Lecture 5: The Distance Scale

5 Measuring the Hubble Constant

[Liddle sec:6.1]

$$\boxed{v = \mathrm{H}_0\, d}$$

Hubble's Law

There are two steps to the classical way of observationally measuring the Hubble constant. Both the distance d and recession velocity v must be measured for a set of distant galaxies. It is important that the galaxies be distant so that their **peculiar** velocities are small compared to the Hubble flow.

5.1 Measuring Galaxy Redshifts

The first task of measuring the recession velocity of distant galaxies is straightforward. Galaxy spectra have many distinctive absorption and emission lines. Some of the absorption lines arise in the atmospheres of stars while others arise from cool gas within the galaxy. The emission lines (when present) arise from hot diffuse gas heated by young stars. All these spectral features can be used to determine the galaxy recession velocity via the Doppler redshift.

$$\Delta\lambda/\lambda = \Delta\nu/\nu = \sqrt{\frac{(1+v/c)}{(1-v/c)}} - 1$$

which for $v \ll c$ reduces to

$$\boxed{\Delta\lambda/\lambda = \Delta\nu/\nu = v/c.}$$

The general expression for the redshift, z, of a galaxy, in terms of the observed (λ_{obs}) and emitted (λ_{em}) wavelengths of a particular spectral feature is, as we have seen,

$$\lambda_{obs} = (1+z)\lambda_{em}.$$

For galaxies at cosmological distances, astronomers refer to galaxies by their redshifts rather than the more archaic 'recession velocity'. Remember that, in general, the observed redshift comprises a combination of the cosmological expansion and any local perturbations from it (so-called 'peculiar velocities').

5.2 Measuring Galaxy Distances

Measuring distances is much more difficult. Direct geometrical methods such as parallax can only be used for very nearby objects. To get to the large distances requires the use of **standard candles**.

A standard candle is simply an object (e.g. a star, supernova or galaxy) whose luminosity we believe we know as a result of measurement that does not depend on its distance. One of the most imporant standard candles are Cepheid variables. These giant pulsating stars vary in brightness with a period that is dependent on their mean luminosity.

Thus if we measure their pulsation period we can deduce their luminosity and then compute their distance from their measured flux, f, using the inverse square law

$$\boxed{f = \frac{L}{4\pi d^2}.}$$

In practice the Cepheid Period-Luminosity relation must first be calibrated, i.e. measured empirically. This can be done directly for a small sample of Cepheids whose distance have been measured geometrically by the HIPPARCOS satellite.

We now know the distance to the cluster of which the Cepheid star is a member. Using all the stars in the cluster we can construct Hertzsprung-Russell diagrams (Luminosity verus Effective Temperature) and so defines the position of the **main sequence**. Then other star clusters (that contain additional Cepheids) can have their distances determined by matching the main sequence of the other stars in the cluster with that of the calibrated H-R diagram. This allows distances to be established for an extensive set of Cepheids.

With the period-luminosity relation of Cepheids firmly calibrated, we are now in a position to measure our first extragalactic distances. However, it is hard to construct a meaningful Hubble diagram from nearby galaxies because their motion is influenced by local galaxy clusters (such as the Virgo cluster) in addition to the Hubble flow. Cepheid variables are too faint to be observed in more distant galaxies and so another step in the distant ladder is needed.

Two important secondary standard candles that are used on the scale of the Hubble flow are Type Ia supernovae and spiral Tully-Fisher galaxies.

• Type Ia supernovae are believed to occur when a **white dwarf** stars in a binary systems accrete so much mass that the degenerate electron pressure which supports them is no longer capable of resisting gravity. The star collapses to form a much denser **neutron star** and the gravitational energy released powers a huge supernova explosion. Type Ia supernovae all have the same peak luminosity (there is a slight dependence on other measurable properties that is taken account of in the most accurate work). The peak SN luminosity has been calibrated by observing SN in galaxies (or clusters of galaxies) that also contain Cepheids.

• Tully-Fisher galaxies are spiral galaxies in which the rotation speed of the stars and gas about the centre of the galaxy can be measured. The rotation speed is found to correlate with total galaxy luminosity and this relation has been calibrated by a few galaxies (or clusters of galaxies) that also contain Cepheids.

Thus measuring distances to distant galaxies depends on several steps in the **distance ladder**. If there is an error in any one step then the distances will be systematically too large or too small.

5.3 Review of the HST Key project and H_0 result.

The HST key project used the uniquely high spatial resolution of the **Hubble Space Telescope** to detect individual Cepheid variables in a sample of nearby ($< 20\,\mathrm{Mpc}$) galaxies. These galaxies were then used to calibrate several secondary distance estimators including Type Ia SN and Tully-Fisher galaxies.

The distance to these nearby Cepheid containing galaxies is determined relative to the distance of the LMC (a small companion of the Milky-Way galaxy) which has a large, well studied population of Cepheids. The distance to the LMC is taken to be 50kpc and is determined by a variety of methods including using RR Lyrae variables and the tip of the red giant branch . The velocities of these nearby galaxies are significantly perturbed from the Hubble flow by local gravitational forces. Thus for this sample one has to make a rather uncertain flow correction before a diagram of Hubble distance versus velocity can be plotted.

The Type Ia SN and Tully-Fisher galaxies can be observed to much larger distances where perturbations from the Hubble Flow become unimportant. Thus once calibrated by the nearby members that contain Cepheids these samples produce much tighter correlations.

The result is

$$72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

http://ned.ipac.caltech.edu/level5/Sept01/Freedman/Freedman_contents.html

The remaining uncertainty is dominated by systematic uncertainties such as:

i) uncertainty in the distance to the LMC
ii) uncertainty in size of dust extinction corrections
iii) uncertainty in effect of varying metallicity on the Cepheid P-L relation

Using a more recent calibration of the Cepheid period-luminosity relation from the Hiparcos satellite, van Leeuwen et al. 2007 obtain $76 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. The current value, derived from the latest HST Cepheid results (Riess et al. 2016) gives $73.24 \pm 1.74\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (http://arxiv.org/abs/1604.01424).

5.4 H_0 from Gravitational Lensing

The phenomenom of gravitational lensing potentially offers a direct way of measuring H_0 independent of the distance ladder.

The path of the light travelling to us from the distant quasar is bent by the gravitational field of an intervening massive galaxy. This can result in light reaching us by two differnt routes, producing two images of the same quasar.

One can measure all the angles and the redshifts of the lensing galaxy and quasar. Then if one **assumes** a value for H_0 one can work out the difference in the two optical path lengths. The length of this path difference is proportional H_0 as it is H_0 that sets the scale of the diagram.

Now if the quasar luminosity varies with time (many of them do) one can study the light curves of the two separate images and measure the time lag, Δt between the curves.

Since the optical path difference is $c\Delta t$ one can solve to find H_0. In practice there is some uncertainty due to the fact that the lens is not a point mass and its mass distribution must be modelled, nevertheless one such measurement (for O957+561) yields

$$H_0 = 77 \pm 26 \text{ km s}^{-1}\,\text{Mpc}^{-1}$$

We will mostly adopt values in the range $70 - 75\,\text{km s}^{-1}\,\text{Mpc}^{-1}$. In some instances we will explicitly note the dependence of certain results on the Hubble constant.

As we shall see in Lecture 7, the Hubble constant (and many other cosmological parameters) can also be determined with high precision from measurements of the fluctuations across the sky in the cosmic microwave background (CMB) temperature.