NPP2 Introduction

Nov 2019

Introduction

Type	Physical Quantity	SI	NU	conversion
angular momentum	$\hbar$	Js	1	$\hbar = 1.054 \cdot 10^{-34}$ Js
speed	c	m/s	1	$c = 299792458$ m/s
energy		J	MeV, GeV	1MeV $= 1.602 \cdot 10^{-13}$ J
momentum		kg m/s	MeV/c, GeV/c	
mass		kg	MeV/c^2, GeV/c^2	
time		s	$\hbar$/MeV, $\hbar$/GeV	
length		m	$c\hbar$/MeV, $c\hbar$/GeV	

"Natural" units

for $\hbar = c = 1$ follows $[\text{mass}] = [\text{energy}] = [1/\text{length}] = [1/\text{time}]$

$\text{GeV} = 10^9\,\text{eV}$

For example the electron mass is given by $m_e = 0.511\,\text{MeV}$

If we need to convert between masses and lengths we use $\hbar c = 0.19733\,\text{fm}\,\text{GeV}$

For example the length scale associated with the electron mass is

$$\frac{\hbar c}{m_e} \approx \frac{0.2\,\text{fm}\,\text{GeV}}{0.5\,\text{MeV}} = 0.4 \cdot 10^{-12}\,\text{m}$$

$\text{fm} = 10^{-15}\,\text{m}$

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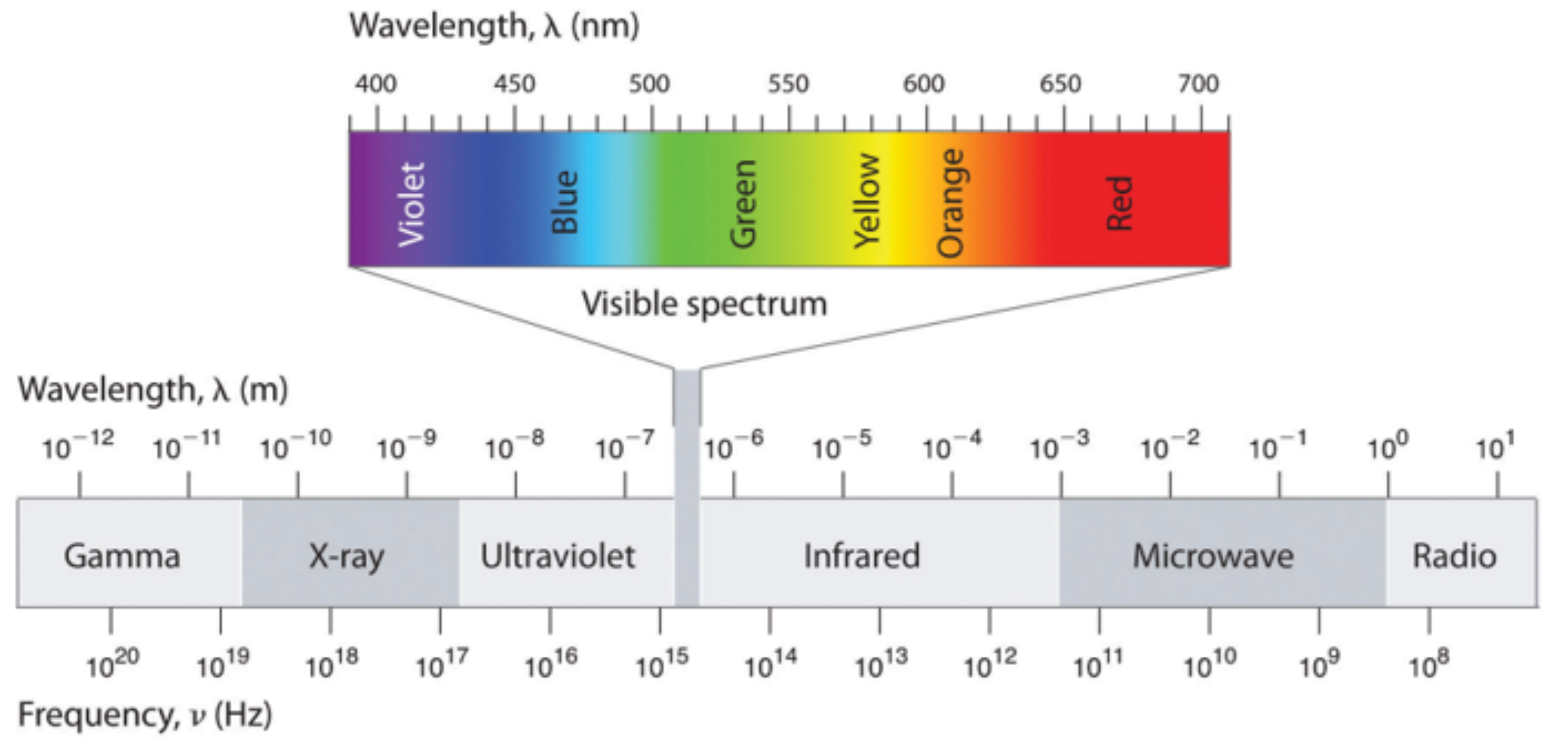
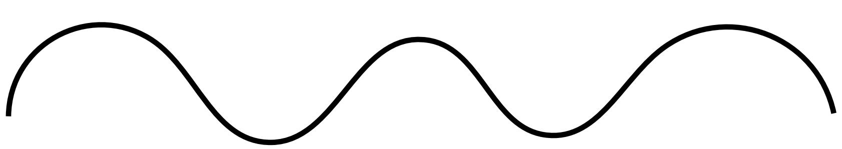
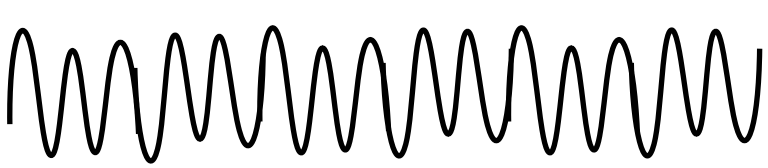


Electromagnetic spectrum

Microscopy is limited by the resolution determined but he wavelength



optical $$r_{\min} \approx \lambda \approx 0.4 \cdot 10^{-6}\,\mathrm{m} = \mu\mathrm{m}$$



electron $$r_{\min} \approx \lambda = \frac{h}{p}$$ ← de-Broglie wavelength

$$\xrightarrow{\text{non-rel.}} \frac{\hbar c}{m_e v} \lesssim 0.4 \times 10^{-12}\,\mathrm{m} = 0.4\mathrm{pm}$$

