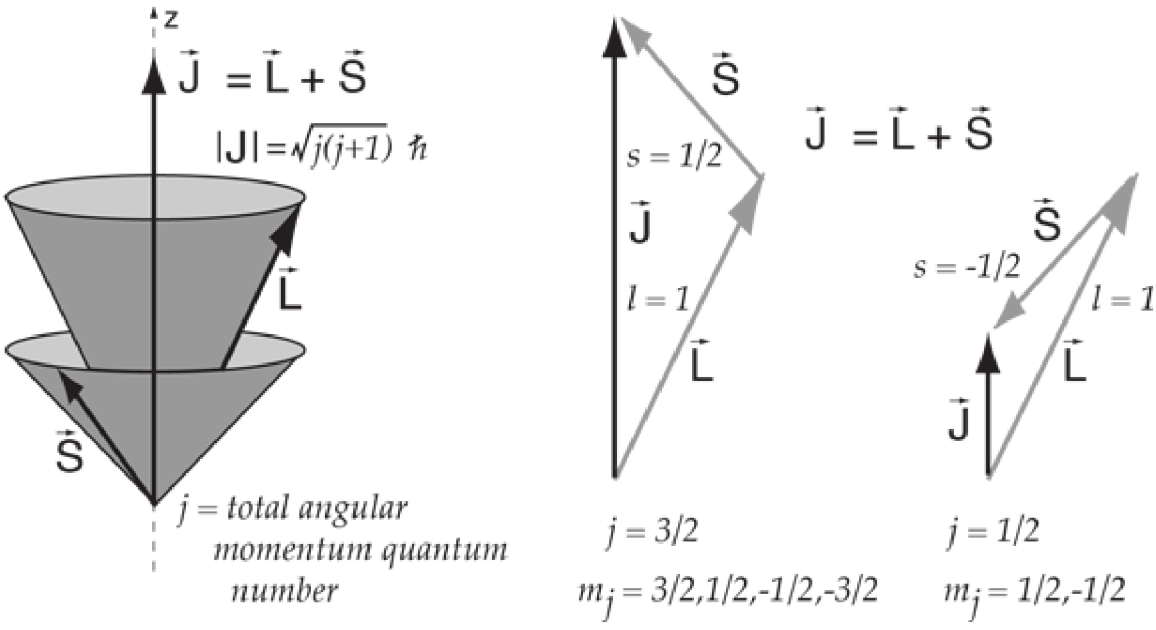
QUANTUM MECHANICS 2 - WORKSHOP 8

Q1: A spin $s = 1$ particle is in an orbit with orbital angular momentum quantum number $l = 2$. The total angular momentum is $\underline{J} = \underline{L} + \underline{S}$, where the associated operators J^2 and J_z, have eigenvalues $j(j+1)\hbar^2$ and $m_j\hbar$.

(a) What are the possible values resulting from measurements of J^2 and J_z? [Hint: do it systematically — j runs from $|l - s|$ to $(l + s)$ in integer steps, and for each possible j give the possible values of m_j.]

Second hint: You might find it useful to think in terms of the vector model as illustrated for the case $l = 1$ and $s = 1/2$ in the following figure.



(b) When considering a thermal cloud of such particles, where you expect the populations of each angular momentum sublevel to be the same, what is the probability to measure one such particle to have $J_z = 0$?

Q2: The Lyman alpha line in Hydrogen involves transitions between $n = 2$ and $n = 1$. The energy of each level depends on n and j only.

(a) How many different line energies are there?

(b) If there were no other factors operating (to, e.g., pump the atoms in a thermal cloud into a particular sublevel), what is the probability to get each line energy?

(c) Emission of a photon leads to a change in angular momentum $\Delta l = 1$ since the photon carries this away. How does this extra condition change the probability for each line energy?

Q3: There are first order corrections to the energy levels in Hydrogen from relativistic corrections, spin-orbit coupling and the finite size of the proton:

$$\begin{aligned}
E^1_{n,\mathrm{r}} &= -\frac{(E^0_n)^2}{2mc^2}\left[\frac{4n}{l+1/2}-3\right], && \text{which holds for all } l;\\
E^1_{n,\mathrm{so}} &= \frac{(E^0_n)^2}{mc^2}\frac{n[j(j+1)-l(l+1)-3/4]}{l(l+1/2)(l+1)}, && \text{which holds for } l\neq 0;\\
E^1_{n,\mathrm{D}} &= 2n\frac{(E^0_n)^2}{mc^2}, && \text{which holds for } l=0.
\end{aligned}$$

add these together for the three separate cases $l \neq 0$ and $j = l - 1/2$, $l \neq 0$ and $j = l + 1/2$, $l = 0$ so $j = 1/2$.