Lecture 6: Weighing the Universe

Determining the mass density of the Universe and hence whether it will expand forever or recollapse, has been one of the major goals of extra-galactic astronomy for many decades.

6.1 The mass density in stars

[Liddle sec:8.1/9.1]

The optical and near infra-red luminosity of galaxies is the result of the summed luminosities of all the stars they contain. It is quite straightforward to carry out a census and calculate the spatial number density of galaxies as a function of luminosity. This distribution is known as the galaxy luminosity function.

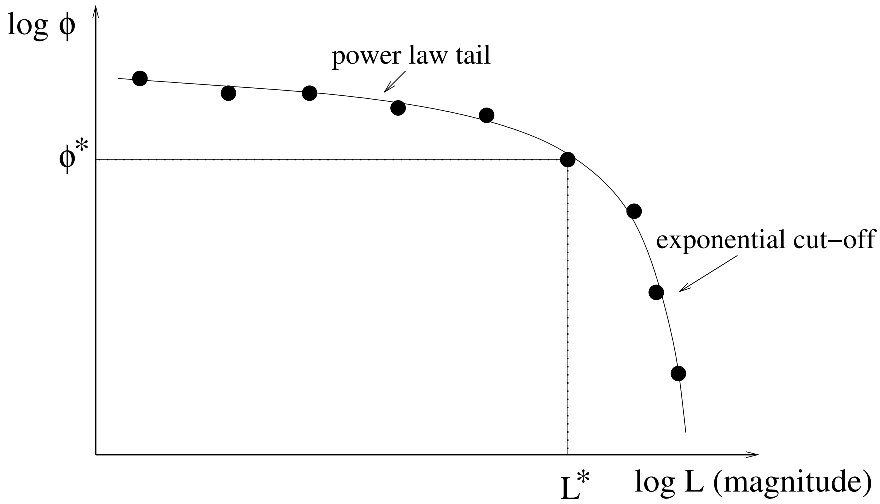

Figure 1: A Schechter function fit to the observed galaxy luminosity function.

One then integrates over luminosity to get the total luminosity density. In the near infra-red this exercise yields

$$j_{\rm NIR} = 4.3 \times 10^8 \, {\rm L}_\odot {\rm Mpc}^{-3}$$

If we knew the mass-to-light ratio of stars we could easily convert this luminosity density into a stellar mass density. Since massive stars produce more light per unit mass than low mass stars, the average stellar mass-to-light ratio depends on the distribution of stellar masses. This is not known precisely and results in an approximately factor of 20% uncertainty in appropriate M/L.

Adopting a conventional value of $M/L = 0.8 \, {\rm M}_\odot/{\rm L}_\odot$ gives

$$\rho_{\rm stars} = 3.4 \times 10^8 \, {\rm M}_\odot {\rm Mpc}^{-3}.$$

Expressed in terms of the critical density this is

$$\Omega_{\rm stars} \equiv \rho_{\rm stars}/\rho_{\rm crit} = 2.1 \times 10^{-3}$$

6.2 The mass in galaxies

There are other techniques which can be used to estimate the total mass in galaxies regardless of whether the mass is in stars or some other form.

6.2.1 Galaxy rotation curves

A galaxy rotation curve is a plot of the circular velocity of stars orbiting around the galaxy as a function of their distance from that galaxy's centre. Observationally such a plot is straightforward to construct. (A long slit spectrum can be used to measure the Doppler shift of the stellar spectra as a function of position.) Using Newton's 2nd law to balance the acceleration towards the centre of the circular motion with the gravitational force we have

$$\frac{V^2(r)}{r} = \frac{GM(<r)}{r^2}$$

Hence

$$M(<r) = \frac{V^2(r)r}{G}.$$

(Here we have assumed the galaxy mass distribution is spherically symmetric. This assumption makes very little difference to the result.)

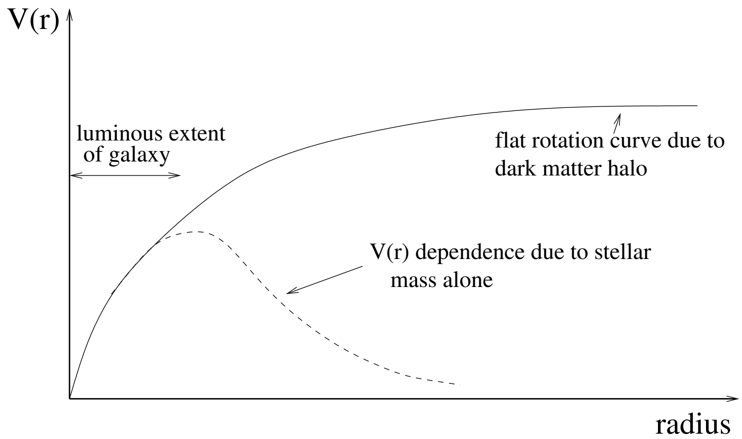

Figure 2: The rotation curve of a typical spiral galaxy.

Typical galaxy rotation curves become flat in their outer parts. This implies that in these outer parts the total mass of the galaxy is increasing as $M \propto r$. On the other hand the total luminosity of a galaxy converges very quickly with very little being contributed by its outer parts. Hence this implies that galaxy mass-to-light ratios are an increasing function of radius $(M/L)_{\rm gal} \propto r$. At scales of $r \approx 100$kpc the inferred mass-to-light ratios are 50 to 100 $M_\odot/L_\odot$.

Such mass-to-light ratios are much greater than can be accounted for by normal stars. This implies that the mass in the outer parts of galaxies is dominated by some form of **dark matter halo**.

6.2.2 Galaxy Clusters

Another constraint on the $(M/L)_{\mathrm{gal}}$ comes from studying the motions of galaxies in rich galaxy clusters. A rich galaxy cluster consists of upto several hundred galaxies orbiting around each other.

