Lecture 14. Primordial Nucleosynthesis

At sufficiently early times, temperatures in the early Universe reached those at the centre of the Sun (1.55×10^7 K), where nuclear reactions take place. But conditions were very different. The energy density was 10^{10} times lower and the photon-to-baryon ratio much higher. In fact, nuclear reactions only occurred at significant rates earlier on when $T \sim 10^9$ K.

14.1. Evidence for the cosmological origin of light elements

(i) Deuterium is destroyed rather than created in stars.

(ii) The Helium abundance is too high relative to abundance of heavier elements (collectively called metals) to be explained by standard stellar nuclear synthesis.

(iii) The distribution of light elements is more homogeneous than that of the heavier elements that are produced in stars.

14.2. Primordial (Big Bang) Nucleosynthesis

At $T \gg 10^{10}$K, proton $\leftrightarrow$ neutron conversion occurs via the following weak interactions

$$\begin{aligned} n &\leftrightarrow p + e^- + \bar{\nu}_e \quad (+0.8 \text{ MeV}), \\ n + e^+ &\leftrightarrow p + \bar{\nu}_e, \\ n + \nu_e &\leftrightarrow p + e^-. \end{aligned} \tag{1}$$

If the timescale for these reactions is short compared to the age of the Universe then thermodynamic equilibrium is reached and

$$\frac{N_n}{N_p} = \exp\left(-\frac{\Delta m\, c^2}{kT}\right) \tag{2}$$

where $\Delta m = m_n - m_p = 1.3$ MeV$/c^2$ is the difference between neutron and proton masses.

The timescale for such two body $n \leftrightarrow p$ interactions is proportional to the square of Fermi's weak coupling constant G_F^2. The dimension of G_F is $[E]^{-2}$, and thus on simple dimensional grounds we can deduce that the interaction timescale will scale with temperature as

$$\tau_{\text{weak}}^{-1} \propto \hbar^{-1} \mathrm{G}_\mathrm{F}^2 (kT)^5,$$

or more simply,

$$\tau_{\rm weak} \propto T^{-5}. \tag{3}$$

For equilibrium to be maintained, this reaction timescale must be shorter than the expansion timescale of the Universe, which is the timescale over which the conditions such as density and temperature change. The expansion timescale is given by $\tau_{\rm exp} = a/\dot{a} = H^{-1}$. As we are in the radiation dominated era, from Lecture 13,

$$\rho_{\rm rel} c^2 = \frac{g_*(T)}{2} \frac{4\sigma T^4}{c}. \tag{4}$$

At the temperature of interest, $kT \sim 1$ MeV, relativistic particles include photons, electrons, positrons and neutrinos (and the τ and μ leptons are not present at $kT \sim 1$ MeV), so that

$$g_*(T) = [2]_{\text{two } \gamma \text{ polarisations}} + \frac{7}{8} \times \left([2]_{e^+ + e^-} \times [2]_{\text{two spins}}\right) + \frac{7}{8} \times [2N_\nu]_{\nu+\bar{\nu}} = 10.75.$$

It is a good approximation at this early epoch to assume $\Omega = 1$ (why?), we can relate the energy density in radiation to the expansion timescale using

$$\rho_{\rm rel} = \rho_{\rm crit} \equiv \frac{3H^2}{8\pi G}.$$

Hence

$$\tau_{\rm exp} = H^{-1} = \left[\frac{3}{8\pi G \rho_{\rm rel}}\right]^{1/2} = \left[\frac{3c^3}{8\pi G 5.375 \times 4\sigma}\right]^{1/2} T^{-2}, \tag{5}$$

which gives

$$\tau_{\rm exp} = 2.0 \left(\frac{T}{10^{10}\,\rm K}\right)^{-2} \text{ s}.$$

When $kT > 0.7$ MeV, $\tau_{\rm weak} < \tau_{\rm exp}$, but $\tau_{\rm weak}$ quickly becomes much longer than $\tau_{\rm exp}$ as kT drops below 0.7 MeV (i.e., as T drops below 8×10^9 K). This occurs at time $t \approx 1.5$ s.

