Relativistic Electrodynamics

RED.5

In the following, you should use the relativistic relation between an external 3-force applied to a particle and its relativistic momentum

$$\underline{F} = \frac{d}{dt}\left(\gamma m \underline{v}\right) \tag{1}$$

in the case where $\underline{\boldsymbol{F}}$ is parallel to $\underline{\boldsymbol{v}}$:

1) Show that under the assumption that $\underline{F}$ is parallel to $\underline{v}$ then

$$F = m\,\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\,\frac{dv}{dt} = m\,\gamma^3\,\frac{dv}{dt}, \tag{2}$$

where m is its rest mass and v is its speed. **[3 marks]**
Note in particular that $F \neq ma$.

Consider now a charged particle moving in a straight line in a uniform electric field $\underline{E}$ with speed v. You can assume the E-field and v are both in the x-direction.

2) Show that the magnitude of the acceleration of the charge q is given by

$$a = \frac{dv}{dt} = \frac{qE}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2} = \frac{q\,E}{\gamma^3\,m}. \tag{3}$$

[2 marks]

3) Explain the significance of the dependence of the acceleration on the speed of the particle. **[1 mark]**

Assume now that at $t = 0$ the position is $x(t = 0) = 0$ and $v(t = 0) = 0$.

4) Find the speed of the particle and its position at $t > 0$. **[3 marks]**

5) Comment on the limiting values of v and x as $t \to \infty$. **[1 mark]**