University of Durham

EXAMINATION PAPER

Examination session:	Year:	Examination code:
May/June	2018	PHYS2591-WE01

Title:
Foundations of Physics 2B

Time allowed:	3 hours		
Additional material provided:	None		
Materials permitted:	None		
Calculators permitted:	Yes	Models permitted:	Casio fx-83 GTPLUS or Casio fx-85 GTPLUS
Visiting students may use dictionaries:	No		

Instructions to candidates:

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper.
- Answer **one** other question from **each** section. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** the answers that are not to be marked.
- The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK.**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklets for Sections B and C, in order, inside your booklet for Section A, before they are collected by the invigilator.

Information

Section A: Thermodynamics
Section B: Optics
Section C: Condensed Matter Physics

A list of physical constants is provided on the next page.

Revision:	

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Bohr magneton:	$\mu_{\mathrm{B}} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Electron mass:	$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_{\mathrm{A}} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	$\mathrm{AU} = 1.50 \times 10^{11}$ m
Parsec:	$\mathrm{pc} = 3.09 \times 10^{16}$ m
Solar Mass:	$M_{\odot} = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_{\odot} = 3.84 \times 10^{26}$ W

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SECTION A: THERMODYNAMICS

Answer question 1 and **either** question 2 **or** question 3.

1. (a) With the aid of a suitable diagram, explain how if the Kelvin statement of the *second law of thermodynamics* is violated, then the entropy statement must be violated also. [4 marks]

   (b) A continuous distribution of particles is described by a distribution function, $f(\varepsilon)$ and its density of states, $g(\varepsilon)$. What information do these quantities provide? For the particular case of bosons, the distribution function is the Bose-Einstein Distribution

$$f(\varepsilon) = \frac{1}{\exp\left(\frac{\varepsilon-\mu}{k_{\mathrm{B}}T}\right) - 1},$$

   where μ is the chemical potential and other symbols have their usual meanings. Explain how Bose-Einstein Condensation can arise from this. [4 marks]

   (c) The isobaric expansivity is defined as $\beta_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$, where all symbols have their usual meanings. If the value of this coefficient is negative for some increase in temperature, how must the sample's volume have changed? One of the Maxwell Relations is $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$, where S is the entropy. Use this fact, and the *third law of thermodynamics,* to show that $\beta_P \to 0$ as $T \to 0$. [4 marks]

   (d) The Helmholtz function is defined by $F = U - TS$, where all symbols have their usual meanings. Under what thermodynamic conditions is F conserved? At constant temperature, if the entropy change becomes the maximum possible, what happens to F? [4 marks]

   (e) Sketch an appropriately labeled pV diagram for the following cycle, operating on one mole of ideal gas, that starts in the thermodynamic state (p_i, V_i, T_i):

   - Isobaric expansion such that the volume changes from V_i to $4V_i$;
   - Adiabatic expansion that lowers the temperature back to T_i;
   - Isothermal compression to the initial state.

   Calculate the unknown thermodynamic coordinates, assuming that the gas is diatomic, and hence that $\gamma = 7/5$. [4 marks]

2. (a) A block of copper with heat capacity $C_p = 7800$ J K^{-1} at 400 °C is brought into contact with a block of iron, with heat capacity $C_p = 9000$ J K^{-1} at 20.0 °C. Determine the entropy change of the Universe as the blocks come into thermal equilibrium. [4 marks]

(b) The atoms of one mole of a certain solid are arranged in the lowest three available energy states in the ratio 7 : 2 : 1. What is the entropy of this solid?

[Hint: Use the *Stirling Approximation*, $\ln X! \approx X \ln X - X$.]

[6 marks]

(c) An object has a temperature dependent heat capacity given by $C = AT$, for some constant A. It is initially at a temperature T_1, and is then brought into thermal contact with a heat reservoir at temperature T_R in an environment at temperature T_E. If $T_1 > T_R > T_E$, determine the irreversibility of the process. [5 marks]

(d) By using the following TdS equation

$$TdS = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV,$$

where all symbols have their usual meanings, show that the heat capacity at constant volume can be expressed as

$$C_V = -T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_S .$$

Hence, evaluate this expression for one mole of ideal gas, and comment on your result if the gas is mono-atomic. [5 marks]

