

Statistical Physics: Weekly Problem 3 (SP3)

- (1) (a) In the *microcanonical ensemble*, or  $(N, U, V)$  macrostate, are the various microstates that are consistent with the  $(N, U, V)$  macrostate equally probable, or do they have different in general probabilities? [1 mark]
- (b) Similarly, in the *canonical ensemble*, or  $(N, T, V)$  macrostate, describing a system in thermodynamic equilibrium, are the various accessible microstates equally probable, or do they have, in general, different probabilities? [1 mark]
- (2) In a system of  $N$  weakly interacting particles in thermal equilibrium at temperature  $T$ , the probability that a particle will be in (single-particle) state  $i$  with energy  $\epsilon_i$  is proportional to

$$p_i \propto \exp \left[ -\frac{\epsilon_i}{k_B T} \right],$$

i.e. the Boltzmann probability.

The two lowest-lying energy levels of a hydrogen atom have energies  $\epsilon_0 = -13.6$  eV and  $\epsilon_1 = -3.4$  eV. Ignoring degeneracies, at what temperature would we find one hundredth as many hydrogen atoms in the first excited state as in the ground state?

( $k_B = 8.617 \times 10^{-5}$  eV K<sup>-1</sup>) [2 marks]

- (3) A paramagnetic solid consists of  $N$  ions with spin 1/2 and magnetic moment  $\mu_B$ . The system lies in a magnetic field  $B$  and each magnetic moment is oriented either parallel to the field (up), with energy  $\epsilon_{\uparrow} = -\mu_B B$ , or antiparallel (down) with energy  $\epsilon_{\downarrow} = +\mu_B B$ .
- (a) What is the probability that an ion will have its magnetic moment oriented parallel to  $B$ ? [2 marks]
- (b) What is the internal energy  $U$ , entropy  $S$  and the temperature  $T$  of the system in the limit where all the magnetic moments are parallel to  $B$ ? [3 marks]  
Hint: do not try to think of complicated equations. Use your physical intuition.
- (c) The system of ions is brought into a state where the internal energy  $U$  is positive. Show that the temperature of the system is negative. Is a negative temperature “hotter” (i.e. of higher energy) or “colder” than infinite temperature? [1 mark]