Lecture 7: The Cosmic Microwave Background

7.1 The Properties and origin of the CMB

[Liddle sec:9.1/10.1 and 9.2/10.2]

The Cosmic Microwave Background was discovered in 1966 and can be seen in all directions in the sky.

It has a perfect Black-Body spectrum with $T = 2.728 \pm 0.004\,\mathrm{K}$ and is almost isotropic.

There is a dipole variation of T across the sky of $\Delta T = 5.4\mathrm{mK}$. This is interpreted as a Doppler shift due to our motion through the CMB

$$\frac{\Delta T}{T} = \frac{v}{c} \quad \Longrightarrow \quad v \approx 600\ \mathrm{km\,s^{-1}}$$

If this is subtracted the CMB is uniform to $\frac{\Delta T}{T} \leq 10^{-5}$.

For black-body radiation

$$I(T,\nu)\,d\nu = \frac{8\pi h\nu^3}{c^3}\left[\exp\left(\frac{h\nu}{k_B T}\right) - 1\right]^{-1} d\nu$$

Using the observed CMB temperature and standard results for the BB spectrum we find that the mean energy of CMB photons is

$$E_{\gamma,0} = 3kT_0 = 7.05 \times 10^{-4}\,\mathrm{eV} = 1.13 \times 10^{-22}\,\mathrm{J}$$

and the their total energy density

$$\epsilon_{\mathrm{phot},0} = \frac{4\sigma T_0^4}{c} = 4.19 \times 10^{-14}\,\mathrm{Jm^{-3}}.$$

This means the CMB photon number density is

$$n_{\gamma,0} = 3.71 \times 10^8\,\mathrm{m^{-3}}.$$

This is huge compared to the density of baryons (neutrons and protons). Taking $\Omega_{b,0} \approx 0.04$, $n_{\mathrm{b},0} = 0.255\,\mathrm{m^{-3}}$ and so

$$n_\gamma / n_{\mathrm{b}} \approx 1.46 \times 10^9.$$

As the universe expands the number of photons is conserved, but their frequencies and energies are redshifted according to $\nu \propto 1/\lambda \propto 1/a$ and therefore $T \propto 1/a \propto (1+z)$.

Note that under the simultaneous transformation of $\nu \propto 1/\lambda \propto 1/a$ the spectrum remains exactly of the BB form, ie simply BB radiation of higher temperature. For example, consider what the spectrum would look like at $a = 1/2$: $\nu \to 2\nu$ and $T \to 2T$ so that the spectrum becomes $I(T',\nu')d\nu' \equiv I(2T, 2\nu)d(2\nu) = 16\,I(T,\nu)d\nu$; ie. the new spectrum is also a blackbody, but with 16 times the energy density of the original spectrum.

7.1.1 The Temperature and Redshift of "Recombination" [Liddle sec:9.3/10.3]

As we go back to higher redshift (earlier times) the radiation becomes progessively hotter until eventually it is hot enough to ionize hydrogen. Above this redshift, $z_{\rm rec}$, the universe will be ionized and photons will scatter efficiently off the e^- and p. The universe will be opaque and the plasma will be in thermodynamic equilibrium.

Hence the CMB is thermal radiation from the Big Bang that decoupled when the universe (re)combined and has travelled from this **last scattering surface** to us without any further interactions.

We can attempt to estimate of the redshift of this recombination epoch by calculating at what redshift the mean photon energy was equal to the 13.6eV necessary to ionize hydrogen. Using

$$3kT = 13.6\,\mathrm{eV}$$

and

$$T = 2.728(1+z)\,\mathrm{K}$$

gives $T = 52\,600$K and $z_{\rm rec} = 19\,300$. However this estimate is in error as we have not taken into account that there are far more photons in the Universe than there are hydrogren atoms. In the previous section we estimated that there are about 10^9 photons for every baryon. Thus even if the typical photon energy is well below 13.6eV there can still be enough photons in the high energy tail of the Boltzmann distribution to ionize every hydrogen atom. We can make a rough estimate by approximating the BB spectrum at high energy as proportional to $\exp(-E/kT)$ by finding the energy at which this factor to reduces the abundance by 10^9, i.e.

