Lecture 11:
Stellar structure –
Review, stellar models, and stellar mass limits

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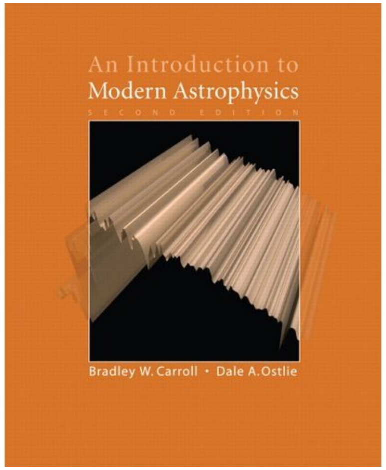
Chapters 10, 11, and 14 of Carroll and Ostlie



Aims of lecture

Key concept: the structure of stars

<u>Aims:</u>

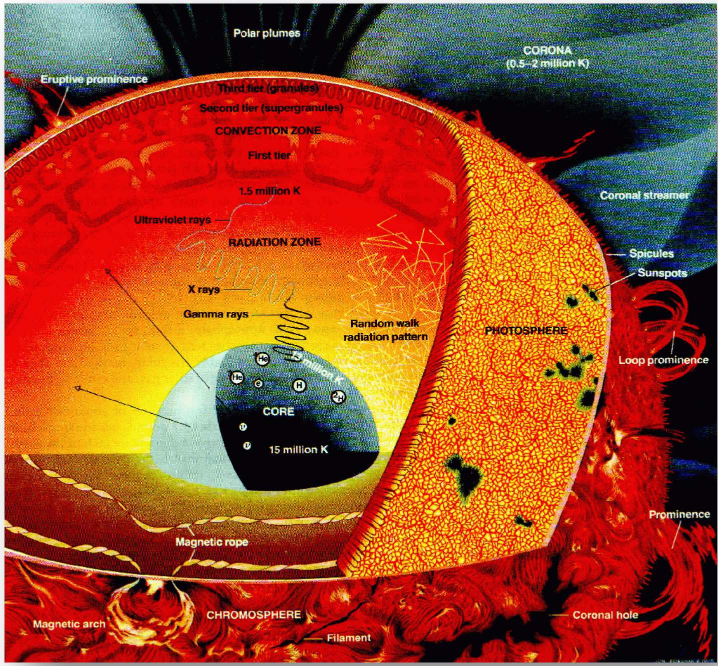
- Brief review of many of the concepts from the course
- Basic understanding of the conditions inside the Sun
- Understand what drives the minimum and maximum masses of stars and be able to show:

$$\frac{M_{max}}{M_{sun}} = \sqrt[\alpha-1]{\frac{4\pi c G M_{sun}}{\kappa L_{sun}}}$$

Maximum mass of stars

Fundamental properties of stars

Emission from stars – black body: $L = 4\pi R^2 \sigma T_e^4$

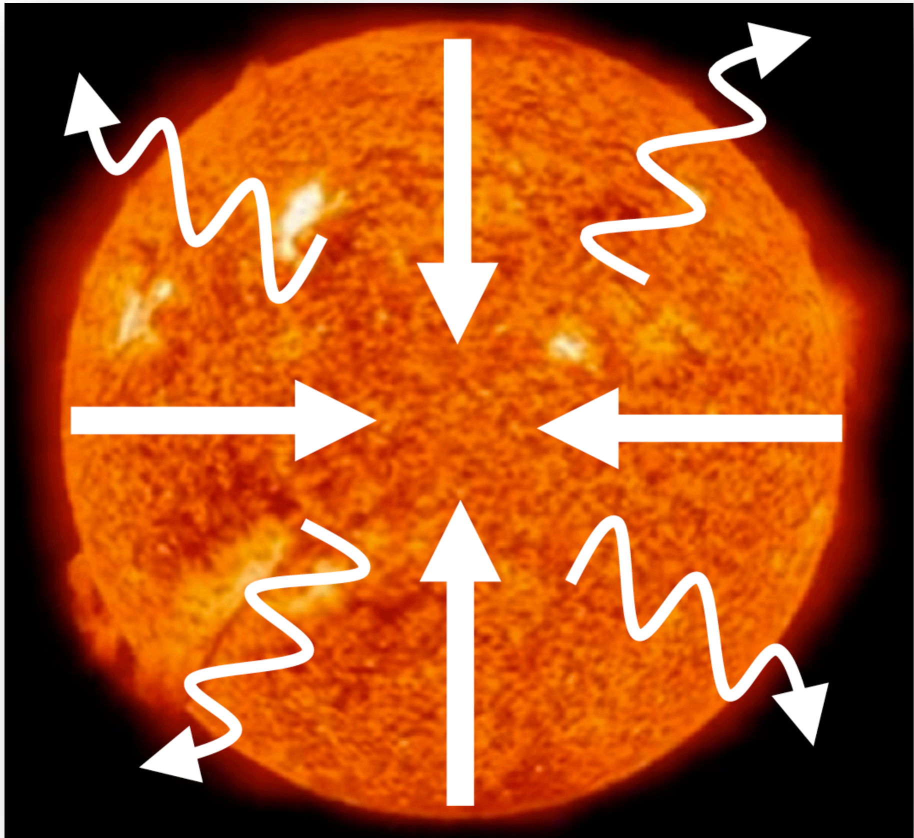


Basic stellar structure – hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

Gravitational energy from collapse/contraction

Energy is released from gravitational collapse, whether slow (contraction) or fast