When $\tau_{\rm weak} > \tau_{\rm exp}$, the weak interactions in Eq. (1) become too slow to significantly change N_n and N_p in an expanding space. Thus the N_n/N_p ratio **freezes out** at

$$\frac{N_n}{N_p} = \exp\left(-\frac{1.3\text{ MeV}}{0.7\text{ MeV}}\right) = 0.16.$$

After that, free neutrons can still decay into protons. The half life for this process is $t_{1/2} \approx 614$ s. Thus in this period the neutron-to-proton ratio evolves as

$$\frac{N_n}{N_p} \approx \exp\left(-\frac{1.3\text{ MeV}}{0.7\text{ MeV}}\right) \times \left(\frac{1}{2}\right)^{t/t_{1/2}}, \tag{6}$$

where $t > 1.5$ s is the age of the Universe at the time considered. The above approximation applies only if t is significantly smaller than $t_{1/2}$, which is the case for our discussion here.

The neutrons and protons can combine by the following process to form deuterons

$$n + p \leftrightarrow \mathrm{D} + \gamma \quad (+2.2\text{MeV}).$$

However, above $kT \gtrsim 0.1$ Mev ($T \gtrsim 1.1 \times 10^9$K), there are too many photons in the tail of the black-body photon spectrum with energy above the 2.2MeV deuteron binding energy, such that the deuteron abundance does not build up. This prevents other nuclear reactions, which all take deuteron as starting point, from happening. This phenomenon is known as the **deuterium bottleneck**.

When $kT < 0.1$ MeV many other reactions begin to occur, of which the most important are

$$\mathrm{D} + \mathrm{D} \leftrightarrow {}^3\mathrm{He} + n \leftrightarrow {}^3\mathrm{H} + p, \quad {}^3\mathrm{H} + \mathrm{D} \rightarrow {}^4\mathrm{He} + n \quad (+17.6\mathrm{MeV})$$

The ^{4}He that is formed is stable because of its large binding energy. These reactions are fast enough for nearly all the remaining neutrons to be incorporated into ^{4}He.

As there are two neutrons per ^{4}He nucleus, the number of ^{4}He nuclei formed is $N_n/2$. Then, as the ^{4}He mass $m_{\mathrm{He}} \approx 4m_n \approx 4m_p$, this results in a ^{4}He mass fraction (in total baryon mass) of

$$Y = \frac{4 \times \frac{N_n}{2}}{N_n + N_p} = \frac{2N_n}{N_n + N_p}, \tag{7}$$

where $N_n/N_p \approx 0.13$ is given by equation (14.1) evaluated at time $t = 132$ s, which is the age of the Universe at $kT = 0.1$ MeV ($g_* = 3.36$ in this energy range). Hence we find $Y \approx 0.24$.

Apart from Helium, the main nuclear residue is small amounts of deuterium, D, that escape from being mopped up in the formation of ^{4}He (plus trace amounts of ^{3}He). There are also very minute fractions of other light elements such as ^{7}Li ($\sim 10^{-10}$) and ^{7}Be ($\sim 10^{-11}$).

14.3. Dependence of abundances on physics parameters

Y depends on N_n/N_p at $kT = 0.1$ MeV, and therefore is sensitive to physics parameters, e.g.,

• If the number of neutrino species is larger than $N_\nu = 3$, then at a given temperature ρ_{rel} is larger. Hence the Hubble constant is larger and at a given temperature the Universe is younger. Thus freeze-out occurs at a higher T and so Y increases.

• Y increases with $\tau_{1/2}$, as longer half-life implies a smaller weak coupling constant ($\mathrm{G_F}$) and therefore slower reactions and earlier freeze-out.

Similarly, Y also changes if other parameters of fundamental physics take different values from their measured values today. For example, a change in Newton's constant G can, as a change in N_ν, modify τ_{exp} and therefore change the freeze-out temperature where $\tau_{\mathrm{exp}} = \tau_{\mathrm{weak}}$. Likewise, a slight change in $\Delta m = m_n - m_p$ can also lead to changes of Y.

