Level 2 Stars

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Problem Set S.5

The period of radial pulsations of a star in the instability strip can be shown to be approximately,

$$\Pi \approx \sqrt{\frac{3\pi}{2\gamma G\rho}},$$

where Π is the period, γ is the ratio of the specific heats (and may be assumed to have a value of 5/3) and ρ is the density.

a) What physical model for the period of radial pulsations would allow you to derive the above formula? State what simplifying assumptions you would make. [3 marks]

Solution

The model assumes that the period of radial pulsations is related to the time taken for an adiabatic sound wave to cross the diameter of the star. [2 marks]

For a simple derivation it is necessary to assume the star has a constant density. [1 mark]

b) Beta Doradus is a Cepheid variable star. It has a pulsation period of 16 days, a mass 6.5 times that of the Sun, and a surface temperature of 6000 K. Altair is a Delta Scuti variable star. It has a pulsation period of 2.7 hours, a mass 1.8 times that of the Sun and a surface temperature of 7000K. What is the ratio of the luminosities of the two stars? Note, in your calculation you should use the formula given above. [7 marks]

Solution

The key to the answer of this question is calculating the radius of each star since that will then allow the luminosities of the stars to be calculated using the well-known luminosity equation. The radius can be calculated from the mass and the density and the density can be obtained from the pulsation periods of the stars.

$\rho = \dfrac{3\pi}{\Pi^2 2\gamma G}$ therefore

$$\rho_{BD} = \frac{3\pi}{\Pi^2 2\gamma G} = \frac{3\times\pi}{\left(16\times24\times3600\right)^2\times2\times\left(5/3\right)\times6.67\times10^{-11}} = 0.022kgm^{-3}$$ *[1 mark]*

$$\rho_{AL} = \frac{3\pi}{\Pi^2 2\gamma G} = \frac{3\times\pi}{\left(2.7\times3600\right)^2\times2\times\left(5/3\right)\times6.67\times10^{-11}} = 450kgm^{-3}$$ *[1 mark]*

The radii can now be easily calculated:

$$R_{BD} = \left(\frac{M_{BD}}{4/3\,\pi\rho_{BD}}\right)^{1/3} = \left(\frac{6.5\times1.99\times10^{30}}{4/3\times\pi\times0.022}\right)^{1/3} = 5.2\times10^{10}m$$ *[1 mark]*

$$R_{AL} = \left(\frac{M_{AL}}{4/3\,\pi\rho_{AL}}\right)^{1/3} = \left(\frac{1.8\times1.99\times10^{30}}{4/3\times\pi\times450}\right)^{1/3} = 1.2\times10^{9}m$$ *[1 mark]*

There is now enough information to calculate the luminosities of the stars:

$$L_{BD} = 4\pi R_{BD}^2\sigma T_{BD}^4 = 4\times\pi\times\left(5.2\times10^{10}\right)^2\times5.67\times10^{-8}\times6000^4 = 2.5\times10^{30}W$$ *[1 mark]*

$$L_{AL} = 4\pi R_{AL}^2\sigma T_{AL}^4 = 4\times\pi\times\left(1.2\times10^{9}\right)^2\times5.67\times10^{-8}\times7000^4 = 2.5\times10^{27}W$$ *[1 mark]*

The ratio of luminosities is therefore:

$$\frac{L_{BD}}{L_{AL}} = \frac{2.5\times10^{30}}{2.5\times10^{27}} = 1000$$ *[1 mark]*

[1 mark for getting the correct result for each of the 7 steps above for a maximum of 7 marks; if obtained the correct numerical answer and took the same overall approach, but didn't calculate the individual densities, radii, and luminosities, then still get 7 marks]