Overview

12 December 2019 13:43

In the last lecture we introduced the **harmonic solution** and the **scalar approximation.**

In this lecture we will **cover:**

- Phase, Phasors and Wavefronts
- Plane Waves
- Spatial Frequency

<u>Phase, Phasors and Phase Fronts</u>

RECAP: In the SCALAR APPROXIMATION the HARMONIC SOLUTION is:

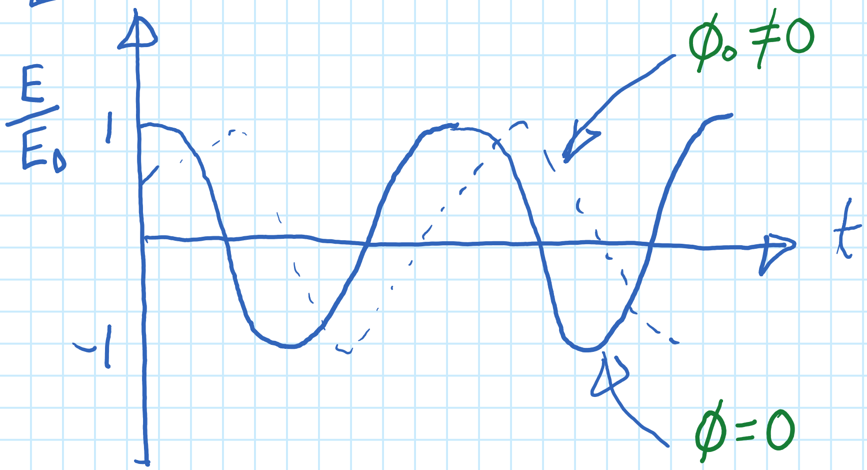
$$E = E_0 e^{i(\underline{k} \cdot \underline{r} - \omega t + \phi_0)}$$

What is phase?

12 December 2019 12:47

Lecture ① we drew

SCALAR NOW



(at $\underline{r} = 0$)

$\underline{k} \cdot \underline{r} - \omega t + \phi_0$ is the <u>PHASE</u> of the wave

ϕ_0 is the <u>relative phase</u> between the two waves

e.g. $\phi_0 = 0, 2\pi \ldots 2n\pi$ IN PHASE

$\phi_0 = \pi, 3\pi \ldots 2n+1\pi$ OUT OF PHASE

Spatial variation of phase

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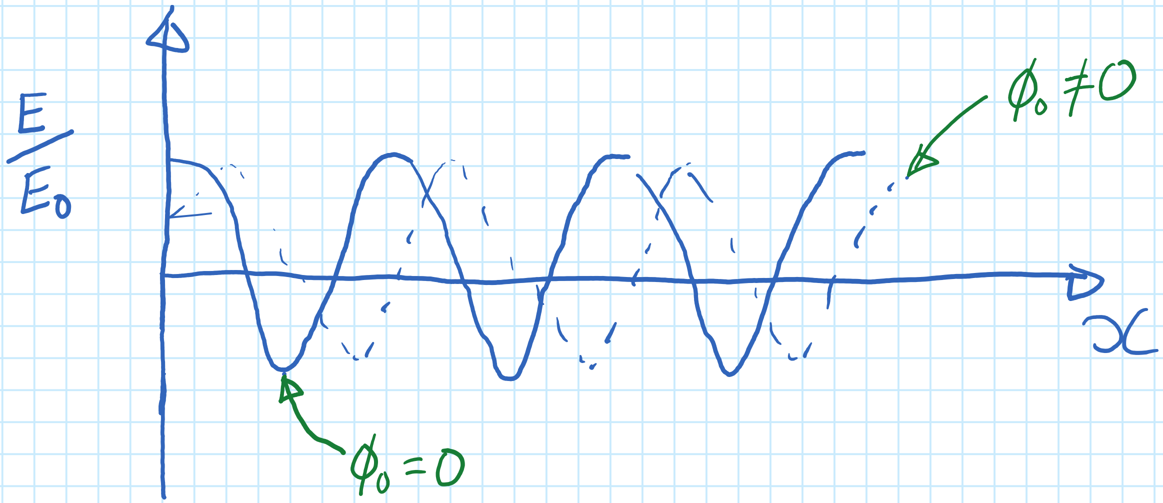
In optics we often fix t (or average over t) but want to know how phase varies with r

e.g. let $\underline{k} = \{k_x, 0, 0\}$ then sketch at $t=0$ becomes



Phase and Complex Notation

18 January 2020 15:43

Optics L2L 1.11

- Make a two-component vector from amplitude E_0
  phase ϕ
- Can be represented using a two-component number: COMPLEX NUMBER

