Young's Interferometer

31 January 2020 14:23

Optics f2f section 3.6

- Interference with paraxial waves
  - 2 holes
  - 3 holes
  - N holes
  - $\infty \longrightarrow$ FRESNEL INTEGRAL $\rightarrow$ light propagation

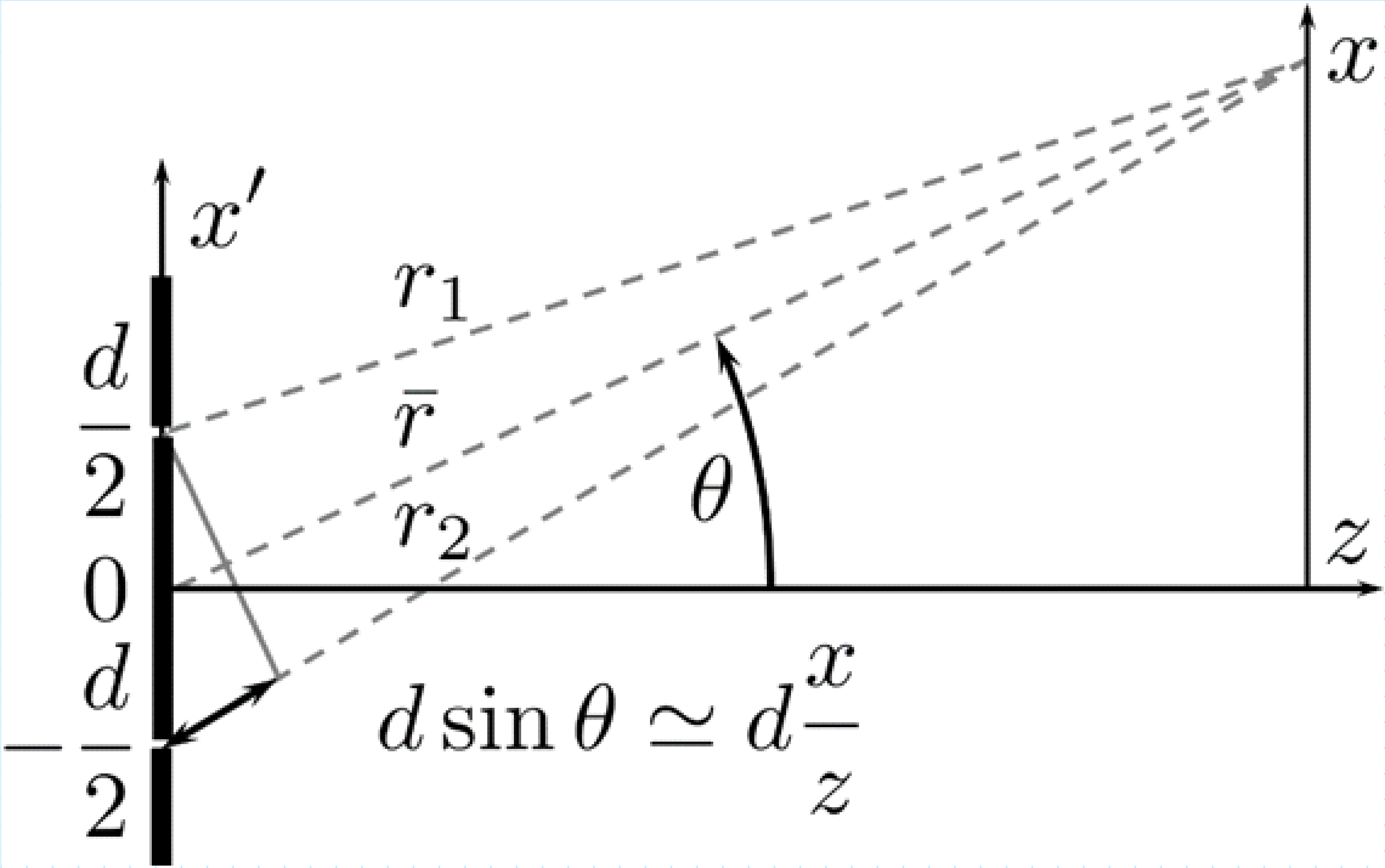
- Also conceptually & historically important

  (still 4 papers in last 12 months!)

