Durham University

Lecture 15: Dark Matter (A)

Cosmology II

Michaelmas Term 2020

Position of this lecture

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}r^2 + S_k(r)^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right]$$

L10

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3}$$

$$\dot\rho + 3\frac{\dot a}{a}\left(\rho + \frac{P}{c^2}\right) = 0$$

$$d_{\mathrm{A}}(z) = \frac{S_k[r(z)]}{1+z} = \frac{l}{\Theta}$$

L12

$$d_{\mathrm{L}}(z) = (1+z)S_k[r(z)] = \sqrt{\frac{L}{4\pi f}}$$

L11

$$r(z) = c\int_?^? \frac{\mathrm{d}z}{H(z)} = c\int_?^? \frac{\mathrm{d}a}{a^2H(a)}$$

$$r(z) = c\int_{z_{\mathrm{rec}}}^{\infty} \frac{\mathrm{d}z}{H(z)} = c\int_0^{a_{\mathrm{rec}}} \frac{\mathrm{d}a}{a^2H(a)}$$

L16, 17

$$\rho_r = \frac{g_*(T)}{2}\frac{4\sigma T^4}{c^3}$$

$$g_*(T) = \sum g_{\mathrm{boson}} + \frac{7}{8}\sum g_{\mathrm{fermion}}$$

L13

$$\frac{N_n}{N_p} = \exp\left[-\frac{\Delta mc^2}{k_{\mathrm{B}}T}\right]$$

$$Y_{\mathrm{He}} = \frac{2N_n}{N_n + N_p}$$

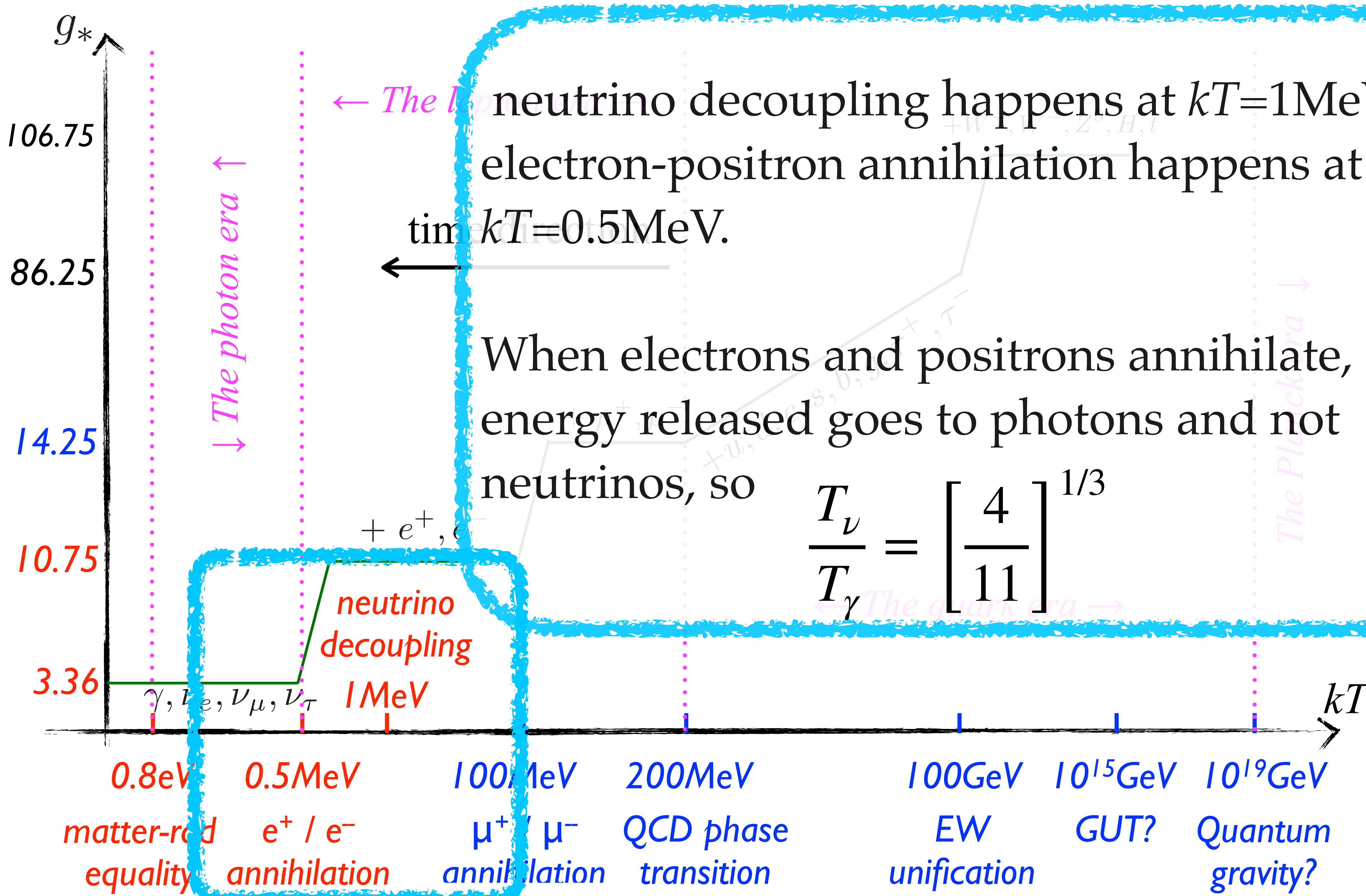
L14

$$\Omega_\nu = \frac{\sum_\nu m_\nu c^2}{94\ \mathrm{eV}}\left[\frac{H_0}{100\ \mathrm{km/s/Mpc}}\right]^{-2}$$

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$$

L15

Recap from L13 (relevant materials)

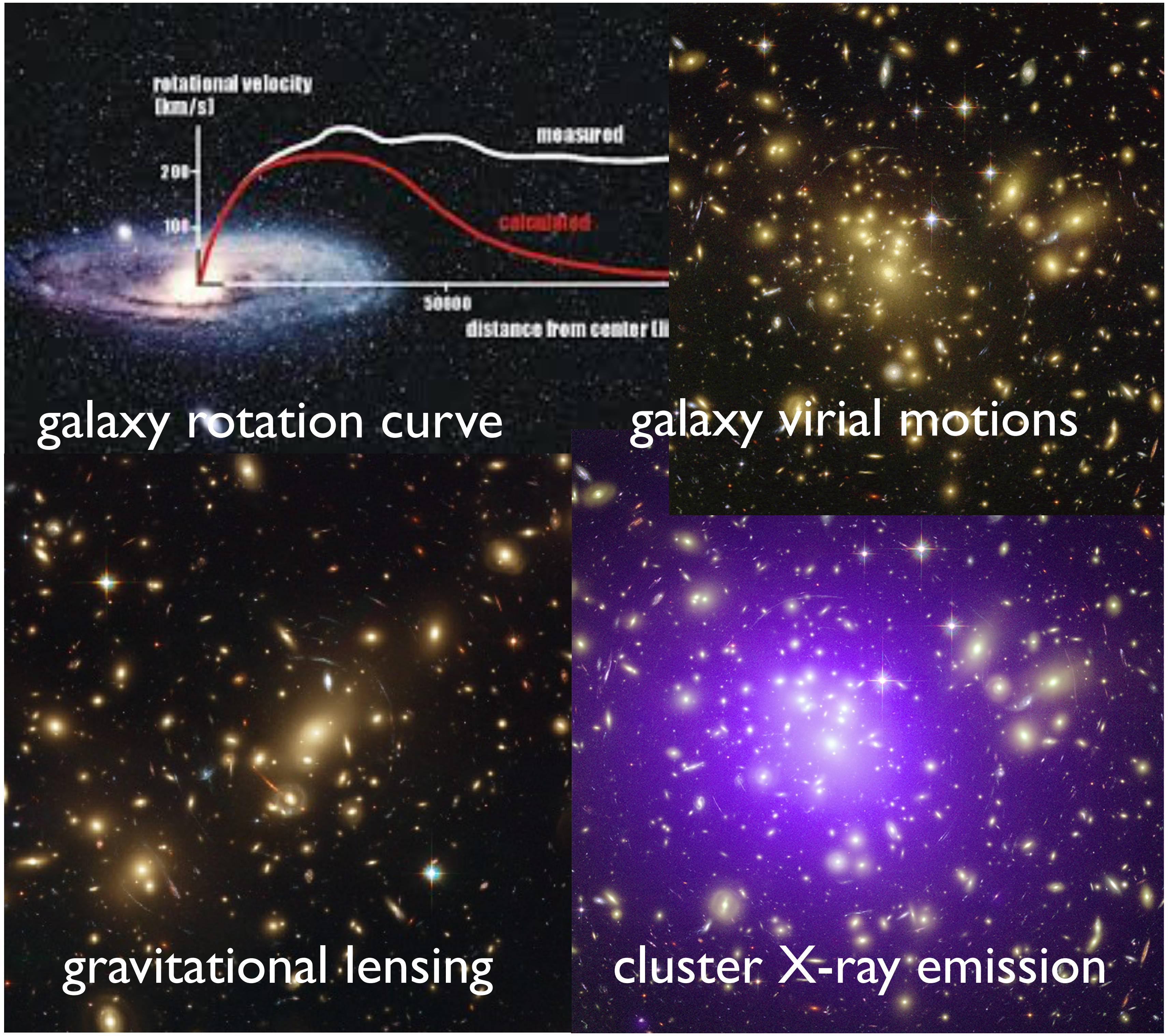


neutrino decoupling happens at kT=1MeV, electron-positron annihilation happens at kT=0.5MeV.

