Chapter 6

The Dark Matter Halo

CO p. 896-897

Measurements of the rotation curve using HI 21-cm emission, analysis of the motions of stars in the solar neighbourhood with Oort's constants, and the Oort limit[1], all suggest the presence of a large amount of invisible 'dark matter' in the MW. Given such a startling conclusion, it may be a good idea to look for other evidence for dark matter in galaxy haloes.

6.1 High velocity stars

A number of high-velocity stars near the Sun have measured velocities[2] up to $v_\star \approx 500$km s^{-1}. Their existence provides us with a probe of the galaxy's mass, if we assume that these stars are still bound to the MW: it requires that the speed of the star, $v_\star$, is lower than the local escape speed[3].

[1] Not discussed in detail.

[2] The quoted velocity is wrt to the centre of mass velocity of the MW. Do not confuse these with Oort's high velocity stars, which are typically low mass, low metallicity stars in the *Galactic Halo*. The velocities of Oort's stars are of order 200km s^{-1}. The present high velocity stars are typically *A*-type stars, presumably born in the disc, that have acquired their high velocity following a super nova explosion. For a recent discussion based on GAIA, see Deason et al, '20

[3] The escape speed in a given potential, is the minimum speed a particles needs to have to be able to escape to infinity.

6.1.1 Point mass model

For a *point mass model* (all the mass in the centre), it is easy to find the relation between escape speed, v_e, and circular speed, V_c. For such a model, the circular speed at radius $R_\odot$ is $V_c^2 = GM/R_\odot$, where M is the mass of the MW[4]. The gravitational potential is $\Phi = -GM/R_\odot = -V_c^2$. A star moving with the *escape speed* has zero specific energy[5],

$$0 = E = \frac{1}{2}v_e^2 + \Phi = \frac{1}{2}v_e^2 - V_c^2\,. \qquad (6.1)$$

Therefore $v_e = 2^{1/2}\,V_c \approx 311\text{km s}^{-1}$ (Using $V_c = 220\text{km s}^{-1}$.) So for a point mass model of the MW, most high velocity stars are not bound to the MW. This analysis also shows that we cannot resolve the discrepancy by simply increasing M. Indeed, although increasing M would increase v_e - it would also increase V_c - yet V_c is *measured*. The only way to increase v_e but not V_c is by changing the *mass distribution* - as we show below.

6.1.2 Dark halo model

Given the failure of the point mass MW model, let's assume there to be a dark halo, which is spherically symmetric (to make the calculations easy). Let's further assume that the MW's rotation curve is flat, $V_c \approx$ constant, out to some radius R_h. In this case, given that $V_c^2 = GM/R$ is constant out to R_h,

$$\begin{aligned} M(R) &= \frac{V_c^2\,R}{G} \quad \text{when } R < R_h \\ &= \frac{V_c^2\,R_h}{G} \quad \text{when } R \geq R_h\,. \end{aligned} \qquad (6.2)$$

The gradient of the gravitational potential is the force per unit mass, which is V_c^2/R for $R \leq R_h$, therefore

$$\frac{d\Phi}{dR} = \frac{V_c^2}{R} = \frac{GM}{R^2}\,. \qquad (6.3)$$

[4]To compute the escape speed at the location of the Sun, we will take $R_\odot$ the distance of the Sun to the centre of the MW, $R_\odot \approx 8$ kpc.

[5]*Specific energy* is energy per unit mass.

Integrating this equation between $R \leq R_h$ and R_h yields $\Phi(R) = \text{constant} - V_c^2 \ln(R_h/R)$. We can determine the value of the constant at $R = R_h$, since then $\Phi(R = R_h) = -G\,M/R_h = -V_c^2$. Hence

$$\Phi(R) = -V_c^2\ [1 + \ln(R_h/R)]\ . \tag{6.4}$$

Using Eq.(6.1) for the escape speed, we obtain

$$v_e^2 = 2\,V_c^2\,[1 + \ln(R_h/R)]\ . \tag{6.5}$$

For the Sun, $R \approx 8.5$kpc, $V_c = 220$km s^{-1}, $v_e \geq 500$km s^{-1} requires $R_h \geq 40$kpc corresponding to a halo mass of at least

$$M(R = R_h) \geq 4.4 \times 10^{11} M_\odot\,. \tag{6.6}$$

Even this lower limit to the mass is significantly higher than the MW's stellar mass of $M_\star \approx 7 \times 10^{10} M_\odot$ from Chapter 3. A recent application of this method put $M_h \approx 10^{12} M_\odot$, see Deason et al, '20. An indendent measure of the MW's halo mass is based on the motion of the Andromeda galaxy in the *Local Group*.