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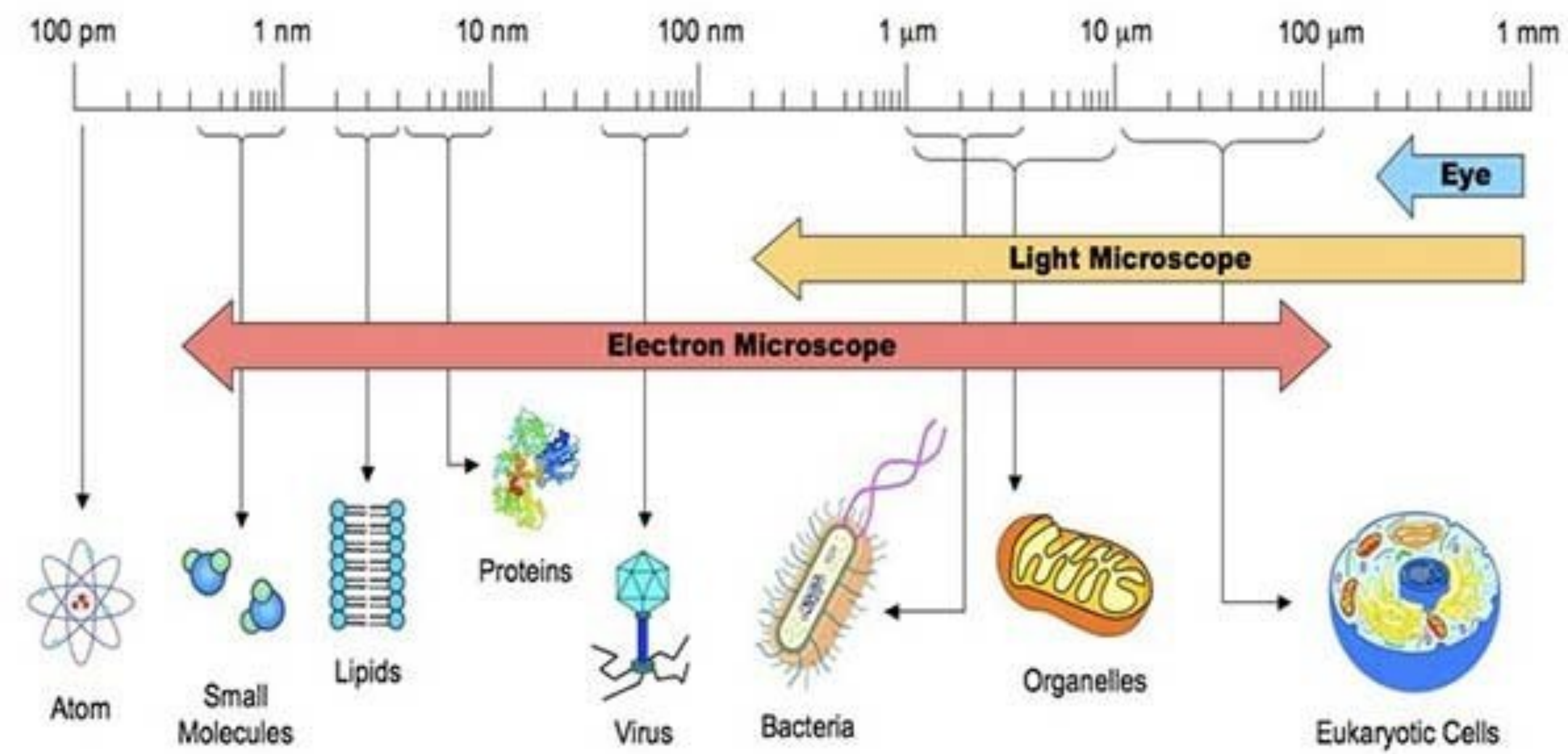
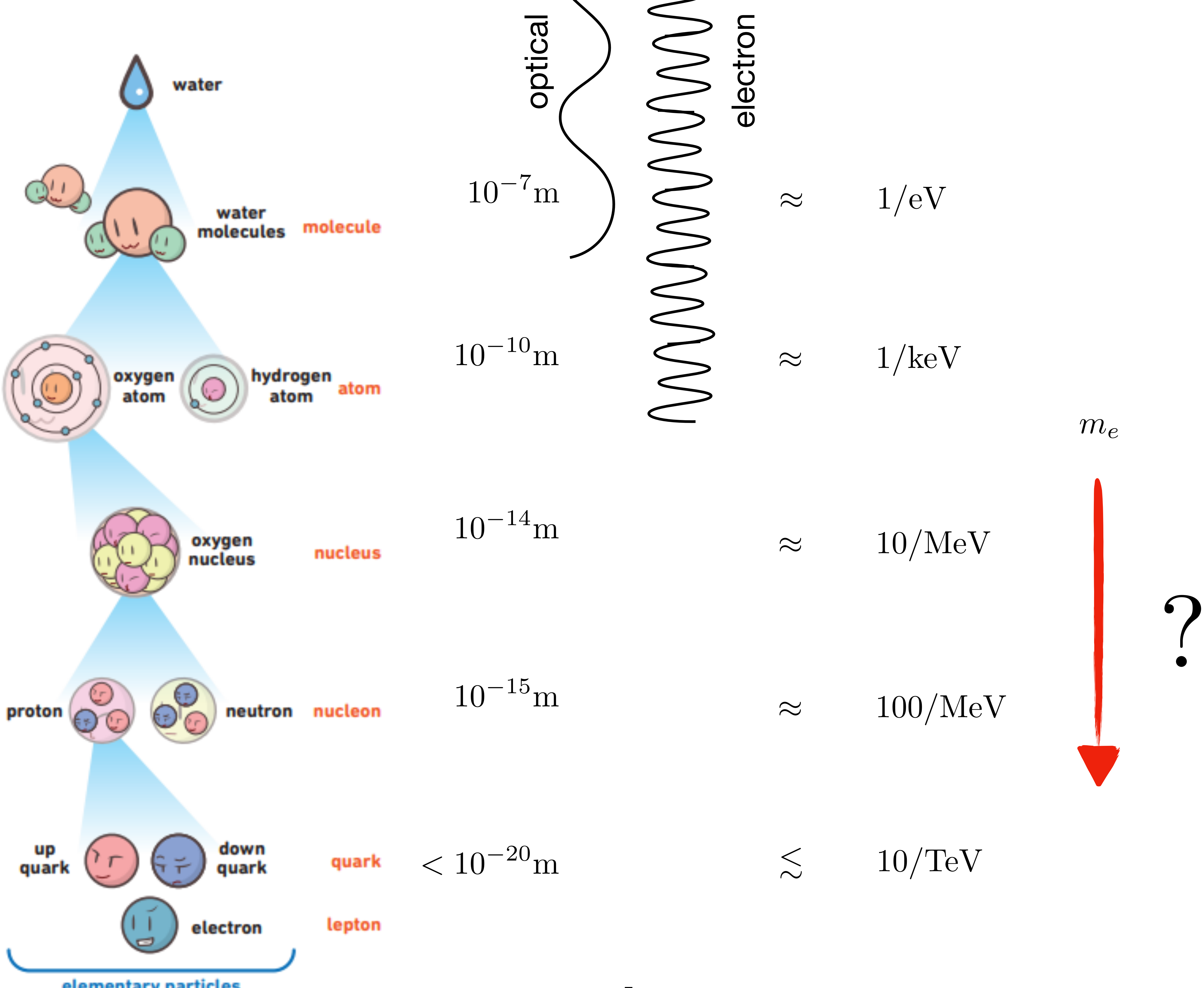


Single atoms can be resolved with an electron microscope

What about smaller scales ?

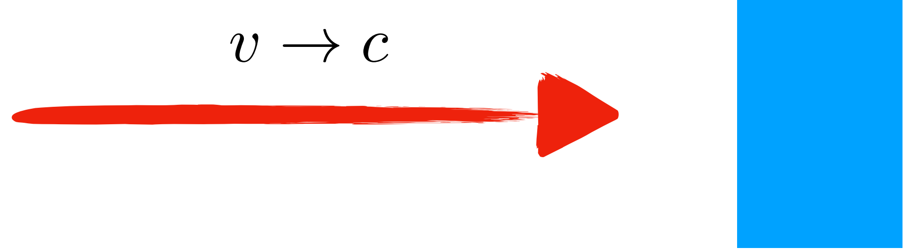


water
water molecules
molecule
oxygen atom
hydrogen atom
atom
oxygen nucleus
nucleus
proton
neutron
nucleon
up quark
down quark
quark
electron
lepton
elementary particles
optical
electron
$10^{-7}\mathrm{m} \approx 1/\mathrm{eV}$
$10^{-10}\mathrm{m} \approx 1/\mathrm{keV}$
$10^{-14}\mathrm{m} \approx 10/\mathrm{MeV}$
$10^{-15}\mathrm{m} \approx 100/\mathrm{MeV}$
$< 10^{-20}\mathrm{m} \lesssim 10/\mathrm{TeV}$
m_e
?

