Cosmology: Problem Cos.1.2

(a) For a matter-only Universe of current density parameter $\Omega_0 > 1$ and Hubble constant H_0, use the Friedmann equation to show that the expansion of the Universe will halt at some future epoch, and evaluate the expansion factor, a_{stop} at this epoch if $a = a_0 = 1$ now.

(b) Derive an expression for $\Omega(a)$ at arbitrary a as a function of a and Ω_0. Show that $\Omega(a) \to \infty$ as $a \to a_{stop}$.

(c) The elapsed cosmic time between the present and the cessation of the expansion at $a = a_{stop}$ is given by:

$$\Delta t_{stop} = \int_1^{a_{stop}} \frac{da}{\dot{a}}$$

If $\Omega_0 >> 1$, show that $\Delta t_{stop} \simeq \frac{2}{H_0 \Omega_0}$. [Hint: consider which parts of the integrand are effectively constant over the small range in a over which the integral is evaluated, in order to simplify the integration.]

(d) If a burst of light is suddenly emitted by a galaxy now (at $a = 1$) and reaches us at the instant the expansion comes to a halt, calculate the redshift with which we will observe the burst. If $\Omega_0 >> 1$, state the approximate co-moving distance to this galaxy in units of c/H_0.