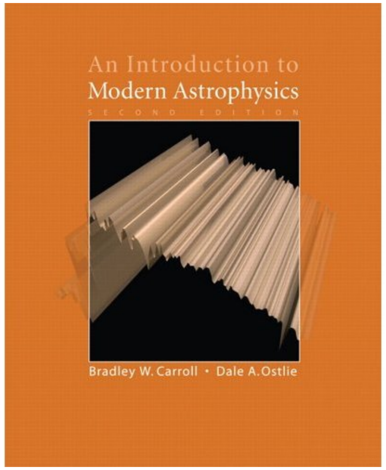
Lecture 3: Observed properties –

Stellar binaries, orbits, masses

Professor David Alexander
Ogden Centre West 119

Chapter 7 of Carroll and Ostlie



Aims of lecture

Key concept: mass measurements from binary system dynamics

Aims:

- Be able to define the various classes of binary system
- Know what properties can be measured from binary systems and how inclination angle effects the measured properties and masses

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{\alpha_2}{\alpha_1} = \frac{v_2}{v_1}$$

Mass ratio relationships

$$P^2 = \frac{4\pi^2 a^3}{G\left(m_1 + m_2\right)}$$

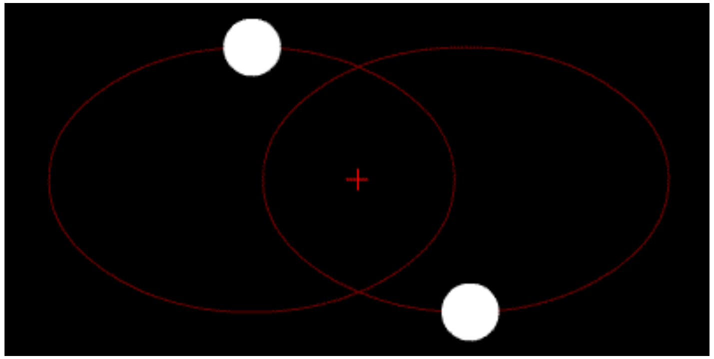
Keplers generalised 3rd law

Binary stars: basic properties

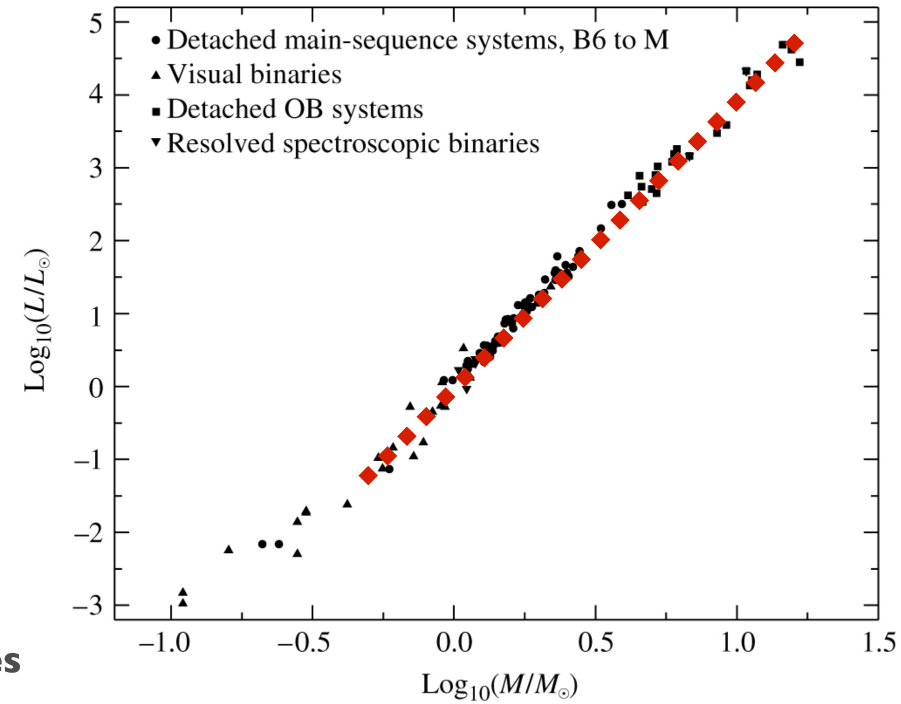
About 70% of stars in our galaxy are members of binary systems.

Planets have even been found around binary (or multiple) star systems!

Binary stars provide key insight into the formation and evolution of stars, most importantly by directly providing masses



We will first look at the observed manifestations of binary stars and then explore how we can determine their masses



Binary stars: classification

Binary stars are classified depending on how they are identified:

(1) Visual binary
(2) Astrometric binary
(3) Eclipsing binary
(4) Spectroscopic binary

These classes are not mutually exclusive (a system can be in >1 class); e.g., eclipsing spectroscopic binaries – an eclipsing and spectroscopic binary

