Mathematical Methods in Physics

Weekly Problems 5

5.1

Show the following properties of the Fourier transform:

a) $\mathcal{F}[f(t+a)](\omega) = e^{ia\omega} \hat{f}\,(\omega)\,,$

[*Hint: Perform a change of variable*]

b) $\mathcal{F}[e^{at}\, f(t)](\omega) = \hat{f}\,(\omega + ia)\,,$

where a is a constant.

5.2

Compute the Fourier transform of the function

$$f(t) = \frac{\delta(a-t) + \delta(a+t)}{1+t^2},$$

where $a > 0$.

5.3

Consider the function

$$f(t) = \begin{cases} (a+t)/a^2 & \text{for } -a \leq t < 0 \\ (a-t)/a^2 & \text{for } 0 \leq t < a \\ 0 & \text{for } |t| > a \end{cases}$$

for a positive constant a. Compute the Fourier transform $\hat{f}(\omega)$ of $f(t)$. Then, consider the limit $a \to 0$ and show that, in this limit, $\hat{f}(\omega)$ is equal to $1/\sqrt{2\pi}$, which is the Fourier transform of a Dirac δ-function.

[*Hint: Bear in mind that the first two terms of the Taylor expansion of the cosine function when the argument, x, tends to zero are:* $\cos x \simeq 1 - x^2/2 + \ldots$]