University of Durham

EXAMINATION PAPER

May/June 2017 Examination code: PHYS2591-WE01

FOUNDATIONS OF PHYSICS 2B

SECTION A. Thermodynamics
SECTION B. Condensed Matter Physics
SECTION C. Modern Optics

Time allowed: 3 hours

Additional material provided: None

Materials permitted: None

Calculators permitted: Yes **Models permitted:** Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

Visiting students may use dictionaries: No

Instructions to candidates:

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **one** other question from **each** section. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklets for Sections B and C, in order, inside your booklet for Section A, before they are collected by the invigilator.

Information

A list of physical constants is provided on the next page.

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Bohr magneton:	$\mu_\mathrm{B} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Electron mass:	$m_\mathrm{e} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_\mathrm{p} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Astronomical Unit:	AU $= 1.50 \times 10^{11}$ m
Parsec:	pc $= 3.09 \times 10^{16}$ m
Solar Mass:	$M_\odot = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26}$ W

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SECTION A. THERMODYNAMICS

Answer question 1 and **either** question 2 **or** question 3.

1. (a) Describe, using appropriate terminology, the thermodynamic processes in a heat engine. As the engine operates in a cycle, what does this mean for its heat, work and internal changes? [4 marks]

   (b) Two identical blocks, of heat capacity C, are initially at temperatures T_1 and T_2, respectively. They are joined and after some time reach thermal equilibrium at temperature $T_E = (T_1 + T_2)/2$. Determine the overall entropy change of the Universe and show that this doesn't violate the second law of thermodynamics, no matter what the two starting temperatures. [4 marks]

   (c) The Clausius-Clapeyron equation, which describes the boundary for a phase change between states i and f, is given by

$$\left(\frac{\partial p}{\partial T}\right)_{i \to f} = \frac{L}{T(V_f - V_i)},$$

   where L is the latent heat added/removed in the phase change and all other symbols have their usual meanings. Determine the shape of the boundary between the liquid and solid phases if the volume difference can be approximated by ΔV. Usually ΔV is small and can be assumed constant. What does this tell you about the gradient of the line? [4 marks]

   (d) Starting from the general definition of heat capacity whilst some general thermodynamic property α is held constant, C_α, determine the entropy change between temperatures T_1 and T_2 assuming C_α is a constant and $T_1 < T_2$. Show that the third law of thermodynamics indicates that the assumption that C_α is a constant is incorrect. What does this mean for the validity of Meyer's equation, $C_p - C_V = R$ for one mole of ideal gas at low temperature? [4 marks]

2. (a) When the field B acting on a magnetic moment, m, changes by an amount dB, work $\delta W = -mdB$ is required. The Gibbs function in this case is $G = U - TS + mB$. Show that the following Maxwell Relation results

$$\left(\frac{\partial B}{\partial T}\right)_m = -\left(\frac{\partial S}{\partial m}\right)_T.$$

[3 marks]

(b) By considering the entropy as a function of magnetic moment and temperature, show that the following equation must hold

$$TdS = C_m dT - T\left(\frac{\partial B}{\partial T}\right)_m dm.$$

[4 marks]

(c) By combining the equation given in (b) with a second equation,

$$TdS = C_B dT + T\left(\frac{\partial m}{\partial T}\right)_B dB,$$

derive an expression for the difference in heat capacities, $C_m - C_B$. If a paramagnetic material can be described by $m \approx B/T$, show that

$$C_m - C_B = -\frac{mB}{T}.$$

[7 marks]

(d) For a certain magnetic material

$$G = am^2T + bT^2 + cT^3,$$

where a, b and c are constants.

i) Calculate the equation of state for the magnetic moment, $m = m(B, T)$, and comment on the significance of the result. [3 marks]

ii) Determine an expression for the system's internal energy, U. [3 marks]

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3. A heat engine has one of mole of a diatomic ideal gas as its working fluid. It operates in a cycle as follows:

   - Isochoric heating from T_L to $5T_L$;
   - Adiabatic expansion to triple the original volume;
   - Isochoric cooling back to T_L;
   - Isothermal heat rejection at T_L, that compresses the gas back to its original state.

   (a) Sketch a fully labelled pV diagram, including the values of the thermodynamic coordinates, (p_i, V_i, T_i) for each state, and indicating where heat enters and leaves this cycle. [7 marks]

   [Hint: Recall that for a diatomic gas $\gamma = 7/5$.]

