PHYS2641 – Laboratory Skills and Electronics

Electronics

Lecture 2

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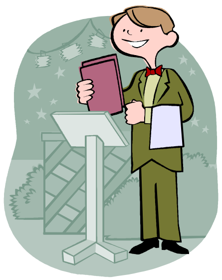
Last week

- Jupyter Notebook system used for electronics labs
- Potential divider circuits & passive filters
- Transfer function, Bode plots: Electronics labs Jupyter Notebooks & underlying Python/matplotlib code

Today’s menu



Aims:

1. Open- & closed-loop control systems
2. PID control systems
3. ‘Operational amplifiers’
4. Simple op-amp circuits

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Control systems

A control system varies a specific parameter ('output') according to an input:

- Central Heating system – you set the desired temperature on thermostat (input), central heating system heats radiators until desired temperature (controlled parameter) is reached

- Respiration – body sets desired level for CO_2, control centres adjust heart rate and breathing until that level is reached

- Cruise control – you set the desired speed, cruise control adjust throttle to accelerate/decelerate as required

Open-loop control

In an open loop control system, the output isn't monitored – the system is pre-calibrated to give a particular output for a particular input

> **e.g.** you could have a central heating system without a thermostat, in which you could set the amount of time the radiators come on each day, giving you some control over the temperature of the house…
>
> But…what happens when the external temperature drops? Internal temperature drops below desired level.

Open-loop control systems cannot cope with a varying input parameter space ('environment')

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Closed-loop control

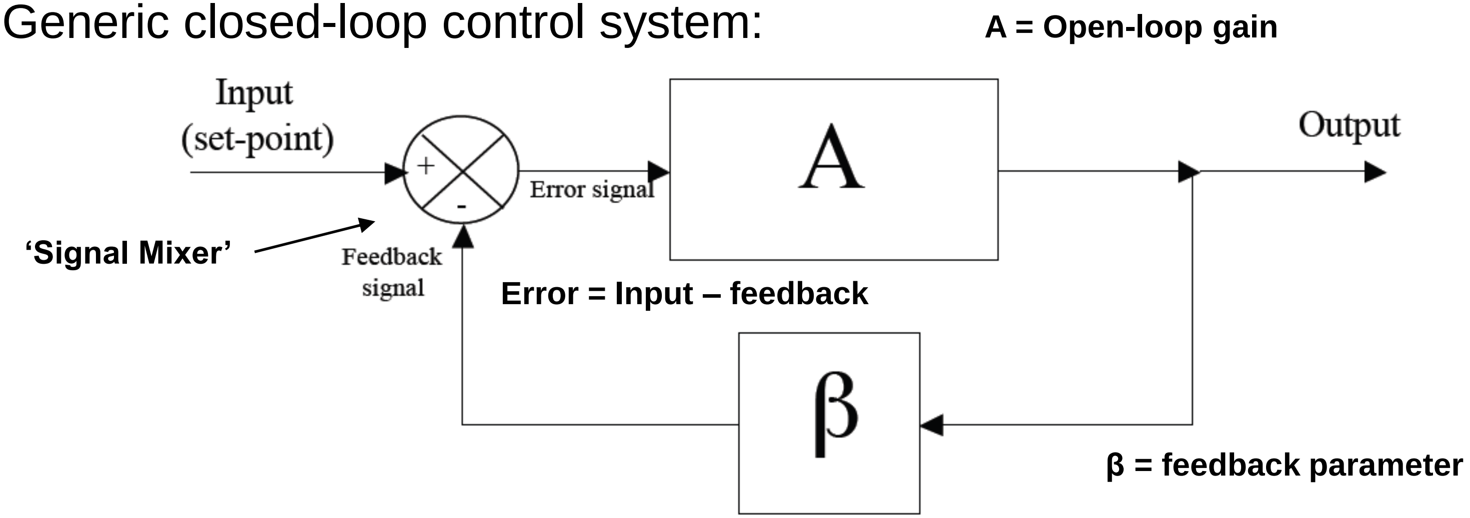
In a *closed-loop control system*, the output ***is*** monitored – the system automatically adjusts to keep it at (or near) the desired level

> **e.g.** in a central heating system ***with a thermostat*** the radiators are switched on until the set temperature is reached
>
> If the external temperature drops, the radiators are switched on for longer
>
> If the external temperature rises, they are switched on for less time

Closed-loop control systems ***can*** cope with a varying input parameter space – within limits!

Closed-loop control (2)

Generic closed-loop control system:



If output is **at** set-point, error = **zero**, so output **doesn't change** (ideally)

If output is **below** set-point, error is **positive**, so output **increases**

If output is **above** set-point, error is **negative**, so output **decreases**

Feedback tends to **reduce** the error signal – this is ***negative feedback***

Positive feedback reinforces the error signal and leads to **instability**

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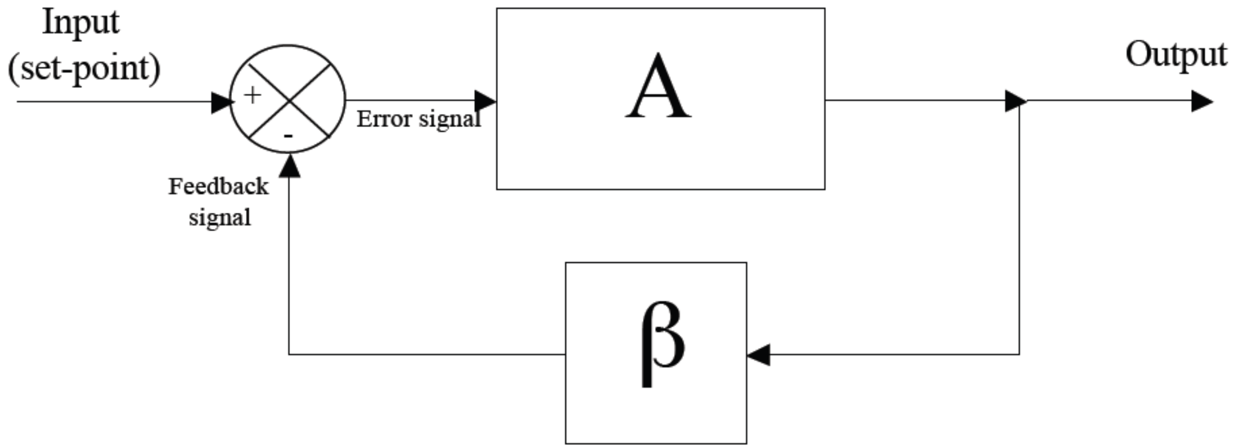
Closed-loop control (3)

