Foundations of Physics 2B/3C 2019-2020

Thermodynamics – Weekly Problem, Th. 2

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a) A volume of ideal gas is initially confined to one half of a container by a partition. Experiments show that when the partition is removed, thereby allowing the gas to expand freely to fill the total volume of the cylinder, the temperature of the gas doesn't change. Use the concept of differentials to show that the *internal energy* of an ideal gas can only depend upon its temperature and not its volume or pressure i.e., $U = U(T)$.
[*Hint*: When the partition is remove, the gas expands freely. As a consequence, no heat is required to be input to accomplish this. Similarly, as the expansion is a free expansion, no work is done either by or on the gas.]

[4 *marks*]

b) One mole of an ideal gas changes adiabatically from the state described by (p_i, V_i, T_i) to the state (p_f, V_f, T_f). Calculate the sign and magnitude of the process work via an explicit integration in the case that the final volume is less than the initial volume $V_f < V_i$, i.e. calculate

$$W_{i\to f} = \int_{V_i}^{V_f} \delta W.$$

[3 *marks*]

c) During a recent exhibition of historic manuscripts at Open Treasure in Durham Cathedral, one such item is found to be giving off an exotic form of gas. Preliminary experiments in Durham have shown that the equation of state is possibly given by

$$p = aT \ln V + \frac{bT}{V},$$

where a and b are constants of appropriate dimensions. Calculate the work done in compressing a sample of this gas, isothermally at a temperature T_0, to one hundredth of its original volume so that it can be sent away to the National Physical Laboratory for further analysis.

[3 *marks*]