Chapter 9

Galaxy statistics

9.1 Introduction

To better understand how galaxies form and evolve, we require a large and unbiased *sample* of them. Acquiring such a sample is difficult for many reasons. Firstly, we detect galaxies based on their *flux* and as a consequence we can detect luminous galaxies to large distances but intrinsically faint galaxies only nearby. So if we simply studied all galaxies above a given flux limit, then intrinsically bright galaxies would be over represented compared to intrinsically faint galaxies. Secondly, measuring reliable distances is difficult with Cepheid distance measurements possible to out to ~ 20 Mpc but not much further.

Galaxy redshift surveys use the redshift of a galaxy to infer the distance by assuming that the Universe expands at a known rate,

$$\lambda = \lambda_0(1 + \frac{v}{c}) = \lambda_0(1 + H_0\frac{r}{c}) , \qquad (9.1)$$

where H_0 is the Hubble constant and λ the observed wavelength of a line in the galaxy's spectrum with laboratory wavelength λ_0; r is the (Hubble) distance to the galaxy[1]. Surveys select objects on the night sky identified in photographics plates - or more recently CCD images - based on size, luminosity and colour to distinguish galaxies from stars or other foregrounds.

[1]The relation between physical size l and and angular extent θ defines the distance r_A: $\theta = l/r_A$. The relation between flux and luminosity defines the distance r_L: $F = L/(4\pi r_L^2)$. Locally, these distances are the same, $r_A = r_L = r$, but on cosmological scales they are not because space is curved.

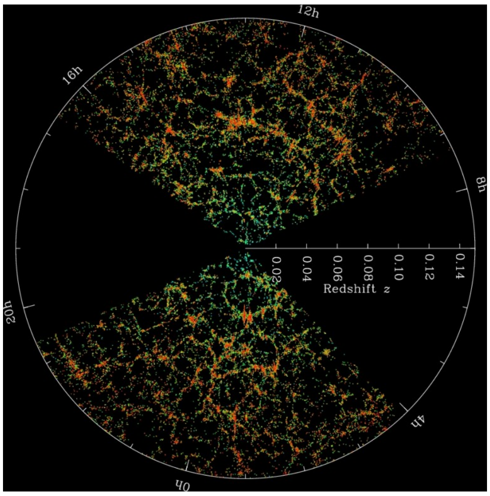

Figure 9.1: Large-scale structure in the distribution of galaxies around us (we are at the centre of the map), from the *dark energy survey.* Galaxies are surveyed in two pie-shaped regions on the sky, and are plotted as a function of right-ascension and distance. Each coloured dot is a galaxy, with red dots representing galaxies with little current star formation (ellipticals), and green and blue dots representing star forming galaxies (spiral and irregular galaxies). The striking 'fingers of god' correspond to galaxy clusters. Notice also the characteristic 'filamentary' distribution of galaxies, and the large dark regions with few galaxies - called 'voids'. For reference, the distance to $z = 0.14$ is ≈ 600 Mpc.

