

Mathematical Methods in Physics

Weekly Problems 6

6.1

Calculate the integral

$$\int_{-\infty}^{\infty} \delta(4t^2 - 1) e^{it} dt.$$

6.2

Show the following property of the Laplace transforms

$$\mathcal{L}[f(t-a)H(t-a)](s) = e^{-as}\bar{f}(s),$$

where a is a positive constant.

[Hint: Perform a change of variable.]

6.3

Use the properties of the Laplace transforms and the fact that

$$\mathcal{L}[t^n](s) = \frac{n!}{s^{n+1}}, \quad \mathcal{L}[\sin bt](s) = \frac{b}{s^2 + b^2}, \quad \mathcal{L}[\cosh bt](s) = \frac{s}{s^2 - b^2},$$

to calculate

a) $\mathcal{L}[H(t-2)(t-2)^3](s),$

b) $\mathcal{L}[2e^{3t} \sin 3t](s),$

c) $\mathcal{L}[4te^{-t} + t \cosh 3t](s).$

d) $\mathcal{L}^{-1} \left[\frac{1}{s(s+1)} \right] (t).$