14.4. Comparing with Observations

At low redshift, the predicted ^{4}He abundance can be compared with the Helium abundance in stellar atmospheres. The comparison of theory and observations is complicated by the fact that Helium is also a product of nuclear reactions in stars. Hence recent generations of stars are enriched in Helium and other elements as the material they formed from was polluted by material ejected from stars in SN explosions. To try and circumvent this problem, both Y and

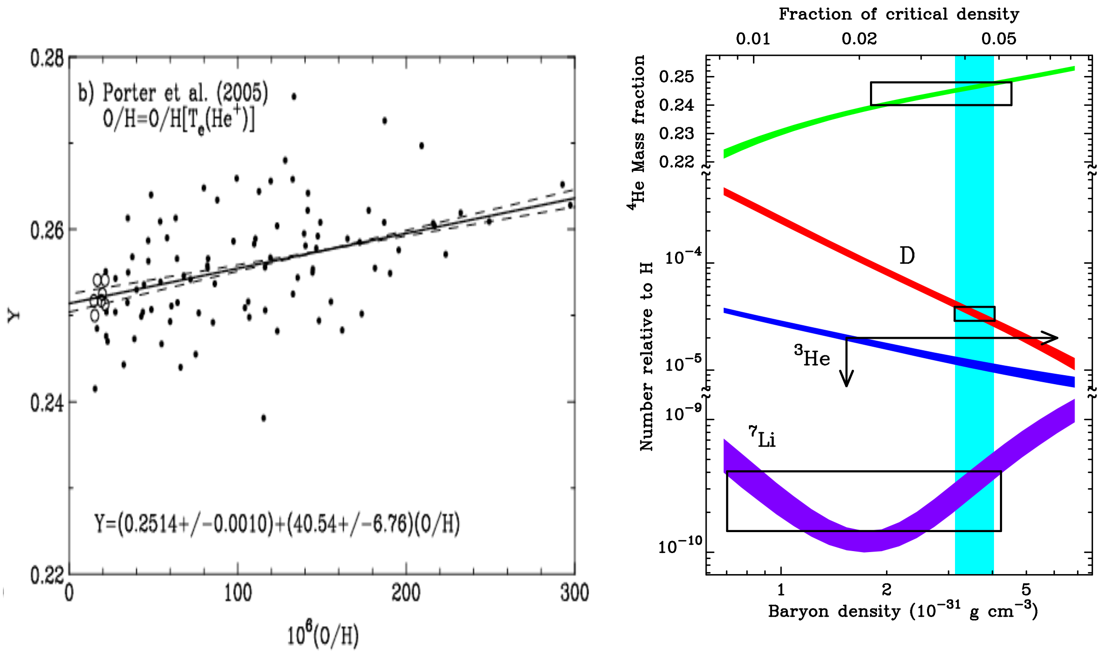

Figure 1: *Left panel*: Measurement of the primordial Helium-4 abundance Y (see the text for more details). *Right panel*: Comparison between the theoretically predicted abundances of ^{4}He (green), D (red), ^{3}He (blue) and ^{7}Li (purple) and observational constraints (black boxes); the vertical cyan bind is the range of baryonic density allowed by CMB observations; note that the theoretical element abundances depend on Ω_{baryon}.

a measurement of the metal abundance, Z (e.g., Fe/H or O/H) are measured for a set of stars. Then Y is plotted against Z. One then extrapolates the correlation to $Z = 0$ to estimate the primordial helium abundance. The inferred value is around $Y = 0.25$ (left panel of Fig. 1).

At high redshift the absorption lines in quasar spectra offer another way of probing elemental abundances. These absorption lines are the result of diffuse clouds of gas along the line of sight between us and the distant bright quasar. They are mainly composed of hydrogen and thus their strongest absorption is due to the Lyman-alpha transition. Recently, it has been possible to detect the much weaker corresponding absorption due to deuterium. The ratio of line strengths directly measures the abundance ratio of deuterium to hydrogen.

