University of Durham

EXAMINATION PAPER

May/June 2013 Examination code: 043641/01

LEVEL 3 PHYSICS: ADVANCED PHYSICS 3

SECTION A. MODERN ATOMIC AND OPTICAL PHYSICS
SECTION B. OPTICAL PROPERTIES OF SOLIDS
SECTION C. SOFT CONDENSED MATTER PHYSICS

Time allowed : 3 hours

Examination material provided : None

Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **one** other question from **each** section. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.

ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK

Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.

CALCULATORS: The following types ONLY may be used: Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Electron mass:	$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}$ J s
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31 \times 10^{3}\ \mathrm{J\,K^{-1}\,kmol^{-1}}$
Avogadro's constant:	$N_{\mathrm{A}} = 6.02 \times 10^{26}\ \mathrm{kmol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	AU $= 1.50 \times 10^{11}$ m
Parsec:	pc $= 3.09 \times 10^{16}$ m
Solar Mass:	$M_{\odot} = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_{\odot} = 3.84 \times 10^{26}$ W

SECTION A. MODERN ATOMIC AND OPTICAL PHYSICS
Answer Question 1 and **either** Question 2 **or** Question 3.

1. (a) The two main broadening mechanisms that have to be minimised in an atomic clock are *natural* broadening and *transit-time* broadening. Discuss, briefly, how these broadening mechanisms are minimised in the ^{87}Sr optical lattice clock and the underlying physical reason for the broadening common to both mechanisms. [4 marks]

   (b) Show graphically that the expectation value of the dipole moment operator $\hat{d} = -ez$ for the $2p1$ orbital in the hydrogen atom is zero. The $2p1$ orbital has the functional form

$$\psi_{2p1}(r,\theta,\phi) = Y_{1,+1}R_{2,1} = \frac{1}{4\sqrt{2\pi}a_0^{3/2}}\frac{r}{a_0}\sin\theta e^{i\phi}e^{-r/2a_0}$$

   [4 marks]

   (c) Show graphically how a π-transition between the $2p0$ and the $1s0$ orbitals of the hydrogen atom does not lead to the emission of a photon along the z-axis. [4 marks]

   (d) The $4s\ ^2S_{1/2} \rightarrow 4p\ ^2P_{3/2}$ "D2" cooling transition used in a magneto-optical trap (MOT) of ^{40}K at 767 nm has a linewidth of $\Gamma = 2\pi(6.0\,\text{MHz})$. The Doppler velocity of the MOT is given by

$$v_{\text{D}} = \left(\frac{\hbar\Gamma}{M}\right)^{\frac{1}{2}}.$$

   Calculate the capture velocity v_{c} of the MOT. [4 marks]

   (e) Discuss, briefly, how the first-order Doppler shift is eliminated in the ^{27}Al^{+} ion quantum logic clock. [4 marks]

2. In the LS-coupling scheme, the expectation value of the spin-orbit (S-O) interaction Hamiltonian for a one-electron atom is

$$\langle H_{\mathrm{S-O}} \rangle = (g_s - 1)\frac{\hbar^2}{2m_{\mathrm{e}}^2 c^2}\frac{e^2}{4\pi\epsilon_0}\left\langle \frac{1}{r^3} \right\rangle \langle \mathbf{s}\cdot\mathbf{l} \rangle,$$

where

$$\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{l\left(l+\frac{1}{2}\right)(l+1)}\left(\frac{Z}{na_0}\right)^3.$$

(a) Draw a vector diagram showing how the spin angular momentum $\mathbf{s}$ couples with the orbital angular momentum $\mathbf{l}$ to give the total electronic angular momentum $\mathbf{j}$ and calculate the values of $\mathbf{j}$ for the $2p\ {}^2P$ and $3d\ {}^2D$ states of the hydrogen atom. [4 marks]

(b) Show that the expectation value of the spin-orbit coupling is given by

$$\langle \mathbf{s}\cdot\mathbf{l} \rangle = \frac{1}{2}\{j(j+1) - l(l+1) - s(s+1)\}.$$

[4 marks]

(c) A spectrum of the hydrogen atom reveals two lines at 456.68585 THz and 456.67488 THz that have been assigned to transitions that couple two different spin-orbit states of the $2p\ {}^2P$ state to the lowest energy spin-orbit state of the $3d\ {}^2D$ state. Calculate the spin-orbit splitting of the $3d\ {}^2D$ state in units of GHz. [12 marks]

