University of Durham
EXAMINATION PAPER

Examination session:	Year:	Examination code:
May/June	2019	PHYS3631-WE01

Title:
Foundations of Physics 3B

Time allowed:	3 hours		
Additional material provided:	None		
Materials permitted:	None		
Calculators permitted:	Yes	Models permitted:	Casio fx-83 GTPLUS or Casio fx-85 GTPLUS
Visiting students may use dictionaries:	No		

Instructions to candidates:

- Attempt **all** questions. The short-answer questions at the start of each section carry 50% of the total marks for the paper. The remaining 50% of the marks are carried by the longer questions, which are equally weighted.
- The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK.**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklets for Sections B and C, in order, inside your booklet for Section A, before they are collected by the invigilator.

Information

Section A: Statistical Physics
Section B: Condensed Matter Physics part 1
Section C: Condensed Matter Physics part 2

A list of physical constants is provided on the next page.

Revision:	

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Bohr magneton:	$\mu_\mathrm{B} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Electron mass:	$m_\mathrm{e} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_\mathrm{p} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}$ J s
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	$\mathrm{AU} = 1.50 \times 10^{11}$ m
Parsec:	$\mathrm{pc} = 3.09 \times 10^{16}$ m
Solar Mass:	$M_\odot = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26}$ W

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SECTION A: STATISTICAL PHYSICS

1. (a) A box contains 12 doughnuts, of which 9 have strawberry filling and 3 have chocolate filling. A person who prefers strawberry filling takes two doughnuts. What is the probability they get their preferred two strawberry doughnuts? [4 marks]

   (b) A set of single particle energy states, labeled j, have energy ϵ_j and degeneracy g_j. $N = \sum_j n_j$ Fermions are distributed amongst the states such that the internal energy of the system is a constant $U = \sum_j n_j \epsilon_j$. The number of microstates available are

$$\Omega_{FD} = \prod_j \frac{g_j!}{n_j!(g_j - n_j)!}.$$

   Derive an expression for the distribution of Fermions which maximises the entropy of the system. [4 marks]

   (c) The ground state of a hydrogen atom has energy -13.6 eV and the first excited state has energy -3.4 eV. Ignoring any other energy levels, at what temperature would we expect to find 0.1% of the hydrogen atoms in the excited state? [4 marks]

   (d) The 1-dimensional quantum harmonic oscillator of frequency ω has energies $\epsilon_j = (j + 1/2)\hbar\omega$. Show that the partition function of this system is

$$Z = \frac{1}{2\sinh\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)}.$$

   You may assume $1 + x + x^2 + ... = 1/(1 - x)$, for $|x| < 1$. [4 marks]

   (e) A free particle is in a 3-dimensional cubic box of side a. Determine the density of states in k-space, $g(k)\delta k$. Note that in one dimension the states are equally spaced in k-space with separation π/a. [4 marks]

   (f) A system contains non-degenerate states with energies 0, ϵ, 2ϵ, 3ϵ, ... 3 particles are distributed amongst these states such that the internal energy of the system is 3ϵ. What is the probability of finding the most likely distribution of particles in the states if the particles are (i) classical, (ii) Fermions and (iii) Bosons? [4 marks]

   (g) State what the equipartition theorem predicts for the heat capacity of a diatomic molecule. Sketch a graph of the specific heat capacity of a diatomic molecule as a function of temperature. Label and explain each of the physical features in the graph. [4 marks]

2. A molecule has 3 non-degenerate vibrational modes with frequencies ω, 2ω and 3ω.

   (a) Calculate the vibrational partition function of the molecule. [3 marks]

   (b) Calculate the probability of each of the modes being excited when the molecule is in contact with a heat bath at temperature T. [3 marks]

   (c) Determine the high and low temperature limits of the probabilities in (b) and sketch a graph showing the probabilities as a function of temperature. [8 marks]

   (d) What is the internal energy of the molecule at temperature T and what are the high and low temperature limits of the internal energy? [3 marks]

   (e) Calculate the free energy of the system. Describe what the difference is between internal energy and free energy. [3 marks]

SECTION B: CONDENSED MATTER PHYSICS part 1

3. (a) Show that the group velocity of an electron at the bottom of an energy band in the nearly-free electron model is given by

$$v_{\text{group}} = \frac{\hbar k}{m^*},$$

where $\hbar k$ is the crystal momentum of the Bloch electron and m^* is the effective mass of the electron. Use this to show that a completely filled band makes no contribution to the current carried by electrons in a crystal. [4 marks]

(b) If a magnetic field $\underline{B}$ is applied perpendicular to the plane of an electron orbit the diamagnetic response can be shown to be described as the change in magnetic moment,

$$\Delta \underline{m} = -\frac{(e^2 r^2 \underline{B})}{4m^*},$$

where r is the radius of the electron orbit and m^* is the effective mass of the electron. Explain how this change in magnetic moment leads to the Langevin result for the diamagnetic susceptibility of a solid,

$$\chi_{\text{d}} = -\frac{(\mu_0 N Z e^2)}{6m^*} < r^2 >,$$

composed of N atoms per unit volume each with Z electrons. [4 marks]

