Homework Set 1

In a matter-dominated universe with spatial curvature k, the comoving distance r and angular diameter distance d_A to an object with redshift z are given by

$$r(z) = \frac{c}{H_0}\int_0^z \frac{dz'}{(1+\Omega_{m0}z')^{1/2}(1+z')}, \quad (1)$$

$$d_\mathrm{A}(z) = \frac{S_k(r)}{(1+z)}, \quad (2)$$

where Ω_{m0} is the present-day density parameter for matter, and

$$S_k(r) = \begin{cases} \sin(\sqrt{k}\,r)/\sqrt{k} & \text{if } (k>0) \\ r & \text{if } (k=0). \\ \sinh(\sqrt{-k}\,r)/\sqrt{-k} & \text{if } (k<0) \end{cases}$$

(i) Show, using the Friedmann equation, that

$$-kc^2 = H_0^2\,(1-\Omega_{m0})\,.$$

[2 marks]

(ii) Using Eq. (1), find an expression for the comoving separation of two galaxies along the same line of sight with slightly different redshifts z_1 and $z_2 = z_1 + \Delta z$ (you can use the assumption that $\Delta z \ll 1$ to get an analytic formula; please do not try to evaluate the integration numerically). [1 mark]

Assuming $\Omega_{m0} = 1$, determine the comoving separation of two galaxies along the same line of sight with redshifts $z_1 = 0.200$ and $z_2 = z_1 + \Delta z$, where $\Delta z = 0.03$. [2 marks]

(iii) Show that the comoving separation of two galaxies both at redshift z_1, but separated on the sky by an angle of θ, is

$$\Delta r = \theta S_k(r)$$

[1 mark]

If $z_1 = 0.200$ and $\theta = 10$ degrees, what is the transverse co-moving separation of the galaxies if $\Omega_{m0} = 1$? [2 marks]

(iv) The galaxy correlation function, $\xi(r)$, measures the probability of finding two galaxies separated by a comoving distance r and is predicted to have a peak at a characteristic separation. Analysis of a survey of galaxies around $z = 0.2$ finds this peak to occur at a line-of-sight separation corresponding to $\Delta z = 0.03$ and transverse angular separation (for galaxies with the same redshift) of $\theta = 10$ degrees. Explain how this information can be used to constrain Ω_{m0} and explain why it suggests that $\Omega_{m0} \neq 1$. Numerical estimate of Ω_{m0} is **not** required. Assume the Hubble parameter $H_0 = 75$ km s^{-1} Mpc^{-1}. [2 marks]

[*Hint: the Universe is to a good approximation isotropic, so that* $\xi(r)$ *should not depend on the relative orientation of galaxy pairs.*]

Optional question: try to derive Eq. (1) yourself.