Summary of lecture 1 (21st November 2019)

Introduction, revision of basic quantities, networks and RC filters

Transfer Function:

A system which has a frequency-dependent response (such as filters, most amplifiers etc.) is described in terms of its ***transfer function***, $H(\omega)$. The transfer function is a complex function which describes the relationship between the input voltage (V_{in}) and the output voltage (V_{out}) and is in turn described by its gain and phase-shift response:

$$\text{Gain} = |H(\omega)| = \left|\frac{V_{out}}{V_{in}}\right| = \sqrt{\text{Re}\left[H(\omega)\right]^2 + \text{Im}\left[H(\omega)\right]^2}$$

$$\text{Phase shift} = \arg\left[H(\omega)\right] = \tan^{-1}\left(\frac{\text{Im}\left[H(\omega)\right]}{\text{Re}\left[H(\omega)\right]}\right)$$

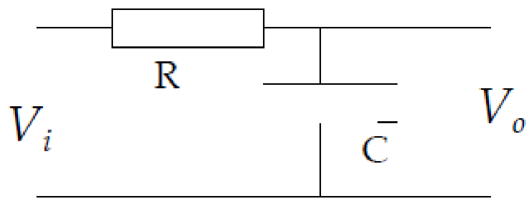
These can be visualised ***over many decades of frequency*** using 'Bode plots'.

RC Filters:

One configuration of an RC filter circuit:
The 'Transfer function' for this filter is

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}.$$

As this preferentially lets through signals at low-frequency, this is called a 'Low-pass' filter.



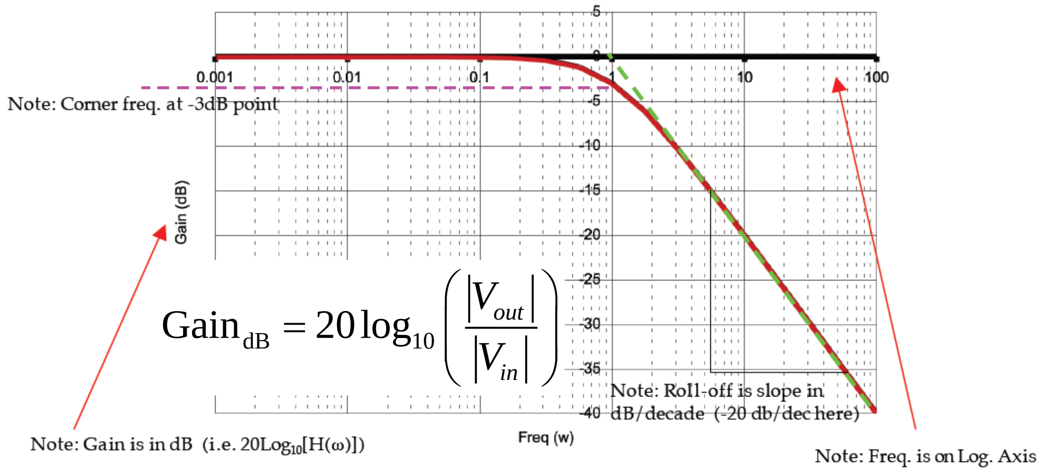
By switching the positions of the resistor and capacitor we can make a 'High-pass' filter, with a transfer function $|H(\omega)| = \dfrac{\omega CR}{\sqrt{1+\omega^2 C^2 R^2}}$.

In both cases, the 'corner' (or 'cut-off') frequency is that when the output power falls by a factor of two. This means that the output voltage drops to $V_{in}/\sqrt{2}$. As $|H(\omega_c)| = \dfrac{1}{\sqrt{2}}$, then

$\sqrt{1+\omega_c^2 C^2 R^2} = \sqrt{2}$ and $\omega_c = 1/RC \rightarrow f_c = 1/2\pi RC$.

Bode Plots:

Example for a low-pass filter. Recall the '20-Log' rule for voltage gain!



$$\mathrm{Gain}_{\mathrm{dB}} = 20\log_{10}\left(\frac{|V_{out}|}{|V_{in}|}\right)$$