(a) $a = 4\pi\epsilon_0\hbar^2/\mu Z e^2$. $\mu_{\rm Fe} \approx m_e = \mu_H$ so $a_{\rm Fe} = a_H/Z = a_H/26$, so it is $26\times$ smaller. [1 mark]

$E_1 = -13.6Z^2/n^2 = -9193$ eV [1 mark]

$n = 1 \to 2 = -9193(1/1 - 1/4) = -6895$ eV
$n = 1 \to 3 = -9193(1/1 - 1/9) = -8172$ eV
$n = 1 \to 4 = -9193(1/1 - 1/16) = -8619$ eV [1 mark]

(b) Reduced mass is $1833 \times 200 m_e^2/[(1833+200)m_e] \approx 180 m_e$, so $a \propto a_H/\mu$; hence, it is $180\times$ smaller. [1 mark]

Energy is $E_1 \propto (1/\mu)(1/a^2)$ but $a \propto 1/\mu$ and so $E_1 \propto \mu$; hence, it is $180\times$ bigger than E_1 for hydrogen. [1 mark]

(c)

$$\begin{aligned}\left\langle \frac{p^2}{2\mu} \right\rangle =& \frac{1}{2\mu}\frac{1}{\pi a^3} \iiint e^{-r/a} \frac{-\hbar^2}{r^2}\frac{d}{dr}\left[r^2 \frac{d}{dr}(e^{-r/a})\right] r^2 \sin\theta \, dr \, d\theta \, d\phi \\ =& \frac{-\hbar^2}{2\mu}\frac{4}{a^3} \int e^{-r/a} \frac{d}{dr}\left[r^2 \frac{d(e^{-r/a})}{dr}\right] dr \\ =& \frac{-2\hbar^2}{\mu}\frac{1}{a^3} \int e^{-r/a} \frac{d}{dr}\left[r^2 \left(-\frac{1}{a}\right) e^{-r/a}\right] dr\end{aligned}$$

[2 marks]

$$\begin{aligned}=& \frac{2\hbar^2}{\mu a^4} \int e^{-r/a} \frac{d}{dr}(r^2 e^{-r/a}) = \frac{2\hbar^2}{\mu a^4} \int e^{-r/a} \left(2re^{-r/a} - \frac{1}{a} r^2 e^{-r/a}\right) dr \\ =& \frac{2\hbar^2}{\mu a^4} \left[2 \int r e^{-2r/a} \, dr - \frac{1}{a} \int r^2 e^{-2r/a} \, dr\right]\end{aligned}$$

[1 mark]

$$= \frac{2\hbar^2}{\mu a^4} \left[2 \frac{1!}{(2/a)^2} - \frac{1}{a}\frac{2!}{(2/a)^3}\right] = \frac{2\hbar^2}{\mu a^4} \left[\frac{a^2}{2} - \frac{a^2}{4}\right] = \frac{\hbar^2}{2\mu a^2} \text{[1 mark]}$$

(d) $\langle V \rangle = -\hbar^2/\mu a \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} (\pi a^3)^{-1} e^{-2r/a} (1/r) r^2 \sin\theta \, dr \, d\theta \, d\phi$
$= -\hbar^2/(\mu a^4 \pi) 4\pi \int_{r=0}^{\infty} e^{-2r/a} r \, dr = -4\hbar^2/(\mu a^4)[1!/(2/a)^2]$
$= -\hbar^2/\mu a^2 = -2\langle T \rangle$ [1 mark]