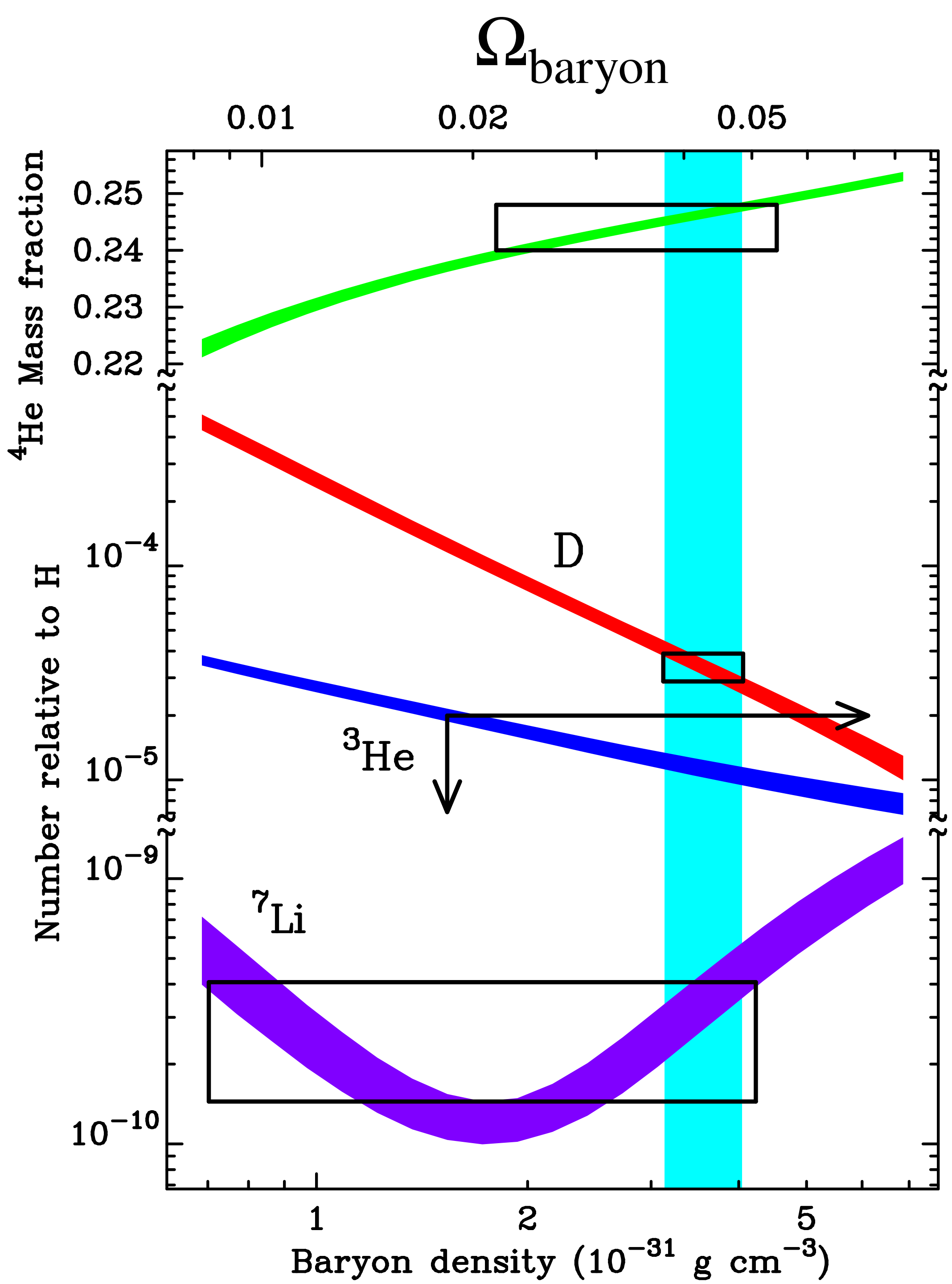
When electrons and positrons annihilate, the energy released goes to photons and not neutrinos, so

$$\frac{T_\nu}{T_\gamma} = \left[\frac{4}{11}\right]^{1/3}$$

Evidence of dark matter (Lecture 6)



Evidence of non-baryon dark matter (15.1. 15.2)



BBN: suggests that baryon density parameter today is Ω_{baryon}~0.04-0.05 (Lecture 14)

Type Ia supernovae: suggest that the total matter density parameter today is Ω_m~0.3 (Lectures 9 & 11)

