Mathematical Methods in Physics

Weekly Problems 1

1.1

If

$$\begin{array}{lll} a_{11} = 1 & a_{12} = -1 & a_{13} = 0 \\ a_{21} = -2 & a_{22} = 3 & a_{23} = 1 \\ a_{31} = 2 & a_{32} = 0 & a_{33} = 4, \end{array}$$

a) show that (summation convention applies)

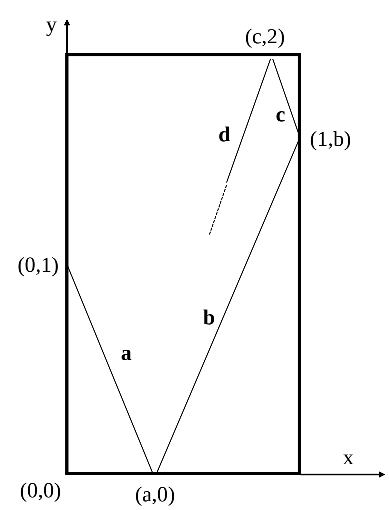
$$a_{ii} = 8, \quad a_{i2}a_{i3} = 3, \quad a_{i1}a_{2i} = -6,$$

b) evaluate (summation convention applies)

$$a_{1j}\delta_{1j}, \quad a_{12}\delta_{ii}, \quad a_{1i}a_{2k}\delta_{ik}.$$

1.2

Consider a typical billiard table with length-to-width ratio of 2:1 and a coordinate system with the x-axis along the short side of the table and the y-axis along the long side. The ball is hit at position $(0, 1)$ such that it follows the path sketched on the right. Assume perfect cushions, which means that on impact the parallel component of the velocity of the ball is conserved, while the component of the ball velocity perpendicular to the cushion is reversed.



a) Give the direction of the vectors **a**, **b**, **c**, **d** (the length is not important for this part, i.e. **a**, **b**, **c**, **d** do not need to be unit vectors).

b) Find the position of the point $(a, 0)$ to aim at in order to sink the ball in the pocket at (0,0).

1.3

Find the vector components of $\mathbf{v} = (1, -3, 2)$ which are parallel and perpendicular to the plane $-2x + 3y + 2z = 3$.

[*Hint: In this problem you should make use of the notion of orthogonal projection.*]