3. A focused laser beam of wavelength λ propagating along the z-axis has an intensity profile of

$$I(r,z) = \frac{2P}{\pi w(z)^2} \exp\left(-\frac{2r^2}{w(z)^2}\right),$$

where P is the laser beam power. The beam waist is $w(z) = w_0(1+z^2/b^2)^{1/2}$, where w_0 is the beam waist at the point of maximum intensity I_{max}, $b = \pi w_0^2/\lambda^2$ and $r^2 = x^2 + y^2$. For an atom interacting with the laser beam, the ratio of I_{max} to the saturation intensity I_{sat} is given by

$$\frac{I_{\text{max}}}{I_{\text{sat}}} = \frac{2\Omega^2}{\Gamma^2},$$

where Ω is the Rabi frequency and Γ is the linewidth of the dipole allowed atomic transition whose wavelength λ_0 is closest to λ, and

$$I_{\text{sat}} = \frac{\pi}{3}\frac{hc}{\lambda_0^3 \tau}.$$

Here, τ is the lifetime of the upper state of the aforementioned transition in the atom. The maximum potential experienced by the atom is

$$U_{\text{max}} = \frac{\hbar\Omega^2}{4\delta},$$

where δ is the detuning.

(a) Show that the integral of $I(r,z)$ over any plane of constant z equals the total power of the beam P.

[Hint: You may need to use the integral $\int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\pi}/\sqrt{a}$.]

[4 marks]

(b) From the equation for $I(r,z)$ above, show that

$$I_{\text{max}} = \frac{2P}{\pi w_0^2}.$$

[2 marks]

(c) In ^{133}Cs, the dipole allowed transition of relevance is the $6s\ ^2S_{1/2} \rightarrow 6p\ ^2P_{3/2}$ transition at $\lambda_0 = 852$ nm, which has an upper state lifetime of $\tau = 31$ ns. Calculate the maximum potential U_{max}, in units of Kelvin, experienced by an atom when it interacts with a focused laser beam of $\lambda = 1.06\ \mu$m, $P = 1$ W and $w_0 = 10\ \mu$m. [6 marks]

(d) Show that the maximum component of the force in the x-direction is given by

$$|F_{\text{max}}| = \frac{2U_{\text{max}}}{w_0\sqrt{e}}.$$

[8 marks]

SECTION B. OPTICAL PROPERTIES OF SOLIDS

Answer Question 4 and **either** Question 5 **or** Question 6.

4. (a) A material has a complex refractive index $\tilde{n} = 1.5 + 0.3i$. What is the value of the (power) reflection coefficient when light impinges on this material from air at normal incidence? [4 marks]

   (b) GaAs has an absorption coefficient $\alpha = 1.3 \times 10^6\ \mathrm{m}^{-1}$ at a frequency of 3.75×10^{14} Hz. What is the value of the extinction coefficient at this frequency, and by what fraction will the intensity of light of this frequency fall in travelling through 1.5 μm of GaAs? [4 marks]

   (c) Ignoring excitonic complications, give a brief description of the contrasting behaviour of the onset of band-edge (across the forbidden gap) absorption in direct gap and indirect gap semiconductors. [4 marks]

   (d) The absorption spectrum of benzene shows a series of lines at the wavelengths 267 nm, 261 nm, 254 nm, 248 nm, 243 nm, 238 nm and 233 nm. Explain briefly the nature of the S_0-S_1 transition and estimate both the energy (in eV) of the S_1 state relative to the S_0 state and the dominant vibrational frequency (in Hz) of the benzene molecule. [4 marks]

   (e) Explain in simple physical terms why some materials exhibit birefringence. [4 marks]

5. A material has a relative permittivity given by the expression

$$\tilde{\epsilon}(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega) = 4.00 - \frac{A}{(\omega^2 + i\omega B)}$$

where A and B are constants. What sort of material is likely to have $\tilde{\epsilon}$ of this form and what therefore is the particular significance of the constant A? [4 marks]

Given that the real part of the permittivity $\epsilon_1(\omega) = 3.97$ and that the imaginary part of the permittivity $\epsilon_2(\omega) = 9.00 \times 10^{-4}$ at angular frequency $\omega = 1.00 \times 10^{16}\ \mathrm{s}^{-1}$ use an appropriate high frequency approximation to obtain estimates of the values of A and B. Numerically justify the use of your approximation. [11 marks]

What are the numerical values of the real and imaginary components of the permittivity at $\omega = 10^{14}\ \mathrm{s}^{-1}$? [5 marks]

6. Write down the expression for and sketch the form of the polarisation, P, as a function of electric field, E, for a material with linear and second order susceptibilities, $\chi^{(1)}$ and $\chi^{(2)}$ respectively. What are the units of P and $\chi^{(2)}$? [6 marks]

   A non-magnetic material with a weak second order non-linearity has $\chi^{(1)} = 3$ and carries an optical pulse with a time averaged power of 3 MW and circular beam diameter of 0.1 mm. If the peak ratio of the second order to linear contributions to P is 1×10^{-5} estimate the value of $\chi^{(2)}$ for this material. [12 marks]

   Describe any approximations employed in obtaining your result. [2 marks]

SECTION C. SOFT CONDENSED MATTER PHYSICS
Answer Question 7 and **either** Question 8 **or** Question 9.

