2.1 Decay Constants

$$\frac{dN}{dt} = -\lambda N \quad \Rightarrow \quad \underbrace{N(t) = N_0 e^{-\lambda t}}$$

$\underbrace{dN = -\lambda N\, dt}$ is the number of decays in a time interval dt.

$$\langle t \rangle \equiv \frac{\int_{N_0}^{0} t\, dN}{\int_{N_0}^{0} dN} = \frac{1}{(-N_0)} \left(- \int_{0}^{N_0} t\, \underbrace{dN} \right)$$

$$= \frac{1}{N_0} \int_{\infty}^{0} t\, (-\lambda N\, dt)$$

$t = 0 \quad N_0$

$t = \infty \quad 0$

$$= \frac{1}{N_0} \int_{0}^{\infty} t \lambda \underbrace{N}\, dt$$

$$= \frac{1}{\cancel{N_0}} \int_{0}^{\infty} t \lambda \cancel{N_0} e^{-\lambda t}\, dt$$

Integration by parts

$$= \lambda \int_{0}^{\infty} t e^{-\lambda t}\, dt$$

$$\tau \equiv \langle t \rangle = \lambda \frac{1}{\lambda^2} = \frac{1}{\lambda}$$

mean time

How long does it take until half of the original amount N_0 is left?

$$N(t_{1/2}) \overset{!}{=} \frac{\cancel{N_0}}{2} = \cancel{N_0}\, e^{-\lambda t_{1/2}}$$

half-life

$$\Rightarrow \quad \frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\log(1) - \log(2) = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{\log(2)}{\lambda} = \log(2) \cdot \tau$$

Further define the <u>activity</u> (frequency of decays)

$$A = -\frac{dN}{dt} = \lambda N$$

measured in Becquerel (1 Bq = 1 decay/s)

Curie (1 Ci = $3.7 \cdot 10^{10}$ Bq)

$$A(t) = \lambda N_0 e^{-\lambda t}$$

decreases with time.

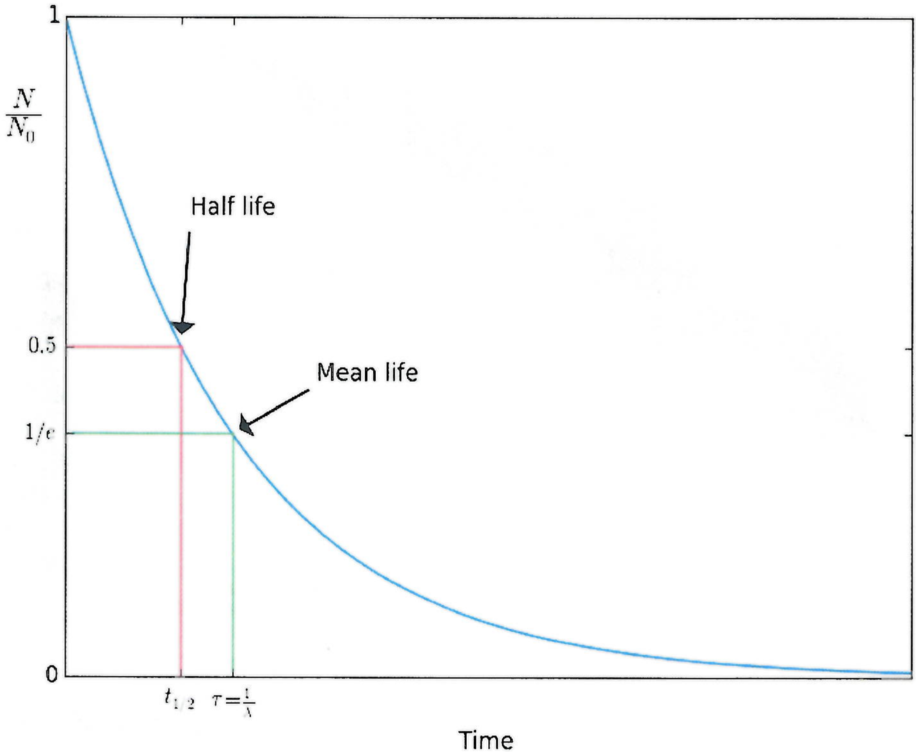


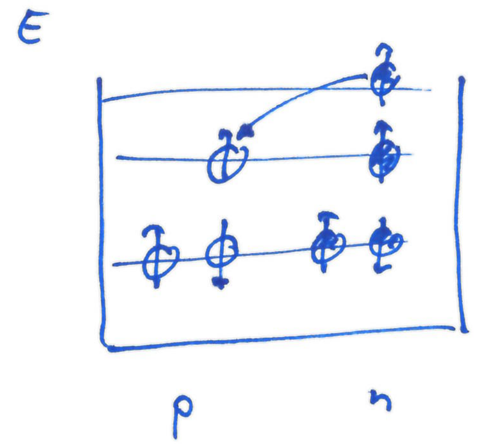
Figure 3: Fraction of atoms remaining as a function of time.

