Many waves: diffraction

03 February 2020 10:22

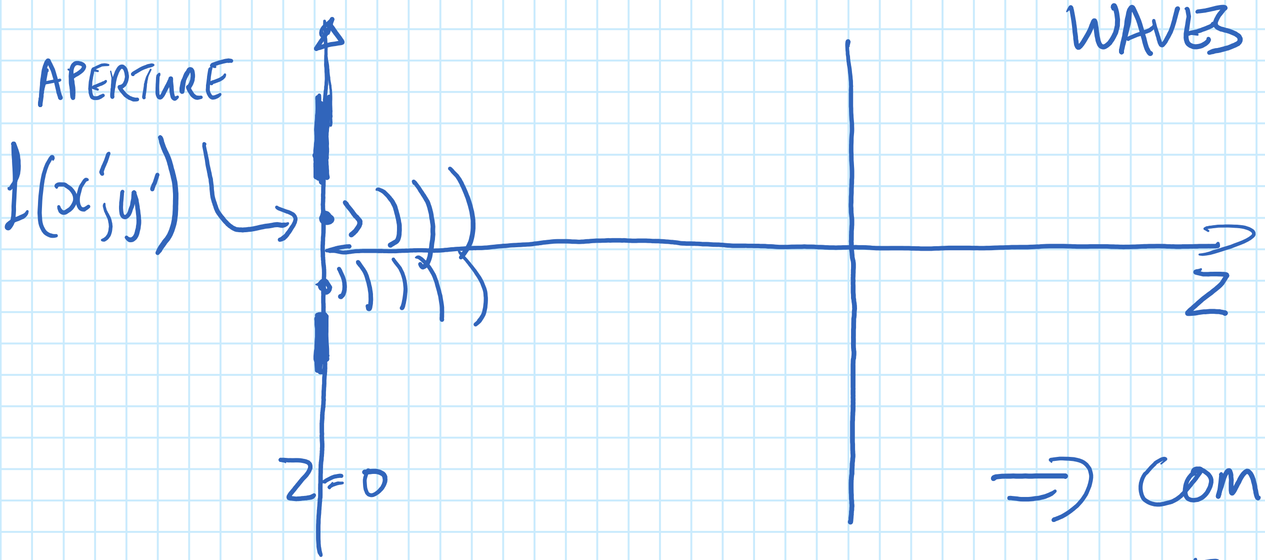
Optics f2f

Chapter 5

So FAR: We have up to N slits in a screen

Each slit is the source of a PARAXIAL SPHERICAL WAVES

CONCEPT: can treat any point on an APERTURE as a source of PARAXIAL SPHERICAL WAVES (FRESNEL ~ 1815)



REPLACE SUM WITH INTEGRAL OVER APERTURE

$\Longrightarrow$ COMPLETE SCALAR DESCRIPTION OF LIGHT PROPAGATION

Fresnel diffraction theory

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Optics f2f chapter 5

Related to HUYGENS PRINCIPAL: each point on the wavefront emits a secondary wave

→ Physically wierd (WHY?) see f2f

My preferred view:

- Paraxial spherical waves are a solution of the SCALAR WAVE EQUATION in the PARAXIAL APPROX
- SCALAR WAVE EQUATION is linear

⇒ any field E can be described by (infinite) sum of PARAXIAL SPHERICAL WAVES (P.S.W.)

(analogous to basis states in QM)

Fresnel diffraction integral

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f2f 5.3

P.S.W. originating from $(x', y', 0)$ is

$$E = \frac{A f_{jm}}{ikZ} e^{ikZ} e^{ik[(x-x_j)^2 + (y-y_m)^2]/2Z}$$

Here (j, m) is a point on the aperture defined by f_{jm}

A is a constant used for normalisation (LATER)

THEN: total field at $z = Z$ is

$$E = A e^{ikZ} \sum_{jm} f_{jm} e^{i[(x-X_j)^2 + (y-y_m)^2]/2Z}$$

$\longrightarrow$ sum over all points on aperture (same idea as grating last lecture)

Fresnel diffraction integral

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DISCRETE APERTURE $\longrightarrow$ CONTINUOUS APERTURE

e.g diffraction grating (infinitely narrow holes) — real hole diameter a, slits with width a

f_{jm} (grid) $\longrightarrow$ $f(x', y')$ aperture function

$\sum_{jm}$ $\longrightarrow$ $\iint_{-\infty}^{\infty} dx'\, dy'$ FRESNEL DIFFRACTION INTEGRAL

$$\therefore E^{(2)} = \underbrace{\frac{E^{(0)}}{i\lambda z}}_{A \text{ (see f2f)}} e^{ikz} \iint_{-\infty}^{\infty} f(x', y')\, e^{ik\frac{\left[(x-x')^2 + (y-y')^2\right]}{2z}}\, dx'\, dy'$$

Alternate form

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The Fresnel Diffraction Integral (FDI)
can be written in terms of r_p (paraxial distance)

$$r_p = z + \frac{(x-x')^2 + (y-y')^2}{2z}$$

$$\therefore \; E^{(z)} = \frac{E_0}{i\lambda z} \iint_{-\infty}^{\infty} f(x', y') \, e^{i r_p} \, dx' \, dy' \quad \Bigg| \; \text{BY SUBSTITUTION}$$

Field at P (x, y, z) is sum of fields from P' (x', y', z) with phase given by propagation distance r_p

(infinite phasor sum!)

