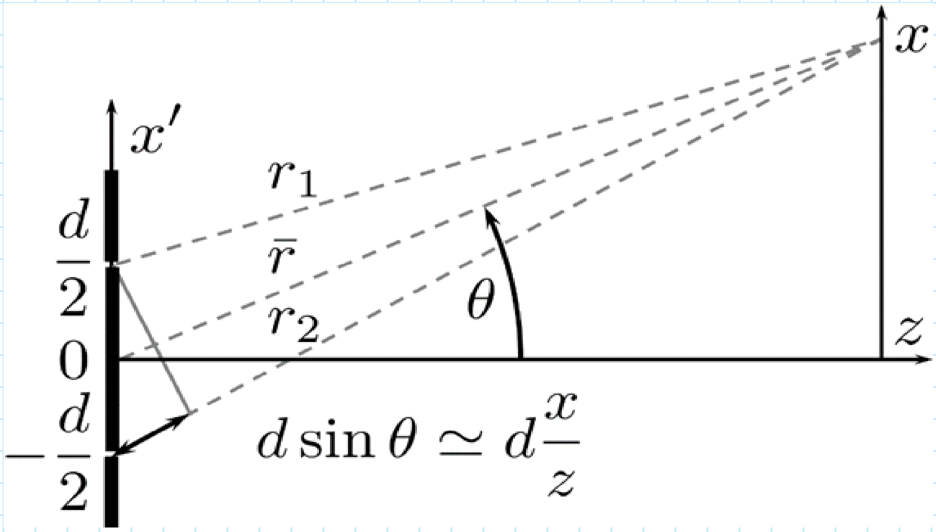
Lecture 7: From two to N slits: phasor sums and diffraction gratings

02 February 2020 16:52

RECAP: TWO HOLES



Slit spacing d

We wrote

$$E = \bar{E}_s \left(e^{ikr_1} + e^{ikr_2} \right)$$

PHASOR SLIT ① PHASOR SLIT ②

Paraxial regime (Fresnel approximation)

$$r_{1,2} = \bar{r} \mp \frac{xd}{2z} + \frac{d^2}{8z} \qquad \text{with} \qquad \bar{r} = z + \frac{x^2}{2z}$$

$\hookrightarrow$ today we neglect these terms

$z >> d$

Three slits

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Optics f2f 3.8

So for two slits we would have

$$E = E_s e^{ik\bar{r}}\left(e^{\frac{-ikdx}{2z}} + e^{\frac{-ikdx}{2z}}\right)$$

For 3 slits at $(0, \pm d)$ this becomes

$$E = E_s e^{ik\bar{r}}\left(\underbrace{e^{-i\frac{kdx}{z}}}_{\text{BOTTOM SLIT}} + \underbrace{e^{i\frac{kdx}{z}}}_{\text{TOP SLIT}} + \underbrace{1}_{\text{MIDDLE SLIT}}\right)$$

(factor of 2 goes as slit is at d not d/2)

MIDDLE SLIT
why? because paraxial distance is just $\bar{r}$

Three slits

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Intensity can be calculated as for two slits (EXERCISE FOR YOU) ie $I \propto EE^*$

The result is

$$I = I_s\left[1 + 2\cos\left(\frac{kdx}{z}\right)\right]^2$$

Let's look a E & I:

First zero: $\cos\left(\frac{kdx}{z}\right) = -\frac{1}{2}$ $\therefore$ $\frac{kdx}{z} = \frac{2\pi}{3}$

