Statistical Physics: Weekly Problem 1 (SP1)

Consider four distinguishable particles where each particle can be in single-particle states k, with energy $\epsilon_k = k\,\epsilon$, where $k = 0, 1, 2, \ldots$. The system is in the microcanonical ensemble (macrostate) with total energy $U = 3\epsilon$.

(a) What is the total number of the possible microstates and what is the value of the Boltzmann entropy (in units of k_B)? [4 mark]

(b) What is the average number of particles n_k in each state k and what is the probability of occupying each state k? [3 mark]

(c) Plot the probability (p_k) versus $k\,\epsilon$. Make a rough estimate of the width Δ of the distribution in units of ϵ (the energy where the distribution reaches $1/e$ of the maximum). What is the value of Δ? What is the physical meaning of Δ? [3 mark]