University of Durham

EXAMINATION PAPER

Examination session:	Year:	Examination code:
May/June	2018	PHYS2621-WE01

Title:
Stars and Galaxies

Time allowed:	3 hours		
Additional material provided:	None		
Materials permitted:	None		
Calculators permitted:	Yes	Models permitted:	Casio fx-83 GTPLUS or Casio fx-85 GTPLUS
Visiting students may use dictionaries:	No		

Instructions to candidates:

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper.
- Answer **any three** of the four optional questions. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** the answers that are not to be marked.
- The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK.**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklets for Sections B and C, in order, inside your booklet for Section A, before they are collected by the invigilator.

Information

Section A: Observational Techniques
Section B: Stars
Section C: Galactic Astronomy

A list of physical constants is provided on the next page.

Revision:	

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Bohr magneton:	$\mu_\mathrm{B} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Electron mass:	$m_\mathrm{e} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_\mathrm{p} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	$\mathrm{AU} = 1.50 \times 10^{11}$ m
Parsec:	$\mathrm{pc} = 3.09 \times 10^{16}$ m
Solar Mass:	$M_\odot = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26}$ W

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SECTION A: OBSERVATIONAL TECHNIQUES

Question 1 is compulsory. Question 2 is optional.

1. (a) State two reasons why modern optical telescopes are made with the largest possible mirror size. [2 marks]

   State two reasons why modern optical telescopes are sited on remote mountain tops. [2 marks]

   (b) By considering the inverse square law, derive the relation between the absolute magnitude, M, apparent magnitude, m, and distance, d, of a star, defining all terms used. [2 marks]

   What is the absolute magnitude of a star which has an apparent magnitude, $m = 11.5$, and an observed parallax of 0.25 arcsec? [2 marks]

   (c) Define the gain of a CCD. [1 mark]

   A star produces 10 000 analogue-to-digital units (ADU) on a CCD detector. If the read-out noise, dark current and sky background are all negligible, what is the gain of the CCD if the error on this measurement is ± 50 ADU? [3 marks]

2. In May 2019, the 6.5 meter aperture diameter James Webb Space Telescope ($JWST$) will be launched into space. One of the primary goals of $JWST$ is to characterise the atmospheres of extra-solar planets orbiting nearby stars.

   (a) State two advantages and two disadvantages of operating astronomical observatories in space, rather than on the ground. [4 marks]

   (b) Show that the peak energy per unit area collected by a telescope in space is proportional to D^4, where D is the diameter of the primary mirror. [2 marks]

   (c) The mid-infrared spectrograph onboard $JWST$ observes at a wavelength of $\lambda = 9.0\,\mu$m using a diffraction grating with 200 lines mm^{-1}. The grating has a width of 50 mm. If a first order spectrum is recorded, calculate the spectral resolution assuming that the slit width matches the diffraction limited width of the point spread function (PSF). [4 marks]

   [Hint: the resolution of a practical spectrograph is given by

   $$R = \frac{n\rho\lambda W}{\chi D_{\mathrm{T}}},$$

   where ρ is the ruling density of the lines on the grating, W is the grating width, λ is the wavelength of observation, χ is the slit width and D_{T} is the diameter of the telescope.]

When an exoplanet passes in front of a star, a fraction of the starlight we observe will have passed through the atmosphere of the exoplanet. If the exoplanet has oxygen in its atmosphere and therefore potentially habitable, the resulting spectrum will have an absorption lines at characteristic wavelengths.

   (d) Ozone (O_3) is a molecular form of oxygen which is of particular interest. This molecule has a strong absorption line at $\lambda = 9.01\,\mu$m.

   If a continuum signal-to-noise ratio of 100 per spectral pixel is required in order to determine whether the O_3 absorption line is detected in the spectrum of an exoplanet, calculate the limiting magnitude of a star that can be observed in a 10 hour exposure.

   Assume that each spectral pixel has a width of $\Delta\lambda = 1.0$ nm and that the absorption line is narrow and covered by a single spectral pixel. Assume that the sky background, dark current and read noise are negligible and that the detector has a gain of 1. $JWST$ has 30% throughput. A star of magnitude zero has a flux density of 3.90×10^{-8} W m^{-2} μm^{-1}. [8 marks].

