University of Durham

EXAMINATION PAPER

May/June 2017 Examination code: PHYS3641-WE01

ADVANCED PHYSICS 3

SECTION A. Soft Condensed Matter Physics
SECTION B. Optical Properties of Solids
SECTION C. Modern Atomic and Optical Physics

Time allowed: 3 hours

Additional material provided: None

Materials permitted: None

Calculators permitted: Yes **Models permitted:** Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

Visiting students may use dictionaries: No

Instructions to candidates:

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **one** other question from **each** section. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklets for Sections B and C, in order, inside your booklet for Section A, before they are collected by the invigilator.

Information

A list of physical constants is provided on the next page.

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Bohr magneton:	$\mu_\mathrm{B} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Electron mass:	$m_\mathrm{e} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_\mathrm{p} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Astronomical Unit:	AU $= 1.50 \times 10^{11}$ m
Parsec:	pc $= 3.09 \times 10^{16}$ m
Solar Mass:	$M_\odot = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26}$ W

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SECTION A. SOFT CONDENSED MATTER PHYSICS

Answer Question 1 and **either** Question 2 **or** Question 3.

1. a) Two binary liquid mixtures have different mixing enthalpies characterised by their parameters χ_1 and χ_2 respectively, with $\chi_1 > \chi_2 > 0$. The free energy of mixing ΔG for a given mixture composition ϕ is given by $\Delta G = \chi\phi(1-\phi) + k_{\mathrm{B}}T[\phi\ln(\phi) + (1-\phi)\ln(1-\phi)]$. Find an expression for the critical temperature T_c of each mixture and determine which of the two mixtures will have the lowest T_c. [4 marks]

b) We place spherical particles of different radii R and identical density ρ on a flat horizontal surface in a humid atmosphere. The surface is then turned upside down. All the particles with a radius smaller than a certain value R^* remain attached to the surface while the larger particles fall off. Explain the observation and derive an expression for R^*. [4 marks]

[Hint: $F \approx 2\pi\gamma R(\cos\theta_1 + \cos\theta_2 - D/r)$
where the symbols have their usual meanings]

c) A flat gel surface is compressed with a cylindrical metal rod of radius a on which a loading force F_L is applied. The indentation depth δ_i is given by $\delta_i = F_L/(2aE^*)$ where E^* is the reduced Young modulus of the gel. The rod is subsequently pulled back until it detaches from the gel. Knowing the surface energies of the gel γ_g, the metal γ_m, and the interfacial energy γ_{gm} between the gel and the metal, calculate the height δ_w the cylinder will need to withdraw above the initial gel surface before detachment occurs. [4 marks]

[Hint: $F_{adh} = -\sqrt{8\pi a^3 E^* W_{gm}}$
where the symbols have their usual meanings]

d) Colloidal particles of radius R are stably dispersed into a given liquid. When polymers with degree of polymerisation N, monomer size a and with a low affinity for the particles are added to the solution, the particles start to aggregate. Knowing that $R = 10a\sqrt{N}$, explain the origins of force driving the aggregation. [4 marks]

e) A medium is composed of molecules that interact through forces deriving from a Lennard-Jones-type pair potential $V_{LJ}(r)$. The potential exhibits a single minimum with a value $V_{LJ}(r_{min}) = -\epsilon$, with $\epsilon > 0$. Depending on the temperature, the medium appears elastic, viscoelastic or viscous when a step-stress is applied. Explain this observation with the help of a graph. [4 marks]

2. Two metals, Cu and Ni are heated to a temperature T_1 above their respective melting temperatures $T_{m,Cu}$ and $T_{m,Ni}$, and mixed to form a liquid alloy.

   a) The mixture is homogeneous for all compositions ϕ at T_1. At a temperature T_2 ($T_{m,Cu}, T_{m,Ni} < T_2 < T_1$) an interval of compositions between ϕ_1 and ϕ_2 ($0 < \phi_1 < \phi_2 < 1$) form nano-domains heterogeneous in composition. Explain the thermodynamic origins of this process using a graph, including the physical significance of ϕ_1 and ϕ_2. Assume that the free energy of the system is G_1 for ϕ_1 and G_2 for ϕ_2. [4 marks]

   b) A metastable mixture with a composition ϕ^* and free energy G^* is made at T_2, with $\phi_1 < \phi^* < \phi_2$. The mixture remains initially homogenous but eventually forms nano-domains of composition ϕ_1 and ϕ_2 after a certain time t^*. Explain this observation and calculate the decrease of energy ΔG^* of the system during this process, as a function of G_1, G_2 and G^*. [3 marks]

