Mathematical Methods in Physics

Weekly Problems 3

3.1

Consider the matrix

$$A = \left(\begin{array}{ccc} 0 & -i & i \\ i & 0 & -i \\ -i & i & 0 \end{array} \right).$$

Is it (i) Hermitian, (ii) anti-Hermitian i.e. $A^{\dagger} = -A$, (iii) singular, (iv) unitary?

3.2

Find a transformation that diagonalise the following matrix, that is find D, S and S^{-1} such that $D = S^{-1}AS$

$$\left(\begin{array}{cc} 2 & -2 \\ 1 & 4 \end{array}\right).$$

3.3

Consider the matrix
$$B = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
.

- a) What kind of special matrix is matrix B? Find its eigenvalues and eigenvectors. Choose the eigenvectors in such a way that they form a orthonormal set of vectors (i.e. they need to be unit vectors, mutually orthogonal.)
- b) Diagonalize the matrix B, i.e find a matrix S such that $D = S^{-1}BS$ is diagonal.