Mathematical Methods in Physics

Workshop 4

4.1

For the function

$$f(x) = 1 - x, \qquad 0 \le x \le 1,$$

a) Calculate the Fourier series of its odd extension, that is the Fourier series of the function $f(x)$ defined on the interval $-1 \le x \le 1$ such that $f(-x) = -f(x)$.

b) Calculate the Fourier series of its even extension, that is the Fourier series of the function $f(x)$ defined on the interval $-1 \le x \le 1$ such that $f(-x) = f(x)$.

4.2

Consider the function $f(x) = |x|$ in the range $-\pi \le x < \pi$.

a) show that its Fourier series is:

$$|x| = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\cos((2n+1)x)}{(2n+1)^2} \qquad -\pi \le x < \pi. \tag{1}$$

b) By integrating the result obtained in part a) from 0 to x, find the function $g(x)$ whose Fourier series is

$$\frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{(2n+1)^3}.$$

c) Deduce the value of the sum S of the series

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} \cdots.$$

4.3

A repeated sinusoidal voltage from an oscillator takes the form $\sin(\omega t)$ for $0 \le t \le \pi/(2\omega)$; it then drops istantaneously to zero and starts again.

a) Find the complex Fourier series of the resulting periodic function, that is

$$\sum_{n=-\infty}^{\infty} c_n\, e^{i2\pi nt/L} \qquad c_n = \frac{1}{L}\int_0^L \sin(\omega t)\, e^{-i2\pi nt/L} dt.$$

b) Write the result in part a) in terms of sine and cosine functions and verify that it coincides with the (non-complex) Fourier series of the function.