Gain for a closed-loop control system:



$$V_{err} = \left(V_{in} - V_{fb}\right) = \left(V_{in} - \beta V_{out}\right), \qquad V_{out} = AV_{err}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{A}{1 + A\beta}}$$

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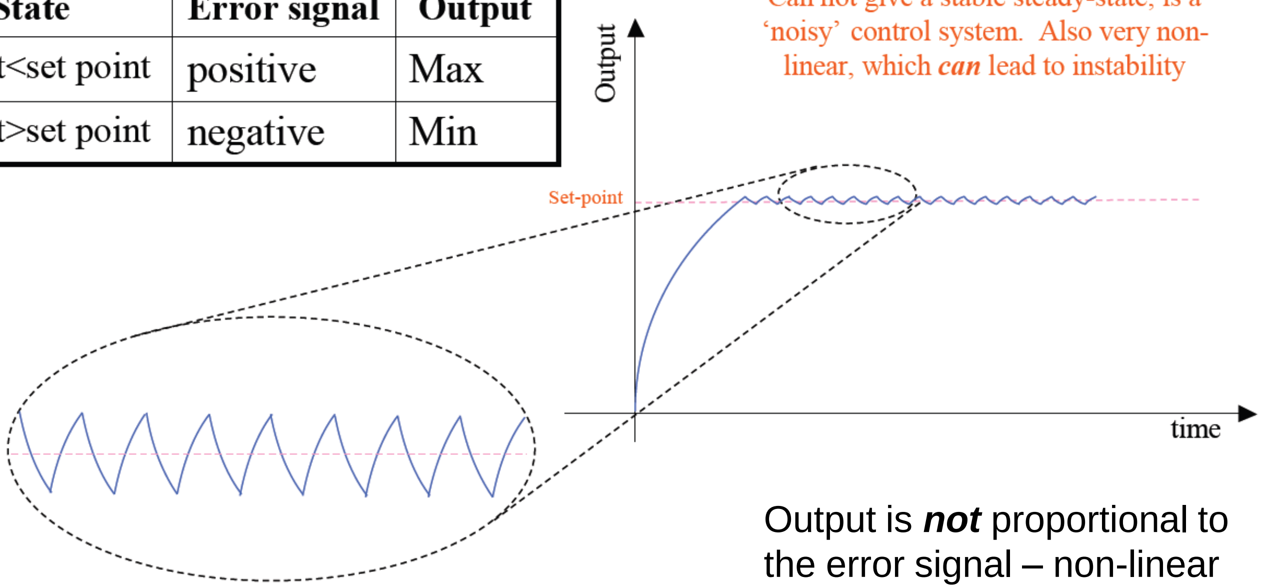
Closed-loop control (4)

There are different types (or ‘topologies’) of control system

The simplest system is an **ON-OFF** (sometimes called ‘Bang-Bang’) control system (e.g. the thermostat in a CH system)

State	Error signal	Output
Output<set point	positive	Max
Output>set point	negative	Min

Can not give a stable steady-state; is a ‘noisy’ control system. Also very non-linear, which ***can*** lead to instability



Output is ***not*** proportional to the error signal – non-linear

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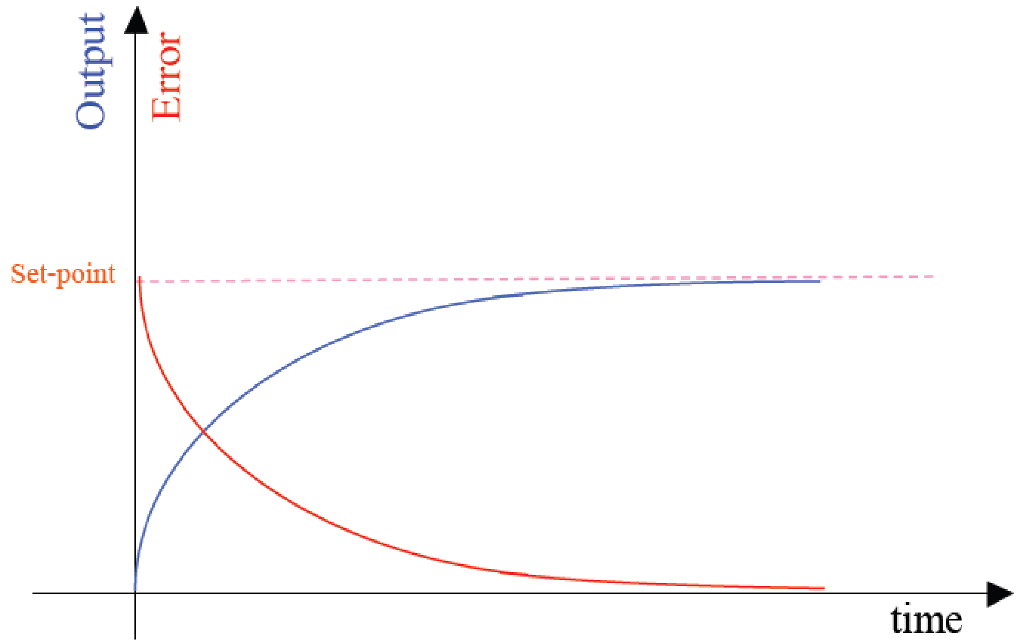
‘Proportional’ control

A common control system is the ***proportional control system***, where the output is linearly proportional to the error signal



Can give a steady-state response, but is often slow to reach it

Steady-state response doesn’t occur at the set-point, but slightly above or below it – ***steady-state error***

Can improve the response speed **and** the steady-state error by increasing the gain of the system; but the system can become **under-damped** (i.e. output **overshoots**) and eventually **unstable** (i.e. output **oscillates**)