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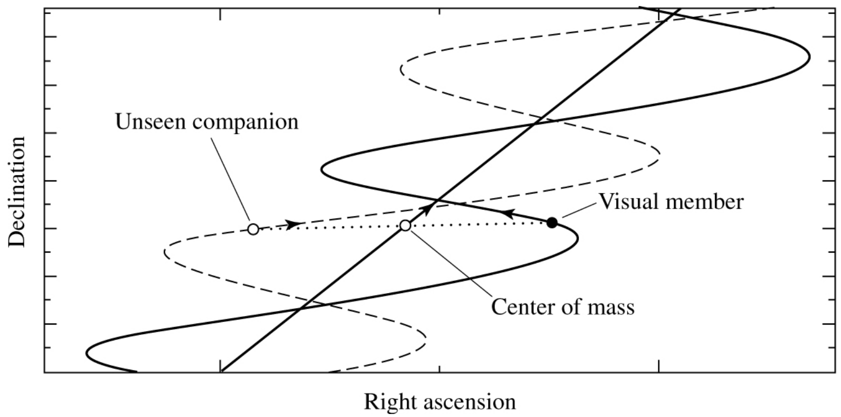
In a visual binary both stars are spatially resolved. Assuming that the orbits of both stars are not long then it is possible to monitor the duration of the orbit and the orbital parameters directly.

This image here shows the Castor star system, taken with a small telescope. The Castor star system was the first multiple-star system to be identified (in 1678).

Castor is actually a sextuple star system. Each of these stars has a faint companion (astrometric and spectroscopic binary) and another faint binary orbits both pairs at a greater distance (which is an eclipsing binary): therefore all binary classifications are found in the Castor system!

Binary stars: classification

Astrometric binary:



If one member of the binary is much brighter than the other then it may not be possible to directly see the fainter star. However, the existence of the unseen star can be inferred from the motion of the brighter star.

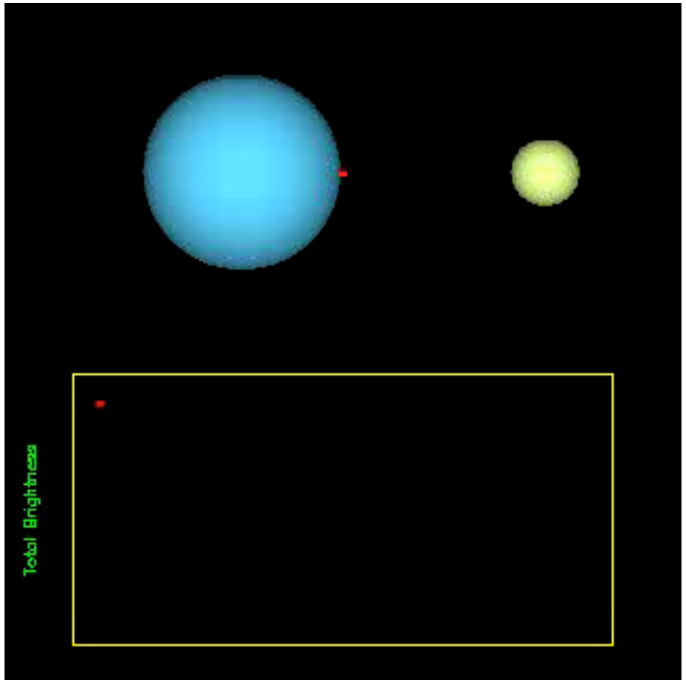
A famous example is Sirius B, the white dwarf discovered from the motion of Sirius A in 1844 (white dwarfs are explored in a later lecture).

Eclipsing binary:

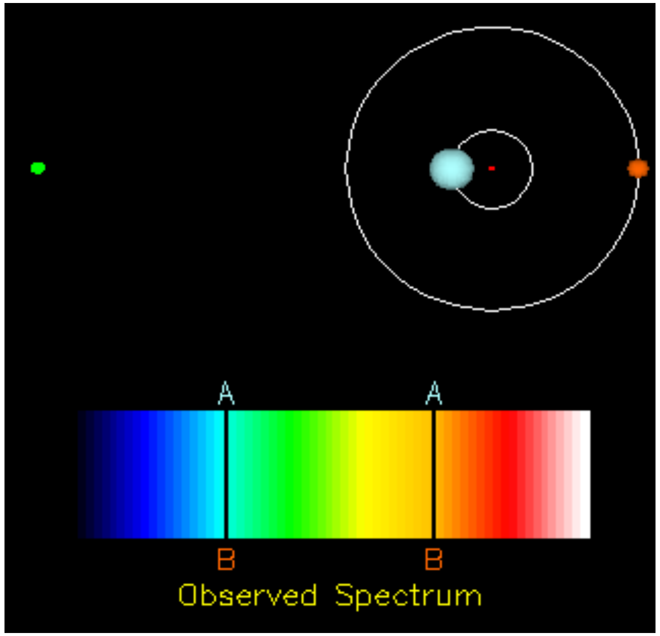
For binaries with orbital planes orientated approximately along the line of sight to the observer, one star may periodically pass in front of the other, blocking the light of the eclipsed component. Such a system is recognised by regular variations in the amount of observed light.

