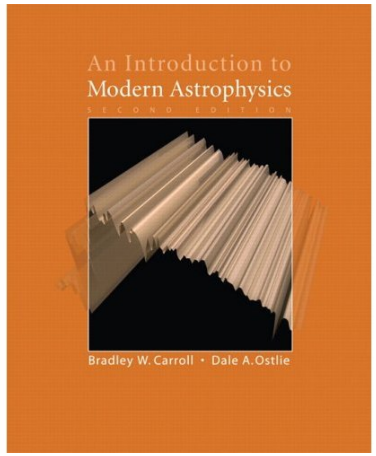
Lecture 7:
Stellar power source –
Nucleosynthesis and neutrinos

Professor David Alexander
Ogden Centre West 119

Chapter 10 and 11 of Carroll and Ostlie



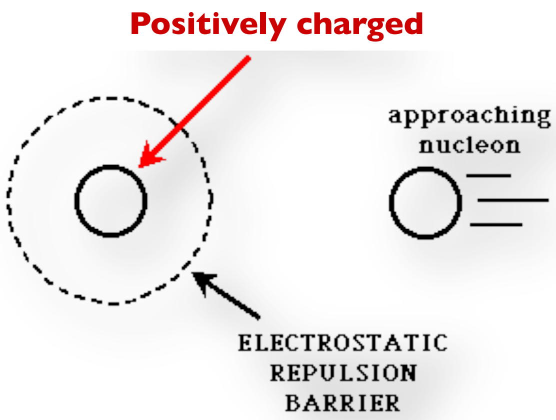
Aims of lecture

Key concept: nucleosynthesis

Aims:

- Know and be able to apply the nuclear reaction conservation rules
- Understand and be able to write down the PP I chain and to know the basic steps behind the other PP chains, CNO cycle, and 3α processes
- Have a basic understanding of the later nuclear fusion processes
- Understand how neutrinos can test our model of the solar core

