CM4 Solutions: The Atwood Machine (again)

1. **(1 mark total)** With the quantities as defined in the question, the constraint is $y_1 + y_2 + x = 0$. **[1 mark]**

2. **(2 marks total)** The kinetic energy is given by $T = \frac{1}{2}(m_1\dot{y}_1^2 + m_2\dot{y}_2^2)$.
   Using the constraint from part (i), the potential energy is $V = m_1 g y_1 + m_2 g y_2 + \frac{k}{2}(y_1 + y_2)^2$.
   With $L = T - V$, the required result follows. **[2 marks]**

3. **(1 mark total)** For y_1, the E-L equation implies $\ddot{y}_1 = -\frac{k}{m_1}(y_1 + y_2) - g$.
   For y_2, the E-L equation implies $\ddot{y}_2 = -\frac{k}{m_2}(y_1 + y_2) - g$. **[1 mark]**

4. **(2 marks total)** In equilibrium, $\ddot{y}_1 = \ddot{y}_2 = 0$.
   Hence, with $m_1 = m_2 = m$, the EoM imply that in equilibrium, $y_1 + y_2 = -mg/k$. **[1 mark]**
   The constraint equation then gives the equilibrium extension as $x = -(y_1 + y_2) = mg/k$. **[1 mark]**

5. **(3 marks total)** The matrices, when evaluated, are

$$\hat{\tau} = \frac{m}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

   and

$$\hat{v} = \frac{k}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

   which can then be substituted into the equations of motion. **[1 marks]**

   This leads to a generalised eigenvalue problem, the solutions to which are found from the following determinant:

$$\begin{vmatrix} k - \omega^2 m & k \\ k & k - \omega^2 m \end{vmatrix} = 0.$$

   These are $\omega_1 = 0$ and $\omega_2 = \sqrt{2k/m}$, with corresponding normalised eigenvectors

$$\underline{b}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

   and

$$\underline{b}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

   which are found by substituting the mode frequencies back into the matrix form of the EoM. **[2 marks]**

6. **(1 mark total)** Solution 1 represents a translation of the system with one mass rising at the same rate that the other drops. Solution 2 represents in-phase oscillations, when both masses rise and fall together at the frequency of oscillation associated with the reduced mass of the system. **[1 mark]**