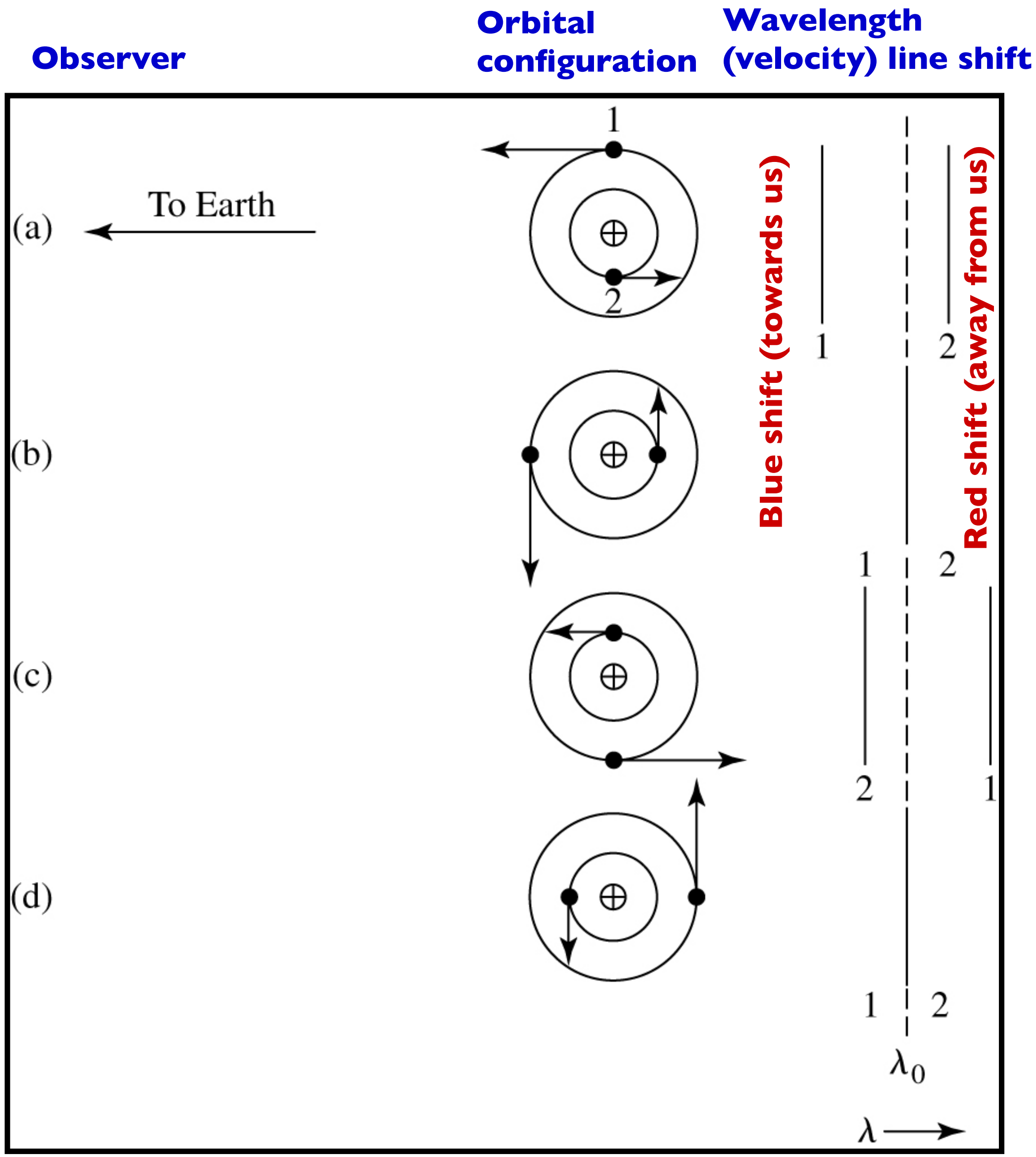
This technique is also used to search for exo-planets from the small dip in brightness due to the eclipse of a planet in front of a star.

We can also determine the radii and even temperature ratios of stars when they are eclipsing binaries (lecture 1)



Binary stars: classification





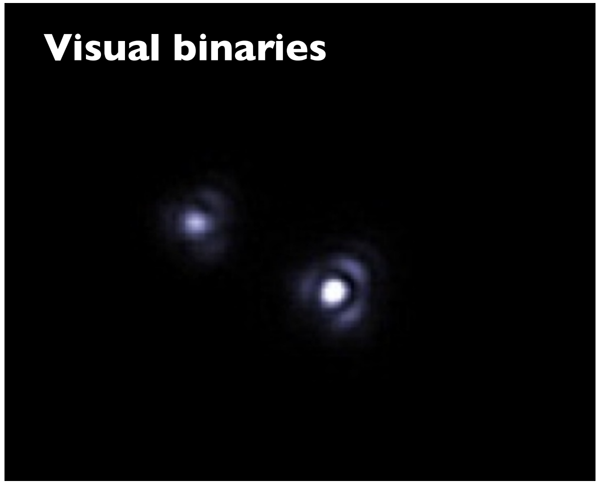
Spectroscopic binary:

If the period of a binary system is not prohibitively long, and if the orbital motion has a component along the line of sight, a periodic shift in the spectral lines will be observable.

