Foundation of Physics 2B/3C Optics 2019-20

O.WP.4 Single and double slit

February 27, 2020

1. For a light field that is uniform in the y direction, the Fraunhofer intensity distribution along the x axis at a distance z is

$$I^{(z)} = \frac{I_0}{\lambda z}\left|\int_{-\infty}^{\infty} \mathrm{f}(x')\mathrm{e}^{-\mathrm{i}2\pi x x'/(\lambda z)}\mathrm{d}x'\right|^2 , \qquad (1)$$

where $I_0|\mathrm{f}(x')|^2$ is intensity distribution in the $z = 0$ plane. An opaque screen containing a rectangular aperture of width a is illuminated normally by uniform monochromatic light with intensity I_0 and wavelength λ.

   (a) Using eqn (1), write an expression for the far-field intensity distribution in terms of a, x, z, I_0, and λ. [2 marks] [Hint: $\int_{-a/2}^{a/2} \mathrm{e}^{-\mathrm{i}2\pi x x'/(\lambda z)}\mathrm{d}x' = a\mathrm{sinc}(\pi a x/\lambda z)$, where $\mathrm{sinc}(\alpha) = \sin(\alpha)/\alpha$.]

$$I^{(z)} = I_0\frac{a^{2[\mathbf{1}]}}{\lambda z}\mathrm{sinc}^2\left(\frac{\pi a x}{\lambda z}\right)^{[\mathbf{1}]} , \qquad (2)$$

   (b) For a laser wavelength, $\lambda = 0.50$ μm, if the observation plane is placed at a distance $z = 1.0$ m beyond the slit, then the first zero in the diffraction pattern is observed at $x = 1.0$ cm. What is the slit width, a? [2 marks] The first zero of $x = (\lambda/a)z$[**1**] so to get $x = 1.0$ cm we need $a = 50$ μm.[**1**]

   (c) The aperture is replaced by a double slit with slit separation d, and the same slit width, a, as before. Write an expression for the Fraunhofer intensity distribution in this case. [2 marks] [Hint: $\int_{d/2-a/2}^{d/2+a/2} \mathrm{e}^{-\mathrm{i}2\pi x x'/(\lambda z)}\mathrm{d}x' = a\mathrm{e}^{-\mathrm{i}\pi d x/(\lambda z)}\mathrm{sinc}(\pi a x/\lambda z)$.] The integral becomes

$$\int_{(-d/2-a/2)}^{(-d/2+a/2)} \mathrm{e}^{-i2\pi x x'/(\lambda z)}\mathrm{d}x' + \int_{(d/2-a/2)}^{(d/2+a/2)} \mathrm{e}^{-i2\pi x x'/(\lambda z)}\mathrm{d}x'$$

   Using the hint, we get the same as before multiplied by $[\mathrm{e}^{\mathrm{i}\pi d x/(\lambda z)} + \mathrm{e}^{-\mathrm{i}\pi d x/(\lambda z)}]$[**1**] which can be written as 2 times a cosine, giving

$$I^{(z)} = 4I_0\frac{a^2}{\lambda z}\mathrm{sinc}^2\left(\frac{\pi a x}{\lambda z}\right)\cos^2\left(\frac{\pi d x}{\lambda z}\right)^{[\mathbf{1}]} , \qquad (3)$$

(d) What is the spacing between the interference maxima (fringes) at $z = 1.0$ m if $d = 0.20$ mm? [2 marks] The maxima are $\pi dx/(\lambda z) = m\pi$, $x = m(\lambda/d)z$, so their spacing is $(\lambda/d)z$[1] $= (0.50 \times 10^{-6}/0.20 \times 10^{-3})1.0 = 2.5 \times 10^{-3}$ m or 2.5 mm.[1]

(e) How many interference fringes are there within the central maxima of the sinc-squared pattern? [2 marks] The fringes are separated by 2.5 mm and the first zero of the sinc-squared pattern is at 10 mm, therefore the 4th maxima in the cosine-squared is suppressed by the first zero in the sinc-envelop.[1] [Note that this follows from $d = 4a$.] There are 7 maxima between within this range.[1]