University of Durham

EXAMINATION PAPER

May/June 2016 Examination code: PHYS2621-WE01

STARS AND GALAXIES

SECTION A. Observational Techniques
SECTION B. Stars
SECTION C. Galactic Astronomy

Time allowed: 3 hours

Additional material provided: None

Materials permitted: None

Calculators permitted: Yes **Models permitted:** Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

Visiting students may use dictionaries: No

Instructions to candidates:

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **any three** of the four optional questions. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.

Information

A list of physical constants is provided on the next page.

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Electron mass:	$m_\mathrm{e} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_\mathrm{p} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	$\mathrm{AU} = 1.50 \times 10^{11}$ m
Parsec:	$\mathrm{pc} = 3.09 \times 10^{16}$ m
Solar Mass:	$M_\odot = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26}$ W

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SECTION A. OBSERVATIONAL TECHNIQUES

Question 1 is compulsory. Question 2 is optional.

1. (a) (i) Calculate the diffraction limited angular resolution of a telescope with a 4-metre diameter primary mirror observing at a wavelength of 550 nm. Give your answer in milli-arcseconds. [2 marks]

   (ii) A second telescope with a 4-metre diameter primary mirror is placed 25.0 metres away to make an optical interferometer. Calculate the maximum angular resolution that can be achieved by combining the light at a wavelength of 550 nm from the two telescopes. Give your answer in milli-arcseconds. [2 marks]

   (b) The European Extremely Large Telescope will have a 40-metre diameter primary mirror and the cassegrain focus will have a focal ratio of f / 17.

   (i) What will be the field of view of an instrument at the cassegrain focus if the detector size is 10×10 cm? Give your answer in arcseconds. [3 marks]

   (ii) What will be the plate scale if the detector has 4096×4096 pixels? Give your answer in arcsec pixel^{-1}. [1 mark]

   (c) (i) A certain CCD has a gain of 10 photo-electrons per analogue-to-digital unit (ADU). After reading out, the CCD records 100 ADUs from a star. What is the signal-to-noise ratio of the detection of the star in the photon-noise limited case? [2 marks]

   (ii) If the CCD camera is upgraded from 12 bit to 16 bit readout, how much longer does it take for a star of fixed magnitude to saturate the detector if the gain is kept constant? Assume that the only source of photons is from the star. [2 marks]

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2. (a) Sketch the optical configuration of a transmission grating spectrograph, describing the purpose of each optical component. [4 marks]

(b) State four factors affecting the resolution of a practical spectrograph. [4 marks]

(c) From the basic grating equation, derive the relation

$$\frac{d\theta}{d\lambda} = \frac{n}{dcos\theta},$$

where d is the grating spacing, n is the spectral order and θ is the angle to the grating normal through which light of wavelength λ is diffracted. [2 marks]

(d) Use this to derive the expression for the reciprocal linear dispersion

$$\frac{d\lambda}{dx} = \frac{dcos\theta}{fn},$$

where f is the spectrograph camera focal length. [2 marks]

(e) A grating in a spectrograph is used in second order with the spectrum observed at angle $\theta = 0°$ to the grating normal. If the spectrum is imaged with a camera of focal length 500 mm, how many lines per mm are required in the grating to give a reciprocal linear dispersion of 33.0 Å mm^{-1}? Note that 1Å$= 10^{-10}$ m. [3 marks]

(f) How large must this grating be in order to achieve a theoretical resolving power of $R = 10,000$? [2 marks]

(g) What is the smallest velocity difference that can be discerned at this resolving power? [3 marks]

SECTION B. STARS

Question 3 is compulsory. Questions 4 and 5 are optional.

3. (a) Calculate the radius of a star with a luminosity that is 10^3 times that of the Sun and an effective black-body temperature of 6×10^3 K. Briefly comment on whether you would expect this star to lie on the main sequence. [4 marks]

   (b) Calculate the temperature at which radiation pressure exceeds gas pressure in the core of a star. In your calculation assume a particle density of $n = 1.00 \times 10^{32}$ m^{-3}. [4 marks]

   [Hint: The radiation constant is $a = 7.57 \times 10^{-16}$ J m^{-3} K^{-4}]

   (c) What is opacity? A strong continuum break (at 365 nm) is seen in the optical spectrum of a star. Briefly describe the opacity process that causes this continuum break and explain why there is a deficit of photons at wavelengths shorter than the break wavelength. [4 marks]

   (d) The Schwarzschild condition for convection is satisfied in different regions for different masses of stars. Draw a cross section through a star with a mass equivalent to that of the Sun and highlight where convection is believed to occur in this star. Why does convection occur in this region? [4 marks]

   (e) Calculate the horizontal-branch lifetime of a star of one solar mass. Assume that the efficiency of the triple-alpha fusion process is 0.070%, that 10% of the mass of the star is used during the horizontal-branch lifetime, and that the star is 5.0 times more luminous than it would be on the main sequence. Give your answer in years. [4 marks]

   (f) Describe the origin of the pressure that prevents a white dwarf and a neutron star from collapsing. How does the physical origin of this pressure differ between a white dwarf and a neutron star? [4 marks]

   (g) By equating the gravitational acceleration with the centripetal acceleration, show that the minimum rotation period of a spherical spinning pulsar of uniform density can be written as

   $$P_{\rm min} = \left(\frac{3\pi}{G\rho}\right)^{\frac{1}{2}},$$

   where ρ is the density. [4 marks]

