Appendix A: Multiplying matrices, column vectors and row vectors

This appendix is just a brief reminder of how to multiply two matrices, a column vector by a matrix, a column vector by a row vector, or a matrix by a row vector. The rule is always the same: one multiplies each element of a row by each element of a column and sum up the products.

1. Multiplying two square matrices. The result is a square matrix:

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \alpha' & \beta' \\ \gamma' & \delta' \end{pmatrix} = \begin{pmatrix} \alpha\alpha' + \beta\gamma' & \alpha\beta' + \beta\delta' \\ \gamma\alpha' + \delta\gamma' & \gamma\beta' + \delta\delta' \end{pmatrix}. \tag{A.1}$$

2. Multiplying a column vector by a matrix. The result is a column vector:

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \alpha a + \beta b \\ \gamma a + \delta b \end{pmatrix}. \tag{A.2}$$

3. Multiplying a column vector by a row vector (the row vector on the left of the column vector). The result is a number:

$$\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} a' \\ b' \end{pmatrix} = aa' + bb'. \tag{A.3}$$

4. Multiplying a matrix by a row vector (the row vector on the left of the column vector). The result is a row vector:

$$\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} a\alpha + b\gamma & a\beta + b\delta \end{pmatrix}. \tag{A.4}$$

5. Multiplying a row vector by a column vector (the row vector on the right of the column vector). The result is a matrix:

$$\begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} a' & b' \end{pmatrix} = \begin{pmatrix} aa' & ab' \\ ba' & bb' \end{pmatrix}. \tag{A.5}$$

Recall, also, that multiplying a matrix by a scalar (a number) amounts to multiplying each element of the matrix by that scalar. E.g.,

$$c \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} c\alpha & c\beta \\ c\gamma & c\delta \end{pmatrix}. \tag{A.6}$$