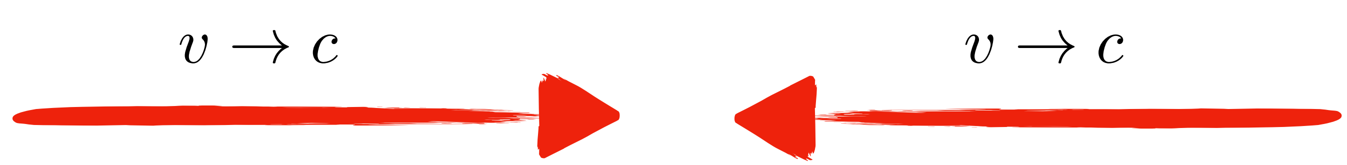
Introduction

To go to even smaller scales we need either very energetic beams of particles, since

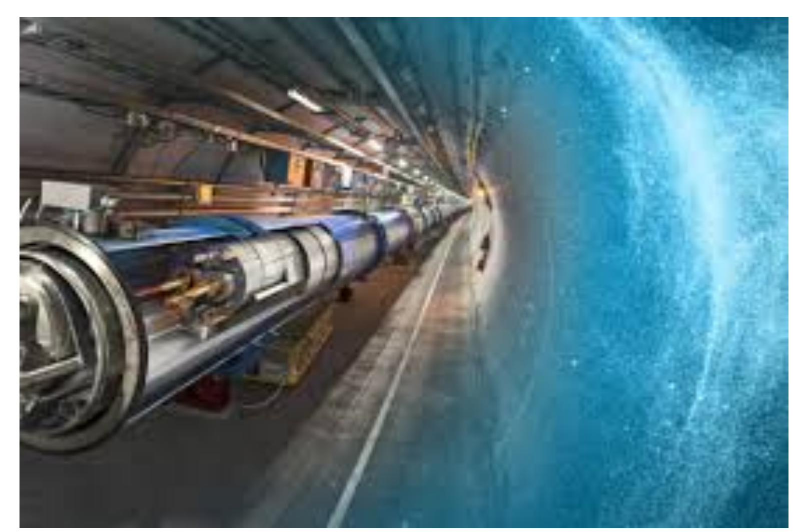
$$\lambda = \frac{\hbar}{p}$$



Or, even better,high-energy head-on-head collisions



The best “microscopes” we have are colliders.



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1. Units
2. Kinematics
3. Discrete symmetries

Nuclear Physics

1. Nuclear masses and binding energies
2. Nuclear stability
3. Shell model

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1. Units
2. Kinematics
3. Discrete symmetries

Repetition of natural units, 4-momentum conservation and symmetries such as Parity and Charge conjugation

Nuclear Physics

1. Nuclear masses and binding energies
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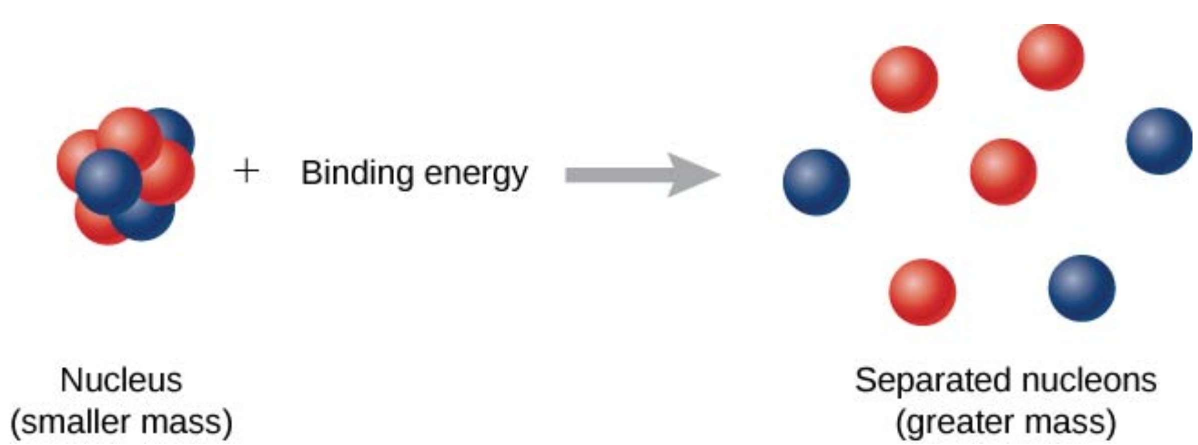
Nuclear Physics

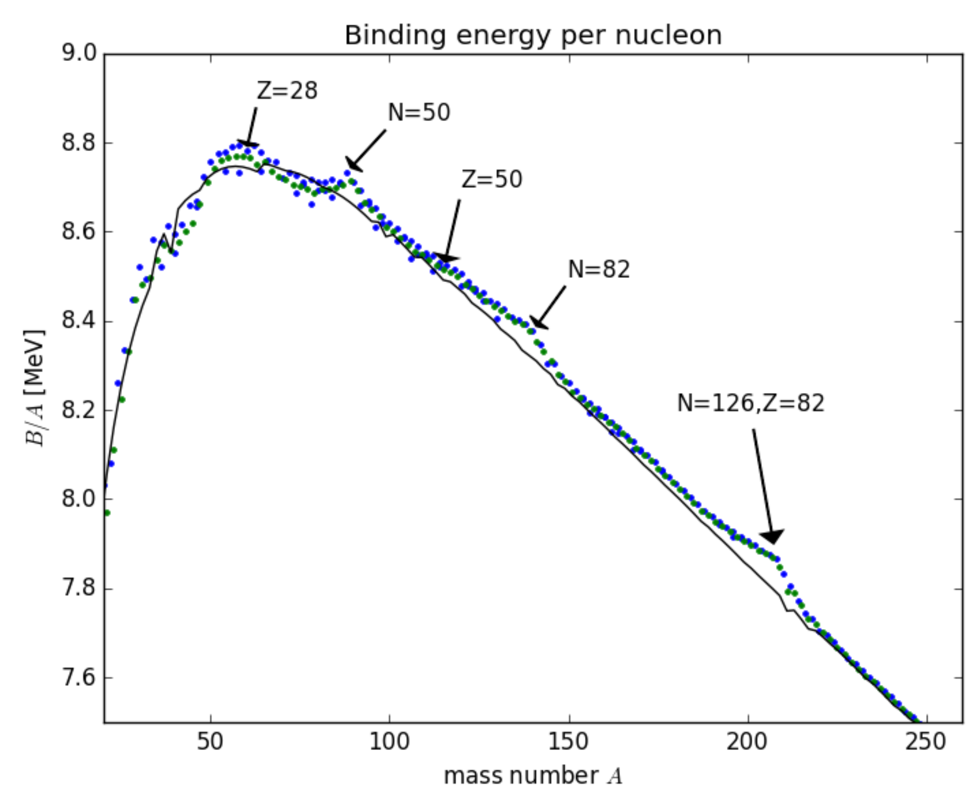
Nuclear Physics 1-3 Nuclear Masses

How can we calculate the nuclear binding energy?