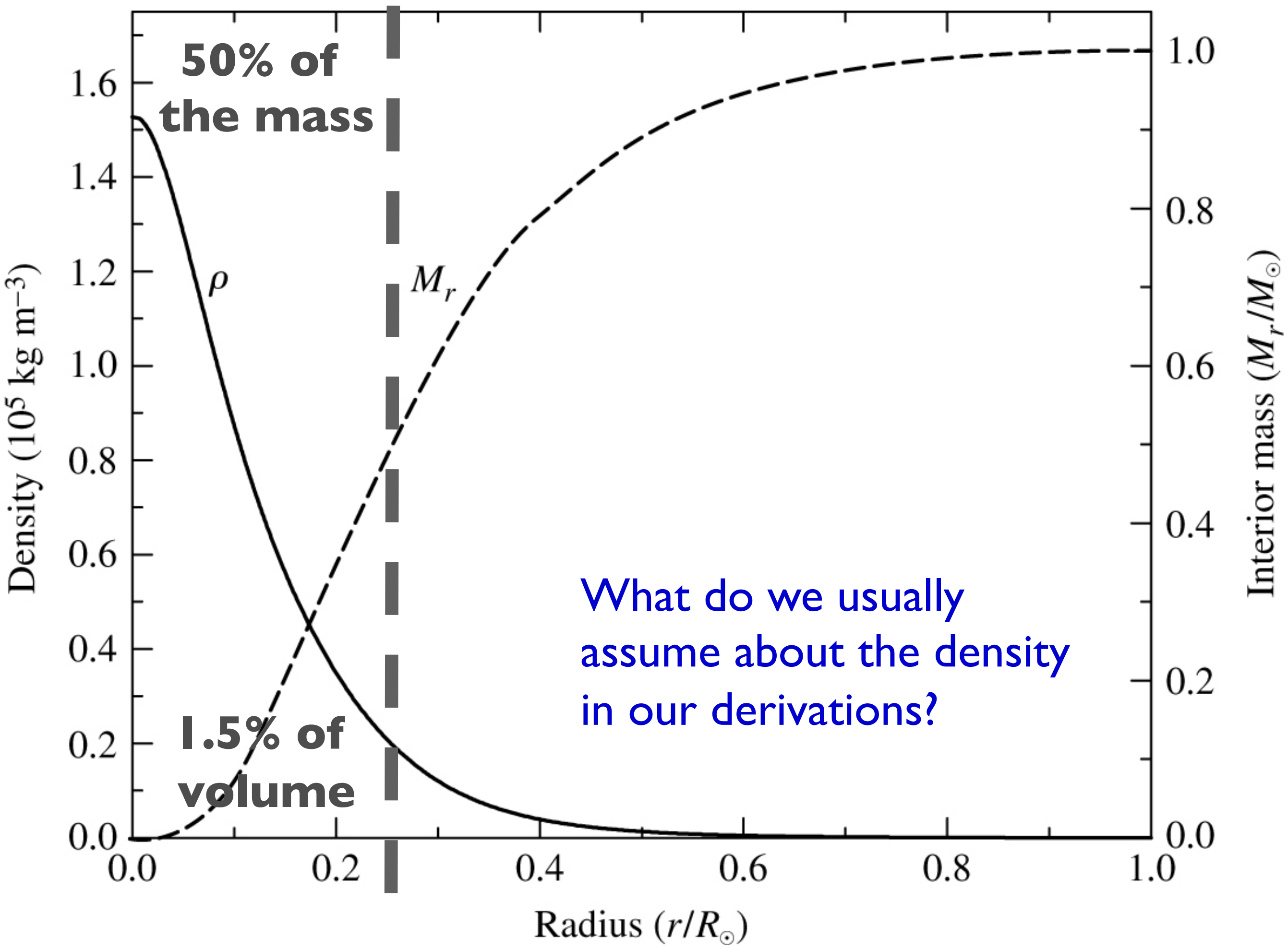


Energy liberated from collapse/contraction:

$$E \sim \frac{3GM^2}{10}\left[\frac{1}{R} - \frac{1}{R_{initial}}\right]$$

Gravitational component – distribution of mass

Predictions for the Sun from a detailed computer simulation



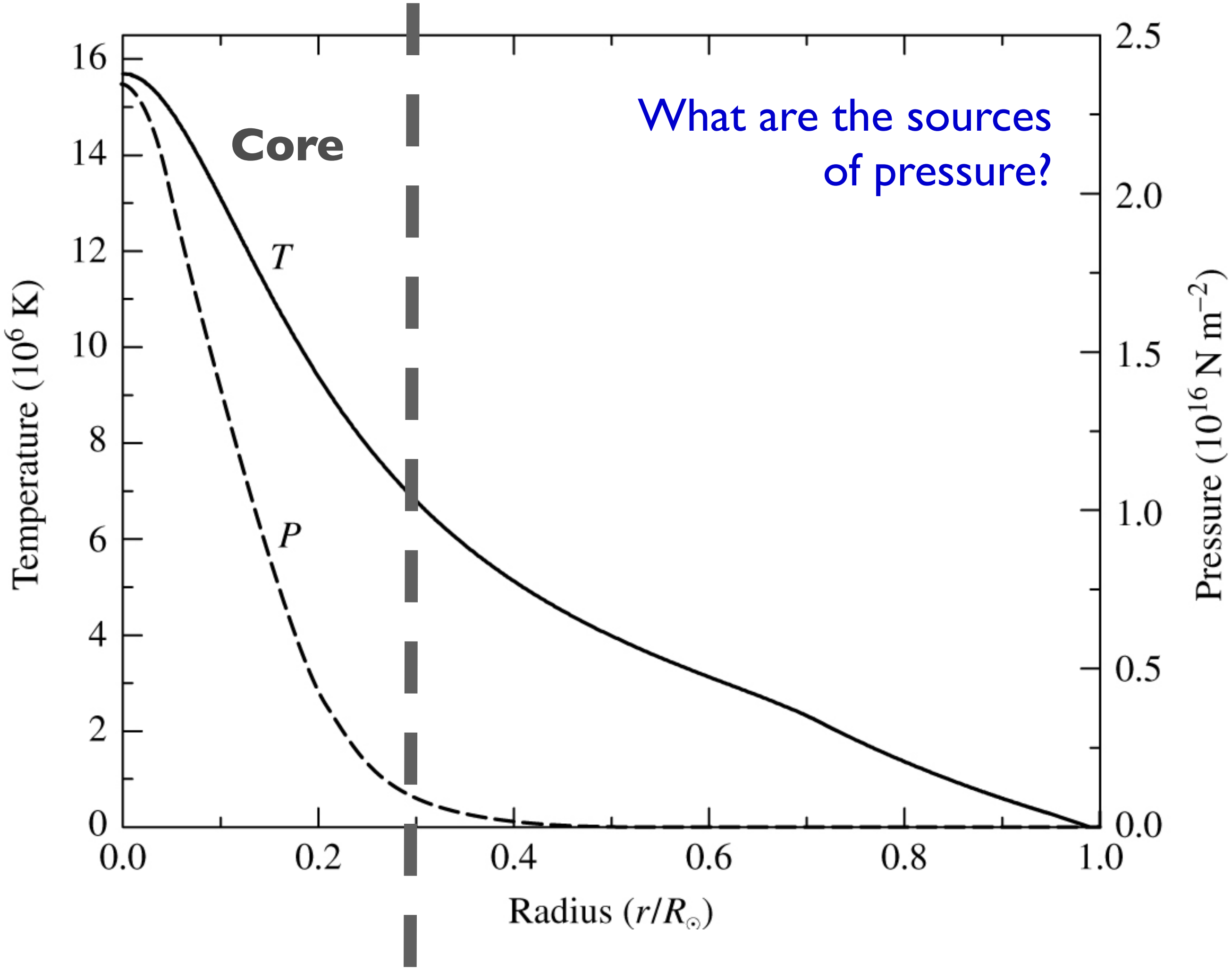
What do we usually assume about the density in our derivations?

Equation of mass continuity:

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

Pressure component – sources of pressure

Predictions for the Sun from a detailed computer simulation

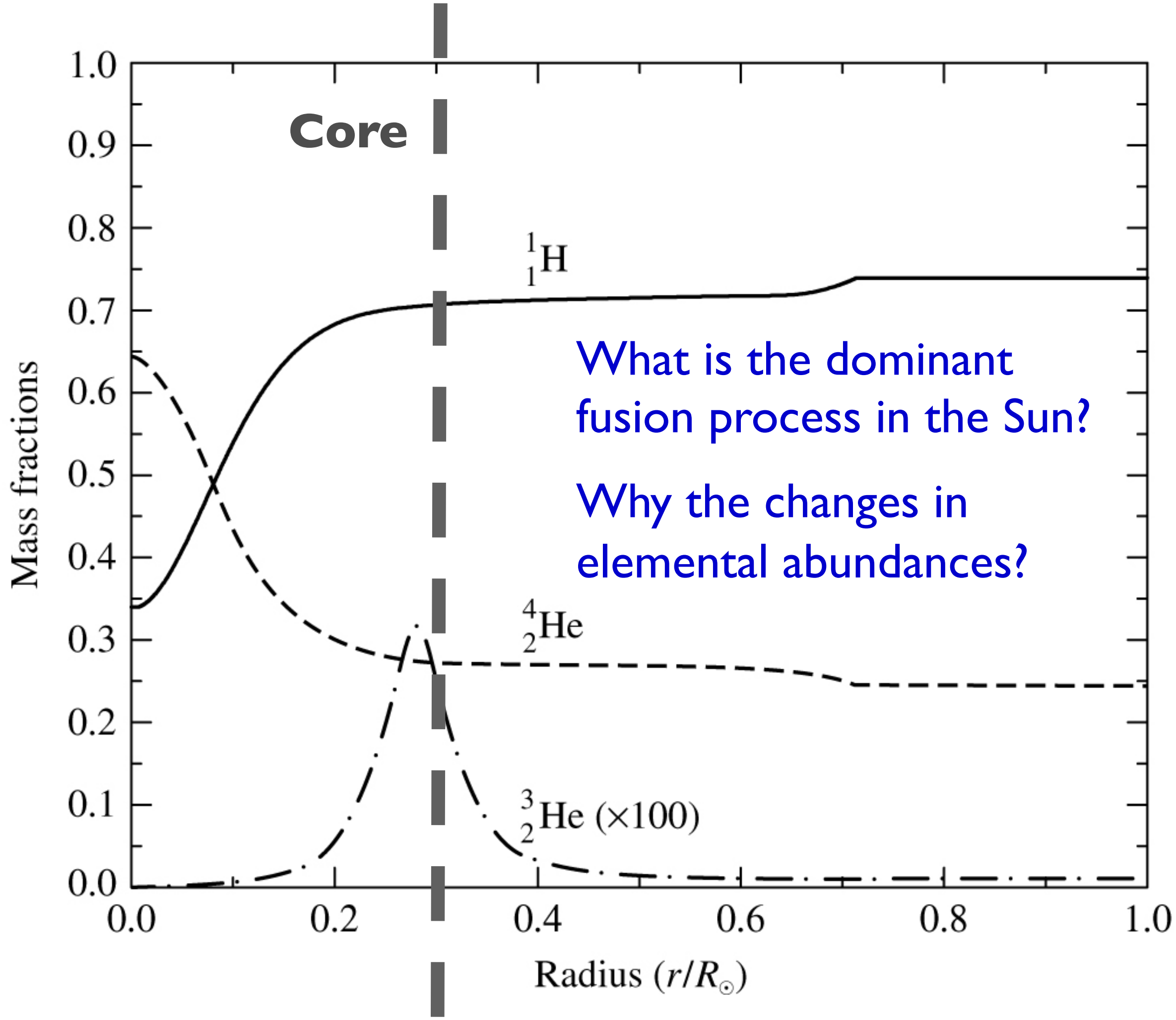


Pressure equations:

$$P = \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4$$

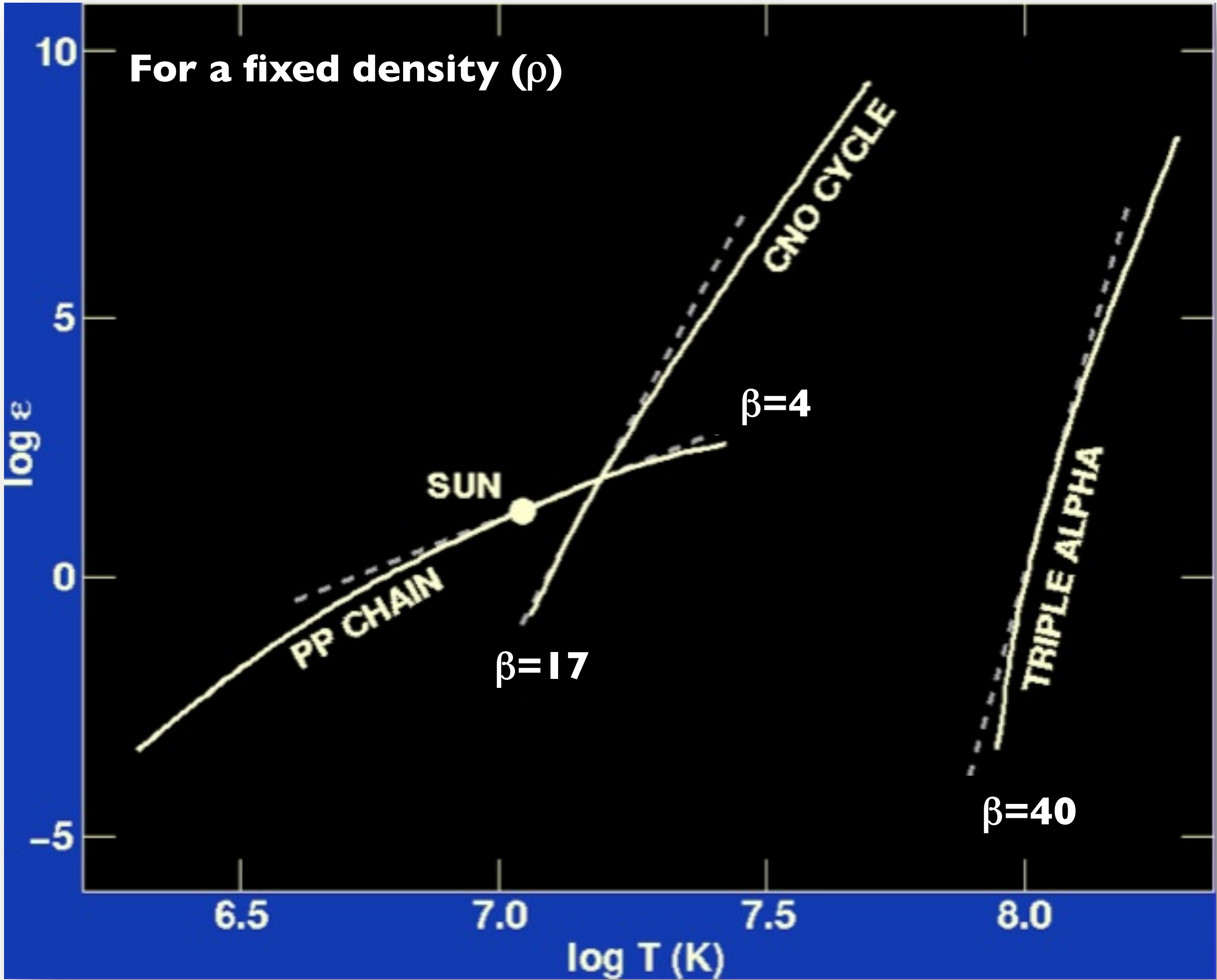
Driver of the pressure support – nuclear fusion

Predictions for the Sun from a detailed computer simulation



Energy release from nuclear reactions: $\varepsilon_{ix} = \varepsilon'_0 X_i X_x \rho^\alpha T^\beta$

