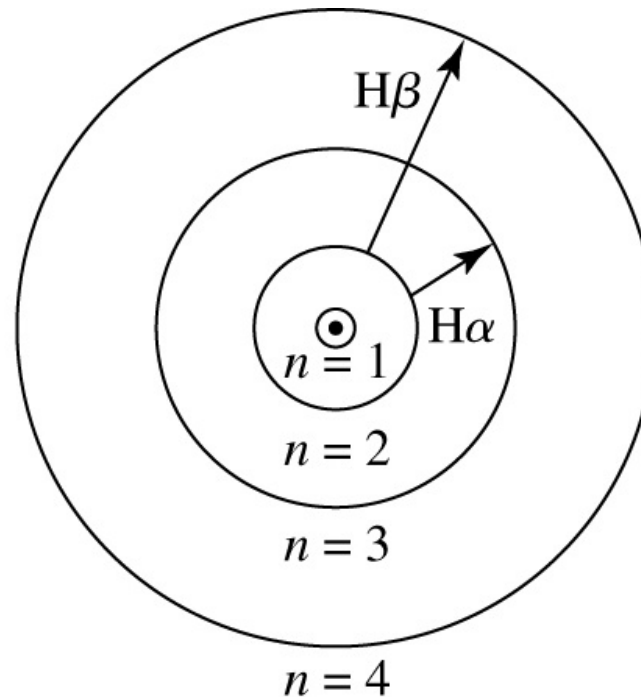
# Production of absorption lines



# Production of spectral lines: excitation levels

The number of electrons in each orbital is calculated assuming the Maxwell-Boltzmann velocity distribution (i.e., assumes that the gas is in thermodynamic equilibrium)

Excitation



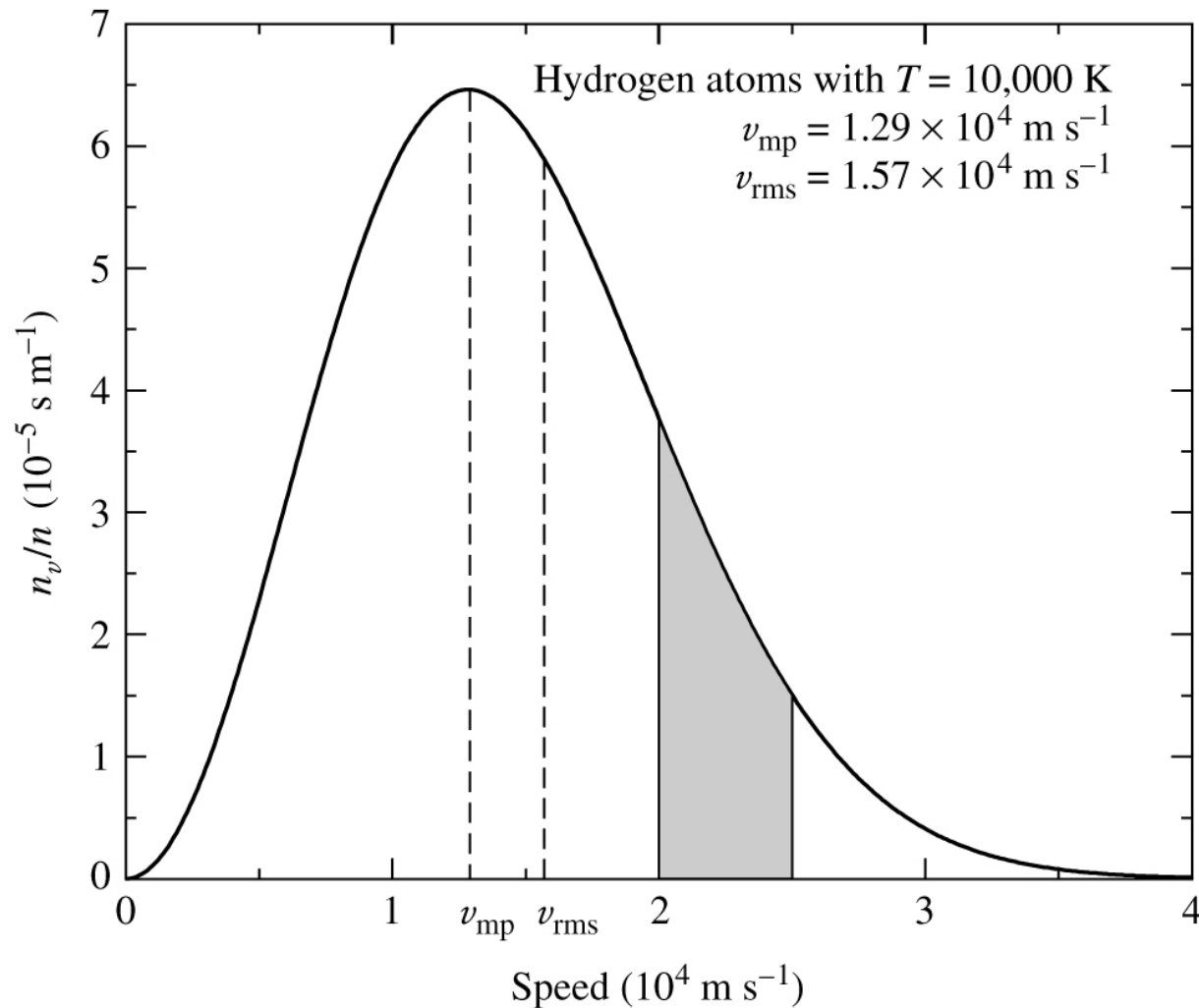
**Ratio of probability between energy states ( $s_a$  and  $s_b$ ), corresponding to energies  $E_a$  and  $E_b$**