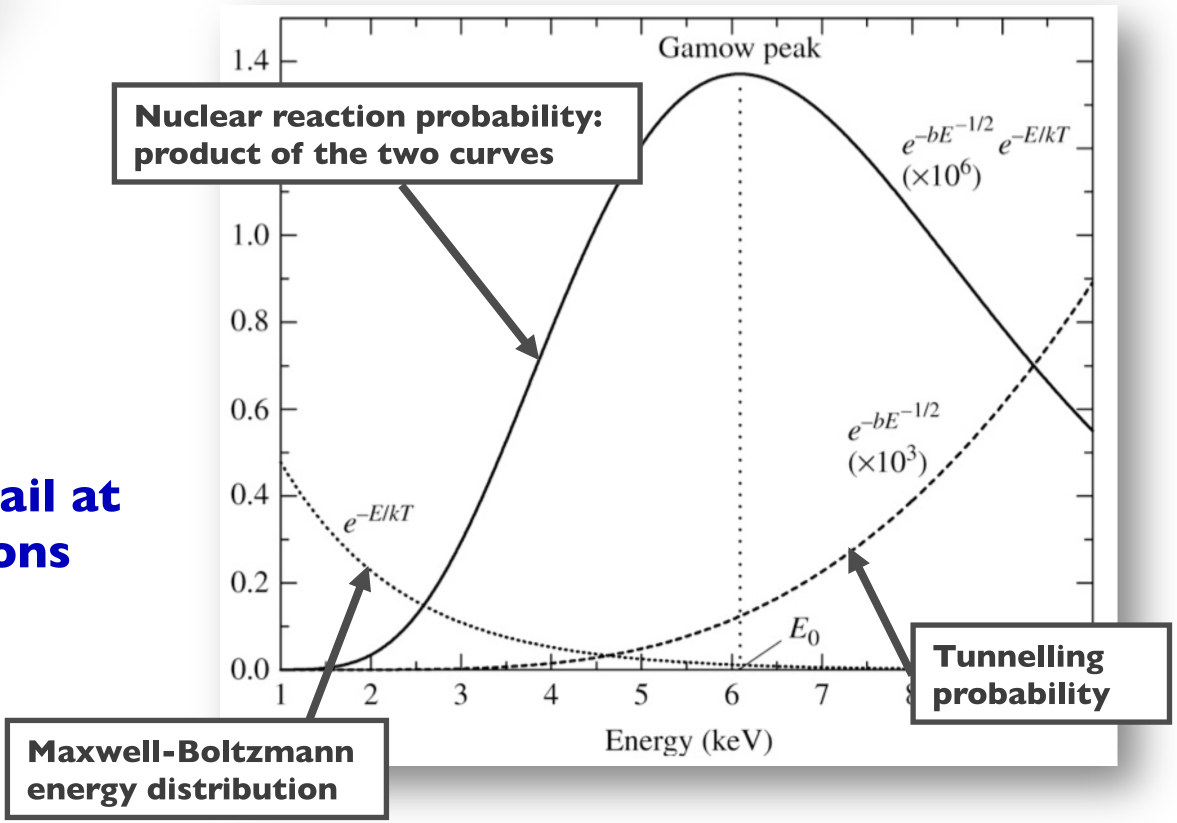
Nuclear fusion



Probability of nuclear reaction is product of:

(1) Probability of two particles close enough for the nuclear force to be important

(2) Probability that a nuclear reaction will occur



Let's look in more detail at the nuclear reactions (nucleosynthesis)

Nuclear reaction conservation laws

There are three conservation laws for nuclear reactions:

1. The total electric charge is the same before and after a reaction.
2. The total nucleon number is conserved. Nucleon number is found by counting p=+1 and n=+1.
3. The total lepton number is conserved. Leptons are electrons, positrons ($e^{+/-}$) and neutrinos and anti-neutrinos (ν , $\bar{\nu}$). It is found by counting:

   $e^- = +1$, $e^+ = -1$, $\nu = +1$, $\bar{\nu} = -1$.

We will now explore the proton-proton (PP) chain, which is the dominant nuclear fusion process in stars like the Sun, and the CNO cycle

Both of these nuclear fusion processes convert Hydrogen into Helium but they differ in how they achieve this and in their dependencies on temperature (as we will see)

Hydrogen-Helium: Proton-proton chain

Remember must conserve:
(1) total electric charge,
(2) total nucleon number, &
(3) total lepton number

~10^10 yrs $\quad {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu_e$

~6 secs $\quad {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma$

69%

~10^6 yrs

$${}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H}$$

(PP I)

31%

$${}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{7}_{4}\mathrm{Be} + \gamma$$

99.7%

$${}^{7}_{4}\mathrm{Be} + e^{-} \rightarrow {}^{7}_{3}\mathrm{Li} + \nu_e$$

$${}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \rightarrow 2\,{}^{4}_{2}\mathrm{He}$$

(PP II)

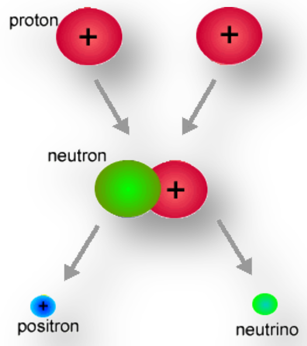
0.3%

$${}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{8}_{5}\mathrm{B} + \gamma$$

$${}^{8}_{5}\mathrm{B} \rightarrow {}^{8}_{4}\mathrm{Be} + e^{+} + \nu_e$$

$${}^{8}_{4}\mathrm{Be} \rightarrow 2\,{}^{4}_{2}\mathrm{He}$$

(PP III)



There are 3 possible paths in the PP chain: PPI, II, and III – the probability of each path is shown

Hydrogen-Helium: Proton-proton chain

The first reaction shown is the basis for the whole PP chain. Qualitatively, it resembles the β-decay of the neutron (i.e. a weak interaction); in this case, a proton 'decays' near another proton to form a deuteron (the nucleus of deuterium). The rate for this process is too slow to be measured in the laboratory (~10^{10} yrs), but we can calculate it. The ^{2}H+^{1}H reaction is so fast (~6 secs) that its rate is unimportant.

The ^{3}He+^{3}He reaction has been studied extensively, and it completes the PP chain in about 69% of PP chain terminations (PP I). The total energy release is 26.7 MeV.

