# University of Durham

## EXAMINATION PAPER

May/June 2017 Examination code: PHYS2621-WE01

**STARS AND GALAXIES**

**SECTION A**. Observational Techniques
**SECTION B**. Stars
**SECTION C**. Galactic Astronomy

**Time allowed:** 3 hours

**Additional material provided:** None

**Materials permitted:** None

**Calculators permitted:** Yes **Models permitted:** Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

**Visiting students may use dictionaries:** No

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**Instructions to candidates:**

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **any three** of the four optional questions. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklets for Sections B and C, in order, inside your booklet for Section A, before they are collected by the invigilator.

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**Information**

A list of physical constants is provided on the next page.

 

## Information

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| Elementary charge: | $e = 1.60 \times 10^{-19}$ C |
| Speed of light: | $c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$ |
| Boltzmann constant: | $k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$ |
| Bohr magneton: | $\mu_\mathrm{B} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$ |
| Electron mass: | $m_\mathrm{e} = 9.11 \times 10^{-31}$ kg |
| Gravitational constant: | $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ |
| Proton mass: | $m_\mathrm{p} = 1.67 \times 10^{-27}$ kg |
| Planck constant: | $h = 6.63 \times 10^{-34}$ J s |
| Permittivity of free space: | $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$ |
| Magnetic constant: | $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$ |
| Molar gas constant: | $R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$ |
| Avogadro's constant: | $N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$ |
| Gravitational acceleration at Earth's surface: | $g = 9.81\ \mathrm{m\,s^{-2}}$ |
| Stefan-Boltzmann constant: | $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$ |
| Astronomical Unit: | AU $= 1.50 \times 10^{11}$ m |
| Parsec: | pc $= 3.09 \times 10^{16}$ m |
| Solar Mass: | $M_\odot = 1.99 \times 10^{30}$ kg |
| Solar Luminosity: | $L_\odot = 3.84 \times 10^{26}$ W |

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**SECTION A**. OBSERVATIONAL TECHNIQUES

Question 1 is compulsory. Question 2 is optional.

1. (a) What is the maximum distance from the Earth that an extrasolar planet (one not in our Solar System) can be resolved from its host star with NASA's 6.50 m diameter James Webb Space Telescope? Assume a wavelength for the observations of $\lambda = 550$ nm and that the planet is 1.00 AU from the star. [4 marks]

   (b) Suggest four ways that noise in an astronomical CCD image can be minimised (before the data has been processed). [4 marks]

   (c) State the reflection grating equation of a spectrograph, defining the symbols. [2 marks]

   Calcium has two prominent spectral lines, at 393.3 nm and 396.9 nm. A certain grating produces a first order spectrum where these two lines are diffracted at angles of 42.19° and 42.28° respectively with respect to the grating normal. Calculate the ruling density of the grating in units of lines $\mathrm{mm}^{-1}$. [2 marks]

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2. The Very Large Telescope (VLT) has an 8.0 m diameter primary mirror and is located on a high mountain site in Chile. To correct for atmospheric turbulence and hence record images at the diffraction limit, the VLT is equiped with laser guide star adaptive optics.

   (a) State four reasons for building optical and infrared astronomical observatories on remote, high altitude sites. [4 marks]

   (b) Show that the energy per unit area collected by a telescope operating at the diffraction limit is proportional to $D^4$, where $D$ is the diameter of the primary mirror. [2 marks]

   Approximately 50 stars are known to orbit the black hole located in the galactic center. In June 2018, one of these stars, named *S2*, will pass within 11 light hours of the black hole. When observed from Earth, Newtonian mechanics predicts a periapsis position (the point of closest approach) for *S2* that appears 6 milli-arcseconds away from that predicted by Einstein's theory of General Relativity. To measure the periapsis position and so test "Einstein versus Newton", diffraction limited observations of the galactic center will be taken at infrared wavelengths using the VLT.