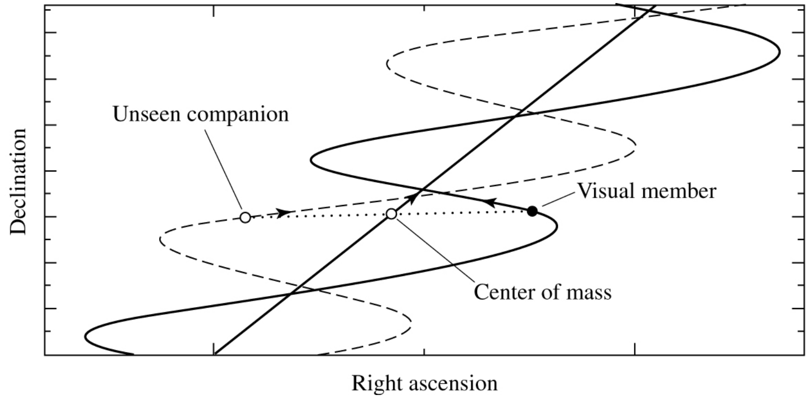
If the luminosities of the two stars are comparable, both spectra will be observable (double lined). If one is much fainter than the other then only one set of lines is seen (single lined).

Stars must be quite close together for rapid changes in velocity to be seen

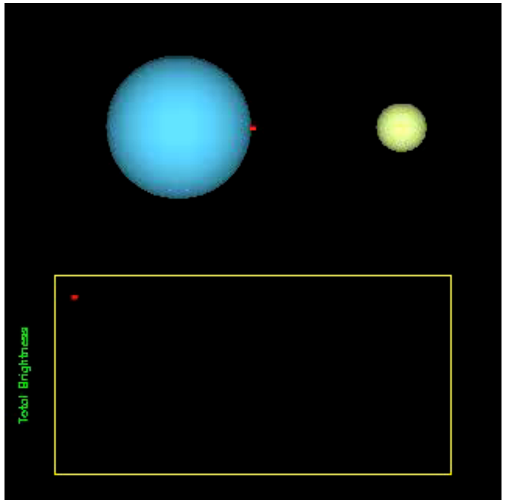
The different ways to identify binary stars



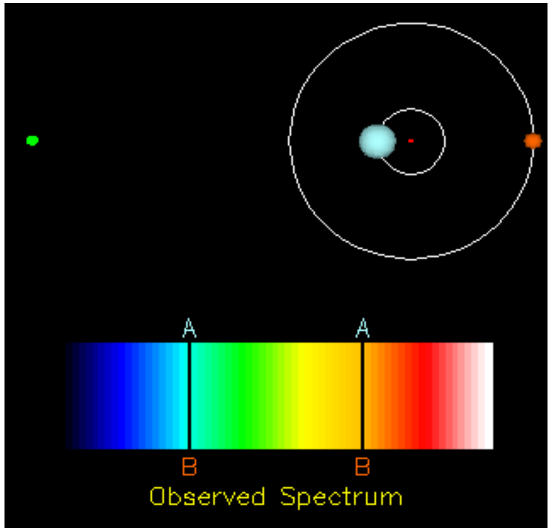
Astrometric binary



Eclipsing binary

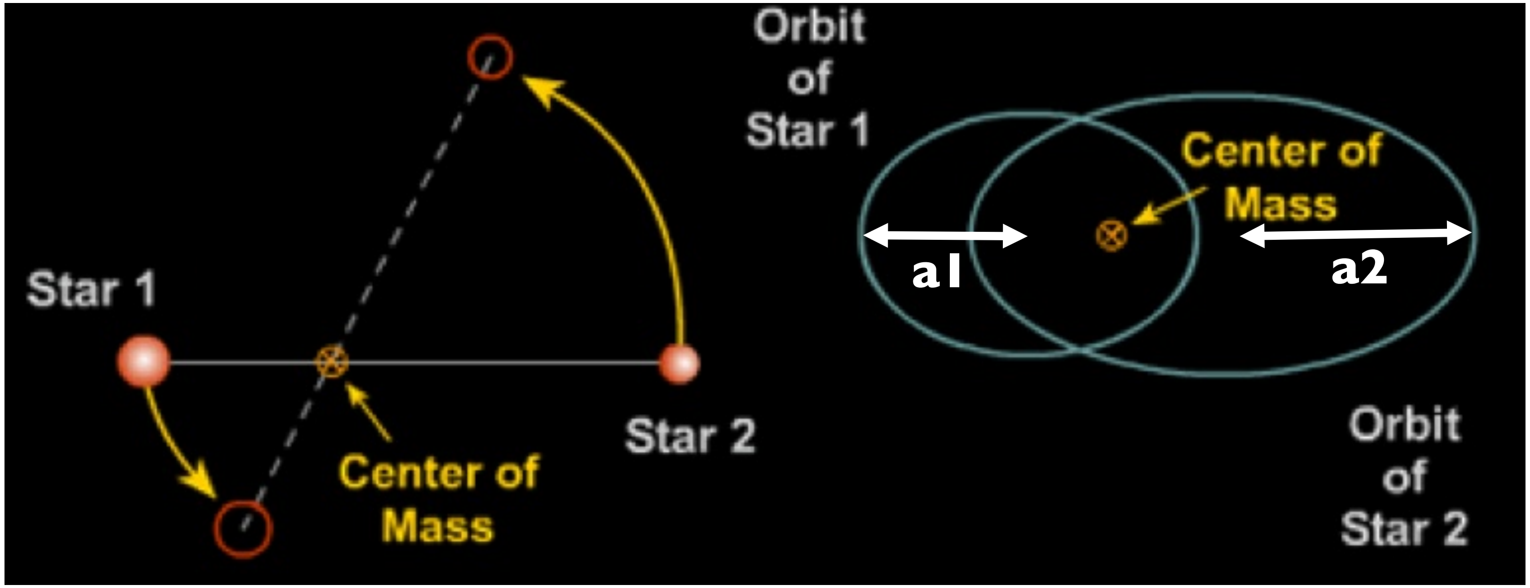


Spectroscopic binary



These are just the observed manifestations of a binary system – there are no fundamental differences between the different classes of stellar binaries