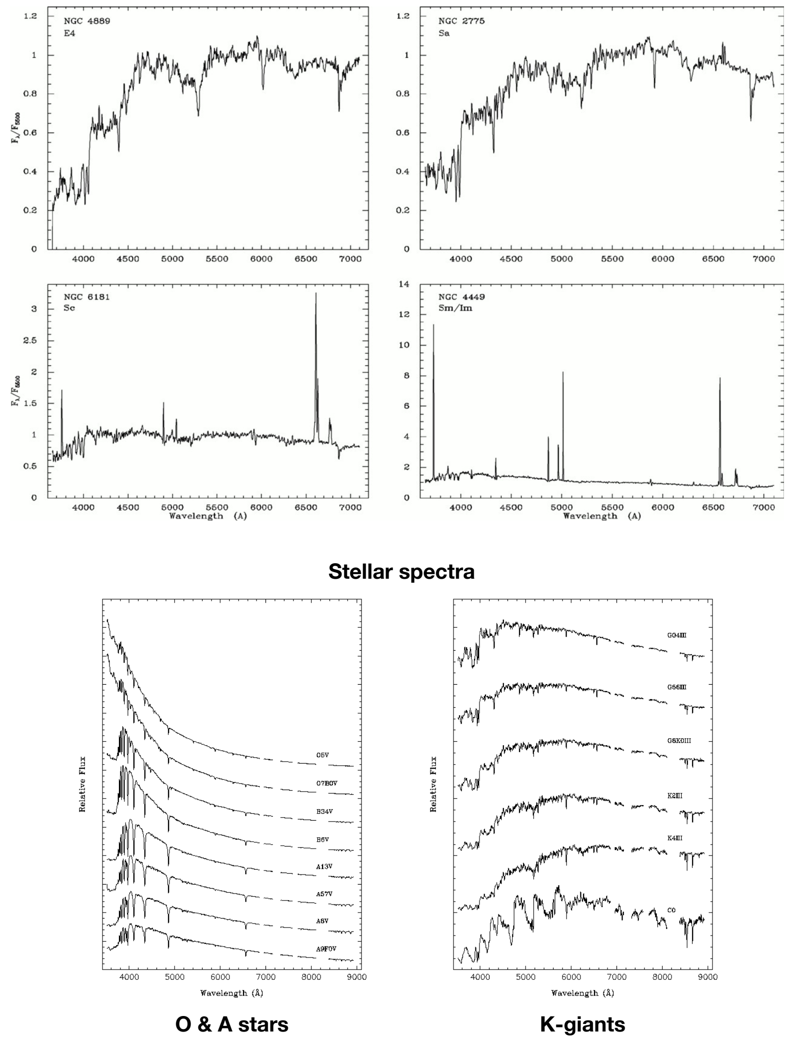

Figure 9.2: Top panels: Sample galaxy spectra from Kennicutt 1992. Galaxy type is indicated in each panel (top left: elliptical, top right and bottom left: spiral galaxies, and bottom right: irregular galaxy.). Notice in particular the deep absorption troughs (corresponding to light from K-giant stars) and the absence of blue continuum in the spectrum of the elliptical, and the striking emission lines (in particular Hα at $\lambda \approx 6800$Å) and the light from A-type stars in the spectra of spirals and irregulars. Bottom panels: sample spectra of O & A type stars (left) and of K-gaints (right).

By measuring a spectrum, they determine the galaxy's Doppler shift, v/c, and from that infer the distance to the galaxy. This allows us to make a 3 dimensional map of galaxies around us, see Fig. 9.1. Given the distance to the galaxy, we can infer its luminosity by measuring its flux. The galaxy's spectrum allows us to infer many other properties for the galaxy as well, for example characterise its stellar population and from that its stellar mass, $M_\star$, measure its star formation rate (basically by counting the combined luminosity of all its H II regions), and measure the mean metallicity of its stars. Compare the galaxy spectra with spectra from individual stars in Fig. 9.2. Future surveys such a *Euclid* aim to observe ~ 2 billion galaxies [2]. We discuss some striking correlations of galaxy properties next.

9.2 The galaxy luminosity and stellar mass function

Counting galaxies as a function of luminosity allows us to compute the *galaxy luminosity function* - the number density of galaxies as a function of luminosity. Combined with modelling the stellar population of these galaxies, we can compute the *galaxy stellar mass function* (GSMF) - the number density of galaxies as a function of stellar mass, see Fig. 9.3. Notice how the number density of low mass galaxies increases with decreasing mass as a power-law. At masses above $M_\star \sim 10^{10.5}\mathrm{M}_\odot$, the number density of galaxies drops very rapidly. To obtain the mass in galaxies of a given mass, we would need to multiply Φ by $M_\star$ - showing that galaxies with mass $\sim 10^{10.5}\mathrm{M}_\odot$ - i.e. with masses similar to the MW - dominate the mass budget[3].

The shape of the GSMF is well captured by the following fit due to[4] Schechter in 1976,

$$\Phi = \frac{dn}{d\log M_\star} = \Phi_0 \left(\frac{M_\star}{M_{\star,c}}\right)^\alpha \exp(-M_\star/M_{\star,c})\,, \tag{9.2}$$

[2]The number of galaxies in the observable Universe that are brighter than the Milky Way is estimated at ~ 200 billion.

[3]Or, with other words, most stars in the Universe today are in galaxies with mass similar to that of the MW.