$$x = \frac{1}{3}\frac{\lambda}{d}z$$

Maximum $\cos\left(\frac{kdx}{z}\right) = 1$ $I = 3I_s$

$$\frac{kdx}{z} = 0, 2n\pi$$

$$x = \frac{n\lambda}{d}z$$

Three slits

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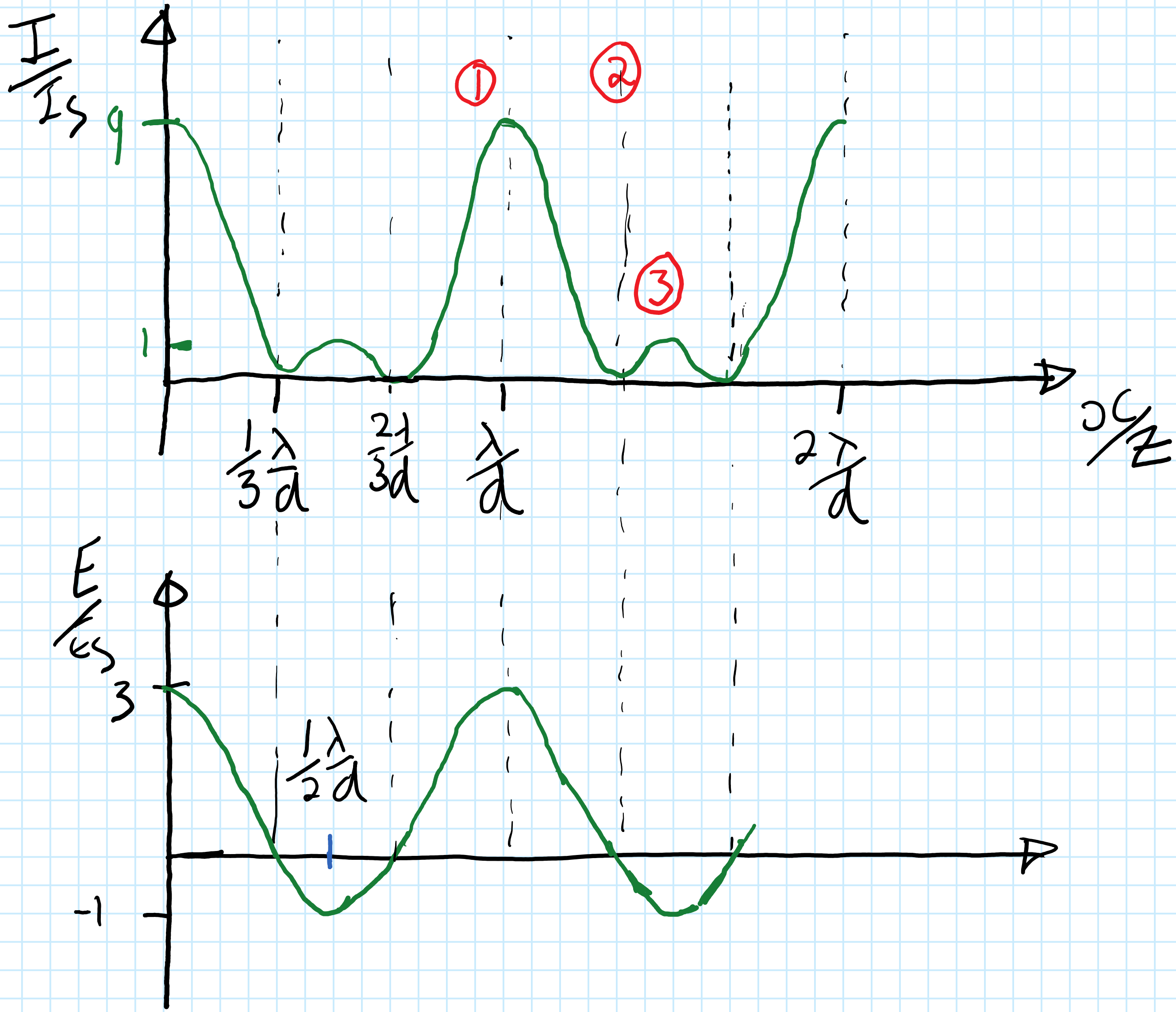

① PRINCIPAL MAXIMUM

PHASORS

② ZERO

③ SUBSIDIARY MAXIMUM

cos( ) = 0

middle outside

N slits - the grating

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PHASOR sum FOR N HOLES with spacing d is

$$E = \bar{E}_s e^{ik\bar{r}} \sum_{m=-(N-1)/2}^{(N-1)/2} e^{imk\frac{d}{z}}$$

Errors in F&F 3.29

$$= \bar{E}_s e^{ik\bar{r}} e^{-i(N-1)\frac{kdx}{2z}} \sum_{n=0}^{N-1} e^{in\frac{kdx}{z}}$$

!

