# Mathematical Methods II
# Weekly problem set 2

(1) A simple harmonic oscillator experiences an oscillating driving force $f(t) = ma\cos(\omega t)$. Its equation of motion is then

$$\frac{d^2x}{dt^2} + \omega_0^2 x = a\cos(\omega t). \tag{1}$$

At $t = 0$ the initial displacement and velocity are zero. The goal of this exercise is to find the function $x(t)$ that satisfies the equation of motion and the boundary condition, using two different techniques.

(a) *Trial function method.* First, solve the complementary equation. Then, find a particular solution using a trial function motivated by the structure of the inhomogeneous term. Finally, fix the integration constants using the given boundary conditions.

(b) *Laplace transform method.* Solve Eq. (1) using the Laplace transform method. You may find the following Laplace transforms useful

$$\mathcal{L}[\sin(\alpha x)](s) = \frac{\alpha}{\alpha^2 + s^2}, \qquad \mathcal{L}[\cos(\alpha x)](s) = \frac{s}{\alpha^2 + s^2}. \tag{2}$$

(c) Study the behavior of $x(t)$ for $\omega \sim \omega_0$. What is the physical interpretation? *Hint:* you may find the following relation useful

$$\cos(ax) - \cos(bx) = 2\sin\left(\frac{a+b}{2}x\right)\sin\left(\frac{b-a}{2}x\right).$$

(2) The hyperbolic sine is defined as

$$\sinh(x) = \frac{e^x - e^{-x}}{2}. \tag{3}$$

(a) Use

$$\mathcal{L}[e^{\alpha x}](s) = \frac{1}{s-\alpha} \tag{4}$$

to find the Laplace transform of $\sinh(x)$.

(b) Using the derivative rule for the Laplace transform, compute

$$\bar{f}(s) \equiv \mathcal{L}\left[\frac{d\sinh(x)}{dx}\right]. \tag{5}$$

(c) Check that the inverse Laplace transform of $\bar{f}(s)$ Eq. (5) is the hyperbolic cosine

$$\cosh(x) = \frac{e^x + e^{-x}}{2}. \tag{6}$$