L2 Foundation of Physics 2B Optics 2019-20

Workshop O.W.1

The purpose of this workshop is to practice some basic mathematics that we use throughout the L2 Optics course. The last question is more conceptual - see if you can answer it without any maths at all!

1. *Binomial expansions*:

   Assuming that $\alpha \ll 1$, use the binomial expansion to write an approximate expression for $(1+\alpha)^n$ accurate to first order in α. [2 marks]

   Use the same expansion to rewrite $r = [(x-x')^2+(y-y')^2+z^2]^{1/2}$ in terms of z, $(x-x')^2$, and $(y-y')^2$, assuming that z is approximately an order of magnitude larger than $|x-x'|$ and $|y-y'|$. [2 marks]

2. *Geometrical distances in optics*:

   In optics, we often consider light propagation from an input plane at $z = 0$ to an output plane at a distance z. In order to distinguish between these two planes the transverse coordinates in the input and output planes are label x' and y' and x and y, respectively. Draw a right-handed coordinate system with the z axis going from upper left to lower right. On your sketch add a line from point (x', y') in the input plane to the point (x, y) in the output plane. [4 marks]

   Write an expression for the distance r between the point (x', y') in the input plane and the point (x, y) in the output plane. [1 mark]

   Write an approximate expression for this distance in the **paraxial regime**, where the change in the transverse displacement is small compared to the propagation distance. [1 mark]

3. *sine, cosine and exp*:

   Re-write $\cos kz$ in terms of $\mathrm{e}^{\pm \mathrm{i}kz}$. [2 marks]

   Re-write $\sin kz$ in terms of $\mathrm{e}^{\pm \mathrm{i}kz}$. [2 marks]

   Re-write $\mathrm{e}^{\pm \mathrm{i}kz}$ in terms of $\cos kz$ and $\sin kz$. [2 marks]

   Rewrite $\mathrm{e}^{\mathrm{i}\pi/2}$, $\mathrm{e}^{\mathrm{i}\pi}$, and i^{i} in the form $a + \mathrm{i}b$. [4 marks]

4. *Adding complex waves* The electric field amplitude of two light fields can be written as $E_1 = E_0\mathrm{e}^{\mathrm{i}(k_x x + k_z z)}$ and $E_2 = E_0\mathrm{e}^{\mathrm{i}(-k_x x + k_z z)}$, respectively.

   Write an expression for the total intensity given by $I = \frac{1}{2}c\epsilon_0|E_1 + E_2|^2$. Write your answer in terms of the intensity of each field individually, $I_0 = \frac{1}{2}c\epsilon_0 E_0^2$. [4 marks]

   What is the maximum and minimum intensity in terms of the individual beam intensities? [2 marks]

   If the two waves propagate at angles $\pm\theta_0/2$ relative to the z axis, then $k_x = k\sin(\theta_0/2)$. What is the distance between the intensity maxima? Express your answer in terms of the wavelength of light, λ. [2 marks]

5. *Consider Young's double slit experiment.* Is the central fringe in the interference pattern bright or dark? Does this depend on the distance between the screen used to observe the pattern and the slit? Is the same true for the diffraction pattern from 3 slits ? Explain how you reached your answers. [8 marks]