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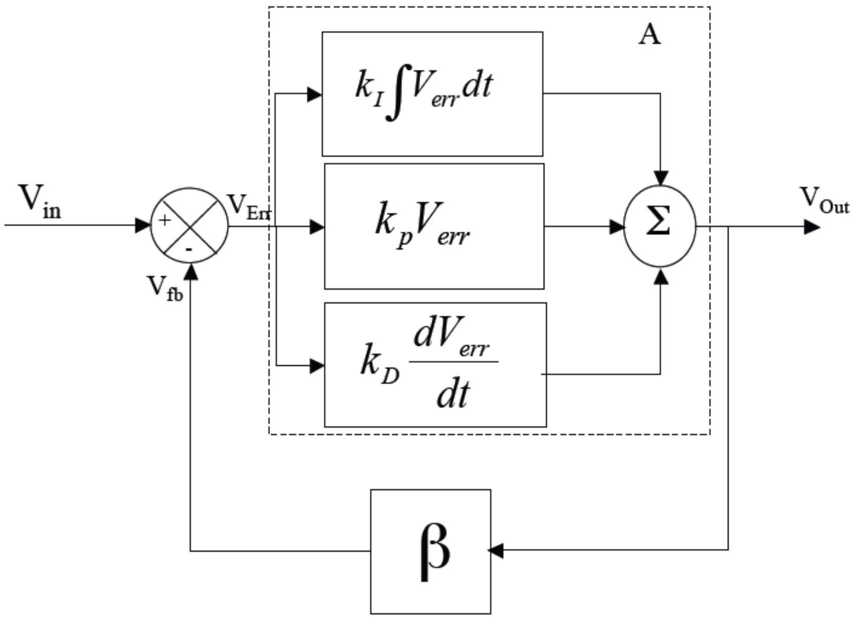
‘PID’ control

An improvement to the proportional control system is the ***proportional+differential+integral*** (PID) control system:

- The **proportional** (P) term **drives the output toward the setpoint**, as previously
- The **integral** (I) term **eliminates steady-state-error**; if there is a small but constant error term, it is integrated with time, causing the output to change and removing the error term
- The **differential** (D) term improves the **response speed** – if the error signal changes rapidly, the output changes accordingly – even for changes of small amplitude

'PID' control (2)



How can we implement this?

V_{out} is now given by an integro-differential equation:

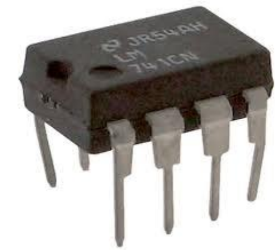
$$V_{Out}=k_PV_{Err}+k_I\int V_{Err}dt+k_D\frac{\partial V_{Err}}{\partial t}$$

$$\Rightarrow\frac{\partial V_{Out}}{\partial t}=k_P\frac{\partial V_{Err}}{\partial t}+k_IV_{Err}+k_D\frac{\partial^2V_{Err}}{\partial t^2}$$

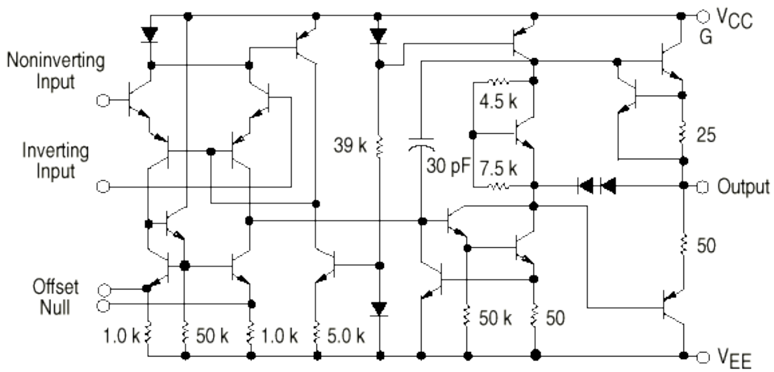
The PID system is a 2nd order system (i.e. can be modelled by a 2nd order differential equation)

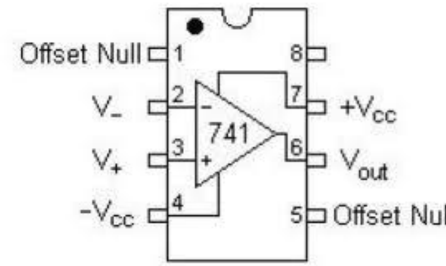
It can be tuned to give under-damped, over-damped or oscillatory output.

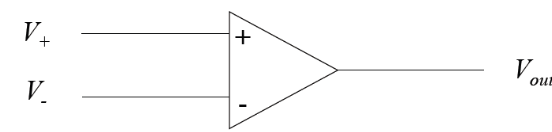
The parameters k_P, k_I, k_D can all be altered to give the required system response (e.g. best response speed or best stability).



The ‘Operational amplifier’