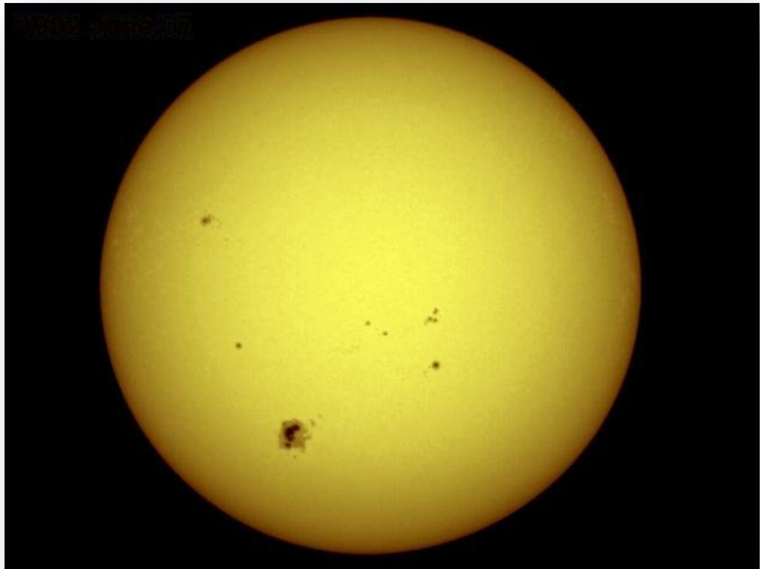
Temperature dependence for nuclear fusion



Note: β is the exponent on T. For just a 10% increase in T there is a 1.5x (β=4), 5.1x (β=17), and 45x (β=40) increase in liberated energy (ε)!

Stellar mass limits

What sets the minimum mass of a star?



What sets the maximum mass of a star?

Maximum masses of stars

An upper limit to the mass of stars can be placed from the violation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

This will occur if the internal pressure exceeds the gravitational force. This happens, when the photon pressure on the gas exceeds the gravitational force (from lecture 8):

$$\frac{dP_{rad}}{dr} = -\frac{\kappa\rho F_{rad}}{c}$$ which is $$\frac{dP}{dr} = -\frac{\kappa\rho L_r}{4\pi r^2 c}$$ (when converted to luminosity)

Equating these two equations therefore gives the maximum luminosity before hydrostatic equilibrium is violated:

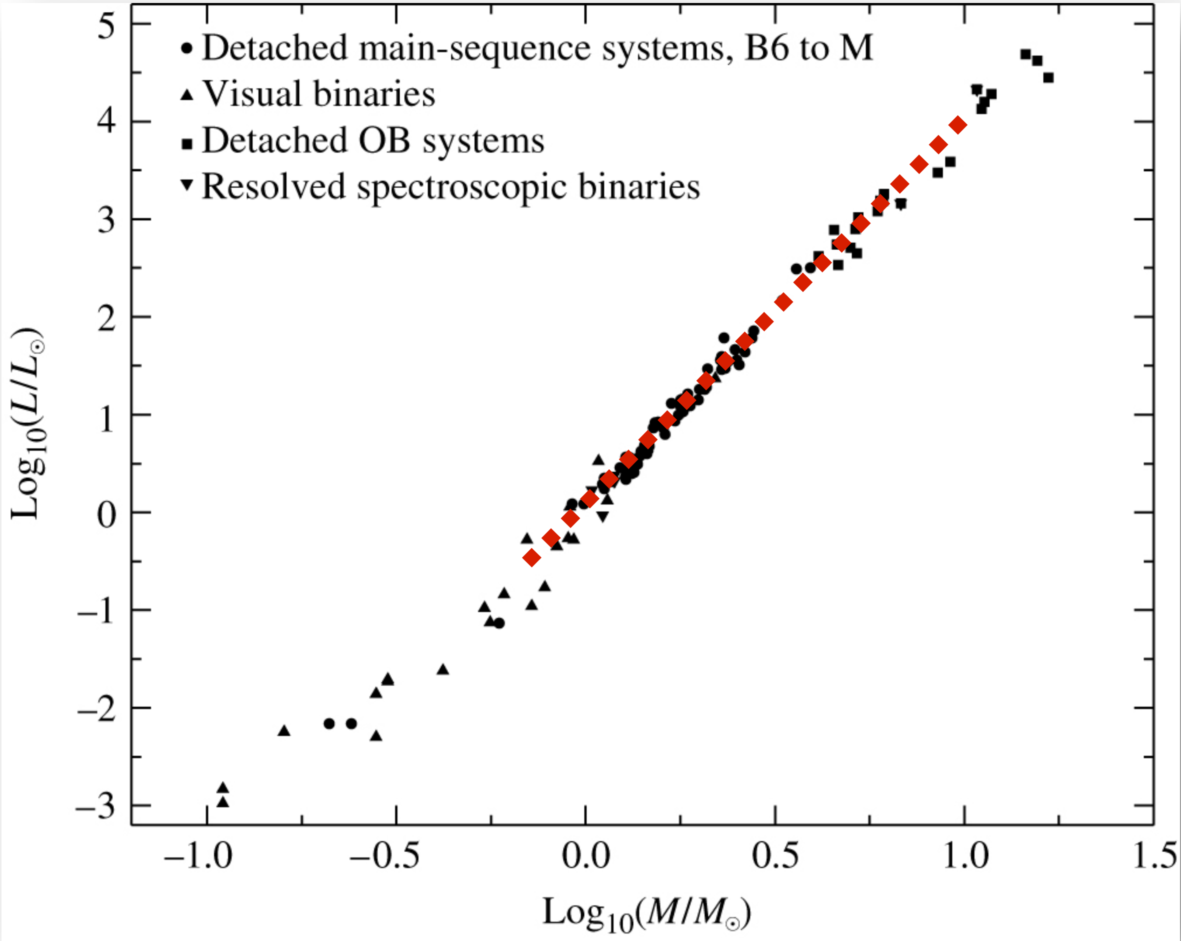
$$L_r = \frac{4\pi c G M_r}{\kappa} = L_{edd}$$