$$\frac{P(s_b)}{P(s_a)} = \frac{e^{-E_b/kT}}{e^{-E_a/kT}} = \boxed{e^{-(E_b - E_a)/kT}}$$

Boltzmann factor: relates energy difference to thermal energy: increases with T (decreases with increase in energy difference)

# Maxwell-Boltzmann velocity distribution

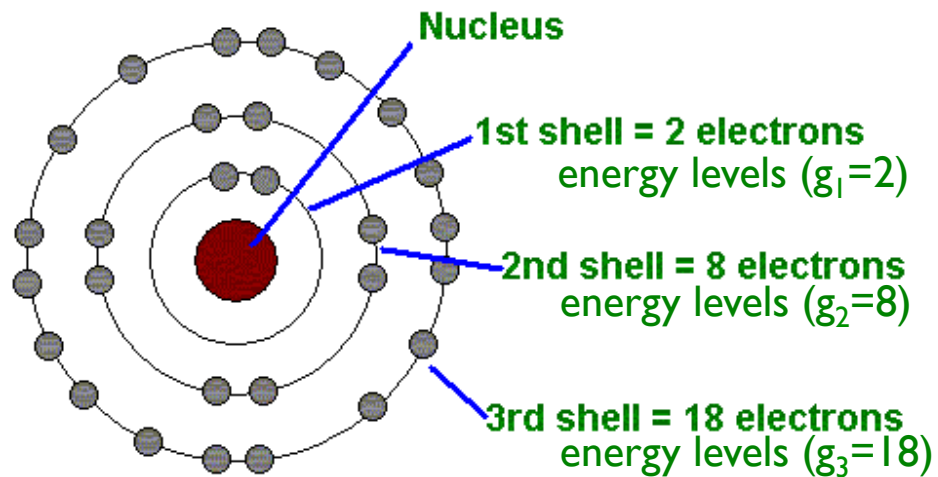
Statistical mechanics: the Maxwell-Boltzmann velocity distribution function



Particle velocities are a function of temperature and mass for a gas in **thermal equilibrium**

# Production of spectral lines: degeneracy

But the orbitals (shells) can be populated with >1 electron (different quantum numbers)...