4. (a) Hayashi tracks trace the paths that protostars take on the Hertzsprung-Russell diagram before joining the main sequence. Draw a Hertzsprung-Russell diagram and highlight the main sequence and the key features of the Hayashi track of a low-mass star (1 $M_{\odot}$). [4 marks]

(b) Derive an expression for the energy (E) available from the gravitational collapse of a spherically symmetrical protostar of uniform density to show that $E = -(3/10)(GM^2/R)$, where M is the mass of the protostar and R is the post-collapse (final) radius. Assume that the protostar is in virial equilibrium but state any other assumptions that you make in your derivation. [8 marks]

$$\left[\begin{array}{c}\text{Hint: the gravitational potential of a point mass is} \\ dU = -\frac{GMdm}{r}, \text{ where } dm = 4\pi r^2 \rho dr \text{ is a shell at radius } r \\ \text{of density } \rho \text{ and thickness } dr.\end{array}\right]$$

(c) Calculate the Kelvin-Helmholtz time for a 2 $M_{\odot}$ protostar with the following properties: an average luminosity of 2 $L_{\odot}$, an initial radius of $R_{\mathrm{i}} = 10^{10}$ m, and a final radius of $R_{\mathrm{f}} = 10^{9}$ m. Give your answer in years. [3 marks]

(d) The protostar core has the following properties: fully ionized hydrogen with a pressure of $P = 3 \times 10^{16}$ N m^{-2} and a density of $\rho = 10^5$ kg m^{-3}. Calculate the temperature of the gas at the core of the protostar. Briefly comment on whether this temperature is high enough for nuclear fusion via the proton-proton chain. [5 marks]

5. (a) The instability strip is the region on the Hertzsprung-Russell diagram, where the majority of pulsating stars are found to lie. Draw a Hertzsprung-Russell diagram and highlight the main sequence and the instability strip. [4 marks]

(b) Assuming a uniform density and using the sound-speed equation for an adiabatic gas, show that the period of a radially pulsating star is given by

$$\Pi = \sqrt{\frac{3\pi}{2\gamma G\rho}},$$

where Π is the pulsation period, γ is the ratio of specific heats and ρ is the density of the star. State any other assumptions that you make. [8 marks]

$$\left[\text{Hint: } \int_0^R \frac{dr}{\sqrt{R^2 - r^2}} = \left[\sin^{-1}\left(\frac{r}{R}\right)\right]_0^R .\right]$$

(c) What region and what physical process drives radial pulsations in stars? Why doesn't the Sun show strong radial pulsations? [4 marks]

(d) Assuming that pulsating stars radiate as black bodies, show that the luminosity of pulsating stars of the same effective temperature and mass is proportional to $\Pi^{4/3}$. [4 marks]

SECTION C. GALACTIC ASTRONOMY

Question 6 is compulsory. Question 7 is optional.

6. (a) Spiral galaxies are usually bluer than elliptical galaxies. Briefly discuss two reasons why this is the case. Why can spiral galaxies appear red when seen edge on? [4 marks]

   (b) Name the three main stellar components of a spiral galaxy. Which of these is the most massive in the Milky Way? [4 marks]

   (c) The Tully-Fisher relation relates the luminosity and rotation speed of a spiral galaxy, $L \propto V_c^4$. What is a 'standard candle' and how can the Tully-Fisher relation be used as a standard candle? [4 marks]

   (d) Describe three lines of evidence that suggest that dark matter dominates the mass content of clusters of galaxies. [4 marks]

   (e) What is gravitational lensing? The lensing signal depends on the distances observer-source ($D_{\rm OS}$), observer-lens ($D_{\rm OL}$) and lens-source ($D_{\rm LS}$) as

$$\text{signal} \propto \frac{D_{\rm OL}\, D_{\rm LS}}{D_{\rm OS}} .$$

   Demonstrate that the signal is largest for a given source when the lens is halfway, i.e., when $D_{\rm OL} = D_{\rm OS}/2$. [4 marks]

7. (a) The observed rotation curve of the Milky Way is flat, with $V_c(R) = 220$ km s^{-1}. Demonstrate that this requires that the mass distribution is

$$\rho(r) = \frac{V_c^2}{4\pi \mathrm{G}\, r^2}\,.$$

[4 marks]

(b) Why does the mass distribution from part (a) suggest that the Milky Way is embedded in a dark matter halo? [2 marks]

(c) Direct detection experiments aim to detect dark matter in the solar neighbourhood through collisions of dark matter particles with the detector. Assuming these particles have the mass of a proton, calculate the number density of dark matter particles at our location in the Milky Way. Express your answer in units of particles per cubic metre. Assume that the Sun is at a distance of $r_\odot = 8.0$ kpc from the centre of the Milky Way. [3 marks]

(d) Take $M_{\rm halo} = 0.80 \times 10^{12}$ $\mathrm{M}_\odot$ for the mass of the Milky Way's halo, and assume it has the density profile from part (a) out to its edge at $r = r_{\rm halo}$. Show that $r_{\rm halo} = 71$ kpc. [4 marks]

(e) Calculate the escape velocity from the Milky Way at the Sun's location, $r_\odot$, for the density distribution from part (a), using $r_{\rm halo} = 71$ kpc for the extent of the halo. Express your answer in km s^{-1}. [5 marks]

(f) Supernovae heat the hydrogen gas in their surroundings to a temperature of $T = 10^7$ K. Determine whether this is hot enough to escape the Milky Way's halo, when the supernova explodes at $r_\odot$. [2 marks]

