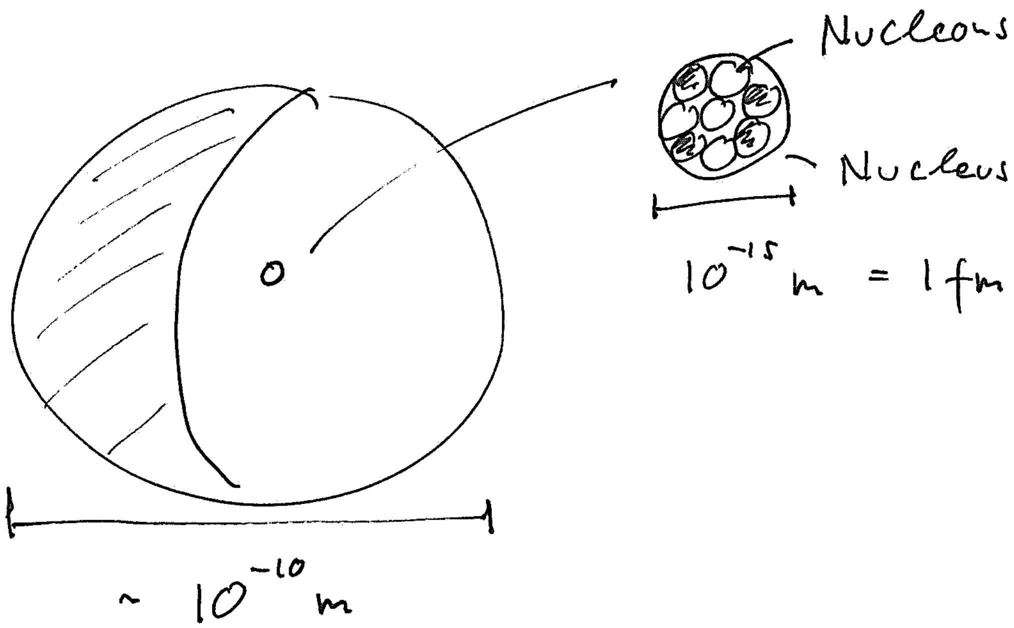
NUCLEAR PHYSICS

1. NUCLEAR MASSES AND BINDING ENERGIES



The nucleus is composed of neutrons and protons.

mass number → A

name of the element → X

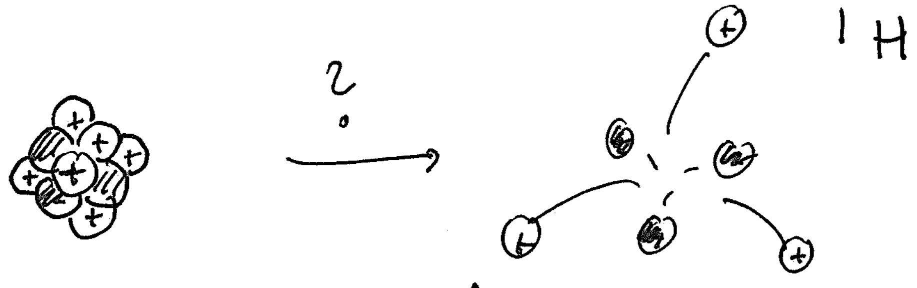
$$ {}^{A}_{Z}X_{N} $$

Z ← number of charges = number of proton

N ← number of neutrons

$$A = Z + N$$

often you will see ${}^{A}_{Z}X$ or ${}^{A}X$



why don't they disintegrate?

It is held together by the strong nuclear force. It is

→ stronger than electromagnetism.

→ it is attractive and short-ranged. (few fm)

→ repulsive for very short distances ($\leq$ fm)

We are interested in the binding energy of the nucleus

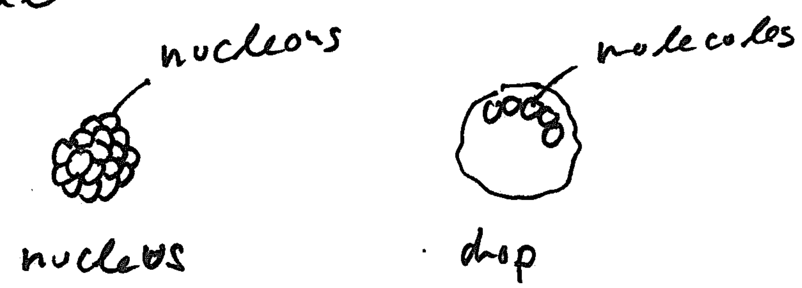
$$-B(A, Z) = M(A, Z) - Z \underset{\underset{M_p + m_e}{\parallel}}{M(^1H)} - N M(n)$$

to be consistent with other formula

We neglect the 1H binding energy ~ 13.6 eV.