   (c) If the flux density of *S2* is $1.10 \times 10^{-17}\,\mathrm{W\,m^{-2}\,\mu m^{-1}}$, show that the telescope will detect approximately 300 photons second$^{-1}$ from this star. Assume the observations will be taken at infrared wavelengths ($\lambda = 2.20\,\mu$m), the filter bandpass is $\Delta\lambda = 0.20\,\mu$m and that the combined efficiency of the atmosphere, telescope and detector are 25%. [4 marks]

   (d) If the position of *S2* can be determined to an accuracy of $\Delta\theta = \theta_{\mathrm{dl}} / (S/N)$ where $\theta_{\mathrm{dl}}$ is the diffraction limit of the telescope and $S/N$ is the signal-to-noise ratio of the star on the detector, estimate the exposure time required to differentiate between the predictions of Einstein versus Newton.

   Assume that the positional accuracy needed is 10% of the difference between the positions predicted by Einstein and Newton, the image of *S2* covers 4 pixels on the detector, the sky-background produces 15000 photo-electrons pixel$^{-1}$ second$^{-1}$, the detector has a gain of 1.0 and the observations are background limited. [8 marks]

   (e) Why it is preferable to make observations of stars in the galactic center at infrared rather than optical wavelengths. [2 marks]



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## SECTION B. STARS

Question 3 is compulsory. Questions 4 and 5 are optional.

3. (a) Calculate the minimum and maximum wavelength that corresponds to the Brackett series (n=4) of Hydrogen. Recall that

$$E = -13.6\ \text{eV}\left(\frac{1}{m^2} - \frac{1}{n^2}\right).$$

[4 marks]

(b) Two stars have the same apparent magnitude and the same physical radius. However, star A has a temperature of 3,000 K while star B has a temperature of 50,000 K. On the basis of these properties, how much further away is star B than star A? [4 marks]

(c) List four observational signatures that indicate a binary star system. [4 marks]

(d) The equation of hydrostatic equilibrium for a star is derived by balancing gravity and pressure. What are the two main contributors to the pressure in a main-sequence star? Which of these dominates in the Sun? [4 marks]

(e) Estimate the time it would take for a photon to escape from the core of a main-sequence star on a "random walk". Assume the stellar core has a radius of $2.0 \times 10^8$ m, a mean-free path of $5.0 \times 10^{-3}$ m, and scattering with a $1.0 \times 10^{-8}$ s delay each time the photon is scattered. What is the dominant form of opacity in the core of the Sun? [4 marks]

(f) What region and what physical process drives radial pulsations in stars? Why doesn't the Sun develop strong radial pulsations? [4 marks]

(g) The main-sequence lifetime of a star with a mass of 10.0 $M_\odot$ and a luminosity of 3,000 $L_\odot$ is $10^8$ years. Assuming that 26.2 MeV of energy is released during each Helium-producing chain, what fraction of the mass of the star is converted from Hydrogen to Helium over the main-sequence lifetime of the star? [4 marks]

[Hint: 1 eV is $1.60 \times 10^{-19}$ J]

4. (a) Draw a Hertzsprung-Russell diagram to illustrate the evolution of a low-mass (1 $M_\odot$) and a high-mass (25 $M_\odot$) star. Clearly label the axes, mark the track of the main sequence, and sketch tracks to indicate the post-main sequence evolution of these stars up to (and including) the giant-branch phase. [4 marks]

(b) List four major factors that drive the evoluton of stars. [4 marks]

(c) Show that the energy ($E$) available from the gravitational collapse of a spherically symmetrical star of constant density is $E = -\frac{3}{10}\frac{GM^2}{R}$, where $M$ is the mass of the star and $R$ is the post-collapse radius. Assume that the star is in virial equilibrium but state any other assumptions that you make in your derivation. [6 marks]

[Hint: the gravitational potential of mass $dm$ in a star of mass $M$ is $dU = -\frac{GMdm}{r}$, where $dm = 4\pi r^2 \rho dr$ for a shell at radius $r$ of density $\rho$ and thickness $dr$.]

