ELECTROMAGNETISM

Level 2 Physics problems – Foundations of physics 2

Solution 6 Cycle 2 Version 1

Professor D P Hampshire – 2nd Year Physics Lecture Course

Information underlined or indicated by red text is required for marks to awarded. The mark scheme is a guide and solutions should not be considered to be unique. Marks are awarded for correct relevant Physics.

1. Displacement current density,

$$\frac{\partial \underline{\boldsymbol{D}}}{\partial t} = -\omega\varepsilon_0 \underline{\boldsymbol{E}}_0 \sin\omega t \qquad 1\text{-}1$$

$$\left|\underline{\boldsymbol{E}}_o\right| = \left|\frac{\partial \underline{\boldsymbol{D}}}{\partial t}\right| \Big/ \varepsilon_0\omega \qquad 1\text{-}2$$

$$\left|\frac{\partial \underline{\boldsymbol{D}}}{\partial t}\right|_{\max} = \sqrt{2}\left|\frac{\partial \underline{\boldsymbol{D}}}{\partial t}\right|_{rms} \qquad 1\text{-}3$$

Plugging in numbers,

$$=> \left|\underline{\boldsymbol{E}}_0\right| = \frac{\sqrt{2}(1\times10^{-4})}{(8.85\times10^{-12})(2\pi\times10^{9})} = 2.54\times10^{-3}\ \mathrm{Vm}^{-1} \qquad 1\text{-}4$$

$$=> \left|\underline{\boldsymbol{B}}_0\right| = \frac{\left|\underline{\boldsymbol{E}}_0\right|}{c} = \frac{2.54\times10^{-3}}{3\times10^{8}} = 8.48\times10^{-12}\ \mathrm{T} \qquad 1\text{-}5$$

1 mark if answers 1-4 and 1-5 are correct.
[Qn 1: 1 mark total]

2.

$$\left|\frac{\partial \underline{\boldsymbol{D}}}{\partial t}\right| = \varepsilon_r\varepsilon_0\omega\left|\underline{\boldsymbol{E}}\right| \qquad 2\text{-}1$$

$$\left|\underline{\boldsymbol{J}}\right| = \sigma\left|\underline{\boldsymbol{E}}\right| \qquad 2\text{-}2$$

$$=> \sigma = \varepsilon_r\varepsilon_0\omega = 5(8.85\times10^{-12})(2\pi\times10^{9}) = 0.278\ \Omega^{-1}\mathrm{m}^{-1} \qquad 2\text{-}3$$

1 mark if answer 2-3 is correct.
[Qn 2: 1 mark total]

3. a) For electromagnetic waves:

$$k^2 = \omega^2\mu\varepsilon + i\mu\sigma_N\omega \qquad 3\text{-}1$$

For a good conductor;

$$\sigma_N >> \varepsilon\omega \qquad 3\text{-}2$$

$$\sigma_N \approx 2 \qquad 3\text{-}3$$

$$\varepsilon\omega \approx (50\times8.85\times10^{-12}\times2\pi\times1.8\times10^{9}) = 5.0 \qquad 3\text{-}4$$

So, the inequality is not satisfied. Therefore, not a good conductor. 3-5

1 mark if inequality 3-2 is quoted and 3-3 and 3-4 values stated. Must state 3-5.

b)

$$k^2 = (7.11 + i2.84) \times 10^4 \text{ m}^{-2} \quad 3\text{-}6$$

$$=> k = \pm(271.8 + 52.3i) \text{ m}^{-1} \quad 3\text{-}7$$

The decay is given by;

$$\frac{E}{E_0} = e^{-\text{Im[k]}\cdot x} = e^{-52.3x} = e^{-(52.3)(0.02)} = 0.35 \quad 3\text{-}8$$

Percentage reduction is 65 % 3-9

1 mark if 3-7 and 3-8 is calculated and answer 3-9 is correct. [Qn 3: 2 marks total]

4. a) Assume that,

$$\underline{\boldsymbol{E}} = \underline{\boldsymbol{E}}_0 e^{i(\underline{\boldsymbol{k}}\cdot\underline{\boldsymbol{r}}-\omega t)} \text{ and } \underline{\boldsymbol{v}} = \underline{\boldsymbol{v}}_0 e^{i(\underline{\boldsymbol{k}}\cdot\underline{\boldsymbol{r}}-\omega t)} \quad 4\text{-}1$$

Inserting this in the given equation gives.

$$-im\omega\underline{\boldsymbol{v}} = -e\underline{\boldsymbol{E}} - \frac{m\underline{\boldsymbol{v}}}{\tau} \quad 4\text{-}2$$

Rearranging,

$$=> \underline{\boldsymbol{v}} = \frac{e\underline{\boldsymbol{E}}}{m\left(i\omega - \frac{1}{\tau}\right)}$$

But,

$$\underline{\boldsymbol{J}} = -Ne\underline{\boldsymbol{v}} = \sigma\underline{\boldsymbol{E}} \quad 4\text{-}3$$

$$=> \sigma = -Ne\left(\frac{e}{m\left(i\omega - \frac{1}{\tau}\right)}\right) = \frac{-Ne^2}{m\left(i\omega - \frac{1}{\tau}\right)} \quad 4\text{-}4$$

1 mark for correct derivation of 4-4

b) At low frequencies (~constant σ),

$$\sigma = \frac{Nq^2\tau}{m} \quad 4\text{-}5$$

$$=> \tau = \frac{m\sigma}{Nq^2}$$

$$= \frac{(9.1 \times 10^{-31})(10^8)}{(10^{30})(1.6 \times 10^{-19})^2}$$

$$= 3.55 \times 10^{-15}\text{s} \qquad 4\text{-}6$$

1 mark for correct answer 4-6

c)

$$|\sigma| = \frac{Nq^2}{m}\left|\frac{1}{\left(\frac{1}{\tau} - i\omega\right)}\right| = \frac{Nq^2}{m}\frac{1}{(\tau^{-2} + \omega^2)^{\frac{1}{2}}} \qquad 4\text{-}7$$

$$\sigma_N(\omega_1) = \frac{\sigma_N(\omega_2 \to 0)}{3} \qquad 4\text{-}8$$

$$=> \frac{1}{(\tau^{-2} + \omega^2)^{\frac{1}{2}}} = \frac{\tau}{3} \qquad 4\text{-}9$$

$|\sigma|$ is reduced by a factor of 3 when,

$$\omega^2 + \frac{1}{\tau^2} = \frac{9}{\tau^2} \qquad 4\text{-}10$$

$$=> \omega = \frac{1}{\tau}(9-1)^{1/2} = 8 \times 10^{14}\text{rad s}^{-1} \qquad 4\text{-}11$$

$$f = \frac{\omega}{2\pi} = 1.3 \times 10^{14}\text{Hz} \qquad 4\text{-}12$$

1 mark for derivation of answer 4-10
1 mark for answer 4-12
[Qn 4: 4 marks total]

5. a)

$$\phi = BA = B\pi a^2 \qquad 5\text{-}1$$

$$EMF = \frac{\partial\phi}{\partial t} = \frac{B\pi a^2}{1/f} = \frac{B\pi a^2}{2\pi/\omega} = \frac{Ba^2\omega}{2} \qquad 5\text{-}2$$

1 mark for answer 5-2

b) $\phi = 0$, EMF = 0 5-3

1 mark for answer 5-3
[Qn 5: 2 marks total]

Total for all questions 10 marks