Binary stars: system properties



In any binary/multiple star system the stars orbit about the centre of mass in elliptical orbits (semi-major axes of a1 and a2 for a binary system)

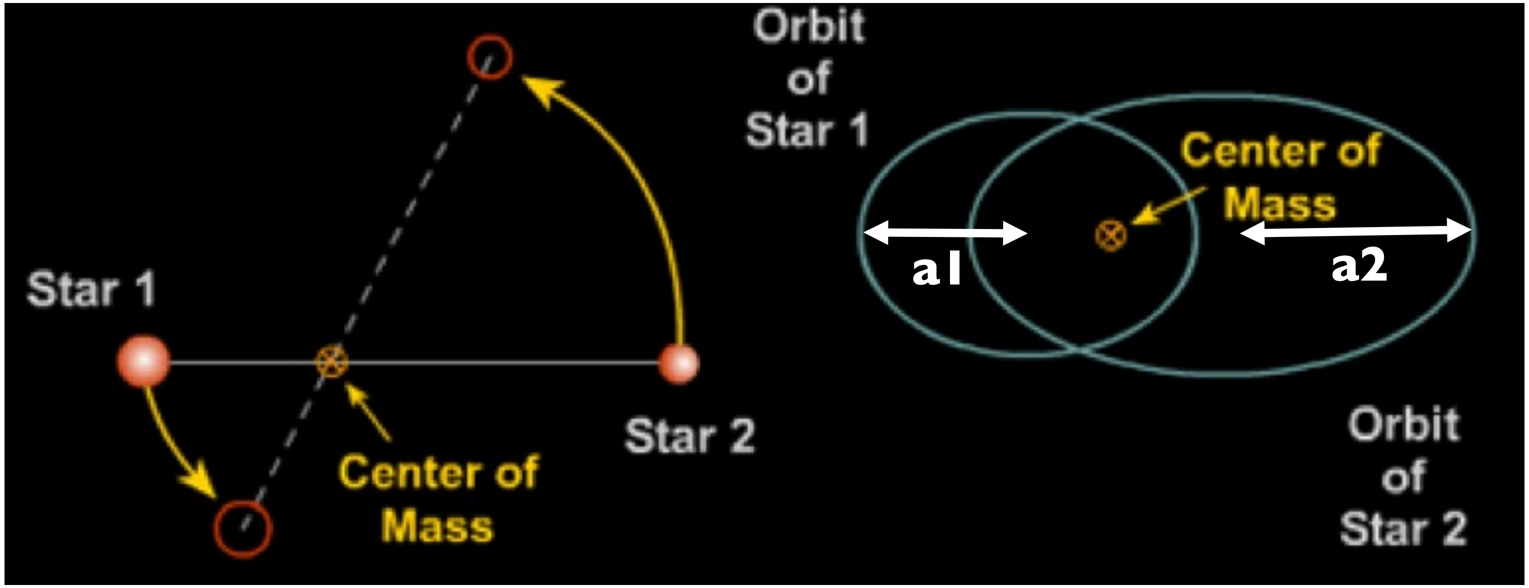
We can determine masses using the generalized form of Kepler's 3rd law:

$$P^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}$$ **Equation 3**

Where $a = a_1 + a_2$. This gives the sum of the masses of the stars, provided that a is known. Other methods, which we explore later, can be used to determine individual masses.

This is the relationship between angular frequency and the mutual gravitational attraction – we will explore more in the workshop

Binary stars: system properties

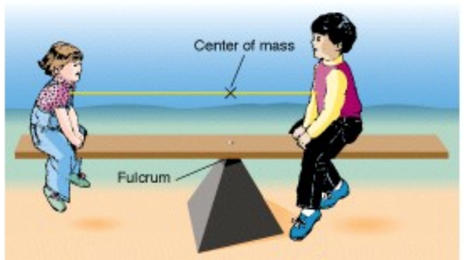


From the orbital data of a visual binary system, it is possible to determine the orientation of the orbits and the mutual center of mass, which provides knowledge of the ratio of the stars' masses. If the distance to the system is also known, the linear separation of the stars can be determined, leading to individual stellar masses.

Consider two stars in orbit about their mutual center of mass, and assume that the orbital plane is perpendicular to the observer's line of sight. It follows that

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

Equation 4



a_2, a_1 = semimajor axes of the ellipses.