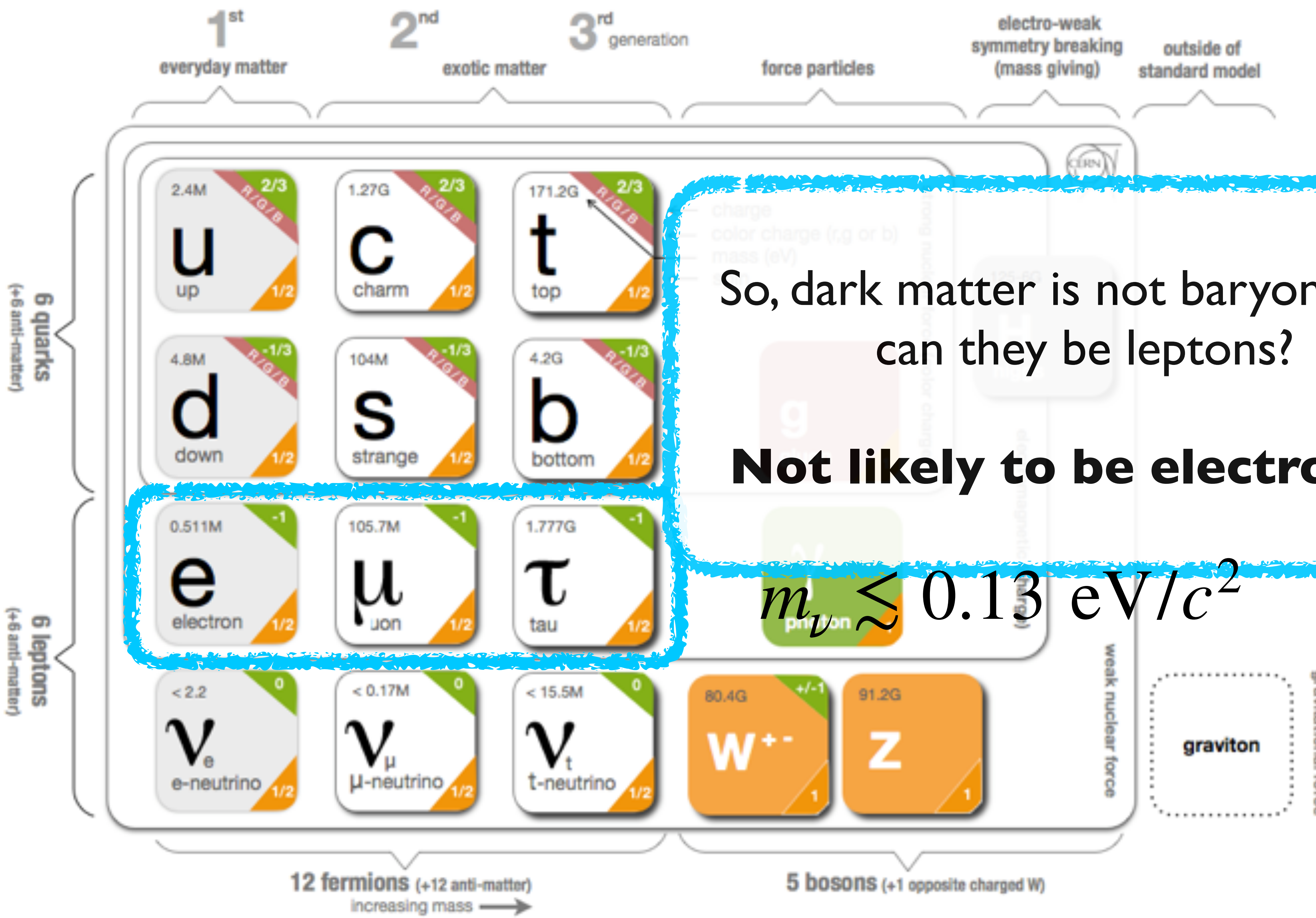
The difference is **dark matter**!

Dark matter (15.3)

What do we know about dark matter?

- $\mathbf{\Omega_{DM0} \sim 0.25}$
- it is **non-relativistic** matter today: $\rho_{DM} \propto a^{-3}$
- weakly interacting with other particles

Hot dark matter: neutrinos



So, dark matter is not baryonic. But can they be leptons?