[4]The Schechter fit was aimed at fitting the luminosity function, but it works equally well for the mass function.

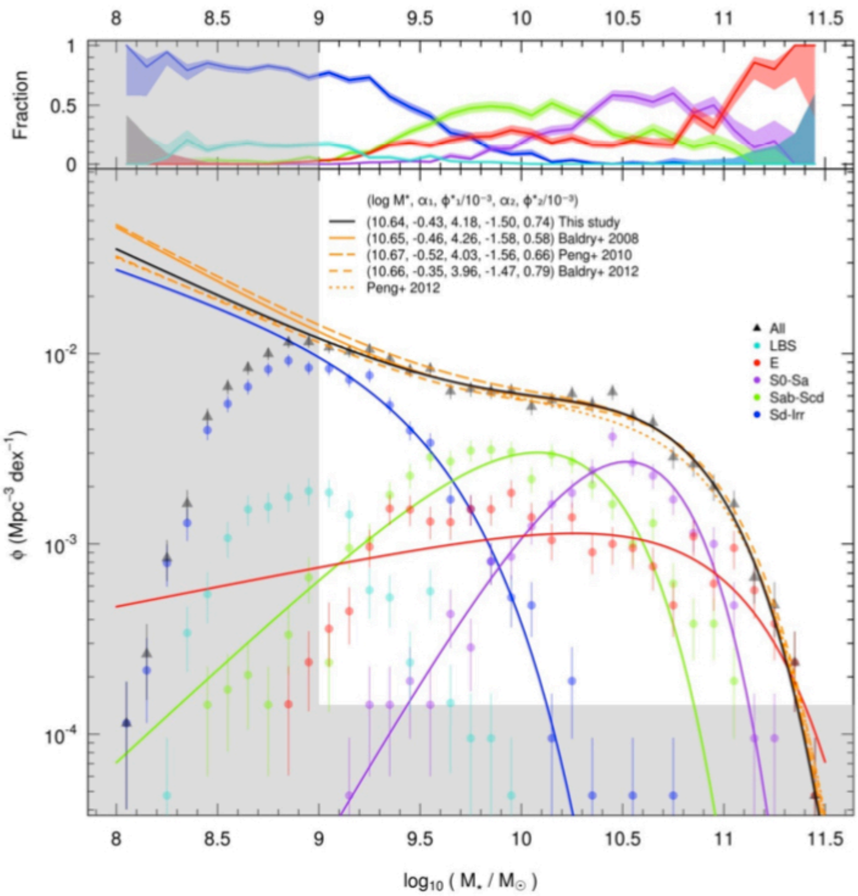

Figure 9.3: Black line: Φ - the number density of galaxies per decade in stellar mass. The different colours decompose this galaxy stellar mass function in terms of different galaxy types. Notice the power-law increase in the number of low-mass galaxies, and the rapid (exponential) drop at high masses. Figure taken from Kelvin 2015.

where n is the number density of galaxies. The function has three fitting parameters:

1. Φ_0 - a measure of the mean number density of galaxies
2. α - a measure of the low-mass slope
3. $M_{\star,c}$ - a characteristic mass above which Φ drops exponentially

9.3 The density-morphology relation

The *density-morphology relation* (Dressler 1980)is the observed correlation between the morphology of galaxies (elliptical versus spiral, or non-star forming versus star forming) and the local density of galaxies (the number of galaxies per unit volume): elliptical galaxies tend to live in dense regions, spirals avoid dense regions - see Fig. 9.4.

There are probably several processes that drive this correlation, but the following three processes probably play a major role:

1. Galaxies in a high density regions undergo frequent tidal interactions with other galaxies - these interactions may destroy fragile discs.
2. The hot gas present in high density regions (clusters) may strip star forming gas from a galaxy, shutting down its star formation.
3. A galaxy in the halo of a more massive galaxy (such as a galaxy in a cluster) - may no longer accrete gas from the intergalactic medium - it is being starved of fuel for star formation.

9.4 Galaxy scaling relations

A *galaxy scaling relation* is a relation between different properties of the galaxy, for example between galaxy mass ($M_\star$) and galaxy size ($R_\star$), or galaxy mass and stellar metallicity ($Z_\star$). The origin of these relations is not always well known. Here we discuss some really striking relations.