   c) Calculate the nucleation energy ΔG_n^* for nano-domains of composition ϕ_1. Assume that the domains are spherical drops and that the interfacial energy between ϕ_1 and ϕ_2 is γ_{12}. [5 marks]

   d) When adding manganese flakes to the solution, t^* changes to t^{**}. Microscopy analysis shows many nano droplets developing on the surface of the flakes. The droplets form a contact angle of 90° with the flakes. Explain the process and calculate the nucleation energy ΔG_n^{**} in the presence of manganese. [4 marks]

   $$\left[\begin{array}{ll} \text{Hint:} & \gamma_{s2} - \gamma_{s1} = \gamma_{12}\cos\theta \qquad \text{(Young)} \\ & \text{where } s \text{ stands for solid and } \theta \text{ has its usual meaning} \end{array}\right]$$

   e) Calculate the ratio t^{**}/t^*, knowing that $\pi\gamma_{12}^3/\Delta G^{*2} = k_B T_2$. Explain qualitatively why the result should be expected. [4 marks]

3. A rheometer is used to apply an oscillatory shear strain $e(t)$ to samples and measure the resulting shear stress $\sigma(t)$. The strain can be modelled by $e(t) = A\sin(\omega t)$ with $A, \omega > 0$.

   a) We first place a polymeric material characterised by a constant relaxation function $G_a(t) = K_0$ inside the rheometer. Calculate the resulting stress $\sigma(t)$ and express it as a function of $e(t)$. Describe the properties of this material. [3 marks]

   b) We now measure a liquid with a relaxation function $G_b(t) = \eta\delta(t)$ with $\delta(t)$ the Dirac function. Calculate the resulting stress and express it as a function of $\dot{e}(t)$, where $\dot{e}(t) = \mathrm{d}e(t)/\mathrm{d}t$. Describe the properties of this liquid. [3 marks]

   c) We now mix the polymeric material with the liquid and obtain a new substance whose relaxation function is $G_c(t) = (\eta + \alpha\dot{e}^2(t))\delta(t)$, where $|\alpha| \ll A$ is a constant. Calculate the resulting stress and express it as a function of $\dot{e}(t)$. Compare this material to those studied in (a) and (b). [4 marks]

   d) Derive an effective viscosity η^* for the material described by $G_c(t)$, and express η^* as function of $\dot{e}_{max}$, the maximum shearing velocity reached by the rheometer during an oscillation cycle. [3 marks]

   e) Make a graph of η^* vs $\dot{e}_{max}$ for α larger, smaller or equal to zero. Discuss in each case the type of material characterised by η^* and provide an example. [4 marks]

   f) How would the results found in (e) change if the strain was given by $e(t) = A\exp(\omega t)$ instead? Justify your answer. [3 marks]

$$\left[\text{Hint: } \sigma(t) = \int_0^t G(t-t')\dot{e}(t')\mathrm{d}t'\right]$$

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SECTION B. OPTICAL PROPERTIES OF SOLIDS

Answer Question 4 and **either** Question 5 **or** Question 6.

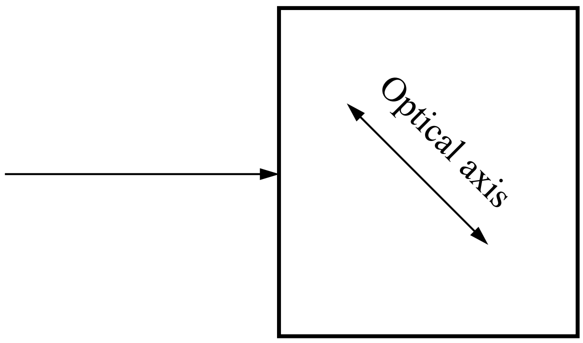
4. (a) Cubic zinc sulphide (ZnS) absorbs 10.0% of light of wavelength 800 nm over a thickness of 1.92 mm. Determine the extinction coefficient of ZnS at this wavelength. [4 marks]

   (b) At several frequencies the refractive index of SiO_2 falls below 1. What does this imply for the phase velocity of electromagnetic radiation propagating in SiO_2 at these frequencies? How can this result be reconciled with the Theory of Relativity? [4 marks]

   (c) Given that caesium is transparent to electromagnetic radiation of wavelengths below 350 nm, find the density of conduction electrons in this metal. [4 marks]

