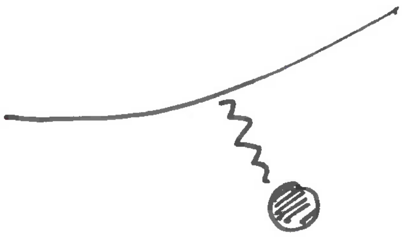
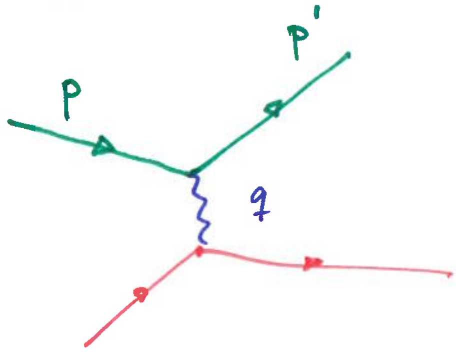
QM → QFT

- Interactions are quantised and described by virtual particles
- The propagator $\frac{1}{q^2 - m^2}$ enters the matrix element





$$\rightarrow \quad \mathcal{M} \propto \frac{Ze\, ze}{q^2 - m^2}$$

For $E = E'$:

$$\mathcal{M} \rightarrow \frac{z Z e^2}{(E-E')^2 - |\vec{q}|^2 - m^2} = \frac{-e^2 z Z}{|\vec{q}|^2 + m^2}$$

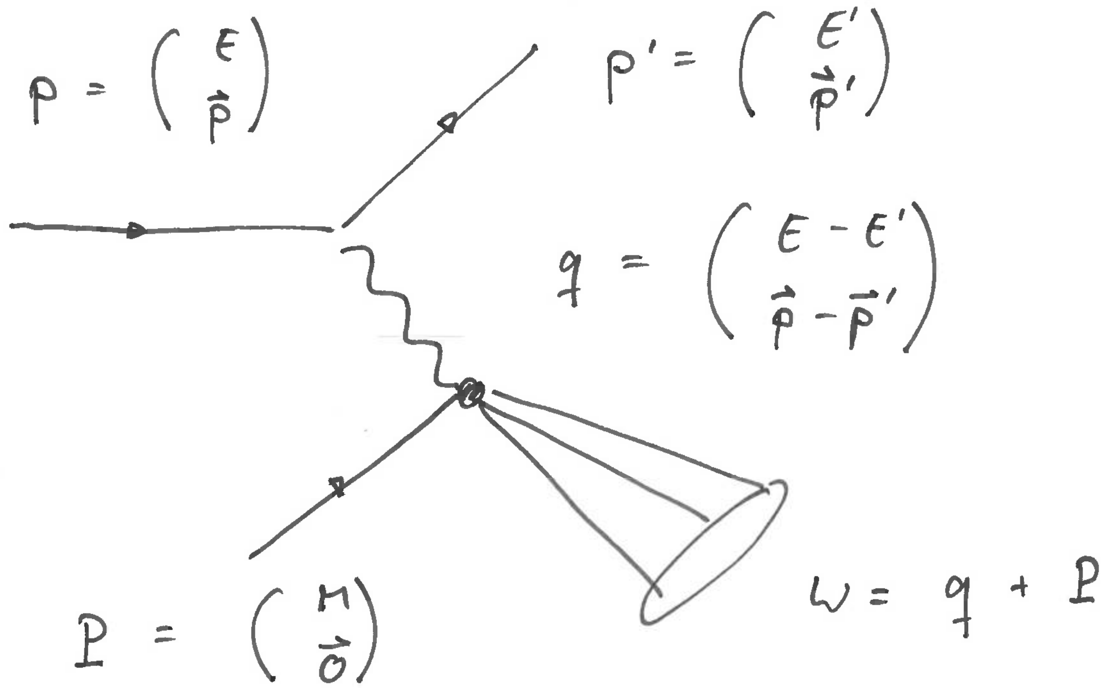
$$M = m \qquad \xrightarrow{m \to 0} \quad \frac{-e^2 z Z}{|\vec{q}|^2}$$

- Elementary particles can and will decay and can be produced from other fields as long as all quantum numbers are conserved

7. Inelastic Scattering

So far all scattering processes have been elastic: the object off which we scatter is left unchanged.

Inelastic Scattering: The target is changed by the scattering.



$$W^2 = (P + q)^2 = (P + p - p')^2$$

$$= P^2 + 2P \cdot q - Q^2$$

$$= M^2 + 2M\nu - Q^2$$

where we have
$$\nu = \frac{P \cdot q}{M}$$

$$= \frac{(M, \vec{0}) \cdot (E - E', \vec{p} - \vec{p}\,')}{M}$$

$$= \frac{M(E - E')}{M} = E - E'$$

given by the energy transfer
(in the lab frame)

For elastic scattering $W^2 = M^2$

$$\Rightarrow \quad 2M\nu - Q^2 = 0$$

If the collision is inelastic

$$2M\nu - Q^2 > 0$$

For elastic scattering we only had one parameter to describe the scattering, because the scattering angle was related to the momentum transfer (i.e. $|\vec{q}|^2 = 2E \sin \theta/2$)
Inelastic scattering needs 2 parameters.

