Foundations of Physics 2B/3C 2019-2020

Thermodynamics – Workshop 2 Problems

Week Commencing 21st October

1. Work and heat flow – understanding sign conventions

This problem begins by getting you to determine the total work which gets done around a Carnot cycle, before an explicit numerical example to help your understanding of the sign conventions.

a) A Carnot cycle operates between hot and cold reservoirs at temperatures T_H and T_L respectively. What is the work done by such a cycle if the working fluid is one mole of ideal gas? Undertake this calculation of the total work done by the cycle in terms of the volume of each state of the system, denoted by V_A, V_B, V_C, V_D respectively.

b) A Carnot engine operates between temperatures of $100\ \mathrm{K}$ and $400\ \mathrm{K}$ taking in heat of $200\ \mathrm{J}$ each cycle. Calculate the following, taking care with the signs:
   i) The efficiency of the cycle;
   ii) The work done during the cycle;
   iii) The heat rejected during each cycle.

   The engine is now run in reverse between the same two reservoirs at T_L and T_H, as a Carnot Fridge and then as a Carnot Heat Pump, taking in an equivalent amount of work to that given out by the engine in part (ii). Calculate the following:
   iv) The coefficient of the system when it is acting as either a Carnot Refrigerator or Heat Pump. What do you notice about the difference between the above two results? Does this happen for any Carnot Fridge and Heat Pump pair, operating between two reservoirs of given temperatures?
   v) Check all you above findings using the temperatures of the engine.

c) A composite engine consists of two Carnot cycles placed in series, having efficiencies η_1 and η_2. The first engine is placed in contact with a reservoir at T_H, whilst the second is contact with a reservoir at T_L. If the exhaust of the first is used as the heat input of the second, show that the combined efficiency is given by

$$\eta_{Tot} = \eta_1 + (1 - \eta_1)\eta_2.$$

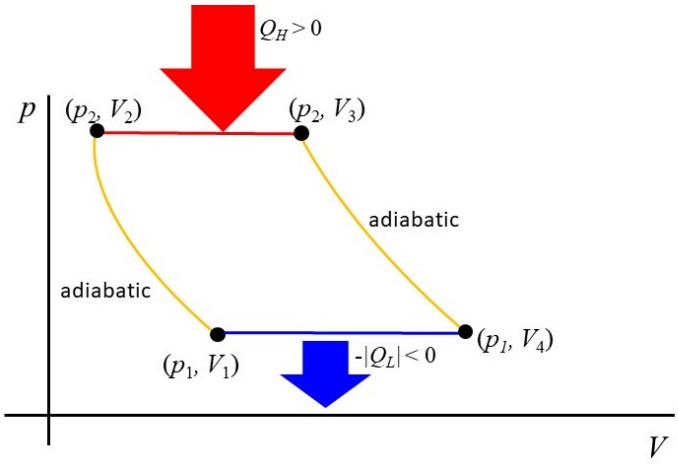
2. Other Engine Cycles

This problem considers efficiencies of engine cycles more generally.

a) One mole of ideal gas, initially at (p_1, V_1) is heated isochorically until its pressure reaches p_2. The gas is then allowed to expand adiabatically to a new volume V_2 with this state pressure p_1, the initial pressure. Finally, the gas is isobarically compressed back to its starting conditions. Sketch a pV diagram for this cycle, showing the directions of heat flow into and out of the engine. Determine its efficiency in terms of the pressure, volume and γ, the ratio between the heat capacities at constant volume and pressure, C_V and C_p.

Turn over for the final question

b) The figure shows the pV diagram for an idealised Brayton cycle. (More information will be provided on other cycles in lecture 5). This cycle consist of two isobaric and two adiabatic processes. Show that its efficiency is given by the following expression,



$$\eta_{Brayton} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}.$$

Here, $r_p = p_2/p_1$, denotes the pressure ratio that the engine operates between.