This can be summed analytically using GEOMETRIC PROGRESSION (Appendix B9)

! $$E = \bar{E}_s e^{ik\bar{r}} \frac{\sin\left(\frac{Nkdx}{2z}\right)}{\sin\left(\frac{kdx}{2z}\right)}$$

! $\therefore$

$$I = \bar{I}_s e^{ik\bar{r}} \frac{\sin^2\left(\frac{Nkdx}{2z}\right)}{\sin^2\left(\frac{kdx}{2z}\right)}$$

Diffraction grating

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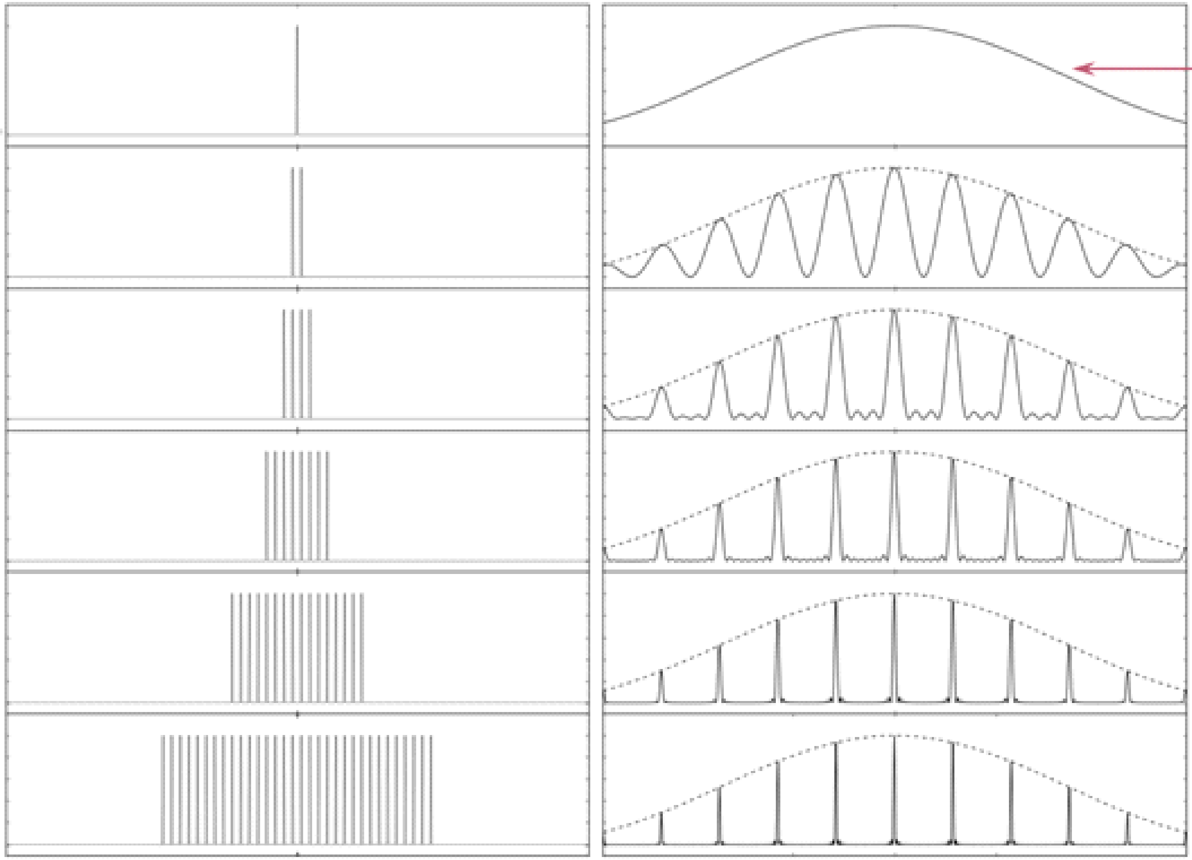
DIFFRACTION next lecture



Envelope is due to finite slit width

More slits leads to sharper lines and better resolution (see slides)

Diffraction grating

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PROPERTIES:

- Principal maxima at $x = \pm m \left(\frac{\lambda}{d}\right) z$

$$I \propto N^2$$

- First zeroes at $x = \pm \left(\frac{\lambda}{Nd}\right) z$

  $N-1$ zeroes between principal maxima.

- $N-2$ secondary maxima between principal maxima.

