Lecture 9:
Stellar structure –

Getting the energy out: sources of opacity

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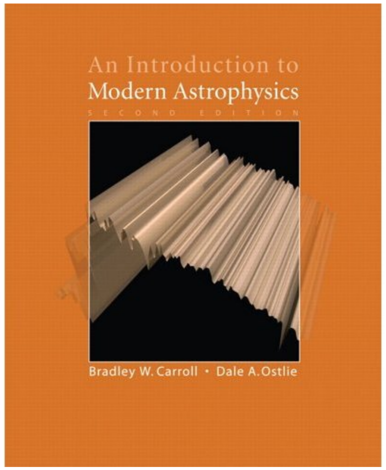
Chapters 9 and 10 of Carroll and Ostlie



Aims of lecture

Key concept: sources of opacity

<u>Aims:</u>

- Be able to describe the different sources of opacity and know when they are important in stars:

  bound bound, bound free, free free, and electron scattering

- Understand the definition of optical depth and the general form of opacity:

$$I_\lambda = I_{\lambda,0} e^{-\kappa_\lambda \rho s}$$

Optical depth: impact of opacity on intensity

$$\kappa = \kappa_0 \rho^\alpha T^\beta$$

General form of opacity

Opacity: a formal definition

Opacity is the resistance by material to the flow of radiation. The amount of opacity is dependent on the chemical composition, density, and temperature (which we will explore later). Importantly, opacity is wavelength dependent.

In most stellar interiors the opacity is determined by all of the processes that scatter and absorb photons, which (1) remove the photon from its original direction and/or (2) sometimes degrade the photon energy (the emission being split into >1 photons of lower energy or the photon energy going into the gas kinetic energy through particle interactions). Remember, the total energy is always conserved in any given process.

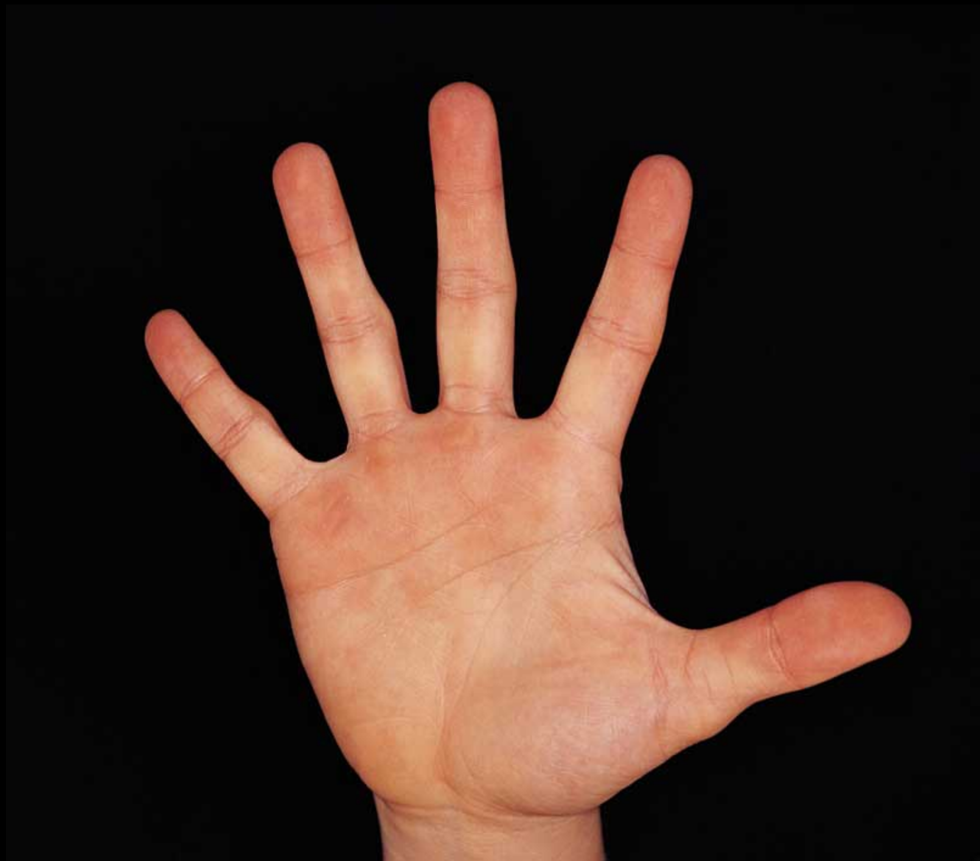
If I_λ is the photon intensity at wavelength λ then the change in intensity (dI_λ) over the distance ds is calculated from the opacity (κ_λ; units of $m^2\ kg^{-1}$, explored here) and the gas density (ρ) as