  (on arXiv)

Young's Interferometer

31 January 2020 14:33



Lecture 6 - Young's Interferometer

31 January 2020 14:34

Holes at $(x', y') = (\pm d/2, 0)$

Diameter $D \ll d$ (finite width later)

Each slit is source of paraxial spherical wave

$\therefore$ at (x, z) $(z \gg d)$

$$E = E_s \frac{e^{ikr_1}}{ikr_1} + E_s \frac{e^{ikr_2}}{ikr_2}$$

$e^{i\omega t}$ terms cancel in I

$\Rightarrow$ neglect

APPROX ①

As $z \gg d$, $\frac{1}{r_1} \simeq \frac{1}{r_2} \simeq \frac{1}{\bar{r}}$

where

$$\bar{r} = z + \frac{(x^2 + y^2)}{2z}$$

(PARAXIAL APPROX)

where $\bar{r}$ is paraxial distance between $(0,0,0)$ & (x,y,z)

THEN

$$E = \bar{E}_s \underbrace{\left(e^{ikr_1} + e^{ikr_2}\right)}_{\text{TWO PHASOR SUM}} \quad \text{with} \quad \bar{E}_s = \frac{E_s}{ik\bar{r}} \qquad (A)$$

APPROX ②

Fresnel approximation for r_1 & r_2

Recall distance between $(x,0)$ & (x,z) is

$$r' = \left[(x-x')^2 + z^2\right]^{1/2} \approx z + \frac{(x-x')^2}{2z}$$

$$= z + \frac{x^2 - 2x'x + x'^2}{2}$$

Lecture 6 Young's Interferometer

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$$x = d/2$$

$$\bar{r} = z + \frac{x^2}{2z}$$

SO: $$r_1 = \bar{r} - \frac{xd}{2z} + \frac{d^2}{8z} \quad (B)$$

$$r_2 = \bar{r} + \frac{xd}{2z} + \frac{d^2}{8z} \quad (C)$$

AND

$$E = E_s e^{ik(\bar{r} + d^2/8z)} \left[e^{ik\frac{dx}{2}} + e^{-ik\frac{dx}{2}} \right]$$

(sub B & C into A

$$= E_s \underbrace{e^{ik(\bar{r} + d^2/8z)}}_{\text{GLOBAL PHASE}} \underbrace{\cos\left(\frac{kdx}{2z}\right)}_{\text{RELATIVE PHASE}}$$

DEMO!

Lecture 6 Young's Interferometer

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$$I \propto EE^* \quad \therefore I = \bar{I}_s \cos^2\left(\frac{Kdx}{2z}\right)$$

where $\bar{I}_s = \dfrac{I_s}{(K\bar{r}^2)}$ on-axis I for single slit

- $\dfrac{dx}{2z}$ is PATH DIFFERENCE in small angle approx
  $d\sin\theta \simeq \dfrac{dx}{z}$

- Fringe spacing $\dfrac{1}{u_0} = \left(\dfrac{\lambda}{d} z\right)$ FRINGES SPREAD OUT

  Spatial frequency → u_0

Lecture 6: Three waves (outlook)

31 January 2020 16:03

Optics f2f 3.6

GENERALISING to three holed is straightforward

$$E = \overline{E}_s e^{ikr}\left(e^{-ik\frac{dx}{z}} + 1 + e^{+ik\frac{dx}{z}} \right)$$

slits at $\begin{pmatrix} +d \\ 0 \\ -d \end{pmatrix}$

bottom middle top

(Here $\frac{d^2}{z}$ terms neglected: FRAUENHOFER APPROX)

3-PHASOR sum

$$I_s = \overline{I}_s \left[1 + 2\cos\left(\frac{kdx}{z}\right) \right]^2$$

not sinusoidal