Relativistic Electrodynamics

RED.1

Consider two inertial frames S and S' and let $\Lambda^{\mu}{}_{\nu}$ be the Lorentz Transformation (a Rank-2 tensor) represented by a 4×4- matrix that relates the contravariant coordinates of an event as measured in S to the contravariant coordinates of the event as measured in S', *i.e.* $x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu}$.

We also define (the rank-2 tensor) $\Lambda_{\mu}{}^{\nu}$ (represented by a 4×4 matrix) that relates the covariant coordinates of an event as measured in S to the covariant coordinates of the event as measured in S', *i.e.* $x'_{\mu} = \Lambda_{\mu}{}^{\nu} x_{\nu}$ as

$$\Lambda_{\mu}{}^{\nu} \equiv g_{\mu\rho}\ \Lambda^{\rho}{}_{\sigma}\ g^{\sigma\nu}$$

where g is the metric tensor.

Give the definition of $\beta(v)$ and $\gamma(v)$, and of the rapidity Ψ. Write down all components for $\Lambda^{\mu}{}_{\nu}$ and $\Lambda_{\mu}{}^{\nu}$ (if you represent the answer as a matrix, then you must indicate which index is for rows, and which for columns) in terms of the rapidity Ψ and rotation angle θ for

(a) a boost along the x-axis, *i.e.* S' moves with velocity v_1 along the x-axis and at $t = t' = 0$ the two frames coincide. [**2 marks**]

(b) a boost along the y-axis, *i.e.* S' moves with velocity v_2 along the y-axis and at $t = t' = 0$ the two frames coincide. [**2 marks**]

(c) a rotation around the z-axis, *i.e.* S' is obtained from S by a rotation around the common z-axis through an angle θ. [**2 marks**]

(d) Show by explicit computation that $\Lambda_{\mu}{}^{\nu} = (\Lambda^{-1})^{\nu}{}_{\mu}$ for cases (b) and (c). You can do this by checking each value of μ, ν. Note that this is not a matrix equation, since μ in the first case signifies the row index, and in the second instance the column index. [**4 marks**]