Application: Radial symmetry, circular aperture and Fresnel zones

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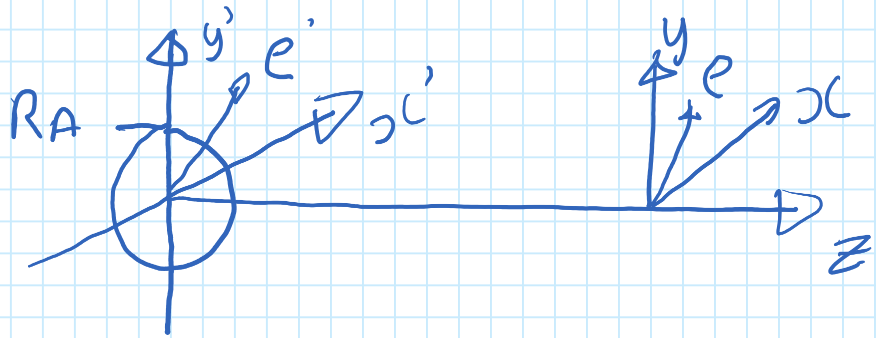
Many important problems have radial symmetry

e.g. circular aperture Re-write FDI using

$\rho' = (x'^2 + y'^2)^{1/2}; \quad \rho = (x^2 + y^2)^{1/2}$

Take the aperture to be uniformly illuminated

i.e $f(\rho) = 1$ for $\rho < R_A$



Circular aperture and Fresnel zones

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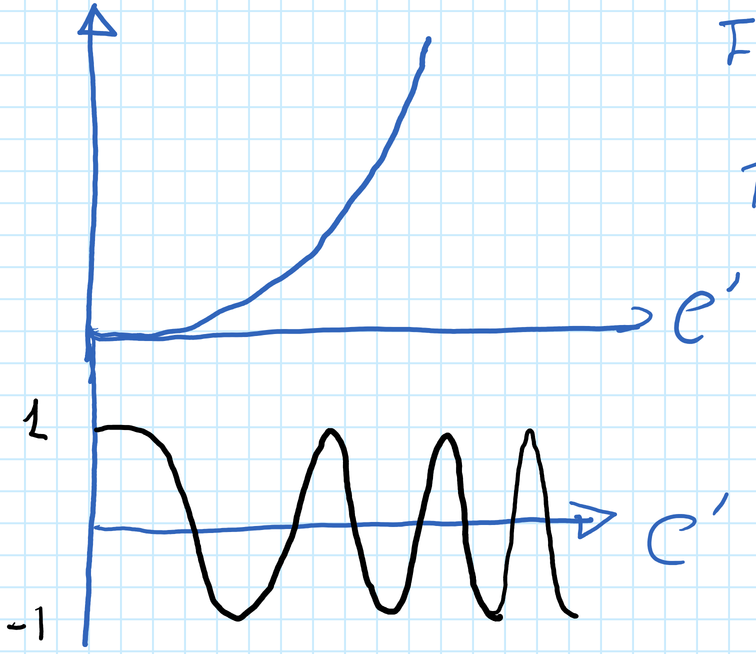
The FDI becomes

$$E^{(z)} = \frac{E_0 e^{ikz}}{i\lambda z} \int_0^{R_a} e^{\frac{ik\rho'^2}{2z}} \underbrace{2\pi \rho'}_{\text{from integral over } \theta} d\rho'$$

Let's look at the integrand:

PHASE varies QUADRATICALLY with ρ' (see lecture 3)



IT IS PHASE MODULO 2π THAT MATTERS!