Visual binary stars: mass determination

But we can't directly measure the semi-major axis; however, in visual binary systems we can measure the angular separation

If the distance from the observer to the binary system is d, the angles subtended by the semimajor axes are:

$$\alpha_1 = \frac{a_1}{d}$$

$$\alpha_2 = \frac{a_2}{d}$$

(small angle approximation as in parallax: see lecture 1)

Where α_1 and α_2 are measured in radians. Then the mass ratio just becomes:

$$\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$$

Equation 4

Let's try out an example

This picture is a bit idealised because most binary systems will be inclined away from our line of sight and therefore the effect of the inclination angle on our calculation needs to be considered.

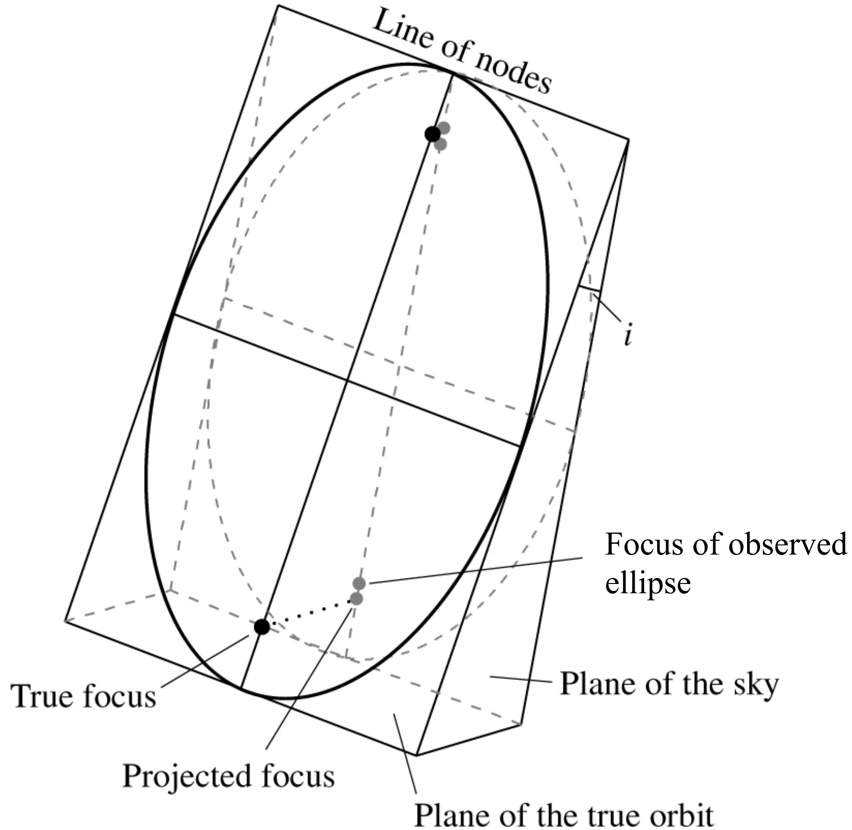
Visual binary stars: inclination

Let i be the angle of inclination between the plane of an orbit and the plane of the sky. As a special case, we will also assume that the orbital plane and the plane of the sky (defined as being perpendicular to the line of sight) intersect along a line parallel to the minor axis, along a line of nodes.

The observer will not measure the actual angles but their projections onto the plane of the sky:

$$\tilde{\alpha}_1 = \alpha_1 \cos i$$
$$\tilde{\alpha}_2 = \alpha_2 \cos i$$

This geometrical effect plays no part in determining the mass ratios, since the cos i term just cancels. However, this projection effect makes a significant difference when using Kepler's 3rd law, since $\alpha = a/d$.



You can show that:

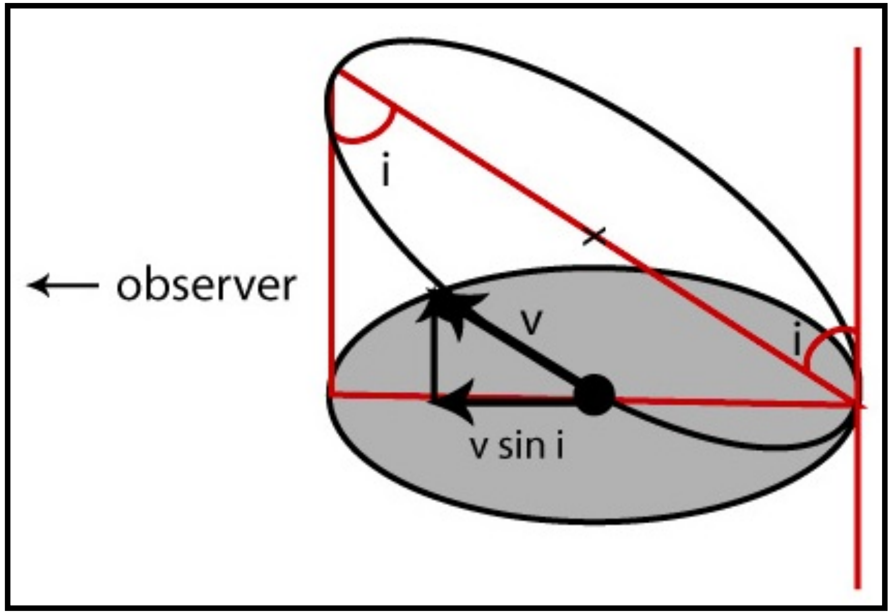
$$m_1 + m_2 = \frac{4\pi^2}{G}\left(\frac{d}{\cos i}\right)^3 \frac{\tilde{\alpha}^3}{P^2}$$

where $\tilde{\alpha} = \tilde{\alpha}_1 + \tilde{\alpha}_2$.

