QUANTUM MECHANICS 2 - WORKSHOP 1

1. An electron is described by a wavefunction

$$\Psi(x,t=0)=\begin{cases} N\sin(\pi x/L) & \text{for } 0<x<L \\ 0 & \text{elsewhere.}\end{cases}$$

i) Show that $N=\sqrt{2/L}$. Calculate the probability of finding the electron in the interval $0<x<L/4$. Give your answer to 3 sig. figs.

ii) Calculate $\langle x^2\rangle = \int \Psi^* x^2 \Psi \, dx$

iii) Calculate $\langle p^2\rangle$, and hence calculate the kinetic energy $\langle T\rangle = \langle p^2\rangle/2m$

iv) Use physical arguments to simply write down, without calculation, the expectation values $\langle p\rangle$ and $\langle x\rangle$ (or do the integrals and then figure out how you should have known the answer in advance!).

v) $(\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$ and $(\Delta p)^2 = \langle p^2\rangle - \langle p\rangle^2$. Hence calculate $\Delta x \Delta p$ for this system and relate your answer to the Heisenberg Uncertainty principle $\Delta x\Delta p \geq \hbar/2$

vi) Calculate $\langle xp\rangle = \int \Psi^* xp\Psi \, dx$

vii) Calculate $\langle px\rangle = \int \Psi^* px\Psi \, dx$

viii) Hence show that $\langle xp\rangle - \langle px\rangle = i\hbar$.

Useful Integrals:

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \sin(ax)\cos(ax)dx = -\frac{\cos^2(ax)}{2a}$$

$$\int x\sin(ax)\cos(ax)dx = \frac{\sin(2ax) - 2ax\cos(2ax)}{8a^2}$$

$$\int x\sin^2(ax)dx = -\frac{\cos(2ax)}{8a^2} - \frac{x\sin(2ax)}{4a} + \frac{x^2}{4}$$

$$\int x^2\sin^2(ax)dx = -\frac{x\cos(2ax)}{4a^2} - \frac{(2a^2x^2-1)\sin(2ax)}{8a^3} + \frac{x^3}{6}$$