QUANTUM MECHANICS 2 - WORKSHOP 5

Q1: The electron in a Hydrogen atom is in the following superposition of the eigenfunctions ψ_{nlm}

$$\psi(r,\theta,\phi) = \frac{1}{\sqrt{21}}(2\psi_{210} + \psi_{321} + 4\psi_{422})$$

What are the possible values and associated probabilities for a measurement of

(a) L^2

(b) L_z

(c) H i.e. energy ($E_n = -13.6/n^2$ eV).

(d) If H is measured to be $-13.6/9$ eV, what is the result of a subsequent measurement of H and L_z? Do we get a deterministic answer if we now measure L_x?

Hint $[H, L^2] = [L^2, L_z] = 0$, but $[L_z, L_x] = i\hbar L_y$

Q2: Any Hydrogen-like ion has a typical size scale $a = 4\pi\epsilon_0\hbar^2/(\mu Z e^2)$, where the reduced mass $\mu = Mm/(M+m)$, M is the nuclear mass with charge $+Ze$ and m is the mass of the electron with charge -e. The energy levels $E_n = -\hbar^2/(2\mu a^2 n^2)$. In Hydrogen, $E_1 = -13.6$ eV, $a = a_H$ and $\mu_H \approx m_e$.

(a) Calculate the energies in eV of the $n = 3 \to 2$ and $n = 4 \to 2$ (Balmer series, Hα and Hβ) for Hydrogen from $E_n = -13.6/n^2$ eV. Give your answer to 3 sig. figs.

(b) The Hα 'line' is actually 2 lines when seen at high resolution, separated by $\Delta E = 4.5 \times 10^{-5}$ eV. We attributed this to electron spin, but some bright student suggests that instead its just due to there being some fraction of Hydrogen present as deuterium, an isotope of hydrogen with nuclear mass of $\approx 2m_p$. Calculate the ratio μ_D/μ_H using $m_p/m_e = 1836$. Write a_D/a_H as a function of μ_D/μ_H and hence find $E_{n,D}/E_{n,H}$, and the difference $E_{n,D} - E_{n,H}$. Calculate this in eV for the $n = 3 \to 2$ transition for deuterium. Can this explain the observed splitting?

Q3: Write down the expression for the momentum operator, p, in one dimension. Hence show that $[H,p]\psi = i\hbar\frac{dV}{dx}\psi$ where $H = p^2/2m + V(x)$