...we account for this degeneracy using statistical weights ( $g_a$  and  $g_b$ )

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a) / kT}$$

**Equation 2**

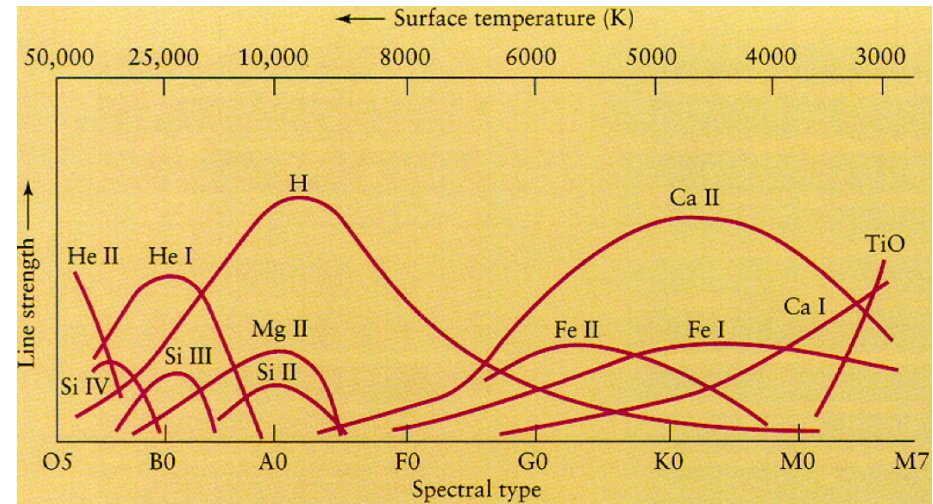
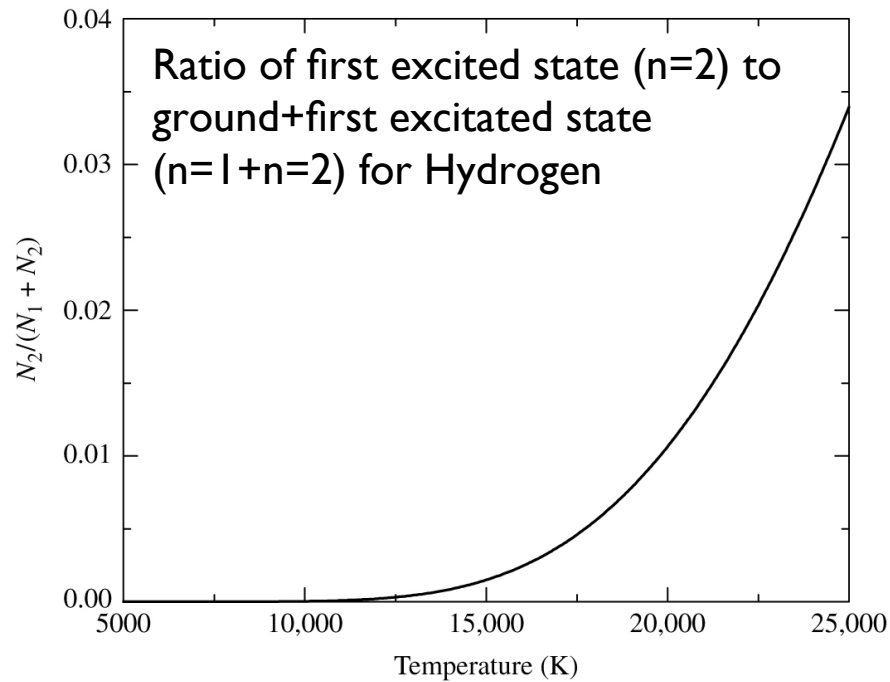
**Hydrogen energy levels**

