

Mathematical Methods II

Workshop 3

- (1) Solve the differential equation

$$x^2 y'' - xy' + y = x, \quad (1)$$

with the boundary conditions $y(1) = 1$ and $y(e) = 2e$. Does Eq. (1) belong to a well-known class of differential equations? Which one?

- (2) Find the general solution of the following differential equation

$$(x+1)^2 y'' + 3(x+1)y' + y = x^2. \quad (2)$$

Does Eq. (2) belong to a well-known class of differential equations? Which one?

- (3) Solve

$$y'' - 2y' + y = 2xe^x, \quad (3)$$

using the method of variation of parameters.

- (4) Consider the generic linear second order equation

$$a_2(x)y'' + a_1(x)y'(x) + a_0(x)y(x) = f(x), \quad (4)$$

and assume that y_1 is a solution of the associated homogeneous problem. Show that the substitution $y(x) = v(x)y_1(x)$ transforms Eq. (4) into an equation of order 1 for v' (this trick is usually called *reduction of order technique*).