Mathematical Methods in Physics

Weekly Problems 2

2.1

Determine whether the sets of given generalised vectors, each in an appropriate vector space, are linearly independent or linearly dependent.

a) In $\mathbb{R}^2 : \{(1,0),(1,2),(1,4)\}$.

b) In $\mathbb{R}^3 : \{(1,2,-3),(3,1,4),(1,1,-1)\}$.

c) In the vector space of the (2×2) matrices, M_{22} :

$$\left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \right\}.$$

d) In the vector space of the polynomials of degree two or less with real coefficients, $\mathcal{P}_2$: $\{1+x, x+x^2, 1+x^2\}$.

2.2

An nth order square matrix $A \neq I$ satisfies $A^2 = A$. Show that

a) $|A| = 0$.

[*Hint: Prove it by contradiction, that is suppose that* $|A| \neq 0$. *This implies that the inverse matrix exists.*]

b) $(I+A)^{-1} = I - A/2$.

[*Hint: Multiply the expression by* $(I+A)$.]

2.3

Use the Gauss-Jordan method seen in the lecture to verify that the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{pmatrix}$$

is

$$A^{-1} = \begin{pmatrix} 9 & -3/2 & -5 \\ -5 & 1 & 3 \\ -2 & 1/2 & 1 \end{pmatrix}.$$

[*Hint: Perform the row operations one at a time in order to minimise the possibility to make mistakes. Also ensure that all the diagonal entries become equal to one.*]

2.4

Using the index notation to show that

$$\mathrm{Tr}\,(ABC) = \mathrm{Tr}\,(CAB),$$

where A, B, C are matrices. Remember, that using the index notation each element of the product ABC can be written as follows

$$(ABC)_{ij} = A_{il}B_{lk}C_{kj}.$$