CM8 Solutions: Inertial Accelerations

1. **(2 marks total)** The gravitational acceleration is GM/R^2 towards the centre of the planet, so the centrifugal force needs to balance this (1 mark).

   The centrifugal acceleration points radially away from the axis of rotation and has size $\omega^2 R$. Hence $\omega = \sqrt{GM/R^3}$ (1 mark).

2. **(2 marks total)** The centrifugal force vanishes at the poles as $\underline{\omega} \times \underline{r} = 0$ (1 mark).

   The Coriolis acceleration has size $|\underline{a}_{Cor}| = 2\omega v$ and it acts to the right of the direction of motion of the particle (1 mark).

3. **(2 marks total)** The time taken for the arrow to reach the target can be approximated as $T = l/v$. During this period, an acceleration of $\ddot{x} = 2\omega v$ to the right is experienced by the arrow (1 mark).

   Thus, the arrow will be displaced by $x_{disp} = \ddot{x}T^2/2$ to the right at the distance of the target, which is an angle $\theta_{disp} = x_{disp}/l = \omega l/v$ to the right of the target. Thus, the north pole archer will aim this same angle to the left of the target centre in order to hit it (1 mark).

4. **(1 mark total)** The Coriolis force acts to the left at the south pole, so the south pole archer will aim an angle $\omega l/v$ to the right of the target centre and miss by a distance $2\omega l^2/v$ to the right (1 mark).

5. **(3 marks total)** The north pole archer will again aim a distance $\omega l^2/v$ to the left of the target. However, this time the arrow will only take a time $T = fl/v$ to reach the target. Thus it will miss the centre by a distance

   $$\Delta x_{NP} = -\frac{\omega l}{v}.v.\frac{fl}{v} + \frac{1}{2}(2\omega v)\left(\frac{fl}{v}\right)^2 = \frac{\omega l^2}{v}(-f + f^2),$$

   i.e. to the left of the centre (1 mark).

   The south pole archer will aim a distance $\omega(fl)^2/v$ to the right of the target, taking into account that they know the new distance to the target. Thus their arrow will miss the centre by a distance

   $$\Delta x_{SP} = \frac{\omega(fl)}{v}.v.\frac{fl}{v} + \frac{1}{2}(2\omega v)\left(\frac{fl}{v}\right)^2 = \frac{2\omega(fl)^2}{v},$$

   to the right of the centre (1 mark).

   For the southerner to win the competition, we need $|\Delta x_{SP}| < |\Delta x_{NP}|$. Thus $2f^2 < f - f^2$, which is satisfied by $f < 1/3$ (1 mark).