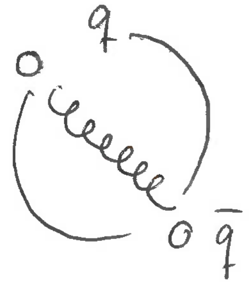
Quarkonia

Positronium

held together by the e/m force



held together by the strong force

Compare the spectra

- small distances ~ low radial wavefunctions (small n)

  the spectra look similar

  For small distances $V(r) \propto \frac{1}{r}$

- large distances ~ spectrum drops like $E_n \propto \frac{1}{n^2}$ for positronium, not so for charmonium

  For large distances $V(r)$ does not scale like $\frac{1}{r}$

- Spin-induced energy splitting is much larger for quarkonia

The split between these states originates from the spin-spin interaction

spin vectors

$$V_{ss}(e^+e^-) = \frac{8\pi}{3} \alpha \frac{\vec{S}_1 \cdot \vec{S}_2}{m_e^2} \delta(\vec{x})$$

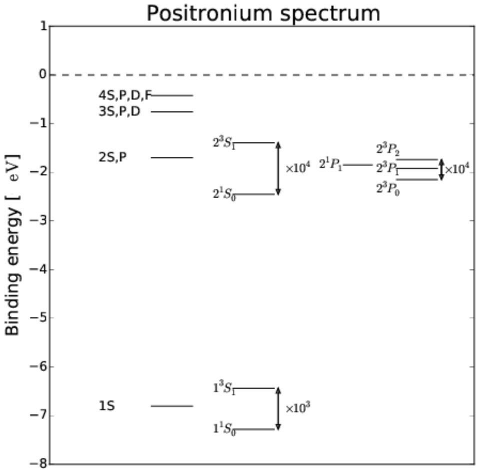
Fine structure constant $\alpha = \frac{e^2}{4\pi}$

overlap with the origin

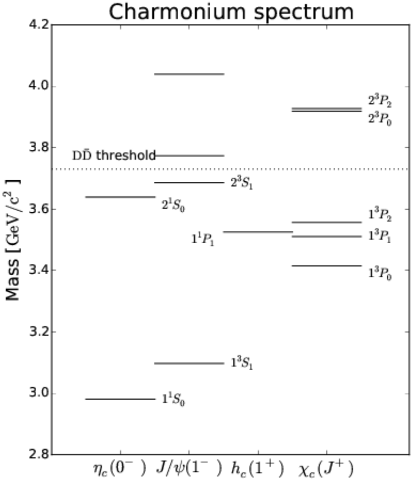
For quarkonium

$$V_{ss}(q\bar{q}) = \frac{32\pi}{8} \alpha_s \frac{\vec{S}_q \cdot \vec{S}_{\bar{q}}}{m_q m_{\bar{q}}} \delta(\vec{x})$$

strong coupling constant



Positronium spectrum
Binding energy [eV]
4S,P,D,F
3S,P,D
2S,P
1S
2^3S_1
2^1S_0
×10⁴
2^1P_1
2^3P_2
2^3P_1
2^3P_0
×10⁴
1^3S_1
1^1S_0
×10³



Charmonium spectrum
Mass [GeV/c^2]
$D\bar{D}$ threshold
2^3P_2
2^3P_0
2^3S_1
2^1S_0
1^3P_2
1^3P_1
1^1P_1
1^3P_0
1^3S_1
1^1S_0
$\eta_c(0^-)$
$J/\psi(1^-)$
$h_c(1^+)$
$\chi_c(J^+)$

Calculate the spin-induced splitting:

we will need $\langle \vec{S}_1 \cdot \vec{S}_2 \rangle$

$$S(S+1) = \langle (\vec{S}_1 + \vec{S}_2)^2 \rangle$$

$$= \langle \vec{S}_1^2 \rangle + 2 \langle \vec{S}_1 \cdot \vec{S}_2 \rangle + \langle \vec{S}_2^2 \rangle$$

$$= \frac{1}{2}\left(\frac{1}{2}+1\right) + 2 \langle \vec{S}_1 \cdot \vec{S}_2 \rangle + \frac{1}{2}\left(\frac{1}{2}+1\right)$$

q and $\bar{q}$ have spin $\frac{1}{2}$

$$= \frac{3}{2} + 2 \langle \vec{S}_1 \cdot \vec{S}_2 \rangle$$

$$\Rightarrow \langle \vec{S}_1 \cdot \vec{S}_2 \rangle = \frac{2S(S+1)-3}{4} = \begin{cases} -\frac{3}{4} & \text{for } S=0 \\ +\frac{1}{4} & \text{for } S=1 \end{cases}$$

we get for the contribution E_{ss} to the states mas

$$E_{ss} = \langle \psi | V_{ss} | \psi \rangle = \frac{8\pi \alpha_s}{9 m_q m_{\bar{q}}} \langle \psi | \delta(\vec{x}) | \psi \rangle \cdot \langle \vec{S}_1 \cdot \vec{S}_2 \rangle$$

$$= \frac{8\pi \alpha_s}{9 m_q m_{\bar{q}}} |\psi(0)|^2 \cdot \begin{cases} -3 & \text{for } S=0 \\ 1 & \text{for } S=1 \end{cases}$$

The mass split is therefore

$$\Delta E_{ss} = E_{ss}(S=1) - E_{ss}(S=0) = \frac{32\pi}{9} \frac{\alpha_s}{m_q m_{\bar{q}}} |\psi(0)|^2$$

here m_q and $m_{\bar{q}}$ are "constituent quark masses," including the binding energy.