This is called the Eddington luminosity – the point at which radiation pressure equals the gravitational force

Maximum masses of stars

We can use the Eddington luminosity to place an upper limit to the mass of a main-sequence star since (from lecture 3) we know that:

$$\frac{L}{L_{sun}} = \left(\frac{M}{M_{sun}}\right)^{\alpha}$$



Where α~3-4, with some dependence on mass

Maximum masses of stars

Calibrating to the mass and luminosity of the Sun:

$$\frac{L}{L_{sun}} = \frac{4\pi c G M_{sun}}{\kappa L_{sun}} \frac{M}{M_{sun}}$$

Plugging back in the luminosity—mass relationship we therefore get:

$$\frac{M_{max}}{M_{sun}} = \sqrt[\alpha-1]{\frac{4\pi c G M_{sun}}{\kappa L_{sun}}}$$

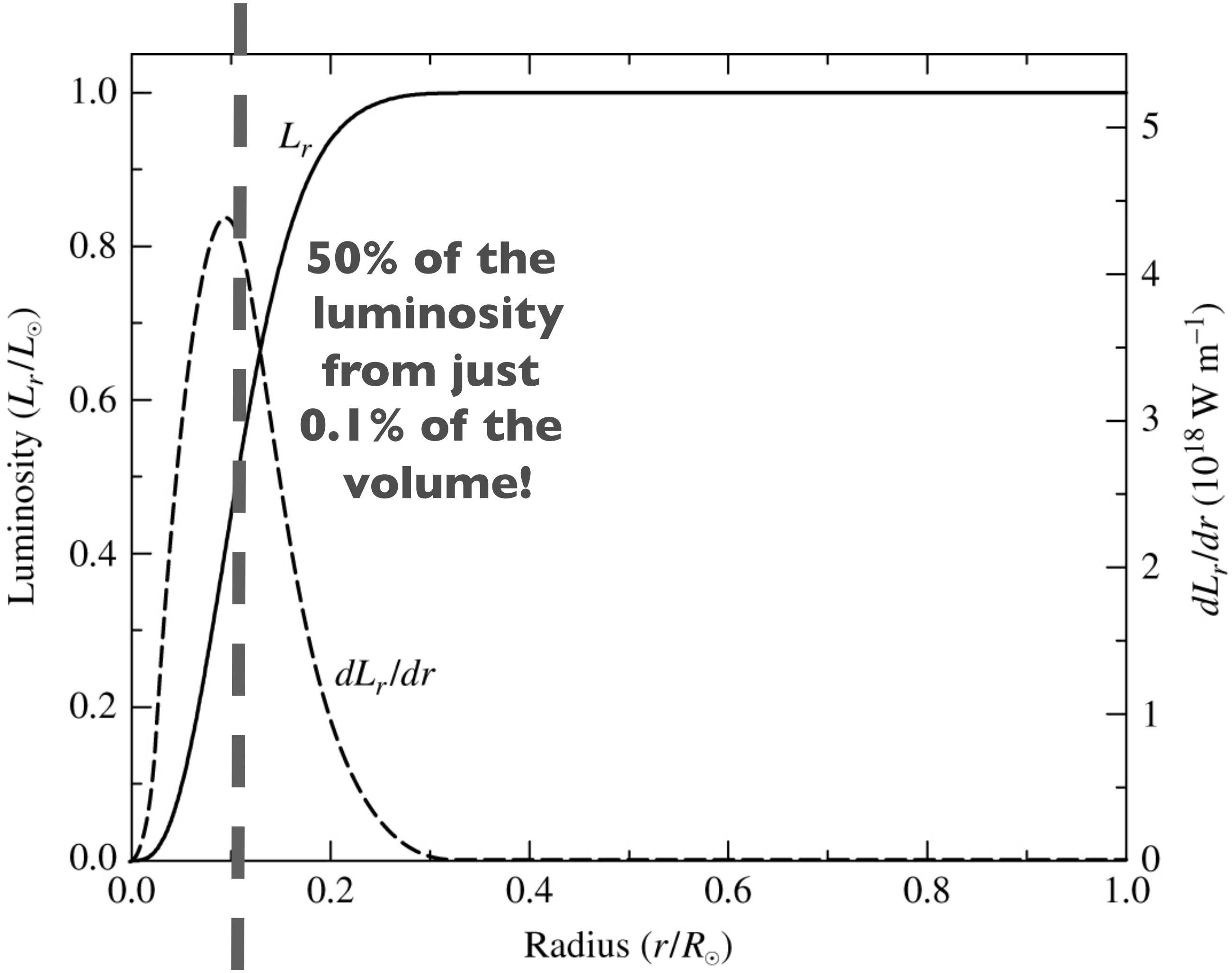
Equation 23

Assuming α=3 for the mass-luminosity relationship, estimate the maximum mass of a star.

What would happen if this mass limit is exceeded?

