(a) The advantages of a CCD over photographic plate are:

The linearity of the response

High quantum efficiency

High dynamic range

No damage from over exposure

Photon arrival rate unimportant

Excellent stability and repeatability

No chemical processing required

[1 mark for each answer to a maximum of 4; *Knowledge/Analysis*]

(b) (i) The effective focal length of the telescope is:

$f = \text{f-ratio} \times \text{diameter}$

$f = 4.00 \times 30$

$\underline{f = 120\,\text{m}}$ [1 mark; *Knowledge/Application*]

(ii) The plate scale of a telescope ($d\theta \,/\, dS$) is related to the effective focal length, (f) by:

$d\theta \,/\, dS = 1 \,/\, f$ [1 mark *Knowledge*]

For this telescope with $f = 120\,\text{m}$,

$d\theta \,/\, dS = 1 \,/\, 120\,\text{radian}\,\text{m}^{-1}$

$d\theta \,/\, dS = 0.00833 \times 206265 \,/\, 1000\ ''\,\text{mm}^{-1}$ [1 mark; *Application*]

$\underline{d\theta \,/\, dS = 1.72''\,\text{mm}^{-1}}$ [1 mark; *Application*]

(c) (i) If Vega is just resolved, then

$\theta = \lambda \,/\, D$

$\theta = 700 \times 10^{-9} \,/\, 55.0 \times 206265 \times (1 \times 10^{3})$

$\underline{\theta = 2.65\,\text{milli-arcseconds}}$ [1 mark *Knowledge/Application*]

(ii) The relation between parallax ($\phi['']$) and distance (d[pc]) is

$d[\text{pc}] = 1 \,/\, \phi$

$d[\text{pc}] = 1 \,/\, 0.12$

$\underline{d[\text{pc}] = 8.33\,\text{pc}}$ [1 mark; *Application*]

(iii) For a distance of $d = 8.33\,\text{pc}$ and angular diameter of $\theta = 2.65\,\text{milli-arcseonods}$, the radius, R, is

$\text{R} = d \times (\theta \,/ 2)$ [1 mark; *Knowledge*]

$\mathrm{R} = 8.33\,\mathrm{pc} \times (2.65\,/\,2)\,\mathrm{mas}$
$\mathrm{R} = (8.33 \times 3.09 \times 16) \times (1.325 \times 10^{-3}\,/\,206265)\,/\,7.00 \times 10^{8}$
$\underline{\mathrm{R} = 2.36\,\mathrm{R}_{\odot}}$ [1 mark; *Application*]

(a) One mark for each of the following effects (up to 4 marks) and one mark for each means of minimising the effect (up to 4 marks) will be awarded.

(i) Sky noise – observe in dark conditions (e.g. no moon). Observe away from light pollution. [2 marks; *Comprehension/Analysis/Knowledge*]

(ii) Atmospheric turbulence – observe with adaptive optics. [2 marks; *Comprehension/Analysis/Knowledge*]

(iii) Atmospheric absorption – observe in the dryest conditions. Observe a target when it is as high in the sky as possible. [2 marks; *Comprehension/Analysis/Knowledge*]

(iv) Atmosheric refraction – observe targets as close to zenith as possible. [2 marks; *Comprehension/Analysis/Knowledge*]

(v) All effects can be reduced by observing on a high site [1 mark; *Comprehension/Analysis/Knowledge*]

(b) Dark current is measured by recording an image with the same exposure time as the science observation, but without opening the camera shutter. [1 mark; *Knowledge*]

Sky background is measured by observing the sky adjacent to the science object. [1 mark; *Knowledge*]

The flat field is measured by observing a uniform field (e.g. twlight sky or dithered image of sky). [1 mark;*Knowledge*]

Corrected data = (raw image - sky background) / (flatfield - dark) [1 mark *Knowledge*]

(c) The maximum time, t, the asteroid falls in a single pixel is

$t = 1'' \,/\, 0.02''\,\mathrm{second}^{-1}$

$\underline{t = 50\,\mathrm{s}}$ [1 mark; *Analysis/Application*]

Full marks awarded if students instead use diagonal of pixel, with $t = \sqrt{2} \,/\, 0.02 = 70.7$

To calculate the number of photons detected, first calculate the flux density ($f_{21.4}$) from a star with $\mathrm{m_V} = 21.4$.

