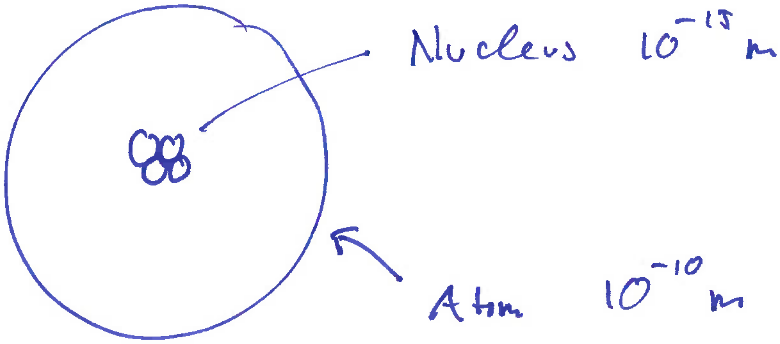
5 Scattering off the Nucleons

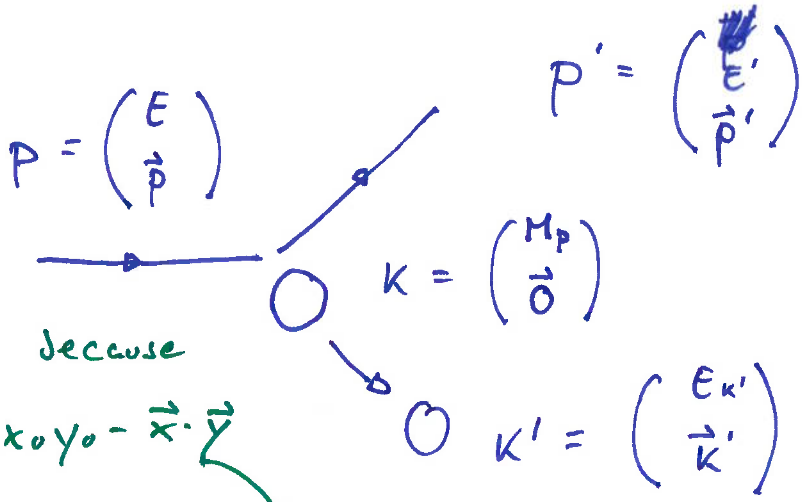


$$q = \frac{\hbar c}{\lambda} = \frac{2 \cdot 10^{-7}\ \text{eV m}}{10^{-15}\ \text{m}} = 200\ \text{MeV}$$

Mass of nucleus few × 900 MeV

we have to move to a fully relativistic description. What changes?

a) $$\vec{q} \to q = p - p' = (\overbrace{E - E'}^{=0}, \vec{p} - \vec{p}\,')$$



sign because
$x \cdot y = x_0 y_0 - \vec{x} \cdot \vec{y}$

$$\vec{q}^{\,2} \to -q^2 = -(E - E')^2 + (\vec{p} - \vec{p}\,')^2 \equiv Q^2$$

Q^2 is called the invariant mass

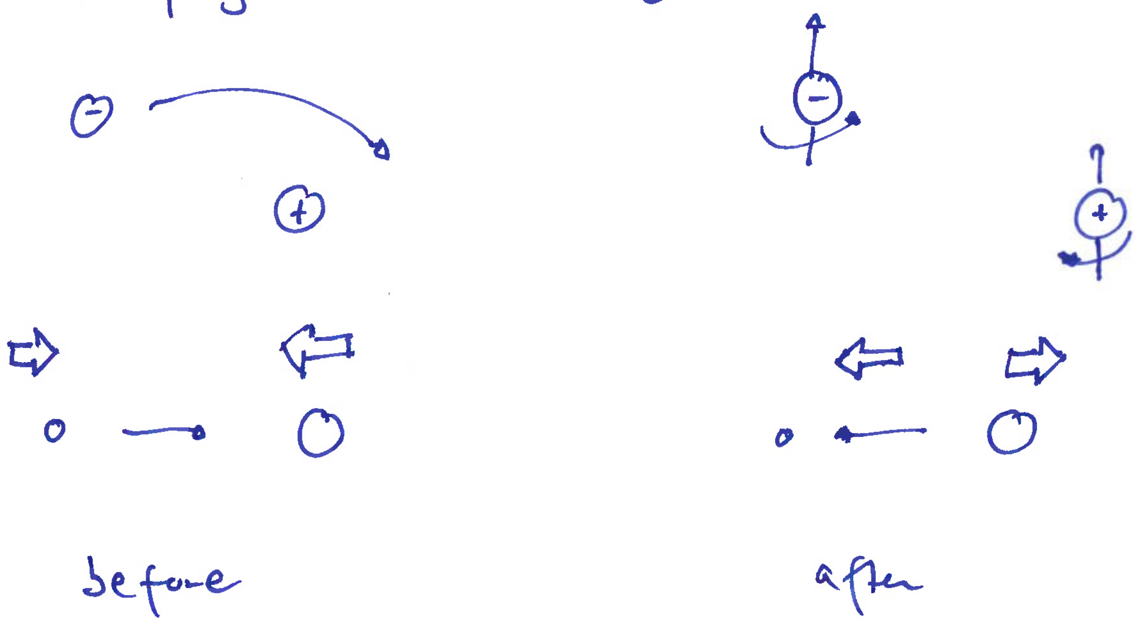
b) Taking nucleon recoil into account gives an extra factor (recoil factor)