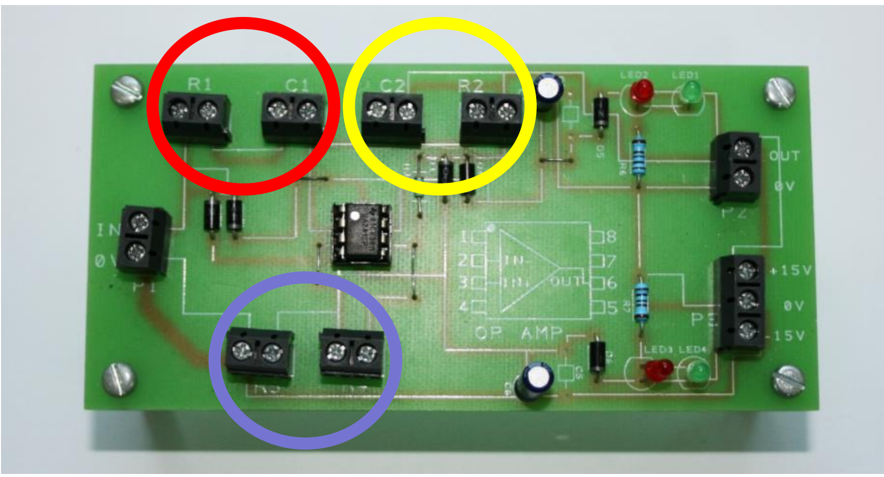


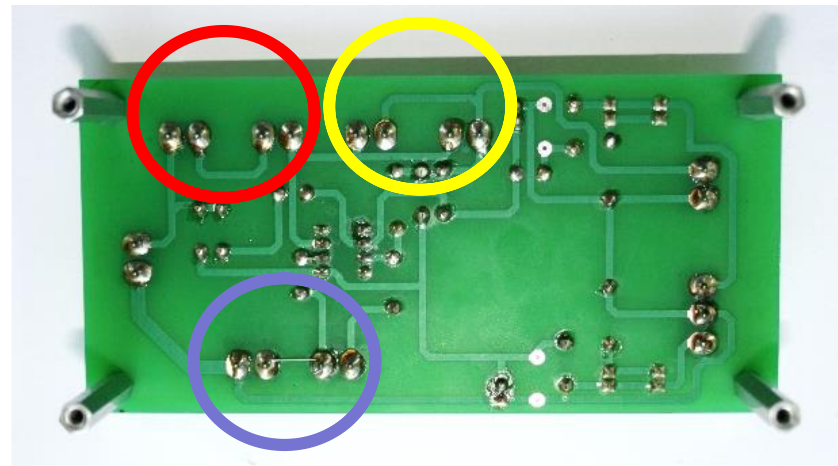
An ‘op-amp’ is a type of integrated circuit (IC), or ‘chip’.

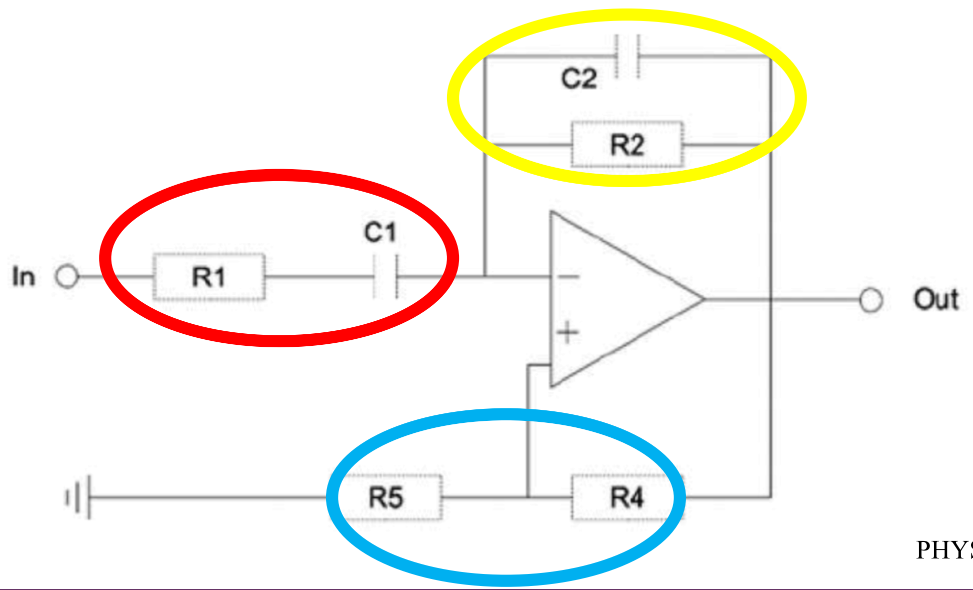
The ‘741’ is the standard/basic op-amp chip – even this is quite a complicated circuit!

Op-amp chips require dc **POWER** [$+V_{CC}$, $-V_{CC}$(V_{EE})] to operate. There are also ‘offset-null’ connections which allow removal of spurious outputs: These terminals are often omitted from simple schematic circuit diagrams

Op-amp PCB







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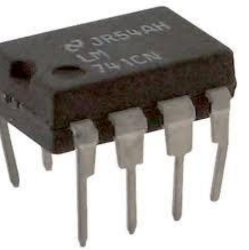
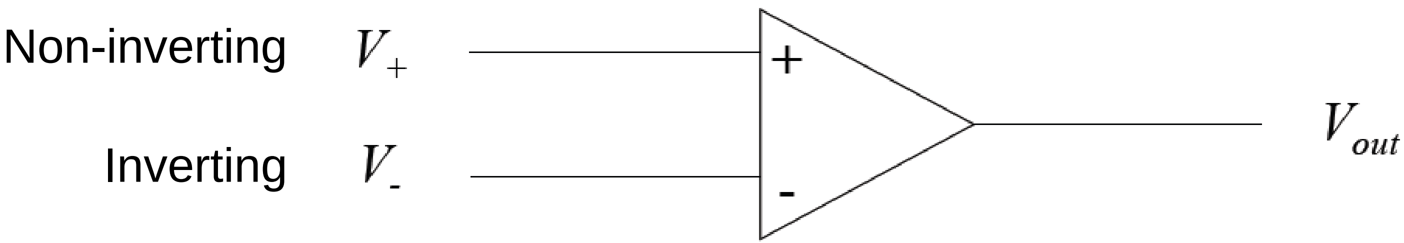
The ‘Operational amplifier’ (2)



An ‘**op-amp**’ has two inputs: V_+ and V_-, also called ‘non-inverting’ and ‘inverting’ inputs



The output voltage is $V_{out} = A\left(V_+ - V_-\right)$

The open-loop gain of an op-amp, A, is very large (e.g. A = $10^5 - 10^8$)

To make useful circuits, we often use ***negative feedback*** to control the op-amp gain