The “liquid drop model”

$$M(A,Z) = \underbrace{N M_n + Z M_p + Z\, m_e}_{\text{Masses}} - \underbrace{a_V A}_{\text{Volume}} + \underbrace{a_s A^{2/3}}_{\text{Area}} + \underbrace{a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{4A} + \frac{\delta}{A^{1/2}}}_{\text{Coulomb, Asymmetry and Pairing}}$$



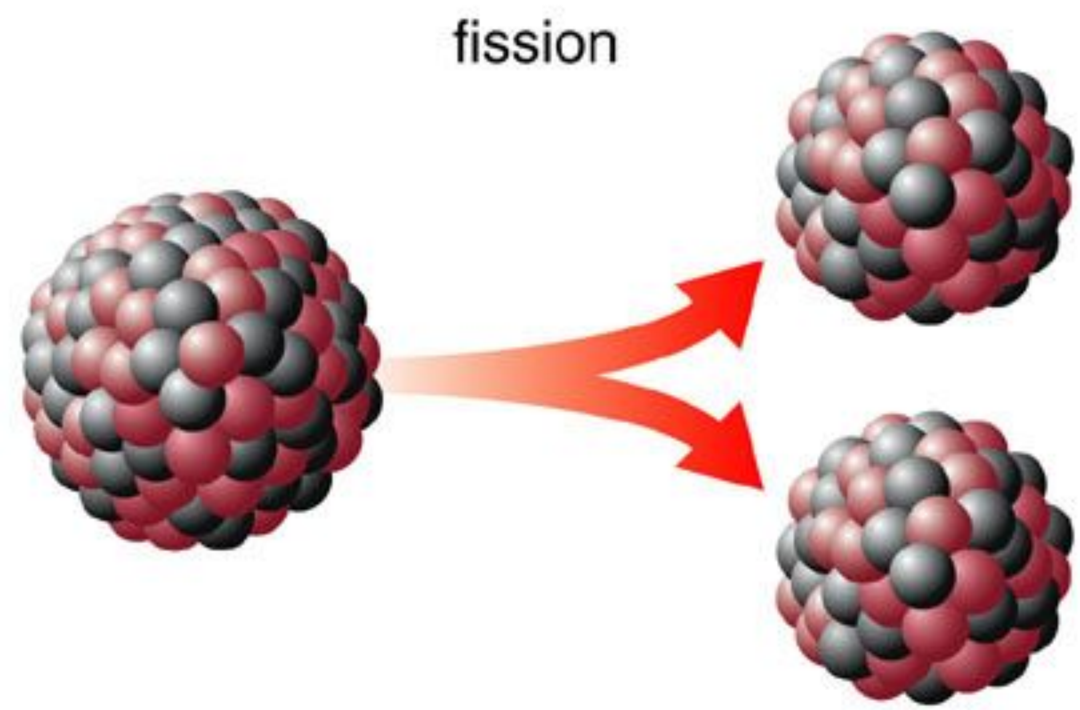


Nuclear Physics 1-3 Nuclear Stability

Nuclear Forces are short range.

Not all nuclei are stable. Some decay with a probability

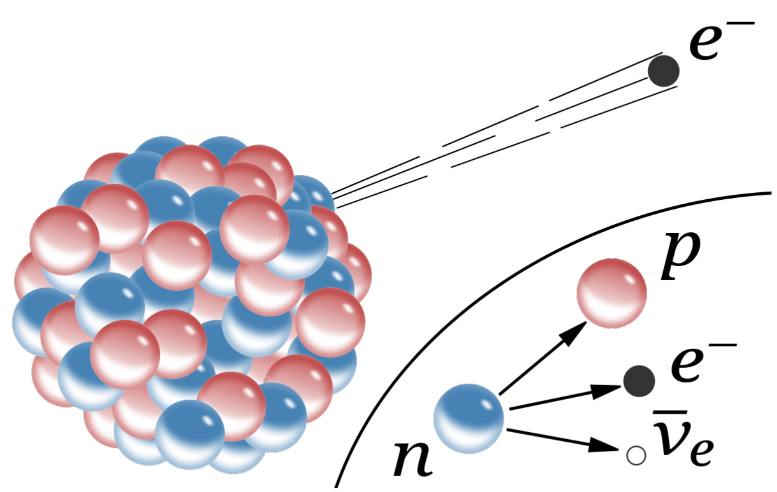
$$\frac{dN}{dt} = -\lambda N \;, \qquad \Rightarrow \qquad N = N_0 e^{-\lambda t}$$

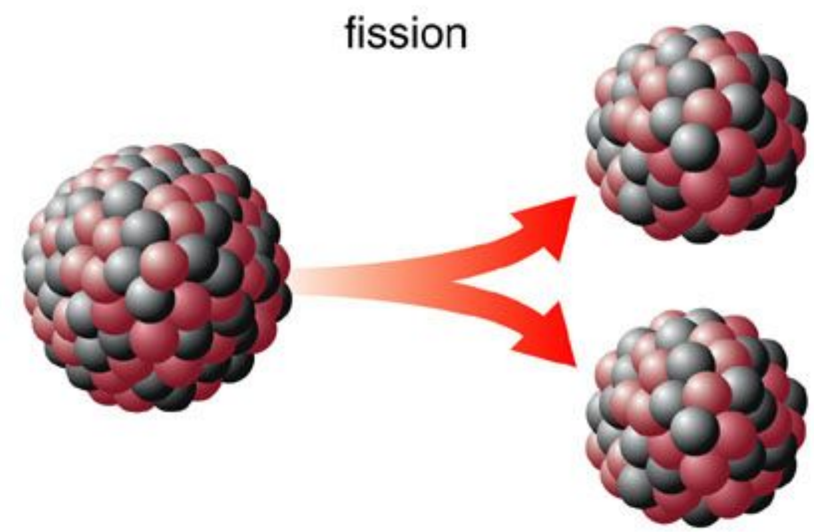


Nuclear Physics 1-3 Nuclear Stability

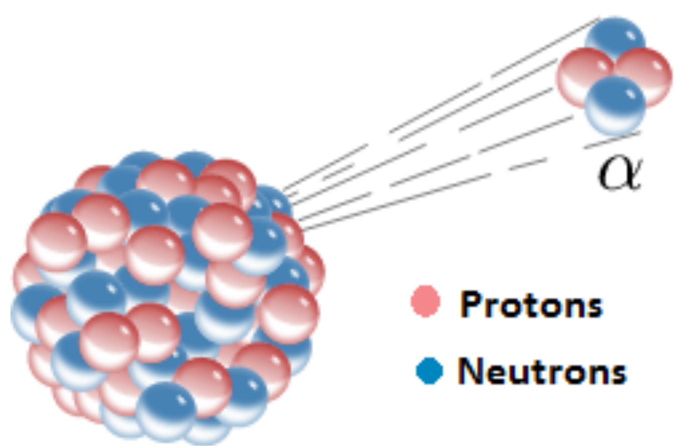
We distinguish three types of Nuclear decays

