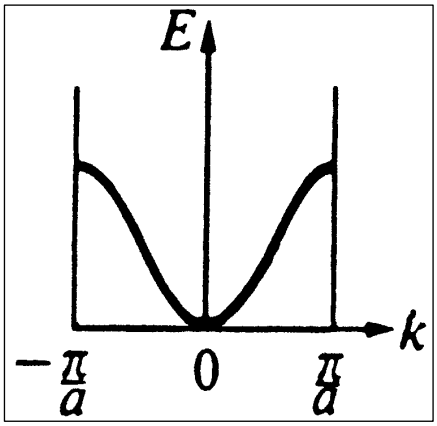
Condensed Matter Physics: Weekly Problem 5

These problems are to be formatively self-assessed by you, the student. Students taking part in the peer-marking pilot scheme will also be required to mark one of their peer's weekly problems. A mark scheme, out of 10, will be provided with each solution to aid your assessment before your timetabled weekly workshop. .

Summary: This problem will look at how the effective mass is related to the $E-(k)$ dispersion relationship (the energy bandstructure) for electrons in Bloch states (electrons moving a weak periodic potential).

In lectures, we saw that the interaction between the electrons and a periodic potential produced an $E(k)$ relationship with the general form as shown in the figure.



In this crystal, the energy bandstructure within the first Brillouin zone can be modelled by the *empirical* relationship:

$$E(k) = \alpha k^2 + \beta k^4$$

Where E is the electron energy, k is the wavevector and α, β are constants.

a. Use what you know about bandstructure to express α as a function of β. **[2 marks]**

b. Obtain an expression for the effective mass in terms of k and α. **[4 marks]**

c. Draw a qualitative sketch illustrating how the nearly-free electron effective mass varies from $k = 0 \rightarrow \pi \,/\, a$.

What is the value of the effective mass at the point $k = \pi/\left(a\sqrt{3}\right)$? What are the implications of this for the electron motion? **[4 marks]**