   (b) Calculate the thermal efficiency of this cycle. Could a more efficient cycle be constructed between the same two reservoirs by using the same four thermodynamic processes (isochoric heating and cooling, adiabatic expansion and isothermal heat rejection) arranged in a different order? [8 marks]

   (c) An engine is constructed which takes in heat simultaneously from four reservoirs. Heat Q is taken in from a reservoir at T_H, heat $2Q$ from a reservoir at $T_H/2$, heat $3Q$ from a reservoir at $T_H/3$ and heat $4Q$ from a reservoir at $T_H/4$. It also rejects heat Q' to a single cold reservoir at a temperature $T_H/10$. Calculate this engine's thermal efficiency. [5 marks]

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SECTION B. CONDENSED MATTER PHYSICS

Answer question 4 and **either** question 5 **or** question 6.

4. (a) Sketch the set of planes with Miller indices $(1\bar{1}0)$ and (201) for a simple cubic lattice. Include the x, y and z axes in your diagram. If the lattice constant, a, is 0.50 nm, determine the spacing between the planes for each of these two sets of Miller indices. [4 marks]

   (b) Describe the physical origin of the forces present in a covalent bond, paying particular attention to the symmetry of the electron wavefunction $\psi\,(\underline{r})$. [4 marks]

   (c) Sketch the general form of the phonon dispersion relation for the first Brillouin zone in a one-dimensional crystal with a two atom basis. Give the names of the different phonon dispersion curves and describe the relative motion of the atoms in each case. Describe the Debye approximation and state the physical principle used in the approximation. [4 marks]

   (d) Briefly describe the Wiedemann-Franz Law. Explain why the Drude model is able to predict the observed temperature variation of the ratio of the thermal to electrical conductivities in different metals. [4 marks]

   (e) In a particular semiconductor the effective masses of electrons and holes are given by $0.1m_{\mathrm{e}}$ and $0.5m_{\mathrm{e}}$ respectively. Sketch the energy-wavevector, $E(k)$, dispersion relation for conduction and valence bands in the region around $k = 0$, labelling your sketch. Indicate on your diagram the most likely position of donor and acceptor energy levels giving a brief explanation for their location. [4 marks]

5. (a) State the Bragg Law and give an explanation for its physical origin. Explain why the relation $\Delta \underline{k} = \underline{G}$ must be true for Bragg reflection, where $\Delta \underline{k}$ is the change in wavevector between the incident and scattered X-ray waves and $\underline{G}$ is any reciprocal lattice vector. What is the impact of the Structure Factor on X-ray diffraction patterns? [7 marks]

(b) An X-ray powder diffraction measurement is performed on a metal powder known to be either gold or silver using an X-ray wavelength of 0.1500 nm. The lattice constants of Au and Ag are 0.4080 nm and 0.4090 nm respectively. The first five peaks in the X-ray diffraction pattern are observed at 2θ values given in the table below.

Peak	2θ (degrees)
1	37.13
2	43.14
3	62.66
4	75.13
5	79.10

Use the data in the table to show that the crystal structure of the metal is face centred cubic (fcc) and determine if the metal is Au or Ag. [10 marks]

(c) Explain qualitatively what changes you would see in the X-ray diffraction pattern if the metal were first heated to 500 K and then 1500 K. [3 marks]

6. (a) Give a definition of the Fermi energy. Describe, with the aid of a sketch, the Fermi surface in the free electron approximation for a three-dimensional solid. Explain why it has this shape. [7 marks]

   (b) Silver has an atomic mass of 108 u and a density of 1.05×10^4 kg m^{-3}. Determine the Fermi energy of Ag, clearly stating any assumptions you have made. Explain why the value you have obtained differs from the typical thermal energy available at room temperature. [5 marks]

   $$[\text{Hint: } 1\text{u} = 1.66 \times 10^{-27} \text{ kg}]$$

   (c) Make a sketch indicating how the free electron energy distribution in Ag varies with energy at room temperature. Explain why your curve has the functional form you have shown. [5 marks]

   (d) What happens to the Fermi energy when the metal temperature is increased by 200 K from room temperature? Explain your answer. [3 marks]

SECTION C. MODERN OPTICS

Answer question 7 and **either** question 8 **or** question 9.