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds$$

We will explore the different sources of opacity in this lecture. Note the wavelength dependence on opacity and photon intensity. Recall from lecture 8 that opacity is a driver of energy transfer.

Demonstration that opacity is wavelength dependent

Optical



X-ray

Optical photons cannot pass through my hand (it is too opaque: optically thick) while X-ray photons can – and you can see my broken metacarpal bones 4 and 5.

Opacity and optical depth

We can therefore determine the final intensity of the beam of photons by integrating through the column density of gas:

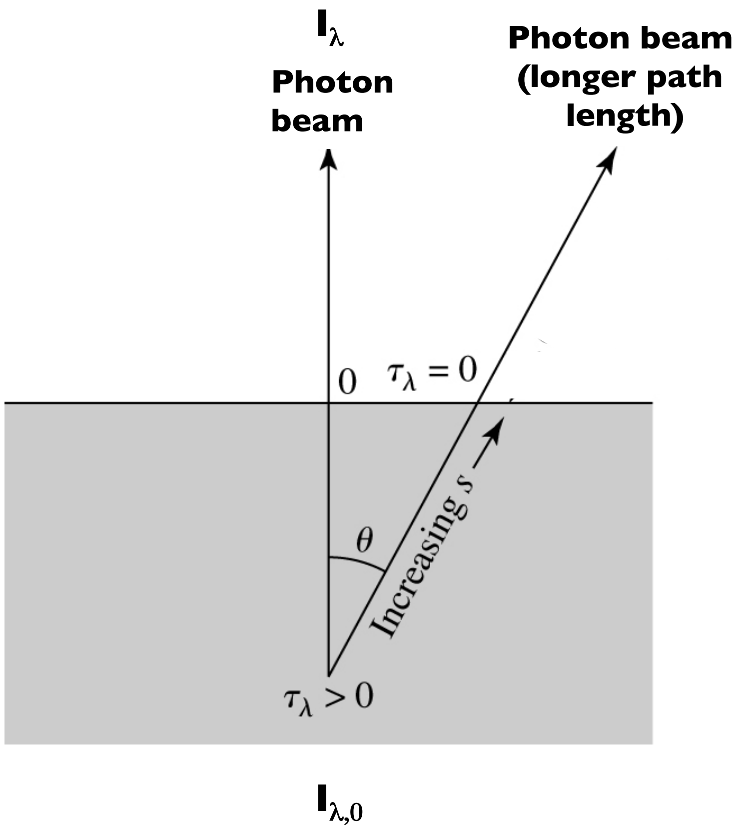
$$\int_{I_{\lambda,0}}^{I_{\lambda,f}} \frac{dI_\lambda}{I_\lambda} = -\int_0^s \kappa_\lambda \rho ds$$

The observed intensity will therefore be

$$I_\lambda = I_{\lambda,0} e^{-\int_0^s \kappa_\lambda \rho ds}$$

Which for the case of a uniform gas density is

$$I_\lambda = I_{\lambda,0} e^{-\kappa_\lambda \rho s}$$ **Equation 19**



The exponential term is often referred to as the optical depth; i.e.,

$$\tau_\lambda = \kappa_\lambda \rho s$$

τ_λ is related to the photon mean-free path – τ_λ>1 then the gas is optically thick; τ_λ<1 then the gas is optically thin

The energy isn't lost - it is scattered or emitted at other wavelengths

Opacity: the various sources

The main sources of opacity in a star are the following:

(1) Bound-bound transitions

(2) Bound-free (and the inverse, free-bound) transitions

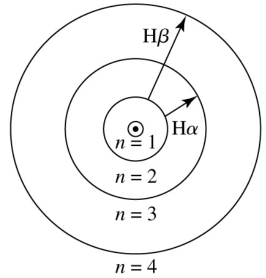
(3) Free-free (and the inverse, Bremsstrahlung) emission

