University of Durham

EXAMINATION PAPER

May/June 2012 Examination code: 042591/01

LEVEL 2 PHYSICS: FOUNDATIONS OF PHYSICS 2B

SECTION A. THERMODYNAMICS
SECTION B. CONDENSED MATTER PHYSICS PART 1
SECTION C. CONDENSED MATTER PHYSICS PART 2

Time allowed : 3 hours

Examination material provided : None

Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **any three** of the four optional questions. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.

ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK

Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.

APPROVED TYPES OF CALCULATOR MAY BE USED.

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Electron mass:	$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}$ J s
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31 \times 10^{3}\ \mathrm{J\,K^{-1}\,kmol^{-1}}$
Avogadro's constant:	$N_{\mathrm{A}} = 6.02 \times 10^{26}\ \mathrm{kmol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	AU = 1.50×10^{11} m
Parsec:	pc = 3.09×10^{16} m
Solar Mass:	$M_\odot = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26}$ W

SECTION A. THERMODYNAMICS

Question 1 is compulsory. Question 2 is optional.

1. (a) Use the Maxwell relation $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$ and by defining an appropriate statement of the third law of thermodynamics, show that the volume expansivity of a solid, β, is zero at 0 K. [4 marks]

(b) When a rubber band of length, L, placed under a tension, f, is stretched by an amount dL, the work that is done is fdL. Use this information and the following statement of the Helmholtz function, $dF = -SdT + fdL$, to derive the Maxwell relation

$$\left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_L.$$

[4 marks]

(c) What are the main differences between the quantum particles, Fermions and Bosons? Sketch the Fermi-Dirac distribution for both $T = 0$ K and $T > 0$ K. [4 marks]

(d) The distribution function for Bosons is given by

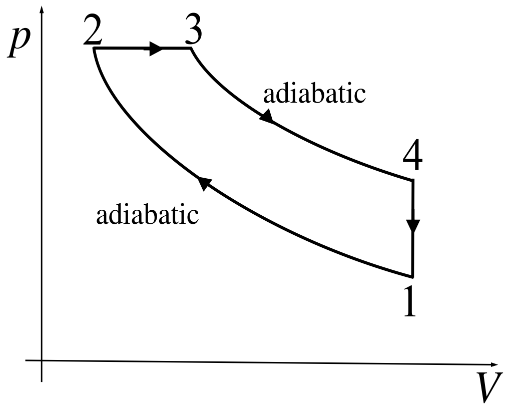
$$f(\epsilon) = \frac{1}{\exp\left(\frac{\epsilon - \mu}{k_{\mathrm{B}}T}\right) - 1},$$

where μ is the chemical potential. How does this distribution function lead to the concept of Bose-Einstein Condensation? Explain how the function given above changes when photons are considered. [4 marks]

2. (a) Describe the four thermodynamic steps of an ideal, reversible heat engine. [4 marks]

(b) Qualitatively describe the difference in entropy change between reversible and irreversible processes. In what type of process can the maximum work be done between two states? How can the *lost work* be expressed when the actual work done in a process is less than the maximum work? [2 marks]

(c) An ideal diesel cycle is shown in the diagram below.



If the working fluid of the engine is an ideal gas, show that the work done by the cycle is

$$W = \frac{1}{1-\gamma}[\gamma p_2(V_2 - V_3) + V_1(p_4 - p_1)],$$

where $\gamma = C_p/C_V$. [6 marks]

(d) Show that the total heat input during the cycle is

$$Q_{\text{in}} = \frac{\gamma}{1-\gamma} p_2(V_2 - V_3).$$

[4 marks]

(e) Hence, show that the cycle efficiency is given by

$$\eta = 1 - \frac{1}{r^{\gamma-1}} \frac{1}{\gamma} \frac{(r_c^\gamma - 1)}{(r_c - 1)},$$

where $r = V_1/V_2$ and $r_c = V_3/V_2$. [4 marks]

SECTION B. CONDENSED MATTER PHYSICS PART 1

Question 3 is compulsory. Questions 4 and 5 are optional.

3. (a) For a simple cubic lattice, sketch the planes with Miller indices (110) and (211). Include the x, y and z axes in your diagram. If the lattice constant a is 0.5 nm, determine the spacing between each of these two families of planes. [4 marks]

   (b) Explain the relationship between a crystal structure and the terms lattice and basis. A square 2D lattice is populated by the basis: $\begin{matrix} M & N \\ O & P \end{matrix}$. Draw the resulting crystal structure. [4 marks]

   (c) Calculate the fraction of space occupied by solid spheres in contact with each other packed in a face centred cubic lattice. [4 marks]

   (d) Describe the motion of atoms in a crystal when propagating sound waves. The dispersion relation for a simple cubic crystal is given by

$$\omega(\underline{K}) = \left(\frac{4C}{M}\right)^{1/2} \left|\sin\left(\frac{\underline{K}\cdot\underline{a}}{2}\right)\right|$$

   where $\underline{K}$ is the phonon wavevector, $|\underline{a}|$ is the lattice constant, C is the interatomic force constant and M is the atomic mass. Obtain an expression for the maximum sound velocity in this system. [4 marks]