SEE "COMPLEX NUMBERS IN OPTICS notes in DUO"
MATHS NOTES & BOOKS
PROBLEM SHEET WP1 & WORKSHOP

Complex notation is a powerful mathematical shorthand

Phase in Complex Notation: Phasors

12 December 2019 13:17

Let's go: $\cos(z) = \dfrac{e^{iz} + e^{-iz}}{2}$ EULER'S FORMULA

z any complex number

$\therefore \cos(\phi) = \frac{1}{2}\left(e^{i\phi} + e^{-i\phi}\right)$ $\phi = \underline{k}\cdot\underline{r} - \omega t + \phi_0$

then $E = E_0 \cos(\phi) = E_+ + E_-$

where $E_+ = \frac{1}{2} E_0 e^{i\phi}$ $E_- = \frac{1}{2} E_0 e^{-i\phi}$

Phase in complex notation: Phasors

12 December 2019 13:21

From now on we use a mathematical shorthand:

$$\boxed{E \longrightarrow E_+ \quad \text{i.e. we write } E = E_0 e^{i\phi} \qquad E = E_0 e^{i(k \cdot r - \omega t + \phi_0)}}$$

What about the $\frac{1}{2}$?

We simply include it in a re-defined E_0

Intensity $\boxed{I = \frac{1}{2} c \varepsilon_0 |E^2|}$

modulus squared of the complex field

Why have you done that?

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The reason we do this is to make the maths easier (no really!)

Phase shifts become simple multiplications

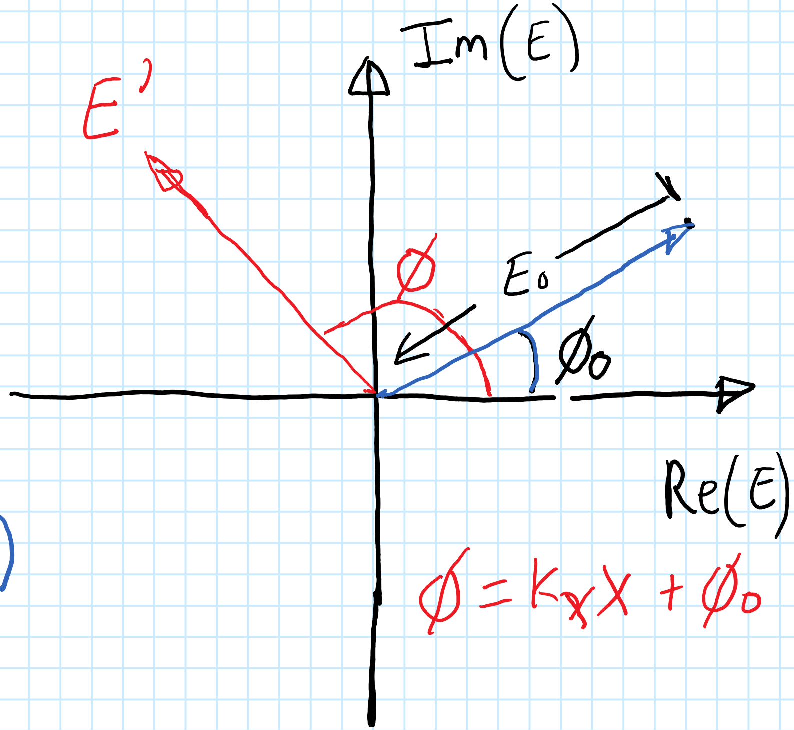
example: consider $\underline{k} = \{k_x, 0, 0\}$

at $t=0$ and $x=0$, field is

$E = E_0 e^{i\phi_0}$

what is field @ $x = x'$?