(4) Electron scattering

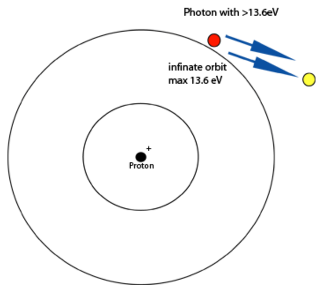
Two classes of opacity: (1) absorption (photon energy lost to KE of the gas or degraded) and (2) scattering (photon reemitted in a different direction – sometimes photon energy degraded)

Opacity: a schematic overview

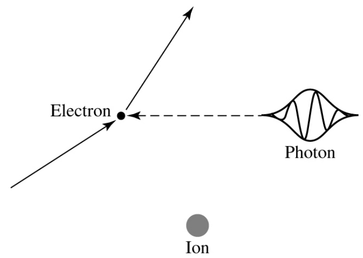
Bound bound



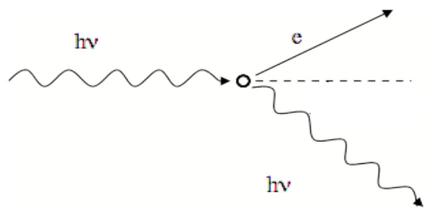
Bound free



Free free



Electron scattering



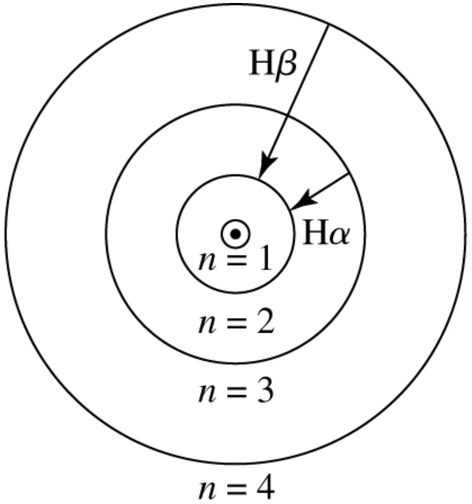
The total opacity is the combination of all of these factors:

$$K = K_{bb} + K_{bf} + K_{ff} + K_{es}$$

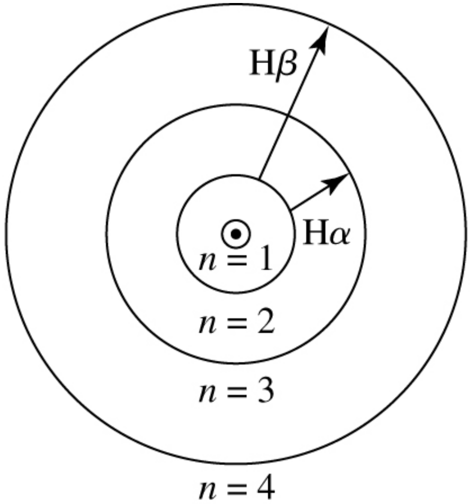
(I) Bound-bound transitions

We came across this in lecture 2: the excitation states of atoms (Boltzmann)

Hydrogen atom:



Emission



Absorption

Electron orbital energies

Typically ~1-10 eV which are ~1000 times less than the average photon energies in the stellar interior (T~10^6 K):

$$h\nu_{max} = 2.82kT$$

Wien equation (energy form)

Bound-bound transitions occur when photons of order ~eV are absorbed (i.e., the typical electron orbital energies); however, some heavier elements have much higher keV transition levels.

