

## 2 Nuclear stability

Most nuclei are not stable

$$N(t) = N_0 e^{-\lambda t}$$

Mean lifetime (average time a nucleus remains in a given set)

$$\tau = \frac{1}{\lambda}$$

Half-life

$$t_{1/2} = \frac{\ln(2)}{\lambda} = \tau \ln(2)$$

Different decays for

- constant  $A$ :  $\beta^-$  decay  $\begin{matrix} A \\ Z \end{matrix} X \rightarrow \begin{matrix} A \\ Z+1 \end{matrix} X' + e^- + \bar{\nu}_e$

$\beta^+$  decay  $\begin{matrix} A \\ Z \end{matrix} X \rightarrow \begin{matrix} A \\ Z-1 \end{matrix} X' + e^+ + \nu_e$

electron capture  $\begin{matrix} A \\ Z \end{matrix} X + e^- \rightarrow \begin{matrix} A \\ Z-1 \end{matrix} X' + \nu_e$

- change  $A$ :

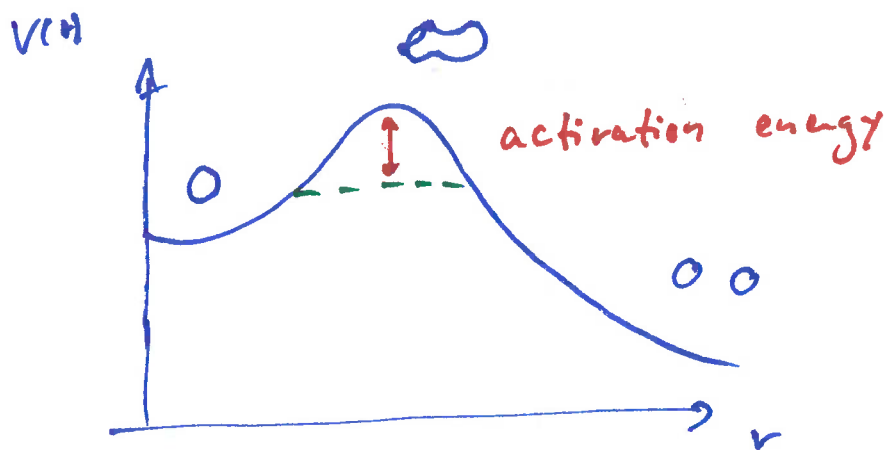
$\alpha$ -decay  $\begin{matrix} A \\ Z \end{matrix} X \rightarrow \begin{matrix} A-4 \\ Z-2 \end{matrix} X' + \begin{matrix} 4 \\ 2 \end{matrix} \text{He}$

fission  $\begin{matrix} A \\ Z \end{matrix} X \rightarrow \begin{matrix} A' \\ Z' \end{matrix} X' + \begin{matrix} A'' \\ Z'' \end{matrix} X''$