Different decay mechanisms.

2.2 β - decay(s)

The free neutron is not stable

$$n \to p + e^- + \bar{\nu}_e \qquad \tau_n = 881.5\,s$$



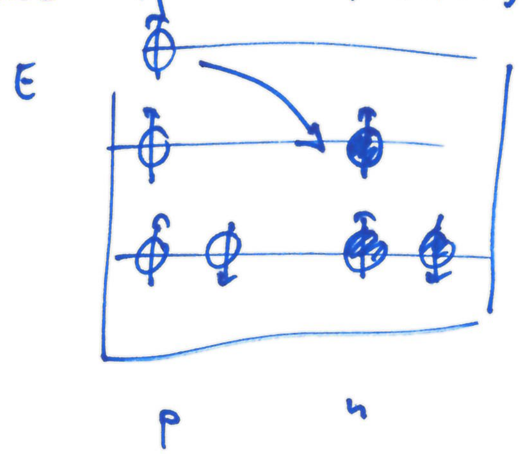
β⁻ decay : Turning a neutron into a proton inside the nucleus.

$$M(Z,A) > M(Z+1,A)$$

mass difference is large enough to afford the creation of e^- and a p.

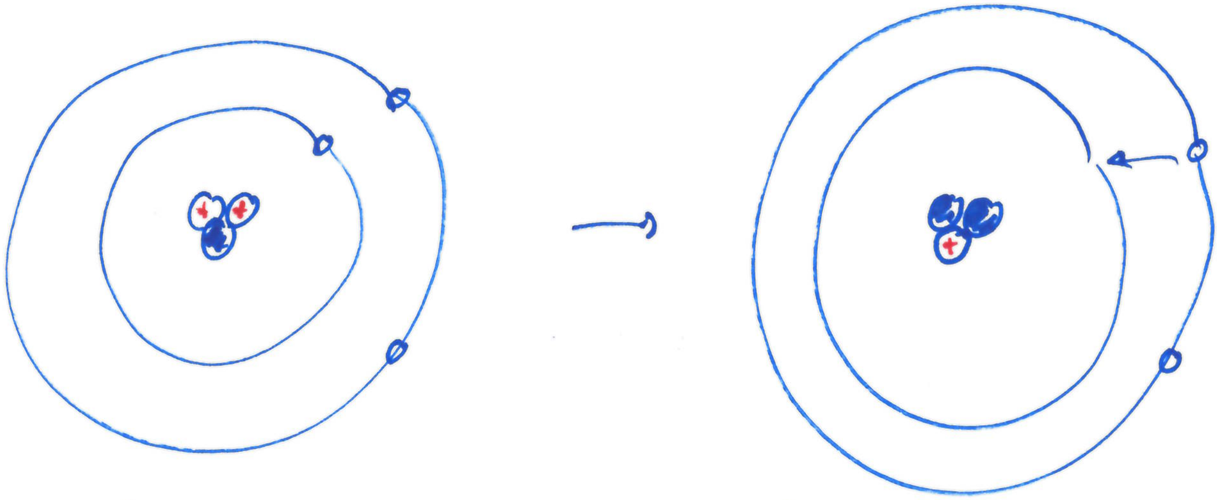
β⁺ decay $\quad p \to n + e^+ + \nu_e$

is also possible if $\quad M(Z,A) > M(Z-1,A) + 2m_e$



Electron capture $\quad p + e^- \to n + \nu_e$

is possible $\quad M(Z,A) > M(Z-1,A) + \epsilon$



$^{A}_{Z}X \longrightarrow {}^{A}_{Z\pm 1}X$ is in an excited state

Electron capture needs overlap of the wavefunctions of the e^- with the nucleus.
$\Rightarrow$ more likely for heavy nuclei.
Whenever β^+ is possible, electron capture is possible as well. The reverse is not true.

These decays all do not change A!

