Relativistic Electrodynamics Workshop 6

November 2015

In this problem we will consider the fields of a moving point charge, and the power radiated by the particle. The electric and magnetic fields of a moving point charge are given by:

$$\vec{E}\left(\vec{r},t\right) = \frac{q}{4\pi\epsilon_0}\frac{R}{\left(\vec{R}\cdot\vec{u}\right)^3}\left((c^2-v^2)\vec{u}+\vec{R}\times(\vec{u}\times\vec{a})\right), \tag{1}$$

$$\vec{B}\left(\vec{r},t\right) = \frac{1}{c}\hat{R}\times\vec{E}\left(\vec{r},t\right). \tag{2}$$

- What is the meaning/definition of each term in the above equations?
- At what time should the terms on the right hand side of the equations be evaluated? Why?
- The equation for the electric field is the sum of two parts, which we call the velocity (or Coulomb) and acceleration fields. What does the R dependence of these two parts look like in the limit that $R\to\infty$?

The Poynting vector is given by

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\left(\vec{r},t\right)\times\vec{B}\left(\vec{r},t\right). \tag{3}$$

The total power radiated is the surface integral over the Poyting vector:

$$P = \oint\vec{S}\cdot d\vec{A} = \int|\vec{S}|R^2 d\Omega \tag{4}$$

We will now consider a couple of cases that are readily solvable.
Case 1: A charge that is moving with constant velocity.
Case 2: A charge that is undergoing acceleration but is instantaneously at rest.

For each each case

- Evaluate the $\vec{E}$ and $\vec{B}$ fields. Do you recover the expected result in the stationary ($a = v = 0$) limit?
- Evaluate the Poynting vector
- Calculate the power radiated to infinity. Compare this result for the two cases and think about how this relates to the R dependence on the velocity and acceleration fields.

In the case of the accelerating charge you should recover the Lamor formula:

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \tag{5}$$