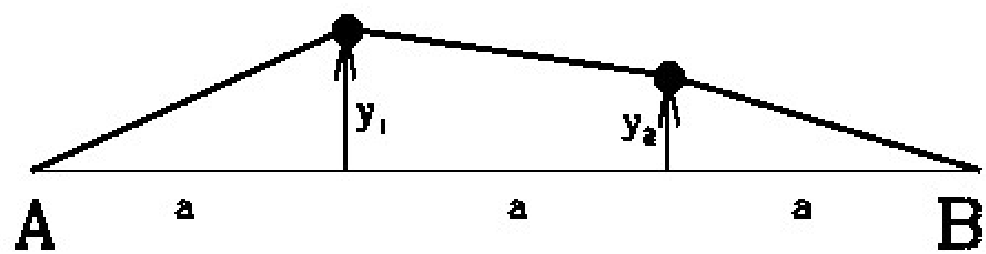
Workshop 4: Particles on a stretched string



Two particles of mass m are attached $1/3$ and $2/3$ of the way along a uniform light elastic string of unstretched length less than $3a$. When fixed at the end points A and B as shown in the plan view above, the string has a tension F along it. We wish to investigate small horizontal displacements in the transverse direction, denoted by the generalised coordinates y_1 and y_2 for the two masses. You should assume that the particles move on a smooth horizontal surface.

1. Write down the kinetic and potential energies of this system, and hence determine its Lagrangian. The expression $V = (1/2)kx^2$ represents the potential energy stored in a string stretched by x from its unstretched length, so cannot be used in this case because the unstretched length is not specified. Instead, determine the potential energy by considering the work done stretching a string under a tension F.

2. Use the Euler-Lagrange equation to find coupled second order differential equations for y_1 and y_2, and rewrite these as a single matrix equation using the notation

$$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

3. Solve this equation to determine the frequencies for the normal modes of oscillation.

4. Calculate the mode vectors corresponding to these frequencies and sketch the motion corresponding to each mode.

5. Write down, in terms of F and m, the solutions for y_1 and y_2 at $t \geq 0$ if the system is disturbed from rest at equilibrium by an impulsive force at $t = 0$ that imparts a velocity $\dot{y}_2 = v$.