   (e) It is suggested that the same spectrograph could be placed on a ground-based telescope which has a diameter of 30 meters. If the ground-based telescope operates at the diffraction limit, how would the spectral resolution change? [2 marks]

SECTION B: STARS

Question 3 is compulsory. Questions 4 and 5 are optional.

3. (a) Calculate the temperature of a white dwarf that has a radius of 6×10^3 km and emits 10% of the luminosity of the Sun. What physical process generates the pressure required to prevent a white dwarf from collapsing? [4 marks]

   (b) What is opacity? Strong absorption lines are seen in the spectrum of a star. Briefly describe the physical process that causes absorption lines in the spectrum. [4 marks]

   (c) Draw a cross section of a star undergoing carbon burning at the core, indicating any other regions in the star where nuclear fusion is occuring. What is the origin of the carbon in the core of the star? [4 marks]

   (d) Estimate the time it would take for a photon to escape from the Sun on a "random walk". Assume a mean-free path of 9.0×10^{-3} m, scattering with a 1.0×10^{-8} s delay each time the photon is scattered, and 7.0×10^8 m for the radius of the Sun. What is the dominant form of opacity in the core of the Sun? [4 marks]

   (e) Briefly describe the physical reaction that leads to the production of neutrinos when the core of a massive star collapses. How is the production of neutrinos different in the Sun? [4 marks]

   (f) What determines the minimum possible mass of a main-sequence star? What limits the maximum possible mass of a main-sequence star? What is the name of the class of objects with masses just below the minimum possible mass of a main-sequence star. [4 marks]

   (g) The rotation period of the Sun is approximately 28 days. Calculate the minimum rotation period of the Sun before it would break up. Assume a uniform density in your calculation and 7.0×10^8 m for the radius of the Sun. [4 marks]

4. (a) What direct observational evidence provides strong support for nuclear fusion as the power source of the Sun? [2 marks]

   (b) Write down the three steps of the dominant proton-proton chain for the fusion of hydrogen into helium in the Sun. Make sure that the nuclear reaction conservation laws are obeyed. [4 marks]

   (c) The Coulomb barrier energy is the energy that particles require to get within the range of strong interactions. Calculate the minimum temperature required for the particle kinetic energy to exceed the Coulomb barrier energy. Assume a pure helium gas and 1.0×10^{-15} m as the range of the strong nuclear force. [5 marks]

   (d) Draw a plot to show how the energy production in the Sun varies as a function of radius (dL/dr versus r). Highlight the region where nuclear fusion occurs. Why is there no significant nuclear fusion outside of this region? How will the dL/dr versus r curve change when the Sun becomes a red-giant star? [9 marks]

5. (a) What is the Jeans mass and how is it applied in the formation of stars? [3 marks]

(b) Show that the Jeans mass (M_J) for a spherical gas cloud of uniform density satisfies the relation

$$M_J = \left(\frac{5k_BT}{G\mu m_p}\right)^{\frac{3}{2}} \left(\frac{3}{4\pi\rho}\right)^{\frac{1}{2}},$$

where T is the temperature of the cloud, μ is the (dimensionless) mean molecular mass, and ρ is the average density of the cloud. [4 marks]

[Hint: recall that the gravitational potential energy is $U = -\frac{3}{5}\frac{GM^2}{R}$]

(c) A spherical cloud of H_2 with uniform density has a radius of 1.0×10^{16} m, a mass of 4.2×10^{30} kg, and a temperature of 50 K. Determine whether this H_2 cloud is Jeans unstable. [4 marks]