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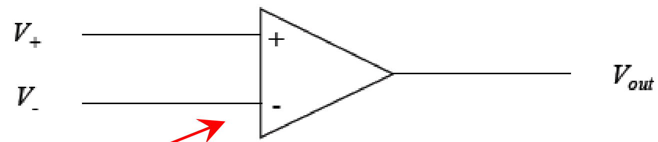
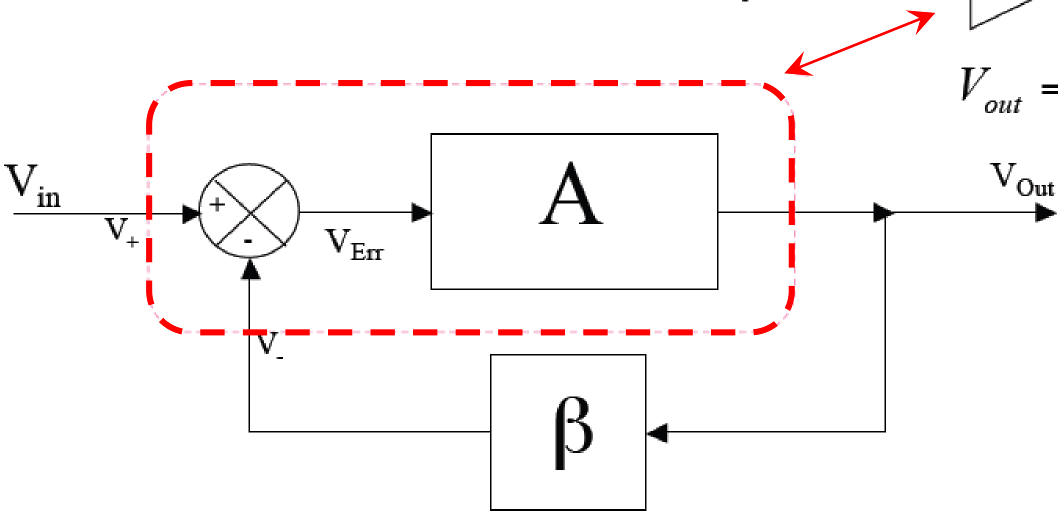
Op-amp with feedback: Gain

Gain equation for closed-loop system:



$$V_{out} = A(V_+ - V_-)$$



$$\frac{V_{out}}{V_{in}} = \frac{A}{1 + A\beta}$$

If A is large, such that Aβ >> 1, then

$$\boxed{\frac{V_{out}}{V_{in}} \approx \frac{A}{A\beta} = \frac{1}{\beta}}$$

i.e. Gain is set only by feedback coefficient (β) and is independent of the amplifier open-loop gain!

How do we specify β?

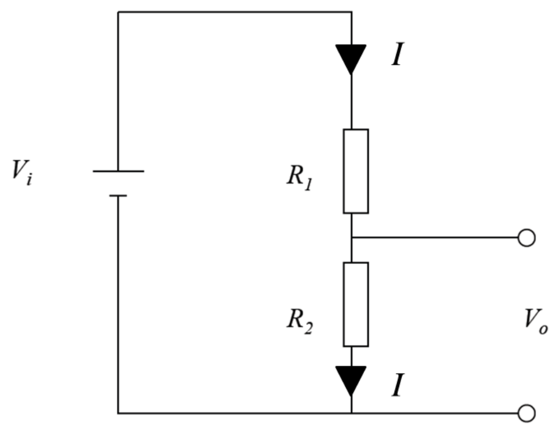
We want to define the ‘feedback’ voltage relative to the ‘output’ voltage

The simplest way to do this is using a ‘potential divider’

– the ‘output’ from the op-amp is the input to the potential divider

– the output from the potential divider is the ‘feedback’ into the op-amp

(recall from last week…)



$$I = \frac{V}{R}$$

$$= \frac{V_i}{R_1 + R_2} = \frac{V_0}{R_2}$$

$$\boxed{V_0 = \frac{R_2}{R_1 + R_2} V_i}$$

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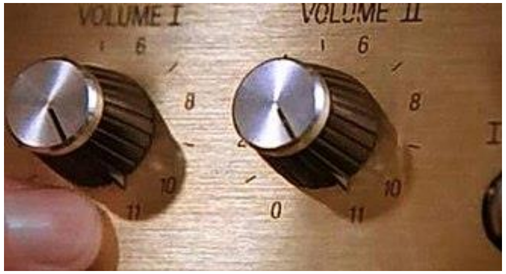
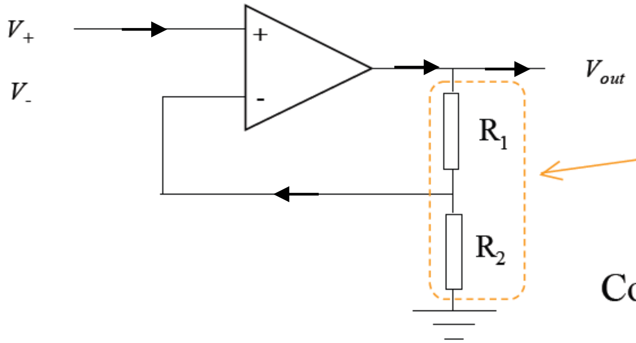
Non-inverting amplifier



Use a potential-divider so that V_{fb} is a fraction of V_{out}



From potential-divider equation:

$$\beta = \frac{R_2}{R_1 + R_2}$$

Connect pot. divider to inverting input to give feedback…

Overall gain is:

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{\beta} = 1 + \frac{R_1}{R_2}}$$

We can set the gain of the circuit by selecting the resistors R_1 and R_2!

Note: the gain is **positive** – the output voltage has the same polarity as the input. This circuit is known as a ***non-inverting amplifier***

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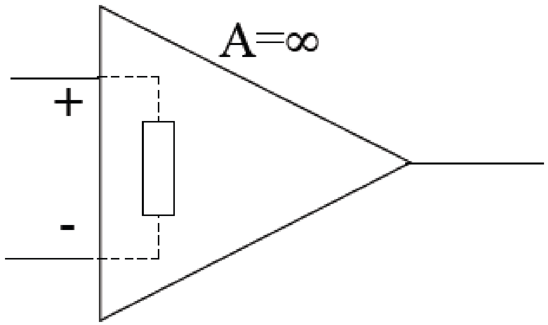
Op-amps: General properties

For an *ideal* op-amp:

- The open-loop gain A is ***infinite***
- The input impedance is ***infinite***
- The output impedance is ***zero***



These give us '**Golden Rules**' for a ***negative-feedback*** system:

1. The output will always attempt to drive the inputs to the same voltage (the steady-state error is zero)
2. No current flows into the inputs
3. Loading does not affect the output

Using these rules we can figure out the behaviour of fairly complex op-amp circuits!