$\mathrm{m_V} - \mathrm{m_0} = -2.5 \times \log(f_{21.4} \,/\, f_0)$

where $\mathrm{m}_{21.4} = 21.4$, $\mathrm{m_0} = 0$, and $f_0 = 3.92 \times 10^{-8}\,\mathrm{W\,m^{-2}\,\mu m^{-1}}$

$f_{21.4} = 3.92 \times 10^{-8} \times 10^{-21.4/2.5}$

$\underline{f_{21.4} = 1.07 \times 10^{-16}\,\mathrm{W\,m^{-2}\,\mu m^{-1}}}$ [1 mark; *Knowledge/Analysis*]

Next, calculate the total power collected by the telescope. If D is the telescope diameter ($\mathrm{D} = 8\,\mathrm{m}$), $\Delta\lambda$ is the bandpass ($\Delta\lambda = 0.05\mu\mathrm{m}$) and *Eff* is the combine efficiency of the atmosphere and CCD ($\mathit{Eff} = 0.5$)

$\mathrm{P} = f_{21.4} \times \pi\,(\mathrm{D}\,/\,2)^2 \times \Delta\lambda \times \mathit{Eff}$

$\mathrm{P} = 1.07 \times 10^{-16} \times \pi\,(8.00\,/\,2)^2 \times 0.05 \times 0.5$

$\underline{\mathrm{P} = 1.36 \times 10^{-16}\,\mathrm{W}}$ [1 mark; *Knowledge/Analysis*]

The total number of detected photons per second is Power / Photon Energy.

The photon energy, E, is:

$\mathrm{E} = h\,\mathrm{c}\,/\,\lambda$

$\mathrm{E} = h\,\mathrm{c}\,/\,0.550\,\mu\mathrm{m}$

$\underline{\mathrm{E} = 3.62 \times 10^{-19}\,\mathrm{J}}$ [1 mark; *Knowledge/Application*]

The total number of photons per second from the asteroid, $\dot{n}_{\mathrm{ast}}$, is:

$\dot{n}_{\mathrm{ast}} = \mathrm{P}\,/\,\mathrm{E}$

$\dot{n}_{\mathrm{ast}} = 1.36 \times 10^{-16}\,/\,3.62 \times 10^{-19}$

$\underline{\dot{n}_{\mathrm{ast}} = 377\,\mathrm{second}^{-1}}$ [1 mark; *Knowledge/Application*]

Finally, calculate the signal-to-noise (S / N). The sky surface brightness per $1 \times 1''$ pixel is the same as the target surface brightness per $1 \times 1''$ pixel, so:

$\dot{n}_{\mathrm{sky}} = \dot{n}_{\mathrm{ast}}$ [1 mark; ; *Application*]

$\mathrm{S}\,/\,\mathrm{N} = \dot{n}_{\mathrm{ast}} \times t\,/\,\sqrt{\dot{n}_{\mathrm{ast}} \times t + \dot{n}_{\mathrm{sky}} \times t}$ [1 mark; ; *Knowledge*]

$\mathrm{S}\,/\,\mathrm{N} = 377 \times 50\,/\,\sqrt{2 \times (377 \times 50)}$

$\mathrm{S}\,/\,\mathrm{N} = 18\,888\,/\,\sqrt{2 \times 18\,888}$

$\underline{\mathrm{S}\,/\,\mathrm{N} = 97}$ [1 mark ; *Application*]

full marks also awarded if students use have used the diagonal of pixel for exposure time calculation, in which case same calculation gives $S/N = 125$

L2, Stars and Galaxies 2019 exam

David Alexander

June, DMA, Q3

7 short questions:

a) A star is identified with a surface temperature of 6,000 K and a radius of 6,000 km. Calculate the luminosity of the star. Evaluate what type of star has been identified. Justify your reasoning. [4 marks]

Solution

$L = 4\pi R^2 \sigma T_e^4$ *and therefore*

$L = 3.3\times 10^{22} W$ *(i.e., 10^{-4} times the luminosity of the Sun).*

[Analysis: 2 marks for the luminosity calculation]

The temperature of the star is similar to that of the Sun but the luminosity is orders of magnitude lower. This indicates that the star is a white dwarf. The small radius of the star also supports this view.

