Foundations of Physics 2B/3C 2019-2020

Thermodynamics – Lecture 3 Recap

- Looked at the meanings of work and internal energy and saw how they led to the first law of thermodynamics.
- Investigated the first law of thermodynamics, mathematically:

$$\boxed{dU = \delta Q + \delta W \quad ; \quad dU = \delta Q - pdV.}$$

- Began to consider the meaning of reversibility and quasi-static when applied to thermodynamics.

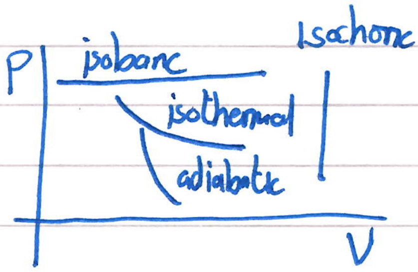
Thermodynamics – Lecture 4 Aims

- Finish considering the meaning of reversibility and quasi-static when applied to thermodynamics and see one derivation of the adiabatic equation of state.
- To be introduced to the concept of heat engines.
- To see the various statements of the Second Law of Thermodynamics and how they are logically equivalent.

8 Reversibility

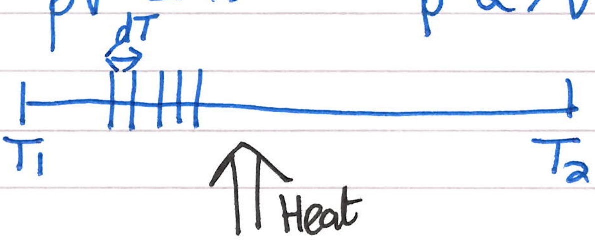
A process is reversible no dissipative work is done (express everything in terms of thermo coordinates)
Quasi-static processes happen very slowly, system always remains in an equilibrium state – there no finite temperature differences   No shock waves

Isobaric – constant pressure
Isochoric – constant volume
Isothermal – constant temperature
adiabatic – Thermally isolated (no heat)



Ideal gas: Isothermal $pV = RT_0$ $\quad p \propto 1/V$
Adiabatic $pV^\gamma = k$ $\quad p \propto 1/V^\gamma$

Isobaric heating
Temperature $T_1 \rightarrow T_2$



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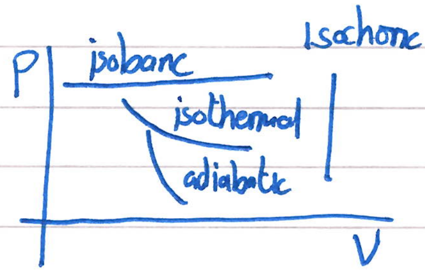

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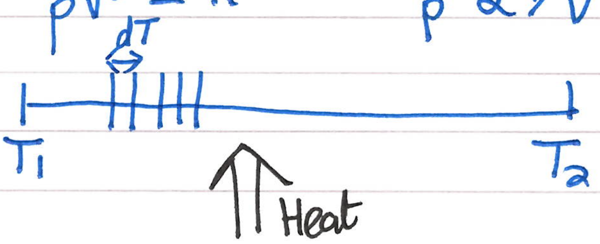


$$\text{Heat} = \int_{T_1}^{T_2} \delta Q = \int_{T_1}^{T_2} C_p dT \quad \left[C_p = \left(\frac{\partial Q}{\partial T}\right)_p\right]$$

Exercise 3 – Which does heat flow in a reversible isothermal expansion of an ideal gas?

1st Law $dU = \delta Q + \delta W = \delta Q - pdV$

Ideal gas has $U = U(T)$ only so isothermal $U_f = U_i \Rightarrow dU = 0$

$\therefore Q_{if} = \int \delta W = \int pdV$ $\quad p = RT_0/V$ for ideal gas

$$Q_{if} = \int_{V_i}^{V_f} \frac{RT_0\, dV}{V} = RT_0 \ln\left(\frac{V_f}{V_i}\right)$$ If $V_f > V_i$ $\ln\left(\frac{V_f}{V_i}\right) > 0$

and $Q_{if} > 0$ [Heat in]

Example 8.1: What is the equation of state for ideal gas on adiabatic?