6.2 The Local Group (CO p. 1059-1060)

The MW is located in a rather average part of the Universe, away from any dense concentrations of galaxies[6]. The 'Local Group' consist of the MW, Andromeda (M31), and a few hundred small, irregular galaxies, all gravitationally bound to each other.

6.2.1 Galaxy population

The MW is orbited by ~ 10 'classical dwarf' satellites, which include, for example, the *Large* and *Small Magellanic Clouds*. These satellites are gravitationally bound to the MW and orbit inside its dark matter halo. The advent of digital sky surveys has resulted in an explosion in the discovery of much fainter dwarf galaxies, also gravitationally bound to the MW, see for example this recent CalTech review. The tally of these ultra-faint galaxies now stands at ~ 100, with likely many more to be discovered.

[6]Such dense concentrations are called *clusters*, see later chapters.

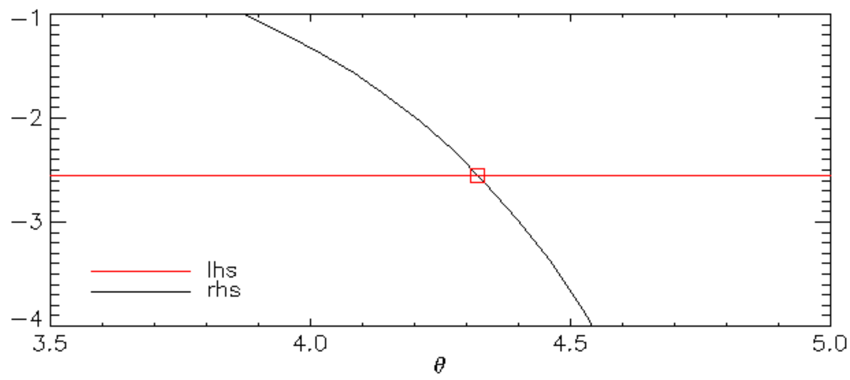

Figure 6.1: The left-hand-side of Eq. 6.11 plotted against the right-hand-side, as a function of the parameter θ. The point with coordinates (4.32, -2.55) is shown by a square.

The tidal force of the MW can rip satellite galaxies apart if they venture too close to the disc and/or bulge. An example is the 'Sagittarius dwarf galaxy' of which we can trace the tidal debris all around the MW.

The Andromeda galaxy, M31, is very similar in mass and luminosity to the MW, and it has its own set of satellites. Interestingly, M31 and the MW are also gravitationally bound to each other. In fact, M31 is on a collision course with the MW, with the impact expected to be about 5 Gyr from now. The tidal force between both galaxies will be so large that we expect both discs to be destroyed in the collision[7].

The bound system of the MW and its satellites, together with M31 and its satellites, and some further smaller galaxies such as the *triangulum galaxy*, constitute the *Local Group*. The motion M31 as seen from the MW can be used to estimate the mass of the MW, using the Local Group *timing argument*.

6.2.2 Local Group timing argument

The dynamics of M31 and the MW can be used to estimate the total mass in the Local Group and in the MW as follows. From the Doppler shifts of

[7]Distances between stars are so large that it is very unlikely that two *stars* will collide when M31 and the MW merge.

spectral lines, we can determine the line-of-sight velocity of M31 with respect to the MW[8],

$$v = -118\text{km s}^{-1} \,. \tag{6.7}$$

The negative sign means that Andromeda is moving *toward* the MW. This may be surprising, given that most galaxies are moving apart with the general Hubble flow. The fact that Andromeda is moving toward the MW is presumably because their mutual gravitational attraction has halted, and eventually reversed their initial velocities. Kahn and Woltjer pointed out in the 1950's that this leads to an estimate of the masses involved.