Ground States $s_1$				Energy $E_1$ (eV)
$n$	$\ell$	$m_\ell$	$m_s$	
1	0	0	+1/2	-13.6
1	0	0	-1/2	-13.6
First Excited States $s_2$				Energy $E_2$ (eV)
$n$	$\ell$	$m_\ell$	$m_s$	
2	0	0	+1/2	-3.40
2	0	0	-1/2	-3.40
2	1	1	+1/2	-3.40
2	1	1	-1/2	-3.40
2	1	0	+1/2	-3.40
2	1	0	-1/2	-3.40
2	1	-1	+1/2	-3.40
2	1	-1	-1/2	-3.40

Quantum numbers

Degeneracy in  $n^{\text{th}}$  orbital for Hydrogen is  $g_n = 2n^2$

# Can we now understand the spectral properties?

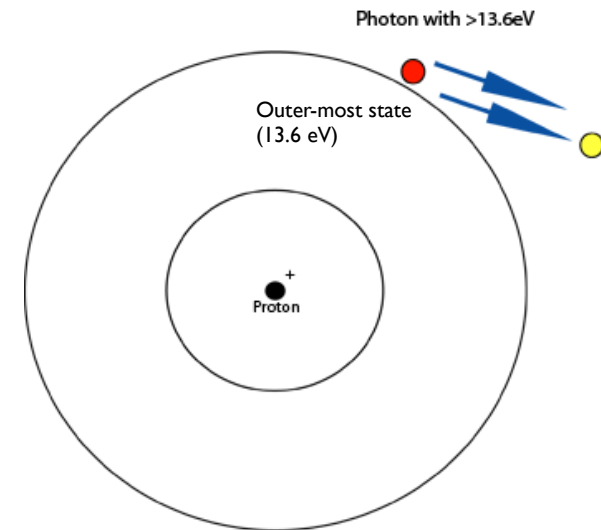


What would the spectral properties of stars look like on the basis of this?  
Does this agree with what we observe?

# Can we now understand the spectral properties?

The relative ratio of states does not explain the weakening of the Balmer lines ( $n=2$ ) for  $T > 10,000$  K

We also need to consider ionisation (electrons ejected from the atom)



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Ionisation can occur when the absorbed energy exceeds the energy required to eject the electron from the atom (the energy depends on the electron orbital):

$$E \geq \chi \qquad \lambda \leq \frac{hc}{\chi}$$

How do the energy requirements differ to the process of excitation?



# Production of spectral lines: ionisation

Saha equation (determines ionisation state of gas)

Ionisation

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left( \frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} e^{-\chi_i/kT}$$

Where:

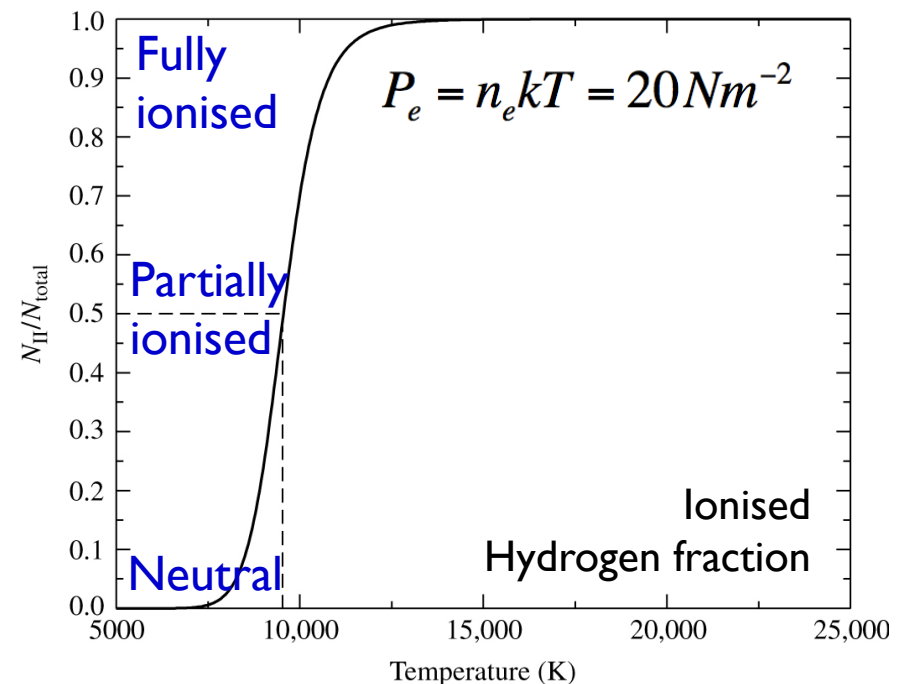
$Z_i, Z_{i+1}$  = partition functions for levels I and I+1 (based on distribution of electrons in orbitals: see eqn 8.7 in CO book)