   (d) Write down the expressions relating polarisation, P, and electric field, E, in terms of linear and second order susceptibilities. Sketch a graph of polarisation vs. electric field for a material with linear susceptibility $\chi^{(1)}$. On the same plot show what happens if the second order susceptibility $\chi^{(2)}$ is non-zero. What are the units of P and $\chi^{(2)}$? [4 marks]

   (e) Calcite has refractive indices $n_o = 1.66$ for ordinary and $n_e = 1.49$ for extraordinary rays. Consider an unpolarized monochromatic light beam normally incident on a calcite crystal with its optical axis oriented 45° from the surface normal (see figure below). Calculate the angle difference between the o-ray and e-ray. [4 marks]



5. The dispersion of phonons within a solid falls into two branches: *acoustic* and *optic*. Explain why acoustic phonons cannot be excited by electromagnetic radiation. What conditions must be met by optic phonons if they are to absorb incident electromagnetic radiation? [4 marks]

   The frequency dependent relative dielectric constant of an ionic crystal in the infra-red region can be written as:

$$\tilde{\epsilon}_r(\omega) = \epsilon_\infty + (\epsilon_{st} - \epsilon_\infty)\frac{\Omega_{TO}^2}{\Omega_{TO}^2 - \omega^2 - i\gamma\omega}$$

   where ϵ_{st} is the low frequency dielectric constant, ϵ_∞ is the value of the dielectric constant for frequencies above the phonon bands, Ω_{TO} is the angular frequency of the transverse optic phonon mode (in rad s^{-1}) and γ is a damping term.

   Given that the static and high frequency dielectric constants of InP are ϵ_{st}=12.5 and ϵ_∞=9.60, respectively and the TO phonon mode frequency, ν_{TO}=9.20 THz:

   (a) Determine the reflectivity of low frequency electromagnetic radiation incident normally on a planar InP surface in vacuum. [4 marks]

   (b) Calculate the upper and lower wavelength bounds of the InP *Reststrahl* band. [4 marks]

   (c) The reflectivity of InP is found to drop to near zero at a frequency just above the *Reststrahl* band. Estimate the wavelength at which this occurs, assuming that damping can be neglected. [4 marks]

   (d) Explain the origin of the damping term, γ, in the equation above and comment on the validity of the assumption made in part (c). [4 marks]

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6. (a) Explain how experimental measurements of the absorption coefficient as a function of photon energy for a semiconductor can be used to deduce whether it has a direct or indirect band gap. [6 marks]

   (b) Enhanced absorption caused by Mott-Wannier excitons can be observed in the band edge absorption spectrum of a direct gap semiconductor. Sketch the band edge absorption spectrum for a direct gap semiconductor with excitonic effects included. [5 marks]

   (c) The binding energy for the Mott-Wannier excitons in germanium is measured to be 1.20 meV for the $n = 1$ state, where n is the principal quantum number. Calculate the binding energy for the $n = 2$ state. How many bound states can be observed if the system is cooled by liquid helium ($T = 4.2$ K)? (The Boltzmann constant is $k_{\mathrm{B}} = 8.62 \times 10^{-5}$ eV·K^{-1}) [9 marks]

SECTION C. MODERN ATOMIC AND OPTICAL PHYSICS
Answer Question 7 and **either** Question 8 **or** Question 9.

7. (a) A ^{133}Cs atom traveling towards a photon source is brought to a standstill by scattering 55295 resonant photons of wavenumber 7.375×10^6 m^{-1}. Ignoring Doppler effects, calculate the initial velocity of the atom. [4 marks]

(b) Sketch the electron distribution of a hydrogen atom at two times separated by one half of an optical cycle for a superposition of the $1s, m = 0$ state and the $2p, m = -1$ state (a σ^- transition). Indicate the electric field direction on the sketch and state the polarisation the absorbing photon must have to drive the transition. [4 marks]

(c) Explain briefly why an electric dipole allowed transition is not used in an atomic clock. State the important features required by the transition used in an optical frequency clock and explain why that transition is employed. [4 marks]

(d) Compare the relative magnitudes of the Zeeman shifts of ^{133}Cs($^2S_{1/2}, I = 7/2, m_{F,\max}$) and ^{87}Rb($^2S_{1/2}, I = 3/2, m_{F,\max}$), where $m_{F,\max}$ is the m_F state with the maximum value and explain your reasoning. [4 marks]

(e) Calculate the matrix elements of the hyperfine interaction operator $\mathbf{I} \cdot \mathbf{s}$ in the hydrogen atom. Use the $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$ basis. [4 marks]