These systems appear to be relaxed and stable. This enables their masses to be estimated by a variety of methods.

Virial Theorem

The mutual gravitational binding energy of the galaxies in the cluster must be greater than their kinetic energies as otherwise these systems would quickly disperse.

The measurable properties of the galaxies in these clusters are their angular positions, luminosities and line of sight velocities. If we assume each galaxy has mass M_i we can use this information to estimate both the kinetic energy

$$\mathrm{KE} = \sum_i 1/2 M_i v_i^2$$

and gravitational potential energy

$$\mathrm{PE} = \sum_i \sum_{j>i} \frac{G M_i M_j}{|r_i - r_j|}.$$

If we a assume a value for $(M/L)_{\mathrm{gal}}$ these quantities can be estimated and compared. One can set a lower limit on $(M/L)_{\mathrm{gal}}$ by demanding that the clusters are bound systems

$$\mathrm{KE} < \mathrm{PE}$$

or estimate $(M/L)_{\mathrm{gal}}$ by assuming the cluster to be relaxed systems that satisfy the **virial theorem**

$$2\,\mathrm{KE} = \mathrm{PE}.$$

The result of such estimates is that $(M/L)_{\mathrm{gal}} \approx 100 M_{\odot}/L_{\odot}$. Combining this with the observed luminosity density implies

$$\Omega_{\mathrm{gal}} \approx 0.2$$

Hydrostatic Equilibrium

Clusters of galaxies also contain a large mass of hot intra-cluster gas. It is the pressure of the gas that supports it and balances the gravitational force that is pulling the gas towards the centre of the cluster.

Approximating the cluster as being spherically symmetric with a gas density profile $\rho(r)$ and pressure profile $P(r)$, the net radially-outward pressure on a shell of gas lying between radius r and $r + dr$ is $-\frac{dP}{dr}dr$. This must balance the net radially-inward gravitational force per unit area of shell of $GM(< r)\rho_{gas}dr/r^2$, giving

$$\frac{1}{\rho_{\rm gas}(r)}\,\frac{dP}{dr} = -\frac{GM(< r)}{r^2}.$$

Thus if we could measure the density, $\rho_{\rm gas}$, and pressure, P, of the gas we could solve this equation and determine the total mass of the cluster interior to any chosen radius, $M(< r)$.

Fortunately this is possible as the hot gas also emits X-rays via thermal bremsstrahlung, $\epsilon(r) = {\rm const.}n(r)^2T^{1/2}$, where T is temperature and n the particle density. Thus a spectrum of the X-ray emission can be used to measure the temperature, T, and then the projected X-ray surface brightness used to determine the particle density $n(r)$. Finally this can be combined with the ideal gas equation to give the pressure $P(r) = n(r)kT$. This equation can also be written as $P = \rho_{gas}kT/\mu m_H$, where μ is the mean mass per particle in the plasma, in units of the proton mass m_H and ρ_{gas} is the mass density; for a pure ionized hydrogen plasma, $\mu = 0.5$. Note in general that T, P, n, ρ_{gas} can all be functions of radius in the cluster.

This method assumes both spherical symmetry and equilibrium. It gives cluster masses comparable to those given by the virial theorem.

Weak gravitational lensing

The gravitational field of the massive galaxy clusters deflects the path of passing light.

This affects the shapes of distant background galaxies, distorting their images into arcs and arclets. These arcs can be analysed and the projected mass of the lens computed.

This method measures the total projected mass of the cluster regardless of whether it is in equilibrium or not. For several clusters both X-ray and lensing inferred masses are available and in general agree well.

6.3 The total mass density

[Liddle sec:6.1]

The above estimates of $\Omega_{\rm gal}$ only estimate the mass density that is associated with galaxies and clusters of galaxies. If the mass does not trace light, i.e. if there is an additional mass component that is more smoothly distributed then there could be a lot of unaccounted mass in between the galaxies. Consequently $\Omega_{\rm gal}$ could just be a lower limit on the total mass density $\Omega_{\rm m}$

6.4 What is the Dark Matter in the Milky Way

Note $\Omega_{\rm gal}$ is much greater than $\Omega_{\rm stars}$. This means that most of the mass in the dark matter halos around the Milky Way and other galaxies is not in the form of stars. Possible candidates include:

i) Stellar remnants such as white dwarfs, neutron stars or black holes.

ii) Failed stars. That is brown dwarf stars or Jupiters that don't have enough mass to reach high enough central temperatures to initiate nuclear reactions.

iii) Cold neutral molecular gas.

iv) Non-baryonic dark matter. For example, massive neutrinos or a more exotic particle physics candidates such as axions or the lightest super-symmetric particle.

Worked Examples

6.1 **The Virial Theorem.** A group of galaxies contains 10 galaxies with line-of-sight velocity dispersion 300 km/s and has a radius of 500 kpc. Estimate the mass of the system, assuming spherical symmetry and that all the galaxies have the same mass.

If the galaxies have an average luminosity of $4 \times 10^{10}\, L_\odot$, what is their average mass-to-light ratio?

6.2 **Hydrostatic Equilibrium.** Estimate the mass of a cluster if it contains an X-ray emitting plasma with $T = 10^8\, K$ and a maximum extent $r_{\rm max} = 1$ Mpc. Assume the gas is isothermal, has a gas density profile

$$\rho_{\rm g}(r) = A \left(\frac{r}{r_{\rm max}} \right)^{-2}$$

(where A is a constant with the units of density) and is composed entirely of ionised hydrogen.

If $A = 1.0 \times 10^{13}\, {\rm M_\odot Mpc^{-3}}$, estimate the mass of ionised plasma in the cluster.