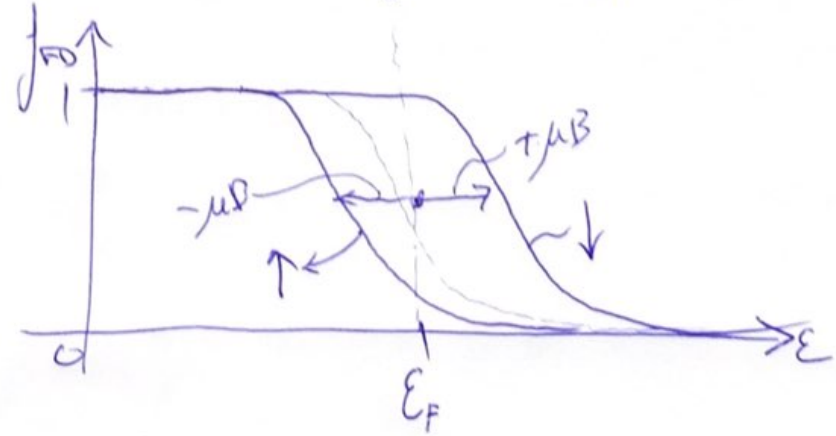
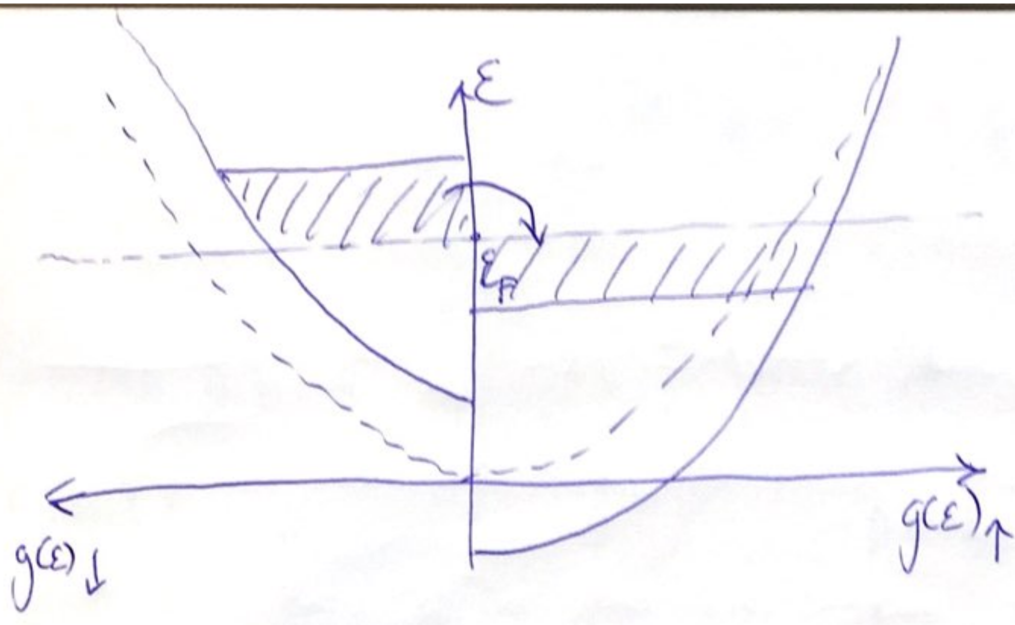
Stat. Phys Applications

Magnetism (paramagnetism). Electrons have spin and respond to a magnetic field. If on a lattice they align parallel or antiparallel to the field. We did this for a two-level system with energy $\varepsilon = \pm \mu B$ we obtained $M = N\mu \tanh(\beta \mu B)$. However if the electrons are not localised then they will follow FD statistics - for an applied B we shift the distributions by $\pm \mu B$





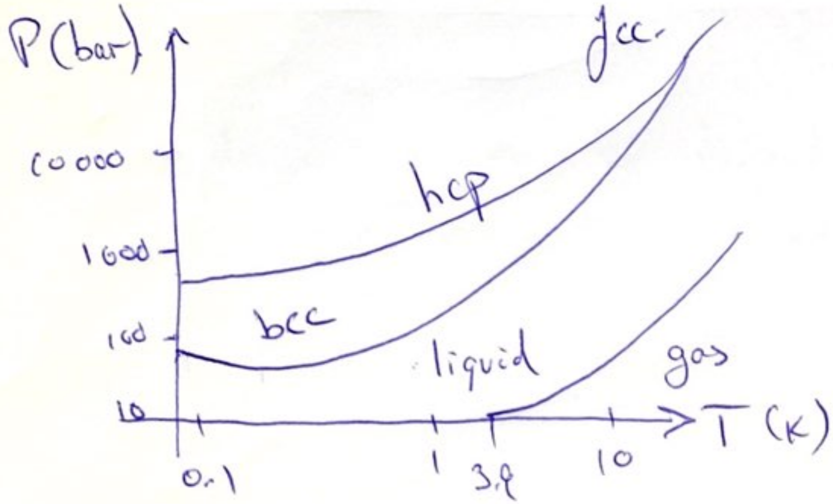
Excess "excited" spin down electrons flip (energy from B) to occupy the otherwise empty spin-up states

This gives excess spin up over spin down thus forms macroscopic magnetic moment. Magnetism is given by the number of flipped electrons within the energy range.

$$\varepsilon_F - \mu B \le \varepsilon \le \varepsilon_F$$

$$M = \tfrac{1}{2}\, g(\varepsilon_F)\, \mu B \cdot 2\mu = \frac{3N}{2\varepsilon_F}\, \mu^2 B.$$

Helium-3 Consists of 5 fermions (2p + n + 2e) and so is a compound fermion. As a gas (calculate λ_D) it is "classical" so MB stats is appropriate, however at low T it's more "interesting".