Let $\phi = k\left(z + \frac{\rho'^2}{2z}\right)$

PLOT $\cos(\phi)$ @ $z = 0$

Fresnel zones

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Zeroes of $\cos(\phi)$ show where contributions of different FRESNEL ZONES change sign

i.e where $\cos(\phi) = 0$ i.e $\frac{2\pi\rho'^2}{\lambda 2z} = \frac{(2m+1)}{2}\pi$

Now value of ϕ for first Fresnel zone can vary (via $e^{i(kz)}$ factor in phase) $\Rightarrow$ write $\frac{2\pi\rho'^2}{\lambda 2z} = \frac{m\pi}{2}$

$\therefore$ we find the edges of the Fresnel zones by

$$\boxed{\rho'^2 = \sqrt{m\lambda z}}$$

Fresnel zones

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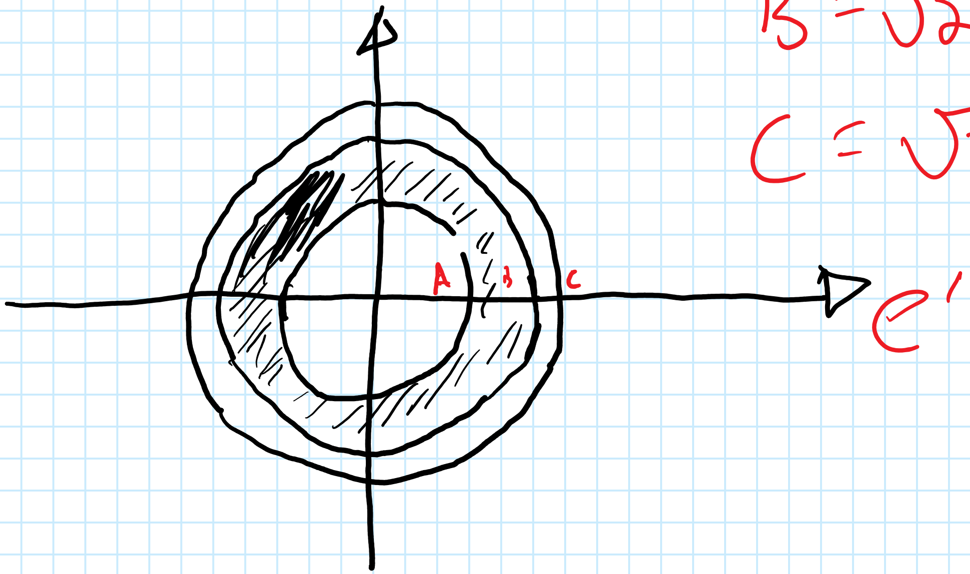


$A = \sqrt{\lambda z}$

$B = \sqrt{2\lambda z}$

$C = \sqrt{3\lambda z}$

LIGHT FROM SUCCESSIVE FRESNEL ZONES IS π OUT OF PHASE $\Rightarrow$ INTERFERES DESTRUCTIVELY

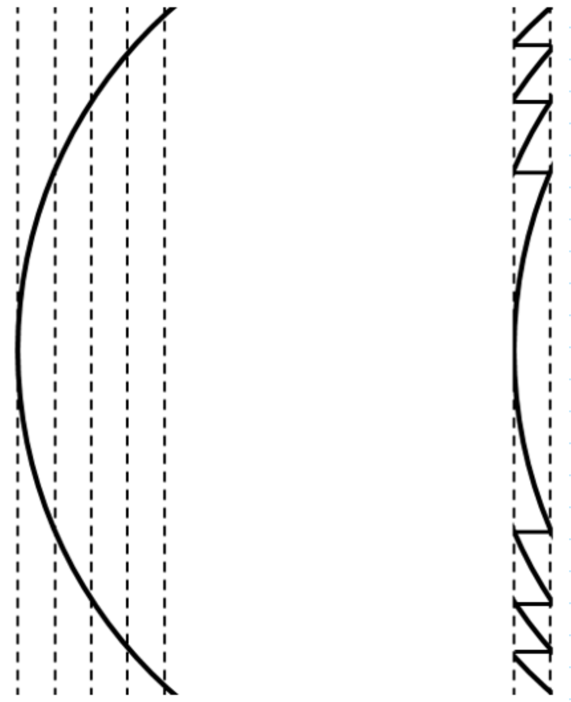
ZONE PLATES & LENSES

By blocking m odd

or phase shifting m odd

we can make a FRESNEL LENS

FOCUSSING USING DIFFRACTION!