Since M31 and the MW are by far the most luminous members of the LG we can neglect in the first instance the others, and treat the two galaxies as an isolated system of two point masses. Since M31 is about twice as bright as the MW, and given that they are so similar, it is presumably also about twice as massive. If we further assume the orbit to be radial[9], then Newton's law gives for the equation of motion

$$\frac{d^2r}{dt^2} = -\frac{GM_{\rm total}}{r^2} \,, \tag{6.8}$$

where $M_{\rm total}$ is the sum of the two masses. Initially, at $t = 0$, we can take $r = 0$ (since the galaxies were close together at the Big Bang).

The solution can be written in the well known parametric form as

$$\begin{aligned} r &= \frac{R_{\rm max}}{2}(1 - \cos\theta) \\ t &= \left(\frac{R^3_{\rm max}}{8\,G\,M_{\rm total}}\right)^{1/2} (\theta - \sin\theta) \,. \end{aligned} \tag{6.9}$$

The distance r increases from 0 (for $t = \theta = 0$) to some maximum value $R_{\rm max}$ (for $\theta = \pi$), and then decreases again. The relative velocity follows from taking the derivative and use the chain rule,

[8]What one measures is the radial velocity wrt to the *Sun*. Since the Sun is on a (nearly) circular orbit around the MW, we need to correct the measured heliocentric velocity of M31 to obtain the radial velocity of Andromeda wrt the MW.

[9]We'll make this assumption for simplicity; GAIA recently measured the *tangential* velocity of M31, see van der Marel et al, 2019.

$$v = \frac{dr}{dt} = \frac{dr}{d\theta} \Big/ \frac{dt}{d\theta} = \left(\frac{2\,G\,M_{\rm total}}{R_{\rm max}}\right)^{1/2} \left(\frac{\sin\theta}{1-\cos\theta}\right) . \tag{6.10}$$

The last three equations can be combined to eliminate $R_{\rm max}$, G and $M_{\rm total}$, to give

$$\frac{v\,t}{r} = \frac{\sin\theta\,(\theta - \sin\theta)}{(1-\cos\theta)^2} . \tag{6.11}$$

v can be measured from Doppler shifts, and $r \approx 710$kpc from Cepheid variables. For t we can take the age of the Universe. Current estimates of t are quite accurate[10], but even using ages of the oldest MW stars as Kahn & Woltjer did, $t \sim 15$Gyr, still gives a relatively accurate and interesting value.

So, taking $v = -118$ kms^{-1}, $r = 710$ kpc, and $t = 15$ Gyr, yields $\theta = 4.32$ radians, as shown graphically in Fig. 6.1, when *assuming M31 is on its first approach to the MW*[11].

Substituting these value in the previous equations yields, amongst others, $M_{\rm total} \approx 3.66 \times 10^{12} M_\odot$. Making the reasonable assumption that the MW's halo mass is half of the M31's (given that M31 is twice as bright), yields a total mass of the MW (stars + halo), of

$$M \approx 1.2 \times 10^{12} M_\odot , \tag{6.12}$$

comfortably higher than the lower limit to M_h of Eq.(6.6).

Notice that this mass is *much* higher than the MW's stellar mass, of $M_\star \approx 7 \times 10^{10} M_\odot$: provided with did our calculations right, the mass in dark matter is ~ 20 times that in stars.

Taking $M_h \approx 1.2 \times 10^{12} M_\odot$, we can estimate the extent of this halo, R_h,

$$R_h = \frac{GM_h}{V_c^2} \approx \frac{G\,10^{12} M_\odot}{(220{\rm km\ s^{-1}})^2} \approx 100{\rm kpc} . \tag{6.13}$$

If, as is more likely, the rotation speed eventually drops below 220km s^{-1}, then R_h is even bigger. Hence the extent of the dark matter halo around the MW (and M31) is truly enormous. Recall that the size of the stellar disc is ~ 15kpc, therefore the halo's radius is probably about 7 times that.

[10]From properties of the micro-wave background radiation.

[11]Equation (6.11) has no unique solution for θ, since it describes motion in a periodic orbit. On its first approach, θ should be the *smallest* solution to the equation.

6.3 Summary

After having studied this lecture, you should be able to

- Show that in a point mass model of the MW, the high velocity stars are not bound.
- Estimate the parameters of a dark halo, assuming the high velocity stars are bound to the MW.
- Describe the properties of the Local Group in terms of the galactic content.
- Estimate the mass and extent of the dark halo of the MW from the Local Group timing argument.