$E' = E e^{ik_x x} = E_0 e^{i(k_x x' + \phi_0)}$



in old notation we could not easily write E' in terms of E as we have to change argument

i.e $\cos(\phi_0) \longrightarrow \cos(k_x x - \phi_0)$

see my extra notes on DUO

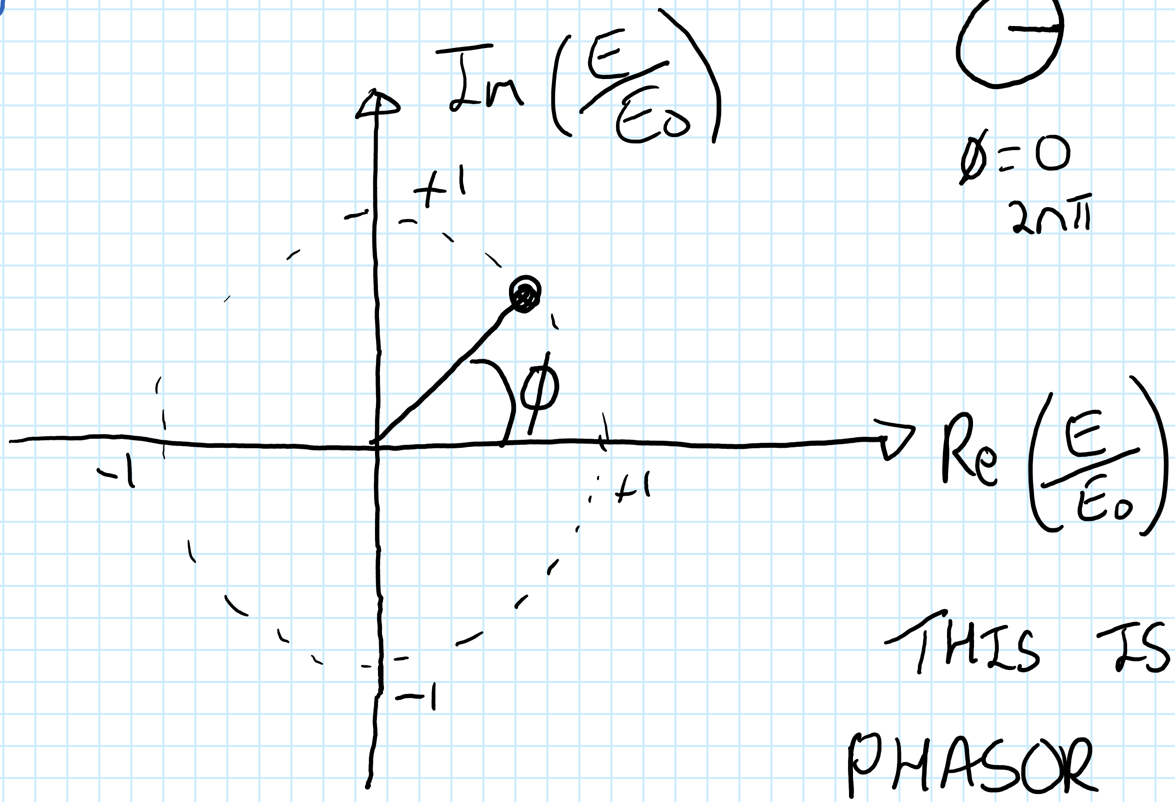
Phasors

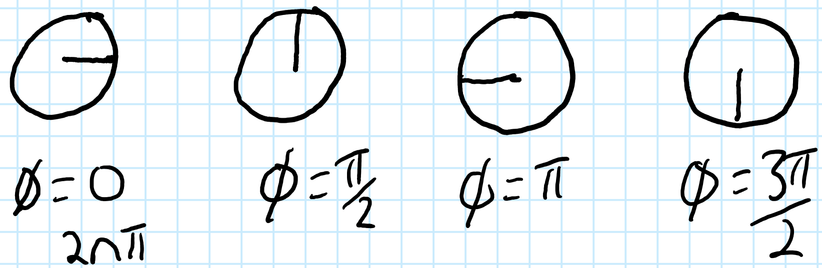
12 December 2019 14:00

OPTICS L2L 1.8

In a lossless medium, E_0 doesn't vary

$\therefore$ we can normalise by E_0 and represent the phase by a <u>UNIT</u> <u>VECTOR</u>





THIS IS CALLED A PHASOR DIAGRAM

Wavefronts, plane and spherical waves

F2F 2.2 & 2.3

12 December 2019 14:09

A wavefront is a contour of constant phase $\phi = \underline{k} \cdot \underline{r} - \omega t + \phi_0 = \text{constant}$