β decay





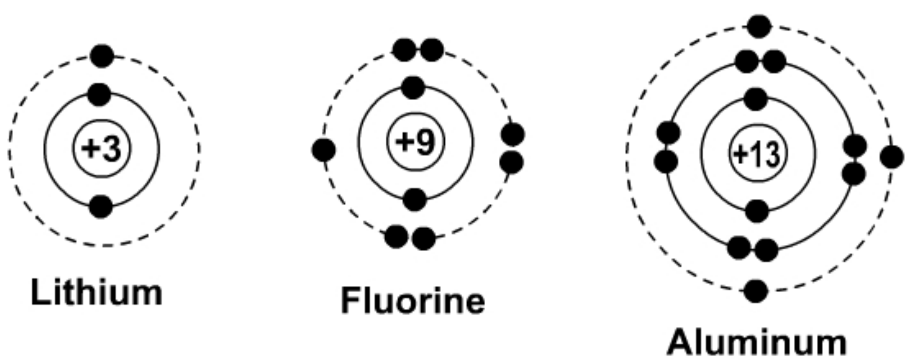
α decay

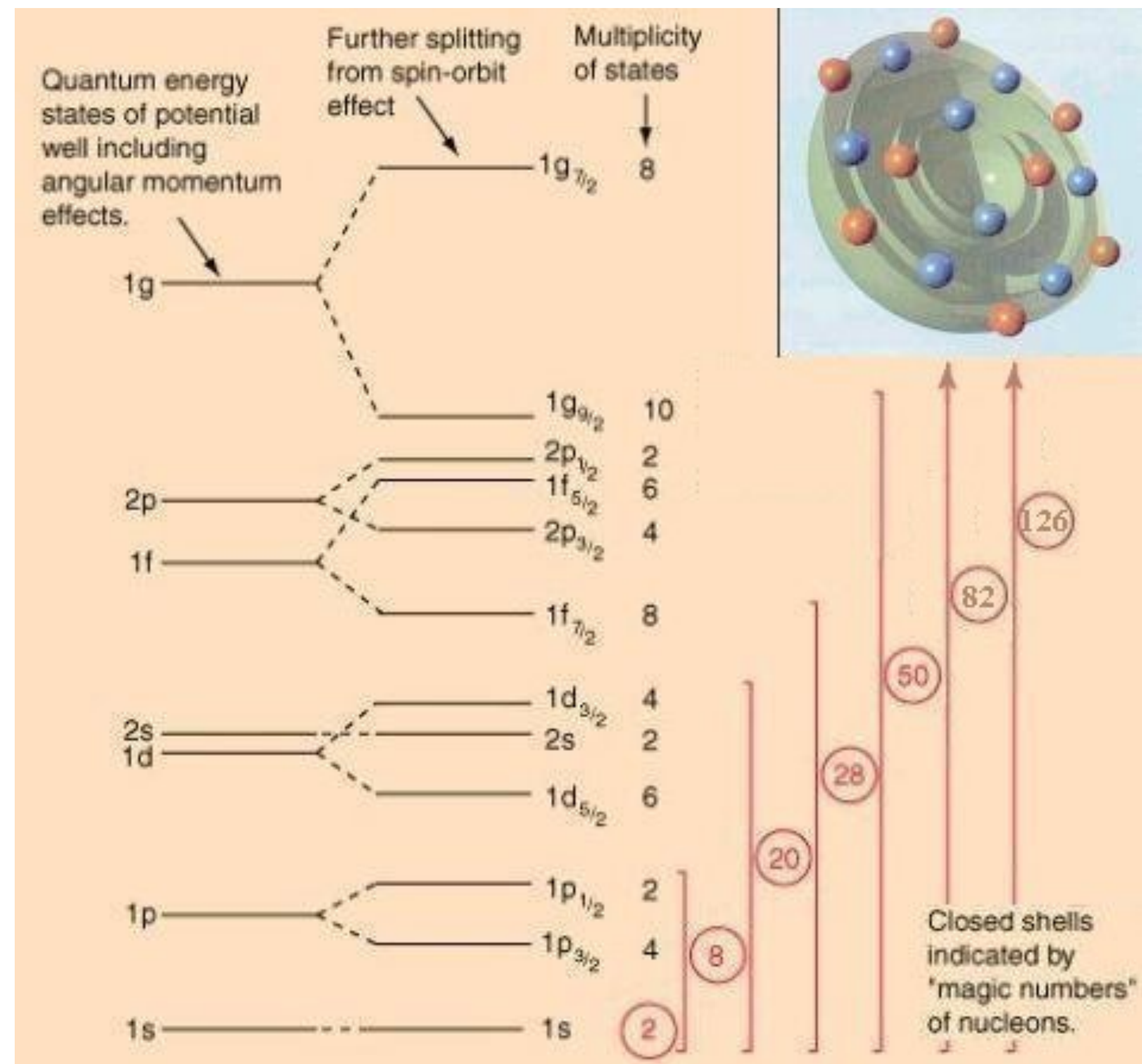


Nuclear Physics 1-3 The Shell model

We discuss how the stability of certain types of nuclei can be understood with the shell model

