Workshop 3: Tossing the Caber

The sport of tossing the caber entails trying to flip a caber (or uniform density thin log) so that it rotates through at least 180° about an axis perpendicular to its length prior to landing. This endeavour can approximately be split into the following phases:

1. the competitor supports the caber from beneath with their hands a height h above the ground, holding it almost vertically, before lumbering forwards;
2. the competitor stops, while still holding the base of the caber, and the caber topples forward;
3. once the caber has toppled an angle θ_i away from the vertical, the competitor applies a vertical impulsive force to its base;
4. the caber experiences no further torques and subsequently moves under the influence of gravity until landing.

We would like to determine if a launch angle, θ_i, will produce a successfully flipped caber.

(a) Using θ as the generalised coordinate, integrate along the caber to determine the kinetic energy for a caber of mass m and length $2l$ during phase 2. Hence, find the Lagrangian for the caber and use the Euler-Lagrange equation to show that

$$\ddot{\theta} = \frac{3g}{4l} \sin\theta.$$

(b) Using the small angle approximation, which is sufficiently accurate for the situation under consideration here, and assuming that the initial angular velocity of the caber is $\dot{\theta}_0$ when $\theta = 0$, find the equation of motion for θ during phase 2.

(c) Calculate the moment of inertia, I, for rotations about an axis through the centre of mass and perpendicular to its length, of a uniform rod with mass m and length $2l$.

(d) If the vertical impulsive force applied through the base of the caber is of the form $F = K\delta(t - t_i)$, then determine the torque being applied to the caber, H. Using your result for part (c) and

$$\int H dt = I \int \ddot{\theta} dt,$$

show that the angular velocity of the caber for $t > t_i$ can be written as

$$\dot{\theta}(t > t_i) = \dot{\theta}(t_i^-) + \frac{3K}{ml} \sin\theta_i,$$

where $\dot{\theta}(t_i^-)$ represents the angular velocity of the caber just before the impulsive force is applied.

(e) Determine the equations of motion for θ and the height of the centre of mass of the caber, z_c, for $t' = t - t_i > 0$. Under the assumption that a successfully tossed caber involves the caber turning through more than 180° before the centre of mass is within a distance l of the ground, write down an inequality that should be satisfied for a successfully tossed caber.

(f) A caber is ~ 5 m long and has a mass of ~ 60 kg. Estimate values for the various other quantities and determine if an angle exists for which the caber is successfully tossed. (You may wish to write a computer code to solve the problem, or have a play with the one in the duo workshop solutions folder.)