Spectroscopic binary stars

Many binary stars are identified spectroscopically. Consider a binary system in which the spectra of both stars are seen (double-lined, spectroscopic binary). The key thing is the inclination angle, i, because it directly influences the stars' observed radial velocities, which cannot exceed:

$$v_{1r}^{\max} = v_1 \sin i$$
$$v_{2r}^{\max} = v_2 \sin i$$



Recall from earlier slide:

The measured radial velocities depend on the positions of the stars at the instant you observe them: zero velocity is measured for motions perpendicular to the line of sight.

Consider how the velocity curve for a binary system would look. How might a change in inclination angle modify the velocity curves?

Spectroscopic binary stars: masses

Although the exact shapes of the curves depend strongly on the orientation of the orbits with respect to the observer, even for a given inclination angle, many spectroscopic binaries actually have nearly circular orbits. This occurs because of tidal interactions in close binaries that tend to circularize the orbits.

Assuming small eccentricity (e<<1), the speeds of the stars are broadly constant:

$$v_1 = \frac{2\pi a_1}{P}$$

$$v_2 = \frac{2\pi a_2}{P}$$

where the stars have masses m_1 and m_2, and semimajor axes a_1 and a_2. P is the period of the orbit. Of course if you have the period and the velocity you can rearrange the equation to determine the semimajor axes.

The ratio of the two masses is given by

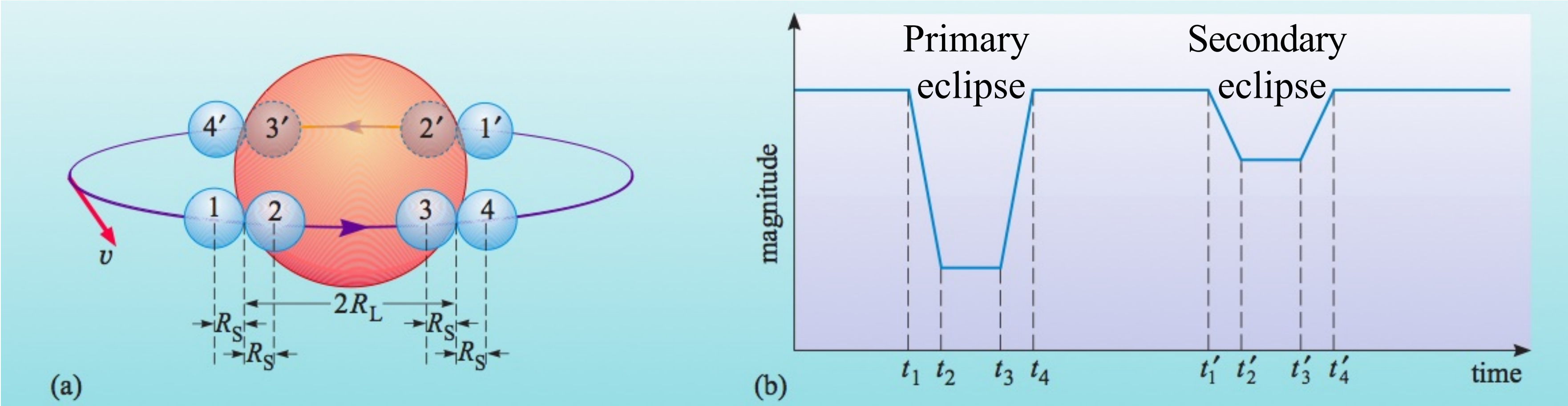
$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

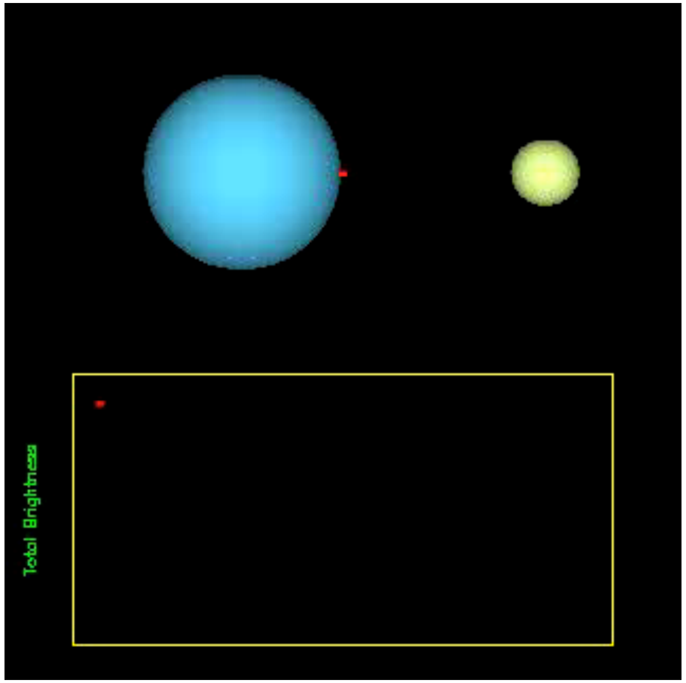
Equation 4

Again, there is no inclination dependence on mass ratios but there is for the total mass (for spectroscopic binaries); however, we won't explore that here

Eclipsing spectroscopic binaries: special case

Eclipsing spectroscopic binaries are a special case of binary star. Since the stars eclipse the inclination of the system must be close to 90 degrees. Therefore, the inclination angle corrections are small and consequently reliable masses can be directly calculated.





