L2 Foundation of Physics 2B Optics 2019-20

Workshops 5 and 6

March 8, 2020

This worksheet it to cover workshops both this week and next week. Due to the strike, there are two types of question labelled (E) and (NE):

(E) These questions cover material you have been taught in the lectures (ie up to lecture 8). This material is Examinable.

(NE) These questions cover material you have not been taught due to the strike. they are therefore Non-examinable.

Non-examinable questions are included here because the university's strike mitigation plan states that you should study this material after the exams to make sure you meet the course learning outcomes. I wanted to give you a chance to look at these questions with demonstrators present. i would encourage you to refer to *Optics f2f* in place of lecture notes (these will be provided later).

E.1 In the course we have so far made (i) the scalar approximation and (ii) the paraxial approximation. Give an example of an optical instruments where each of these approximations break down, and explain your reasoning.

E.2 A scalar plane wave with $\lambda = 514$ nm impinges normally on an opaque screen containing a circular hole. When viewed axially from a distance of 250 mm, the hole uncovers the first Fresnel zone. What is the diameter of the hole?

E.3 A conventional Fresnel zone plate achieves a high intensity on-axis by blocking all of either the odd or the even Fresnel zones, thus eliminating the destructive cancellation of the fields from neighbouring zones. What would happen if it were possible to manufacture a mask that rather than blocking the even zones, allowed the light to pass but retarded the phase by π?

NE.1 What is the Rayleigh range of a laser of wavelength $\lambda = 633$ nm of waist 0.250 mm? What is the diameter of the beam after it has propagated 500 m. (Hint: See Lecture 9 and f2f Example 5.9])

NE.2 (See Lecture 9 and *Optics f2f* sections 5.8 and 5.9). If the input light field, $E_0\mathrm{f}(x', y')$, is cartesian separable, then we can write $\mathrm{f}(x', y') = \mathrm{g}(x')\mathrm{h}(y')$, and the Fraunhofer diffraction formula becomes

$$I^{(z)} = \frac{I_0}{\lambda^2 z^2}\left|\int_{-\infty}^{\infty} \mathrm{g}(x')\mathrm{e}^{-\mathrm{i}2\pi x x'/(\lambda z)}\mathrm{d}x' \int_{-\infty}^{\infty} \mathrm{h}(y')\mathrm{e}^{-\mathrm{i}2\pi y y'/(\lambda z)}\mathrm{d}y'\right|^2 . \tag{1}$$

An opaque screen containing a rectangular aperture of width, a, and height, b, is illuminated normally by uniform monochromatic light with intensity I_0 and wavelength λ.

(a) Using eqn (1), write an expression for the far-field intensity distribution in terms of a, b, x, y, z, I_0 and λ. [4 marks]
[Hint: $\int_{-\infty}^{\infty} \mathrm{rect}(x'/a)\mathrm{e}^{-\mathrm{i}2\pi x x'/(\lambda z)}\mathrm{d}x' = a\mathrm{sinc}(\pi a x/\lambda z)$.]

(b) If $\lambda = 0.5$ μm, $z = 5$ m, and the first zero in the diffraction pattern is observed at $x = 5$ mm, what is the slit width, a? [2 marks]

(c) For a slit of width, a, the Rayleigh distance, where the effect of diffraction becomes as large as the slit width is, $d_\mathrm{R} = a^2/\lambda$. Using the parameters of part (b), what is the value of the ratio, z/d_R? If this consistent with the far-field condition, $z \gg d_\mathrm{R}$? [2 marks]

(d) Explain, why according to the Fraunhofer approximation, displacing the slit by a small distance, d, along the x' axis does not change the intensity distribution? [3 marks]
[Hint: $\int_{-\infty}^{\infty} \mathrm{rect}[(x'-d)/a]\mathrm{e}^{-\mathrm{i}2\pi x x'/(\lambda z)}\mathrm{d}x' = \mathrm{e}^{-\mathrm{i}2\pi x d/(\lambda z)}a\mathrm{sinc}(\pi a x/\lambda z)$.]

(e) How far, in practice, does the diffraction pattern move for $d = 1$ mm? Express your answer as a fraction of the distance to the first zero, and comment on the accuracy of the Fraunhofer approximation. [3 marks]

(f) In contrast to the far-field case, the Fraunhofer diffraction formula is exact in the focal plane of a lens. If a lens with focal length $f = 10$ cm is placed in the aperture plane, what is the distance to the first zero along x in this case? [2 marks]

(g) How far does the diffraction pattern move if the slit is displaced by a distance, $d = 1$ mm, along x' in the lens plane. [2 marks]

NE.3 *Circularly polarized light in the linear basis* (Optics f2f Chapter 4): A circularly polarized plane wave propagating along the z axis can be written as,

$$\boldsymbol{E}_{\mathrm{L}} = \tfrac{1}{\sqrt{2}} E_0 \left(\hat{\boldsymbol{\epsilon}}_x + \mathrm{i}\hat{\boldsymbol{\epsilon}}_y\right) \mathrm{e}^{\mathrm{i}(kz-\omega t)} \ , \tag{2}$$

where $\hat{\boldsymbol{\epsilon}}_x$ and $\hat{\boldsymbol{\epsilon}}_y$ are unit vectors along the x and y axes.

(a) What is the amplitude of the wave? [1 mark]

(b) Sketch the electric field vector at positions $z = 0$, $\lambda/4$, $\lambda/2$, $3\lambda/4$ and λ for $t = 0$. [5 marks]

(c) Is the light is left- or right-circularly polarized. [1 mark]

(d) Sketch the electric field vector at position $z = 0$, for times (i) $t/T = 0$, (ii) 1/8, (iii) 1/4 and (iv) 3/8, where $T = 2\pi/\omega$ is the period of the wave. [4 marks]

(e) In which direction does the electric field vector rotate in time? Assume you are sitting at the origin and looking at the field as it propagates towards you. [1 mark]