The expansion coefficients are given by the overlap integrals:

i) $c_1 = \int_0^L \sqrt{\frac{2}{L}} \sin(\pi x/L) \frac{4}{\sqrt{5L}} \sin^3(\pi x/L)\, dx = \frac{4\sqrt{2}}{\sqrt{5}L} \int_0^L \sin^4(\pi x/L)\, dx$

Wolfram: $\int_0^L \sin^4(\pi x/L) dx = \frac{3L}{8}$ so $c_1 = \frac{4\sqrt{2}}{\sqrt{5}L}\frac{3L}{8} = \frac{3}{\sqrt{10}}$ [2 marks]

$c_2 = \frac{4\sqrt{2}}{\sqrt{5}L} \int_0^L \sin(2\pi x/L)\, \sin^3(\pi x/L) dx = 0$ using Wolfram alpha [1 mark]

$c_3 = \frac{4\sqrt{2}}{\sqrt{5}L} \int_0^L \sin(3\pi x/L)\, \sin^3(\pi x/L) dx = \frac{4\sqrt{2}}{\sqrt{5}L}\frac{-L}{8} = \frac{-1}{\sqrt{10}}$ [1 mark]

$c_4 = \frac{4\sqrt{2}}{\sqrt{5}L} \int_0^L \sin(4\pi x/L)\, \sin^3(\pi x/L) dx = 0$ [1 mark]

ii)

$$\begin{aligned}
\Psi(x,0) &= A\sin^3(\pi x/L) = \frac{A}{4}(3\sin \pi x/L - \sin 3\pi x/L) \\
&= \frac{1}{\sqrt{5L}}(3\sin \pi x/L - \sin 3\pi x/L) = \frac{1}{\sqrt{5L}}\sqrt{\frac{L}{2}}\left(3\sqrt{\frac{2}{L}}\sin \pi x/L - \sqrt{\frac{2}{L}}\sin 3\pi x/L\right) \\
&= \frac{1}{\sqrt{10}}(3\psi_1 - \psi_3)
\end{aligned}$$

[1 mark]

Hence $c_1 = 3/\sqrt{10}$ and $c_3 = -1/\sqrt{10}$ and all the rest of the $c_n = 0$ [1 mark]

iii) Probability of E_1 is $|c_1|^2 = 9/10$ [1 mark]

iv) The energy expectation value

$$\langle H \rangle = \langle E \rangle = \sum_n c_n^2 E_n = (9/10)E_1 + (1/10)E_3 = (9/10)E_1 + (1/10)9E_1 = 18/10 E_1 = 1.8E_1$$

[1 mark]

v) The full time dependent wavefunction is

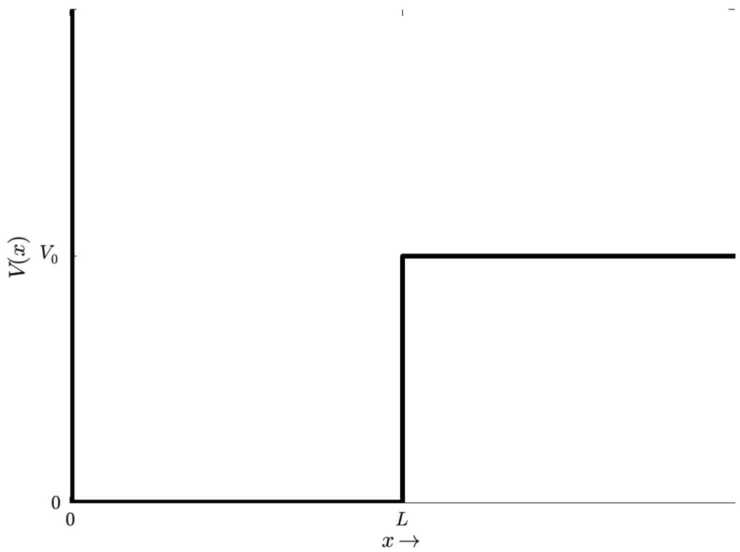
$$\Psi(x,t) = \frac{1}{\sqrt{10}}(3\psi_1 e^{-iE_1 t/\hbar} - \psi_3 e^{-iE_3 t/\hbar})$$

[1 mark]

The *semi-infinite* square well has a potential

$$V = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < L \\ V_0 & x > L, \end{cases}$$

as shown in the figure.



i) Assuming that V_0 is sufficiently large that there exists at least two bound solutions of Schrödinger's equation, then without solving any equations sketch the wave functions corresponding to the ground and first excited states. [4 marks]

ii) Using the Schrödinger equation show that for $0 < x < L$ the wavefunction is of the form $\psi(x) = A \sin kx$ with $k^2 = 2mE/\hbar^2$ and where A is a constant [2 marks]

iii) If V_0 is slightly reduced sketch how this changes the wavefunction of the first excited state. [2 marks]

iv) As V_0 is reduced further eventually there ceases to be a first excited state. What is the value of V_0 at which this occurs? [2 marks]