Cosmology Part I: Workshop I

1. Distance, energy and wavelength

This question checks that you understand some fundamental concepts.

A. The typical distance between bright galaxies today is 2.5 Mpc. Making the (incorrect) assumption that the number of galaxies remains constant, calculate the redshift at which the average separation between galaxies was

i) 1 Mpc and

ii) 20 kpc.

What is the average *co-moving* seperation of galaxies at these redshifts?

B.i) Calculate, in eV, the energy of a Lyman-α photon with rest-wavelength $\lambda = 91.2\,\mathrm{nm}$.

ii) If this photon is emitted by a galaxy at redshift $z = 3$ calculate its observed wavelength and energy.

[$h = 6.63 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C]
[Discuss: does this violate energy conservation?]

iii) If this $z = 3$ galaxy is an edge-on rotating disk galaxy, the two extremities of which have velocities of ± 250 km s^{-1} relative to the nucleus, calculate the range in observed wavelength of the Lyman-α emission.

2. The Friedmann equation describing the expansion of the Universe is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2}$$

where a is the expansion factor, ρ the mass density (which varies with time as the Universe expands) and k is a constant.

A. Write down some definitions.

i) Define the Hubble parameter H in terms of $\dot{a}$ and a.

ii) Describe qualitatively (and sketch) the behaviour of the future expansion for Universes with $k < 0$, $k = 0$ and $k > 0$. The critical density ρ_{crit} at any given epoch is defined as the density the Universe would need to have to ensure that $k = 0$. Derive an expression for the present value of the critical density $\rho_{\mathrm{crit},0}$ in terms of H_0. The density parameter Ω at any epoch is the ratio of the actual density of the Universe to the critical density, $\Omega = \rho/\rho_{\mathrm{crit}}$.

iii) Express the constant k in terms of Ω_0 and H_0. What is the value of the constant k if $\Omega = 1$?

B. If the Universe is matter dominated and the present value of the density equals the critical value $\rho_0 = \rho_{\rm crit,0} \approx 1.0 \times 10^{-26}$kg m^{-3}, estimate the density of the universe at redshift $z = 3$ and also the value of the critical density, $\rho_{\rm crit}(t)$, at this time.

C. If instead the universe currently has $\rho_0 = \frac{1}{2}\rho_{\rm crit,0} \approx 5.0 \times 10^{-27}$kg m^{-3} again estimate the mean and critical density at redshift $z = 3$.

[Hint: As always in this type of problem, the first step is to determine the constant kc^2. From this, you should determine $H(t)^2$ using the Friedmann equation, and then plug this into the definition of the critical density.]

3. The Expansion of the universe. Which of the following distances are becoming larger due to the expansion of the Universe.

i) The distances to distant galaxies

ii) The distances to high redshift quasars

iii) The distances between distant galaxies

iv) The distances between local group galaxies such as the Milky Way and Andromeda galaxies.

v) The distances between stars in our galaxy

vi) The distances between the earth and the sun.

vii) The size of this piece of paper/computer screen.

4. A parametric solution of the Friedmann equation. (advanced). Consider the Friedmann equation in the case that it is dominated by matter (ie. $\rho = \rho_0/a^3$), and $k > 0$. By substituting the expressions below into the Friedmann equation, verify that a parametric solution of the Friedmann equation is:

$$a(\theta) = \frac{4\pi G\rho_0}{3kc^2}(1 - \cos\theta)$$

$$t(\theta) = \frac{4\pi G\rho_0}{3k^{3/2}c^3}(\theta - \sin\theta)$$

where θ is a variable that runs from 0 to 2π.

Show that there two values of θ for which $a = 0$. What is the physical interpretation of this?

[Hint: remember that $\dot{a} = \frac{da}{dt} = da/d\theta \times d\theta/dt$. However, you cannot solve this problem by deriving the parametric equations starting from the Friedmann equation! Instead, start from the parametric equations and show they solve the Friedmann equation.]