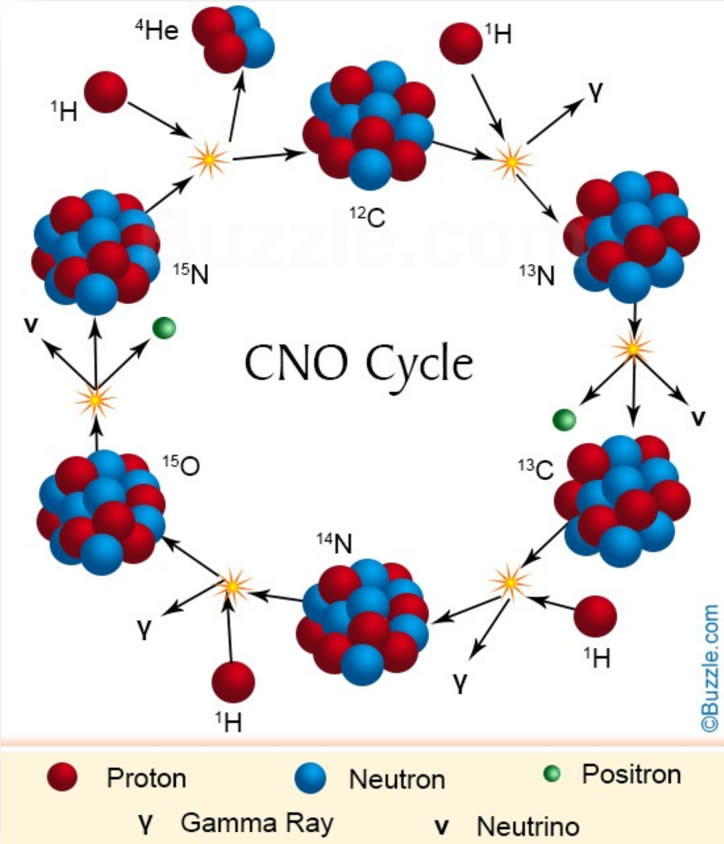
Recall that this 26.7 MeV is the mass difference and origin of the 0.7% efficiency

The reaction involving ^{3}He and ^{4}He leads to some very important neutrino-producing interactions. The cross section is larger than people realized at first, and in fact it (and its associated reactions) occur about 31% of the time in a star like the sun. The majority (~99.7%) of the ^{7}Be produced in this way is then burnt via the PP II chain.

The ^{7}Be+^{1}H reaction occurs only rarely, something like 0.3% of the time (PP III). We will briefly discuss some of the more obscure parts of the PP chain when we look at solar neutrino measurements later in this lecture.

Hydrogen-Helium: CNO nuclear reaction cycle

Carbon Nitrogen Oxygen (CNO) cycle: another Helium producing chain but with CNO as catalysts



The CNO cycle probably only contributes ~1.5% of the total solar luminosity

But for higher temperature stars (typically >1.2x mass of Sun) it is more important

An α-particle is the Helium nucleus

In this set of reactions, first described by Bethe in 1939, the overall conversion of four protons to form an α-particle, two e^+s and two ν s is accomplished with the aid of ^{12}C, the most abundant heavy isotope in normal stellar conditions. The total energy release is the same as for the pp chain (26.7 MeV).

Temperature dependence on dominant chain

The rates of energy release of the pp chain and the CNO cycle vary smoothly with temperature. Over a limited range in temperature one can write:

$$\varepsilon_{pp} = \varepsilon_1 X_H^2 \rho T^4 \qquad \mathbf{W\ kg^{-1}}$$

where:

ε_1 = constant

X_H = fractional hydrogen content

ρ = density

Which factor is the most important for fusion to occur?

For CNO, this is:

$$\varepsilon_{CNO} = \varepsilon_2 X_H X_C \rho T^{17} \qquad \mathbf{W\ kg^{-1}}$$