[Synthesis: 2 marks for correctly identifying the nature of the star – lose 1 mark if sufficient reasoning is not given]

b) In an eclipsing binary system, the time taken for the light to drop from uneclipsed to fully eclipsed is 3.5 hrs. The relative velocity of the smaller star with respect to the larger star is 40 km s^{-1}. Calculate the radius of the smaller star. [4 marks]

Solution

$$R = \frac{v \times t}{2} = 2.5 \times 10^{8} m$$

[Application: 2 marks for method]

[Analysis: 2 marks for the correct numerical answer]

c) Calculate the temperature at which radiation pressure exceeds gas pressure in the core of a star. In your calculation assume a particle density of n=1.00x10^{31} m^{-3}.

[Hint: the radiation constant is given as $a = \frac{4\sigma}{c} = 7.57\times10^{-16}\,Jm^{-3}K^{-4}$]

Solution

$$P = nkT \qquad and \qquad P = \frac{1}{3}aT^4 \qquad where\ a = \frac{4\sigma}{c} = 7.57\times10^{-16}\,Jm^{-3}K^{-4}$$

so

$$nkT = \frac{1}{3}aT^4 \quad therefore \qquad T^3 = \frac{3nk}{a} \qquad and \qquad T = \left(\frac{3nk}{a}\right)^{1/3}$$

$$T = \left(\frac{3\times10^{31}\times1.38\times10^{-23}}{7.57\times10^{-16}}\right)^{1/3} = 8.18\times10^{7}\,K$$

[Application: 3 marks for method]

[Analysis: 1 mark for correct numerical answer]

d) A horizontal-branch star has a mass of 1.00 $M_\odot$, luminosity of 3.00 $L_\odot$, and a lifetime of 1.00 x 10^8 yrs. Assuming 7.59 MeV of energy is produced during each triple-alpha reaction, calculate the fraction of the mass of the star that is converted into carbon from helium over the horizontal-branch lifetime. [4 marks]

[Hint: 1 eV = 1.60 x 10^{-19} J; mass of helium: m_{He} = 6.65 x 10^{-27} kg]

Solution:

The energy release from the star in 10^8 years, assuming a constant luminosity of 3 times that of the Sun, is given by:

$$E = 10^8 \times 3.16 \times 10^7 \times 3 \times 3.84 \times 10^{26} = 3.64 \times 10^{42}\, J$$

The total amount of radiative energy that could be released from the star, assuming that all of its mass is converted from helium to carbon (which is 3 times the mass of helium) is:

$$E_{total} = \left(\frac{M}{3 \times m_{He}}\right) \times 7.59 MeV = \left(\frac{1.99 \times 10^{30}}{3 \times 6.65 \times 10^{-27}}\right) \times 7.59 \times 1.60 \times 10^{-19} \times 10^6 = 1.21 \times 10^{44}\, J$$

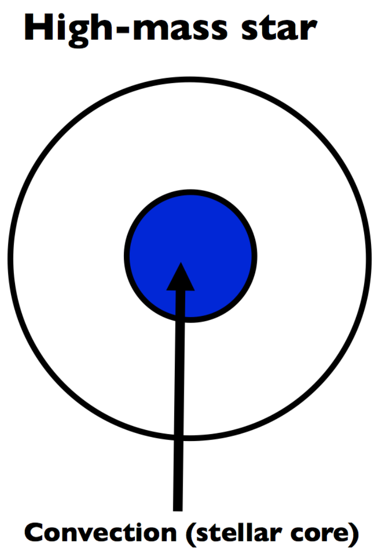
Therefore the fraction of the mass converted during the horizontal-branch lifetime of the star is simply:

$$f = \frac{E}{E_{total}} = \frac{3.64 \times 10^{42}}{1.21 \times 10^{44}} = 0.03 \;\; \textit{(i.e., \textasciitilde 3\%)}$$