The First Law of thermodynamics is, $dU = \delta Q + \delta W = \delta Q - pdV$ & $\delta Q = 0$. [No heat on adiabatic]

From proof 7.1 we have that $C_V = \left(\frac{\partial U}{\partial T}\right)_V$. $\Rightarrow dU = C_V dT$

$$C_V dT + pdV = 0 \quad \Rightarrow \quad dT = -\frac{pdV}{C_V}. \qquad (1)$$

Using $C_p = R + C_V$ $\quad C_V = (C_p - R)$

$$0 = (C_p - R)dT + pdV = C_p dT - RdT + pdV. \qquad (2)$$

For an ideal gas, $pV = RT$, and the total differential of the temperature is

$T = \frac{pV}{R}$ $\quad dT = \frac{1}{R} d(pV)$ $$dT = \frac{1}{R}[pdV + Vdp], \qquad (*)$$ Chain rule $d(pV)$

Substituting $(*)$ into (2) (the term RdT) gives

$$0 = C_P dT - \cancel{pdV} - Vdp + \cancel{pdV} \quad \Rightarrow \quad dT = \frac{Vdp}{C_p}.$$ Two dT

Any two infinitesimal temperature changes must be equal, so $\frac{Vdp}{C_p} = -\frac{pdV}{C_V}$.

Differential equation

separate variables + integrate

$$\int \frac{dp}{p} = -\int \frac{C_p}{C_V}\frac{dV}{V}$$

$$\ln p = -\gamma \ln V + C.$$

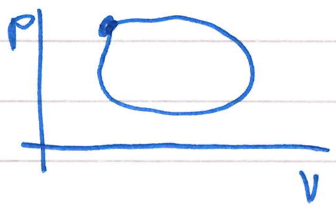
$$\ln p = \ln(V^{-\gamma} \times \text{Const}) \quad \Rightarrow \quad \boxed{pV^{\gamma} = \text{Const.}}$$

Describes ideal gas behaviour under adiabatic condition

9. Heat Engines

Turning work to heat is easy. Reverse more difficult.
Heat engine used to produce work from a temperature difference

Operate in cycle
- Heat taken in at high temperature
- Some heat is converted to work
- Waste heat is rejected at low temperature
- Cycle returned to its initial state



Blob in thermo state (p, V, T) has its thermo coordinates changed as it moves around the cycle

Engines can run in reverse — requires work to be supplied, take heat from cold to hot.

i) Refrigerator – removes heat from an enclosed cold space to the environment

ii) Heat pump – takes from the cold environment to a hot space.

Efficiency – how much work you get out for a given heat input [Product / expense]

Coefficient of performance – how much heat is moved for a given amount of work

$$\eta = \frac{\text{Product}}{\text{Expense}} = \frac{|\text{Work Done}|}{\text{Heat input}} \leftarrow \text{Engines do -ve work}$$

$$COP_L = \frac{|\text{Heat remove from Cold}|}{\text{Work input}} \leftarrow \text{heat out of cold}$$

$$COP_H = \frac{\text{Heat added to hot}}{\text{Work input}}$$

Thermodynamics – Handout 4

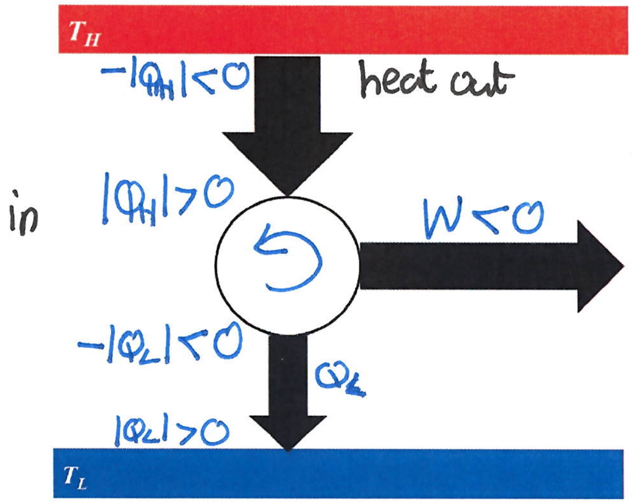


Figure 5: Efficiency of a Heat Engine.

Cycle $\oint dU = 0$ [Exact, engine returns to initial state]

1st Law $dU = \delta Q + \delta W$

$$0 = \oint \delta Q + \oint \delta W$$

$$\oint \delta W = -\oint \delta Q$$

$$W = -(|Q_H| - |Q_L|)$$

Difference in heats

$$\eta = \frac{|\text{Work}|}{\text{Heat in}} = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$

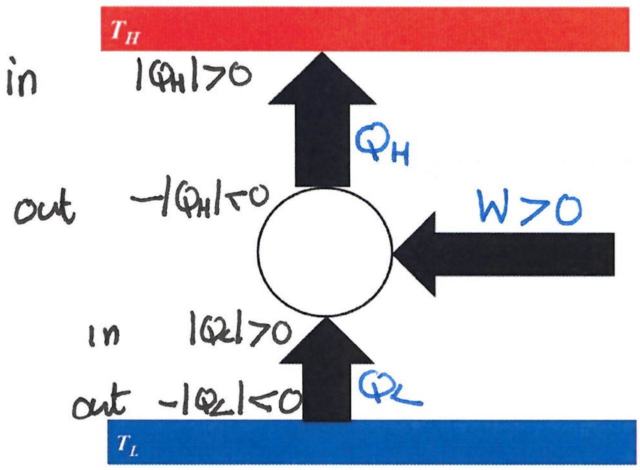


Figure 6: Coefficient of performance of a Refrigerator/Heat pump.

