Mathematical Methods in Physics

Weekly Problems 8

8.1

Compute the line integral

$$I = \int_C \mathbf{a} \cdot d\mathbf{r}$$

of the vector field $\mathbf{a}(x, y, z) = (x + z)\mathbf{i} + (z + y)\mathbf{j} + (x - y)\mathbf{k}$ along the following curves C with endpoints A and B.

a) C is the straight line with $A = (1, 2, 3)$ and $B = (4, 5, 9)$.

b) C is the curve

$$y = x^2 \,, \qquad z = x^3 \,.$$

with $A = (0, 0, 0)$ and $B = (2, 4, 8)$.

8.2

Consider the surface S given by the following parametric equation

$$\mathbf{r}(\phi, z) = \sqrt{z}\cos\phi\,\mathbf{i} + \sqrt{z}\sin\phi\,\mathbf{j} + z\,\mathbf{k}, \qquad 0 \le z \le 2\,, \quad 0 \le \phi \le 2\pi$$

and the vector field

$$\mathbf{a}(x, y, z) = y\,\mathbf{i} - x\,\mathbf{j} + z\,\mathbf{k}.$$

a) Compute the area of the surface, i.e.

$$I_1 = \int_S dS.$$

[*Perform a change of variable in order to solve the integral.*]

b) Compute the integral

$$I_2 = \int_S (\nabla \times \mathbf{a}) \cdot d\mathbf{S}.$$