L2 Foundation of Physics 2B Optics 2019-20

O.3 Summary: Paraxial waves and lenses:

Learning outcomes:

1. To define the **Fresnel approximation** of **paraxial optics** [Optics $f2f$ Sec. 2.13].

2. To write an equation for a paraxial spherical/cylindrical wave and introduce the concept of **wave front curvature** [Optics $f2f$ Sec. 2.14].

3. To introduce the concept of a **lens** [Optics $f2f$ Sec. 2.15–2.18].

Key equations: For both plane and spherical waves the phase change in moving a distance r' (not necessarily the same as the polar coordinate r) in the direction of propagation is kr'. For example, the distance between an input point $(x', y', 0)$ and an observation point (x, y, z) is

$$r' = [z^2 + (x - x')^2 + (y - y')^2]^{1/2} . \tag{1}$$

if $z \gg |x - x'|$ then we can use a binomial expansion to make the **Fresnel approximation**

$$\begin{aligned} r' &= z\left[1 + \frac{(x-x')^2 + (y-y')^2}{z^2}\right]^{1/2} \simeq z\left[1 + \frac{1}{2}\frac{(x-x')^2 + (y-y')^2}{z^2}\right] \\ r_{\rm p} &= z + \frac{(x-x')^2 + (y-y')^2}{2z} . \end{aligned}$$

The paraxial distance, $r_{\rm p}$, is used to write a paraxial form of the spherical wave. For a paraxial spherical wave emitted from the origin, we have $x' = y' = 0$, and

$$E = \frac{E_{\rm s}}{{\rm i}kr}{\rm e}^{{\rm i}kr} \simeq \frac{E_{\rm s}}{{\rm i}kz}{\rm e}^{{\rm i}kz}{\rm e}^{{\rm i}k\rho^2/2z} , \tag{2}$$

where $\rho^2 = x^2 + y^2$. The parameter after the 2 in the denominator of the ρ^2-term tells us the **radius of curvature** of the wavefronts which is equal to the propagation distance from the source, in this case z.

A **lens** imprints a phase that changes the curvature of a wave: The field immediately downstream of a thin lens placed in the $z = 0$ plane is

$$E^{(L)} = E^{(0)}{\rm e}^{-{\rm i}k\rho'^2/2f} , \tag{3}$$

where $E^{(0)}$ is immediately before the lens. A lens converts a paraxial spherical wave with radius of curvature s_1 (diverging) to another paraxial spherical wave with curvature $-s_2$ (coverging), where

$$\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f} . \tag{4}$$

Outlook: In the next lecture, we shall start to consider two waves.