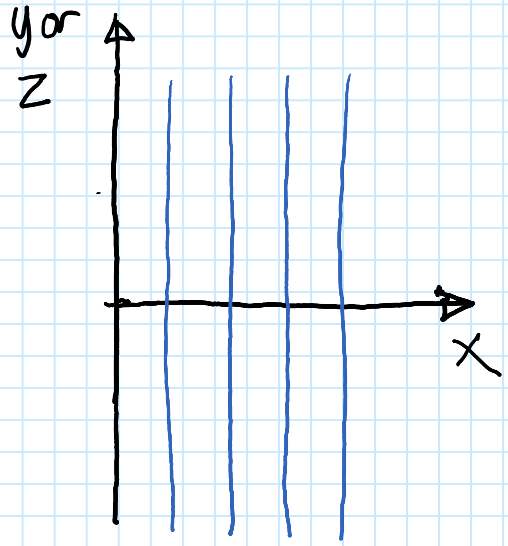
Example: Plane Waves

e.g. let $\underline{k} = \{k_x, 0, 0\}$

$$E = E_0 e^{i(k_x x - \omega t)}$$

ϕ is independent of y & z

$\therefore$ wavefronts are planes in $\{y, z\}$



This is an example of a SCALAR PLANE WAVE

Scalar Plane Waves

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General form: $E = E_0 e^{i(k_x x + k_y y + k_z z)}$

PROPERTIES:

Planar wavefront normal to propagation direction k

Solution of Maxwell's equations with TRANSVERSE PROPERTY: $\underline{k} \cdot \underline{E} = 0$

BUT infinite extent is unphysical

$\hookrightarrow$ STILL USEFUL

Aside: Spatial Frequency

Definition: Spatial frequency is the number of waves per unit length along a given direction

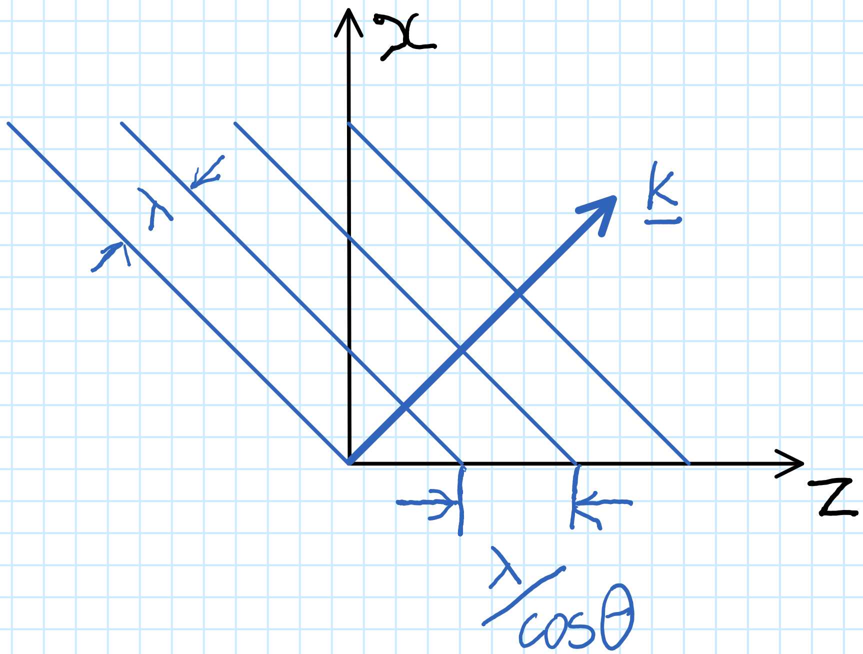
Spatial frequency

12 December 2019 14:39

F2F 1.9

Definition: Spatial frequency is the number of waves per unit length along a given direction

Example: PLANE WAVE propagates at angle θ wrt z axis in xz plane



Along z spacing is $\dfrac{\lambda}{\cos\theta}$

Along x spacing is $\lambda/\sin\theta$

SPATIAL FREQUENCY along z $\quad u_z = \dfrac{\cos\theta}{\lambda}$

" along x $\quad u_x = \dfrac{\sin\theta}{\lambda}$

DIMENSION OF u IS LENGTH^{-1}

Spatial frequency

20 January 2020 16:57

Spatial frequency along a direction is related to the corresponding component of $\underline{K}$

e.g. $K_x = K \sin\theta = \frac{2\pi}{\lambda} u$