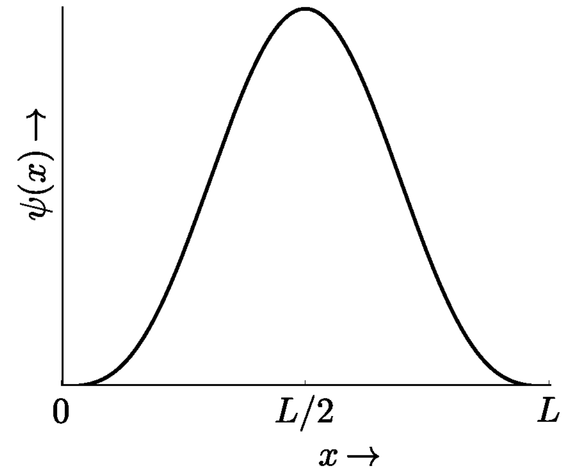
QUANTUM MECHANICS 2 - WORKSHOP 2

1. A particle confined to the interval $0 < x < L$ by an infinite square well has the wavefunction $\psi(x)$ shown in the figure.



This wavefunction can be expressed by the expansion

$$\psi(x) = \sum_{n=1}^{\infty} c_n \, \phi_n(x),$$

where $\phi_n(x) = \sqrt{\dfrac{2}{L}} \sin(n\pi x/L)$ are the usual eigenfunctions of the infinite square well. Which of the coefficients c_1, c_2, c_3 and c_4 are zero? Which has the largest magnitude? Which are positive and which are negative? Give reasons for your answers.

2. Consider the wavefunction

$$\Psi(x, t = 0) = \begin{cases} Ax(L - x) & \text{for } 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

in an infinite square well potential, where $A = \sqrt{30/L^5}$. Eigenfunctions of this potential are $\psi_n(x) = N \sin n\pi x/L$ with $N = \sqrt{2/L}$ and corresponding to energy $E_n = n^2\pi^2\hbar^2/(2mL^2) = n^2 E_1$.

i) The wavefunction can be expanded in terms of the eigenfunctions as $\Psi = \sum_n c_n \psi_n$, where $c_n = \int \psi_n^* \Psi \, dx$. Calculate a general expression for the c_n.

ii) Write down the first 3 terms explicitly, and hence show that the general form for even and odd n is $c_n = 0$ for even n, $c_n = 8\sqrt{15}/(n^3\pi^3)$ for odd.

iii) What is the probability (3 sig figs) that the system is measured to be in the ground state?

iv) Write down an infinite sum for $\langle E\rangle$ in terms of E_1. Evaluate the sum given that $\sum_{\text{odd}\,n} 1/n^4 = \pi^4/96$. Why is this bigger than E_1?

v) Instead calculate the expectation value using $\langle E\rangle = \int \Psi^* H \Psi\, dx$ and show it is the same value.

(Hint: $H = -(\hbar^2/2m)d^2/dx^2$ as $V = 0$ in the well.)

Useful Integrals (c is an integration constant):

$$\int x \sin(ax)\, dx = \frac{\sin(ax) - ax\cos(ax)}{a^2} + c$$

$$\int x^2 \sin(ax)\, dx = \frac{(2 - a^2x^2)\cos(ax) + 2ax\sin(ax)}{a^3} + c$$