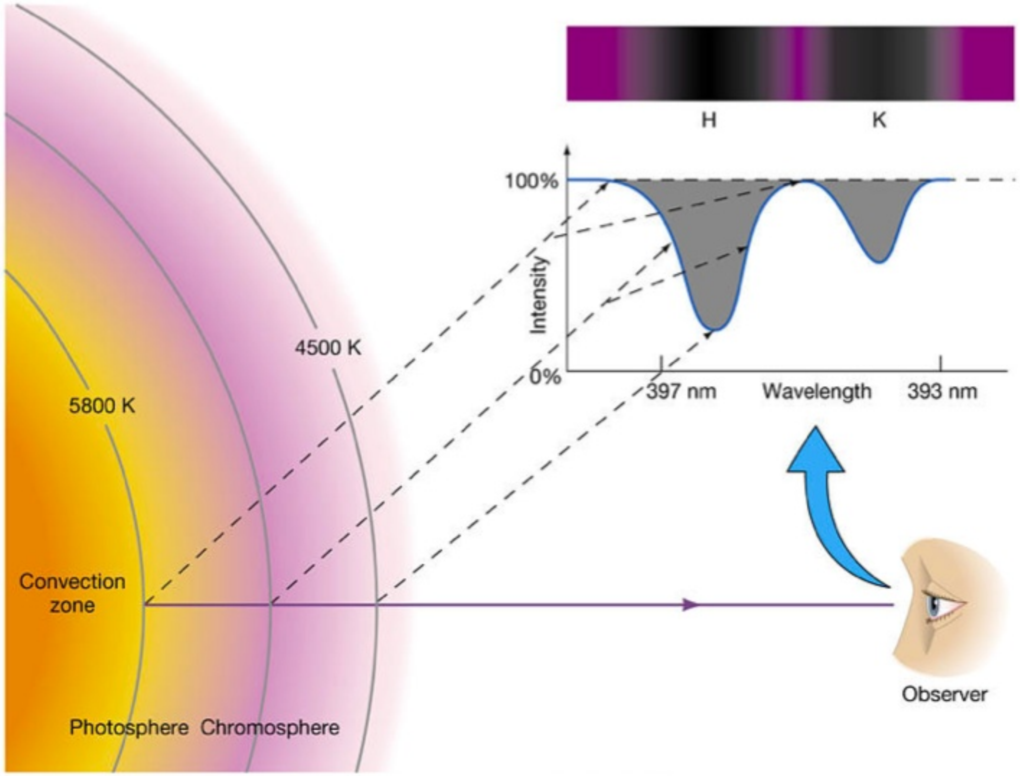
Consequently, bound-bound transitions are most common at "low" temperatures (T<10^5 K; see Boltzmann equation in lecture 2) and they are therefore a common form of opacity in stellar atmospheres but less so in stellar interiors (particularly stellar cores) where the atoms are ionised.

Line opacity – opacity at discrete wavelengths

Demonstration of line opacity

In bound-bound transitions only photons corresponding to allowed electron orbital energies can be absorbed (therefore very specific wavelengths).

If the electron then de-excites down to the original level then the emitted photon will be at the same energy as the absorbed photon but it will be emitted in a random direction (hence the absorption lines seen in stellar spectra; see below and lecture 2).



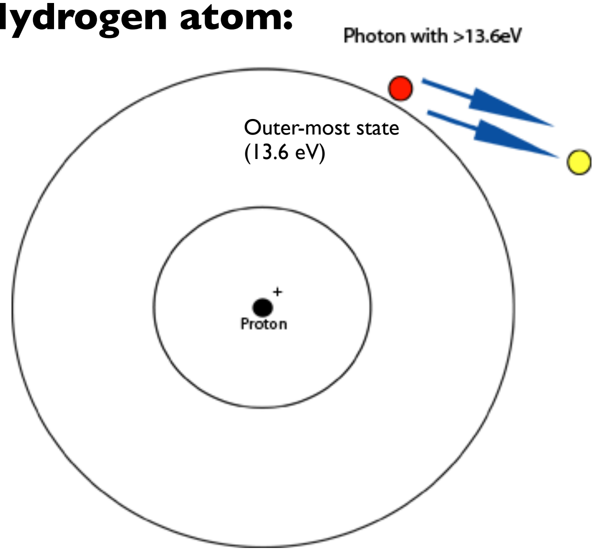
However, often the electron will cascade down the different orbital energies causing multiple photons to be emitted at energies less than the original photon - this is a clear case for degradation of photon energies.

