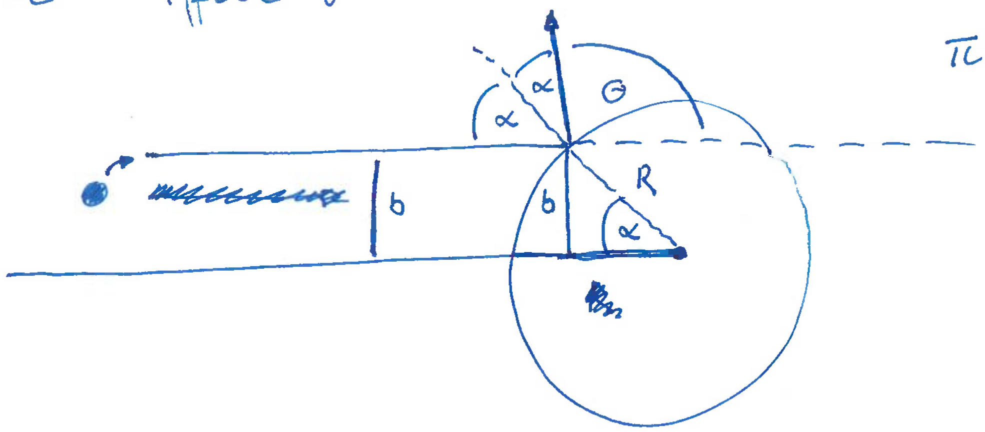
The cross section for a given scattering process is defined as

time derivative

$$\sigma = \frac{\dot{N}_{scattered}}{N_{target} \; N_{incoming}} \; \frac{V}{v_{incoming}}$$

The differential cross section



$\pi = 2\alpha + \theta$

$$b = R \sin\alpha = R \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = R \cos\left(\frac{\theta}{2}\right)$$

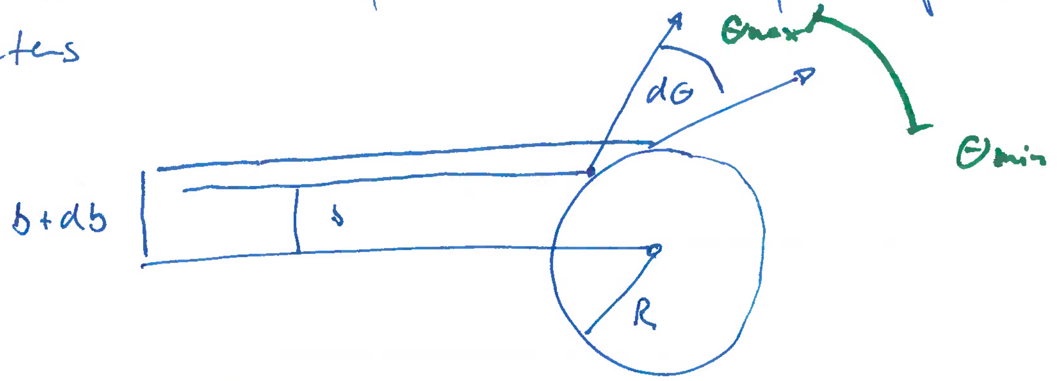
$$\Rightarrow \quad \theta = 2 \operatorname{Arccos}\left(\frac{b}{R}\right)$$

Makes no sense for $b > R$, because $\operatorname{Arccos}(x)$ is imaginary for $x > 1$.