Can we understand the shape of this curve.

Liquid-drop model



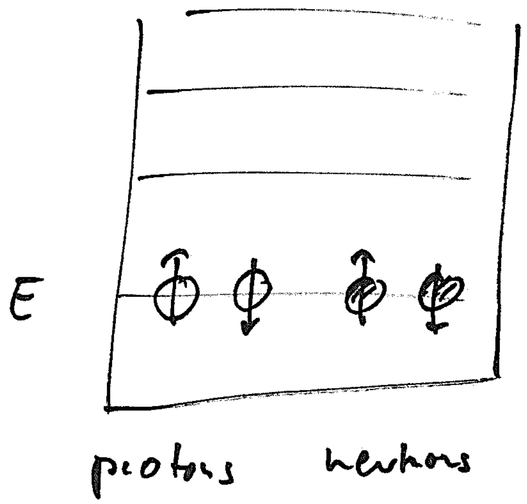
homogeneous, incompressible, constant mass density which falls off sharply at the boundary.

Since it is densely packed $R \propto A^{1/3}$

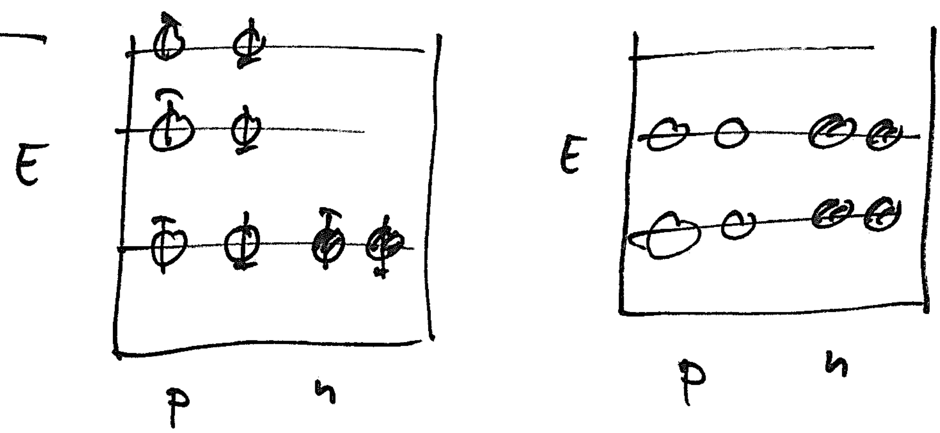
$$B(A, Z) = E_{volume} - E_{surface} - E_{coulomb} - E_{asymmetry} \pm E_{pairing}$$

Volume, Surface and Coulomb terms are classical effects.

Quantum effects are relevant because of the Pauli principle. Both protons and neutrons are fermions [Spin ½]



<u>Asymmetry term</u>



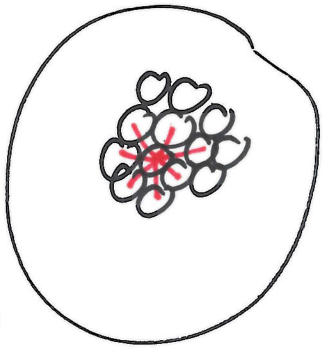
$A = 8$ $Z = 6$ $N = 2$

$A = 8$, $Z = 4$ $N = 4$

Symmetric nuclei are ~~more~~ denser
=> higher binding energy

$$\boxed{E_{Asymmetry} = a_a \frac{(N-Z)^2}{4A}}$$

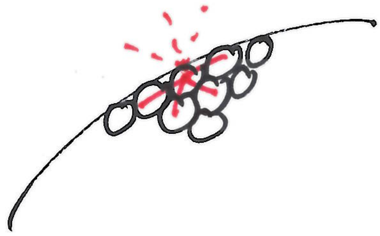
- Volume term



Because the strong force is short-ranged, the potential energy is the same for all nucleons and it depends on the density $\rho = \frac{m}{V} \propto A$