[Application: 2 marks for the overall approach in terms of calculating the energy release from the star (the student doesn't need to use exactly the same approach as that shown here) and the total amount of energy that could be released from the star; 2 marks for the correct numerical answer]

e) The Schwarzschild condition for convection is satisfied in different regions for stars of different masses. Draw a cross section of a high-mass main-sequence star (~15 solar mass) and highlight where convection is believed to occur in this star. Why does convection occur in this region? [4 marks]

Solution



[Application: 2 marks]

Convection occurs when there is a steep temperature gradient. In high-mass stars this occurs in the central regions of the star. [Comprehension: 2 marks]

f) What are protostars and what physical process is largely responsible for generating their radiative output? Why are protostars challenging to identify at optical wavelengths? [4 marks]

Solution

Protostars are pre main sequence stars that are undergoing formation. The physical process that drives their radiative output is gravitational collapse; i.e., the conversion of potential energy into kinetic energy. Protostars are challenging to identify at optical wavelengths because they are often obscured by dust.

[Knowledge: 1 mark for the description of protostars, 1 mark for correct explanation of their luminosity output]

[Comprehension: 2 marks for correctly stating why protostars are difficult to identify at optical wavelengths]

g) List the four forms of pressure that prevent the collapse of a main-sequence star, a white dwarf star, and a neutron star. [4 marks]

Solution

Gas pressure and radiation pressure (predominantly for main-sequence stars)
Electron degeneracy pressure (predominantly for white dwarf stars)
Neutron degeneracy pressure (predominantly for neutron stars)

[Knowledge: 1 mark for each correct answer for 4 marks overall – note, the student does not need to associate the different forms of pressure to the different types of stars to get full marks]

L2, Stars and Galaxies 2019 exam

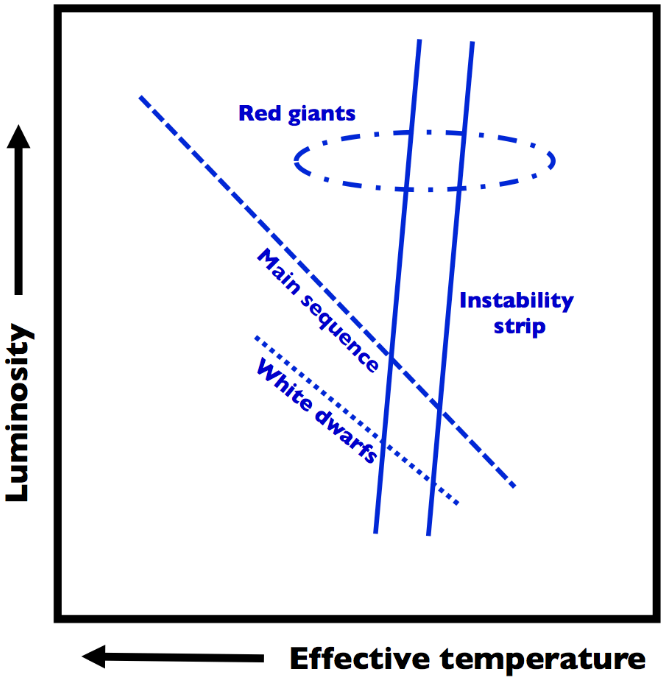
David Alexander

June, DMA, Q4

Long question:

a) The instability strip is the region on the Hertzsprung-Russell diagram, where the majority of pulsating stars are found to lie. Draw a Hertzsprung-Russell diagram and highlight the main sequence, the instability strip, and the regions where red giants and white dwarfs lie. [6 marks]

Solution:



[Application: 1 mark for giving the correct axes, 1 mark for drawing on the correct main sequence, and 2 marks for correctly highlighting the instability strip, which is broadly constant in temperature, 1 mark for the white dwarf region, and 1 mark for the red-giant region]

b) The period of radial pulsations of a star in the instability strip can be shown to be approximately

$$\Pi \approx \sqrt{\frac{3\pi}{2\gamma G\rho}},$$

where Π is the pulsation period, γ is the ratio of the specific heats, and ρ is the density. Assume γ=5/3 in your calculations.

