University of Durham

EXAMINATION PAPER

May/June 2011 Examination code: 042511/01

LEVEL 2 PHYSICS: FOUNDATIONS OF PHYSICS 2

SECTION A. ELEMENTS OF QUANTUM MECHANICS AND ATOMIC PHYSICS
SECTION B. ELECTROMAGNETISM

Time allowed : 2 hours and 30 minutes

Examination material provided : None

Answer the compulsory question that heads each of sections A and B. These **two** questions have a total of 15 parts and carry 60% of the total marks for the paper. Answer **one** optional question from **each** of the two sections. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.

ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK

Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.

APPROVED TYPES OF CALCULATOR MAY BE USED.

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Electron mass:	$m_\mathrm{e} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_\mathrm{p} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Bohr magneton:	$\mu_\mathrm{B} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Nuclear magneton:	$\mu_\mathrm{N} = 5.05 \times 10^{-27}\ \mathrm{J\,T^{-1}}$
Molar Gas constant:	$R = 8.31 \times 10^{3}\ \mathrm{J\,K^{-1}\,kmol^{-1}}$
Avogadro's constant:	$N_\mathrm{A} = 6.02 \times 10^{26}\ \mathrm{kmol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$

SECTION A. ELEMENTS OF QUANTUM MECHANICS AND ATOMIC PHYSICS

Answer question 1 and **either** question 2 **or** question 3 .

1. (a) A quantum mechanical particle moves in a one dimensional potential

$$V(x) = \alpha x^4,$$

with $\alpha > 0$.
What units does α have?
Write down the time independent Schrödinger equation for this system.
Assume now that at a given time the particle is in a state described by the properly normalized wave function $\psi(x)$. Give an expression for the expectation value of the energy.
[4 marks]

(b) Consider two quantum mechanical operators $\hat{A}$ and $\hat{B}$. They have a common set of eigenfunctions $f_n(x)$. This means that for a given n, $f_n(x)$ is an eigenfunction of $\hat{A}$ with eigenvalue α_n but it is also an eigenfunction of $\hat{B}$ with corresponding eigenvalue β_n. The eigenvalues are discrete.
Write down the eigenvalue equations for the operators $\hat{A}$, $\hat{B}$ and $\hat{C} = \hat{A} + \hat{B}$.
What are the eigenvalues and eigenfunctions of $\hat{C}$?
[4 marks]

(c) $\phi_1(x)$ and $\phi_2(x)$ are energy eigenfunctions of a given system with corresponding (different) eigenvalues E_1 and E_2. $\phi_1(x)$ is properly normalized but $\phi_2(x)$ is not properly normalized. At a certain time the properly normalized wave function is given by

$$\psi(x) = \frac{1}{\sqrt{2}}\phi_1(x) + \sqrt{2}\phi_2(x).$$

By what factor must one multiply $\phi_2(x)$ to normalize it properly?
What is the expectation value of the energy?
[4 marks]

(d) At a certain time the state of a hydrogen atom is described by the normalised wavefunction

$$\psi(\underline{r}) = (1/\sqrt{N})(3\phi_{100}(\underline{r}) + 4\phi_{211}(\underline{r}) - 2\phi_{322}(\underline{r}))$$

which is a superposition of normalised energy eigenfunctions

$$\phi_{nlm_l}(\underline{r}).$$

Calculate N and the probability of finding the electron with a non-zero z-component of angular momentum. [4 marks]

(e) The binding energy for the valence electron in the $2p$ state for Lithium ($Z = 3$) is 3.5 eV, close to that of the $n = 2$ state for atomic hydrogen. However, when the outer Lithium electron is in the $2s$ state its binding energy is 5.4 eV. If the ionisation potential for hydrogen is 13.6 eV, using a simple model and calculation, explain why the $2p$ binding energy is close to that of hydrogen. Give a reason for the much higher binding energy for the $2s$ level in Lithium. [4 marks]

(f) Explain, using Hund's rules, why the ground state for Carbon ($Z = 6$) is $^3P_0$. [4 marks]

(g) Neon ($Z = 10$) and Sodium ($Z = 11$) have very different reactive properties. State the electronic configuration of Neon and Sodium and explain their chemical properties. [4 marks]

(h) What is the approximate frequency of the K_β X-ray for Iodine ($Z = 53$). Give a reason why the exact frequency might be different from your answer. [4 marks]

2. Consider a quantum mechanical particle of mass m in a potential

$$V(x) = \frac{1}{2} m\omega^2 x^2 + ax.$$

   (a) Make a sketch of the potential. Calculate the position of the minimum of the potential. In the same figure also draw a qualitative sketch of the ground state wave function and explain why it should have the shape you have drawn. [5 marks]

   (b) Consider the wave function,

$$\psi(x) = N \exp\left(-\beta(x-\gamma)^2\right),$$

   where β and γ are real and $\beta > 0$.

      (i) Calculate the normalization constant N. What would happen if $\beta < 0$?

      (ii) Calculate the expectation value of x.

