Relativistic Electrodynamics, Workshop 4

Applications of Relativity

1. Show that 4-momentum conservation prohibits an on-shell photon creating an electron-positron pair. This is the example on page 20 in the notes.

2. Consider the scattering process where the collision of a proton $p(p_1)$ with 4-momentum p_1 and a neutron $n(p_2)$ with 4-momentum p_2 produces two protons, an antiproton $\bar{p}$ and a neutron,

$$p(p_1)\ n(p_2)\ \rightarrow\ p(q_1)\ n(q_2)\ p(q_3)\ \bar{p}(q_4)\ . \tag{1}$$

   We ignore the difference in the rest mass of the neutron, m, and the rest mass of the proton (and antiproton). Let $\underline{v}_{\rm cm}$ be the velocity of $p(p_1)$ and $n(p_2)$ in the centre-of-mass frame, and $\underline{v}_{\rm L}$ be the velocity of $p(p_1)$ in the rest frame of $n(p_2)$.

   (a) Express the 4-momenta of the incoming proton and neutron in the centre-of-mass frame in terms of m and $\underline{v}_{\rm cm}$. What are the corresponding momenta in the rest-frame of $n(p_2)$, expressed in terms of $\underline{v}_{\rm L}$?

   (b) Use the invariance of the Minkowski scalar product to find a relation between $\gamma(v_{\rm cm})$ and $\gamma(v_{\rm L})$.

   (c) Show that the 4-momenta satisfy the relation

$$p_1\cdot p_2 - p_1\cdot q_1 - p_2\cdot q_1 = q_2\cdot q_3 + q_2\cdot q_4 + q_3\cdot q_4\ . \tag{2}$$

   (d) Find the smallest possible value for $\gamma(v_{\rm cm})$ that allows for this process to take place. What is the corresponding value for $\gamma(v_{\rm L})$?