$n_e$  = number of free electrons per unit volume

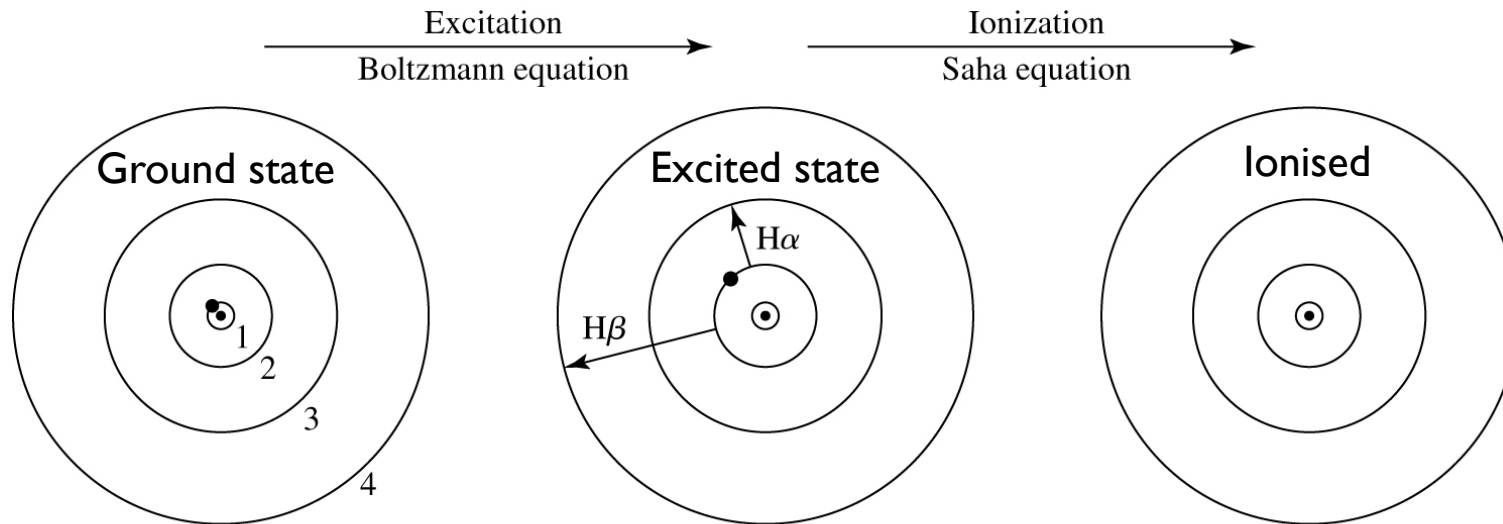
$\chi_i$  = ionization energy required to remove an electron from an atom in state i.

What are the relative numbers of atoms in various stages of ionisation?

