L2 Foundation of Physics 2B Optics 2019-20

O.WP.3 Fresnel diffraction, aperture functions and Fresnel zones

February 13, 2020

The field on-axis at a distance z downstream of a circularly symmetric aperture is given by

$$E^{(z)} = \frac{E_0 \mathrm{e}^{ikz}}{i\lambda z} \int_0^{\infty} f(\rho') \mathrm{e}^{ik\rho'^2/(2\pi)} 2\pi\rho' \mathrm{d}\rho' \ ,$$

where $f(\rho')$ is the aperture function. Let the aperture function $f(\rho')$ be a uniformaly illuminated circular annulus with inner and outer radii ρ'_1 and ρ'_2 respectively.

1. Sketch the aperture function in the $z = 0$ plane labelling ρ'_1 and ρ'_2. [2 marks]
2. Derive an expression for the field on-axis in the plane $z = z$ downstream of the annular aperture. [5 marks]
3. Any aperture function with circular symmetry can be considered to be made up of a series of Fresnel zones. Write an expression for the electric field $E_2^{(z)}$ produced by the second Fresnel zone only, in terms of E_0. [3 marks]
4. Rewrite $E_2^{(z)}$ in terms of the electric field produced by the first Fresnel zone $E_1^{(z)}$. [2 marks]

$$\left[\text{Hint: } \int_{\xi_1}^{\xi_2} \mathrm{e}^{i\xi^2} 2\xi \mathrm{d}\xi = -i \left(\mathrm{e}^{i\xi_2^2} - \mathrm{e}^{i\xi_1^2} \right) \right]$$