Getting the energy out – energy conservation

Predictions for the Sun from a detailed computer simulation

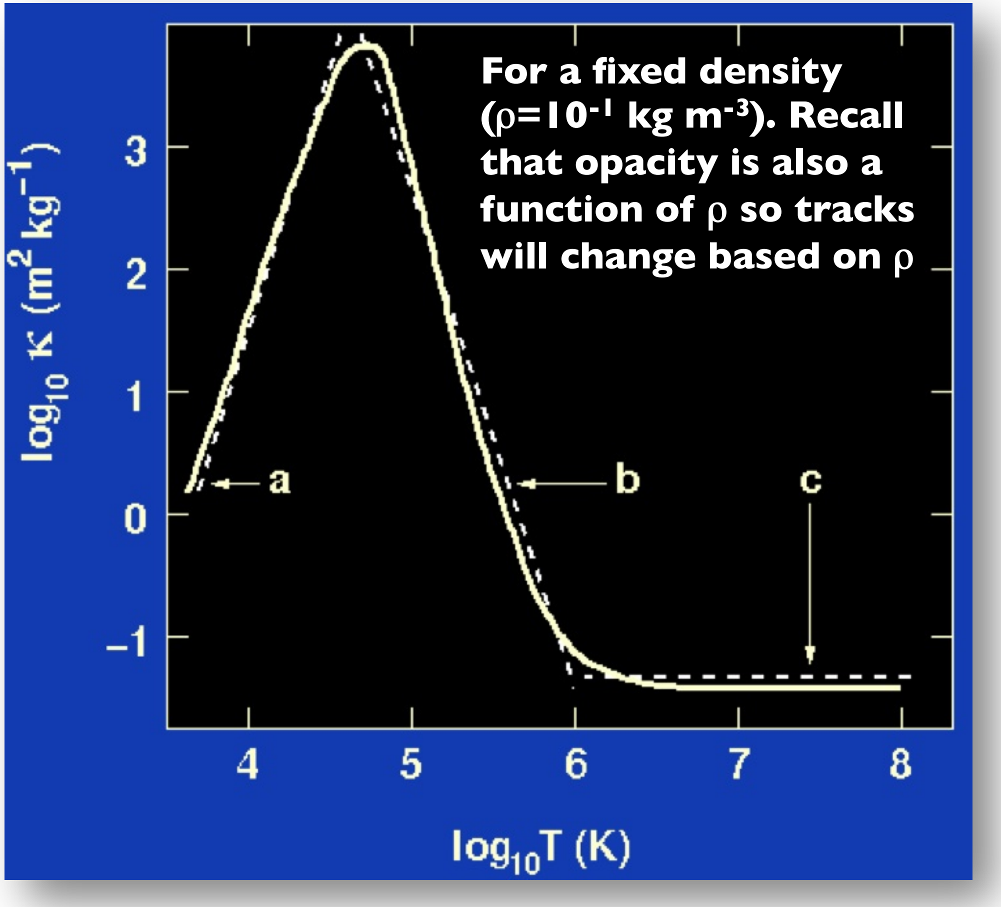


Equation of energy conservation:

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

Getting the energy out – opacity

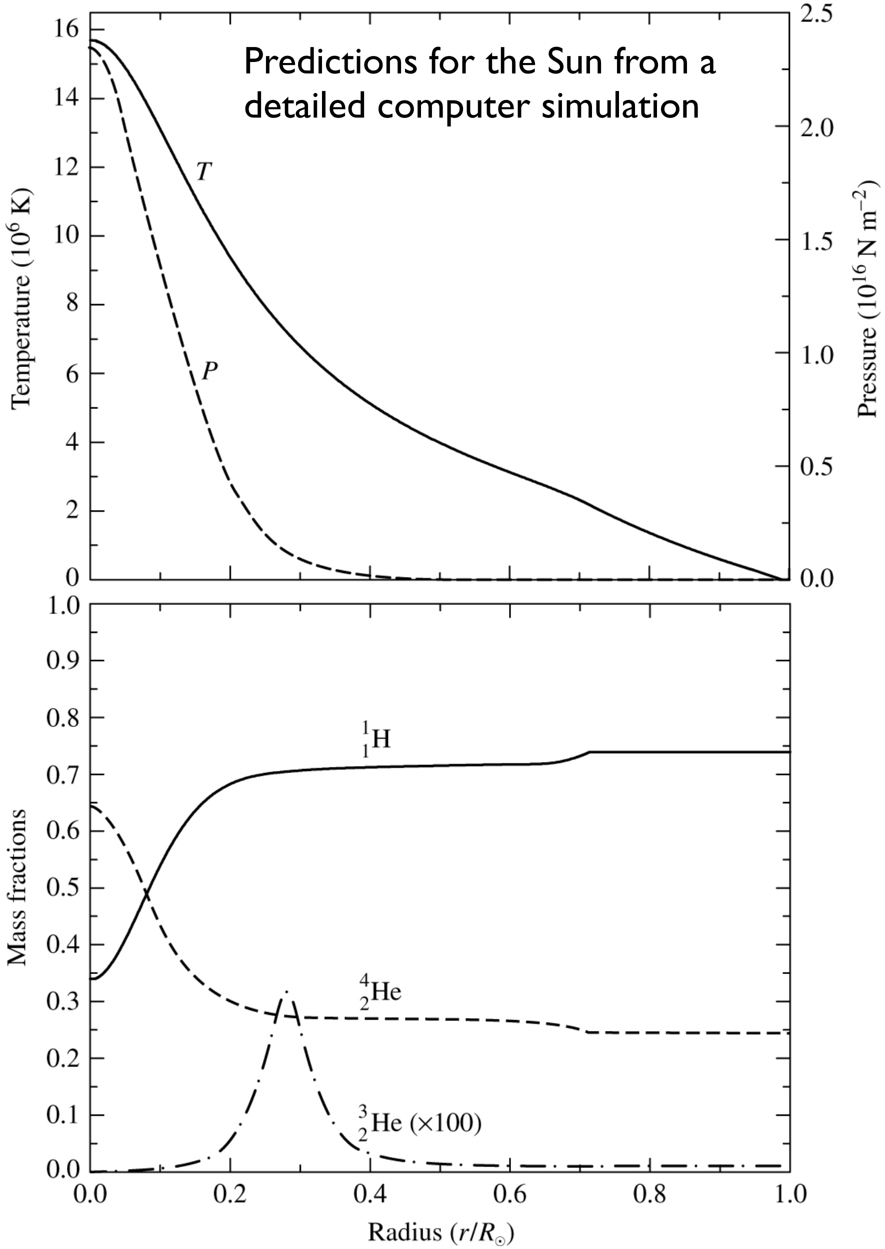
General equation of opacity: $\kappa = \kappa_0 \rho^{\alpha} T^{\beta}$