i) Calculate the ratio of the densities of a white dwarf with an observed pulsation period of 3.0 seconds to that of a red giant star with a period of 3.0 years. [3 marks]

ii) The white dwarf star has a mass of 1.0 $M_\odot$ and a luminosity of 1.0 x 10^{23} W while the red giant star has a mass of 3.0 $M_\odot$ and a luminosity of 1.0 x 10^{31} W. Calculate the ratio of the surface temperatures of the two stars. State any simplifying assumptions you make in your calculation. [7 marks]

Solution:

For the white dwarf:

$$\rho = \frac{3\pi}{\Pi^2 2\gamma G} = \frac{3\pi}{(3.0)^2 \times 2 \times (5/3) \times 6.67 \times 10^{-11}} = 4.7 \times 10^9\, kgm^{-3}$$

For the red giant:

$$\rho = \frac{3\pi}{\Pi^2 2\gamma G} = \frac{3\pi}{(3.0 \times 3.16 \times 10^7)^2 \times 2 \times (5/3) \times 6.67 \times 10^{-11}} = 4.7 \times 10^{-6}\, kgm^{-3}$$

Therefore the ratio of densities is:

$$R(\rho) = \frac{4.7 \times 10^9}{4.7 \times 10^{-6}} = 10^{15}$$

[Analysis: 1 mark for each of the densities and 1 mark for the density ratio (3 marks if go straight to the ratio)]

Simplifying assumption: assume uniform density.

For the white dwarf:

$$R = \left(\frac{M}{4/3 \times \pi \times \rho}\right)^{1/3} = \left(\frac{1.99 \times 10^{30}}{4/3 \times \pi \times 4.7 \times 10^{9}}\right)^{1/3} = 4.7 \times 10^{6}\,m$$

$$T_e = \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4} = \left(\frac{10^{23}}{4 \times \pi \times (4.7 \times 10^{6})^2 \times 5.67 \times 10^{-8}}\right)^{1/4} = 8900\,K$$

For the red giant:

$$R = \left(\frac{M}{4/3 \times \pi \times \rho}\right)^{1/3} = \left(\frac{3.0 \times 1.99 \times 10^{30}}{4/3 \times \pi \times 4.7 \times 10^{-6}}\right)^{1/3} = 6.7 \times 10^{11}\,m$$

$$T_e = \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4} = \left(\frac{10^{31}}{4 \times \pi \times (6.7 \times 10^{11})^2 \times 5.67 \times 10^{-8}}\right)^{1/4} = 2400\,K$$

Therefore the ratio of temperatures is:

$$R(T) = \frac{8900}{2400} = 3.7$$

[Application: 1 mark for stating assuming uniform density, 1 mark for each radius (total of 2 marks), 1 mark for knowing how to manipulate the luminosity equation, 1 mark for each temperature (total of 2 marks), 1 mark for ratio. Full marks will be awarded if the student combines and rearranges the equations to directly calculate the ratio and gets the answer correct]

c) What physical process drives radial pulsations in stars and where in stars do the pulsations originate? Why don't all stars show strong radial pulsations? [4 marks]

Solution:

Radial pulsations originate in the partial ionisation zone. The partial ionisation of the hydrogen and helium gas in this region allows the stellar material to be strongly compressed – the energy produced through compression goes into ionising the gas rather than significantly raising the gas temperature and pressure. [Knowledge: 2 marks]

Stars with low surface temperatures do not pulsate radially because convection in the outer regions dampens the radial pulsations. [Comprehension: 1 mark]

Stars with high surface temperatures do not pulsate radially because the partial ionisation zone is near the surface and consequently there is insufficient material to drive significant radial pulsations. [Comprehension: 1 mark]

Galaxies -Answers to June short questions

(a) Spiral galaxies undergo star formation as evidenced by their blue colours: the blue light comes from massive hot stars, which have short life times (and hence formed recently). The red colours of elliptical galaxies suggest that much less star formation is going on. [2 marks, Analysis]
Therefore at a given $M_\star$, spiral galaxies have higher $\dot{M}_\star$ and hence this corresponds to region I. Conversely, elliptical galaxies correspond to region II. [2 marks, Synthesis]

(b) At high temperature, particles in a gas collide with high kinetic energy. [2 marks, Analysis]
When collisions are energetic, they can destroy fragile molecules.[2 marks, Comprehension]