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3. (a) The Helmholtz function is given by $F = U - TS$, where all symbols have their usual meaning. For a material that has magnetic moment m, which is subject to a change in magnetic field dB, the work that is done is $\delta W = -mdB$. Demonstrate the following Maxwell Relation

$$\left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial m}{\partial T}\right)_B .$$

[4 marks]

(b) A material exhibits paramagetic behaviour, which can be described by the relation $m = CB/(\mu_0 T)$, where C is a constant. If the magnetic field is doubled isothermally, at temperature T_0, determine the entropy change of the above material. [3 marks]

(c) Consider the entropy as a function of temperature and field, i.e. $S = S(T, B)$. Hence, show that the heat of magnetization for a paramagnet, as the field is increased isothermally at temperature T_0, from 0 to B_0, is given by

$$\Delta Q = -\frac{C}{T_0\mu_0}\frac{B_0^2}{2}.$$

[6 marks]

(d) Derive the following expression for the internal energy of a magnetic material,

$$\left(\frac{\partial U}{\partial B}\right)_T = T\left(\frac{\partial m}{\partial T}\right)_B - m.$$

Hence, find an expression for the internal energy of a paramagnet. [4 marks]

(e) For a particular magnetic sample, the Helmholtz function is experimentally found to be

$$F = aT^5 - \frac{kB^2}{2T},$$

for some constants a and k. What type of material is this? Further, determine an expression for its internal energy, U. [3 marks]

SECTION B: OPTICS

Answer question 4 and **either** question 5 **or** question 6.

4. (a) If waves travelling at 1.00 m s^{-1} break on the beach every 100 s, what is the spatial frequency of the wave? Specify the units in your answer. [4 marks]

   (b) Write expressions for the following: (i) a plane wave in the xz plane with wave vector $\underline{k} = (k_x, 0, k_z)$ and (ii) a paraxial spherical wave propagating along the z axis with source point at the origin. [4 marks]

   (c) A paraxial spherical wave propagating along the z axis with source point $(0, 0, -s_1)$ is incident on a thin lens lying in the $z = 0$ plane with focal length f. Write an expression for the field immediately downstream of the lens. What is the position of the image? [4 marks]

   (d) ~~Calculate the beam radius of a red laser pointer with wavelength $\lambda = 633$ nm on the Moon. The initial beam size, on Earth, is $w_0 = 1.00$ mm, and the distance to the Moon is 3.84×10^8 m. [4 marks]~~

   (e) A circular aperture with radius R_{a} placed in the $z = 0$ plane is illuminated by uniform monochromatic light with wavelength λ propagating in the positive z direction. Describe, or sketch, the light intensity in the xy plane at a distance: (i) $z = R_{\mathrm{a}}^2/\lambda$ and (ii) $z = R_{\mathrm{a}}^2/(2\lambda)$. Explain, briefly, the origin of the observed patterns in each case. [4 marks]

5. A Michelson interferometer consists of a beamsplitter that divides an input with amplitude $\mathcal{E}_0$ into two equal amplitude 'arms' with lengths ℓ_1 and ℓ_2. Two perfect mirrors retro-reflect the light such that the two paths interfere at the beamsplitter.

   (a) Write an expression for the output field after the two paths recombine at the beamsplitter. State any assumptions you make. [5 marks]

   (b) Write an expression for the intensity at the output. [3 marks]

   (c) The path difference, $\ell_2 - \ell_1$, is chosen such that the intensity at the output is equal to one-half of its maximum possible value. Write an expression for $\ell_2 - \ell_1$ in terms of the wavelength, λ. [2 marks]

   (d) A gravitational wave arriving at a Michelson interferometer increases the length of one arm by $\Delta\ell$, and decreases the length of the other arm by $\Delta\ell$. Write an expression for the output intensity as a function of $\Delta\ell$, assuming that $\Delta\ell$ is small. [4 marks]

   (e) If the power circulating in each arm is 0.8 MW and the minimal detectable signal is 1 μW, the wavelength is 0.5 μm and the length of each arm is 4 km, estimate the minimum strain, $\Delta\ell/\ell$, that can be detected in principle. [4 marks]

   (f) Give two reasons why Young's double-slit interferometer is less well suited to measure gravitational waves than a Michelson interferometer. [2 marks]

> Hints:
> $\cos(A + B) = \cos A \cos B - \sin A \sin B$.
> For small B, $\sin B = B$, $\cos B = 1$, and $\cos(A + B) = \cos A - B \sin A$.