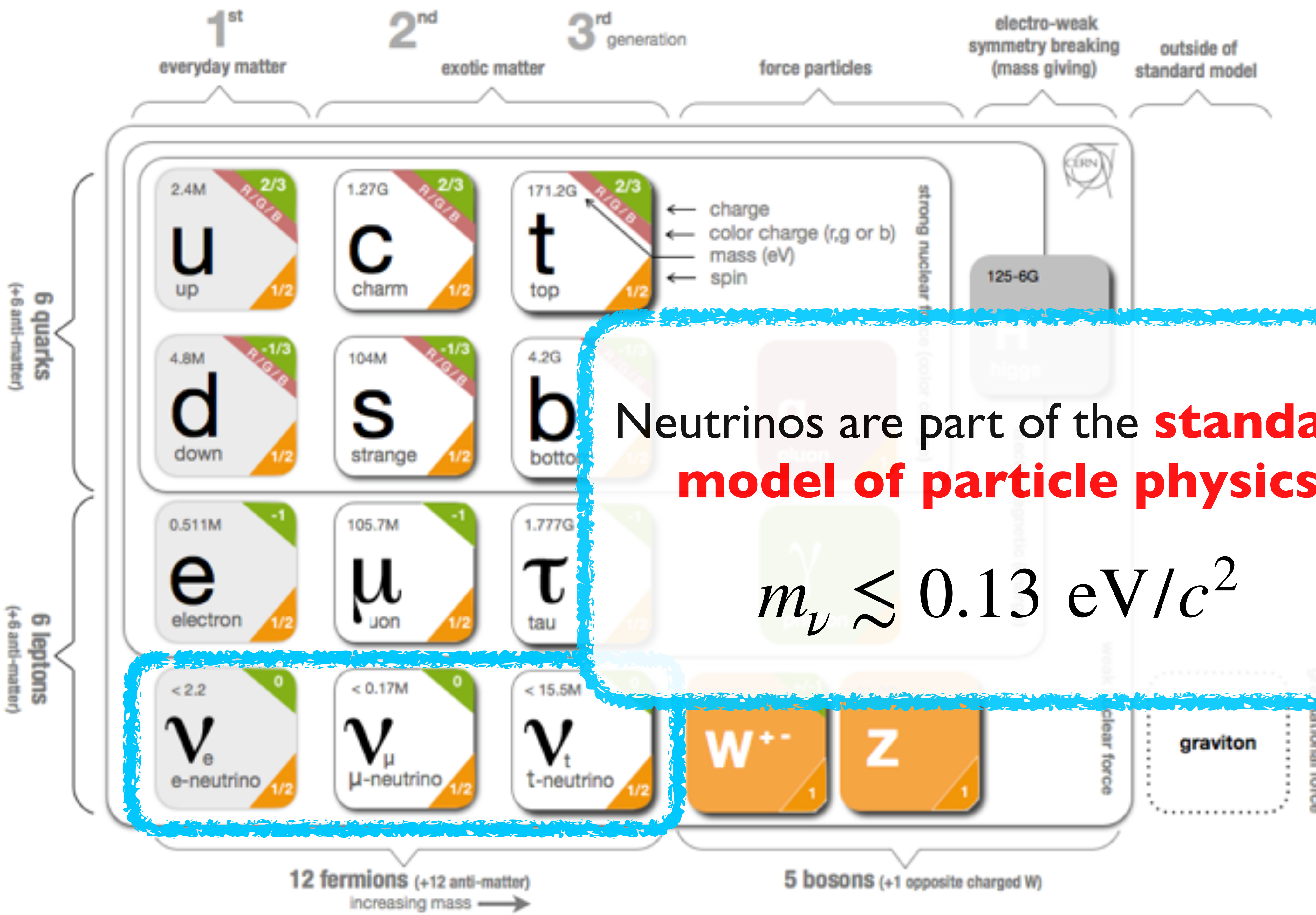
Not likely to be electrons…

$$m_\nu \lesssim 0.13\ \mathrm{eV}/c^2$$

standard model particles

source: STFC site / CERN

Hot dark matter: neutrinos



Neutrinos are part of the **standard model of particle physics**

$$m_\nu \lesssim 0.13\ \mathrm{eV}/c^2$$

standard model particles

source: STFC site / CERN

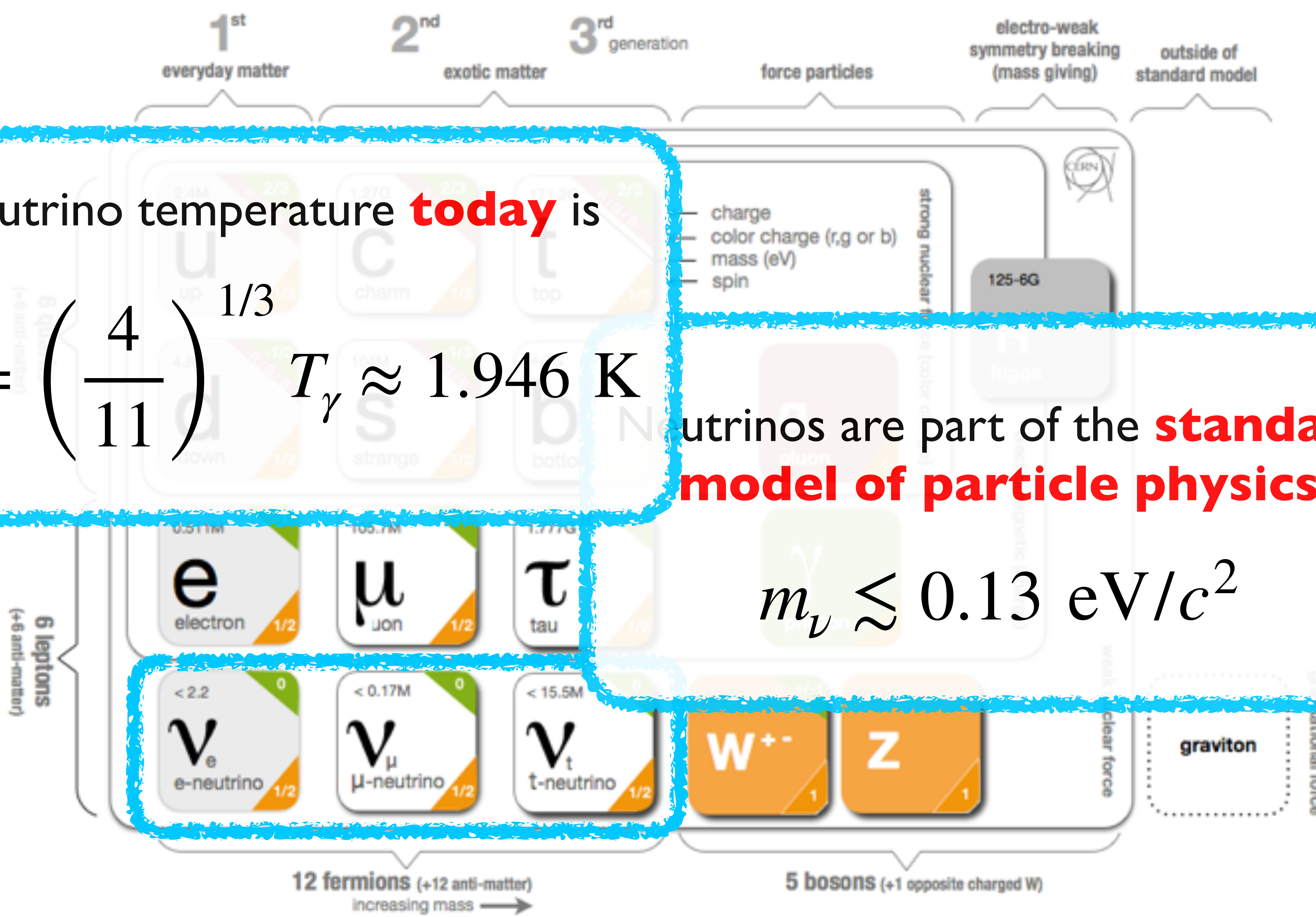
Hot dark matter: neutrinos

Neutrino temperature **today** is

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.946 \text{ K}$$

Neutrinos are part of the **standard model of particle physics**

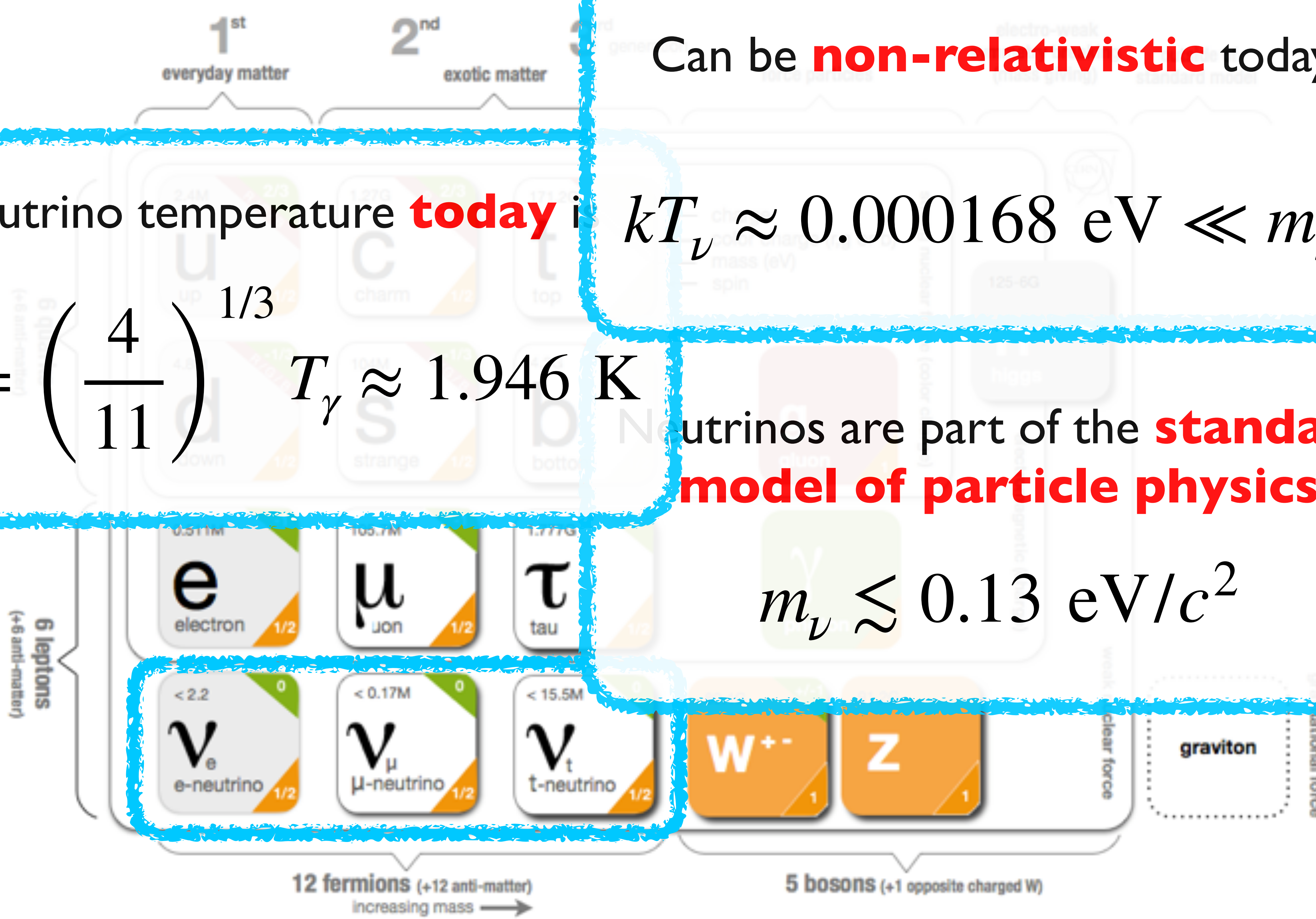
$$m_\nu \lesssim 0.13 \text{ eV}/c^2$$



standard model particles

source: STFC site / CERN

Hot dark matter: neutrinos



