Foundations of Physics 2B/3C 2019-2020

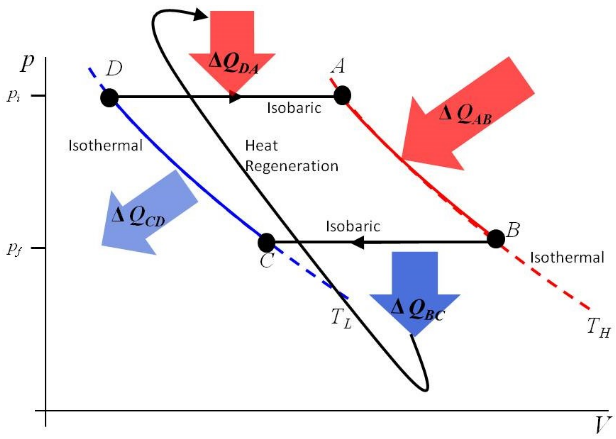
Thermodynamics – Solution, Th. 3

As a guide to completing your self-assessment, please use the following correspondence: Unsuccessful: (0 – 3 marks out of 10); Partially Successful: (4 – 7 marks out of 10); Successful (8 – 10 marks out of 10). Please also give information on any parts which you found difficult, as this will allow me to go over any common issues in the workshops. You can also talk individually to myself, and other staff members at these about any issues you faced when solving the problem.

a) The working fluid of a heat engine consists of ideal gas having heat capacity C_p. The engine has no friction and follows the cycle outlined below:

- Isothermal expansion when placed in contact with a hot reservoir at T_H between the states having pressures p_i and p_f;
- Isobaric cooling from T_H to T_L;
- Isothermal compression when placed in contact with a cold reservoir at T_L back to the original pressure, p_i;
- Isobaric heating from T_L to T_H.

i) Draw a fully labelled pV diagram for the cycle, carefully marking each place where heat can enter or leave the cycle.



Heat is taken in on the hot isotherm, and to increase the volume isobarically between the cold and hot isotherms. Heat is rejected at the cold isotherm, and in the isobaric cooling.

[1 mark for the axis with pressures labelled]
[1 mark for the correct shape and correctly labelled isotherms/isobarics]
[1 mark for indicating all the heats correctly]

(In the ideal Ericsson cycle, heat rejected at the lower pressure isobaric is stored in a regenerator before being re-input to the cycle at high pressure - not needed to be shown in this part of the solution).

ii) Show that the efficiency of this engine cycle is given by

$$\eta = \frac{T_H - T_L}{T_H + C_P(T_H - T_L)/\big(R\ln(p_i/p_f)\big)}.$$

Calculate the heat and work interactions for each thermodynamic step. The thermodynamic coordinates (p_n, V_n, T_n) of each point are, using the Ideal Gas Law for the volumes

$$A = (p_i, RT_H/p_i, T_H);\ B = (p_f, RT_H/p_f, T_H);\ C = (p_f, RT_L/p_f, T_L);\ D = (p_i, RT_L/p_i, T_L).$$

AB – Isothermal expansion. An ideal gas has $U = U(T)$ so $dU = 0$, so $\delta Q = -\delta W$ (first law)

$$\Delta Q_{AB} = -\int_{V_A}^{V_B} -p\,dV = \int_{V_A}^{V_B} \frac{RT_H\,dV}{V} = RT_H \ln\left(\frac{V_B}{V_A}\right).$$

$V_B = RT_H/p_f, V_A = RT_H/p_i$. So $\Delta Q_{AB} = RT_H\ln(p_i/p_f) > 0$, as $p_i > p_f$ i.e. heat in. [1 mark]

BC – Isobaric cooling. We can use heat capacities to calculate this.

$$\Delta Q_{BC} = \int_{T_H}^{T_L} C_p dT = C_p\,(T_L - T_H\,) < 0. \qquad \textbf{[1 mark]}$$

CD – Isothermal compression. Similarly to AB,

$$\Delta Q_{CD} = -\int_{V_C}^{V_D} -pdV = \int_{RT_L/p_f}^{RT_L/p_i} \frac{RT_L dV}{V} = RT_L \ln\left(\frac{p_f}{p_i}\right) < 0$$

DA – Isobaric heating, $\Delta Q_{DA} = \int_{T_L}^{T_H} C_p dT = C_p(T_H - T_L) > 0.$ **[1 mark]**

Efficiency is the ratio of product to expense $\eta = \frac{|\text{Work}|}{\text{Heat In}} = \frac{|-(Q_H - |Q_L|)|}{Q_H}$.

$$Q_H = \Delta Q_{AB} + \Delta Q_{DA} = C_p(T_H - T_L) + RT_H \ln(p_i/p_f) > 0.$$

$$Q_L = \Delta Q_{BC} + \Delta Q_{CD} = C_p(T_L - T_H) + RT_L \ln(p_f/p_i) < 0.$$

[1 mark]

$$\eta = \frac{C_p(T_H - T_L) + RT_H \ln(p_i/p_f) - |C_p(T_L - T_H) + RT_L \ln(p_f/p_i)\,|}{C_p(T_H - T_L) + RT_H \ln(p_i/p_f)}$$

$$= \frac{R(T_H - T_L)\ln(p_i/p_f)}{C_p(T_H - T_L) + RT_H \ln(p_i/p_f)} = \frac{T_H - T_L}{T_H + C_p(T_H - T_L)/R\ln(p_i/p_f)}.$$

[1 mark]

b) An Ericsson engine, which is totally reversible, follows the cycle describe above, but the process called regeneration takes place between the isobaric cooling and heating steps. Here, the heat rejected during the isobaric cooling goes through a regenerator, and never leaves the confines of the engine, before being re-input during the isobaric heating process. This means that no additional net heat energy from an external source is required to be added during the isobaric heating i.e.) the only heat that is input during the cycle is during isothermal heating. Determine how the engine's efficiency changes. Comment on your result.

We now have $Q_H = \Delta Q_{AB}$ **only, since the heat output** ΔQ_{BC} **doesn't leave the confines of the heat engine, and is returned at** ΔQ_{DA} **(no additional external energy is required in this step). The work done is identical to before (cycle area is the same), as the two isobaric heat interactions are equal and opposite, so cancel and we now have**

$$\eta_{Ericsson} = \frac{\left|\Delta Q_{AB} - |\Delta Q_{CD}|\right|}{\Delta Q_{AB}} = \frac{1 - |RT_L \ln(p_f/p_i)\,|}{RT_H \ln(p_i/p_f)} = 1 - \frac{\left(RT_L \ln(p_i/p_f)\right)}{RT_H \ln(p_i/p_f)} = 1 - \frac{T_L}{T_H}.$$

[1 mark]

This is the same as the efficiency of a Carnot cycle! This is to be expected as the first Carnot principle states that all totally internal reversible engines operating between two reservoirs of given temperatures have the same efficiency. **[1 mark]**

Regeneration increases the efficiency of an engine. Engine design seeks to keep as much heat in the engine confines.