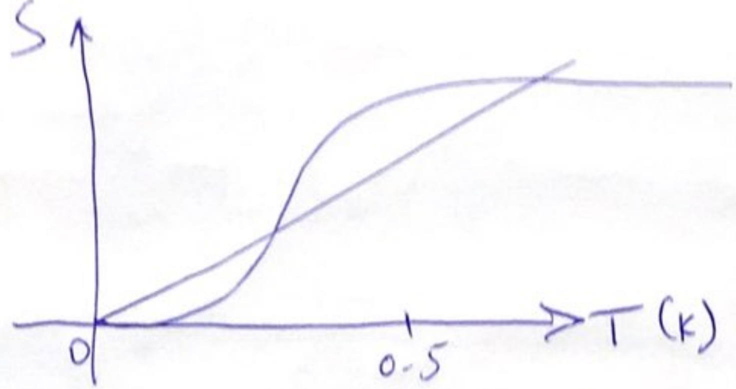


At low T and low P, it remains liquid even down to ~T = 0. Why?

Zero point motion.

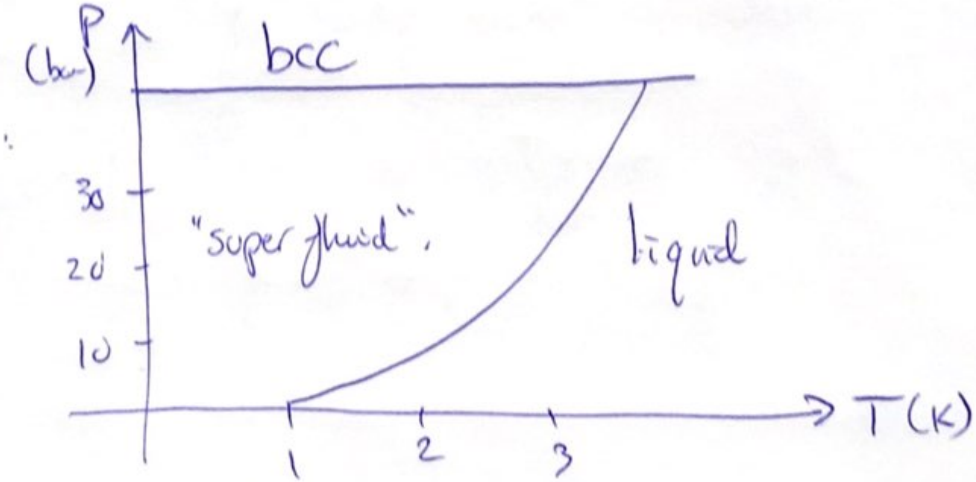
What is the entropy of a system at T = 0? It is S = 0 at T = 0. Therefore ^{3}He at T ~ 0 K is a liquid with no entropy!

Entropy curves



A region of $S^{\text{liquid}} < S^{\text{solid}}$.

Phase diagram at very low temperature:



The ^{3}He atoms "pair up" (very weak attraction)

$$2 \times {}^3He = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow\downarrow + \downarrow\uparrow\rangle$$ ~ this is a Boson.

Bose - Einstein Statistics

The BE distribution function is

$$f_{BE}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon - \mu)} - 1} \quad \text{where } e^{\alpha} = e^{-\beta\mu}$$

for particle number conservation.

This gives us the number of particles $f_{BE}(\varepsilon_i)$ with energy ε_i or the number of bosons with energy between $\varepsilon \to \varepsilon + d\varepsilon$ having.

$$N = \int g(\varepsilon) f_{BE}(\varepsilon)\, d\varepsilon .$$

The chemical potential - the distribution must be non-negative so

$$e^{-\beta\mu} e^{\beta\varepsilon} > 1 \Rightarrow -\beta\mu + \beta\varepsilon > 0 \Rightarrow \mu < \varepsilon, \ \forall \varepsilon.$$

Hence $\mu(T) < 0$. Physically for non-interacting bosons at $\sim T=0$ it is energetically favourable for all (N) particles to occupy the single particle state with the lowest energy. (let $\varepsilon=0$ for this) so.

$$T=0 \Rightarrow f_{BE}(0) = N \Rightarrow \frac{1}{e^{-\beta\mu}-1} = N.$$

For low enough T a macroscopic number of bosons $n_0(T)$ occupy the $\varepsilon=0$ energy state. Let T_B denote the temperature below which $n_0(T<T_B)$ is macroscopic.

Here "macroscopic" means $n_0 \sim N$ or $n_0(T) \gg 1$.

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(6)
(7)

For $T < T_B$ we have

$$\frac{1}{e^{-\beta\mu(T)} - 1} = n_0(T) \Rightarrow e^{-\beta\mu(T)} = 1 + 1/n_0(T)$$

$$\Rightarrow -\beta\mu(T) = \ln\left(1 + 1/n_0(T)\right) \simeq 1/n_0(T) \quad \left[\ln(1+x) \simeq x\right]$$

$$\mu(T) = -k_B T/n_0(T) \simeq 0.$$

Because N is very large (thermodynamic limit) then $n_0(T) \sim N$.