$\boxed{1}$ Statistical Physics

Book: p1-4.

Uses probability with large numbers to gain insight into system properties. Classical and quantum particles, e.g.

$$|\alpha \psi_1 \pm \beta \psi_2|^2 = |\alpha \psi_1|^2 + |\beta \psi_2|^2$$
$$\underbrace{+ \left(\alpha^* \psi_1^* \beta \psi_2 + \beta^* \psi_2^* \alpha \psi_1 \right)}_{\text{large systems !}}$$

Probability Let A and B be independent

$$P(A \text{ or } B) = P(A) + P(B)$$
$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Count events

The number of ways to arrange N objects is $N!$

Pick r objects from N : combinations $\binom{N}{r} = C^N_r$

$$= \frac{N!}{r!\,(N-r)!}$$

where the order of the objects is not important.

e.g. divide 5 objects into 2 piles of 2 objects and 3 objects :

1 2 - 3 4 5	2 3 - 1 4 5	3 4 - 1 4 5
1 3 - 2 4 5	2 4 - 1 3 5	3 5 - 1 3 4
1 4 - 2 3 5	2 5 - 1 3 4	
1 5 - 2 3 4		4 5 - 1 2 3

$$\frac{5!}{2!\,3!} = 10.$$

States of a system.

A system can be thought of as a collection of particles, e.g. a 3 - spin system (i.e. 3 particles with spins ↑ or ↓).

States are :

3 up	2 up 1 down	1 up 2 down	3 down
1. ↑ ↑ ↑	2. ↑ ↑ ↓	5. ↑ ↓ ↓	8. ↓ ↓ ↓
	3. ↑ ↓ ↑	6. ↓ ↑ ↓	
	4. ↓ ↑ ↑	7. ↓ ↓ ↑	

The system has bulk properties that are not equally likely although all arrangements of particles (i.e. up or down) is equally likely.

$$P(3\ \text{up}) = 1/8\ ,\quad P(2\text{up}\ 1\ \text{down}) = 3/8 \quad P(3\ \text{down}) = 1/8$$
$$P(1\text{up}\ \ 2\ \text{down}) = 3/8$$

Distributions

Discrete: x can take on a set of discrete values

$\{x_1, x_2, \ldots\}$ let the probability of x_i be P_i

Normalisation - gives us $\sum_i P_i = 1$.

Mean value $\bar{x} = \langle x \rangle = \sum_i P_i x_i$

Variance: $\sigma^2 = \overline{x^2} - \bar{x}^2 = \sum_i P_i x_i^2 - \left(\sum_i P_i x_i\right)^2$

and so standard deviation is σ.

Binomial Distribution.

Process has 2 outcomes repeated N times. If p is the probability of one out come then $(1-p)$ is the probability of another in a single go. The probability of p happening k times out of N attempts we have

$$P_N(k) = \underbrace{\frac{N!}{k!\,(N-k)!}}_{\text{binomial coefficient}} p^k (1-p)^{N-k}$$

Normalisation $\sum_{k=0}^{N} P_N(k) = 1.$

Average $\underbrace{\sum_{k=0}^{N} P_N(k)\,k}_{=\mu}$, variance $\sum_{k=0}^{N} P_N(k)\,k^2 - \mu^2$

$= N p(1-p).$

Example In cricket a batter hits ball with probability $1/3$.

What's the probability in 4 attempts the batter hits the ball

0 times, or 1 time, or .... 4 times.

$0/4 \to \left(\frac{2}{3}\right)^4 \simeq 20\%$

$1/4 \to \frac{4!}{3!\,1!}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^1 \simeq 40\%$

$2/4 \to \frac{4!}{2!\,2!}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 \simeq 29\%$

$3/4 \to \sim 10\%$

$4/4 \to \sim 1\%$.

Continuous Probabilities - a continuous range of variables x. It's meaningless to ask what's the probability of getting a particular value of x, but instead we ask the probability of x in a given range.

If our outcomes lie in range $[a, b]$

then $$p(a \le x \le b) = \int_a^b f(x)\, dx$$

If the total range is, e.g. $(-\infty, \infty)$ we get normalisation

$$\int_{-\infty}^{\infty} f(x)\, dx = 1.$$

e.g. Gaussian distribution $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

the probability that x lies between $\pm\sigma$ of the mean μ is

$$P(\mu - \sigma \le x \le \mu + \sigma) \simeq 0.68.$$

<u>Probability</u> The outcome of an experiment is not always the same - issues either in our control or not, classically or quantum mechanically. If we perform an experiment N times an event i occurs n_i times - the frequency of occurrence

$$F(n_i, N) = n_i/N$$

This is not the probability, however probability is defined as

$$P_i = \lim_{N \to \infty} F(n_i, N) = \lim_{N \to \infty} \frac{n_i(N)}{N}.$$

We assume that for N big enough then $F(n_i, N) = P_i$

- this is the Ergodic hypothesis.

Sometimes we can assign probabilities a priori (before).
We know what they are before an experiment.

Fair dice $P_1 = P_2 = \cdots = P_6 = 1/6$.

Fair coin $P_h = P_t = 1/2$

Toss a coin 4 times look at each possibly of outcome.

Microstate	Macrostate.	Number of microstates in macrostate.
hhhh	4-heads	1
hhht hhth hthh thhh	3-heads 1-tails	4
hhtt htht htth thht thth tthh	2-tails 2-heads	6
httt thtt ttht ttth	3-tails 1-heads	4
tttt	4-tails.	1

Ignoring order we get macrostates.

We have 16 equally likely microstates.

But we have 5 macrostates which have differing probability.