(c) Ignoring any effects of charge, calculate the energy difference between the spin states of an isolated electron in the presence of a magnetic field of $B = 10.0$ Tesla. [4 marks]

(d) In a ferromagnet, what are spin waves and ferromagnetic magnons? [4 marks]

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4. (a) Calculate the quantum numbers of the atomic spin S, the orbital angular momentum L, and the total angular momentum J of the ground state of the paramagnetic holmium ion Ho^{3+} ($4f^{10}$) stating any assumptions that you make. [8 marks]

(b) Calculate the magnitudes of the S, L and J angular momenta of the Ho^{3+} ion in its ground state (in units of $\hbar$). The effective g-factor, known as the Landé g-factor, is given by

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}.$$

What are the magnitudes of the three magnetic moments corresponding to S, L, and J (in units of the Bohr magneton μ_B)? [4 marks]

(c) State the number of energy levels that contribute to the magnetic moment of a Ho^{3+} ion in the presence of a magnetic field. Calculate the smallest energy separation between two of these levels if the applied magnetic field strength is $H = 2.0 \text{ x } 10^6 \text{ A m}^{-1}$. What frequency of electromagnetic radiation could be used to excite a transition between two of these levels? [4 marks]

(d) Assuming the ions are magnetically isolated from one another in the solid, and the same magnetic field strength as given in part (c), comment on the applicability of Curie's law to the paramagnetic susceptibility at 1 K and 300 K. [2 marks]

[Hint: Use the condition adopted to obtain Curie's law from the Brillouin function form of the magnetisation].

(e) Calculate the maximum measureable magnetic moment of a solid consisting of 1 mole of Ho^{3+} ions. [2 marks]

SECTION C: CONDENSED MATTER PHYSICS part 2

5. (a) Draw separate energy (E)- wavevector (k) diagrams for a direct and indirect band gap semiconductor. Clearly label the conduction band, valence band, and band gap in each diagram.

   Identify which of the above two semiconductors would be suitable as a photodiode for light detection. Explain your reasoning. [4 marks]

   (b) Sketch the variation of critical magnetic field with temperature for Type I and Type II superconductors, clearly identifying the phases in each diagram.

   For a solenoid with N turns per unit length and current I the magnetic field is $B = \mu_0 NI$. If the critical magnetic field of a Type I superconductor is 0.05 T, calculate the maximum current than can be carried in a solenoid with 500 turns per metre in the superconducting state. [4 marks]

   (c) Sketch a graph of the dielectric constant of a solid as a function of frequency and identify the regions corresponding to dipolar, ionic and electronic polarisations. Briefly describe the microscopic origin of electronic and ionic polarisations. [4 marks]

   (d) A ferroelectric material can be described by the Ginzburg-Landau equation

$$G_{FE} = G_{PE} + \frac{1}{2}g_2 P^2 + \frac{1}{4}g_4 P^4,$$

   where G_{FE}, G_{PE} are the free energies in the ferroelectric and paraelectric states respectively and P is the polarisation. $g_2 = \gamma(T - T_0)$, where T is the temperature. γ, T_0 and g_4 are positive constants.

   Derive expressions for the spontaneous polarisation and plot a graph of spontaneous polarisation as a function of temperature. [4 marks]

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6. The density of states $D(E)$ for a free electron metal is given by

$$D(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E},$$

where m is the electron mass and E is the energy.

Consider a semiconductor in equilibrium with chemical potential μ, minimum conduction band energy E_c and maximum valence band energy E_v.

(a) Using the free electron metal density of states write down equivalent expressions for the conduction and valence band density of states in the semiconductor. [2 marks]

(b) Derive expressions for the equilibrium electron and hole concentrations between energies E and $E + dE$, where dE is a small energy increment. [4 marks]

(c) If the chemical potential is shifted towards the valence band maximum explain how the electron and hole concentrations would change. [2 marks]

A group III impurity is added to a group IV intrinsic semiconductor.

(d) Sketch the variation in majority carrier concentration as a function of temperature, clearly labelling the different regimes. [3 marks]

(e) In the saturation regime explain whether the majority carriers are electrons or holes. [2 marks]

(f) Calculate the atomic fraction of impurity atoms that must be added in order to achieve a majority carrier concentration of 10^{20} carriers m^{-3} in the saturation regime. Assume the semiconductor has a diamond cubic crystal structure with 0.54 nm lattice parameter. [3 marks]

(g) Calculate the temperature above which the semiconductor shows intrinsic behaviour. Assume $E_g = 1.1$ eV, $N_c = 2.8 \times 10^{25}$ m^{-3} and $N_v = 1.0 \times 10^{25}$ m^{-3}, where E_g is the semiconductor band gap, and N_c, N_v are the effective density of states for the conduction and valence bands respectively. [2 marks]

(h) If the impurity energy level is 10 meV above the valence band maximum estimate the temperature below which impurity freeze-out takes place. [2 marks]