Mean-free path: $\ell = \dfrac{1}{\kappa\rho} = \dfrac{1}{n\sigma}$

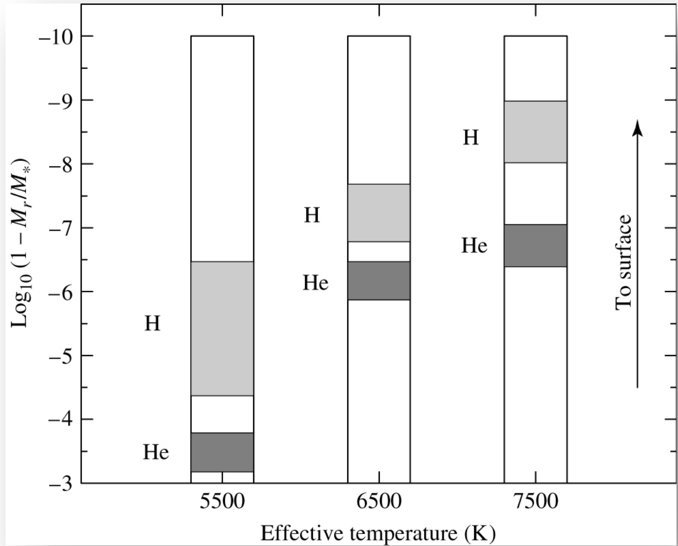
What is the mean-free path?

Opacity and the ionisation state of the gas



Where will the gas be neutral, partially ionised, and fully ionised?

Depth of partial ionisation zones (more in lecture 12)

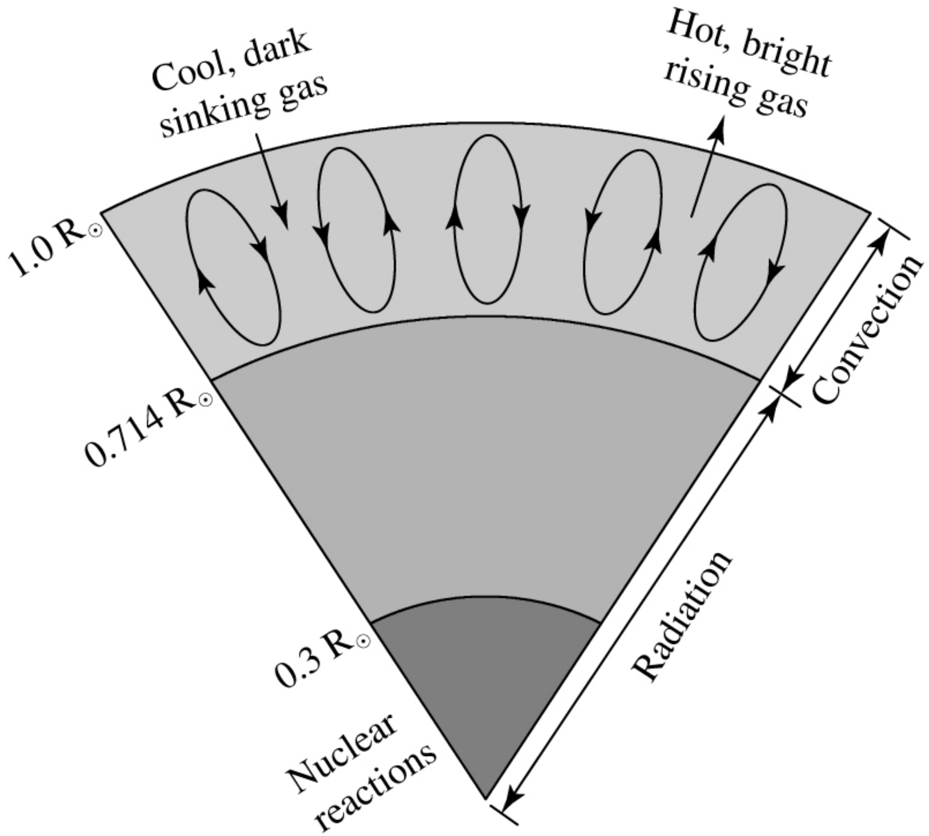


Partial H ionisation: ~10,000K
Partial HeI→HeII: ~15,000K
Partial HeII→HeIII: ~40,000K

Getting the energy out – radiation and convection

Equation of energy transport:

$$\frac{dT}{dr} = -\frac{3}{16\pi ac}\frac{\kappa\rho}{T^3}\frac{L_r}{r^2}$$



Why does convection occur in the outer regions of the Sun?

Where does convection occur in hotter, more massive, stars?

Threshold for convection:

$$\left|\frac{dT}{dr}\right|_{sur} > \left(\frac{\gamma_{ad}-1}{\gamma_{ad}}\right)\frac{T}{P}\left|\frac{dP}{dr}\right|_{sur}$$