Rutherford scattering cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{e^4 z^2 Z^2}{4 E'^2 \sin^4 \theta/2}$$

Mott cross section

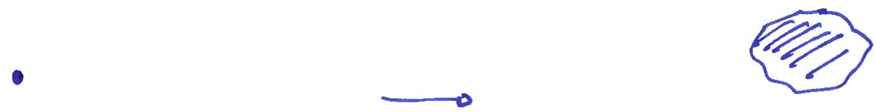
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_R \cos^2 \theta/2$$

3.1 key Points

- When the projectile is relativistic, its spin has to be taken into account; this leads to the Mott cross section.
- Helicity conservation suppresses backward scattering.

4 Nuclear form factors

So far we have only considered point-like potentials.



pointlike charge distribution

extended charge distribution

$$\rho(\vec{x}) = Ze\,\delta^3(\vec{x}-\vec{x}_0)$$

$$\rho(\vec{x}) = Ze f(\vec{x})$$

normalized $\int f(\vec{x})\, d^3\vec{x} = 1$

$$\Delta\phi(\vec{x}) = -\rho(\vec{x})$$

$$\rho(\vec{x}) = \int \rho(\vec{y})\, \delta(\vec{x}-\vec{y})\, d\vec{y}$$

$$= Ze \int f(\vec{y})\, \delta(\vec{x}-\vec{y})\, d\vec{y}$$

$$\Rightarrow \quad \phi(\vec{x}) = Ze \int f(\vec{y})\, \frac{1}{|\vec{x}-\vec{y}|}\, d\vec{y}$$

For this potential the matrix element is given by

$$M_{fi} = \langle \psi_f | \mathcal{H}_{int} | \psi_i \rangle$$

$$\overset{Z=1}{=} \frac{eZ}{V} \int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, \phi(\vec{x}) \qquad \vec{q} = \vec{p} - \vec{p}'$$

$$= \frac{e}{V} \int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, Ze \int f(\vec{y})\, \frac{1}{|\vec{x}-\vec{y}|}\, d^3y$$

$$= \frac{Ze^2}{V} \int d^3y\, f(\vec{y}) \underbrace{\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, \frac{1}{|\vec{x}-\vec{y}|}}_{\text{Calculation yesterday}}$$

$$= \frac{Ze^2}{V} \int d^3y\, f(\vec{y})\, \underline{\frac{4\pi}{|\vec{q}|^2}\, e^{i\vec{q}\cdot\vec{y}}}$$

$$= \frac{4\pi Z e^2}{V\, |\vec{q}|^2} \underbrace{\int d^3y\, f(\vec{y})\, e^{i\vec{q}\cdot\vec{y}}}_{\text{Fourier transform of } f(\vec{y})}$$

$$= \frac{4\pi Z e^2}{V\, |\vec{q}|^2}\, F(\vec{q})$$

where

$$F(\vec{q}) = \int d^3y\; f(\vec{y})\, e^{i\vec{q}\cdot\vec{y}}$$

is called the <u>form factor</u>.

The spin of the projectile does not affect the shape of the target. Therefore the cross section for scattering off an charge distribution reads

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rho(x)} = \left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{Mott,}\\ \text{no recoil}}} |F(\vec{q})|^2$$

Measuring the cross section for an extended target and comparing it to the Mott cross section allows to extract the form factor and therefore the shape of the charge distribution.

