ELECTROMAGNETISM

Level 2 Physics problems – Foundations of physics 2

Solution 1 Cycle 2 Version 1

Professor D P Hampshire – 2nd Year Physics Lecture Course

Information underlined or indicated by red text is required for marks to awarded. The mark scheme is a guide and solutions should not be considered to be unique. Marks are awarded for correct relevant Physics.

1.

Coulomb's law: $\underline{\boldsymbol{F}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{\boldsymbol{r}}$ 1-1

where: q_1, q_2 are the magnitudes of the two charges,
ε_0 is the permittivity of free space,
r is the distance between the two charges,
$\hat{\boldsymbol{r}}$ is a unit vector in the direction of the force,
$\underline{\boldsymbol{F}}$ is the force between the two charges.

1 mark if 1-1 and definitions all correct.
[Qn 1: 1 mark total]

2. a) To make it a bit easier, use the vector identity:

$$\underline{\boldsymbol{A}} \times (\underline{\boldsymbol{B}} \times \underline{\boldsymbol{C}}) = (\underline{\boldsymbol{A}} \cdot \underline{\boldsymbol{C}})\underline{\boldsymbol{B}} - (\underline{\boldsymbol{A}} \cdot \underline{\boldsymbol{B}})\underline{\boldsymbol{C}} \tag{2-1}$$

$$= (6)(\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) - (21)(5\hat{\mathbf{\imath}} + \hat{\mathbf{k}})$$

$$= 6\hat{\boldsymbol{\jmath}} - 30\widehat{\boldsymbol{k}} - 105\hat{\boldsymbol{\imath}} - 21\widehat{\boldsymbol{k}}$$

$$= -105\hat{\boldsymbol{\imath}} + 6\hat{\boldsymbol{\jmath}} - 51\widehat{\boldsymbol{k}} \tag{2-2}$$

b) This is the vector identity for $\underline{\boldsymbol{A}} \times (\underline{\boldsymbol{B}} \times \underline{\boldsymbol{C}})$, so the answer is the same as a)

c) To find the angle between $\underline{\boldsymbol{A}}$ and $\underline{\boldsymbol{B}}$, use the dot-product:

$$\underline{\boldsymbol{A}} \cdot \underline{\boldsymbol{B}} = |\underline{\boldsymbol{A}}||\underline{\boldsymbol{B}}|\cos\theta \tag{2-3}$$

$$\underline{\boldsymbol{A}} \cdot \underline{\boldsymbol{B}} = 21$$
$$|\underline{\boldsymbol{A}}| = \sqrt{4+1+16} = \sqrt{21}, \quad |\underline{\boldsymbol{B}}| = \sqrt{1+25} = \sqrt{26}$$
$$\Rightarrow \cos\theta = \frac{21}{\sqrt{21}\sqrt{26}} = 0.8987$$
$$\Rightarrow \theta = 26.0^\circ \tag{2-4}$$

d) $\underline{\boldsymbol{A}} \cdot (\underline{\boldsymbol{B}} \times \underline{\boldsymbol{C}})$A × (B × C) is a scalar triple product:

$$\underline{\boldsymbol{A}} \cdot (\underline{\boldsymbol{B}} \times \underline{\boldsymbol{C}}) = \begin{vmatrix} 2 & 1 & -4 \\ 0 & 1 & -5 \\ 5 & 0 & 1 \end{vmatrix} = 2(1-0) - 1(0+25) - 4(0-5)$$
$$= -3 \tag{2-5}$$

1 mark if two parts correct, 2 marks if all parts correct.

[Qn 2: 2 marks total]

3. Given that, $\underline{\boldsymbol{E}} = e_1\hat{\boldsymbol{\imath}} + e_2\hat{\boldsymbol{\jmath}} + e_3\hat{\mathbf{k}}$ and $\underline{\boldsymbol{F}} = f_1\hat{\boldsymbol{\imath}} + f_2\hat{\boldsymbol{\jmath}} + f_3\hat{\mathbf{k}}$ also and making use of,

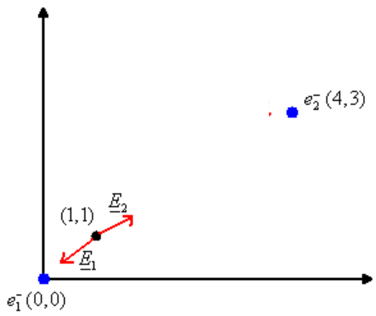
$$\underline{\boldsymbol{A}} \cdot \underline{\boldsymbol{B}} = |\underline{\boldsymbol{A}}||\underline{\boldsymbol{B}}|\cos\theta \tag{3-1}$$

Rearrangement gives answer,

$$\cos\theta = \frac{e_1 f_1 + e_2 f_2 + e_3 f_3}{\sqrt{e_1^2 + e_2^2 + e_3^2}\sqrt{f_1^2 + f_2^2 + f_3^2}} \tag{3-2}$$

1 mark if 3-2 correct.
[Qn 3: 1 mark total]