7. (a) If $F(u) = \mathcal{F}[f(x)]$, where $\mathcal{F}$ indicate a Fourier transform, give expressions for the following in terms of the Fourier transforms of their constituent functions g and h:

   (i) $\mathcal{F}[g(x) + h(x)]$;

   (ii) $\mathcal{F}[g(x)h(x)]$;

   (iii) $\mathcal{F}[g(x)h(y)]$;

   (iv) $\mathcal{F}[g(x)/h(x)]$.

   (b) Consider two plane waves, one with wave vector $\underline{k} = (k_x, 0, k_z)$ and a second with the same amplitude, but with wave vector $\underline{k} = (-k_x, 0, k_z)$. Write down an expression describing the first plane wave and hence derive an expression for the intensity distribution along the x axis of the sum of these two waves. [4 marks]

   (c) Write an expression for a paraxial spherical wave in the xy plane at position z downstream of the source. At what position should a lens of focal length f be positioned in order to collimate the light? What is the phase imprinted by the lens? [4 marks]

   (d) A laser pointer with a beam radius of $w_0 = 1.00$ mm is shone from the Earth to the Moon, a distance of 3.84×10^8 m. Calculate the radius of the beam when it reaches the Moon's surface if it has a wavelength $\lambda = 500$ nm. [4 marks]

   (e) An small aperture with the shape of the letter E is placed in the path of collimated laser beam propagating along the z axis. Sketch the far-field intensity distribution in the xy plane with labels to indicate the main features. [4 marks]

   (f) Sketch the spatial extent of a laser beam inside a plano-convex laser cavity indicating the wave fronts near both mirrors. [4 marks]

8. (a) A light field with wavelength λ propagating along the z axis is incident on a lens with focal length f placed in the $z = 0$ plane. The transverse dependence of the input field is $\mathcal{E}^{(0)} = \mathcal{E}_0 \mathrm{f}(x', y')$, where x' and y' are the transverse coordinates. Write an expression for the intensity distribution in the focal plane. Explain how the units balance. [4 marks]

(b) What is the intensity distribution if $\mathrm{f}(x', y') = \cos(2\pi u_0 x')$? [4 marks]

(c) Sketch the intensity pattern along the x axis for (i) small and (ii) large u_0 indicating the scale. [4 marks]

(d) What is the spacing between the intensity maxima? Is it larger for red or green light? [2 marks]

(e) What non-physical assumption has been made in adopting the form of $\mathrm{f}(x', y')$ in part (b)? How would the intensity distribution in the focal plane be modified for a more realistic form of $\mathrm{f}(x', y')$? [2 marks]

(f) Estimate the minimum value of u_0 you could detect. Give your answer in terms the diameter of the lens. [4 marks]

9. (a) Sketch the optical layout of a $4f$ spatial filter with the optical axis along z. [4 marks]

   (b) Briefly explain how a $4f$ spatial filter is used. [2 marks]

   (c) The input field distribution for a $4f$ spatial filter is given by $\mathcal{E}^{(-2f)} = \mathcal{E}_0 \mathrm{f}(x')$, where x' is the position along the x axis in the input plane. Write an expression for $\mathrm{f}(x')$ when the input field is a plane wave propagating along the z axis. [1 marks]

   (d) For the plane wave input in part (c) sketch the intensity distribution along the x-axis in the Fourier plane assuming the lenses have a focal length f and diameter D. Indicate the scale on the x axis. [4 marks]

   (e) A Ronchi grating is now placed in the input plane such that the input field becomes, $\mathcal{E}^{(-2f)} = \mathcal{E}_0 \mathrm{f}(x') g(x')$, where $g(x') = \mathrm{rect}(2x'/d) * \mathrm{comb}_\infty(x'/d)$, where $\mathrm{comb}_\infty(x'/d)$ is a Dirac comb with spacing d .

   Sketch the input field indicating the width and spacing of the maxima. [2 marks]

   (f) What fraction of the input light in part (e) is transmitted? [1 mark]

   (g) Sketch the intensity distribution along the x axis of the central maxima in the Fourier plane for the arrangement detailed in part (e). [2 marks]

   (h) A strip of paper is placed in the Fourier plane that blocks the central maximum. Sketch the modified field and intensity in the output plane. [4 marks]