[Hint: The Pauli spin matrices are:

$$\mathbf{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \mathbf{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \mathbf{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.]$$

8. (a) Consider the operator $\mathbf{j}^2 = (\mathbf{s} + \mathbf{l})^2$ in the LS-coupling scheme. Show that the spin-orbit interaction energy is given by

$$E_{\mathrm{S-O}} = \beta\langle \mathbf{s}\cdot\mathbf{l}\rangle = \frac{\beta}{2}\{j(j+1) - l(l+1) - s(s+1)\},$$

where β is the spin-orbit coupling constant and l, s and j are the orbital, spin and total electronic angular momentum quantum numbers, respectively. [3 marks]

(b) Give the term symbols, and calculate $E_{\mathrm{S-O}}$, for the spin-orbit states of the ground electronic state of the $^{19}_{9}\mathrm{F}^{8+}$ ion. [3 marks]

(c) Give the term symbols for the spin-orbit states of the first excited electronic state of $^{19}_{9}\mathrm{F}^{8+}$ that are accessible from the ground electronic state by dipole allowed transitions. [3 marks]

(d) Derive a similar expression to $E_{\mathrm{S-O}}$ above for the hyperfine interaction energy, E_{HF}, in terms of the hyperfine coupling constant, A, and in terms of j, I and F, which are the total electronic, nuclear and total angular momentum quantum numbers, respectively. [3 marks]

(e) $^{19}_{9}\mathrm{F}^{8+}$ has a nuclear spin $I = 1/2$. Using the expression for E_{HF} derived above, calculate the hyperfine splittings, ΔE_{HF}, in the ground electronic state and in the first excited state of $^{19}_{9}\mathrm{F}^{8+}$ in terms of A. [8 marks]

9. The ^{87}Sr optical lattice clock involves two stages of laser cooling to reach temperatures of $\sim 1\ \mu$K. The $5s^2\ ^1S_0 \rightarrow 5s5p\ ^1P_1$ "blue" transition at 461 nm is used in the first cooling stage and the $5s^2\ ^1S_0 \rightarrow 5s5p\ ^3P_1$ "red" transition at 689 nm is used in the second cooling stage. The natural linewidths of the two cooling stages are $\Gamma_{\mathrm{B}} = 2\pi \times 32$ MHz and $\Gamma_{\mathrm{R}} = 2\pi \times 7.1$ kHz, respectively. Once the ^{87}Sr atoms are cooled to $\sim 1\ \mu$K, they are loaded into a "magic wavelength" ($\lambda_{\mathrm{m}} = 813$ nm) optical lattice where the $5s^2\ ^1S_0 \rightarrow 5s5p\ ^3P_0$ "clock" transition is interrogated.

   (a) Explain why two transitions are required to reach $\sim 1\ \mu$K. [4 marks]

   (b) The Doppler velocity of a magneto-optical trap (MOT) is given by

$$v_{\mathrm{D}} = \left(\frac{\hbar\Gamma}{M}\right)^{\frac{1}{2}} = (v_{\mathrm{r}} v_{\mathrm{c}})^{1/2},$$

   where v_{r} is the recoil velocity, v_{c} is the capture velocity and M is the mass of the atom. Calculate the capture velocity and the recoil velocity of the blue cooling stage. [6 marks]

   (c) Explain what the term "magic wavelength" means and why the optical lattice is required to be at this wavelength. [4 marks]

   (d) Assume that the magic wavelength optical lattice is "one-dimensional", formed from the interference of a retro-reflected focused laser beam of power $P = 300$ mW propagating along the z-axis. The intensity profile is given by

$$I = \frac{8P}{\pi w(z)^2} \cos^2\left(\frac{2\pi z}{\lambda_{\mathrm{m}}}\right) \exp\left(-\frac{2(x^2+y^2)}{w(z)^2}\right),$$

   where the beam waist, $w(z)$, is 80 μm at the radially smallest part of the lattice at $z = 0$. The trap depth in the optical lattice is given by

$$U_{\max} = \frac{3\Gamma\lambda^3}{16\pi^2|\delta|c} I_{\max},$$

   where it can be assumed that Γ is the lifetime of the strongest dipole allowed transition (of wavelength λ) involving the $5s^2\ ^1S_0$ state. δ is the detuning of λ from λ_{m} and $I_{\max}$ is the maximum intensity of the optical lattice. Evaluate $U_{\max}$. [6 marks]