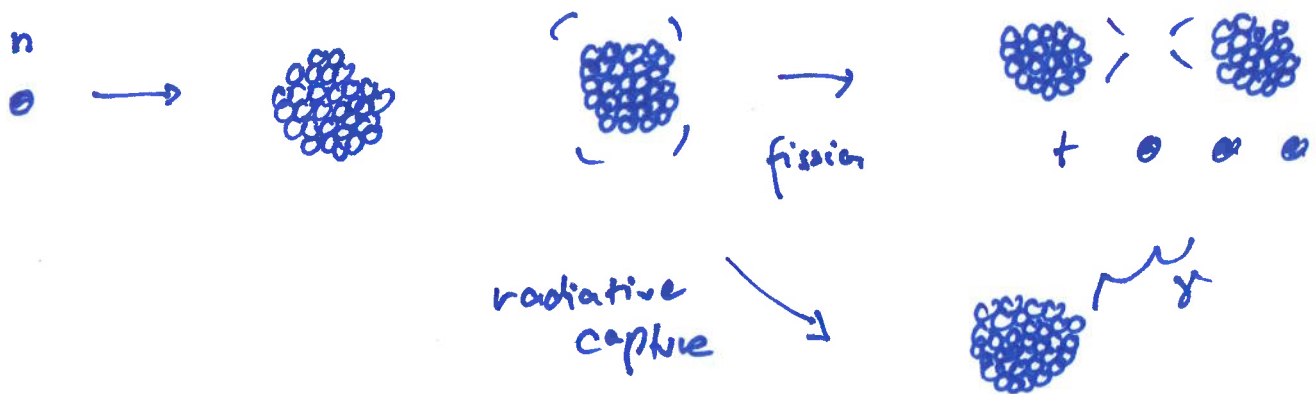
$$A = A' + A'' \quad Z = Z' + Z''$$

Spontaneous fission is a rare process and only happens if the nucleus is very large.

It is a tunnelling process, but can also be triggered by supplying the activation energy.

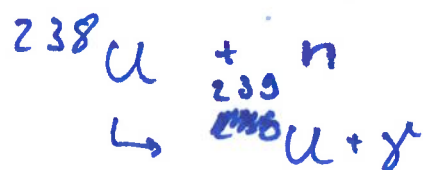


For example with neutrons



fission does not always happen if the activation energy is supplied. Sometimes the extra energy is radiated off.

E.g.



$E(n) \sim 1 \text{ eV}$

usually leads to radiative capture



usually leads to fission

The decay of  $^{235}\text{U}$  through fission also adds 3 neutrons with energies  $\sim 1\text{ MeV}$ . At this energy they are unlikely to trigger any further fission (unless there is a lot of  $^{235}\text{U}$ )

In nature  $^{238}\text{U} : 99\%$   $^{235}\text{U} : \leq 1\%$

Most of the  $1\text{ MeV}$  neutrons produced in  $^{235}\text{U}$  fission are absorbed by  $^{238}\text{U}$  through radiative capture.

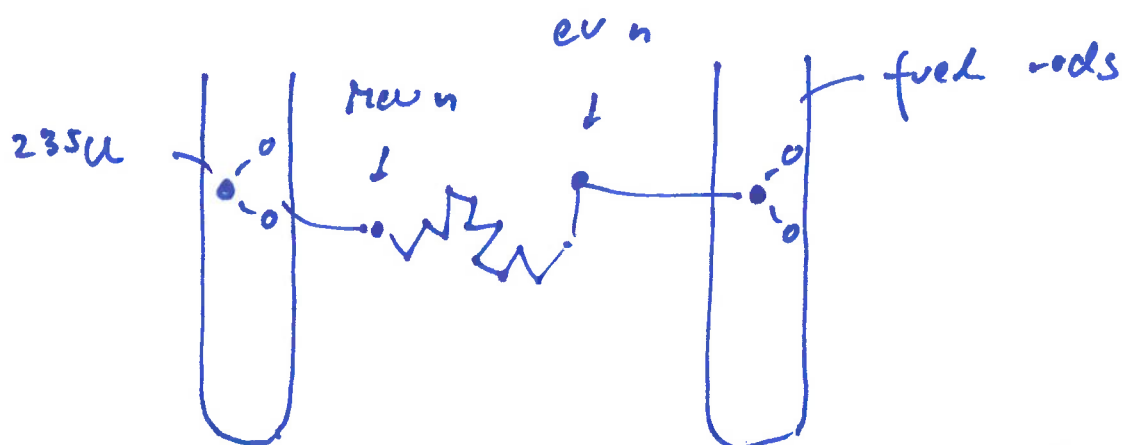
In fission reactors  $^{238}\text{U} : 96\%$   $^{235}\text{U} : 4\%$

The chain reaction can be sustained if the  $1\text{ MeV}$ -neutrons are "cooled down" in a controlled way.