Diffraction grating properties

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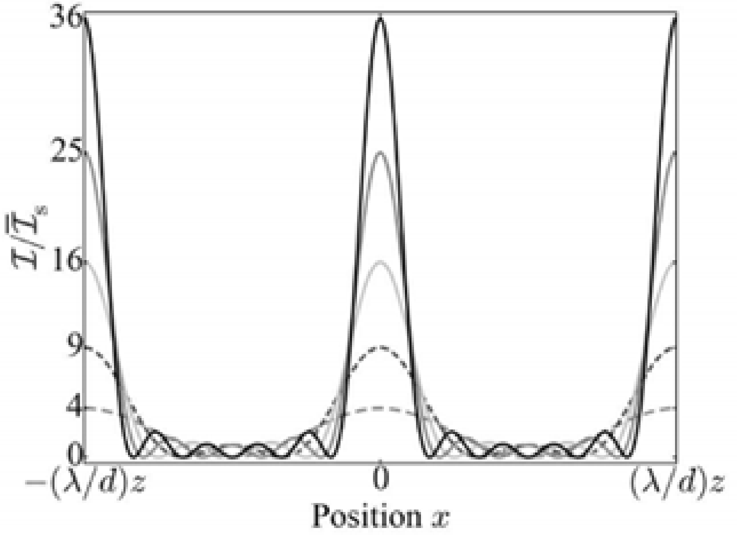

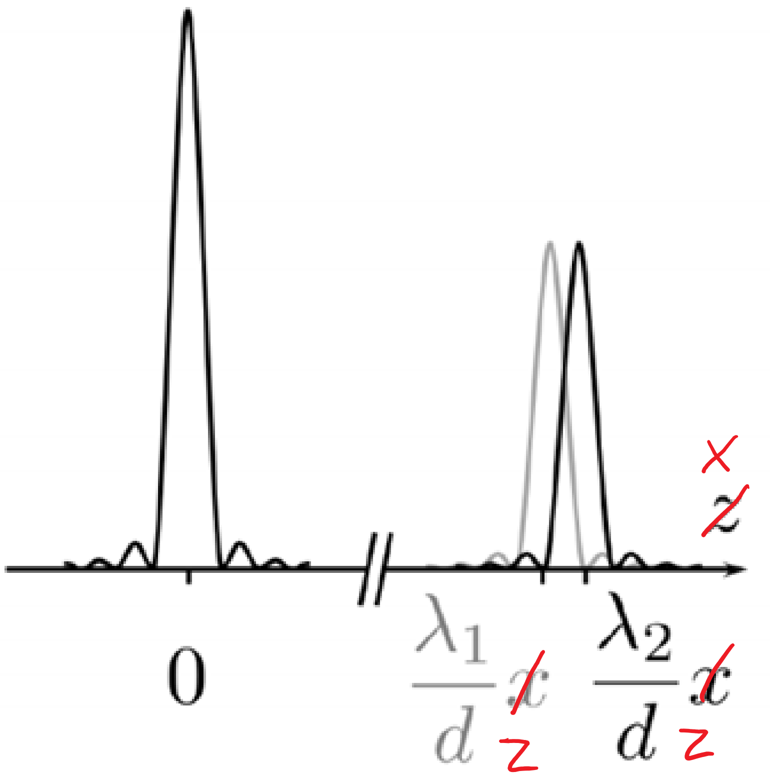

Many waves: the Fabry-Perot etalon

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Optics f2f 3.11

Just idea covered here.

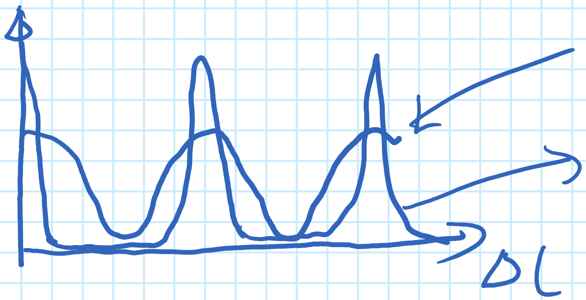
LIGO arms contain extra mirrors to make each arm a Fabry-Perot cavity

$\Rightarrow$ instead of adding 2 waves we add N waves where $N = \underset{\uparrow\ \text{finesse}}{F} \approx \pi\sqrt{\underset{\nwarrow\ \text{reflectivity}}{R}}$

$(R \approx 1)$

In LIGO $F \simeq 450$ $\therefore$ like having arms $4\text{km} \times 300 = 1200\,\text{km}$!

Adding more waves makes the response to Δl sharper



see F2F 3.20