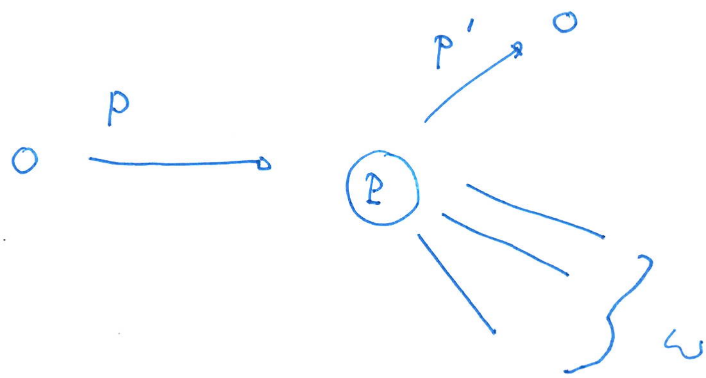
# Inelastic Scattering




Cross Section

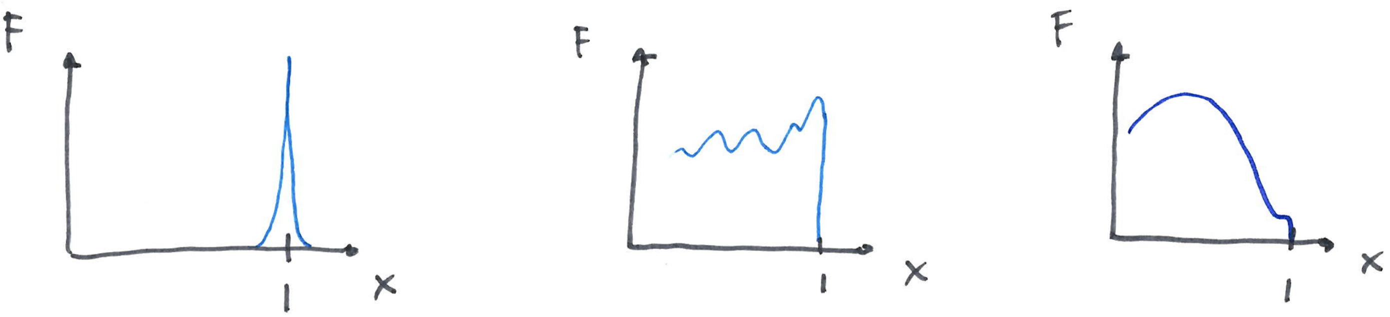
$$\frac{d^2\sigma}{dQ^2\,dx} = \frac{4\pi\alpha^2}{Q^4}\left[\frac{1}{x}\left(1-y-\frac{M^2y^2x^2}{Q^2}\right)F_2(x,Q^2) + y^2 F_1(x,Q^2)\right]$$

$$x = \frac{Q^2}{2P\cdot q}$$

$$x\cdot y = x_0 y_0 - \vec{x}\cdot\vec{y}$$

$$y = \frac{P\cdot q}{p\cdot P}$$ ← fractional energy loss

x-dependence (elasticity) of the structure functions




$Q^2 \ll 1/R^2$ ← proton radius

elastic

$Q^2 \approx 1/R^2$

inelastic

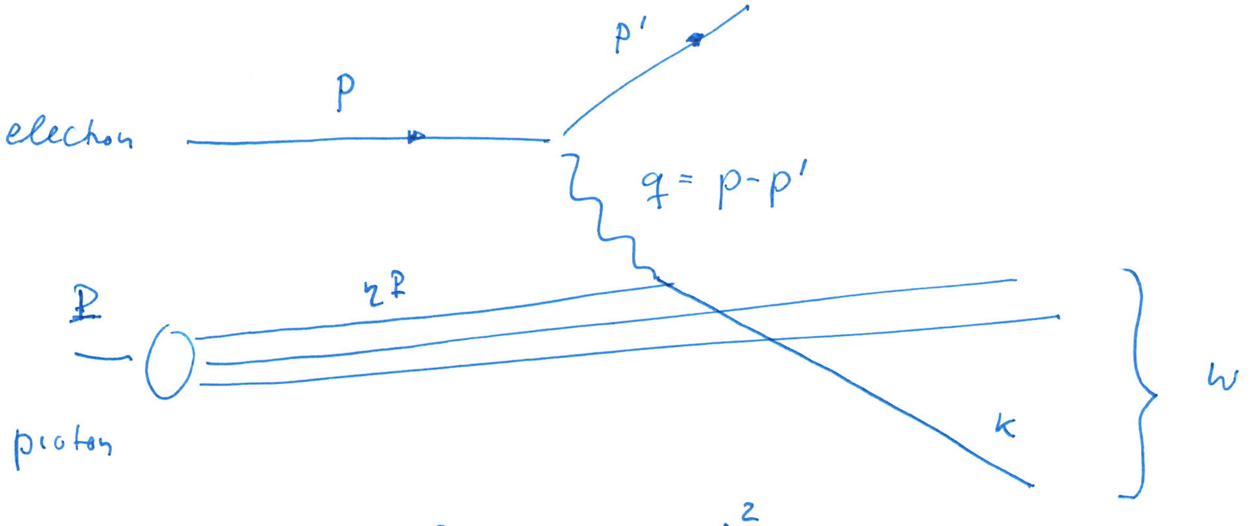
$Q^2 \gg 1/R^2$

deep inelastic scattering

## Parton model

The $Q^2$-independence of the structure functions imply a point-like substructure of the nucleons.