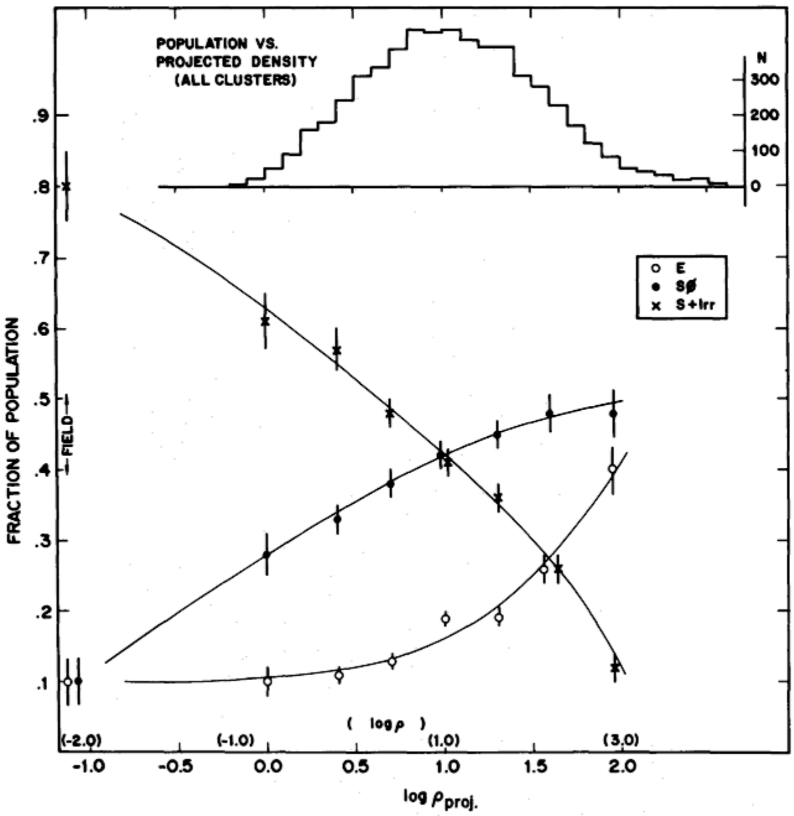

Figure 9.4: The fraction of elliptical (E), S0, and spiral + irregular (S+I) galaxies as function of the logarithm of the projected density($\rho_{\rm proj}$, in galaxies Mpc^{-2}). At low density (small $\rho_{\rm proj}$), most galaxies are of type S or Irr, whereas at high densities, most galaxies are S0 or E: there appears to be a relation between galaxy density and galaxy morphology. Figure taken from Dressler 1980.

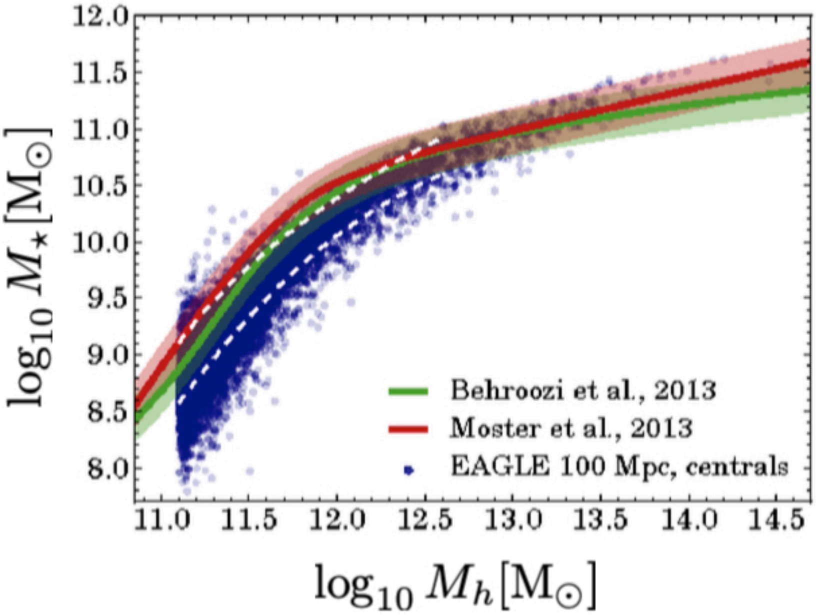

Figure 9.5: Relation between dark matter halo mass, M_h, and galaxy stellar mass, $M_\star$, from Matthee et al, 2017. Red and green drawn lines are relations inferred from the observed galaxy stellar mass function, purple points are galaxies from the EAGLE cosmological hydrodynamical simulation.