~~otherwise~~ $\cancel{\propto A(A-1)}$

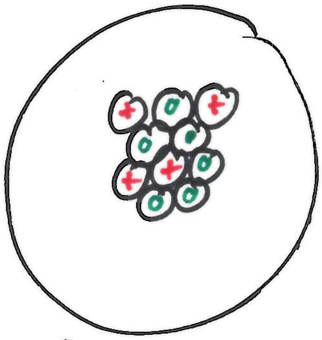
$$\boxed{E_{volume} = a_v A}$$



- Surface term

Subtract the missing neighbours from the surface nucleons. Surface scales like $S = 4\pi R^2 \propto (A^{1/3})^2$

$$\boxed{E_{surface} = a_S A^{2/3}}$$



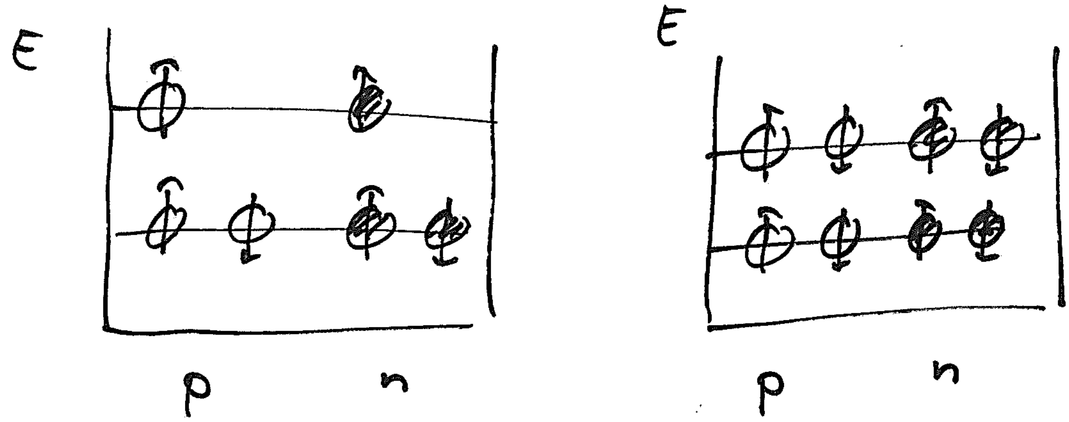
- Coulomb term

Electromagnetism drives the nucleons apart. The potential is $V(r) = \frac{\alpha}{r} z^2$ $\quad \alpha = 4\pi e^2$

expect $\frac{z^2}{r} \propto z^2 / A^{1/3}$

$$\boxed{E_{coulomb} = a_c \, z^2 / A^{1/3}}$$

Pairing term



$$\boxed{E_{\text{pairing}} = \delta / \sqrt{A}}$$

$$\delta = \begin{cases} \delta_p & \text{odd/odd} \\ 0 & \text{even/odd} \\ -\delta_p & \text{even/even} \end{cases}$$

