Mathematical Methods II
Workshop 4

(1) Find all the singular points of the following equations

$$y'' - 2xy' + 2\nu y = 0 \quad \text{(Hermite equation)},$$
$$xy'' + (m + 1 - x)y' + (\nu - m)y = 0 \quad \text{(associated Laguerre equation)},$$

and classify them. (Do not forget to consider potential singularities at infinity). Here m and ν are constants.

(2)

(a) Solve the equation

$$\frac{d^2y}{dx^2} + \omega^2 y = 0\,,$$

where $\omega^2 > 0$ is a constant, using the simplest possible method, or any approach you wish.

(b) Taylor-expand both components of the general solution found above to order $\mathcal{O}(x^5)$.

(c) Now use the series expansion method around $x = 0$ to solve the same differential equation. The two independent solutions are obtained by choosing either $a_0 = 0$ or $a_1 = 0$ for the coefficients of the series.

(d) Compare the results in (b) and (c) and draw conclusions about the series expansion in (c).