   (e) The Bragg scattering condition is given by: $2d\sin\theta = n\lambda$. Describe Bragg scattering using the concept of a reciprocal lattice. What is the equivalent expression for Bragg scattering in the reciprocal lattice formulation? [4 marks]

   (f) Describe how a covalent bond is formed. Pay particular attention to the symmetry of the different components of the wavefunction ψ. [4 marks]

   (g) Show how the Bohr model of the hydrogen atom may be modified to describe donor impurities in semiconductors. If the ionisation energy of atomic hydrogen is given by:

$$-\frac{e^4 m_\mathrm{e}}{2(4\pi\epsilon_0\hbar)^2}$$

   determine an expression for the binding energy of a shallow donor in a semiconductor. Illustrate the position of the energy level with a simple sketch showing the conduction and valence bands. [4 marks]

4. (a) Give an expression for the structure factor S_{hkl} used to determine the intensity distribution for maxima in an X-ray diffraction pattern. Define the terms used. [4 marks]

   (b) Use this expression to perform structure factor calculations for a face centred cubic structure with a single atom basis. Limit your answers to terms where $h^2 + k^2 + l^2 \leq 12$. [6 marks]

   (c) A diffraction pattern was recorded from a thin (50 nm) crystalline metal film deposited on glass using a $\theta - 2\theta$ geometry where the X-ray wavelength was 0.15418 nm. Peaks were observed at 2θ values of $39.00°, 45.34°, 66.06°, 79.46°$ and $83.77°$. From this data confirm that the sample has the face centred cubic lattice. [8 marks]

   (d) Describe qualitatively the differences you would expect to see in the X-ray diffraction pattern if the same metal film had been deposited on a single crystal silicon wafer. [2 marks]

5. (a) Describe how waves propagate in a dispersive medium. [3 marks]

(b) The dispersion relation for waves in a 1 dimensional diatomic system is given by:

$$\omega^2 = C\left(\frac{1}{M_1} + \frac{1}{M_2}\right) \pm C\sqrt{\left(\frac{1}{M_1} + \frac{1}{M_2}\right)^2 - \frac{2\left(1 - \cos \underline{K} \cdot \underline{a}\right)}{M_1 M_2}}$$

where M_1 and M_2 are the masses of the two atoms, C is the spring elastic constant, $\underline{K}$ is the phonon wavevector and $|\underline{a}|$ is the lattice constant. Using this equation obtain two solutions and explain how these relate to the acoustic and optical branches of the phonon dispersion relation giving the limiting values for each branch at the boundaries of the Brillouin zone. Describe the motion of the atoms for each branch. Illustrate your answer with a sketch of the phonon dispersion relation for the first Brillouin zone. [11 marks]

(c) In the salt sodium chloride the spring constant is 20 N m^{-1}. The atomic masses of Na and Cl are 23 u and 35 u. Using this information, determine the maximum frequency for the optical phonon. [3 marks]

(d) What would you expect to see in the optical transmission of sodium chloride as a consequence of this result? [3 marks]

SECTION C. CONDENSED MATTER PHYSICS PART 2

Question 6 is compulsory. Question 7 is optional.

6. (a) Write down the classical (Drude) expression for the Wiedemann-Franz law, which relates a metal's thermal and electrical conductivities to temperature. Given that the thermal conductivity of potasssium at 300 K is 100 W m^{-1} K^{-1}, obtain a theoretical value for the electrical conductivity of potassium at the same temperature. [4 marks]

   (b) A metal has an electron density of 8.45×10^{28} m^{-3} and an electrical conductivity of 6.0×10^{7} Ω^{-1} m^{-1} at 300 K. Using the Drude expression for the electrical conductivity of a free-electron metal,

$$\sigma = \frac{ne^2\tau}{m_\mathrm{e}}$$

   where n is the electron density and τ is the relaxation (or mean free) time, obtain the length of the mean free path of electrons in this metal at 300 K. [4 marks]

   (c) Starting from the free-electron expression for the magnitude of the Fermi wavevector,

$$k_\mathrm{F} = (3\pi^2 n)^{1/3},$$

   find the Fermi energy of electrons in gold. The atomic mass of gold is 196.97 u (where 1 u $= 1.66 \times 10^{-27}$ kg), it has a density of 19 300 kg m^{-3} and can be assumed to be monovalent. [4 marks]

   (d) The Hall coefficient in aluminium is $+1.02 \times 10^{-10}$ m^{3} C^{-1}. Determine the density and type of charge carriers in this metal. Comment on the importance of your result for the theoretical understanding of electrons in solids. [4 marks]

7. (a) The dispersion (E versus k) relation for a one dimensional band in a certain solid described by the nearly-free electron model is given by

$$E(k) = Ck^2 - \frac{Da^2}{2\pi^2}k^4,$$

where a is the period of the lattice and C and D are positive constants. Sketch the form of the band in the first Brillouin zone, labelling the values of energy at the Brillouin zone centre and boundaries. [5 marks]

(b) Using the dispersion relation given above, derive an expression for the group velocity, v_g, of the electrons within the band. Hence determine the relationship between C and D. [5 marks]

(c) Derive the expression for the electron effective mass

$$m^* = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}.$$

Hence show that, for the dispersion relation given above, states close to the zone boundary have a negative effective mass. Explain how this result can be interpreted. [10 marks]