$$\exp(-E/kT) = 10^{-9} \qquad \Rightarrow \qquad E = 13.6\ \mathrm{eV} = \ln(10^9)kT.$$

This gives $T = 7\,600$K. This is still a crude estimate and the true value is much closer to $T = 3\,000$K corresponding to a recombination redshift of $z \approx 1000$. To get this accurate estimate one needs to consider the distribution functions of hydrogen, free protons and electrons and then solve the Saha equation to determine the equillibrium abundance (see [Liddle sec:10.4]).

7.1.2 Distance to last Scattering Surface

We can compute the comoving distance between us and the last scattering surface using (4.6)

$$r_{\mathrm{ls}} = \frac{c}{\mathrm{H}_0} \int_0^{z_{\mathrm{rec}}} \frac{\mathrm{d}z}{(1+\Omega_0 z)^{1/2}(1+z)}. \tag{7.1}$$

If we assume $\Omega = 1$ this integral gives

$$r_{\mathrm{ls}} = \frac{2c}{\mathrm{H}_0}\left[1-(1+z_{\mathrm{rec}})^{-1/2}\right]$$

and so for $z_{\mathrm{rec}} \approx 1000$ $r_{\mathrm{ls}} \approx 2c/\mathrm{H}_0 \approx 8000\,\mathrm{Mpc}$

If $\Omega_0 < 1$ the denominator in (7.1) is smaller and so integral is larger. For $\Omega_0 = 0.2$ $r_{\mathrm{ls}} \approx 3.14c/H_0 \approx 12\,600$ Mpc.

7.2 The Radiation and Matter Dominated Eras

[Liddle sec:4.3/5.3]

The energy density in black-body radiation is

$$\epsilon_{\mathrm{phot}} = \rho_\gamma c^2 = \frac{4\sigma T^4}{c}$$

where σ is Stefan's constant. Thus we note

$$\Omega_{\gamma,0} \equiv \rho_{\gamma,0}/\rho_{\mathrm{crit},0} \approx 4.4\times 10^{-5} \ll 1.$$

However since $T \propto (1+z)$ we have $\rho_\gamma \propto (1+z)^4$, while $\rho_{\mathrm{mass}} \propto (1+z)^3$. Thus

$$\frac{\rho_\gamma}{\rho_{\mathrm{mass}}} \propto (1+z)$$

and at very high redshift $\rho_\gamma > \rho_{\mathrm{mass}}$ and the universe will be radiation dominated. We would estimate that the two energy densities should be equal at

$$(1+z_{\mathrm{eq}}) \approx 22,700\ \Omega_{\mathrm{mass},0}\left(\frac{H_0}{75\,\mathrm{km\,s^{-1}Mpc^{-1}}}\right)^2.$$

However as we shall see in Lecture 15 that as well as the CMB one predicts that there is also an (undetected) neutrino background which contributes to the energy density in the relativistic component.

$$\rho_{\mathrm{rel}} = \rho_\gamma(1+3\times 7/8\times(4/11)^{4/3}) = 1.68\rho_\gamma$$

$$(1+z_{\mathrm{eq}}) \approx 13,500\ \Omega_{\mathrm{mass},0}\left(\frac{H_0}{75\,\mathrm{km\,s^{-1}Mpc^{-1}}}\right)^2.$$

Examples

7.1 The present temperature of the Cosmic Microwave Background radiation is 2.73 K. Estimate the redshift at which the radiation was sufficiently hot to fully ionize Helium.

[The energy required to convert He^{+} to He^{2+} is 54.4 eV. The mass fraction in Helium-4 may be assumed to be 22%.]

7.2 If the energy density in the 2.73 K CMB is currently 1.5×10^{-4} times that in ordinary non-relativistic matter, calculate the redshift at which the combined energy density in the CMB & relativistic particles equals that in non-relativistic particles, and the temperature of the CMB at this redshift. [Assume that the present-day energy density in relativistic particles is 0.68 times that of the CMB.]