GA 4

(1) Consider a spherical density distribution, and denote by $M(< R)$ the mass enclosed in a sphere of radius R. The speed, $V_c(R)$, of a particle on a circular orbit with radius R in such a density distribution, is given by

$$V_c^2(R) = \frac{GM(< R)}{R}\,. \tag{1}$$

Demonstrate that a constant circular velocity, $V_c = V_{c,0}$, implies that the density distribution is $\rho(R) = V_{c,0}^2/(4\pi G R^2)$. [2 marks]

(2) The surface density, $\sigma(R)$, of stars in a spiral galaxy falls with distance R to the centre as

$$\sigma(R) = \sigma_d\,\exp(-R/R_d)\,.$$

Here, σ_d is the central surface density, and R_d is the scale-length. Both are constants. Demonstrate that the disc mass enclosed in a sphere of radius R is

$$M_d(< R) = 2\pi\,\sigma_d\,R_d^2\,(1 - (1+x)\exp(-x))\,,$$

where $x \equiv R/R_d$. [2 marks]

The relation between enclosed mass and circular velocity given in Eq.(1) is only approximately valid for a disc. However you may assume it does hold in what follows.

(3) Take $R_d = 3$ kpc, $M_d(R \to \infty) = 2 \times 10^{10}\ M_\odot$, and assume that the circular velocity of disc and dark halo combined is V_c=220 km s^{-1} at distance $R = 100$ kpc. Evaluate V_c at $R = 5$ kpc and $R = 10$ kpc. [3 marks]

(4) This galaxy is at a distance of $d = 10$ Mpc, and its disc is tilted by 30 degrees (with 90 degrees corresponding to the case where the galaxy is face on). An observer uses a radio telescope to detect gas in the disc moving with the circular velocity using the HI 21-cm line. Sketch the detected wavelength of the line as function of angle from the centre of the galaxy. [3 marks]

[1 pc $= 3.09 \times 10^{16}$ m, $M_\odot = 2.0 \times 10^{30}$ kg, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}]