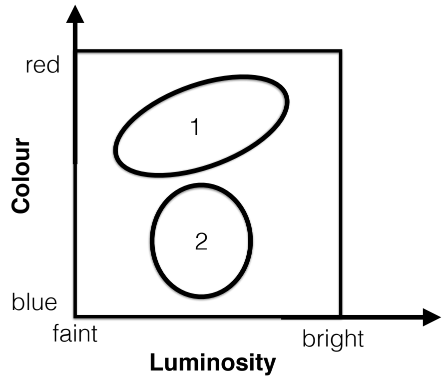
(d) The radiation from a protostar of radius 4.0×10^6 km is absorbed by a thin shell of dust at a radius of 1.0×10^9 km, centred on the protostar. If 10% of the radiation from the protostar is re-emitted by the dust shell, and the shell is observed to have a temperature of 500 K, what is the effective temperature of the protostar? [4 marks]

(e) A protostar core has the following properties: fully ionized hydrogen with a pressure of $P = 6.0 \times 10^{16}$ N m^{-2} and a density of $\rho = 1.0 \times 10^5$ kg m^{-3}. Calculate the temperature of the gas at the core of the protostar. Briefly comment on whether this temperature is high enough for nuclear fusion via the proton-proton chain. [5 marks]

SECTION C: GALACTIC ASTRONOMY

Question 6 is compulsory. Question 7 is optional.

6. (a) In the colour-luminosity diagram of the figure below, galaxies occupy two regions labelled '1' and '2'. These regions correspond relatively well to spiral and elliptical Hubble types. Based on the properties of galaxies, discuss which region(s) likely correspond to spirals and which to ellipticals, explaining your reasoning. [4 marks]



(b) Name the three main *stellar* components of the Milky Way galaxy. Which of these is the Sun part of? [4 marks]

(c) The interstellar medium consists of hot gas ($T \sim 10^7$ K), warm gas ($T \sim 10^4$ K) and cold gas ($T \sim 10^2$ K). Which of these emit X-rays produced by thermal bremsstrahlung and why? Which of these is mostly molecular and why? Briefly explain your reasoning. [4 marks]

(d) Briefly describe three observational manifestations of an Active Galactic Nucleus (AGN). [4 marks]

(e) What is gravitational lensing? How was it used by the MACHO collaboration to study the nature of the dark matter in the Milky Way's halo? What did the collaboration conclude about the nature of the dark matter? [4 marks]

7. Consider a model of a galaxy embedded in a dark matter halo. Assume that the halo's density distribution is spherically symmetric, and depends on distance R from the centre as $\rho(R) = \rho_0\,(R_0/R)^2$, where $R_0 = 8.00$ kpc and ρ_0 is a constant.

   (a) Demonstrate that the mass enclosed in a sphere of radius R, centred on the centre of the halo, is given by $M(R) = (4\pi\,\rho_0\,R_0^3)\,(R/R_0)$. [2 marks]

   (b) What is the 'rotation curve' of a galaxy? Compute and sketch the rotation curve of this galaxy. [6 marks]

   (c) Apply this model to the Milky Way, and determine ρ_0 such that the rotation speed measured at the location of the Sun, $R = R_0$, is $V_c = 2.00 \times 10^2$ km s^{-1}. Express your answer in $\mathrm{M}_\odot$ pc^{-3}. [2 marks]

   (d) The number density of stars in the Solar neighbourhood is approximately 1 pc^{-3}. Assuming that all stars have mass $M_\star = 1\ \mathrm{M}_\odot$, compute the mass density in stars in the Solar neighbourhood, $\rho_\star$, expressing your answer in units of $\mathrm{M}_\odot$ pc^{-3}. Compute the ratio $\rho_\star/\rho_0$. Interpret this result, and discuss whether it is surprising that the circular velocity of the Sun is dominated by dark matter, rather than by the mass in stars. [6 marks]

   (e) Oort's constants can be used to compute the speeds of stars on circular orbits, as seen from the Sun. In particular, the line-of-sight velocity, V_r, of a star at distance, d, with Galactic longitude, l, is given by

   $$V_r = A\,d\,\sin(2l)\,,$$

   where Oort's constant A is given by

   $$A = -\frac{1}{2}\left[\frac{\mathrm{d}V_c}{\mathrm{d}R} - \frac{V_c}{R}\right]_\odot .$$

   The quantity in square brackets is to be evaluated at the location of the Sun. Compute A for our model distribution, and compute and sketch V_r as a function of l for $d = 1$ kpc. [4 marks]

