L2 Foundation of Physics 2B Optics 2019-20

O.WP.2 Spatial frequencies and paraxial plane waves

January 28, 2020

The harmonic wave solution of Maxwell's wave equation is

$$\underline{E}(\underline{r},t) \;=\; \underline{E}_0(\boldsymbol{r},t)\cos(\underline{k}\cdot\underline{r}-\omega t)\ . \tag{1}$$

In optics, it is sometimes convenient to use a paraxial plane wave solution of the form:

$$E = E_0 \mathrm{e}^{\mathrm{i}2\pi(ux+vy)}\mathrm{e}^{\mathrm{i}2\pi z/\lambda}\mathrm{e}^{-\mathrm{i}\pi(u^2+v^2)\lambda z}\ . \tag{2}$$

1. What are u and v, and what are their units? [2 marks]

2. List four steps needed to re-write the harmonic wave solution in the form of a paraxial plane wave. [4 marks]

3. Calculate u and v for a plane wave with wavelength $\lambda = 500$ nm, propagating at an angle $\theta = 30.0^\circ$ to the z axis in the xz plane. [4 marks]