Quark - antiquark potential

The $q\bar{q}$ potential deviates from the positronium potential for large distances and is similar for small distances. Since there is no suppression for large r the potential appears to be unbounded:

$$V(r) = \underbrace{-\frac{4}{3}\frac{\alpha_s(r)}{r}}_{\text{short range}} + \underbrace{kr}_{\text{long-range}}$$

For

$$\alpha_s \simeq 0.15 - 0.25$$

$$k \simeq 1\ \mathrm{GeV/fm}$$

$$m_c \simeq 1.5\ \mathrm{GeV}$$

we get a good fit to the Charmonium spectrum.

Quarkonia decays

There are several ways for quarkonium states to decay:

- E/M decays
- Annihilation
- Decay through $q\bar{q}$ production
- Weak decays

E/M decays

The parity for a quarkonium state $n^{2S+1}L_J$ is given by

$$P(n^{2S+1}L_J) = \underbrace{P(q)\,P(\bar{q})}_{-1}\,(-1)^L = (-1)^{L+1}$$

(fermions and antifermions have opposite parity)

Electric have parity $(-1)^{\ell}$

Magnetic transitions have parity $(-1)^{\ell+1}$

E.g. for $\ell = 1$

$$2\,^3S_1 \longrightarrow 1\,^3P_J \qquad J = 0, 1, 2$$

$\Rightarrow \quad J_i = 1 \qquad J_f : \quad |J_i - \ell| \leq J_f \leq J_i + \ell$

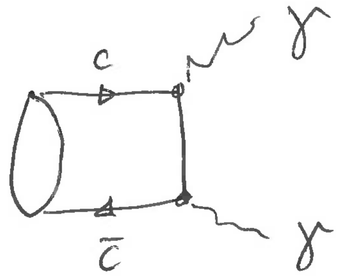
Since $\quad P(2\,^3S_1) = (-1)^{0+1} = -1$

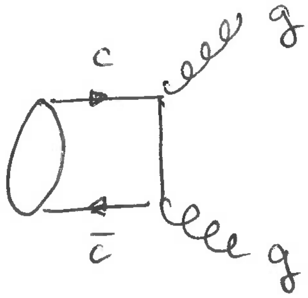
$\qquad\quad P(1\,^3P_J) = (-1)^{1+1} = +1$

E1

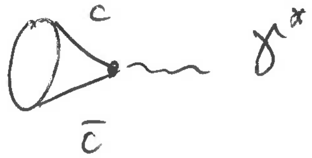
2 – Annihilation

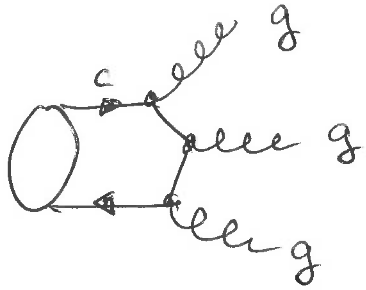
$J=0$ $\quad \eta_c \to \gamma\gamma$ $\quad\quad \eta_c \to gg$



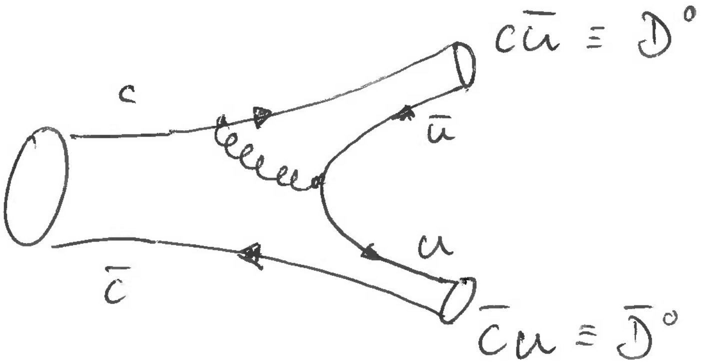


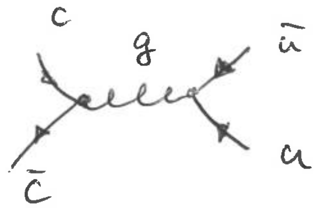
$J=1$ $\quad J/\psi \to \gamma^*$ $\quad\quad J/\psi \to ggg$



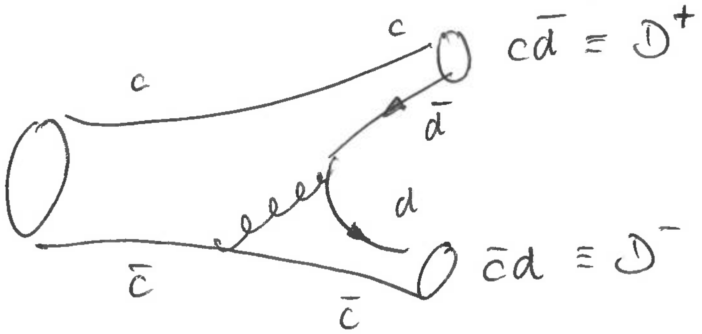


– $q\bar{q}$ ~~An~~ production





$c\bar{c} \to D^0 + \bar{D}^0$



$c\bar{c} \to D^+ + D^-$