Foundations of Physics 2B/3C 2019-2020

Thermodynamics – Workshop 3 Problems

Week Commencing 28th October

1. Entropy Calculations

This problem provides practice at calculating entropy changes. Note problem e) is very similar to c) and d) so you may wish to move onto question 2 first.

a) One mole of an ideal gas undergoes an isothermal expansion when placed in contact with a heat reservoir at $400\ \mathrm{K}$, so that the expanded volume is ten times that of the initial volume. What is the entropy change of the gas?

b) A $2\ \mathrm{kg}$ block of lead having heat capacity $C_p = 0.256\ \mathrm{kJ\ K^{-1}}$ at a temperature of $500\ °\mathrm{C}$ is dropped into the River Wear on an autumn day, when its temperature is $10\ °\mathrm{C}$. What is the entropy change for both the lead block and the river, assuming that the river can be treated is an infinite heat reservoir and its temperature does not change?

c) Two equal amounts of water, having mass $10.0\ \mathrm{kg}$ and at temperatures of $90\ °\mathrm{C}$ and $10\ °\mathrm{C}$ respectively, are mixed and come to equilibrium. Water, has specific heat capacity given by $c_V = 4180\ \mathrm{J\ kg^{-1}\ K^{-1}}$. Calculate the following:
   i) The overall energy change of the system;
   ii) The final temperature reached;
   iii) The entropy changes of the hot water, cold water and the Universe.

d) A heat engine takes in heat in equal quantities from two hot reservoirs at $800\ \mathrm{K}$ and $1000\ \mathrm{K}$. If the cold reservoir of the engine is at $450\ \mathrm{K}$, what is the maximum possible efficiency of the engine?

e) A $10\ \mathrm{kg}$ block of ice, which has latent heat $L_f = 334 \times 10^3\mathrm{J\ kg^{-1}}$, and is at $0.0\ °\mathrm{C}$ falls into the North Sea, at $10\ °\mathrm{C}$, from a fishing trawler and melts. Once the ice has melted, the water warms, reaching thermal equilibrium with the sea water. What is the entropy change of each of the ice, atmosphere and Universe? Comment on your results and their significance.

2. Entropy and heat engines

Here we look at the maximum possible work that can be extracted from a heat engine cycle. It shows the Clausius Inequality in action.

a) A cold body of heat capacity, C, at temperature, T_L is joined to the environment (a reservoir) at a temperature, T_0 by a perfect, reversible engine.
   i) What is the maximum work, W_{max} that can be obtained from the engine?
   ii) Is this negative or positive and why? Is this always the case?

[*Hint*: $(x - \ln x - 1) > 0$ for all $x > 0, x \neq 1.$]

b) The work done by the engine in part (a) is stored in a suitable device, known as a regenerator, before being returned back to the engine at some later point, now acting as a fridge/heat pump, causing a further rise in the body's temperature to T_N.
   i) Show that the following relation must hold,
$$\frac{W_{max}}{C} = T_N - T_0\left[\ln\left(\frac{T_N}{T_0}\right) + 1\right].$$
   ii) Again, determine the sign of the work and convince yourself of its validity.