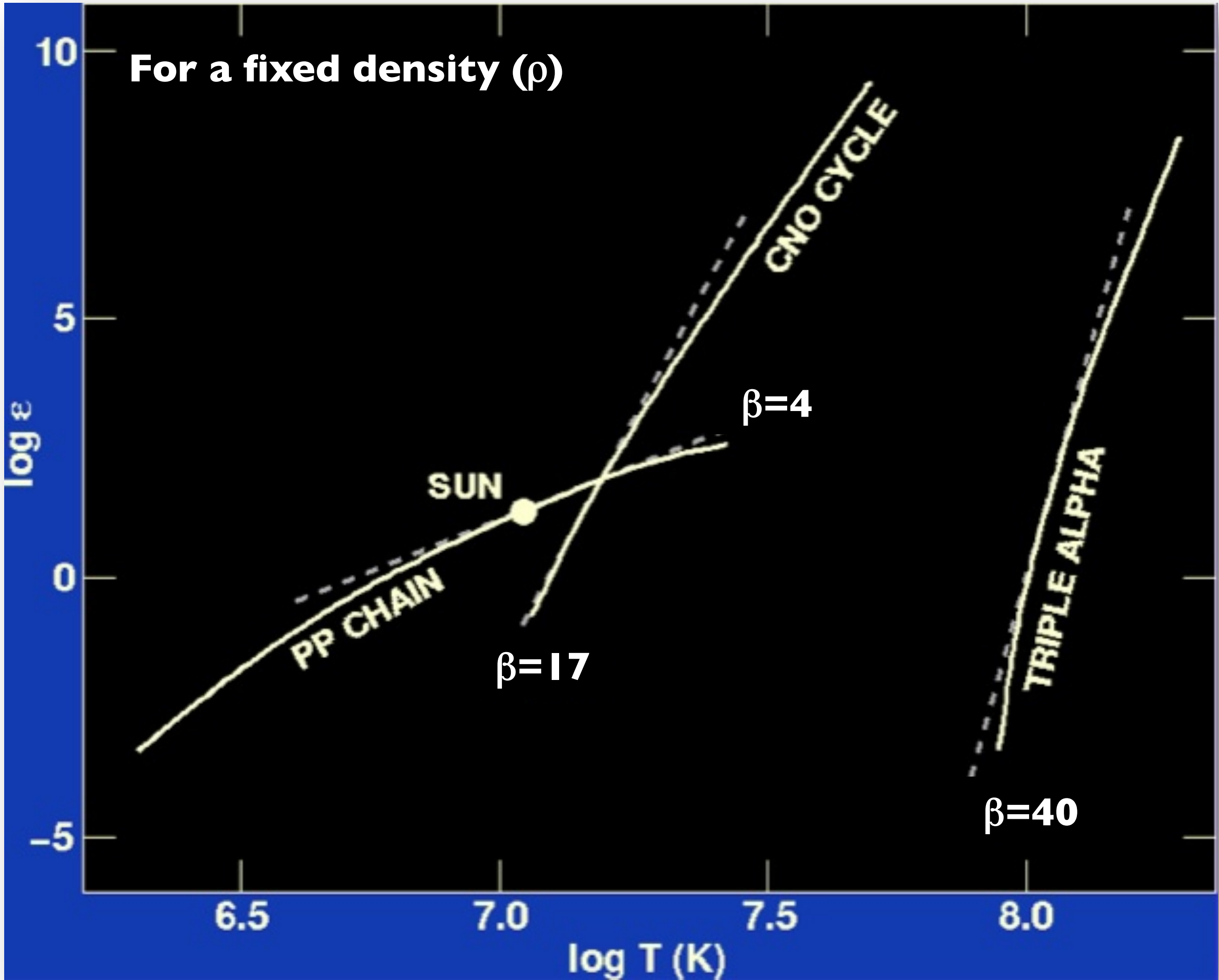
where:

X_C = fractional carbon content

ε_2 = constant

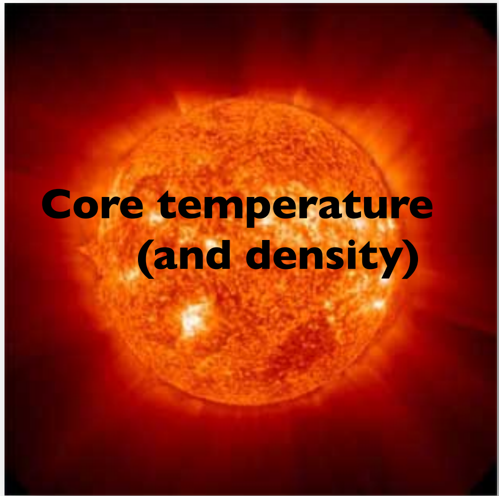
Note the exponents are slightly different to those in the CO book