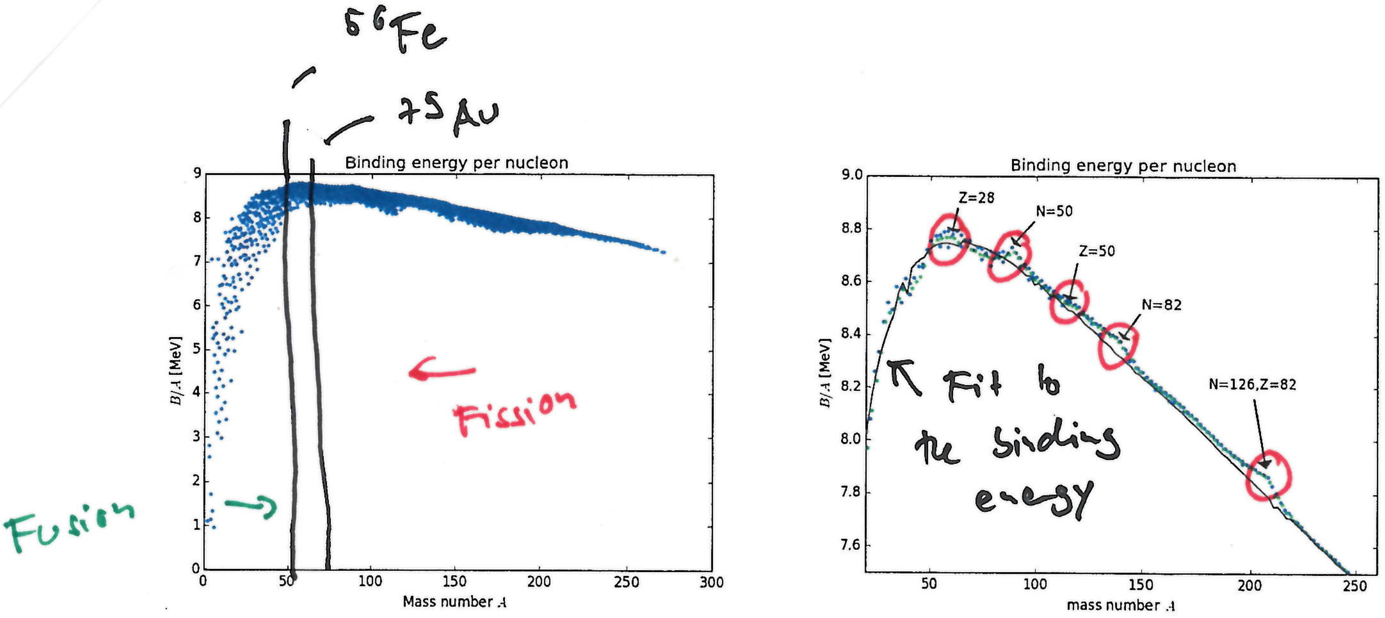


Figure 1: Binding energy per nucleon as a function of the mass number. The first figure shows all

1.1 Key points

- Protons and neutrons are called nucleons
- Nuclei are characterised by their mass $A = Z + N$ and charge Z
- The binding energies are calculated with the semi-empirical mass formula
- The nuclear force is short-ranged and creates a constant potential inside the nucleus
- Nucleons at the surface have fewer neighbours and therefore their binding energy is smaller
- Nucleons prefer to be in pairs in the nucleus

2 NUCLEAR STABILITY

Not all combinations of protons and neutrons are stable.

2.1 Decay constants

Starting with a number N_0 of nuclei at $t=0$, how many are left after a time t?

$$\frac{dN}{dt} = -\lambda N$$

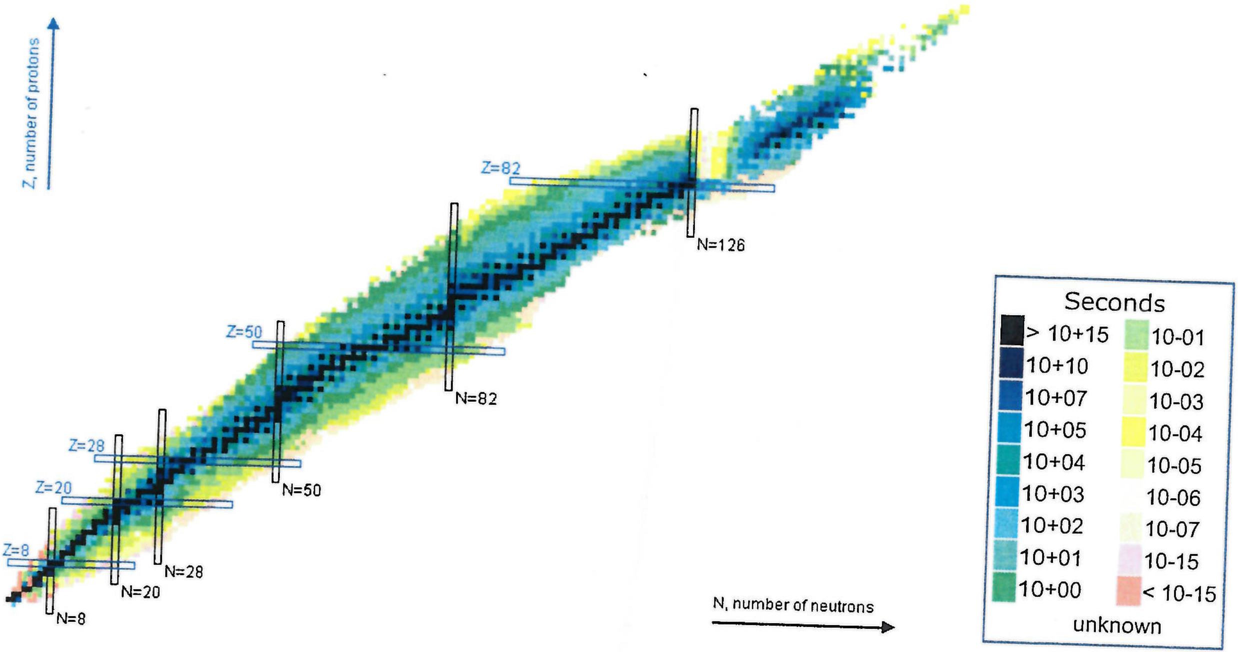
decay constant

proportional to initial number

decrease

change in time

$$\longrightarrow \quad N(t) = N_0 e^{-\lambda t}$$



Z, number of protons
N, number of neutrons
Z=8
Z=20
Z=28
Z=50
Z=82
N=8
N=20
N=28
N=50
N=82
N=126
Seconds
> 10+15
10+10
10+07
10+05
10+04
10+03
10+02
10+01
10+00
10-01
10-02
10-03
10-04
10-05
10-06
10-07
10-15
< 10-15
unknown