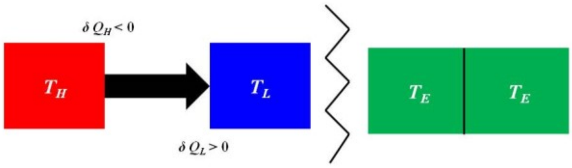
Foundations of Physics 2B/3C 2019-2020

Thermodynamics – Solution, Th. 4

As a guide to completing your self-assessment, please use the following correspondence: Unsuccessful: (0 – 3 marks out of 10); Partially Successful: (4 – 7 marks out of 10); Successful (8 – 10 marks out of 10). Please also give information on any parts which you found difficult, as this will allow me to go over any common issues in the workshops. You can also talk individually to myself, and other staff members at these about any issues you faced when solving the problem.

a) Two blocks of Nickel, each having mass $15.0\ \mathrm{kg}$ and heat capacity $C_p = 6{,}600\ \mathrm{J\ K^{-1}}$, are brought into thermal contact and allowed to come to thermal equilibrium, with no heat energy being lost to the environment. If one block is initially at a temperature of $527\ ^\circ\mathrm{C}$ and the other is at the temperature of the environment, taken to be room temperature at $300\ \mathrm{K}$, determine the following:

i) The blocks' common final temperature;



As the two blocks are identical, one will lose heat energy whilst the other gains an equal and opposite amount, as per the schematic, so the equilibrium temperature will be the midpoint of the two starting ones, namely $550\ \mathrm{K}$ - we have to use a consistent temperature scale and $527\ ^\circ\mathrm{C} = 800\ \mathrm{K}$! This can also be proved using heat capacities, $C_p = \left(\frac{\partial Q}{\partial T}\right)_p$, noting that we are given heat capacity (for the whole block) as opposed to specific heat (per unit mass) in the question, so we don't need to worry about the masses of the block! The hot block loses heat $-Q_H$ to the cold block which gains heat Q_L. Defining T_E as the final equilibrium temperature, and T_H and T_C as the initial block temperatures, and $|Q_H| = Q_L$

$$-\int_{T_H}^{T_E} C_p dT = +\int_{T_C}^{T_E} C_p dT \quad \Rightarrow \quad -(T_f - T_H) = (T_f - T_C) \quad \Rightarrow \quad 2T_E = (T_H + T_C)$$

$$T_E = 550\ \mathrm{K}.$$

[1 *mark*]

ii) The entropy change of each block and the entropy change of the Universe overall, commenting on the significance of each of your results;

We can use the heat capacity for the heat $\delta Q = C_p dT$ to enable us to determine the entropy changes of the blocks, since $\Delta S_{i\to f} = \int_i^f \frac{\delta Q}{T}$. The trick here is to remember that entropies are always worked out in degrees Kelvin, the thermodynamic temperature!

$$\Delta S_H = \int_{800}^{T_E} \frac{C_p dT}{T} = C_p \ln\left(\frac{550}{800}\right) = -2.47\ \mathrm{kJ\ K^{-1}}$$

$$\Delta S_C = \int_{300}^{T_E} \frac{mc_p dT}{T} = C_p \ln\left(\frac{550}{300}\right) = +4.00\ \mathrm{kJ\ K^{-1}}$$

[1 *mark*]

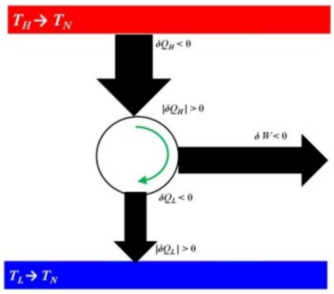
The Universe entropy change is the sum of these, $\Delta S_U = 1.53\ \mathrm{kJ\ K^{-1}}$. This is positive as expected from the second law. Even though the entropy of the hot block decreases (it becomes more ordered as it loses energy) the corresponding entropy increase of the cold block is (always) greater. **[1 *mark*]**

i) The irreversibility of the above process, defined by $I = T_0 \Delta S_U$.

The irreversibility is defined as the entropy change of the Universe multiplied by the temperature of the lowest available reservoir, T_0, which in this case is the environment, and it gives the energy that becomes unavailable for work

$$I = \Delta S_U T_0 = 1.53 \times 300 = 458 \text{ kJ}. \qquad \textbf{[1 mark]}$$

b) If instead, the two blocks are connected via a reversible heat engine, state the entropy change of the system. Then calculate the new equilibrium temperature attained.



No entropy change in a reversible process. **[1 *mark*]**

A schematic of the engine is show in the figure. As the engine is reversible it has zero entropy change. Since this is connecting the two blocks, their total entropy change (sum) must also be zero as they are brought to some equilibrium temperature, T_N, by the engine (different to that in part (a)). The engine will work until the two reservoirs are at the same temperature. In this case the Clausius inequality for the engine cycle actually equals zero.

To calculate the equilibrium temperature, consider the sum of individual entropy changes of the two blocks, and use the fact that the engine has no entropy change.

$$\int \frac{\delta Q_H}{T} + \int \frac{\delta Q_L}{T} = 0 \quad \Rightarrow \quad \int_{800}^{T_N} \frac{C_p dT}{T} + \int_{300}^{T_N} \frac{C_p dT}{T} = 0$$

$$\ln\left(\frac{T_N}{800}\right) + \ln\left(\frac{T_N}{300}\right) = 0 \quad \Rightarrow \quad \ln\left(\frac{T_N^2}{300 \times 800}\right) = 0$$

$$T_N^2 = 300 \times 800 \quad \Rightarrow \quad T_N = 490 \text{ K}$$

(This is lower than part (a), which means that more work can be done!) **[2 *marks*]**

c) The combined block at the equilibrium temperature determined from part (b), is dropped into the River Wear, which has an autumnal temperature of 7 °C. Determine any additional entropy change of the Universe during this process.

The river gains heat equal to that lost by the combined nickel block, $(-\Delta Q_{Ni} = \Delta Q_R)$. Using heat capacity (and realising that for two blocks it gets doubled as the question gave the heat capacity of one block),

$$\Delta S_{Ni} = 2 \int_{490}^{T_R} \frac{C_p dT}{T} = 2 C_p \ln\left(\frac{280}{490}\right) = -7.39 \text{ kJ K}^{-1} \qquad \textbf{[1 mark]}$$

As the river's temperature remains constant when the block is dropped in (it acts as a heat reservoir), $\Delta S_{River} = \frac{1}{T_{River}} \int \delta Q_{River} = \frac{\Delta Q_{River}}{(273+7)}$. We can easily calculate the energy lost by the block as we know its initial and final temperatures,

$$\Delta Q_{Ni} = \int_{490}^{273+7} C_p dT = C_p(280 - 490) = -2.77 \text{ MJ}.$$

Hence $\Delta Q_{River} = +2.77 \text{ MJ}$ and $\Delta S_{River} = +\frac{2.77 \text{ MJ}}{280} = 9.90 \text{ kJ K}^{-1}$.

Hence, $\Delta S_U = 2.51 \text{ kJ K}^{-1}$. **[2 *marks*]**