This is also a very useful property!

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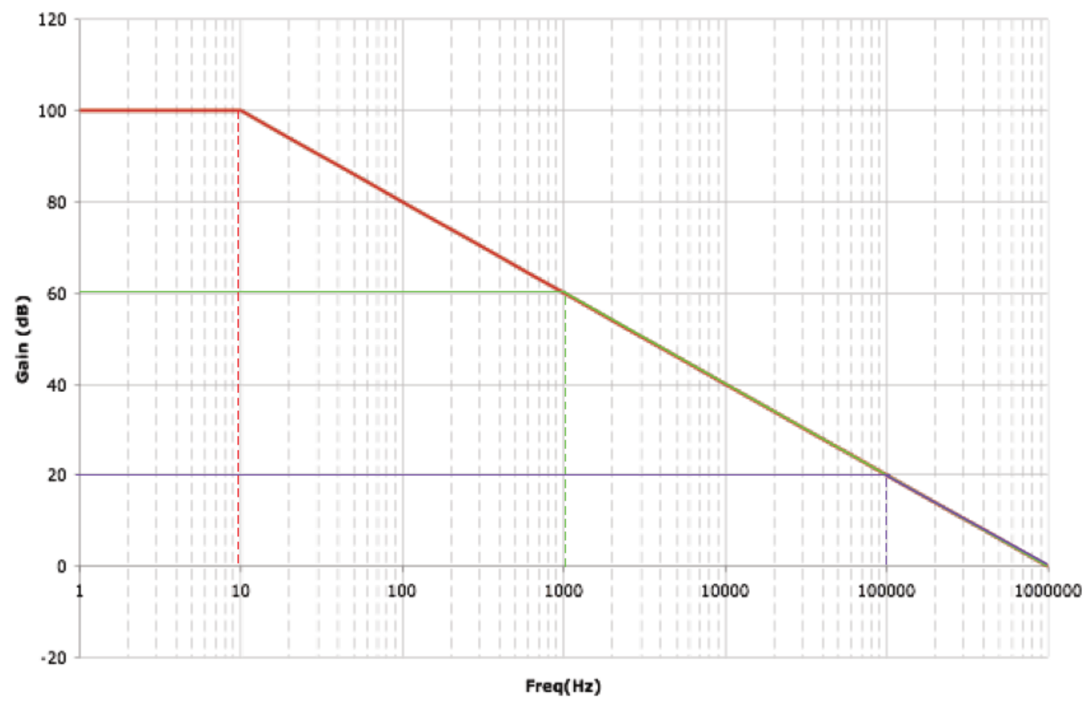
Op-amp frequency response

- Ideal op-amp has infinite gain
- Real op-amps have limited gain, which *drops off with increasing frequency*

This is described by the *'gain-bandwidth product'* (i.e. Gain x Frequency = const.)

There is a trade-off between Gain and Bandwidth

We can use ***feedback*** to *reduce the gain* and thereby *increase the bandwidth*



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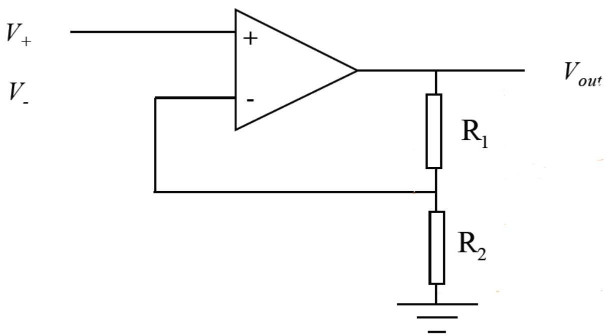
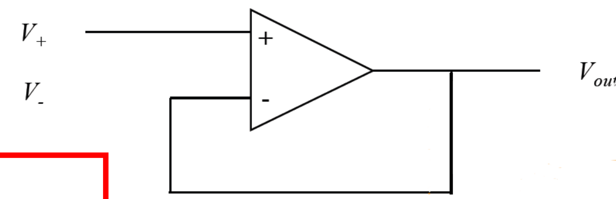
Unity gain buffer

Limiting case of non-inverting amplifier: $R_1=0$, $R_2=\infty$





$$\boxed{\frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_2} = 1}$$

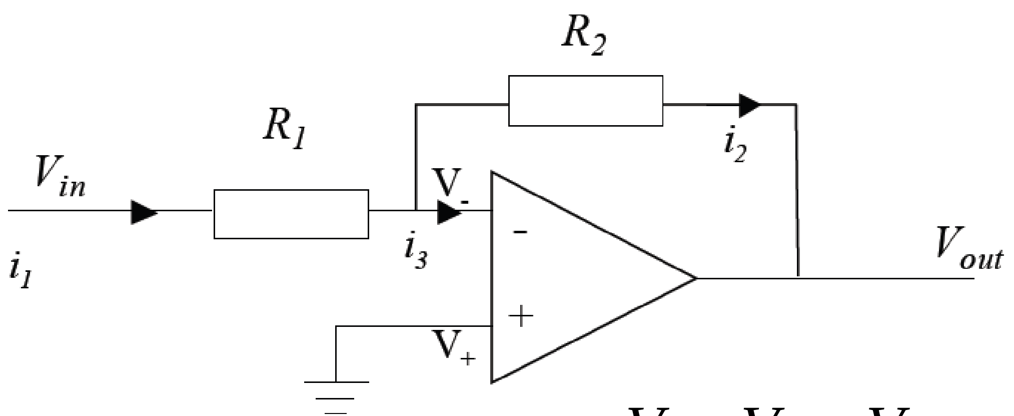
Why is this useful?

Recall ‘rule 3’ – the op-amp output is ***isolated*** from the input: placing a variable load on the output doesn’t affect the input voltage – the input voltage is ***buffered***.

This is useful if you need to connect a high impedance source which can’t deliver much current (e.g. signal-generator) to a low-impedance load (e.g. heater, bulb, speaker etc...) – *network loading* does **NOT** change V_{out}.