Diffraction from a circular aperture

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Return to FDI for a circular aperture:

Calculate on axis intensity @ $(\rho, z) = (0, z)$

$$E^{(2)} = \frac{E_0 e^{ikz}}{i\lambda z} \int_0^{R_a} e^{ik\rho'^2/2z} \, 2\pi \rho' d\rho'$$

$$= \frac{E_0 e^{ikz}}{i\lambda z} \left[\frac{2z\lambda}{i 2\pi 2\rho'} e^{\frac{ik\rho'^2}{2z}} 2\pi\rho' \right]_0^{R_a}$$

$$= -E_0 e^{ikz} \left(e^{\frac{ikR_a^2}{2z}} - 1 \right) = -2i E_0 e^{ikz} e^{\frac{ikR_a^2}{4z}} \sin\left(\frac{kR_a^2}{4z}\right)$$

(see notes on diffraction grating for last step)

Diffraction from a circular aperture

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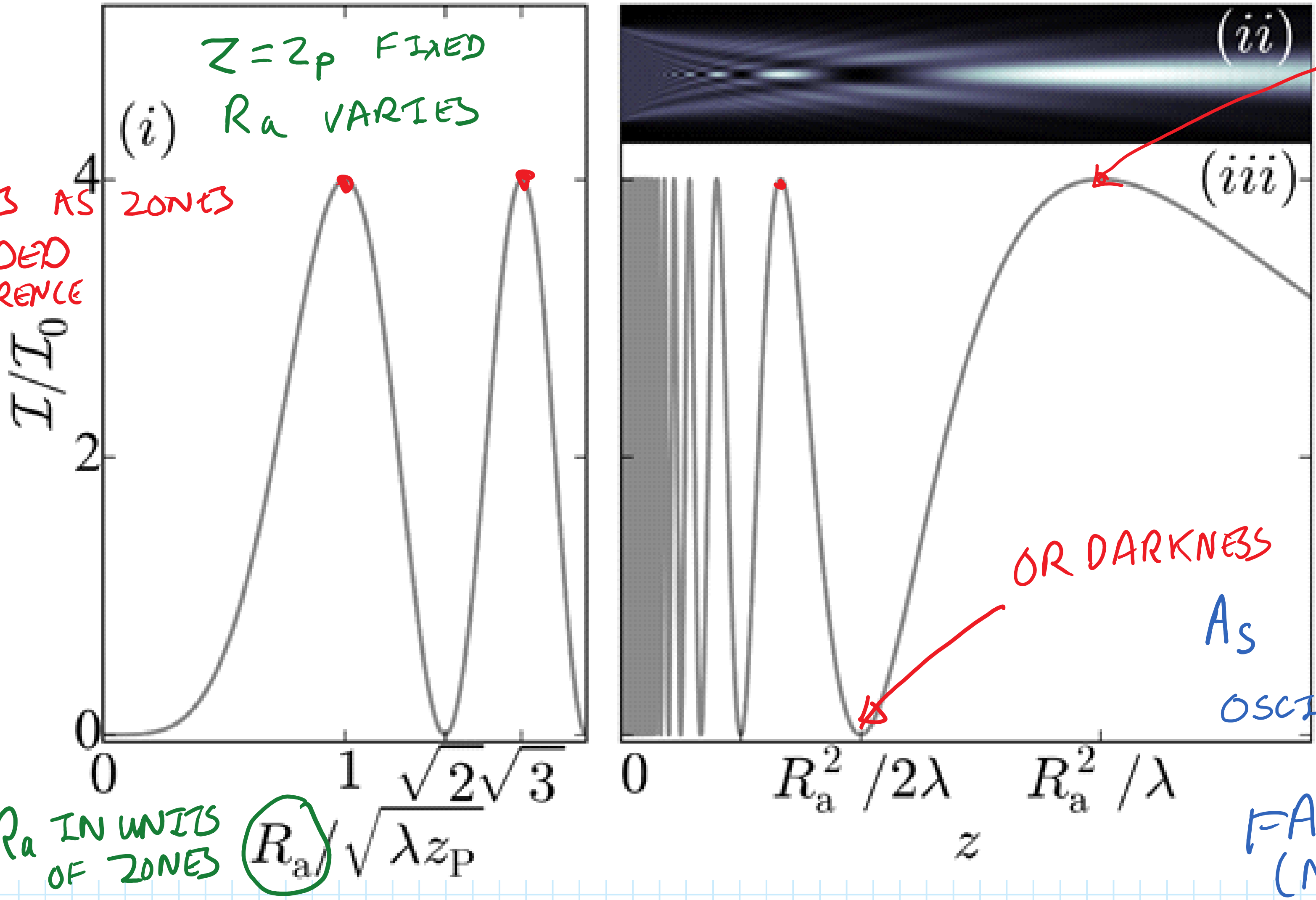
Finally $I^{(2)} = 4 I_0 \sin^2\left(\frac{\pi R_a^2}{2\lambda z}\right)$ SLIDES



$z = z_p$ FIXED

R_a VARIES

OSCILLATES AS ZONES ARE ADDED INTERFERENCE

R_a IN UNITS OF ZONES

CAN HAVE BRIGHT SPOT IN CENTER OF SHADOW (ARAGO SPOT)

OR DARKNESS

As $z \gg \frac{R_a^2}{\lambda}$

OSCILLATIONS GO AWAY

FAR FIELD (NEXT LECTURE)