How does the impact parameter change for infinitesimal changes of the angle

$$\frac{db}{d\theta} = -\frac{R}{2} \sin\left(\frac{\theta}{2}\right)$$

The cross section for a slice of impact parameters



$$\sigma(b) = \pi b^2$$

$$\sigma(b+db) = \pi (b+db)^2$$

$$d\sigma = \pi (b+db)^2 - \pi b^2 = 2\pi b\, db + \mathcal{O}(db^2)$$

$$\Rightarrow \quad \frac{d\sigma}{d\theta} = \frac{d\sigma}{db}\left|\frac{db}{d\theta}\right| = 2\pi\, b \left|\frac{db}{d\theta}\right|$$

$$= 2\pi \left(R \cos\left(\frac{\theta}{2}\right)\right)\left(\frac{R}{2} \sin\left(\frac{\theta}{2}\right)\right)$$

$$= \frac{1}{2} \pi R^2 \sin(\theta)$$

$$\sigma_{measured} = \int_{\theta_{min}}^{\theta_{max}} \frac{d\sigma}{d\theta}\, d\theta$$

More generally, the cross section can depend on 2 angles and on the energy of the projectile E

$$\sigma_{measured} = \int_{\Omega} \int_{E_{min}}^{E_{max}} \frac{d^2\sigma}{d\Omega\, dE}\, dE\, d\Omega$$

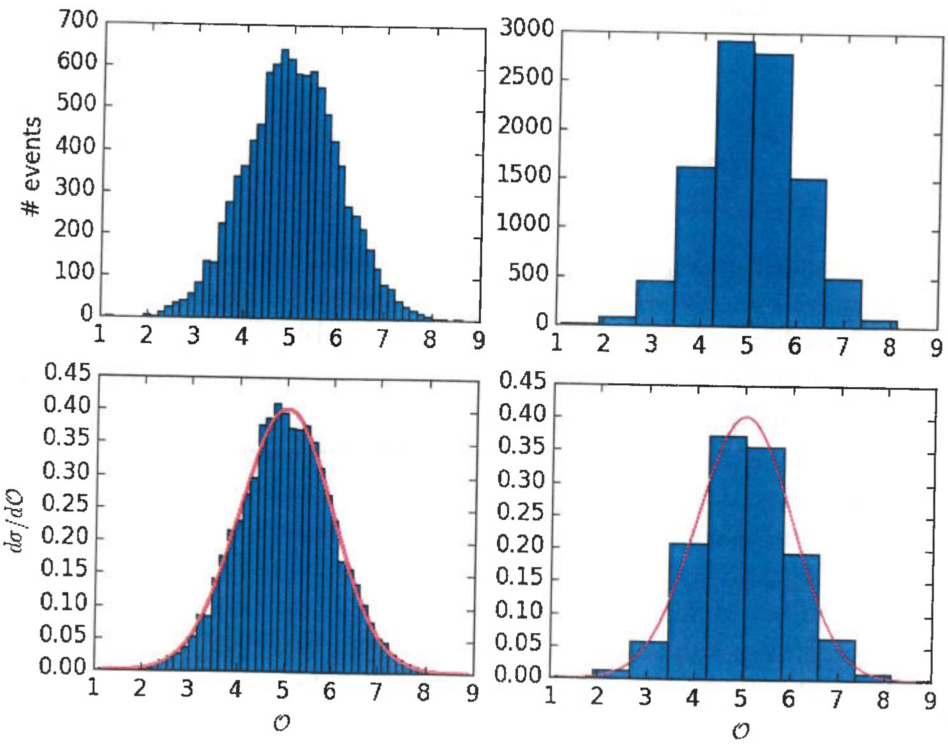
$$d\Omega = \sin\theta\, d\theta\, d\phi$$

Theoretical predictions are possible for a distribution $\frac{d\sigma}{d\Omega}$ of continuous observables

$$\mathcal{O} = \phi, \theta, E.$$

Experiments measure only a finite number of events. Cross sections are therefore usually represented as histograms, where the number of bins depends on the statistics (= number of scattered particles observed)

$$n_{predicted} = \int_{\mathcal{O}_1}^{\mathcal{O}_2} \frac{d\sigma}{d\mathcal{O}} \, d\mathcal{O}$$



1.1 Key points

- The higher the energy of the projectile, the deeper one can "see" inside the structure of matter
- The cross section is a very useful quantity to characterise a physical scattering process
- More information about the dynamics can be shown by measuring the cross section differentially with respect to different observables.
  Differential cross sections are represented as histograms.