Electrons can be collisionally de-excited (atoms colliding): the photon energy goes into kinetic energy of the gas

(2) Bound-free transitions

We came across this in lecture 2: photon ionisation of atoms (photo-ionisation: Saha)

Hydrogen atom:



$E \geq \chi_n$ therefore $\lambda \leq \dfrac{hc}{\chi_n}$

where χ_n is the ionisation energy

Excess photon energy will contribute to the electron kinetic energy:

$$h\nu = \chi_n + \frac{1}{2} m_e V^2$$

In bound-free transitions the original photon is removed from the radiation field, which is given to the electron and atom (hence the opacity).

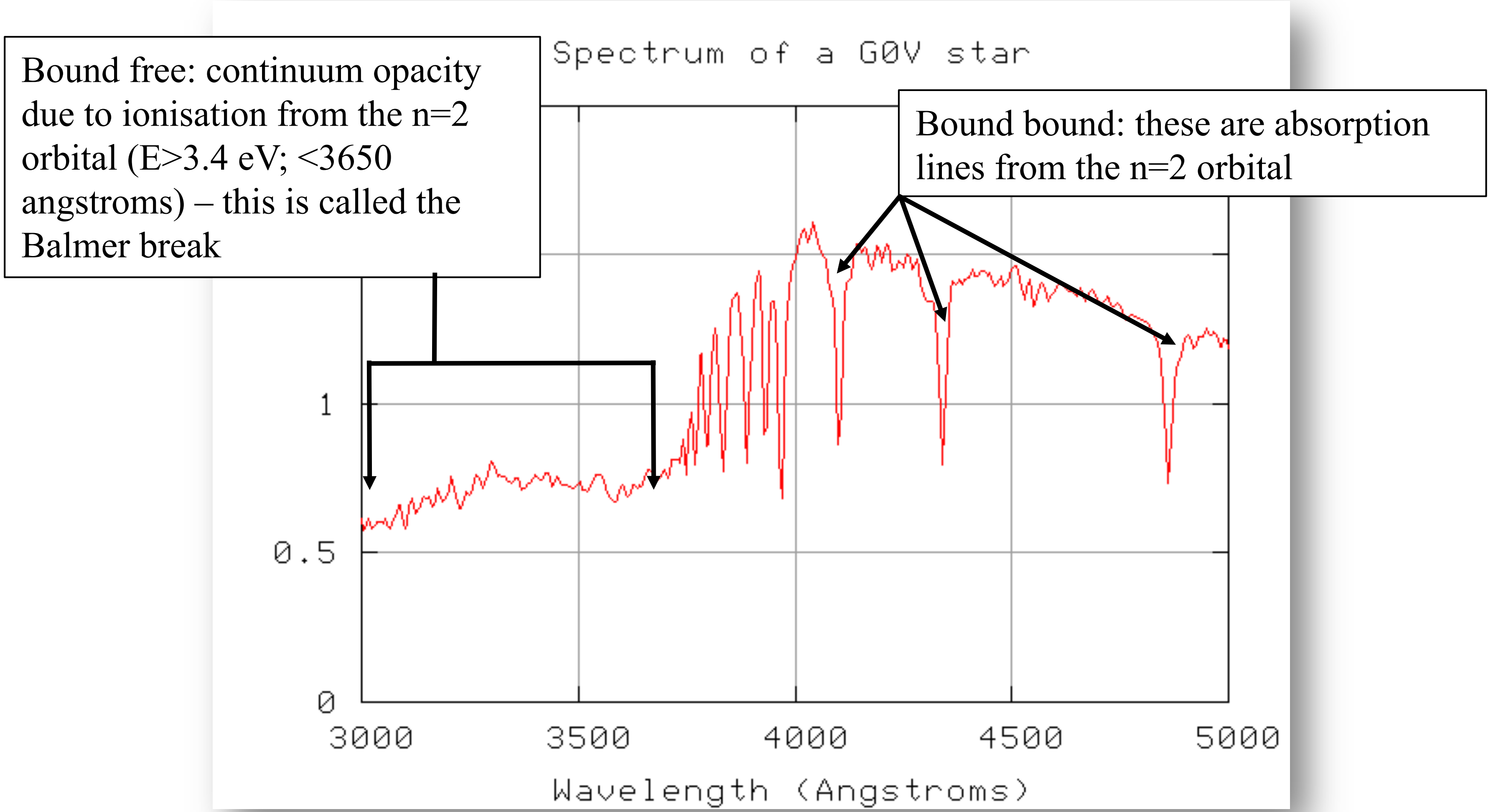
Bound-free transitions are most common at moderate temperatures when a large fraction of the atoms are only partially ionised ($T \sim 10^4$-10^6 K; see Saha equation in lecture 2).