Temperature dependence for different reactions



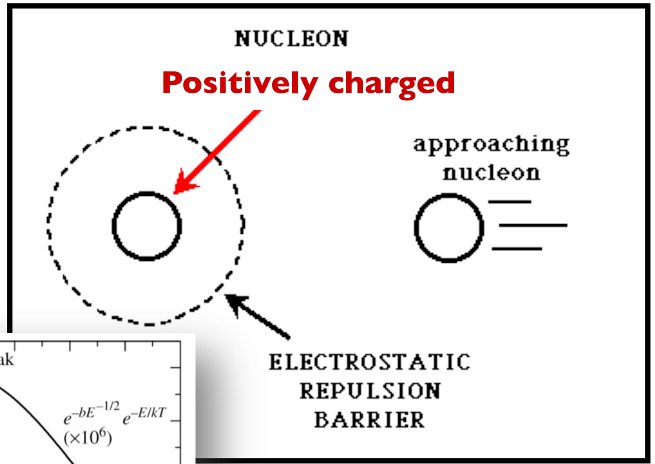
Note: β **is the exponent on T. For just a 10% increase in T there is a 1.5x (β=4), 5.1x (β=17), and 45x (β=40) increase in liberated energy (ε)!**

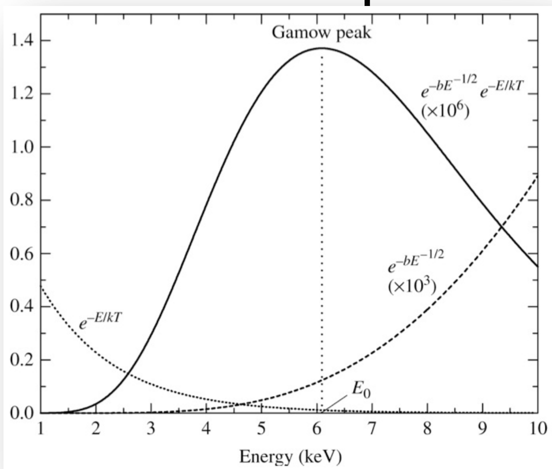
The battle for fusing heavier elements



VS

Coulomb barrier





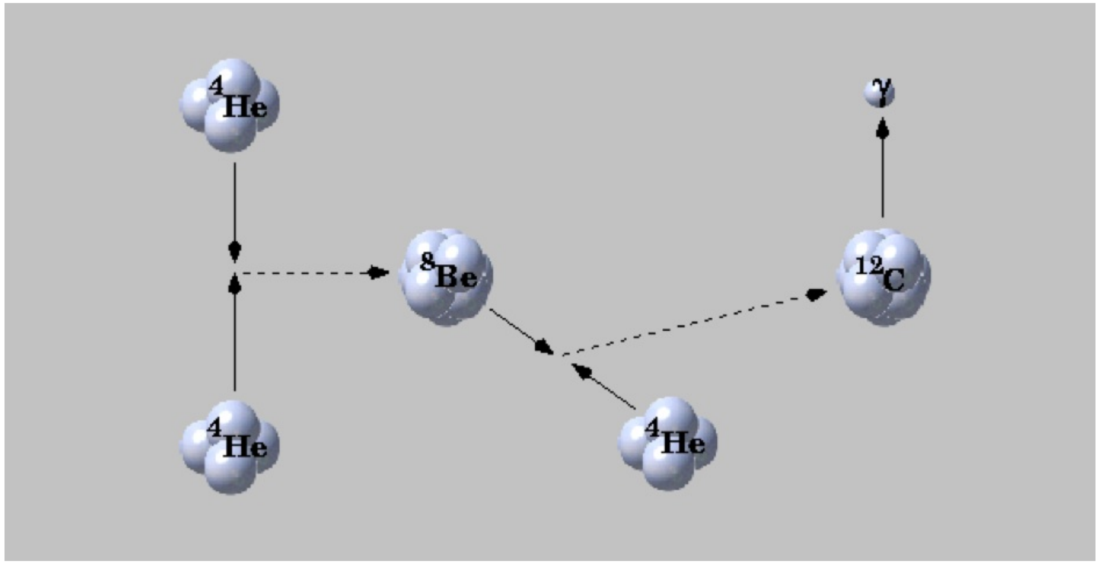
Nuclear reaction probability

Fusing of heavier elements means the Coulomb barrier becomes significantly higher which requires higher core temperatures.

The most massive stars will ultimately achieve stellar core temperatures of >10^9 K, more than lower-mass stars: we will investigate this in the stellar evolution lectures. For now we only briefly explore the later nuclear fusion processes.

Helium-Carbon: triple alpha process

This is not important for main sequence stars, but become important at temperatures between 10^8 and $2x10^8$ K. At first, it was difficult for astrophysicists to envisage He-burning, because the product of 2 He nuclei, ^8Be, is highly unstable. The answer was the triple-α process, and is summarized as:



The rate of energy generation is given by:

$$\varepsilon_{3\alpha} = \varepsilon_3 X_{He}^3 \rho^2 T^{40} \qquad \mathbf{W\ kg^{-1}}$$

where:

X_{He} = fractional helium content

Note extreme temperature exponent! ρ^2 because triple-alpha is effectively a 3-body collision – He particles need to fuse before Be decays (~10^{-16} s!)