The output ***current*** is now sourced from the op-amp power supply rather than the input voltage source

Inverting amplifier



Rule 1 states that the two inputs are driven to the same voltage, so

$\boxed{V_- = V_+ = 0, \quad V_- \text{ is a 'virtual earth'}}$

We can now calculate i_1: $$i_1 = \frac{V_{in} - V_-}{R_1} = \frac{V_{in}}{R_1}$$

Rule 2 states that i_3 must be zero.

So, $i_2 = i_1$ and we can calculate V_{out}: $$V_{out} = V_- - i_2 R_2 = -V_{in}\frac{R_2}{R_1}$$

$$\boxed{\text{Gain} = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

Note the '-' sign: the output voltage has *opposite polarity* to the input voltage – this is an ***inverting amplifier***

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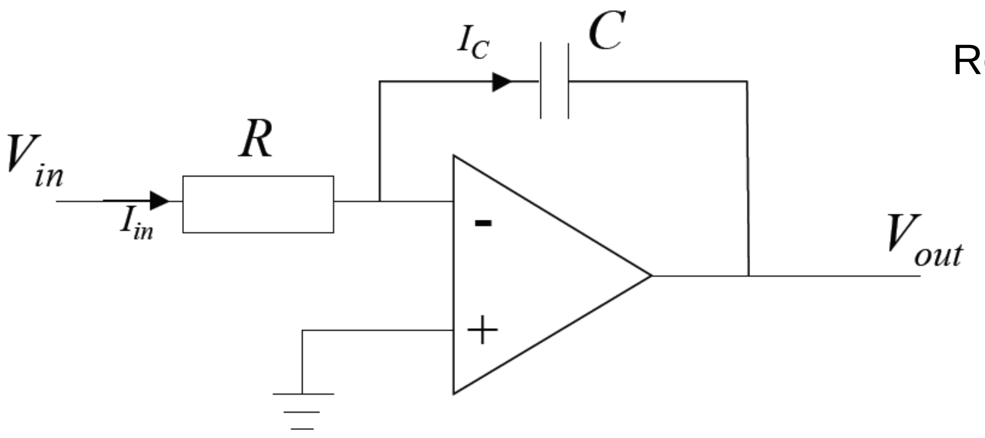
Integrator



Recall for a capacitor:

$$Q = CV \qquad I_C = \frac{\partial Q}{\partial t}$$

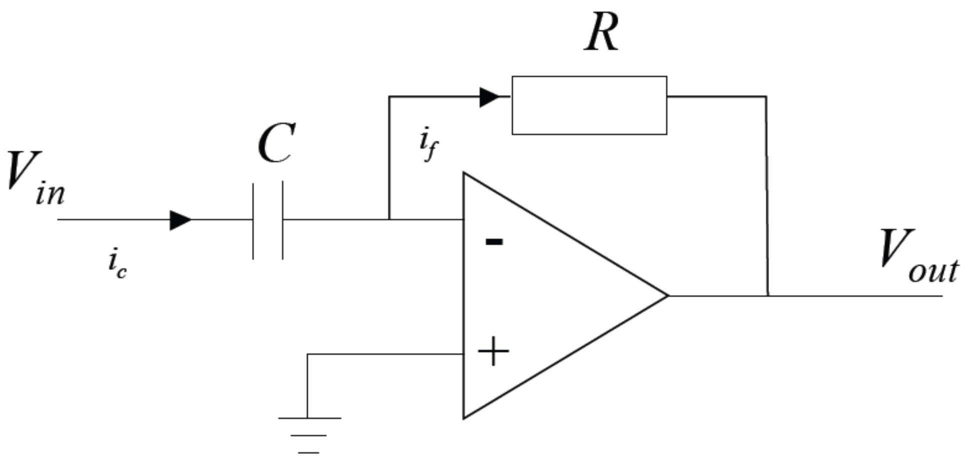
$$I_C = C\frac{\partial V}{\partial t}$$

Using the 'golden rules': $$I_{in} = \frac{V_{in}}{R}, \quad I_C = I_{in} : \qquad \frac{V_{in}}{R} = -C\frac{\partial V_{out}}{\partial t}$$

$$\boxed{V_{out} = -\frac{1}{RC}\int V_{in}\,dt + V_{initial}}$$

The circuit acts as a voltage-integrator, with a scaling factor of -1/RC

Differentiator



Using golden rules:

$$V_C = V_{in}$$
$$i_f = i_c$$

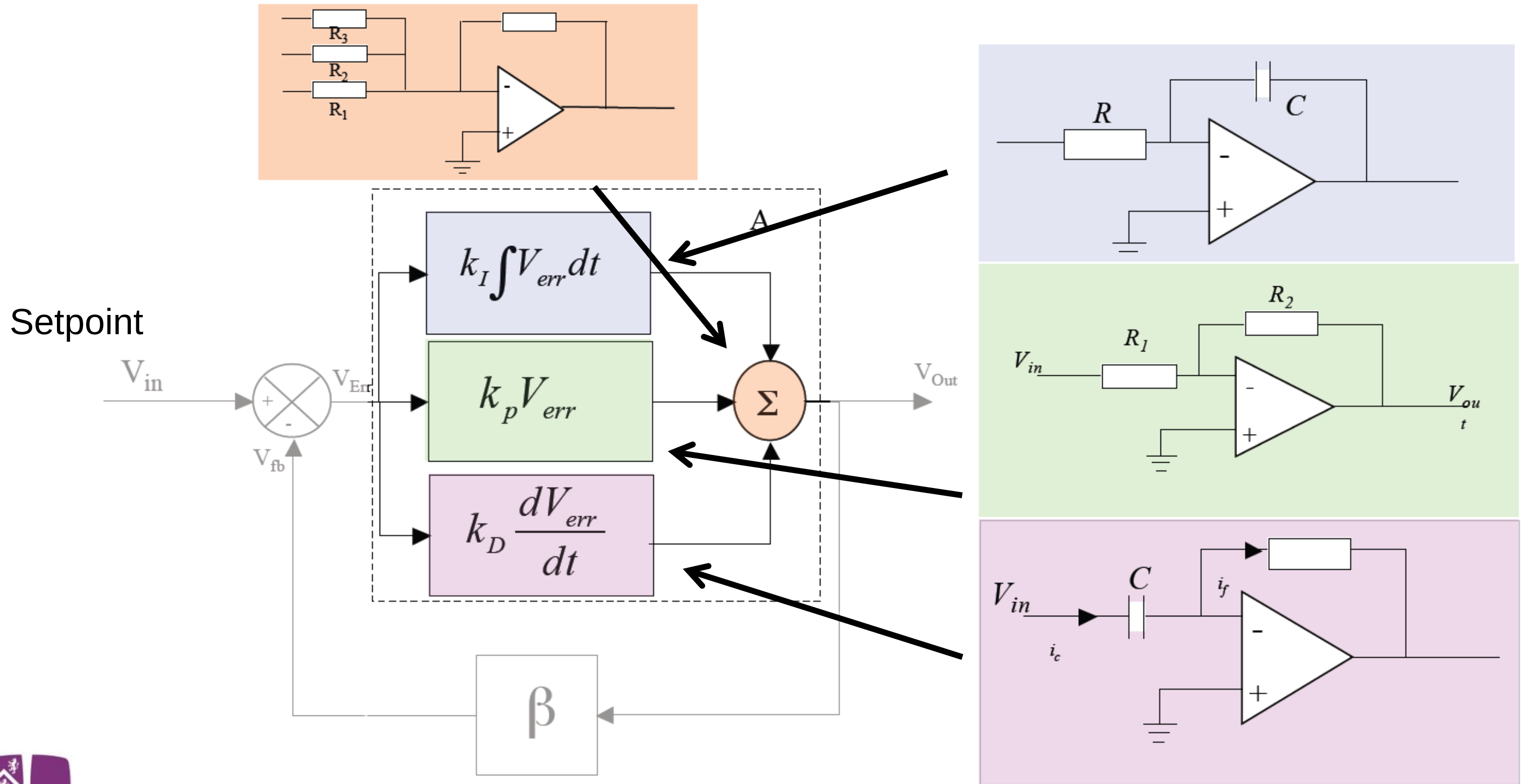
We know that $i_c = C\frac{\partial V_c}{\partial t}$ and $V_{out} = V_- - i_f R = -i_f R$

$$\boxed{V_{out} = -RC\frac{\partial V_{in}}{\partial t}}$$

The circuit acts as a voltage-differentiator, with a scaling factor of –RC

Op-amp PID control system

We now have everything needed to implement a PID control system!



Summary

- ‘PID’ controllers allow us to accurately control the value of an output in relation to a control setpoint – useful for a wide range of stabilisation and control applications

- ‘Operational amplifiers’ and analysis of various circuits using the op-amp Golden Rules

- PID control can be implemented using a series of simple op-amp circuits

Next week: Limitations of real op-amps; Comparators; Stability and oscillation

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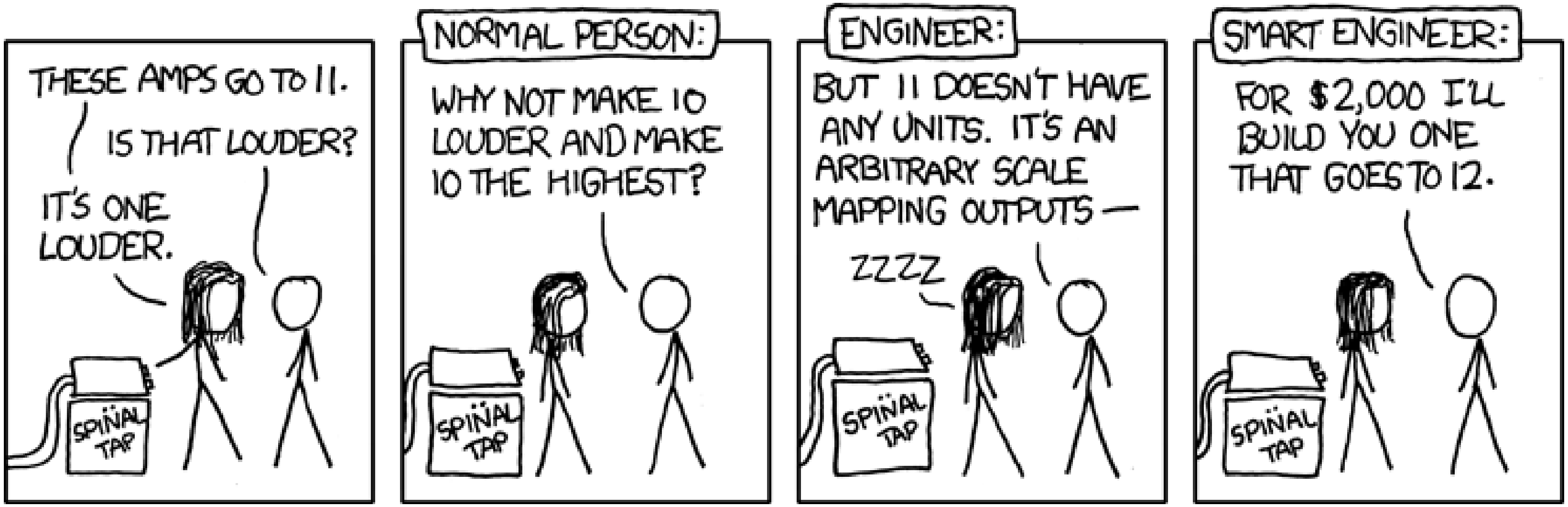
THESE AMPS GO TO 11.
IS THAT LOUDER?
IT'S ONE LOUDER.
SPINAL TAP
NORMAL PERSON:
WHY NOT MAKE 10 LOUDER AND MAKE 10 THE HIGHEST?
SPINAL TAP
ENGINEER:
BUT 11 DOESN'T HAVE ANY UNITS. IT'S AN ARBITRARY SCALE MAPPING OUTPUTS —
ZZZZ
SPINAL TAP
SMART ENGINEER:
FOR $2,000 I'LL BUILD YOU ONE THAT GOES TO 12.
SPINAL TAP