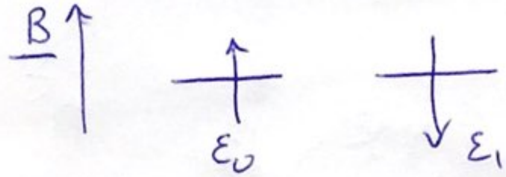
Example (continued).

$\varepsilon_0 = -\varepsilon/2$

$\varepsilon_1 = +\varepsilon/2$

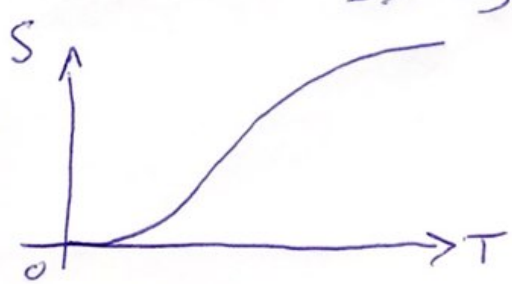


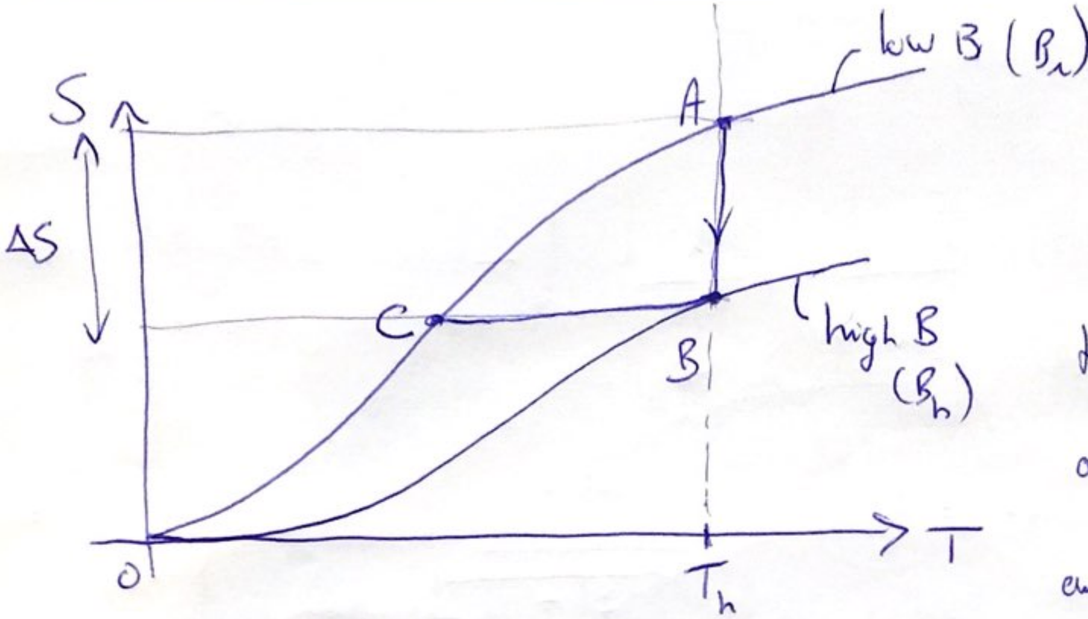
Note: $\varepsilon = 2\mu B$

$$U = -\frac{N\varepsilon}{2} + N\varepsilon \, \frac{e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}} \quad ; \quad F = -\frac{N}{\beta}\ln Z = -\frac{N\varepsilon}{2} - \frac{N}{\beta}\ln\left(1+e^{-\beta\varepsilon}\right)$$

Recall entropy $F = U - TS \Rightarrow S = \beta k_B (U - F)$

$$\Rightarrow S = N k_B \left[\frac{(\beta\varepsilon)e^{-(\beta\varepsilon)}}{1+e^{-(\beta\varepsilon)}} - \frac{N}{\beta}\ln\left(1+e^{-(\beta\varepsilon)}\right) \right]$$





Let T_h be a high temperature, and apply a magnetic field B_l and increase to B_h at constant temperature.

As we go from low B to high B the spins orientate with the field. hence entropy decreases. The system gives away heat $T_h \Delta S$, the energy of the system decreases, spins occupy the low energy level.

$B \rightarrow C$: thermally isolate so no change in heat $(dS = 0)$, and changing magnetic field from B_h to B_l. (constant entropy).

Note that S depends on $2\mu B / k_B T \propto B/T$

If S does not change then B/T cannot change.

$$\frac{B_h}{T_h} = \frac{B_l}{T_l} \Rightarrow \boxed{T_l = \frac{B_l}{B_h} T_h}$$

The temperature must drop from T_h to T_l as $B_h \rightarrow B_l$.

Along $B \rightarrow C$ magnetisation remains the same energy levels $\mathcal{E}$ decreases.
This is cooling by adiabatic demagnetisation.