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6. An aperture containing 5 narrow slits with spacing, d, is placed in the $z = 0$ plane and illuminated normally using uniform monochromatic light with amplitude, $\mathcal{E}_0$, and wavelength, λ. The slits can be assumed to be effectively infinitely long in the y direction and centred around $x = 0$.

   (a) Write an expression for the field at a point (x, z), where $z \gg d$. State any approximations you make. [6 marks]

   (b) Draw, or describe, a phasor diagram to represent the field at $x = [\lambda/(2d)]z$. What is the ratio of the intensity at this position compared to the maximum intensity? [4 marks]

   (c) How many positions of zero intensity are there between $x = 0$ and $x = [\lambda/(2d)]z$? Sketch the phasor diagrams, or specify the phasor angles, in each case. [3 marks]

   (d) Two glass plates that shift the phase by π are placed in front of the slits at input coordinates, $(x', 0) = (\pm d, 0)$. Draw, or describe, the modified phasor diagrams at $x = 0$ and $x = [\lambda/(2d)]z$. [2 marks]

   (e) Describe, briefly, the similarities and differences between the diffraction patterns without and with the glass plates. Alternatively, sketch a graph of both patterns on the same axes. [2 marks]

   (f) Instead of a simple phase shift, different glass plates are used to rotate the plane of polarization of light emerging from the slits at $x' = \pm d$ by 90°. Describe the expected pattern. What is the peak intensity? [3 marks]

SECTION C: CONDENSED MATTER PHYSICS
Answer question 7 and **either** question 8 **or** question 9.

7. (a) Sketch examples of planes with Miller indices $(\bar{1}10)$ and (002). Include the x, y and z axes in your diagram. If the lattice constant a is 0.4 nm, determine the spacing between the planes for each of these two sets of Miller indices. [4 marks]

   (b) Describe the two types of bonding that will be present in solid CO_2. For each type briefly describe the physical origin of the bonding. [4 marks]

   (c) With the aid of a simple sketch explain the Debye and Einstein approximations as applied to the phonon dispersion curve for a crystal with a two-atom basis. For each approximation, state which physical property is assumed to be constant. [4 marks]

   (d) What assumptions are made about the electron in the Drude model? Describe the drift velocity profile of an electron under an applied electric field in the Drude model. What simple parameter is used to quantify this motion? [4 marks]

   (e) Describe the physical origin of the energy gap in a crystalline solid as modelled by the quantum nearly-free electron model. Draw a diagram of the $E(k)$ energy-wavevector curve for the first Brillouin zone to illustrate your answer. [4 marks]

8. (a) State the Bragg Law and explain, with the aid of a simple sketch, how this describes the diffraction of X-rays from a crystal lattice. [4 marks]

   (b) State the structure factor expression and explain its role in determining the distribution and intensity of observed peaks in an X-ray diffraction pattern. [5 marks]

   (c) Reports of a new element called Durhamium have recently been published. The claim is that this new element has a body-centred cubic structure (bcc) and that its lattice constant is 0.250 nm at room temperature. Its diffraction pattern is measured using X-rays of wavelength 0.15 nm. Calculate the expected 2θ values for the first four diffraction peaks clearly showing your method of working, where θ is the Bragg scattering angle. [8 marks]

   (d) This element has the unusual property of undergoing a transition to a simple cubic structure at a temperature of 310 K with no change to the lattice constant a. Explain qualitatively what the X-ray diffraction pattern would look like when measured at 315 K. [3 marks]

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9. (a) ~~Explain, with the aid of a sketch, the role of the energy density of states function and the Fermi-Dirac distribution function in determining the electron energy distribution in a metal as described by the free-electron model. What is the significance of the Fermi Energy?~~ [7 marks]

(b) A piece of copper is known to have a Fermi Energy of 7.0 eV. Determine its free electron concentration and the velocity of electrons at the Fermi energy. Compare this result with the classical value at 300 K and explain any differences. [6 marks]

(c) The atomic mass of copper is 63.5 u, where u = 1.66×10^{-27} kg. By making some reasonable approximations determine the density of copper in kg m^{-3}. [3 marks].

(d) A second sample of copper is a very thin film effectively confining the electrons in a layer approximating two dimensional behaviour. Describe qualitatively what changes you would expect in the electron energy distribution. [4 marks]