(c) X-rays are produced by thermal Bremsstrahlung, which occurs when an electron passes near a nucleus and is deflected by the Coulomb interaction. [2 marks, Comprehension]
Simply stating the process is thermal Bremsstrahlung yields 1 mark.
Therefore the gas is mostly ionised. [2 marks, Analysis]

(d) Several were mentioned. Correct answers include: (a) presence of a very bright optical source (QSO), (b) presence of a bright radio source, (c) presence of a bright X-ray source. There are other correct answers. 1 mark per correct manifestation up to 3 marks. [3 marks, Knowledge]
The answer is not simply yes or no: the student should state that the MW's central BH is currently not bright in any of these wavelength ranges and so is not currently an AGN - but it might have been in the past. [1 mark, Synthesis]

(e) Accretion of mass onto a (supermassive) black hole. [2 marks, Knowledge]
The radiation from the AGN exerts radiation pressure onto the accreting gas. [1 mark, Synthesis]
When the outward force due to radiation pressure is greater than the inward pull due to gravity, gas can no longer accrete. Therefore a given

luminosity implies a minimum black hole mass. [1 mark, Synthesis]
Stating that the mass is limited by the Eddington limit yields 1 mark.

Long Questions June - answers

(a) The light distribution is such that there is much more light in the centre than in the outskirts. If mass followed light, then we would expect to see a falling rotation curve (Keplerian rotation). The missing mass is dark matter. [2 marks, Synthesis]

(b) The gravitational acceleration g at distance d on a test mass from a spherically symmetric density distribution only depends on the total enclosed mass M, and is radially inwards with magnitude $g = GM/d^2$. [1 mark, Application]
The acceleration along a circular orbit (radius d, circular velocity V_c) is radially outwards with magnitude $a = V_c^2/d$. [1 mark, Application]
These two must be equal for a test mass to remain on a circular orbit, hence $M = V_c^2 d/G$. [1 mark, Synthesis]
Substituting numerical values yields $M_{\rm DM} = 7.44 \times 10^{10} M_\odot$. [1 mark, Evaluation]

(c) In polar coordinates, $L = 2\pi \int_0^d R\Sigma(R)dR$. [2 marks, Analyse]
Defining $x \equiv R/R_c$ and $\delta \equiv d/R_c$, this can be re-written as

$$L = 2\pi\Sigma_\odot R_c^2 \exp(\delta) \int_0^\delta x \, \exp(-x)dx.$$

Using the hint, we find

$$L = 2\pi\Sigma_\odot R_c^2 \exp(\delta)(1 - \exp(-\delta) - \delta \exp(-\delta)).$$

[2 marks, Evaluation]

(d) Substitution in the relation given in (c) yields $L = 1.82 \times 10^{10} \mathrm{L}_\odot$. [2 marks, Evaluation]

(e) If all stars had the same mass and luminosity as the Sun, then $M_\star = 1.82 \times 10^{10} \mathrm{M}_\odot$. [2 marks, Synthesis]

(f) The mass-luminosity relation of stars is approximately $L \propto M^3$ - as the mass increases, luminosity increases much faster. [1 mark, Synthesis]
Therefore lower mass stars would have higher total mass for given luminosity. [1 mark, Synthesis]

(g) Comparing the enclosed masses, we find that the dynamical mass of $M_{\rm DM} = 7.44 \times 10^{10} M_\odot$ is larger than the stellar mass, $M_\star = 1.82 \times 10^{10}{\rm M}_\odot$. This therefore suggests that dark matter is present. This answer yields 1 mark, Synthesis.
However, the actual contribution that stars make to V_c is possibly uncertain enough to undermine the case for dark matter. Students need some arguments to make this case. (*i*) the same mass in a disc contributes more to the circular velocity than if the mass were in a spherical distribution - we would need to determine by how much. (*ii*) as has been hinted at in part (f), if the stars had on average slightly lower mass, then the stellar mass may be comparable to the dynamical mass. We also neglected the contribution from the bulge, and there may be mass in gas in addition to stars. 2 marks per reasoned argument, up to a maximum of 4 marks. [4 marks, Synthesis]