<u>Deep inelastic scattering</u>

For inelastic scattering

$$\frac{d\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{Mott,} \\ \text{no recoil}}} \left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right]$$

The form factors W_1, W_2 of inelastic scattering now depend on 2 parameters and are called <u>structure functions</u>.

we introduce a new parameter

$$x = \frac{Q^2}{2M\nu}$$

which measures the elasticity of the scattering process

$$2M\nu - Q^2 = \begin{cases} 0 & \text{el. scattering} \\ >0 & \text{inel. scattering} \end{cases}$$

$$\Rightarrow \quad 1 \geq x > 0$$

For elastic scattering $x = 1$.

Define dimensionless structure functions

$$F_1(x, Q^2) = M W_1(Q^2, \nu)$$

$$F_2(x, Q^2) = \nu W_2(Q^2, \nu)$$

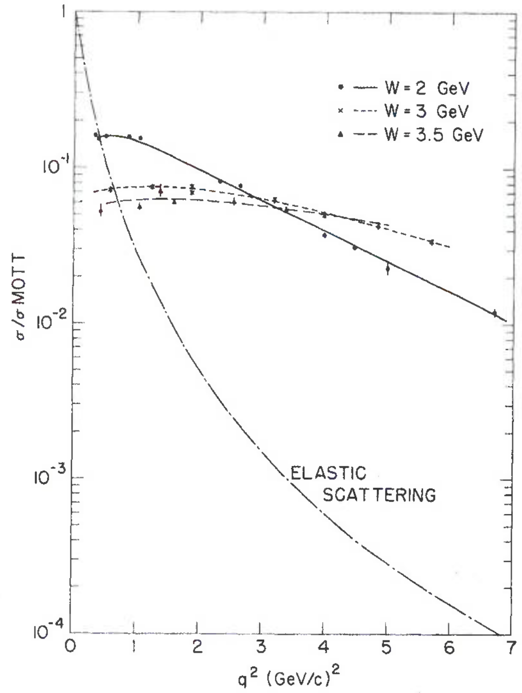
and

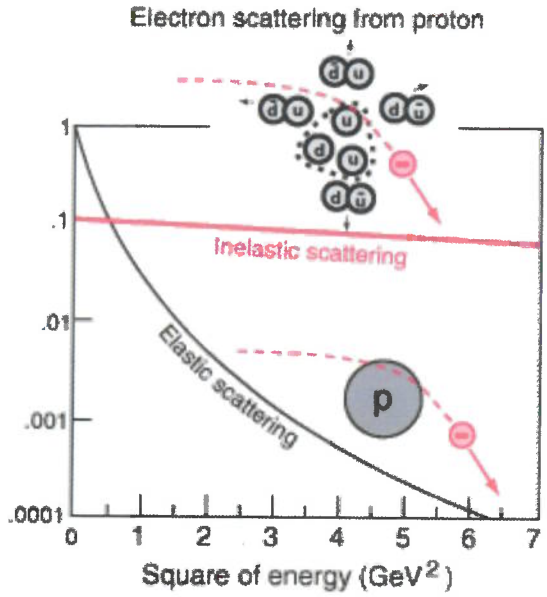
$$\frac{d^2\sigma}{d\Omega\, dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\frac{1}{x}\left(1 - y - \frac{M^2y^2}{Q^2}\right) F_2(x, Q^2) + y^2 F_1(x, Q^2)\right]$$

and

$$y = \frac{P \cdot q}{p \cdot q}$$

measures the fractional energy loss of the probe.