9.4.1 The stellar mass - halo mass relation

We discussed at length the evidence that galaxies form inside massive dark matter halos, with galaxy stellar mass much smaller than halo dark matter mass, $M_\star \ll M_h$. Directly measuring M_h is not easy, and the methods we described so far are not able to measure M_h for a large number of galaxies. *Gravitational lensing*, discussed in Chapter 11, can in principle be used to measure M_h. But an indirect method is called *abundance matching.* This method uses the fact that we can compute very accurately the number density of dark matter halos as a function of their mass. If we make the reasonable assumption that *more massive galaxies inhabit more massive halos*, than we can relate $M_\star$ to M_h using *abundance matching* - meaning that we identify galaxies of a given $M_\star$ to halos with a given M_h, provided they have the same number density.

So, combining the observed number density of galaxies with given $M_\star$ from §9.2 with the theoretical number density of *halos* with given M_h, yields the $M_\star - M_h$ relation of Fig. 9.5. Reproducing the observed relation is a stringent test of galaxy formation theories. In addition, some observed properties of galaxies can be understood in terms of this $M_\star - M_h$ relation.

9.4.2 The Tully-Fisher relation (CO p. 952-956)

Spiral galaxies have flat rotation curves: the circular velocity is independent of radius away from the centre. The *Tully-Fisher* relation is an observed relation between that constant circular speed, V_c, and the luminosity of the galaxy, L, of the form

$$L = L_0 \left(\frac{V_c}{V_0}\right)^4 . \tag{9.3}$$

The value of the exponent depends (weakly) on the choice of filter in which the luminosity is measured; the ratio L_0/V_0^4 is a normalisation constant[5].

We can determine the normalisation constant L_0/V_0^4 as follows. Suppose we determine the distances to (nearby) spiral galaxies using the period-luminosity relation of Cepheid variables. Knowing the distance, we can compute the luminosity of the galaxy once we've measured its flux. If we also measure the rotation speed, we have both L and V_c, and hence can compute

[5] Notice that we only need to know the ratio L_0/V_0^4, we never need to know them separately.

L_0/V_0^4:

$$\text{measure distance } d, \text{ flux } F, \text{ rotation speed } V_c \rightarrow L = (4\pi d^2)F \rightarrow \frac{L_0}{V_0^4} = \frac{L}{V_c^4}\,. \tag{9.4}$$

This turns the TF relation into a **standard candle**! Indeed, suppose we measure the flux and rotation speed of a distant spiral galaxy. Assuming it is on the TF relation, we can estimate L from measuring V_c, and hence determine d:

$$\text{measure } V_c \text{ and flux, } F \rightarrow L = L_0 \left(\frac{V_c}{V_0}\right)^4 \rightarrow d = \left(\frac{L}{4\pi F}\right)^{1/2}\,. \tag{9.5}$$

Is is not surprising that more massive galaxies are both brighter and have a higher value of circular speed. What is surprising is that observed galaxies follow the relation with so little scatter. Ultimately, the TF relation is closely related to the $M_\star - M_h$ relation, but there is more to it than that.

The usual textbook explanation for the origin of the TF relation goes as follows. The circular velocity depends on enclosed mass, $M(< R)$, as,

$$V_c^2 = \frac{\mathrm{G}\, M(< R)}{R}\,. \tag{9.6}$$

Now define the *mass-to-light ratio*, Υ, using the galaxy's luminosity, L,

$$\Upsilon \equiv \frac{M(< R)}{L}\,, \rightarrow V_c^2 = \mathrm{G}\Upsilon\,\frac{L}{R}\,. \tag{9.7}$$

The mass-to-light ratio Υ will depend on the number and types of stars in the galaxy (which determines L), and the amount of dark matter (which mainly determines M). Eliminate R, the radius of the galaxy, by using the intensity I - the luminosity per unit area,

$$I = \frac{L}{\pi\, R^2} \rightarrow R = \left(\frac{L}{\pi\, I}\right)^{1/2}\,. \tag{9.8}$$