I analogy with the shell model for atoms

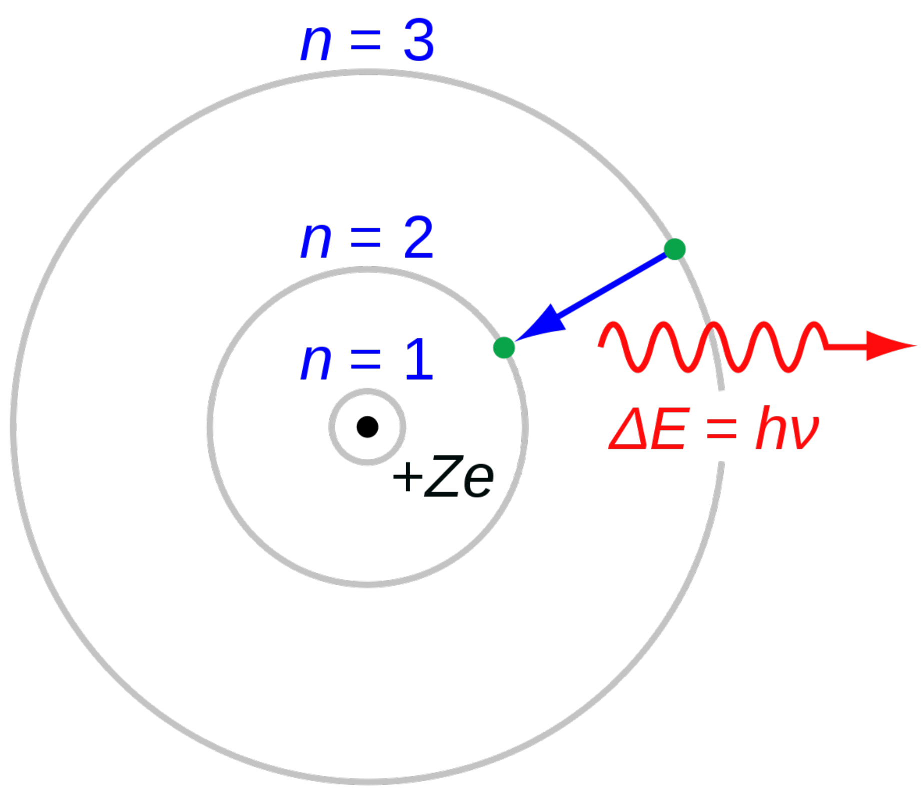


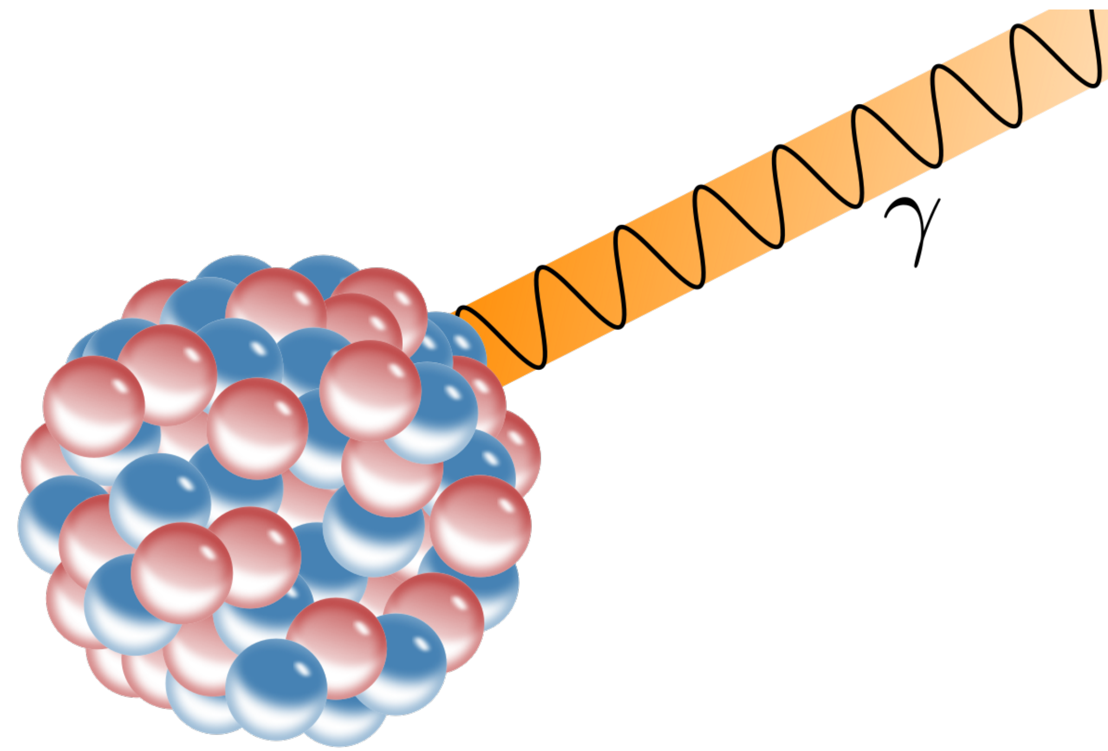


If all occupied shells are full, the nuclei are particularly stable: "Magic numbers"

Nuclear Physics 1-3 The Shell model

In analogy to the atomic shell model, nuclei can be excited





γ emission

And we discuss which transitions are allowed by angular momentum and parity conservation

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5. Nuclear form factors
6. Scattering off nucleons
7. From Quantum Mechanics to Quantum Field Theory
8. Inelastic Scattering

Particle Physics

1. Feynman diagrams
2. Quarkonia
3. Light Quark Mesons
4. Baryons
5. e^+ e^- collisions
6. Weak Interaction

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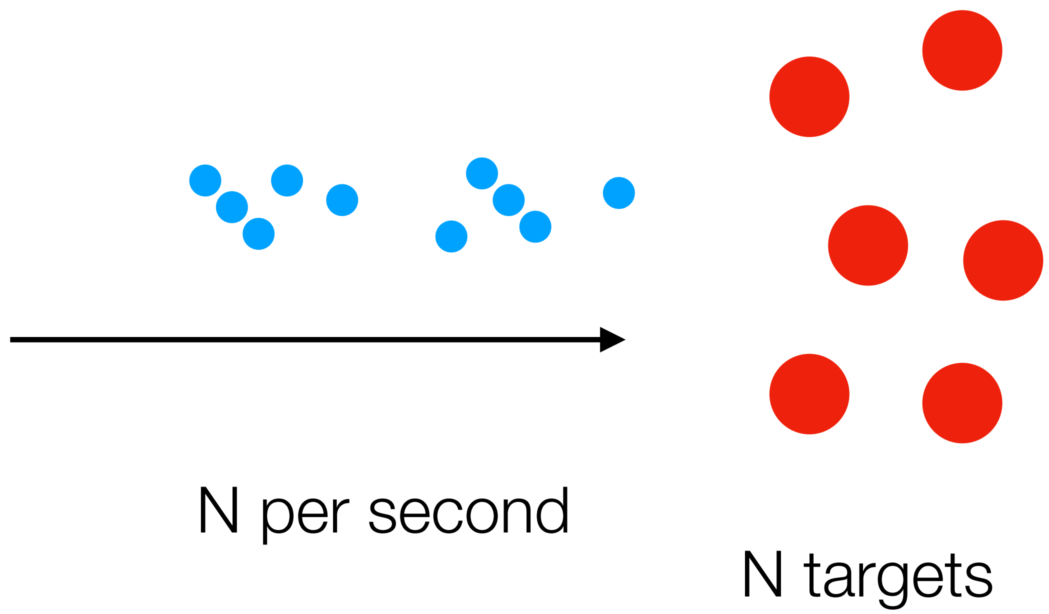
Scattering 1-3 Cross sections

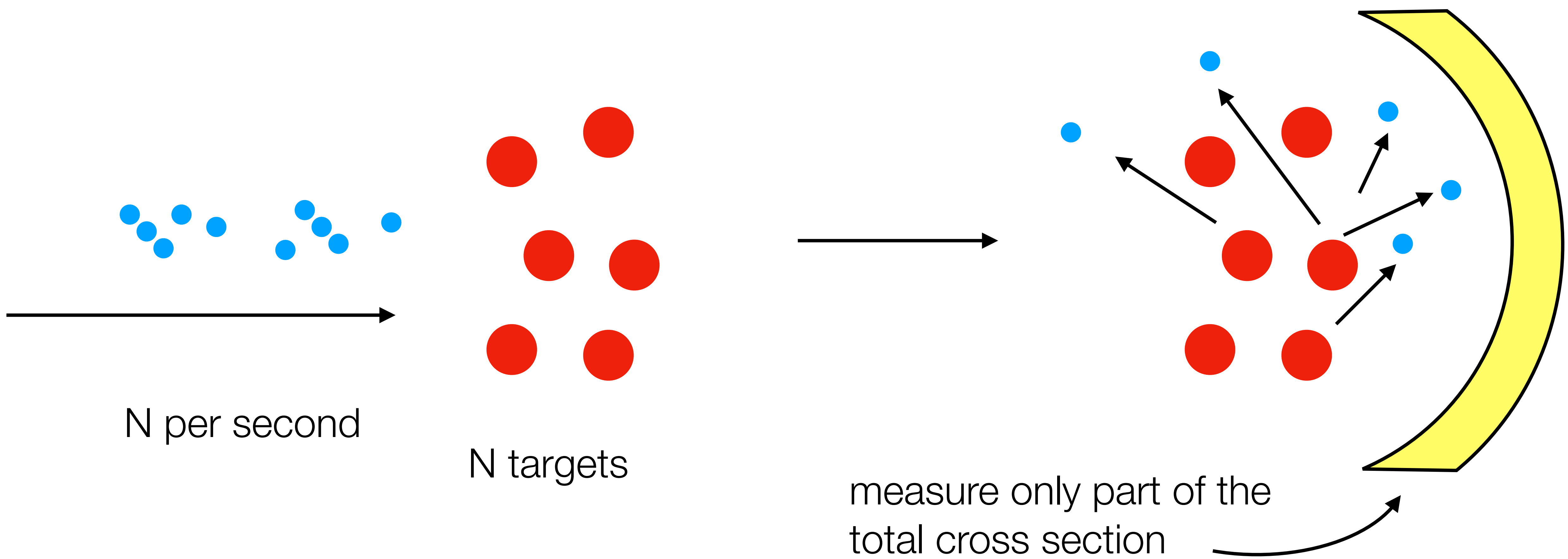
Experimental situation

$$\sigma_{tot} = \frac{\text{number of reactions per time unit}}{\text{beam particles per unit time} \times \text{number of scattering centres per unit area}}$$



Scattering 1-3 Cross sections

Experimental situation

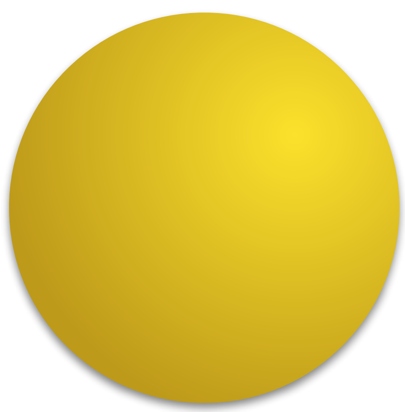


$$\sigma_{measured} = \int_{\Omega_c} \int_{E_{min}}^{E_{max}} \frac{d\sigma}{d\Omega\, dE} dE\, d\Omega$$

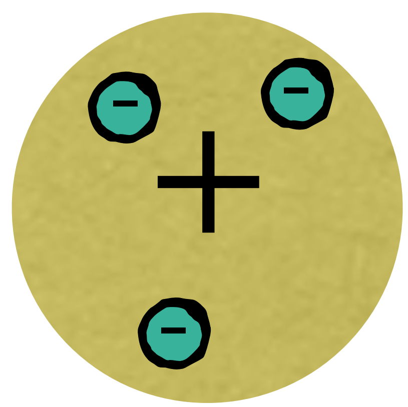
Scattering 1-3 Cross sections

Rutherford scattering cross section

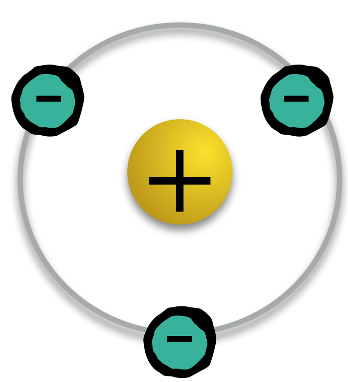
How do we know the shape of the atoms?