The reverse process is called free-bound transitions ("recombination"): an electron is captured and photons are emitted as it cascades down through the orbital energies

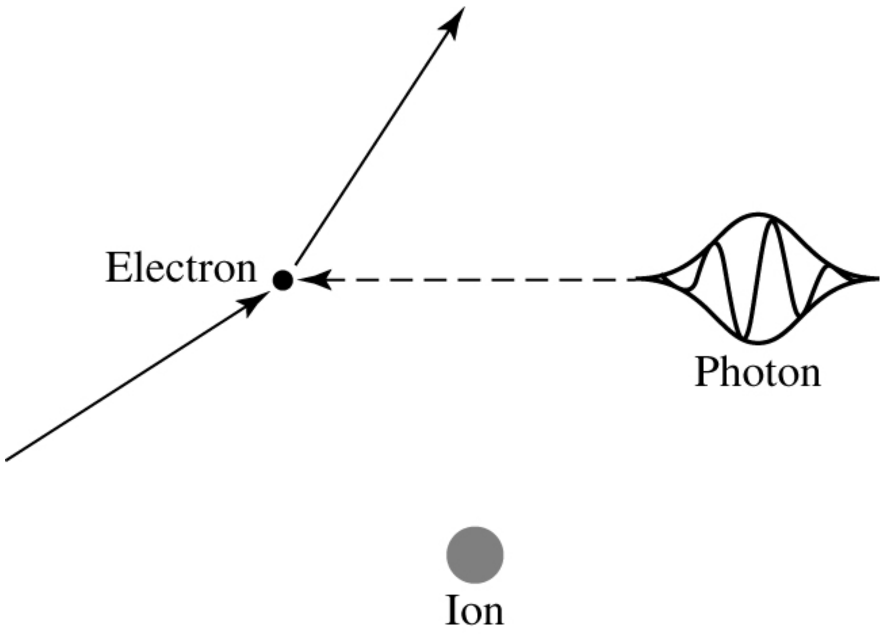
Continuum opacity: opacity at continuous wavelengths

Demonstration of continuum (and line) opacity

Unlike bound-bound transitions, bound-free transitions are not at discrete wavelengths. Therefore the opacity reduces the continuum emission from the star (the black-body emission) for energies above the ionisation energy (below the corresponding wavelength)



(3) Free-free transitions



The interaction between a free electron, a photon, and an ionised atom (i.e., positive charge)

This causes the electron to gain velocity at the expense of the photon (i.e., the photon is removed from the radiation field, hence the opacity)

The ionised atom is required to conserve energy and momentum

The energy from the photon is given to the kinetic energy of the electron (increasing its velocity from v_i to v_f):

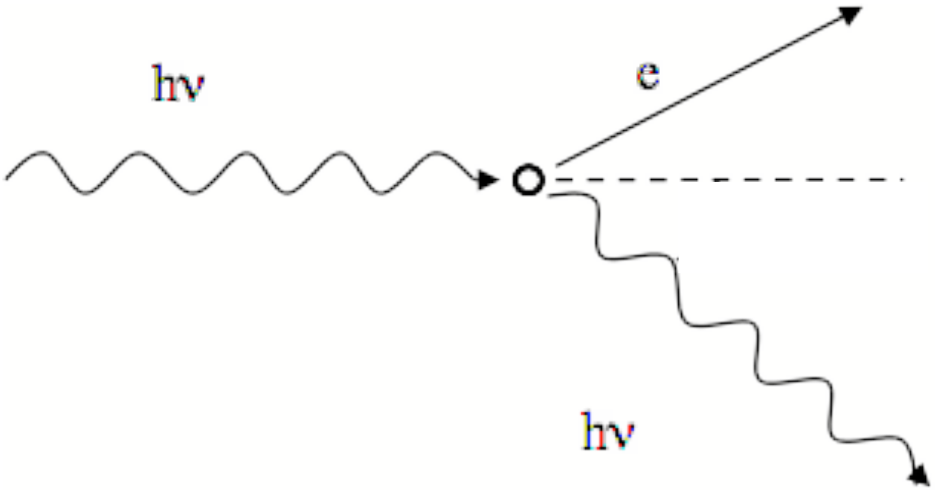
$$\frac{1}{2}m_e v_i^2 + h\nu = \frac{1}{2}m_e v_f^2$$