In a nuclear bomb  $^{238}\text{U} \leq 80\%$   $^{235}\text{U} \geq 20\%$

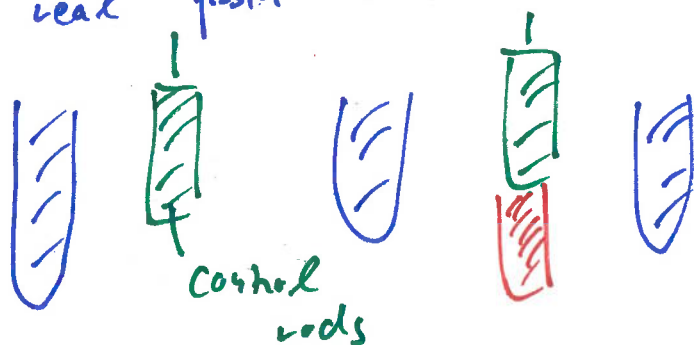
Self-sustained, because a critical mass of  $^{235}\text{U}$  is reached. Critical mass: Enough fissionable material such that each new neutron (on average) triggers another fission.

Cooling of neutrons in a fission reactor proceeds through moderator material by scattering



Good moderators are rich in neutrons (Graphite,  $D_2O$ )

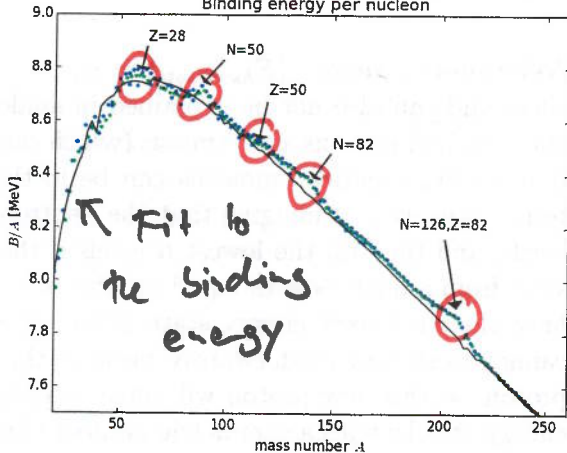
In a real fission reactor

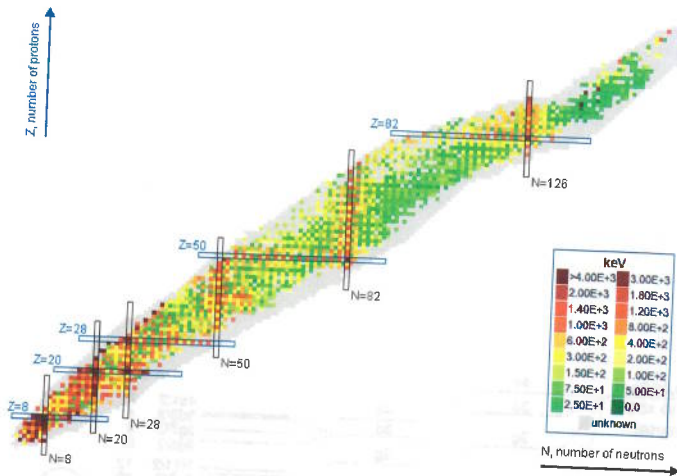


### Key points

- Nuclei can exchange a neutron for a proton in  $\beta^-$  decay
- Nuclei can exchange a proton for a neutron in  $\beta^+$  decay
- Nuclei can capture electrons in the atom to change a proton into a neutron
- The  ${}^4_2\text{He}$  nucleus is very stable and can be emitted by an unstable nucleus ( $\alpha$ -decay)
- Very heavy nuclei decay spontaneously into smaller parts (fission)
- A suitable arrangement of fissile material and moderator can sustain a controlled chain reaction.

Binding energy per nucleon





0: First excited state energy. The highest energies (and hence the most stable states) are located around proton and neutron numbers equal to 2,8,20,28,50,82,126

3 Shell model

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