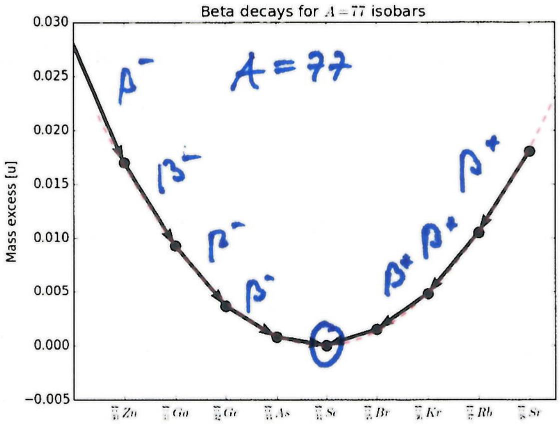
$$M(A,Z) = (A-Z)M_n + ZM_p + Zm_e - B(A,Z)$$
$$= \ldots - a_V A + a_S A^{2/3} + a_C Z^2/A^{1/3} + a_a \frac{(A-2Z)^2}{4A} \pm \delta/A^{1/2}$$
$$= a + bZ + cZ^2$$

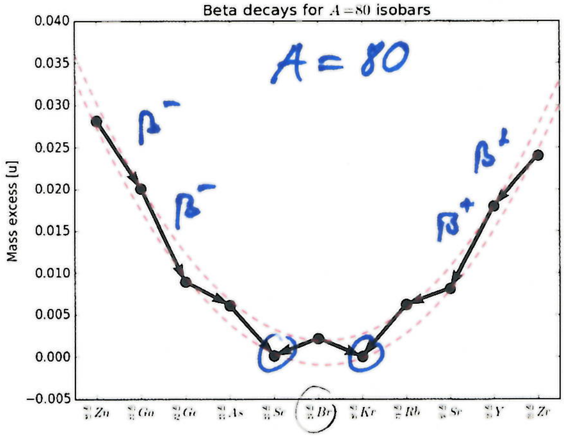
for constant A.

This results in a parabolic shape, nicely explaining the observed 'β-decay chains'

$^{80}_{35}Br$

35 protons
45 neutrons



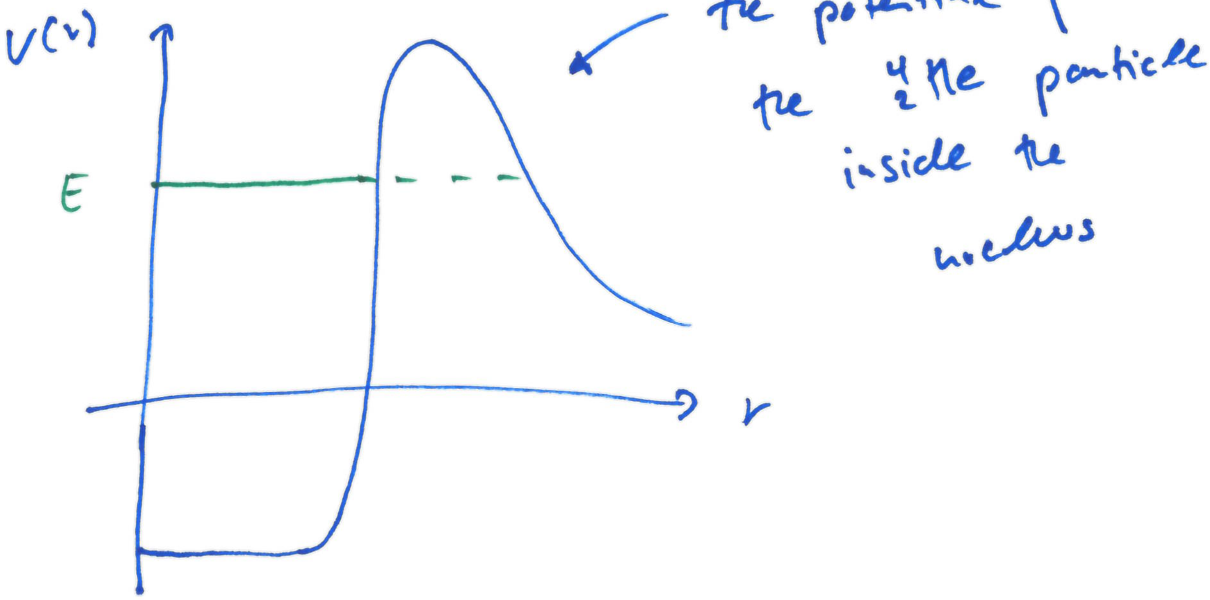


2.3 α-decay

α-decay changes A, the nucleus emits a ${}^4_2He$ nucleus. As in β decays this is only possible if the binding energy of the small nucleus fulfils a condition

$$M(Z,A) > M(Z-2, A-4) + M(2,4)$$

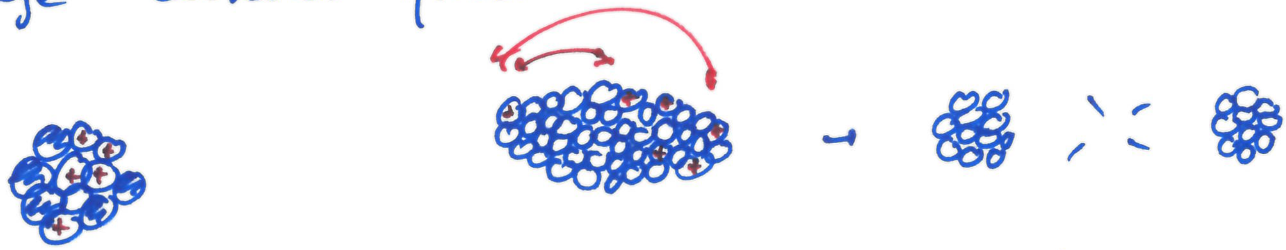
α-decay is a tunnelling effect.