$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} = \left.\frac{d\sigma}{d\Omega}\right|_{\substack{Mott,\\ \text{no recoil}}} \cdot \frac{E'}{E}$$

c) For $Q^2 > M_p^2$ of the target, the magnetic moment of the target is <u>not</u> zero

$$\mu = g \frac{e}{2M_p} \cdot \frac{1}{2}$$

As a consequence there is an additional term in the cross section from the magnetic interaction between projectile and target



$$h = \frac{\vec{S}\cdot\vec{p}}{|\vec{S}||\vec{p}|}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{with recoil}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} \left[1 + 2\tau \tan^2\frac{\theta}{2} \right]$$

where $\tau \equiv \frac{Q^2}{4M^2}$

Combine this with the discussion of the previous chapter to include form factors to arrive at the most general differential cross section for elastic, relativistic scattering processes, the Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) \cdot \underline{\tan^2}\frac{\theta}{2} \right]$$

includes recoil factor

We now have 2 form factors

$$G_E(Q^2) \simeq G_E(|\vec{q}|^2) = \int e^{i\vec{q}\cdot\vec{r}} \rho(\vec{r})\, d^3r$$

Fourier transform of the electric charge density $\rho(\vec{r})$

$$G_M(Q^2) \simeq G_M(|\vec{q}|^2) = \int e^{i\vec{q}\cdot\vec{r}} \mu(\vec{r})\, d^3r$$

Fourier transform of the magnetic moment density $\mu(\vec{r})$

If the nucleons were pointlike, we would find

$$G_E(Q^2) \;\&\; G_M(Q^2) = \text{constant}$$

Since they are not, we expect some fall off for $Q^2 \gg 1/R^2$ for which the wave only interacts with part of the target.

For the proton

$$G_E^p(Q^2=0) = 1 \qquad G_M^p(Q^2=0) = 2.79$$

expected +1
for a point-like Dirac particle

For the neutron

$$G_E^N(Q^2=0) = 1 \qquad G_M^p(Q^2=0) = -1.91$$

This shows some internal structure.
How to measure G_E and G_M?

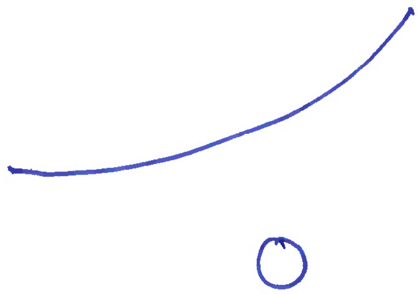
- The intersection with the y-axis of $\left(\frac{d\sigma}{d\Omega}\right)_{exp} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{Mott}$ is $\underbrace{\dfrac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}}_{=A}$

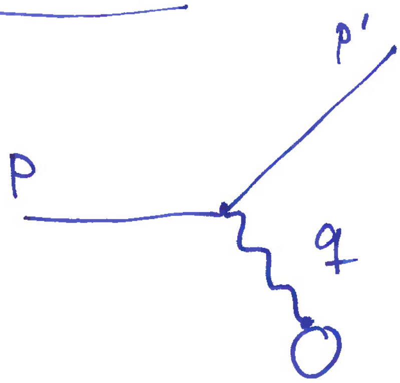
- The slope is given by

$$B = \frac{\cancel{A} + B\tan^2\theta/2 - \cancel{A} + B\tan^2\theta'/2}{\tan^2\theta/2 - \tan^2\theta'/2}$$

$$= 2\tau G_M^2(Q^2)$$

6 From QM to QFT





In QFT the potential does not act continuously, but it is quantised as well.

One cannot "see" these photons making up the E/M field. They are not detectable even though they transmit a force, they are called <u>virtual particles</u>.

$$p \cdot p = E^2/c^2 - |\vec{p}|^2 \neq m^2c^2$$

This is allowed in a quantum theory because of the uncertainty principle

$$\Delta E \, \Delta t \geq \hbar/2$$

The matrix element for an exchange particle of mass m

$$\mathcal{M} = \frac{1}{q^2 - m^2}$$ propagator

$$\sigma \simeq |\mathcal{M}|^2 = \left| \frac{1}{q^2 - 0} \right|^2 = \frac{1}{Q^4}$$