L2 Foundation of Physics 2B Optics 2019-20

O.8 Summary

February 10, 2020

Learning outcomes:

1. To study the sum of many waves [Optics $f2f$ Chapter 5].
2. To apply the Fresnel diffraction integral to the case of a circular aperture or obstacle [Optics $f2f$ Sec. 5.4 - 5.5].

Key equations: The light distribution in any plane z can be written as a **sum of paraxial spherical waves** originating from the input plane at $z = 0$:

$$E^{(z)} = \frac{E_0}{\mathrm{i}\lambda z} \iint_{-\infty}^{\infty} \mathrm{f}(x', y') \mathrm{e}^{\mathrm{i}kr_\mathrm{p}} \mathrm{d}x' \mathrm{d}y' \ , \quad \text{where} \quad r_\mathrm{p} = z + \frac{(x - x')^2 + (y - y')^2}{2z} \ , \tag{1}$$

is the **paraxial distance** from the source point $(x', y', 0)$ to the observation point (x, y, z). This is known as the **Fresnel diffraction integral**.

For the case of the a **circular aperture** [Optics $f2f$ Sec. 5.4 - 5.5] with radius R_a the field on axis at $(\rho, z) = (0, z)$ in cylindrical coordinates:

$$\mathcal{E}^{(z)} = \frac{\mathcal{E}_0}{\mathrm{i}\lambda z} \mathrm{e}^{\mathrm{i}kz} \int_0^{R_\mathrm{a}} \mathrm{e}^{\mathrm{i}k\rho'^2/2z} 2\pi\rho' \mathrm{d}\rho' = -\mathcal{E}_0 \mathrm{e}^{\mathrm{i}kz} \left(\mathrm{e}^{\mathrm{i}kR_\mathrm{a}^2/2z} - 1 \right) \ , \tag{2}$$

and the intensity is $\mathcal{I}^{(z)} = 4\mathcal{I}_0 \sin^2[\pi R_\mathrm{a}^2/(2\lambda z)]$, the intensity oscillates between 0 and $4\mathcal{I}_0$! This is explained in terms of **Fresnel zones**.

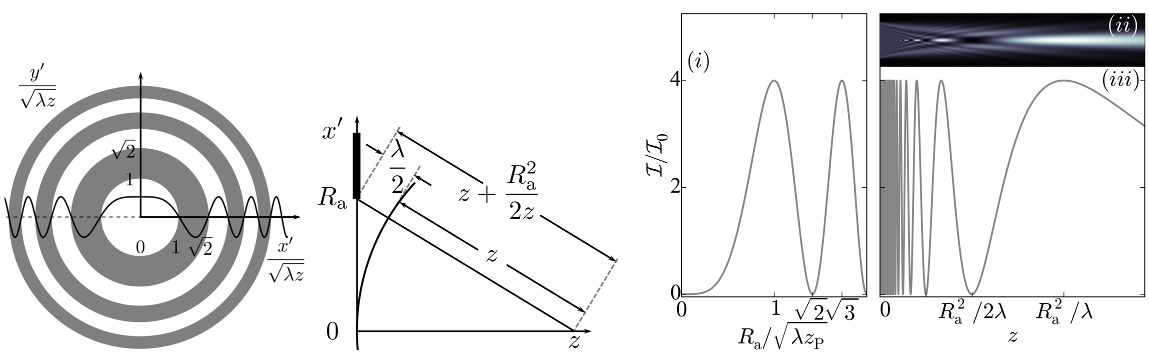

Figure 1: Left: Path difference variation across a circular aperture and how that changes the phase of waves arriving from different points. Middle: Geometry of the 1st Fresnel zone. The field changes sign at the edge of the 1st zone where $R_\mathrm{a} = \sqrt{\lambda z}$. Right: The intensity pattern in the xz plane downstream of a circular aperture.

Outlook: In the next lecture, we shall look at Fraunhofer diffraction [Optics $f2f$ Sec. 5.7].