Rarely, also a single proton ~~...~~ or a single neutron is emitted.

2.4 Nuclear fission

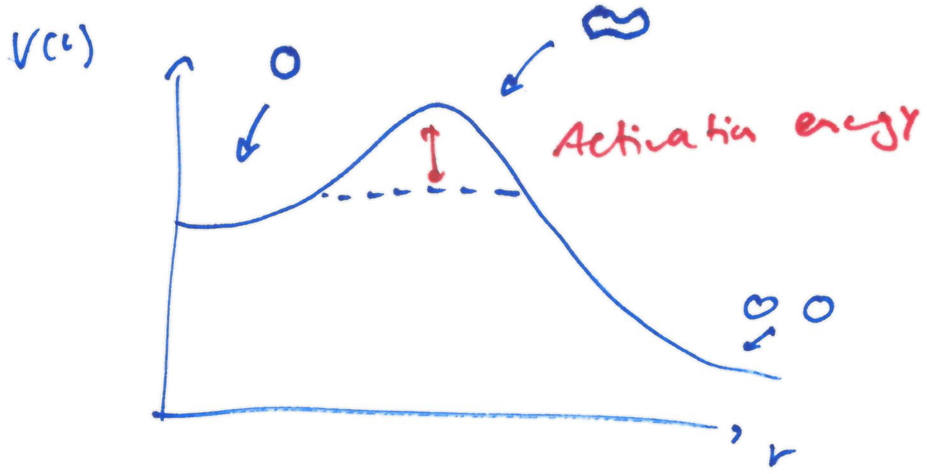
Instead of splitting off a small nucleus, very heavy nuclei can also split into two heavy nuclei = fission. This is a result of the long-range Coulomb force.



Spontaneous fission

Strong force $\propto A$

E/M force $\propto Z^2$



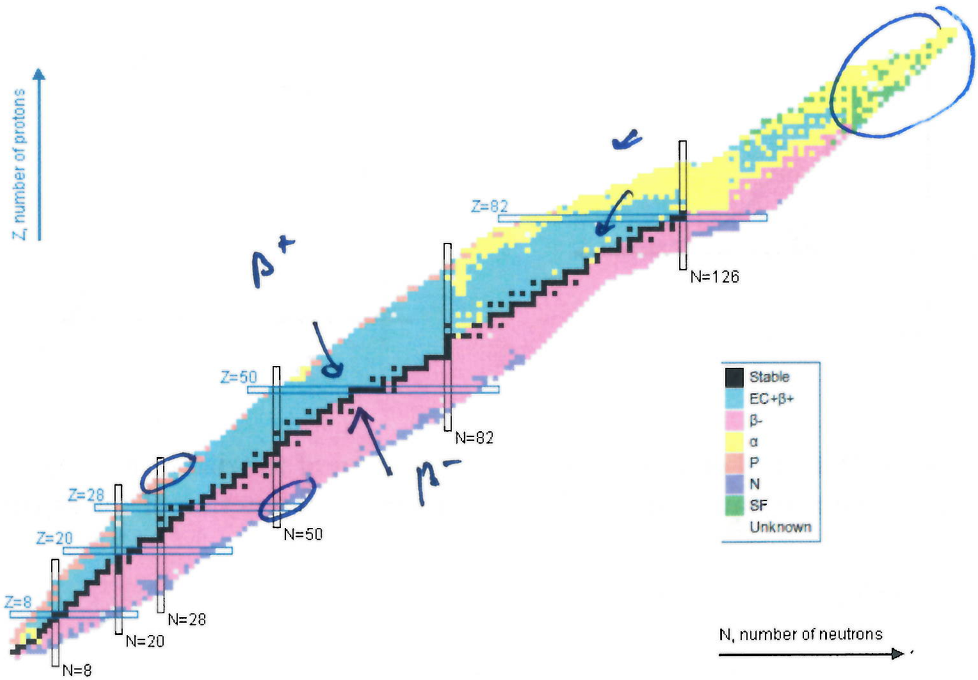


Figure 6: Decay modes in the N-Z plane. Data from [1]