The inclination angle must be close to 90 degrees for an eclipse to occur, particularly if the secondary star is not in close orbit. If the secondary star is in close orbit than i will still be quite close to 90 degrees: **so *v* is reliably measured**

We can also use eclipses to estimate stellar radii (recall from Lecture 1):

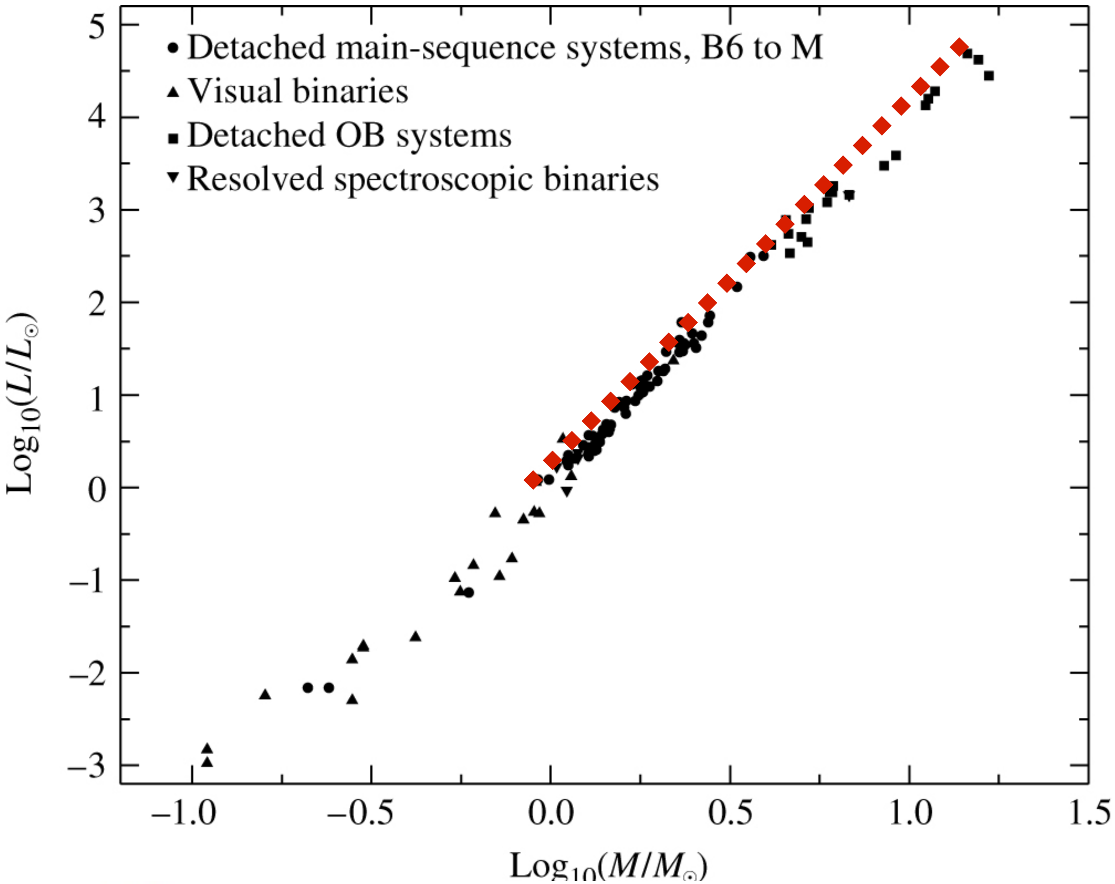
$$2R_S = v \times (t_2 - t_1)$$

$$2R_L = v \times (t_4 - t_2) \qquad \textbf{Recall } v = v_L + v_S$$

$$2R_S + 2R_L = v \times (t_4 - t_1)$$

Stellar mass-luminosity relationship

Using the best mass constraints (from binary systems), astronomers have found a tight relationship between mass and luminosity for main-sequence stars – this very important finding ultimately helps to provide constraints on the energy source (and lifetimes) of stars



Equation 5

$$\frac{L}{L_\Theta} \approx \left(\frac{M}{M_\Theta}\right)^\alpha$$ where α~3-4, with some evidence for a mass dependency

The observed properties of stars

From these series of lectures you should now know:

- **What properties can be measured of stars**
- **The basic emission mechanism of stars and how we can calculate the luminosity, radius, and temperature (if we have 2 of the 3)**
- **How the observed spectral properties of stars are driven by temperature through excitation and ionisation**
- **The Hertzsprung-Russell diagram and how to sketch it**
- **How to identify a stellar binary system**
- **How to calculate the masses of stars in a binary system and how inclination effects the measured properties**
- **The connection between luminosity and mass (for main-sequence stars)**