# University of Durham

## EXAMINATION PAPER

May/June 2013 Examination code: 042621/01

**LEVEL 2 PHYSICS: STARS AND GALAXIES**

**SECTION A**. OBSERVATIONAL TECHNIQUES
**SECTION B**. STARS
**SECTION C**. GALACTIC ASTRONOMY

**Time allowed : 3 hours**

**Examination material provided : None**

Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **any three** of the four optional questions. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.

**ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK**

Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.

CALCULATORS: The following types ONLY may be used: Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

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**Information**

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|---|---|
| Elementary charge: | $e = 1.60 \times 10^{-19}$ C |
| Speed of light: | $c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$ |
| Boltzmann constant: | $k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$ |
| Electron mass: | $m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg |
| Gravitational constant: | $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ |
| Proton mass: | $m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg |
| Planck constant: | $h = 6.63 \times 10^{-34}$ J s |
| Permittivity of free space: | $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$ |
| Magnetic constant: | $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$ |
| Molar gas constant: | $R = 8.31 \times 10^{3}\ \mathrm{J\,K^{-1}\,kmol^{-1}}$ |
| Avogadro's constant: | $N_{\mathrm{A}} = 6.02 \times 10^{26}\ \mathrm{kmol^{-1}}$ |
| Gravitational acceleration at Earth's surface: | $g = 9.81\ \mathrm{m\,s^{-2}}$ |
| Stefan-Boltzmann constant: | $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$ |
| Astronomical Unit: | AU = $1.50 \times 10^{11}$ m |
| Parsec: | pc = $3.09 \times 10^{16}$ m |
| Solar Mass: | $M_{\odot} = 1.99 \times 10^{30}$ kg |
| Solar Luminosity: | $L_{\odot} = 3.84 \times 10^{26}$ W |

**SECTION A**. OBSERVATIONAL TECHNIQUES

Question 1 is compulsory. Question 2 is optional.

1. (a) Give two reasons why most modern telescopes are reflectors rather than refractors. [2 marks]

   A 4-m reflecting telescope has f-ratio $f/3$ at prime focus. What is the maximum field-of-view in arcseconds that could be achieved by a detector that is 1000 pixels across and where each pixel is square of side $15\mu$m? [2 marks]

   (b) Briefly describe the functions of the entrance slit and collimator in a simple transmission grating spectrograph. [2 marks]

   By drawing a diagram, derive the *grating equation*, that is, the condition for constructive interference from a reflective diffraction grating. Show the incident ($\alpha$) and reflected ($\beta$) angles of light rays, the diffraction order, $m$, and the line density, $\rho$. [2 marks]

   (c) Derive the relation between the absolute magnitude, $M$, apparent magnitude, $m$, and distance, $r$, of a star, defining all terms used. [2 marks]

   Using this relation or otherwise, show that the count, $N$, of stars per unit sky area to magnitude, $m$, rises as $N(< m) \propto 10^{0.6m}$ in a model for the Milky Way where the star density is uniform everywhere. [2 marks]

2. An astronomical slit spectrograph has a resolving power of $R = 7000$. What is the smallest wavelength difference that can be discerned at a wavelength of 700 nm? [3 marks]

   The same spectrograph is used to observe an active galaxy nuclear emission line with flux $1 \times 10^{-20}$ W m$^{-2}$. What is the total power that reaches the spectrograph detector when it is used on a telescope with a 6-m diameter mirror? You may assume that the telescope and spectrograph combination have 100% quantum efficiency. [4 marks]

   If the emission line is just contained within a single pixel on the spectrograph CCD detector, what is the signal-noise ratio ($S/N$) of the measurement of the emission line strength in a $t = 1000$ s exposure if the sky flux is 10 times higher than the total emission line flux? You may assume that the dark noise is negligible and that the read-out noise is $\pm 5$ electrons per pixel. [6 marks]

   Is the $S/N$ of the observation approximately sky-noise or read-out noise limited? [2 marks]

   If the slit width doubles what is the new resolving power, $R$, of the spectrograph? [2 marks]

   What is the new $S/N$ of the measured emission line strength? [3 marks]

## **SECTION B**. STARS

Question 3 is compulsory. Questions 4 and 5 are optional.

3. (a) Calculate the ratio of electron abundances in the n=1 and n=2 orbits of hydrogen for T=57,000 K. What other physical process besides excitation can affect this ratio and needs to be considered to explain the observed spectral properties of stars? [4 marks]

   [Hint: recall that the statistical weights $g_{\rm n} = 2n^2$; $E_{\rm n=1} = -13.60$ eV; $E_{\rm n=2} = -3.40$ eV]

   (b) State the equation of hydrostatic equilibrium as used in stellar structure calculations, giving all definitions. Name an astrophysical object that is not in hydrostatic equilibrium. [4 marks]

   (c) List the steps of the triple-alpha process. Explain what must occur in order to avoid the rapid ${}^{8}_{4}$Be decay. [4 marks]

   (d) Draw cross sections through both a low mass (1 $M_{\odot}$) and high mass (8 $M_{\odot}$) main-sequence star to highlight where convection is believed to be occuring. Why does convection occur in each of these regions? [4 marks]

   (e) At $r$ = 1 AU the measured mass density in the solar wind is $n$ = $10^7$ protons m$^{-3}$ and the velocity of the material is $v$ = 500 km s$^{-1}$. Estimate the amount of mass lost in the solar wind over the lifetime of the Sun ($10^{10}$ years). Give your answer in kilograms. [4 marks]

   (f) Briefly describe the physical process that leads to the production of neutrinos when the core of a massive star collapses. Why were the neutrinos detected several hours before the optical emission in SN 1987A? [4 marks]

   (g) What defines the characteristic radius of a black hole? Calculate this radius for a black hole of 5.0 $M_{\odot}$. [4 marks]

