L2 Foundation of Physics 2B Optics 2019-20

O.2 Phase, complex notation and plane waves

1. To introduce the concepts of **phase** and **relative phase**.
2. To introduce complex notation [Optics $f2f$ Sec. 1.11] and phasors
3. To introduce scalar waves plane waves [Optics $f2f$ Sec. 2.3] and wavefronts.
4. To relate the components of the wave vector to spatial frequency [Optics $f2f$ Sec. 1.9].

Summary: In **complex notation** we write the harmonic wave solution (in the scalar approximation) as

$$E = E_0 \mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} \ . \tag{1}$$

This wave is characterized by an **amplitude** E_0 and **phase** $\phi = i(\underline{k}\cdot\underline{r} - \omega t)$. Complex notation is used because it is easy to shift the phase of the wave by e.g. by a quantity ϕ_0:

$$\begin{aligned} E' &= Ee^{i\phi_0} = E_0\mathrm{e}^{\mathrm{i}(\underline{k}\cdot\underline{r}-\omega t)}e^{i\phi_0} \qquad (2)\\ &= E_0\mathrm{e}^{\mathrm{i}(\underline{k}\cdot\underline{r}-\omega t+\phi_0)} \ . \qquad (3)\end{aligned}$$

For a **plane wave**, E_0 is independent of position and time, i.e. a constant. As a plane wave has infinite spatial extent it is a mathematical idealisation!

For a plane wave propagating at angle θ relative to the z axis in the xz plane, the **spatial frequency** along x is

$$u = \frac{\sin\theta}{\lambda} \ , \tag{4}$$

which for small θ is much smaller than the spatial frequency along z (or in the direction of propagation) [Figs. 1.6 or 2.5 in Optics $f2f$]. As $k_x = k\sin\theta$ we find that the x-component of the wave vector is equal to 2π times spatial frequency along the x-axis:

$$k_x = 2\pi u \ , \tag{5}$$

i.e. we can think of the components of the wave vector as the rate of change of phase along a particular direction, with units rad.m^{-1}.

In optics we measure **intensity** rather than field. Why? Intensity is proportional to the **modulus squared** of the complex form of the field:

$$\mathcal{I} = \tfrac{1}{2}\epsilon_0 c|E|^2 \ . \tag{6}$$

Outlook: In the next lecture, we shall consider the **paraxial regime**.