$\boxed{10}$

Classical : $n_i/g_i = e^{-\alpha} e^{-\beta \varepsilon_i}$

Fermionic : $n_i/g_i = \dfrac{1}{e^{\alpha} e^{\beta \varepsilon_i} + 1}$

Bosonic : $n_i/g_i = \dfrac{1}{e^{\alpha} e^{\beta \varepsilon_i} - 1}$.

Plot them.

with $\beta = 1/k_B T$ and $B = e^{\alpha}$ then

$$f(T) = \frac{1}{B e^{\beta \varepsilon_i} \begin{cases} +1 \\ +0 \\ -1 \end{cases}}$$

Fermi-Dirac (FD)

Maxwell-Boltzmann (MB)

Bose-Einstein (BE)

Example. What is the low density limit for the number of microstates in the FD and BE systems? The dilute limit is defined when $n_i \ll g_i$.

$$g = 100,\ n = 3 \qquad \frac{\cancel{1 \cdot 2 \cdot 3 \cdot 4 \cdots 96 \cdot 97} \cdot \overbrace{98 \cdot 99 \cdot 100}^{\sim 100^3}}{3!\ \cancel{97!}}$$

FD: Recall that $\Omega(\varepsilon_i) = \dfrac{g_i!}{n_i!\,(g_i - n_i)!}$

$$= \frac{\overbrace{\cancel{1 \cdot 2 \cdot 3 \cdots (g_i - n_i)}}^{(g_i - n_i)!}\,(g_i - n_i + 1) \cdots g_i}{n_i!\,\cancel{(g_i - n_i)!}} = \frac{\overbrace{(g_i - n_i + 1)(g_i - n_i + 2) \cdots g_i}^{n_i}}{n_i!}$$

$$\simeq \frac{g_i^{n_i}}{n_i!} \Rightarrow \Omega_{FD} \simeq \prod_i \frac{g_i^{n_i}}{n_i!}$$

BE : We had $\Omega(\varepsilon_i) = \dfrac{(n_i + g_i)!}{n_i!\, g_i!}$

$$= \frac{\overbrace{1.\,2.\,3 \ldots g_i}^{g_i!} \cdot \overbrace{(g_i+1)(g_i+2)\ldots(g_i+n_i)}^{\simeq\, g_i^{n_i}}}{n_i!\, \cancel{g_i!}} \simeq \frac{g_i^{n_i}}{n_i!}$$

$$\Rightarrow \Omega_{BE} \simeq \prod_i \frac{g_i^{n_i}}{n_i!}$$

We have $\Omega_{BE} \simeq \Omega_{FD} \rightarrow$ Classical we have $\prod_i \dfrac{g_i^{n_i}}{n_i!}$ except for

a factor of $N!$

Look up Gibb's Paradox.

Reading: Ch4 from book.

In large systems energy levels go from discrete to continuous, so instead of g_i being the degeneracy of individual levels we want to count the number of levels in a small range, i.e. $g(\varepsilon)$ this is levels between ε and $\varepsilon + d\varepsilon$.

Additionally in a solid we usually know the relation between energy and momentum $(p = \hbar k)$, e.g $\varepsilon = \hbar^2 k^2/2m = p^2/2m$.

<u>Example</u> Recall the 1-D infinite square well, we have

$$\psi_k(x) = A \sin kx \quad , \quad E = \hbar^2 k^2/2m$$

In solving the Schrödinger eqn. for this system we have

$$ka = n\pi, \quad k = n\pi/a, \quad n = 1, 2, 3, 4, \dots$$

Let's count the number of independent states between k and $k + \delta k$.
Let this be $n(k)$.

$$g(k)\delta k = \frac{(n(k+\delta k) - n(k))}{\delta k}\,\delta k$$

$$g(k)\delta k = \frac{dn(k)}{dk}\,\delta k.$$

So $n(k) = k/(\pi/a) = \frac{a}{\pi}k \Rightarrow g(k)\delta k = a/\pi\,\delta k.$

The number of states in k-space between k and $k + \delta k$

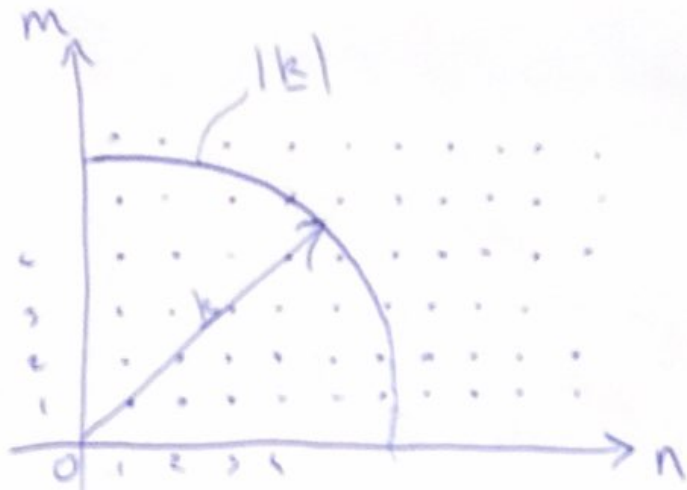
is $g(k)\,\delta k = \frac{a}{\pi}\delta k$, valid $k \gg \pi/a$. (i.e. when continuous).

<u>3D Box.</u> In the infinite 3D square well we found independent solutions $\sin(k_x x)\sin(k_y y)\sin(k_z z)$ with

$$k_x = \frac{n\pi}{a}, \quad k_y = \frac{m\pi}{a}, \quad k_z = \frac{l\pi}{a}$$

where $l, m, n = 1, 2, 3, 4 \ldots$ (independently).

<u>Count states:</u>



positive

$$n(k) = \frac{\text{Volume in 3D space (octant)}}{(\pi/a)^3}$$

$$= \frac{1}{8}\, \tfrac{4}{3}\pi k^3 \left(\frac{a}{\pi}\right)^3$$

So $$g(k) = \frac{dn}{dk} = \frac{a^3}{(2\pi)^3}\, 4\pi k^2$$

So in k-space the density of states is given by

$$g(k)\,\delta k = \frac{a^3}{(2\pi)^3}\, 4\pi k^2\, \delta k.$$

Note: spin not included. If spin degenerate multiply by 2.