Level 2 Stars

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Problem Set S.6

a) Initially, stars are formed through the condensation of dense molecular clouds that have a density of about 100 Hydrogen molecules cm^{-3} (i.e., H_2) and a temperature of about 10 K. Estimate the minimum mass such a cloud must have in order to become unstable. [2 marks]

Solution

Density of cloud = 100 x 2 x 1.67 x 10^{-27} kg cm^{-3}
= 3.34 x 10^{-19} kg m^{-3} *[1 mark]*

Using the Jeans Formula:

$$M_J \geq \left(\frac{5kT}{G\mu m_H}\right)^{\frac{3}{2}}\left(\frac{3}{4\pi\rho}\right)^{\frac{1}{2}}$$

$$M_{cloud} \geq \left(\frac{5\times1.38\times10^{-23}\times10}{6.67\times10^{-11}\times2\times1.67\times10^{-27}}\right)^{\frac{3}{2}}\left(\frac{3}{4\pi\times3.34\times10^{-19}}\right)^{\frac{1}{2}}$$

$$\geq 1.5\times10^{32}\ kg \quad or \quad > 73\ M_{sun}$$ *[1 mark]*

b) Given that typical stars have masses similar to that of the Sun, what does your answer from (a) suggest about the formation of stars? [2 marks]

Solution

The Jeans mass of 73 M_{sun} is much higher than that of the Sun which suggests that stars will form in associations with other stars. Indeed, as the initial gas cloud contracts individual regions will exceed the local Jeans mass and, as a result, they will break up to form smaller, star-sized clouds. *[2 marks]*

c) After some collapse, the cloud fragments to form protostars of average mass double that of the Sun. Assuming that the temperature of the cloud remains constant at 10 K, what is the ratio of densities between the initial cloud (in part a) and the fragmented cloud? [3 marks]

Solution

The density at the point at which the cloud fragments to 2 solar masses is:

$$\rho = \left(\frac{5kT}{G\mu m_H}\right)^3 \left(\frac{3}{4\pi}\right)\left(\frac{1}{M_J}\right)^2 \qquad \textit{[1 mark]}$$

Which gives a density of:

$$\rho = \left(\frac{5\times 1.38\times 10^{-23}\times 10}{6.67\times 10^{-11}\times 2\times 1.67\times 10^{-27}}\right)^3 \left(\frac{3}{4\pi}\right)\left(\frac{1}{2\times 2\times 10^{30}}\right)^2 = 4.43\times 10^{-16}\, kgm^{-3}$$

[1 mark]

Therefore the ratio of densities between the initial cloud collapse and the point of fragmentation is:

$$\frac{\rho_{Frag}}{\rho_{Initial}} = \frac{4.43\times 10^{-16}}{3.34\times 10^{-19}} = 1300 \qquad \textit{[1 mark]}$$

d) The radiation from a protostar of radius 4×10^6 km is absorbed by a surrounding spherical dust cloud of radius 10^{10} km. If all of the radiation (as measured by the luminosity) from the protostar is re-emitted by the dust cloud and the cloud is observed to have a temperature of 100 K, what is the effective temperature of the protostar? [3 marks]

Solution

If all of the protostar's luminosity is absorbed by the cloud, then:

$$L_* = L_{cloud}$$

so:

$$4\pi R_*^2 \sigma T_*^4 = 4\pi R_{cloud}^2 \sigma T_{cloud}^4 \qquad \textit{[1 mark]}$$

i.e.:

$$T_*^4 = \frac{R_{cloud}^2 T_{cloud}^4}{R_*^2}$$

$$T_* = \left(\frac{\left(10^{13}\right)^2 \times 100^4}{\left(4\times 10^9\right)^2}\right)^{1/4} \qquad \textit{[1 mark]}$$

$$T_* = 5000\ K \qquad \textit{[1 mark]}$$