University of Durham
EXAMINATION PAPER

Examination session:	Year:	Examination code:
May/June	2019	PHYS3721-WE01

Title:
Modern Atomic and Optical Physics 3

Time allowed:	3 hours		
Additional material provided:	None		
Materials permitted:	None		
Calculators permitted:	Yes	Models permitted:	Casio fx-83 GTPLUS or Casio fx-85 GTPLUS
Visiting students may use dictionaries:	No		

Instructions to candidates:

- Attempt **all** questions. The short-answer questions at the start of each section carry 50% of the total marks for the paper. The remaining 50% of the marks are carried by the longer questions, which are equally weighted.
- The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK.**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklet for Section B inside your booklet for Section A, before they are collected by the invigilator.

Information

Section A: Atomic Clocks
Section B: Fourier Optics

A list of physical constants is provided on the next page.

Revision:	

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Bohr magneton:	$\mu_\mathrm{B} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Electron mass:	$m_\mathrm{e} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_\mathrm{p} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	$\mathrm{AU} = 1.50 \times 10^{11}$ m
Parsec:	$\mathrm{pc} = 3.09 \times 10^{16}$ m
Solar Mass:	$M_\odot = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26}$ W

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SECTION A: ATOMIC CLOCKS

1. (a) In the presence of a magnetic field B, the energy shifts for the components of the ground state of an alkali metal atom having angular momentum quantum number m_F along the field direction are

$$\Delta E = \mp\frac{A}{2} \pm \frac{A}{2}\sqrt{1 + \frac{8 m_F \mu_{\rm B} B}{A(2I+1)} + \frac{4\mu_{\rm B}^2 B^2}{A^2}}.$$

A is the zero-field hyperfine splitting, I is the nuclear spin, $\mu_{\rm B}$ is the Bohr magneton and B is the magnetic field. Derive the values of m_F for which the Zeeman shift is linear. [4 marks]

(b) It takes 42000 resonant photons of wavelength 780.24 nm to stop a ^{87}Rb atom of mass $m = 86.909$ u that is traveling towards the photon source. Calculate the initial velocity of the atom. [4 marks]

(c) The clock transition in a ^{133}Cs fountain clock has an angular frequency of $\omega_0 = 2\pi \times 9.1926 \times 10^9$ s^{-1}. The time between the end of the first pass and the beginning of the second pass through the microwave cavity is

$$T = \left(\frac{8h}{g_0}\right)^{\frac{1}{2}},$$

where $g_0 = 9.80665$ m s^{-2}. For a fountain of height $h = 0.50000$ m, calculate the width of the central Ramsay fringe. Calculate the clock uncertainty given that the line centre can be determined to one part in 10^4. [4 marks]

(d) Contrast and compare the choice of clock transition used in the Cs fountain clock and the Sr optical lattice clock. [4 marks]

(e) Show, graphically or otherwise, how a π-transition between 2p0 and the 1s0 orbitals of the hydrogen atom does not lead to the emission of a photon along the z-axis. [4 marks]

(f) The 4s ^{2}S$_{1/2}$ $\rightarrow$ 4p ^{2}P$_{3/2}$ cooling transition used in an optical molasses of ^{40}K at 767 nm has a linewidth of $\Gamma = 2\pi(6.0$ MHz). The Doppler velocity of the optical molasses is given by

$$v_{\rm D} = \left(\frac{\hbar\Gamma}{M}\right)^{\frac{1}{2}}.$$

Calculate the capture velocity $v_{\rm C}$ of the optical molasses. [4 marks]

(g) Describe briefly how a Zeeman slower works to slow atoms in an atomic beam. [4 marks]

(h) Describe briefly how an atomic-beam apparatus can be used to measure the frequency of a hyperfine transition in an atom with total electronic angular momentum $J = 1/2$. [4 marks]

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2. The $3s\,^2S_{1/2} \rightarrow 3p\,^2P_{3/2}$ transition in ^{23}Na at an angular frequency of $\omega_0 = 2\pi \times 5.09 \times 10^{14}$ Hz (589 nm) is used for laser cooling. The lifetime of the $3p\,^2P_{3/2}$ state is 16.2 ns.