   [5 marks]

$$\left[\text{Hint} : \int_{-\infty}^{\infty} dx \, \exp(-\kappa x^2) = \sqrt{\frac{\pi}{\kappa}}\,\right]$$

   (c) Insert the wave function given in part (b) into the time independent Schrödinger equation. Determine the constants β and γ such that $\psi(x)$ is a solution of the time independent Schrödinger equation. Determine the corresponding energy eigenvalue. [8 marks]

   (d) For the same wave function as in (b) with the constants determined in (c) calculate the expectation value of the energy. [2 marks]

3. (a) The ground state wavefunction for atomic hydrogen is given by

$$\psi_{100}(\underline{r}) = (\pi a_0^3)^{-1/2} \exp{(-r/a_0)}$$

where a_0 is the Bohr radius. For this state calculate the radial position at which the radial probability density is a maximum and the expectation value for the radial coordinate in this state. Sketch the radial probability density function for the ground state of atomic hydrogen and explain the difference between the position expectation value and the position at which the probability is maxmimal. [10 marks]

$$\left[\text{Hint: } \int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}\right]$$

(b) The Balmer β ($n = 4$ to $n = 2$) spectral line of atomic hydrogen has a wavelength of 486.13 nm. Explain, with numerical results, what will be observed if the light from a discharge tube containing hydrogen and deuterium is examined in a high resolution spectrometer. [4 marks]

(c) The first two excited states of Sodium ($Z = 11$) are ${}^2P_{3/2}$ and ${}^2P_{1/2}$. A weak magnetic field is used to split the magnetic sub-levels of these states. Draw a labelled diagram showing the energies of the sub-levels relative to the unperturbed state. If the magnetic field has a strength of 0.1 T calculate the energy splitting for the lower energy state in eV. [6 marks]

$$\left[\text{Note: The Landé g-factor is } g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}.\right]$$

SECTION B. ELECTROMAGNETISM

Answer question 4 and **either** question 5 **or** question 6 .

4. (a) Write down Maxwell's four equations in their most general differential form. [4 marks]

   (b) Explain why the relative permeability of a metallic wire that is magnetic is found experimentally to be frequency dependent. [4 marks]

   (c) The general dispersion relation for an electromagnetic wave propagating in a non-magnetic, conducting medium is given by:

   $$k^2 = \mu_0 \varepsilon \omega^2 + i\omega\mu_0\sigma_N,$$

   where k is the wavevector, ω is the angular frequency and σ_N is the electrical conductivity. A material has a relative permittivity of 70 and an electrical conductivity of 10^5 Ω^{-1} m^{-1}. Can this material be considered a good conductor at a frequency of 10^{10} Hz ? [4 marks]

   (d) Use a diagram to explain the relationships between the electric field, the magnetic field and the direction of propagation of a plane electromagnetic wave propagating in free space. [4 marks]

   (e) Provide a brief outline of Ampère's model, which describes how a magnetic material generates a magnetic field. [4 marks]

   (f) Consider an infinite line of equally spaced electrons where there is 1 nm between neighbouring electrons. Calculate an approximate value for the magnitude of the electric field at a distance of 2 metres from the line of electrons. [4 marks]

   (g) Explain the difference between polar and non-polar dielectrics. [4 marks]

5. The wave equation for an electromagnetic wave propagating in vacuum is given by:

$$\nabla^2 \underline{E} = \mu_0 \epsilon_0 \frac{\partial^2 \underline{E}}{\partial t^2}.$$

(a) Calculate the group velocity of an electromagnetic pulse obeying this wave equation. [3 marks]

(b) It is known that one possible solution for the wave equation within the interior of an infinitely long hollow square metallic tube (known as a waveguide) of width a is:

$$\underline{E} = \underline{E}_0 \sin\left(\frac{5\pi x}{a}\right) \sin\left(\frac{5\pi z}{a}\right) \exp[i(ky - \omega t)],$$

where x, y and z are the conventional Cartesian coordinates, ω is the angular frequency, k is the wavevector, $\underline{E}_0$ gives the magnitude and polarisation of the wave and the axis of the tube is parallel to the y-axis. Given that $a = 0.2$ m, find the minimum frequency at which this particular wave can propagate along the waveguide (Hint: Derive the dispersion relation and find the condition that the wavevector k is real). [6 marks]

(c) Above this minimum frequency, determine whether a pulsed wave propagates without changing shape. [4 marks]

(d) Write down another solution to the wave equation which describes a different wave that can propagate through the waveguide with a higher minimum frequency. [4 marks]

(e) Discuss whether the energy of photons associated with the wave you have proposed is higher or lower than the original wave considered above. [3 marks]

6. (a) Use Maxwell's equations to derive the boundary conditions for the $\underline{E}$-field parallel and orthogonal to the interface between two dielectrics. [4 marks]

(b) For an electromagnetic wave at normal incidence to the planar interface between two dielectric media, the power reflection coefficient, R, is given by:

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 ,$$

where n_1 and n_2 are the refractive indices of the media. At what values of n_1/n_2 does the power transmitted across the interface equal one third of that reflected from it ? [6 marks]

(c) Sketch the form of R against n_1/n_2. [4 marks]

(d) A thick sheet of material with refractive index 3 is in vacuum with an electromagnetic wave fired at it at normal incidence. By including the effect of internal reflections, calculate, to an accuracy of about 1%, approximately what fraction of the incident energy is reflected. [6 marks]