L2 Foundation of Physics 2B Optics 2019-20

O.WP.1 Harmonic waves, Complex notation

January 17, 2020

(a) The harmonic wave solution of Maxwell's wave equation is

$$\underline{E}(\underline{r},t) \;=\; \underline{E}_0 \cos(\underline{k}\cdot\underline{r}-\omega t)\ . \tag{1}$$

1. What are $\underline{k}$ and ω? [2 marks]
2. Write expressions for $\underline{k}$ and ω in terms of the wavelength of light λ. [2 marks]
3. For the case where $\underline{E}_0$ is a constant, sketch the magnitude of the electric field as a function of position for a harmonic wave with wavelength $\lambda = 0.45$ μm. Include a scale on both axes. [Hint: Assume that $\underline{k}$ is parallel the z axis.] [2 marks]
4. Now plot the magnitude of the field as a function of time. Include a scale on both axes. [2 marks]
5. Now plot the intensity as a function of time and show the time-averaged intensity $\mathcal{I}_0 = \frac{1}{2}c\epsilon_0\mathcal{E}_0$. Include a scale on both axes with the vertical axis in units of $\mathcal{I}_0$. [2 marks]

(b) Make a plot (Argand diagram) showing the location of each of theze complex numbers in the complex plane: (i) $-1+2i$ (ii) $-1-2i$ (iii) $2e^{i\pi/2}$, (iv) $2e^{i\pi/4}$. Label your axes and provide a scale. [5 marks]

(c) Sketch the phasor corresponding to a phase $\phi = 3\pi/4$ [2 marks]