Statistical Physics: Workshop Problems 6

(1) (a) Explain in what circumstances Maxwell-Boltzmann, Fermi-Dirac and Bose-Einstein statistics are appropriate making reference to the number of particles that can occupy energy states and their degeneracies.

(b) Give a physical discussion on why differences in the three statistics become unimportant at high temperatures.

(c) At what temperature (order of magnitude) will quantum statistics have to be used for a system containing $10^{18}/\mathrm{cm}^3$ of neutrons?

(2) Consider a material where each of the constituent atoms may occupy one of two sites. For each atom the sites have energy ϵ_i and $-\epsilon_i$, where i labels the atom.

(a) If the energy levels are the same two values for all of the atoms (ϵ and $-\epsilon$ say) calculate the contribution of the atoms to the heat capacity of the material.

(b) Now return to the case where the energies are atom dependent (ϵ_i and $-\epsilon_i$). Find the behaviour of the low temperature heat capacity (i.e. $k_B T \ll \epsilon$). Express the result as both a sum and an integral (do not attempt to evaluate either).

(3) The quantum energy levels of a rigid rotor of mass m and length a are

$$\epsilon_j = j(j+1)\frac{h^2}{8\pi^2 m a^2}$$

where $j = 0, 1, 2, ...$ The degeneracy of each level is $g_j = 2j + 1$.

(a) Find the general expression for the partition function and show that at high temperatures it can be approximated by an integral (and evaluate the integral).

(b) Evaluate the high temperature energy and heat capacity.

(c) Find a low temperature approximation for the partition function and hence the low temperature internal energy and heat capacity.

(4) The electronic energy levels of a hydrogen atom are given by $E_n = -E_0/n^2$ where E_0 is a constant and $n = 1, 2, 3...$ and the degeneracy of the states are $g_n = 2n^2$.

(a) Write down the expression for the electronic partition function for a single isolated hydrogen atom at temperature T (do not try to evaluate the sum). Does the expression diverge for $T = 0$ and what about for $T \neq 0$?

(b) Is this divergence caused by the chosen zero of energy? [Add some constant E' onto the expressions for energy levels].

(c) Calculate average thermal energy,

$$\langle E \rangle = \frac{\sum_i E_i e^{-E_i/k_B T}}{Z},$$

of the system.

(d) We seem to have non-physical results. Can you explain what is going on? Consider what happens when an atom is confined to a large finite volume L^3 in a quantum calculation of the full partition function (you don't need this full calculation, just think degeneracies and energy levels).