Combining the above equations yields

$$L = \frac{1}{\mathrm{G}^2\,\Upsilon^2 \pi\, I}\, V_c^4\,. \tag{9.9}$$

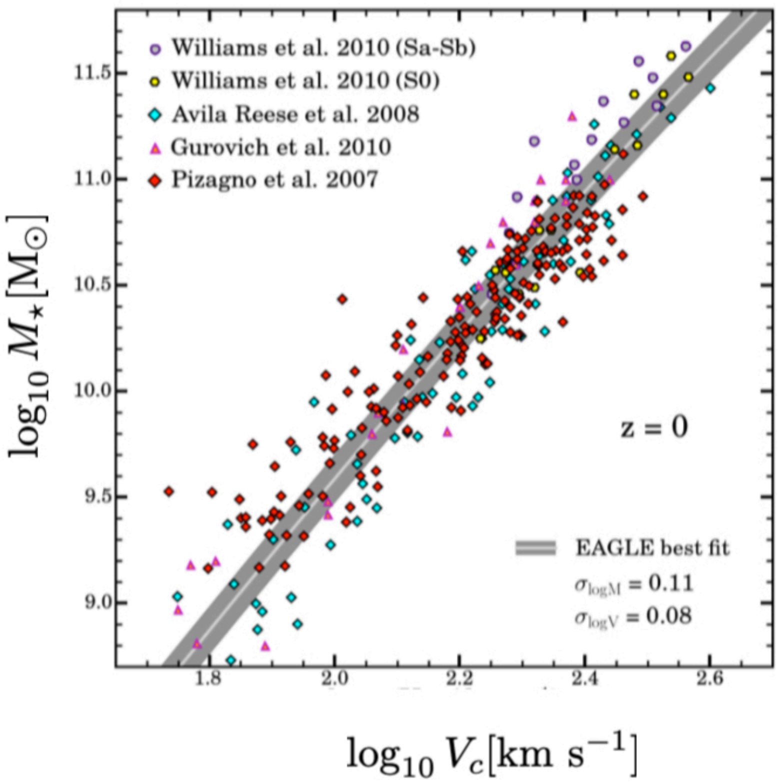

Figure 9.6: Observed 'Tully-Fisher' relation, which relates a galaxy's stellar mass, $M_\star$ (or galaxy luminosity), to circular speed, V_c. Coloured symbols are observed galaxies, grey band is a prediction from a simulation of galaxy formation. Figure from Ferrero et al, 2014.

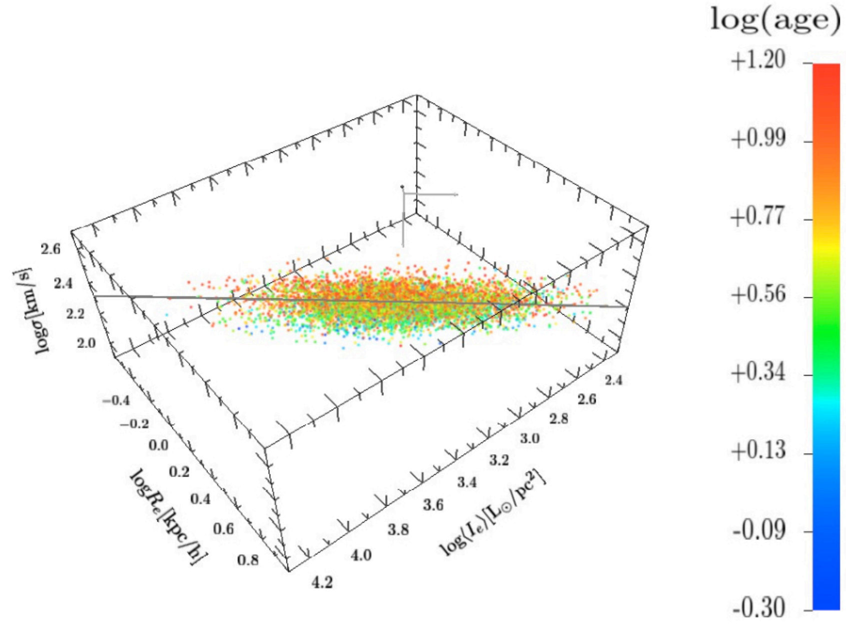

Figure 9.7: Observed correlation between the size of galaxies, R_e, their stellar velocity dispersion, σ, and their central intensity, I_e. Every dot is an elliptical galaxy, the colour of the dot is a measure of the galaxy's stellar age. Striking is that galaxies do not scatter randomly in this figure, but lie on a plane - the fundamental plane. Figure from Magoulas et al, 2012.