(d) Neutron stars may result from the collapse of a massive star. What reaction causes the production of neutrinos in this physical process? [3 marks]

(e) Assuming that the energy available from the gravitational collapse of a 2 $M_\odot$ core is released in the form of neutrinos, and that the energy of a typical neutrino is 6 MeV, how many neutrinos are produced from the collapse? How many neutrinos per square metre would be detected by an observer at a distance of 10 kpc? [3 marks]

[Hint: 1 eV is $1.60 \times 10^{-19}$ J]

5. (a) How are degenerate stars fundamentally different from main-sequence stars? [2 marks]

(b) Show that the pressure ($P$) from non-relativistic degenerate electrons is $P = \frac{\hbar^2}{m_\mathrm{e}} n_\mathrm{e}^{5/3}$, where the number density of electrons, $n_\mathrm{e}$, is given by $n_\mathrm{e} = (Z/A)(\rho/m_\mathrm{H})$, $Z$ is the atomic number, and $A$ is the mass number. Note that $P \sim \frac{1}{3} n_\mathrm{e} p v$, where $p$ is the electron momentum, with momentum in the $x$ direction given by $p_\mathrm{x} = \hbar n_\mathrm{e}^{1/3}$. [5 marks]

(c) The pressure at the centre of a carbon-dominated white dwarf with a mass of 1 $M_\odot$ and a radius of 5,000 km is $P \sim 10^{22}$ N m$^{-2}$. Calculate whether electron degeneracy pressure is sufficient to hold up the white dwarf. What other element could dominate the composition of a white dwarf? [5 marks]

(d) White dwarfs cool through the release of kinetic energy. Estimate the cooling time for a carbon-dominated white dwarf of 1 $M_\odot$ with a constant temperature of $1 \times 10^4$ K and a constant luminosity of 0.001 $L_\odot$. Why is this estimate a lower limit on the true cooling time? [5 marks]

(e) Briefly explain why the radius decreases with increasing mass in white dwarfs. [3 marks]



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## **SECTION C**. GALACTIC ASTRONOMY

Question 6 is compulsory. Question 7 is optional.

6. (a) Suppose that the tidal interaction between two galaxies results in a burst of star formation in both of them. How would that affect their optical colours? What does the red colour of elliptical galaxies imply about their star formation activity? [4 marks]

   (b) Dust absorbs some of the light emitted by stars. Briefly discuss two consequences of this on the optical flux detected from those stars. Where does the absorbed energy go? [4 marks]

   (c) The gas in a star forming region is photo-ionised by stars. Why are the massive stars the dominant source of ionising photons? Which process allows you to observe such star forming regions in the H$\alpha$ line? [4 marks]

   (d) Estimate the accretion rate onto a supermassive black hole that outshines all the stars in the Milky Way. Express your answer in units of $M_\odot$ yr$^{-1}$. Take $L = 1.0 \times 10^{10} L_\odot$ for the luminosity of the Milky Way. [4 marks]

   (e) What are clusters of galaxies? Briefly describe how X-ray observations, and gravitational lensing, have been used to determine their mass. Briefly discuss one basic assumption underlying each of these two methods. [4 marks]

  

7. (a) What is the 'rotation curve' of a galaxy? The Milky Way's rotation curve is said to be flat. What does that mean? Demonstrate that a flat rotation curve requires that density, $\rho$, depends on radius, $R$, as $\rho(R) \propto 1/R^2$. [5 marks]

Discuss two lines of evidence that most of this mass is in the form of dark matter. [4 marks]

(b) The Sun orbits the centre of the Milky Way on a circular orbit with radius $R_\odot = 8.00$ kpc and orbital speed $V_c(R = R_\odot) = 220$ km s$^{-1}$. Compute the mass, $M_{\rm enc}$, enclosed by the orbit in solar masses. [3 marks]

(c) The intensity profile of the Milky Way disc as a function of galactic radius, $R$, is given by $\Sigma(R) = \Sigma_0 \exp(-R/R_h)$, where $\Sigma_0$ is the central intensity with units of $L_\odot$ pc$^{-2}$. $\Sigma_0$ and $R_h = 3.0$ kpc are constants. Calculate the value of $\Sigma_0$ such that the disc luminosity is equal to $L = 1.0 \times 10^{10}\ L_\odot$. Compute the luminosity of the disc, $L_{\rm enc}$, enclosed by the solar orbit in units of $L_\odot$. [4 marks]

Combine these results to show that $M_{\rm enc}/L_{\rm enc} = 12\ M_\odot/L_\odot$. Would it be possible that $M_{\rm enc}$ is all in stars like the Sun? Could more massive stars provide the required mass? [4 marks]

[Hint: the indefinite integral $\int x \exp(-x)\, dx = -(x+1)\exp(-x)$.]

  