Neutrino temperature **today** is

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.946 \text{ K}$$

Can be **non-relativistic** today!

$$kT_\nu \approx 0.000168 \text{ eV} \ll m_\nu c^2$$

Neutrinos are part of the **standard model of particle physics**

$$m_\nu \lesssim 0.13 \text{ eV}/c^2$$

standard model particles

source: STFC site / CERN

Hot dark matter: neutrinos

Neutrinos are called **hot dark matter** because they are highly relativistic at decoupling (**since when the total neutrino number in the universe stays nearly constant**).

But can neutrinos account for all dark matter, $\mathbf{\Omega_{DM0} \sim 0.25}$?

Let's derive its abundance.

Hot dark matter: neutrinos

$$\Omega_\nu = \frac{8\pi G \rho_\nu}{3H^2}$$

$$\rho_\nu = n_\nu m_\nu$$

neutrino
number density

neutrino
particle mass

$$\boxed{\Omega_\nu = \frac{\sum m_\nu c^2}{94\ \text{eV}} \left[\frac{H_0}{100\ \text{km/s/Mpc}} \right]^{-2}}$$

key eqn of L15

$$n_\nu(a) \propto a^{-3}$$

$$n_\nu(a) = n_\nu(a_{dec})\frac{a_{dec}^3}{a^3}$$

'dec' stands for neutrino decoupling

$$n_\nu(a_{dec}) = \left[\frac{\text{neutrino energy density}}{\text{mean energy per neutrino}}\right]_{dec}$$

$\frac{g_{*,\nu}}{2}\cdot\frac{4\sigma T_\nu^4}{c}$, $g_{*\nu} = 6\times\frac{7}{8}$

$3kT_\nu$

$$= \left[\frac{\frac{6}{2}\cdot\frac{7}{8}\cdot\frac{4\sigma T_\nu^3}{c}}{3\,kT_\nu}\right]_{dec}$$

$$= \frac{7\sigma}{2kc}T_{\nu,dec}^3$$

// total number density of all three flavors of neutrinos and their anti-neutrinos.