Exercise 4 – Determine Fridge and Heat Pump coefficients of performance (See DUO)

Fridge: heat from cold to hot

$$COP_L = \frac{\text{Product}}{\text{Expense}} = \frac{|Q_L|}{W}$$

(Work supplied removes heat)

Heat pump: heat into hot from cold

$$COP_H = \frac{Q_H}{W}$$

Example 9.1: To maintain a fridge at 4 °C, heat must be removed at 360 kJ/min using 2 kW power. The coefficient of performance is

Write as rates

$$COP_L = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{360}{2}\left(\frac{1\ \text{kW}}{60\ \text{kJ/min}}\right) = 3.$$

COP can be > 1

Heat rejected to room $\dot{Q}_H = \underbrace{\dot{W} + \dot{Q}_L}_{\text{Energy conservation}} = 2\ \text{kJ s}^{-1} \times 60 + 360 = 480\ \text{kJ/min}.$

Heat taken from the fridge plus the work supplied show up in room's internal energy

Heat pump has $COP_H = 2.5$ to keep house at 20 °C. When −2 °C outside, the house loses 80,000 kJ/h.

Supply heat to house equivalent to what is lost

Work to the heat pump

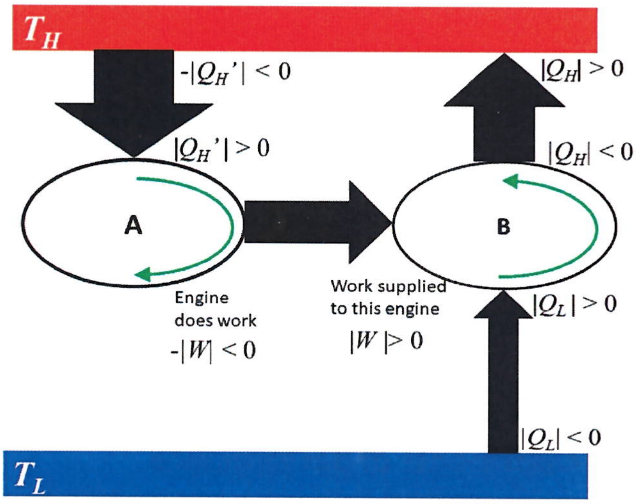
$$W_{in} = \frac{|\dot{Q}_H|}{COP_H} = \frac{80,000}{2.5} = 32,000 \text{ kJ/h} \quad (8.9 \text{ kW}).$$

Energy conservation to work out heat extract from environment

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 80,000 - 32,000 = 48,000 \text{ kJ/hour}.$$

Electricity supplied.

Resistive heater would require 80 kJ to warm the house



- See lecture 5.

Figure 7: If the Kelvin statement of the *Second Law* is violated, so is the Clausius.

Most efficient engines are Carnot cycles – Ideal engine model in an abstract form, tells us max possible heat to work.
Considered a system passing through many equilibrium states, so is reversible (+ quasi static)
Carnot knew work from a temperature difference, and heat from hot to cold under its own action

Devised a system where only heat transfer was between bodies of near equal temperature
- Heat in ~~heat~~ at constant temp
- Adiabatic expansion
- Heat rejected at constant temp
- Adiabatic compression

Perfect engine has $\eta = 1 - \frac{T_L}{T_H}$, $COP_L = \frac{T_L}{T_H - T_L}$

$COP_H = \frac{T_H}{T_H - T_L}$

Perfect reversible cycles only depend on temperature.

10 Second Law of Thermodynamics

Encapsulates understanding of heat engines + places a direction on processes. First Law (energy conservation) even if satisfied some processes just don't happen.
Energy having quality as well as quantity.

Refrigerator

Clausius – It is impossible to devise a process whose sole result is the transfer of heat from a cold to a hot reservoir
Nature is asymmetric

Kelvin Planck – It is impossible to construct a device that operates in a cycle, whose sole result is the transfer ~~to~~ of heat to work

Engine

Tax on conversion of heat to work