This relation has the form of the TF relation, provided $\Upsilon^2 I$ is (approximately) constant. But why would that be true? Since L is related to stars, but M (mostly) to dark matter, the product $\Upsilon^2 I$ can only be approximately constant if the stellar mass (which sets L) and the size of the galaxy (which sets R) are correlated with the mass of the halo (which sets M). Or in other words, provided the properties of the galaxy are closely related to those of its halo.

9.4.3 The Faber-Jackson relation and the fundamental plane in ellipticals (CO p. 987)

To find an equivalent scaling relation for ellipticals, we start from

$$\sigma^2 = \frac{5}{3}\frac{GM(< R)}{R}\,, \tag{9.10}$$

motivated by Eq. (8.4). Following the TF reasoning predicts

$$L \propto \sigma^4\,, \tag{9.11}$$

called the **Faber-Jackson** relation. Similar to the TF relation, it acts as a distance indicator: suppose we first determine the proportionality constant by measuring σ and L for nearby galaxies (requiring a distance measurement based on Cepheids, say, and measuring the galaxy's flux). We can now measure distances to distant ellipticals, by measuring σ from a galaxy spectrum, computing L by assuming the galaxies lies on the FJ relation, and then obtain distance from the computed value of L and the measured flux.

Elliptical galaxies do follow the FJ-relation approximately, but the *scatter* around the relation is significantly larger than the scatter around the TF relation. This motivated Djorgovski & Davis (1987) to introduce a dependence of Υ on L to improve the correlation (i.e. reduce the scatter). They assumed that

$$\Upsilon \propto L^\alpha\,, \tag{9.12}$$

for some value of the exponent α that is to be determined. Combining Eq. (9.10) with the relations from the previous section, yields the following relation between R, L and σ,

$$R \propto \sigma^{2/(1+2\alpha)}\, I^{-(1+\alpha)/(1+2\alpha)}\,. \tag{9.13}$$

Figure 9.7 plots elliptical galaxies in the 3D space of $R-\sigma-I$ - where they tend to lie close to a plane, of the form

$$R \propto \sigma^{1.25} I^{-0.89}\,. \tag{9.14}$$

This plane is called the **fundamental plane**. Equation (9.13) reduces to Eq. (9.14) for $\alpha \approx 0.24$. The origin of this relation is perhaps even less well understood than the origin of the TF relation. But the fact that ellipticals follow this fundamental relation can (again) be used to measure distances.

9.5 Tully-Fisher and Fundamental plane relations as standard candles

The importance of the TF and fundamental plane relations are twofold

- they show that the growth of a galaxy is closely related to that of its dark matter halo (although detailed understanding of the underlying physics of why the scatter is so small is currently lacking)
- both can be used to measure distances to galaxies

The reason they can be used to measure distances is because they relate parameters of the galaxy that are easy to measure and are *distance independent* , to properties of the galaxy that *do* depend on distance.

- For the TF relation: V_c is independent of distance, d, but when L is inferred from V_c using the TF relation, d follows by combining the measured flux and the computed luminosity.
- For the FP relation: σ and I can be measured independently of distance[6]. Combining these with the FP relation yields R. The distance d follows from measuring the angular extent of the galaxy.

[6]Recall that surface brightness is distance independent

9.6 Summary

After having studied this lecture, you should be able to

- explain what the galaxy luminosity and stellar mass functions are, and sketch them
- describe how the stellar-mass halo mass relation is determined
- describe the density-morphology relation for galaxies
- describe the Tully-Fisher relation, and explain how it is used as a distance indicator
- describe the Faber-Jackson and fundamental plane relations, and explain how they are used as a distance indicator