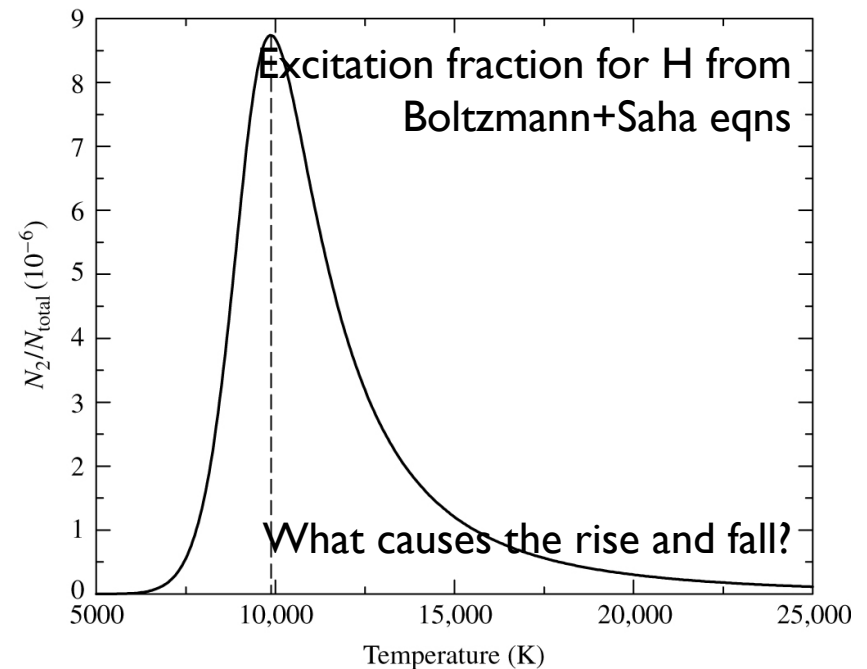
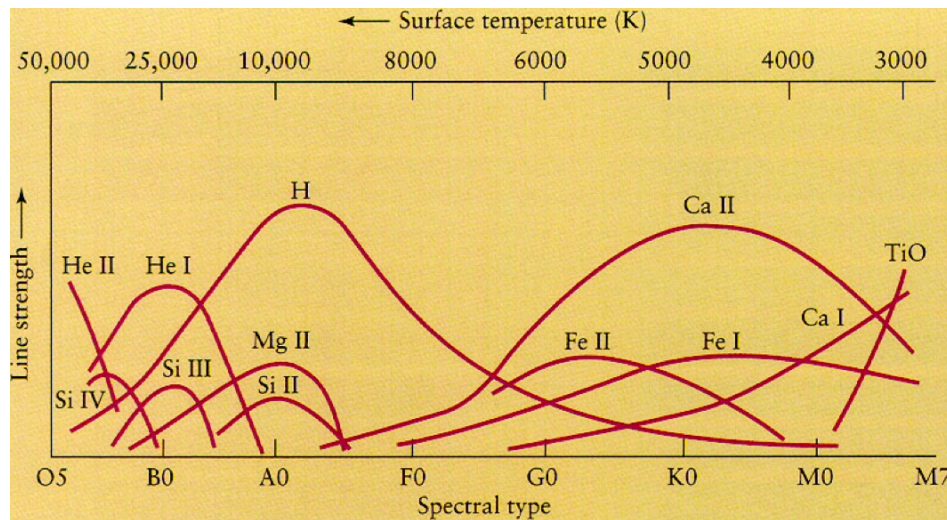
At  $T > 9600$  K >50% of H is ionised: a small (~3000 K) range of T required to ionise Hydrogen (note used  $p_e$  here rather than  $n_e$ )