4. At position (1,1), we use vector addition (i.e. superposition) of the two electric fields:



Field from electron at (0,0):

$$\underline{\boldsymbol{E}} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\boldsymbol{r}} \tag{4-1}$$

$$r^2 = 1^2 + 1^2 = 2, \qquad \hat{\boldsymbol{r}} = \frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}.$$

$$=> \underline{\boldsymbol{E_1}} = \frac{-e}{4\sqrt{8}\pi\varepsilon_0}(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}) \tag{4-2}$$

Field from electron at (4,3):

$$r^2 = 3^2 + 2^2 = 13 \quad \hat{\boldsymbol{r}} = \frac{-3\hat{\imath}-2\hat{\jmath}}{\sqrt{13}}$$

$$=> \underline{\boldsymbol{E_2}} = \frac{e}{4\sqrt{2197}}(3\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}) \tag{4-3}$$

Principle of superposition,

$$\underline{\boldsymbol{E}}_{total} = \underline{\boldsymbol{E}}_1 + \underline{\boldsymbol{E}}_2 \tag{4-4}$$

$$= \frac{e}{4\pi\varepsilon_0}\left(-\frac{\hat{\boldsymbol{\imath}}}{\sqrt{8}} - \frac{\hat{\boldsymbol{\jmath}}}{\sqrt{8}} + \frac{3\hat{\boldsymbol{\imath}}}{\sqrt{2197}} + \frac{2\hat{\boldsymbol{\jmath}}}{\sqrt{2197}}\right)$$

$$=> \underline{\boldsymbol{E}}_{total} = -4.18 \times 10^{-10}\hat{\boldsymbol{\imath}} - 4.46 \times 10^{-10}\hat{\boldsymbol{\jmath}} \tag{4-5}$$

1 mark if the electric field 4-2 or 4-3 correct. 2 marks if 4-5 is correct.
[Qn 4: 2 marks total]

5. a) From Gauss' law,

$$\int \underline{\boldsymbol{E}}.d\underline{\boldsymbol{S}} = \int \frac{\rho}{\varepsilon_0} dV \quad \text{5-1}$$

From symmetry the $\underline{\boldsymbol{E}}$-field is entirely radial.

Area of surface = $4\pi r^2$.
Charge enclosed = (ρ) × (volume of shell) = $\frac{4\pi\rho}{3}[R^3 - (R-t)^3]$
So from Gauss, $4\pi r^2 E = \frac{4\pi\rho}{3\varepsilon_0}[R^3 - (R-t)^3]$

$$=> \underline{\boldsymbol{E}} = \frac{\rho[R^3 - (R-t)^3]}{3\varepsilon_0}\frac{1}{r^2}\hat{\boldsymbol{r}} \quad \text{5-2}$$

1 mark if 5-2 correct.

b) From Gauss' law,

$$\int \underline{\boldsymbol{E}}.d\underline{\boldsymbol{S}} = \int \frac{\rho}{\varepsilon_0} dV \quad \text{5-3}$$

Area of surface = $4\pi r^2$

Charge enclosed = (ρ) × (volume of shell enclosed by surface)
$= \frac{4\pi\rho}{3}[r^3 - (R-t)^3]$

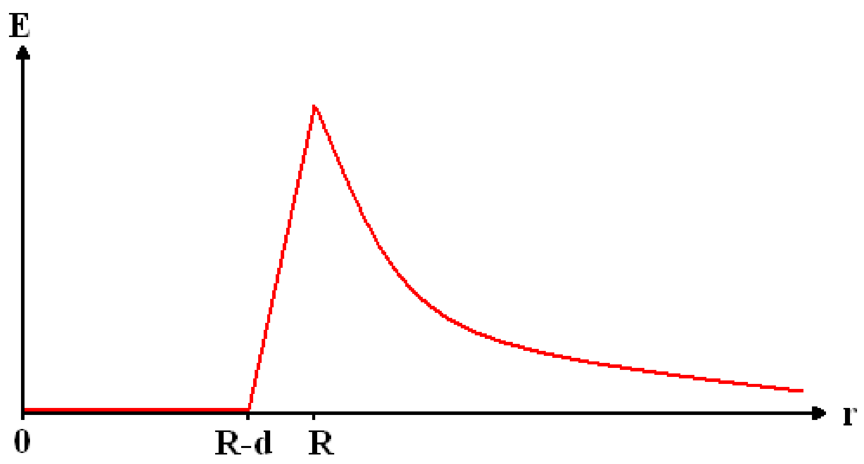
5-4

So $4\pi r^2 E = \frac{4\pi\rho}{3\varepsilon_0}[r^3 - (R-t)^3]$ $=> \underline{\boldsymbol{E}} = (\frac{\rho r}{3\varepsilon_0} - \frac{\rho(R-t)^3}{3\varepsilon_0 r^2})\hat{\boldsymbol{r}}$

1 mark if 5-4 correct.

c) A surface drawn with a radius r < R-d will not enclose any charge, and Gauss' law gives the field as $\underline{\boldsymbol{E}} = 0$.
1 mark statement correct.

d)



1 mark if diagram correct.
[Qn 5: 4 marks total]
Total for all questions 10 marks.