Free-free transitions can occur across a continuous range of wavelengths. Free-free transitions are most effective at moderate temperatures in stellar interiors when atoms are **partially** ionised since there is a high availability of electrons close to ions. Therefore there is a connection of free-free with bound-free transitions since it enhances the electron density in the vicinity of an ion.

The reverse process is Bremsstrahlung emission ("braking radiation"): the electron is decelerated and emits a photon with a wavelength equal to the electron energy loss

Continuum opacity: opacity at continuous wavelengths

(4) Electron scattering



The photon is scattered due to an interaction with an electron (and so is removed from the original path, hence the opacity)

The scattering is based on the Thomson cross section (cross section of an electron); see below

Thomson cross section:

$$\sigma_T = \frac{1}{6\pi\varepsilon_0^2}\left(\frac{e_c^2}{m_e c^2}\right)^2 = 6.65\times10^{-29}\,m^2$$

m_e is the electron mass (9.109×10^{-31} kg)
e_c is the elementary electrical charge (1.6022×10^{-19} C)
ε_0 is the permittivity of free space (8.8542×10^{-12} F m^{-1})

The related Compton scattering process occurs at the highest temperatures (T>10^8 K) when an electron loosely bound to an atom: key difference is the electron gains energy at expense of the photon

Electron scattering is important at high temp (T≥10^6 K) when other forms of opacity are removed; i.e., when atoms are completely ionised and there is a high electron density: no temp dependence

Continuum opacity: opacity at continuous wavelengths

The Rosseland mean opacity

Removing the wavelength dependence of opacity

As mentioned earlier, the amount of opacity is dependent on wavelength, chemical composition, density, and temperature. Since the radiation field at any given region in a star is close to that of a black body, with the temperature increasing with decreasing radius towards the centre, it is possible to average over all frequencies and produce a mean opacity that is dependent on temperature, density, and chemical composition but not wavelength. **This is performed for convenience in stellar models so don't have to track individual photons:**

$$\frac{1}{\kappa} = \int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu}{dT}\,d\nu \Bigg/ \int_0^\infty \frac{dB_\nu}{dT}\,d\nu$$

This is referred to as the "Rosseland mean opacity", named after Svein Rosseland. The opacity defined in this way is characterised as:

$$\kappa = \kappa(\rho, T, composition)$$ **[no wavelength/frequency dependence]**

$$\kappa = \kappa_0 \rho^\alpha T^\beta$$ **Equation 20**

Rosseland mean opacity: the functional form

Mean opacities for different processes:

κ_{bb} There is no simple equation for bound-bound transitions that describes all contributions to the opacity by individual spectral lines (but see last slide)

$\kappa_{bf} = \kappa_0 \rho T^{-3.5}$ where $\kappa_0 = 4.34 \times 10^{21} Z(1+X) \frac{g_{bf}}{t}$ $m^2\ kg^{-1}$

$\kappa_{ff} = \kappa_0 \rho T^{-3.5}$ where $\kappa_0 = 3.68 \times 10^{18} (1-Z)(1+X) g_{ff}$ $m^2\ kg^{-1}$

$\kappa_{es} = \frac{\sigma_T n_e}{\rho}$ which is $\kappa_{es} = 0.02(1+X)$ $m^2\ kg^{-1}$