2 CALCULATION OF CROSS SECTIONS

Fermis golden rule:

$$\frac{\dot{N}_{scattered}}{N_{incoming}\, N_{target}} \equiv W = 2\pi \, |M_{fi}|^2 \, \rho(E')$$

$$\rho(E') = \frac{dn}{dE'}$$ Density of final states

$$M_{fi} = \langle \psi_f \,|\, \mathcal{H}_{int} \,|\, \psi_i \rangle$$ transition matrix element

Using the result from last section

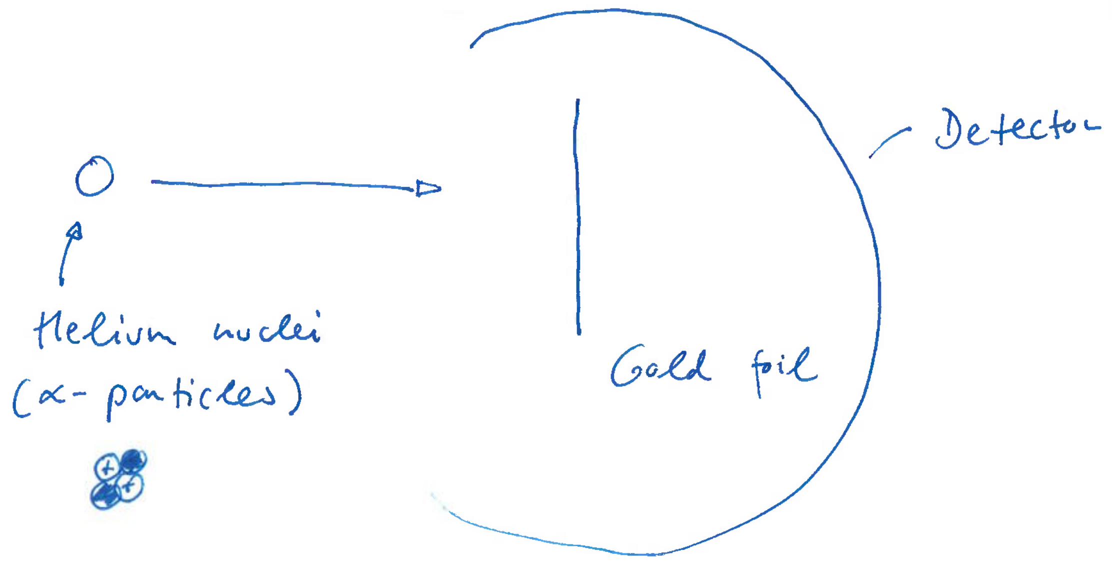
$$\sigma = \frac{\dot{N}_{scattered}}{N_{target}\, N_{incoming}} \quad \frac{V}{v_{incoming}}$$

$$\frac{\sigma\, v_{incoming}}{V} = \omega$$

$$\Rightarrow \quad \boxed{\sigma = \frac{2\pi}{V} \left| M_{fi} \right|^2 \rho(E')\, V}$$

2.1 Rutherford Scattering

1909



We need to calculate 3 ingredients

$$\mathcal{M}_{fi} = \langle \psi_f \mid \mathcal{H}_{int} \mid \psi_i \rangle$$

Initial and final states are plane waves

$$\psi_i = \frac{1}{\sqrt{V}} e^{i \vec{p} \cdot \vec{x}}$$

$$\psi_f = \frac{1}{\sqrt{V}} e^{i \vec{p}' \cdot \vec{x}}$$

$$\int_{-\infty}^{\infty} d^3x \; \psi^* \psi \quad \text{diverges}$$

V is a finite volume necessary to normalise plane waves.

Needs to drop out eventually!

$$\mathcal{H}_{int}(\vec{x}) = ze\,\phi(\vec{x}) = ze \frac{Ze}{|\vec{x} - \vec{x}_0|}$$

$\frac{1}{r}$

z = charge of the projectile

Z = charge of the target

e = elementary charge