It is a great triumph of the Big Band model that the predicted abundances agree so well with the observed values. In fact, the observed abundances, particularly that of deuterium, can be used to set a limit on the baryon density Ω_{baryon} (upper x-axis of the right panel of Fig. 1) of

$$0.032 < \Omega_{\text{baryon}} \left(\frac{H_0}{75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}} \right)^2 < 0.048.$$

For reasonable values of the Hubble constant, Ω_{baryon} is less than Ω_m inferred from the CMB. Therefore much of the **dark matter** in the Universe must be **non-baryonic**. Also, Ω_{baryon} is significantly larger than Ω_{stars}, implying the existence of **dark baryons**.

Key Takeaway Points of Lecture 14

• The majority of light elements, such as hydrogen and helium, in the present-day Universe were synthesised soon after the Big Bang. There are observational evidences for this.

• At early times, baryons existed in the form of protons and neutrons, which interact with each other via weak reactions. When temperature was high, their abundances satisfy the following equilibrium ratio:

$$\frac{N_n}{N_p} = \exp\left(-\frac{\Delta m c^2}{kT}\right).$$

Note that this ratio increases as temperature T decreases.

• Freeze-out happens when T is too low to maintain this equilibrium ratio, after which N_n/N_p stays constant (apart from some decay of free protons into neutrons) $[N_n/N_p]_{\rm FO}$.

• After the deuteron bottleneck has been passed, nuclear reactions took place, but not for long, because T and density dropped quickly.

• In this short window, almost all free neutrons ended up in helium-4. This makes it possible to estimate the helium mass fraction through

$$Y = \frac{4 \times \frac{N_n}{2}}{N_n + N_p} = \frac{2N_n}{N_n + N_p}.$$

• Primordial nucleosynthesis is a topic in cosmology where theory meets well with observations. It can also place powerful constraints on fundamental physics parameters.

Lecture 14 Example (Long Cosmology question in Exam 2018)

The mass of a proton is $m_p = 938.27$ MeV/c^2, and the mass of a neutron is $m_n = 939.57$ MeV/c^2. In the very early Universe, neutrons and protons were in thermal equilibrium maintained by weak interactions, and their number densities satisfied the following equilibrium relation

$$\frac{N_n}{N_p} = \exp\left(-\frac{\Delta mc^2}{kT}\right),$$

where $\Delta_m = m_n - m_p$ and T is the temperature of the Universe.

(a) Explain why this thermal equilibrium could not be maintained forever and the meaning of freeze out. [4 marks]

(b) Assuming that freeze out took place at a temperature T_{fo} given by $kT_{fo} = 0.8$ MeV, and that all neutrons at freeze out ended up in Helium-4 nuclei during Big Bang nucleosynthesis (BBN), find the mass fraction of Helium-4 nuclei, $Y_{\rm He}$, immediately after BBN. [4 marks]

(c) The relevant difference between neutron and proton mass is roughly 0.14%. A cosmologist, in a rush to do the calculation of part (b), incorrectly uses the value that a neutron is 1.4% more massive than a proton. What is the calculated value for $Y_{\rm He}$? [4 marks]

(d) Realising that the result is too small compared with the correct value $Y_{\rm He} = 0.24$, the cosmologist sets off to find what has gone wrong but insists that the Δm value used in part (c) is correct. The cosmologist understands that the freeze-out temperature is the temperature at which the expansion time scale $\tau_{\rm exp}$ and the weak interaction time scale $\tau_{\rm weak}$ were equal to each other, and that $\tau_{\rm exp} = 1/H(T)$ where $H(T)$ is the Hubble expansion rate at temperature T, given by

$$H(T)^2 \propto g_* T^4$$

where g_* is the number of effective bosonic relativistic degrees of freedom. The textbook value is $g_* = 10.75$ at the time of BBN, but the cosmologist thinks that the correct value of g_* should be different. Assuming that the cosmologist does have the correct value of $\tau_{\rm weak}$, can you use a simple argument to show that this idea to solve the discrepancy in $Y_{\rm He}$ won't work? [8 marks]