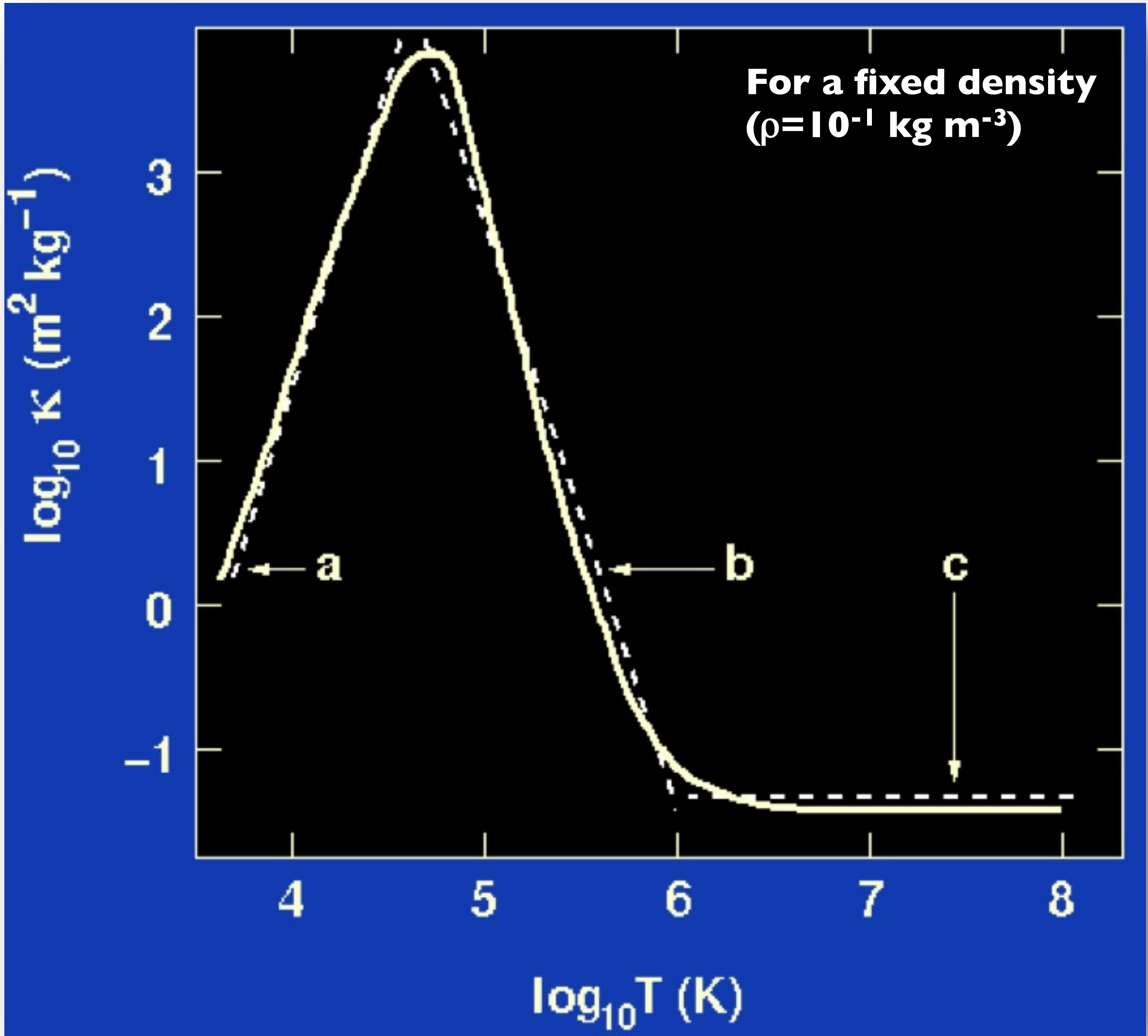
where X and Z are the mass fractions of Hydrogen and metals (anything heavier than Helium), respectively, and g and t are the Gaunt and guillotine factors which are corrections related to the state of the atom (see section 9.2 of CO book for details).

No need to remember the details of these equations

The total opacity is the combination of all of these factors:

$$\kappa = \kappa_{bb} + \kappa_{bf} + \kappa_{ff} + \kappa_{es}$$

Opacity: temperature and density dependence



Rise from a:

$$\kappa = \kappa_0 \rho^{0.5} T^4$$

Drop to b:

$$\kappa = \kappa_0 \rho T^{-3.5}$$

Flat gradient to c:

$$\kappa_{es} = \frac{\sigma_T n_e}{\rho}$$

This figure shows the opacity for a star of a given density ($\rho = 10^{-1}$ kg m^{-3}): solid curve is from detailed opacity calculations and the dotted lines are approximate power-law forms: need to calculate these curves for a range of densities to fully model opacity.