$$n_{\nu_e+\bar\nu_e}^{dec} = n_{\nu_\mu+\bar\nu_\mu}^{dec} = n_{\nu_\tau+\bar\nu_\tau}^{dec} = \frac{1}{3}\cdot\frac{7\sigma}{2kc}\cdot T_{\nu,dec}^3 = \frac{7}{6}\frac{\sigma}{kc}T_{\nu,dec}^3$$

$$n_{\nu_e+\bar\nu_e}(a) = n_{\nu_\mu+\bar\nu_\mu}(a) = n_{\nu_\tau+\bar\nu_\tau}(a) = \frac{7}{6}\frac{\sigma}{kc}T_{\nu,dec}^3\left(\frac{a_{dec}}{a}\right)^3$$

$$= \frac{7}{6}\frac{\sigma}{kc}T_\nu^3(a)$$

$T_\nu \propto \frac{1}{a}$

$$\Omega_{\nu 0} = \frac{8\pi G}{3H_0^2}\rho_{\nu 0}$$

$$= \frac{8\pi G}{3H_0^2}\left[n_{\nu_e+\bar{\nu}_e} m_{\nu_e} + n_{\nu_\mu+\bar{\nu}_\mu} m_{\nu_\mu} + n_{\nu_\tau+\bar{\nu}_\tau} m_{\nu_\tau}\right]_{a=1}$$

$$= \frac{8\pi G}{3H_0^2}\frac{7}{6}\frac{\sigma}{kc}T_{\nu 0}^3 \underbrace{(m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau})}_{\sum m_\nu}$$

$$= \frac{8\pi G}{c^2 3H_0^2}\frac{7}{6}\frac{\sigma}{kc}T_{\nu 0}^3 \sum m_\nu c^2$$

unit of energy : eV

$$= \sum m_\nu c^2 \times \left[\frac{H_0}{100\ \mathrm{km/s/Mpc}}\right]^{-2} \times \boxed{\frac{8\pi G}{3\times 100^2 \times c^2}\cdot\frac{7}{6}\frac{\sigma}{kc}T_{\nu 0}^3}$$

$T_{\nu 0} \cong 1.946$ K

$100\ \mathrm{km/s/Mpc} \cong 3.24\times 10^{-18}\ \mathrm{s}^{-1}$

$\cong 7\times 10^{16}\ \mathrm{J}^{-1}$

$\cong \frac{1}{90\,\mathrm{eV}}$

$$\cong \frac{\sum m_\nu c^2}{90\,\mathrm{eV}}\left[\frac{H_0}{100\ \mathrm{km/s/Mpc}}\right]^{-2}$$

For $\sum m_\nu c^2 \simeq 0.1\,\mathrm{eV}$, $H_0 = 70\ \mathrm{km/s/Mpc}$,

we have: $\Omega_{\nu 0} \cong \frac{0.1}{90}\times 0.7^{-2} \cong 0.002 \ll 0.25$!

Cold dark matter (15.3.2)

$$m_\nu \lesssim 0.13\ \mathrm{eV}/c^2$$

experiment bound
on neutrino mass

$$\Omega_\nu = \frac{\sum m_\nu c^2}{94\ \mathrm{eV}} \left[\frac{H_0}{100\ \mathrm{km/s/Mpc}} \right]^{-2} \ll 0.25$$

Neutrinos cannot be the whole picture of dark matter. And unfortunately no other fundamental particles in the standard model can be a viable candidate for dark matter either.

Cold dark matter (15.3.2)

Beyond standard models offer a plethora of new particles. One example is supersymmetry (SUSY), a Grand Unified theory.

SM fermions	SUSY bosonic partners

Leptons → Sleptons
neutrino → sneutrino
Quarks → Squarks

SM bosons	SUSY fermionic partners

Photon → Photino
Higgs → Higgsino
Graviton → Gravitino
W → Wino

Cold dark matter (15.3.2)

- very heavy masses: 100 GeV/c^2
- interact with other particles species weakly (typical strength of weak interactions)
- **WIMPs** (Weakly Interacting Massive Particles)

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W → Wino	

Cold dark matter (15.3.2)

Like neutrinos, these SUSY particles were also in thermal equilibrium with the rest of the plasma in the very early universe. When decoupling, they are non-relativistic, so that they are called **cold dark matter**.

A similar logic as for neutrinos can be used to calculate the abundance of WIMPs, but the calculation is beyond the scope of this course. We just note that WIMPs can naturally have an abundance of $\boldsymbol{\Omega_{WIMP0} \sim 0.25}$.

A summary of dark matter

So we have two classes of candidates

- **hot dark matter** (e.g., neutrinos): already detected but only a small contribution to all dark matter

- **cold dark matter** (e.g., WIMPs): undetected in particle colliders or terrestrial experiments so far, but the preferred and widely accepted model, whose predictions agree with cosmological observations very well

Key topics and learning outcomes

Key topics

- hot dark matter
- cold dark matter

Learning outcome

- Be aware of evidences of dark matter and that dark matter cannot be just dark baryons
- Know that neutrinos can be a (hot) dark matter candidate
- Be able to calculate the density parameter of neutrinos given their masses