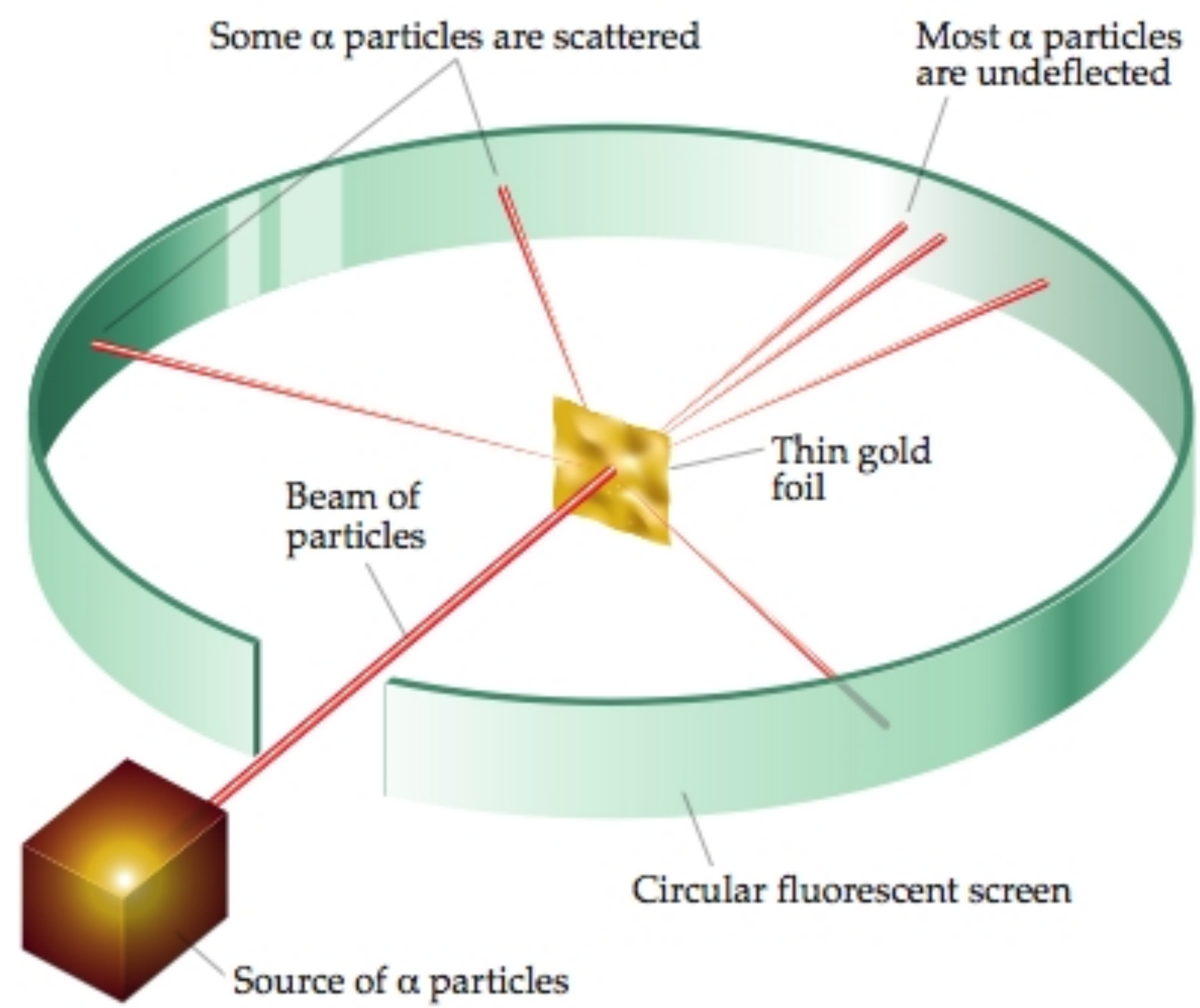
Dalton



Thomson



Bohr



Scattering 1-3 Cross sections

How to calculate a cross section

Fermis golden Rule

$$\sigma = \frac{2\pi}{v_a} \left|\mathcal{M}_{fi}\right|^2 \rho(E')V$$

The transition Matrix element

$$\mathcal{M}_{fi} = \langle \psi_f | \mathcal{H}_{int} | \psi_i \rangle$$

with a potential

$$\mathcal{H}_{int}(\mathbf{x}) = ze\phi(\mathbf{x})$$

the differential energy density

$$d\rho(E') = \frac{V}{(2\pi)^3} d\Omega |\mathbf{p}'|^2$$

and the initial and final state are plane waves

$$\psi_i = \frac{1}{\sqrt{V}} e^{i\mathbf{p}\cdot\mathbf{x}}$$

$$\psi_f = \frac{1}{\sqrt{V}} e^{i\mathbf{p}'\cdot\mathbf{x}}$$

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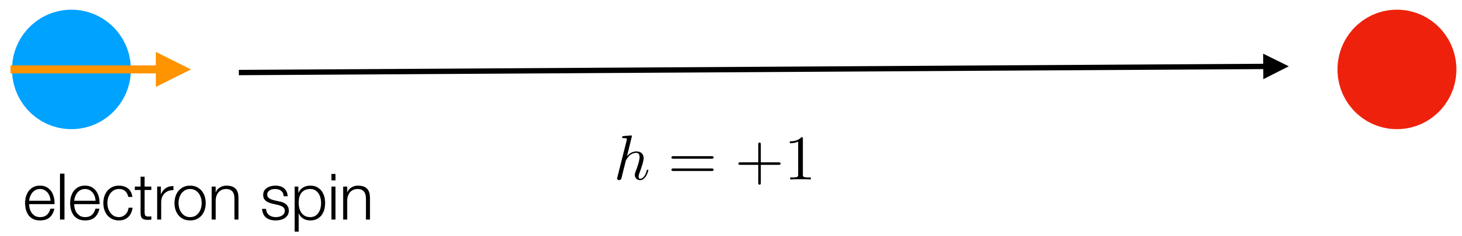
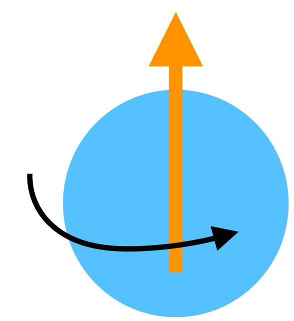
Scattering 4-6 Form Factors and Nucleons

The Rutherford cross section makes several assumptions. The target is fixed, does not recoil and carries no spin. The electron carries only electric charge.