# Production of spectral lines: ionisation+excitation



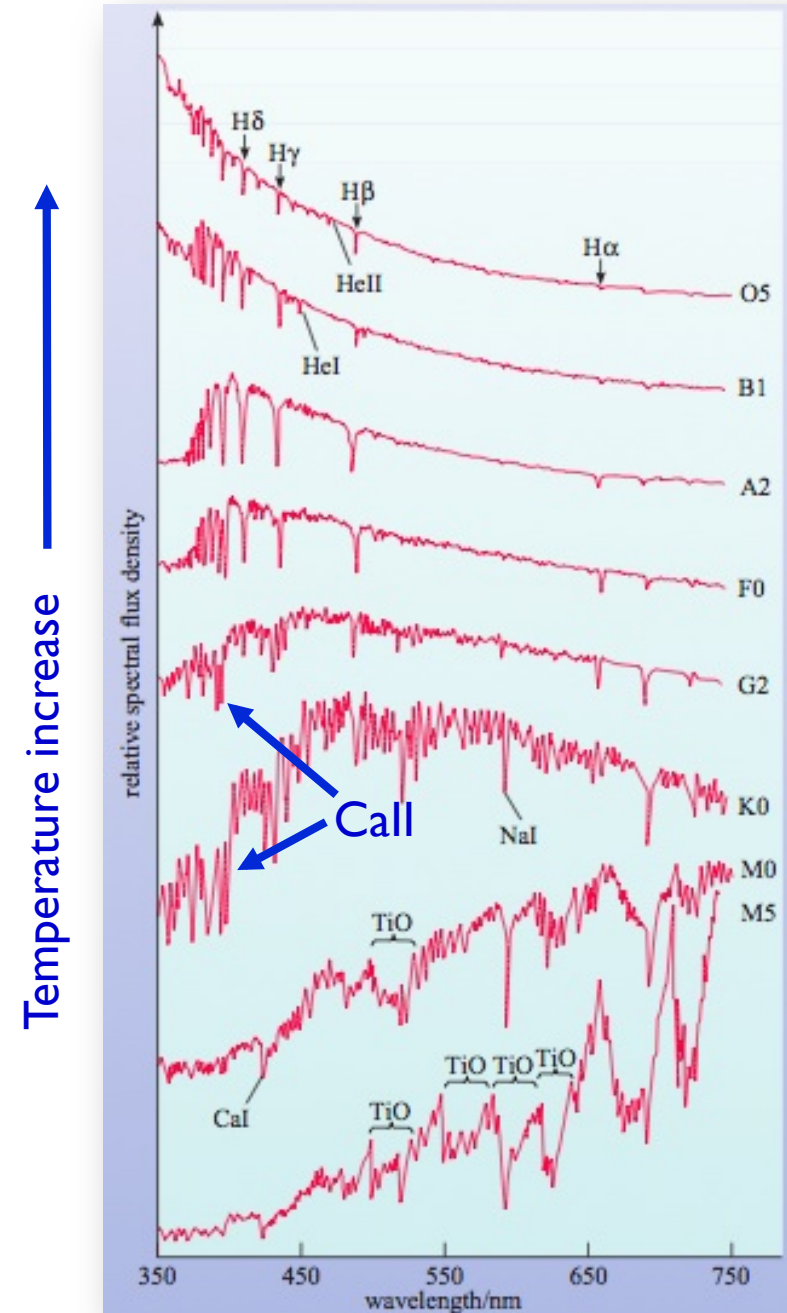
Increasing temperature →



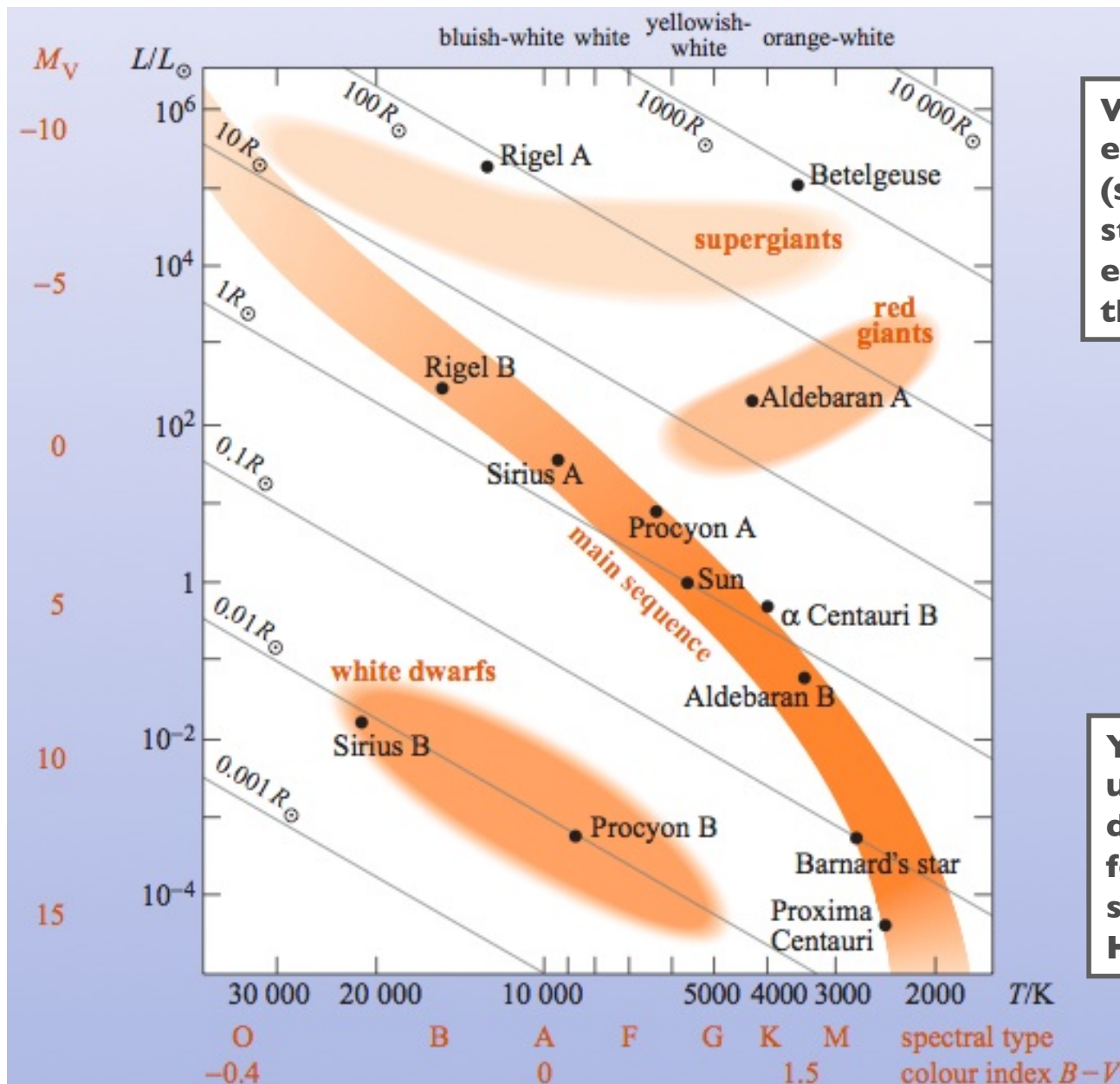
16 Calculate Ca II and H ionisation and excitation strengths for the Sun – see example 8.1.5 in CO

# We can now explain the spectral properties

From lecture 1 and 2 we can now understand the different continuum shapes and the absorption features, which are driven by temperature and the properties of the atoms - we have only considered Hydrogen



# Putting it together: Hertzsprung-Russell diagram



We now understand the emission and absorption (spectral) properties of stars - at least the emission that we see from the stellar surface!

Stars obey this relationship for black body radiation:

$$L = 4\pi R^2 \sigma T_e^4$$

You should also understand why we often don't see strong Hydrogen features even though most stars are dominated by Hydrogen.