Assume the proton is made up from massless partons and the virtual photon interacts with a single parton that carries a momentum fraction $0 < \eta < 1$:




$$k^2 = 0 = (\eta P + p - p')^2 = (\eta P + q)^2$$

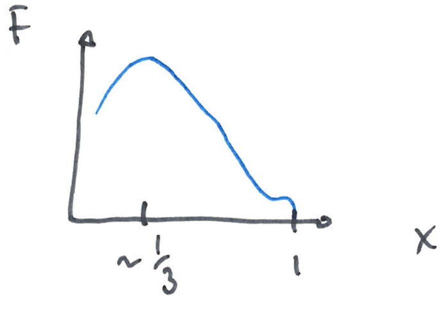
$$= \eta^2 M^2 + 2\eta P\cdot q - Q^2$$

$$Q^2 \gg M^2 \quad \approx 2\eta P\cdot q - Q^2$$

$$\Rightarrow \quad \eta = \frac{Q^2}{2P\cdot q} = x$$

Physical interpretation of the parameter $x$: it is the fraction of the proton momentum carried by the parton that is struck in the scattering.

From




Peak at $1/3$

$\Rightarrow$ 3 objects in the proton?

For spin-$\frac{1}{2}$ particles

$$F_2(x, Q^2) = 2x\, F_1(x, Q^2)$$

For spin 0 : $F_2 = 0$.

Express structure functions

$$F_2(x) = x \sum_{\text{constituents}} Q_c^2\, f_c(x)$$

parton distribution function

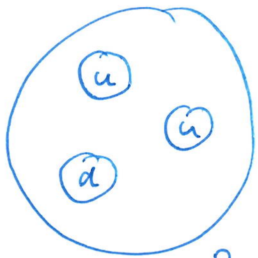
where $dn_c(x) = f_c(x)\, dx$

number of constituents of type c with momentum fraction in an infinitesimal interval $x \to x + dx$

$$\Rightarrow \quad \frac{d^2\sigma}{dQ\,dx} = \sum_{\text{const.}} f_c(x)\, \frac{d\sigma}{dQ^2}(p, xP, p')$$

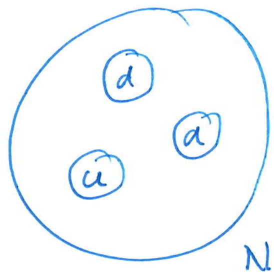
<u>Partons</u>

<u>Quarks</u> Spin $\frac{1}{2}$




Proton

$\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1$



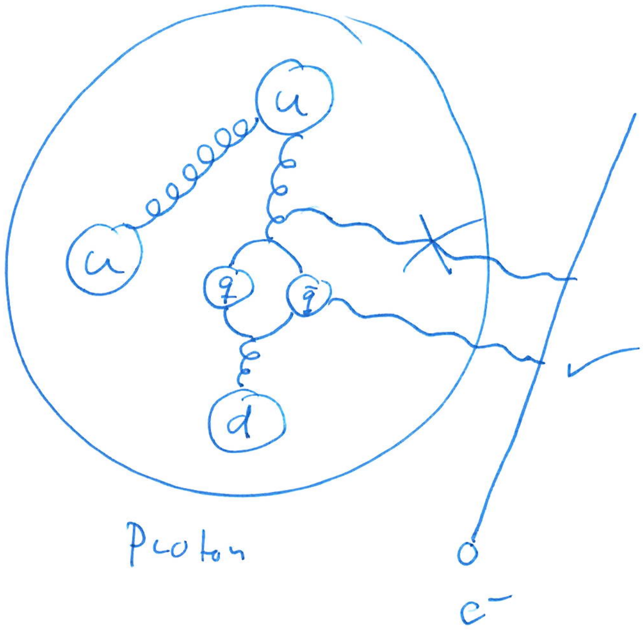

Neutron

$-\frac{1}{3} - \frac{1}{3} + \frac{2}{3} = 0$

| Symbol | name | charge |
|---|---|---|
| u | up | $+2/3$ |
| d | down | $-1/3$ |
| s | strange | $-1/3$ |
| c | charm | $+2/3$ |
| b | bottom | $-1/3$ |
| t | top | $+2/3$ |

## Gluons   Spin 1

Carriers of the strong force.
They play the same role for the strong force as photons play for the e/m force.
They are uncharged, but photons they interact with themselves.




## Sea quarks

Gluons keep the nucleons together by being exchanged between the quarks. During the exchange they can split into $q\bar{q}$ pairs. These quarks are called sea quarks.

## Parton distribution functions

For the proton at rest

$$\sum_{\text{constituents}} Q_q = +1 \qquad \sum_c x = +1$$

including sea quarks

$$\int_0^1 \sum_q Q_q f_q(x)\, dx = 1 \qquad \text{charge conservation}$$

and

$$\int_0^1 \sum_c x f_c(x)\, dx = 1 \qquad \text{momentum conservation}$$

Let us split the PDFs into valence and sea quark contributions

$$f_q(x) = f_q^v(x) + f_q^s(x)$$

$$\Rightarrow \int_0^1 dx \, [f_u(x) - f_{\bar{u}}(x)] = +2 \quad \text{for the proton}$$

$$\int_0^1 dx \, [f_d(x) - f_{\bar{d}}(x)] = +1$$

$$\int_0^1 dx \, [f_s(x) - f_{\bar{s}}(x)] = 0$$