Later fusion stages of heavier elements

Other elements beyond C can be fused – Ne, Mg, O, Si; see summary here:

Nuclear Fuel	Process	$T_{threshold}$ 10^6K	Products
H	PP	~4	He
H	CNO	15	He
He	3α	100	C,O
C	C+C	600	O,Ne,Ma,Mg
O	O+O	1000	Mg,S,P,Si
Si	Nuc eq.	3000	Co,Fe,Ni

Nuclear fusion occurs at [T~$4x10^6$-$3x10^9$ K]: sufficient energies not achieved at lower T for fusion and the fused particles are destroyed at higher T (photo disintegration – more in the stellar evolution lectures)

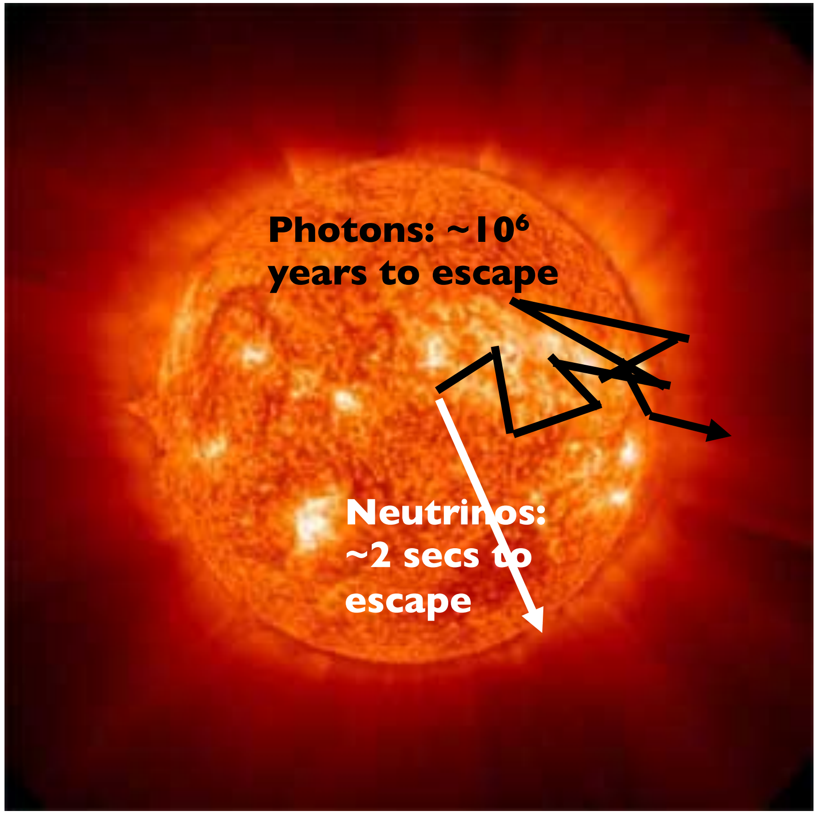
Testing nuclear fusion in the stellar core

The model developed so far has a good theoretical basis but can we test it? Any theoretical model can only be considered robust if it can be observationally tested.

We cannot directly explore the conditions in the stellar core using radiation since photon transport takes of order ~10^6 years (which we explore in the next lectures) and the photons change in energy and wavelength. There is evidence that the physics is broadly correct from various experiments but can we directly probe the stellar core?

Yes we can with the most wraith-like particles we know: neutrinos

Probing the stellar core using neutrinos



The cross section of a neutrino is so small that it can typically pass unimpeded through 1 light year of lead!

By comparison, as we will learn in the next lecture, photons take ~10^6 years to escape from the Sun!

The cross section of a typical neutrino:

$$\sigma \sim 10^{-48}\ \mathrm{m}^2!$$

(note not m^{-2})

Source of neutrino	Flux at Earth ($m^{-2}\ s^{-1}$)	Average energy (MeV) **- neutrino**
PPI: $p + p \rightarrow {}^2D + e^+ + \nu$	6.0×10^{14}	0.263
PPII: ${}^7Be + e^- \rightarrow {}^7Li + \nu$	4.9×10^{13}	0.80
PPIII: ${}^8B \rightarrow {}^8Be + e^+ + \nu$	5.7×10^{10}	7.2

Detecting neutrinos: Nobel prize winning science

Brookhaven experiment: 100,000 gallons of Chlorine (cleaning fluid) in a tank in a mine ~1 mile underground.