4. (a) What property of Cepheid variables makes them extremely useful in estimating the distances to other galaxies? [2 marks]

(b) Assuming a uniform density and the adiabatic sound speed, show that the period of a radially pulsating star is given by

$$\Pi = \sqrt{\frac{3\pi}{2\gamma G\rho}},$$

where $\Pi$ is the period, $\gamma$ is the ratio of specific heats and $\rho$ is the density. State any assumptions you make. [8 marks]

$$\left[\text{Hint: } \int_0^R \frac{dr}{\sqrt{R^2 - r^2}} = \left[\sin^{-1}\left(\frac{r}{R}\right)\right]_0^R.\right]$$

(c) What region and what physical process drives radial pulsations in stars? What determines the upper and lower boundaries in temperature for the instability strip on the Hertzsprung-Russell diagram? [6 marks]

(d) Estimate the mass of a star with a luminosity of 18.5 times that of the Sun, a surface temperature of $1.03\times10^4$ K, a measured pulsation period of $6.19\times10^3$ s, and a ratio of specific heats of $\gamma = 5/3$. [4 marks]

5. (a) Sketch the evolution of a 2 $M_{\odot}$ star on the Hertzsprung-Russell diagram, highlighting the *main sequence*, *red giant*, *horizontal branch*, *asymptotic giant branch*, and *white dwarf* phases. [6 marks]

   (b) The pressure at the centre of a white dwarf is $P \sim 10^{22}$ N m$^{-2}$. Derive an equation that expresses the degeneracy pressure from non-relativistic electrons, noting that $P \sim \frac{1}{3} n_{\rm e} p v$, where $p$ is the electron momentum, with momentum in the $x$ direction given by $p_{\rm x} = \hbar n_{\rm e}^{1/3}$, and where the number density of electrons, $n_{\rm e}$, is given by $n_{\rm e} = (Z/A)(\rho/m_{\rm H})$, where $Z$ is the atomic number and $A$ is the mass number. Use this equation to demonstrate that electron degeneracy pressure is sufficient to hold up a 1 $M_{\odot}$ white dwarf with a radius of 5,000 km. [10 marks]

   (c) The relationship between mass and radius for main-sequence stars is $R \propto M^{1/2}$. Contrast this relationship with that found for white dwarfs and explain any differences. [4 marks]

## SECTION C. GALACTIC ASTRONOMY

Question 6 is compulsory. Question 7 is optional.

6. (a) Describe the Hubble sequence (tuning fork diagram) of galaxies. Contrast two defining characteristics of spirals and ellipticals. [4 marks]

   (b) What are clusters of galaxies? Describe two ways that can be used to determine the mass of a cluster of galaxies. Discuss the main assumptions made in each of the two methods. [4 marks]

   (c) Which physical process is responsible for producing the 21 cm line of neutral hydrogen? How is this line used to measure the rotation curves of spiral galaxies? [4 marks]

   (d) Quasars are powered by the accretion of mass onto a central black hole, with luminosity, $L$, and mass accretion rate, $\dot{M}$, related by $L = 0.1\,\dot{M}\,c^2$. The immense luminosity then requires $\dot{M}$ to be large – but why does it also require the black hole to be very massive? What evidence is there that the Milky Way also contains a central black hole? Why is the Milky Way's black hole not displaying quasar activity? [4 marks]

   (e) What is gravitational lensing? A source (S) at distance $D_{\rm OS} = 500.0$ Mpc from an observer (O), is gravitationally lensed by a lens (L) with mass $M = 1.0 \times 10^{13} M_\odot$ at distance $D_{\rm OL} = 250.0$ Mpc. Observer, lens and source are co-linear. Calculate the angle in arc seconds at which the observer sees the Einstein radius $R_{\rm E}$, where

$$R_{\rm E}^2 = \frac{{\rm G}M}{c^2}\,\frac{D_{\rm OL}\,(D_{\rm OS} - D_{\rm OL})}{D_{\rm OS}}\,.$$

   [4 marks]

7. The Milky Way galaxy has a flat rotation curve, with circular velocity $V_c(R) = 220$ km s$^{-1}$, where $R$ is the radius from the centre of the galaxy.

   (a) Demonstrate that a flat rotation curve implies a $\rho \propto 1/R^2$ density distribution. [4 marks]

   (b) The Milky Way is orbited by an almost identical companion, Andromeda, on a radial orbit. The distance $r$ to Andromeda can be determined using the period-luminosity relation of Cepheid variables shown in the figure below. Observations show a Cepheid with period $P = 10$ days to have a flux $F = 2.0 \times 10^{-16}$ W m$^{-2}$. Determine $r$ in kilo parsecs. [4 marks]

   (c) The radial velocity of Andromeda is measured from the spectral lines of an HII region. What are such regions, and how do we know HII regions are associated with star formation? [2 marks]

   (d) The H$\alpha$ line has laboratory wavelength $\lambda_0 = 6.563 \times 10^{-7}$ m. The observed wavelength is blue-shifted to $\lambda = 6.560 \times 10^{-7}$ m. Assuming this blueshift is due to the speed $v$ with which Andromeda is approaching the Galaxy, calculate $v$. [3 marks]

   (e) In the local group timing argument $r$ and $v$ are used to estimate the Milky Way's total mass, $M = 1.2 \times 10^{12} M_\odot$. What assumptions is this argument based on, and which measurement is still required to determine $M$? [3 marks]

   (f) The Milky Way's total mass $M = 1.2 \times 10^{12} M_\odot$ is much larger than the stellar mass, $M_\star$. Assuming this mass is distributed in a spherical halo of dark matter, estimate the dark halo's radius in kilo parsecs. Is this a minimum or maximum radius? [4 marks]