   (a) Explain briefly why a resonant laser beam exerts a force on atoms. [4 marks]

   (b) The maximum force experienced by an atom is $F_{\mathrm{max}} = R_{\mathrm{scatt}}\hbar k$, where k is the photon wavevector and R_{scatt} is the scattering rate. Calculate the value of the maximum force experienced by a ^{23}Na atom in a single, resonant laser beam. [3 marks]

   (c) Calculate the detuning required to bring a laser beam into resonance with a ^{23}Na atom travelling at $v = 500$ m s^{-1}. [2 marks]

   (d) In two counter-propagating laser beams, the force along the direction of the laser beams experienced by an atom due to each laser beam, F_+ and F_-, is given by

$$F_{\pm} = \pm\frac{\hbar k\Gamma}{2}\frac{I/I_{\mathrm{sat}}}{1 + 2I/I_{\mathrm{sat}} + 4(\delta \mp kv)^2/\Gamma^2},$$

   where I/I_{sat} is the ratio of the laser intensity to the saturation intensity, δ is the detuning, k is the wavevector, v is the atom velocity and Γ is the linewidth. The total force an atom experiences in both laser beams is F_{molasses}.

   (i) On the same diagram, sketch F_+, F_- and F_{molasses} versus v. [3 marks]

   (ii) For small values of v, i.e. $|kv/\Gamma| < 1$, show that F_{molasses} is linearly proportional with respect to v. [8 marks]

$$\left[\begin{array}{c}\text{Hint: You may find the following relationship useful:}\\ 1/(A - Bx) - 1/(A + Bx) = 2Bx/(A^2 - B^2x^2)\end{array}\right]$$

   (e) Describe how atoms can be cooled to below the Doppler limit in two counter-propagating laser beams, where one beam has vertical linear polarisation and the other has horizontal linear polarisation. [10 marks]

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SECTION B: FOURIER OPTICS

3. (a) Two functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ have Fourier transforms $\mathrm{F}(u)$ and $\mathrm{G}(u)$, respectively. What are the Fourier transforms of
(i) $\mathrm{f}(x)+\mathrm{g}(x)$,
(ii) $\mathrm{f}(x)-\mathrm{g}(x)$,
(iii) $\mathrm{f}(x)\times\mathrm{g}(x)$, and
(iv) $\mathrm{f}(x)*\mathrm{g}(x)$? [4 marks]

(b) An aperture in the form of a one-dimensional diffraction grating with period d is studied. The transmission function of the aperture is

$$T(x)=0.5+0.4\cos\left(2\pi\frac{x}{d}\right).$$

A lens is used to view the Fourier transform of the grating. Show that the Fourier plane will contain three spots, and calculate their relative intensities. [4 marks]

(c) Three circular pinholes of radius a in an opaque mask have their centres at $(x,y)=(-b,0)$, $(0,0)$ and $(b,0)$ respectively, with $b\geq 2a$. Sketch the form of the Fraunhofer intensity pattern observed along the x and y axes when the mask is illuminated normally with plane radiation of wavelength λ. [4 marks]

(d) Explain, briefly, what you understand by the concept of apodization. Show that the peak intensity of the diffraction pattern of an apodized function has to be less than that obtained with the unapodized aperture. [4 marks] [Hint: use the central ordinate theorem.]

(e) A mode-locked pulse train is achieved by using an Ar^{+} ion laser of cavity length $L=2.00$ m. The gain bandwidth has a gaussian profile and a standard deviation of $2\pi\times 1.0$ GHz. Calculate the separation of the pulses in the mode-locked train, and estimate the temporal duration of the pulses. [4 marks]

(f) A Michelson interferometer is illuminated with two monochromatic lasers of different wavelengths. Sketch the form of the interferogram (the total intensity at the output) as a function of path-length difference, and explain your result. [4 marks]

(g) A gaussian laser beam has a waist of 0.750 mm and a wavelength of 514 nm. Calculate the Rayleigh range and the propagation distance (in multiples of Rayleigh ranges) at which the central intensity falls to a tenth of its initial value. [4 marks]

4. (a) Describe the principles of a spatial filtering experiment in optics. Draw a diagram to show the optical layout, clearly labelling the object, image and transform planes. [6 marks]

(b) An aperture in the form of the negative of the letter `I` is used as the object in a spatial filtering experiment, i.e. the object is opaque everywhere except for the shape of the letter which is transparent. Interpret what is seen in (i) the transform plane, and (ii) the image plane. [6 marks]

(c) Design a filter that can be inserted in the transform plane in order to remove the vertical bar from the letter `I` in the image. Show your design in a sketch. [4 marks]

(d) Appraise whether it is possible to remove just one horizontal line in the image, i.e. form a T, by inserting a mask in the transform plane. Explain your reasoning. [4 marks]

(e) A different wavelength laser is used to perform the experiment. Reason as to how the location of the features in the transform and image planes change. [4 marks]

(f) A transmission diffraction grating is inserted into the transform plane. Evaluate what image is seen in the image plane, explicitly stating what properties of Fourier transforms are exploited in this set up. [6 marks]