Over a period of ~30 years (from late 1960s) ~2,000 neutrinos were captured, enough to prove that nuclear fusion is occurring in the Sun.

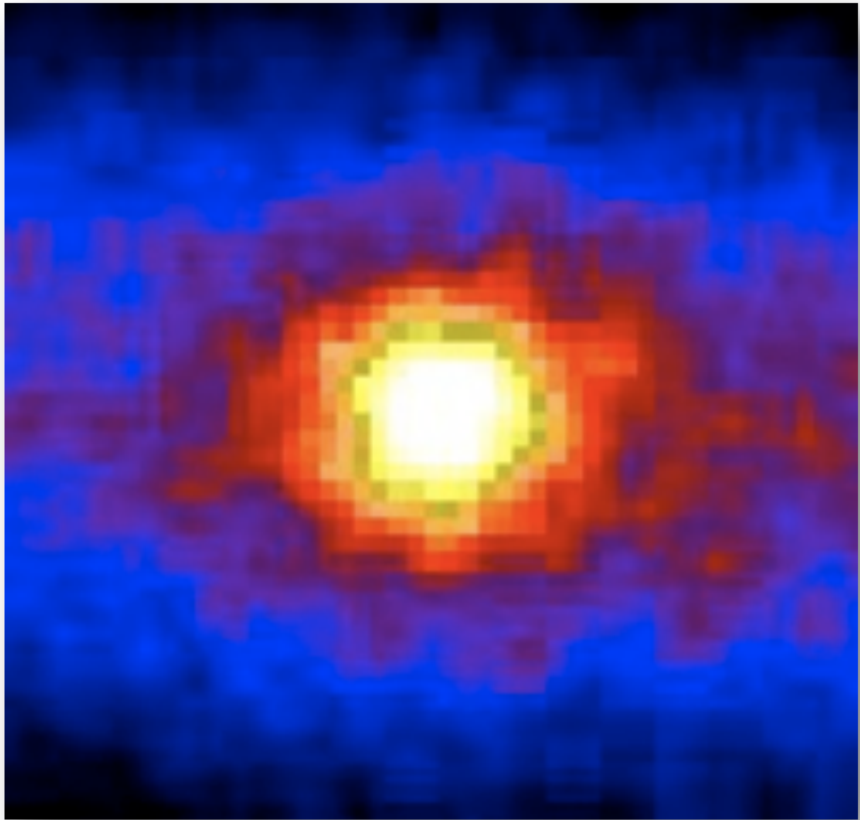
Raymond Davies Jr shared the Nobel physics prize for high-energy astrophysics in 2002.

$$\nu + {}^{37}\mathrm{Cl} \longrightarrow e^- + {}^{37}\mathrm{Ar}$$

Only 10s of Ar were captured out of ~2×10^{30} Cl atoms in the tank.

This experiment was sensitive to high-energy neutrinos produced in the rare PPIII chain.

Imaging the sun in neutrinos



An image of the sun in neutrinos! Proof that the detected neutrinos are produced in the Sun.

This is from Super Kamiokande which used photomultiplier tubes to detect Cherenkov radiation caused by electrons (scattered by neutrinos) moving faster than the speed of light in water.

Traced the same PPIII reaction as the Brookhaven experiment.

These experiments have demonstrated that our model for nuclear fusion in the Sun is correct: the neutrino flux depends on the 25th power of the central temperature!

The implied central temperature of the Sun from these experiments is $T=15.7 \times 10^6$ K (to an astonishing 1% accuracy)!

The power source of stars

From these series of lectures you should now know:

- **Hydrostatic equilibrium and how it applies to stars**
- **The conditions at the cores of stars**
- **How to calculate the energy available from the gravitational collapse of a star, utilising the viral theorem**
- **The origin of the large energy release in nuclear fusion and the basic conditions required for nuclear fusion to occur**
- **The difference between the classical and quantum temperature**
- **The nuclear reaction conservation laws, the proton-proton chain, and a basic understanding of the later fusion chains**
- **How the detection of neutrinos provides our strongest test of the conditions at the cores of stars**