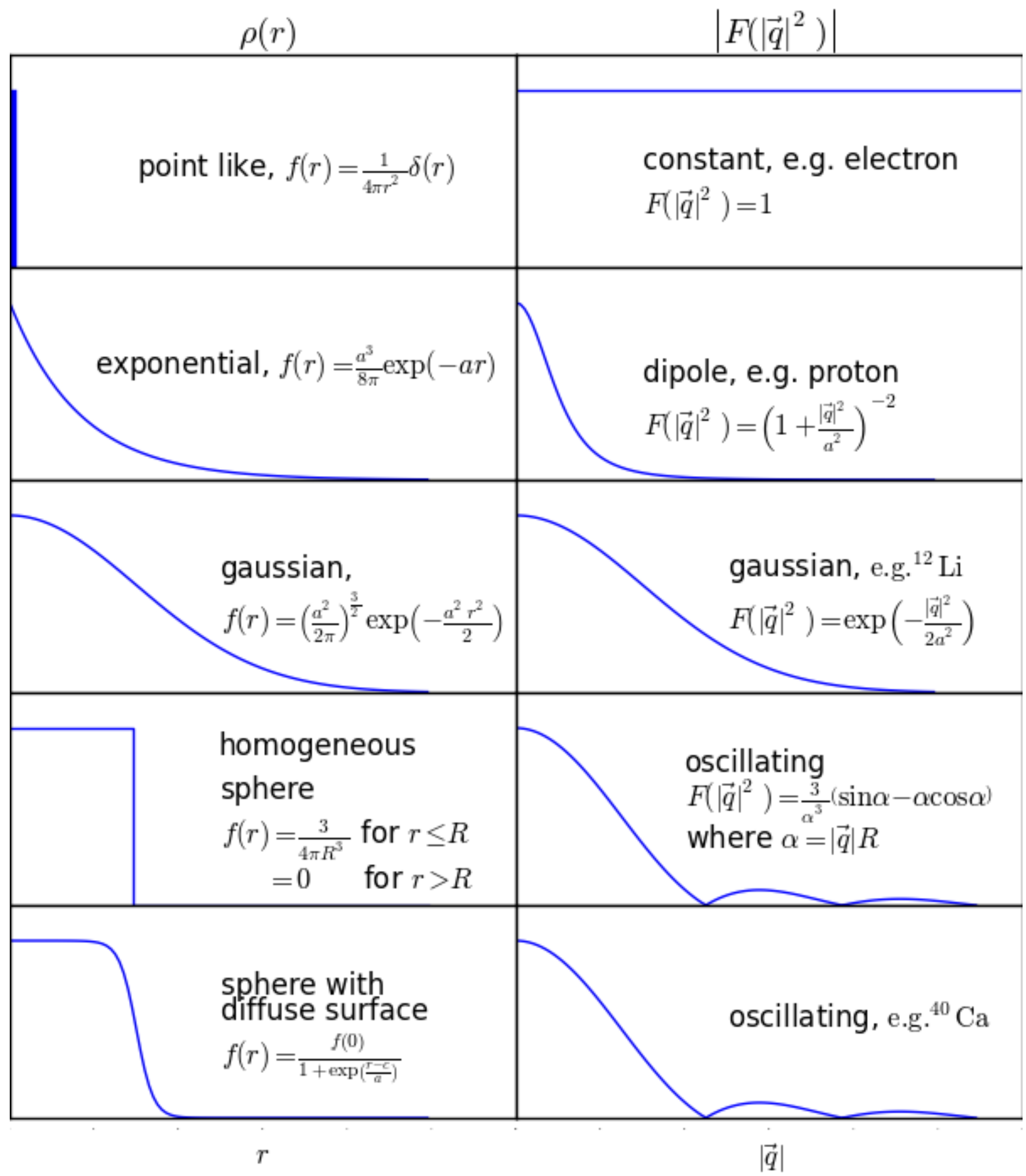


Figure 14: Form factors for several charge distributions

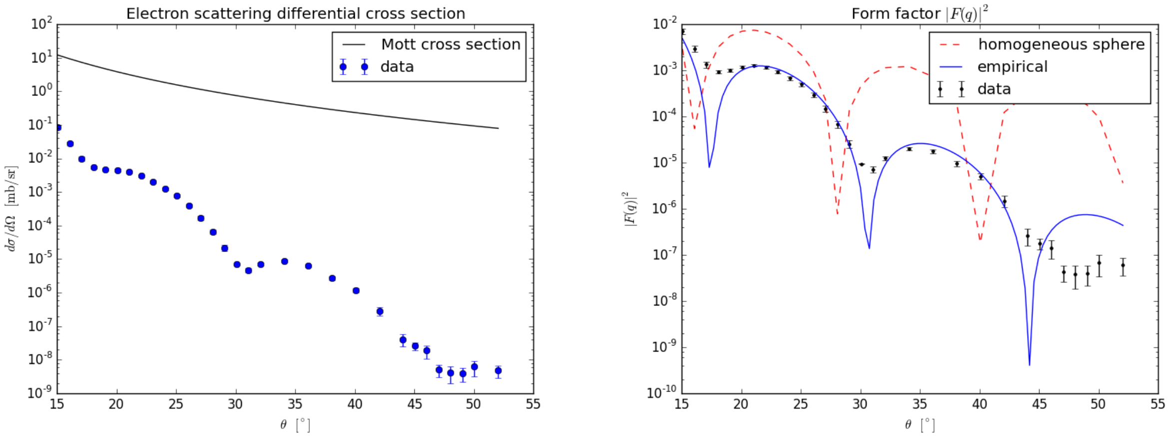
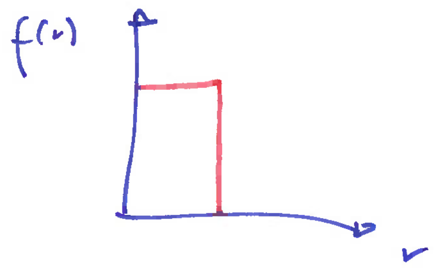


Figure 13: The left panel shows the measured cross section and Mott cross section for the scattering of an electron of energy 757.5 MeV off a calcium nucleus. The right-hand side panel shows the ratio of the two cross sections which is the form factor and the prediction using two different model: an homogeneous sphere of radius 4.13 fm and a charge distribution based on eq. 37 with $r_0 = 3.66$ fm and $a = 0.54$ fm.

For the example of a spherically symmetric homogeneous sphere of radius R

$$f(r) = \begin{cases} \frac{3}{4\pi R^3} & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$



Fourier transform of $f(r)$

$$F(|\vec{q}|^2) = \frac{3}{a^3}\left(\sin(a) - a\cos(a)\right) \quad \text{with} \quad a = |\vec{q}|R$$

From the zeros of this function we can obtain the radius of the sphere

$$\left(\sin(a) - a\cos(a)\right) = 0 \quad \text{for} \quad a = 4.5,\ 7.725, \ldots$$

$$a_0 = |q_0|R \simeq 4.5 \quad \Rightarrow \quad R \simeq \frac{4.5}{q_0}$$

$$q_0 = 2E\sin\frac{\theta_0}{2}$$

In practice the charge distribution of nuclei are not really homogeneous, and are described by a Fermi function