7. (a) An expression for the elastic (shear) modulus of a rubber can be written,

$$G = \frac{ck_{\mathrm{B}}T}{N_x}.$$

Define the terms and briefly summarise (without derivation) the physics underlying the expression. [4 marks]

(b) (i) Sketch graphs of stress against strain rate for a Newtonian, shear thinning and shear thickening fluid. [1 mark]

(ii) Give real world examples of a shear thinning and a shear thickening fluid and explain how the mesoscopic structure of the fluid leads to non-newtonian behaviour. [3 marks]

(c) (i) Explain the physical origin of the terms in the Flory free energy,

$$F(R_g) = \frac{q^2N^2}{\epsilon_0 R_g} + \frac{k_{\mathrm{B}}TR_g^2}{N},$$

for a charged flexible polymer of N monomers each carrying a charge q and extended to a radius of gyration R_g. [2 marks]

(ii) Use the expression in (i) to derive the form of $R_g(N)$, and comment on its physical meaning in the limit of large N. [2 marks]

(d) (i) Give the Arrhenius formula for the dependence of viscosity η on temperture T in a liquid. Identify all the quantities in this expression. [3 marks]

(ii) Give the Vogel Fulcher formula for the dependence of viscosity on temperature in the glassy state and similarly identify the quantities. [1 mark]

(e) Sketch three curves for the way volume changes with temperature upon steadily cooling a liquid: Firstly for a material that is able to form a crystal at some temperature T_{m}; secondly for a material that is unable to form a crystal, but instead falls out of thermal equilibrium at a glass transition temperature T_{g1}; and finally for a material that is again unable to form a crystal, but that is cooled more slowly than the previous example. [4 marks]

8. (a) Give three examples of physical properties of the system that determine if a mixture of two polymers will phase separate. In each case give the conditions under which the property increases the likelihood of phase separation. [6 marks]

(b) A peptide chain comprises m Alanine, A, monomers followed by n Lysine, K, monomers forming a polymer of the form $A_m K_n$. The Alanine can be considered to be hydrophobic while the Lysine caries a positive charge making the polymer an amphiphile.

(i) Assuming that the theta condition is met, what is the expected radius of gyration, R_g, of the Alanine block as a function of m and the monomer length l_0? [2 marks]

[Hint: $R_g^2 = \frac{\left\langle R^2_{end-to-end}\right\rangle}{6}$.]

(ii) Why would this be a poor approximation for the Lysine block? [1 mark]

(c) The polymer above self-assembles when dissolved at low concentration in salt water.

(i) Derive an equation for the radius, r, of a spherical micelle formed by the polymer assuming a surface area per charged monomer of a_0 and a volume per hydrophobic monomer of v_0. [3 marks]

(ii) Assuming that the hydrophobic block cannot be extended to a length of more than l_c per monomer, derive an inequality describing the limit set on n for spherical micelles to form. [2 marks]

(iii) Find an equivalent inequality for the limit set on n for cylindrical micelles to form and explain why no such limit exists for bilayers. [4 marks]

(iv) Explain under what circumstances micelles form in preference to bilayers when both are geometrically accessible. [2 marks]

9. (a) Plot a suitable graph and describe the main features of the general relationship between the viscosity of a polymer melt of flexible chains, and its molecular weight (for all reasonably monodisperse melts). [4 marks]

   (b) For each regime of behaviour in the graph you produced in (a), sketch a suitable graph and describe the behaviour of the stress relaxation modulus $G(t)$. Explain in qualitative terms the physics that gives rise to the behaviour in each case. [6 marks]

   (c) A polystyrene melt with M_w=100,000 has a terminal relaxation time of 0.1 s. Estimate the relaxation time of a polystyrene melt with M_w=250,000 at the same temperature. [3 marks]

   (d) If the entanglement molecular weight of polystyrene is 13,500 (for a polymer of this length the Rouse and reptation times are equal), estimate the ratio $\frac{\tau_{rept}}{\tau_{Rouse}}$ for the melt with M_w=250,000. [3 marks]

   (e) Describe briefly three experimental techniques that have been used to measure the reptation time in entangled polymers. [4 marks]