ELECTROMAGNETISM

Level 2 Physics problems – Foundations of physics 2

Question 4 Cycle 2 Version 1

Professor D P Hampshire – 2nd Year Physics Lecture Course

These problems are formatively self-assessed. Students who showed the chutzpah to volunteer for the peer-marking pilot scheme will also mark one of their peer's scripts.

Not necessary for homework but for 'fun' read Feynman Lectures in Physics Chapters 8, 13, 14 and 18.

1. Consider an infinitely long straight wire of radius R carrying a current I which is assumed to be uniformly distributed throughout its circular cross-section. Use Ampère's law to obtain an expression for the magnetic field both inside and outside the wire. [1 mark]

2. A long, straight solid cylinder, oriented with its axis in the z direction, carries a current whose current density is $\underline{\boldsymbol{J}}$. The current density, although symmetrical about the cylinder axis, is not constant but varies according to the relation;

$$\underline{\boldsymbol{J}} = \begin{cases} \dfrac{2I_o}{\pi a^2}\left[1-\left(\dfrac{r}{a}\right)^2\right]\hat{\mathbf{k}} \text{ for } r \leq a, \\ 0 \text{ for } r \geq a, \end{cases}$$

where a is the radius of the cylinder, r is the radial distance from the cylinder axis, and I_o is a constant having units of amperes.

a) Show that I_o is the total current passing through the entire cross-section wire. [1 mark]

b) Show that for $r \geq a$, the magnitude of the magnetic field is given by [1 mark]

$$B = \frac{\mu_0 I_0}{2\pi r}$$

c) Show that for $r \leq a$, the magnitude of the magnetic field is given by; [2 marks]

$$B = \frac{\mu_o I_o r}{2\pi a^2}\left(2 - \frac{r^2}{a^2}\right)$$

3. Very briefly describe four experiments that would test Maxwell's four equations in integral form. [2 marks]

4. Consider a standard plane electromagnetic wave in a vacuum which can be described by;

$$\underline{\boldsymbol{E}} = \begin{pmatrix} E_{ox} \\ E_{oy} \\ E_{oz} \end{pmatrix} \exp i(k_x x + k_y y + k_z z - \omega t)$$

and,

$$\underline{\boldsymbol{B}} = \begin{pmatrix} B_{ox} \\ B_{oy} \\ B_{oz} \end{pmatrix} \exp i(k_x x + k_y y + k_z z - \omega t).$$

a) Prove that $\underline{\boldsymbol{\nabla}} \cdot \underline{\boldsymbol{E}} = 0$ and $\underline{\boldsymbol{\nabla}} \cdot \underline{\boldsymbol{B}} = 0$ can only be always true provided that $\underline{\boldsymbol{E}}$ and $\underline{\boldsymbol{B}}$ are perpendicular to the direction of propagation given by $\underline{\boldsymbol{k}}$. [1 mark]

b) Prove (by using the appropriate Maxwell equation) that for a plane electromagnetic wave in free space that $\underline{\boldsymbol{k}} \times \underline{\boldsymbol{E}} = \omega \underline{\boldsymbol{B}}$ and that $k^2 \underline{\boldsymbol{E}} = \omega \underline{\boldsymbol{B}} \times \underline{\boldsymbol{k}}$. [2 marks]