$$f(r) = \frac{f_0}{1 + e^{\frac{r - r_0}{a}}}$$

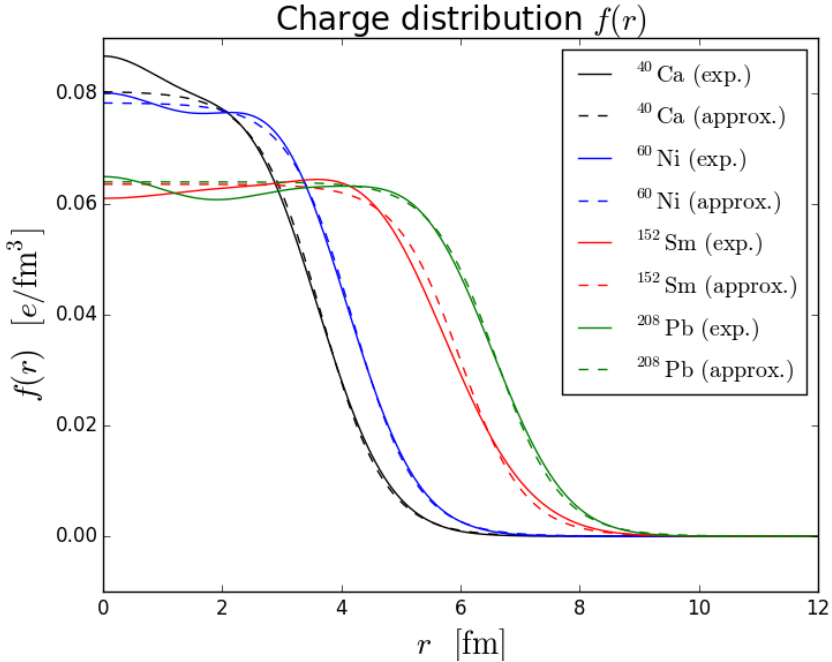


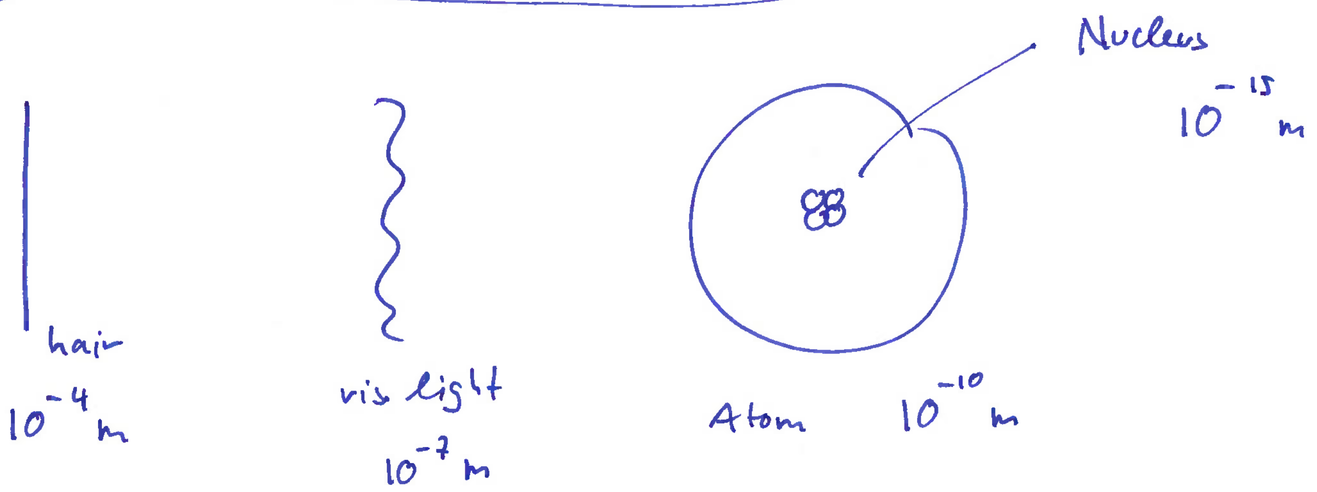
Figure 15: Experimental charge distribution for four nuclei and the approximation based on eq. (37).

4.1 key points

- Scattering off a charge distribution is reduced by a factor called the form factor with respect to a point-like particle.
  The form factor is the Fourier transform of the charge distribution. The cross section is reduced by $|F(\vec{q})|^2$

- Looking at the ratio between the point-like and the actual cross section one can infer the form factor and therefore the shape of the charge distribution. Using this technique to measure the charge distribution in the nucleus of an atom.

5 Scattering off the nucleons



$$q = \frac{\hbar c}{\lambda} = \frac{2 \cdot 10^{-7}\, eV \cdot m}{10^{-15}\, m} = 200 \text{ MeV}$$

Mass of nucleus $\gtrsim$ 900 MeV