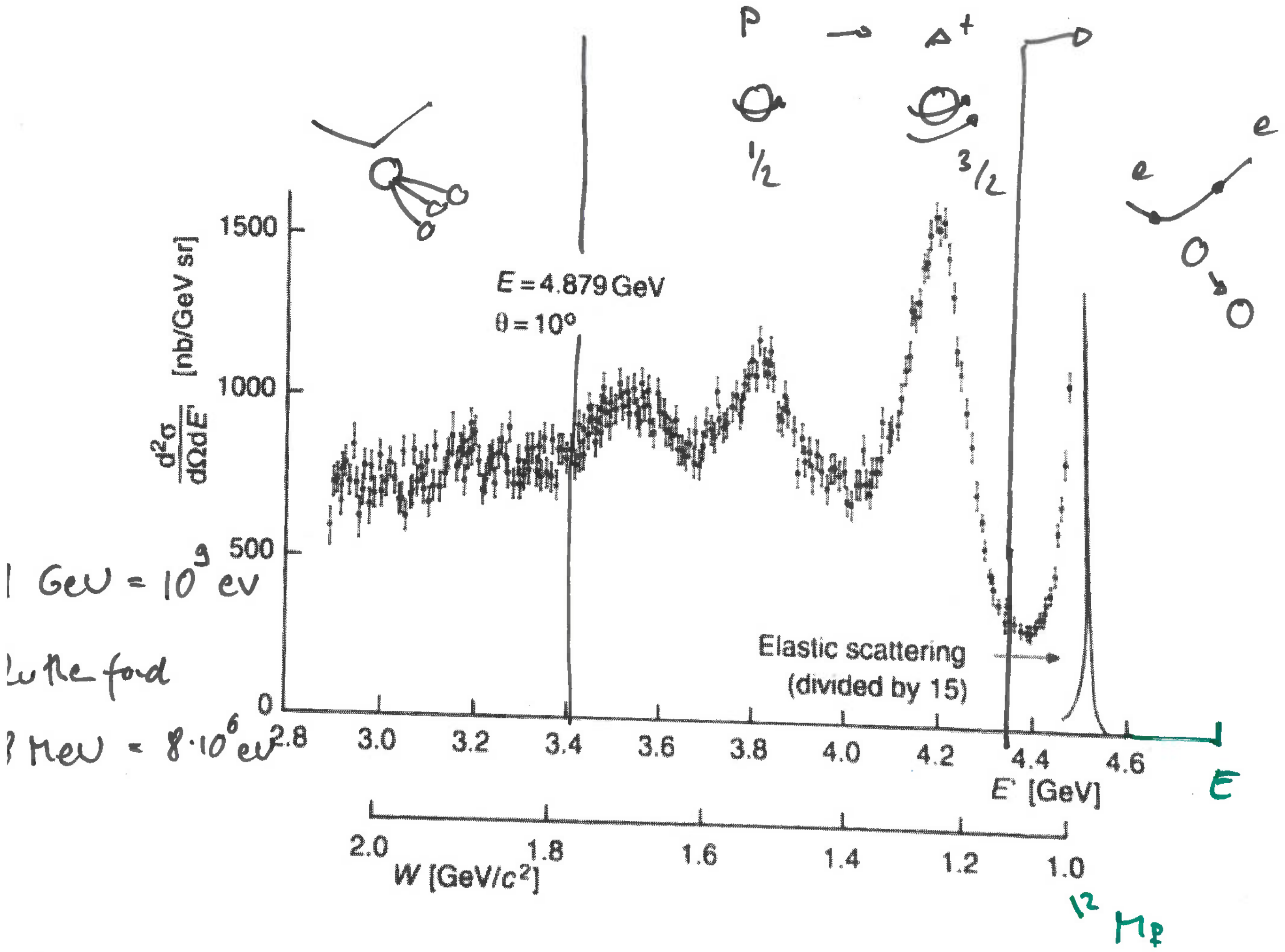
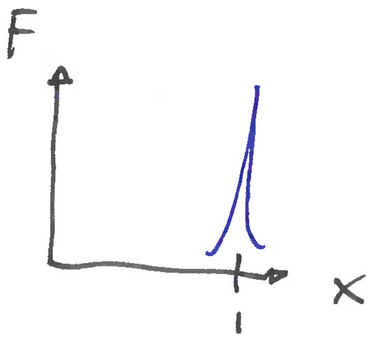
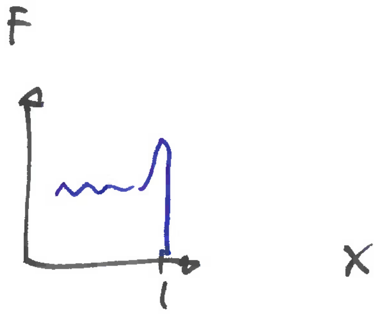


Figure 20: Differential cross section for an electron off a proton for a fixed scattering angle of 10^o and a beam energy of 4.9 GeV. Data from [7] and figure from [2].

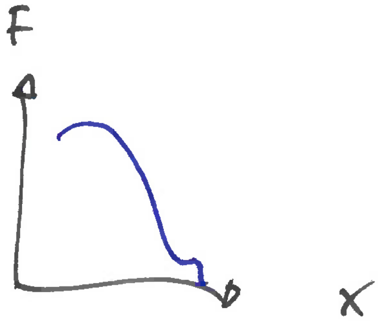
x-dependence of the structure functions



$Q^2 R^2 \ll 1$ elastic



$Q^2 R^2 \simeq 1$ inelastic



$Q^2 R^2 \gg 1$ deep inelastic