Atomic Clocks

Weekly Problem 1

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Learning outcomes: *Calculate the lifetime of an excited state due to spontaneous emission.*

Calculate the lifetime of the 2p0 state of the hydrogen atom as it decays to the 1s0 state by spontaneous emission of a photon of wavelength 121.5668 nm. [10 marks]

Hints: You will need to calculate the expectation value of the dipole moment operator $\langle \mathbf{d} \rangle$ for the transition. The dipole moment lies along the z axis, but nonetheless the problem is 3-dimensional, so choose your coordinate system carefully. You may find the following integral useful

$$\int r^4 e^{-br} dr = -\frac{1}{b^5} \left[e^{-br} \left(b^4 r^4 + 4b^3 r^3 + 12 b^2 r^2 + 24 br + 24 \right) \right]$$

where $b = 3/(2a_0)$ and $a_0 = 5.29177 \times 10^{-11}$ m.