

Thermodynamics – Weekly Problem, Th. 5

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The following Maxwell Relation may be useful when solving this problem. It is derived by considering the Gibbs function, $G = U + pV - TS$,

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$$

- a) Consider the entropy as a function of temperature and pressure, $S = S(p, T)$.

- i) First, derive the second TdS equation given below,

$$TdS = C_p dT - T \left(\frac{\partial V}{\partial T}\right)_p dp.$$

[3 marks]

- ii) Hence show that in an isothermal process, at T_0 , the heat change can be related to the isobaric expansivity $\beta_p = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p$, via the following relationship

$$\delta Q = -T_0 \beta_p V dp.$$

If ten moles of water, at 1.0 °C, that occupy a volume of $1.8 \times 10^{-4} \text{ m}^3$ and have a volume expansivity coefficient $\beta_p = -50 \times 10^{-6} \text{ K}^{-1}$, what is the heat transferred to the water, if the pressure is increased from 1.0 to 10 atm isothermally?

[3 marks]

- b) Derive the general relation given below, and evaluate the expression for one mole of monoatomic ideal gas, for which $\gamma = 5/3$,

$$C_p = T \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_S.$$

[Hint: recall that the equation of state for an adiabatic process for an ideal gas is pV^γ .

[4 marks]