It is therefore a purely electric interaction. Spin generates a magnetic field. It corresponds to a rotating charge



Rutherford scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{e^4 z^2\, Z^2}{4E^2 \sin^4 \frac{\theta}{2}}$$

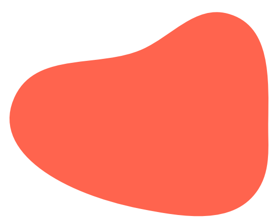
Mott scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott,no recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \cos^2 \frac{\theta}{2}$$

Scattering 4-6 Form Factors and Nucleons

For larger energies the target appears to be extended…

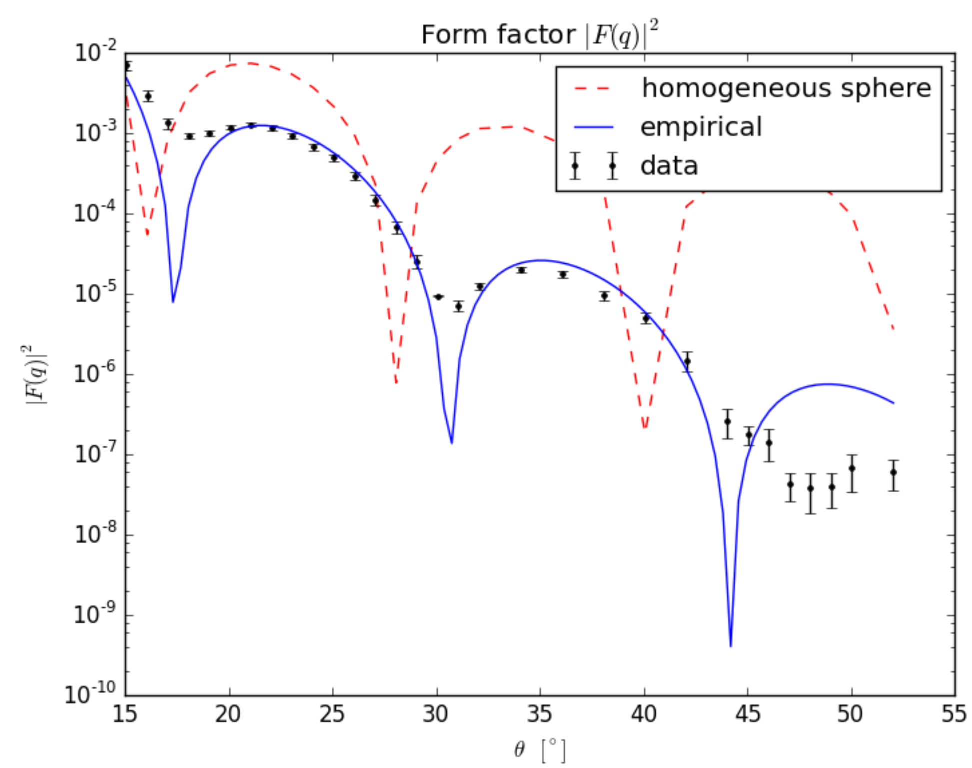
$$f(y) = \delta^3(\mathbf{x_0} - \mathbf{y})$$



$f(y)$

…and the target recoil can become relevant

We discuss how to derive the shape of the target from the scattering pattern



Scattering 4-6 Form Factors and Nucleons

The target atom nucleus does not look like a hard sphere, because it is a composite object made out of protons and neutrons.

More energetic projectiles can resolve this structure. In this case the spin of the target becomes relevant as well and scattering is described by the Rosenbluth formula

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta}{2}\right] \ , \quad \tau = \frac{Q^2}{4M_p^2}}$$

$G_E(Q^2)$ is related to the Fourier transform of the electric charge

$G_M(Q^2)$ is the form factor associated with the magnetic moment density.

Measuring the “shape” of the target for different momenta allows to determine spin and charge of the target.

Scattering 4-6 Form Factors and Nucleons

Scattering Process	Cross Section
pointlike charge [no spin] on pointlike charge [no recoil, no spin]	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{e^4 z^2 Z^2}{4E^2 \sin^4 \frac{\theta}{2}}$
pointlike charge [**spin**] on pointlike charge [no recoil, no spin]	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{R}} \cos^2 \frac{\theta}{2}$
pointlike charge [**spin**] on **extended** charge [no recoil, no spin]	$\left(\frac{d\sigma}{d\Omega}\right)_{\rho} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} \lvert F(\underline{q}^2)\rvert^2$
pointlike charge [**spin**] on pointlike charge [**recoil**, no spin]	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} \cdot \frac{E'}{E}$
pointlike charge [**spin**] on pointlike charge [**recoil**, **spin**]	$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left(1 + 2\tau \tan^2 \frac{\theta}{2}\right)$
pointlike charge [**spin**] on **extended** charge [**recoil**, **spin**]	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rosenbluth}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{G_E^2(\underline{q}^2) + \tau G_M^2(\underline{q}^2)}{1+\tau} + 2\tau G_M^2(\underline{q}^2) \tan^2 \frac{\theta}{2}\right]$

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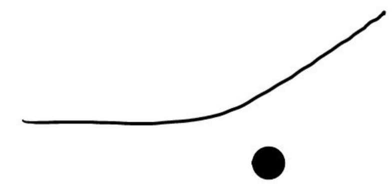
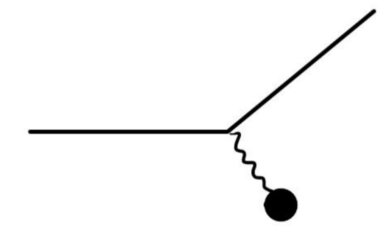
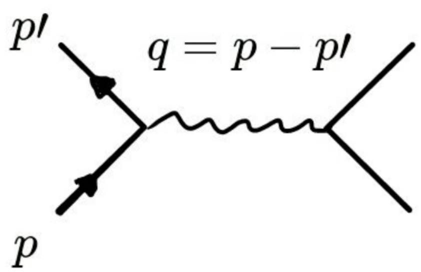
Scattering 7-8 QFT and inelastic scattering

QFT tells us that a virtual photon is exchanged in electromagnetic scattering. Forces are mediated by particles.





The propagator explains the scaling of the cross section

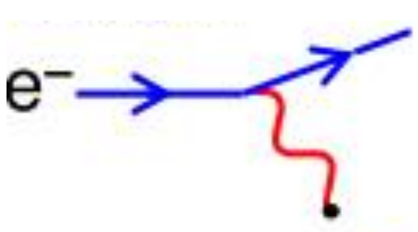


$$\rightarrow \quad \mathcal{M} \simeq \frac{1}{q^2 - m^2} \qquad \Rightarrow \qquad \sigma \simeq |\mathcal{M}|^2 \simeq \left(\frac{1}{q^2 - 0}\right)^2 = \frac{1}{Q^4}$$

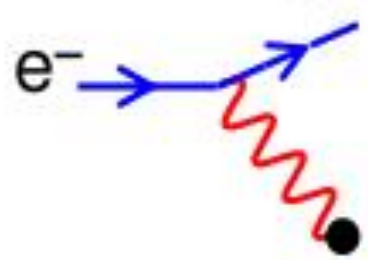
The photon wavelength determines the resolution of the scattering process

$\lambda \gg R_{\text{Proton}}$ | $\lambda \sim R_{\text{Proton}}$ | $\lambda < R_{\text{Proton}}$ | $\lambda \ll R_{\text{Proton}}$

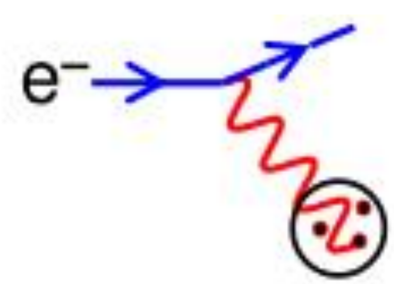
Rutherford



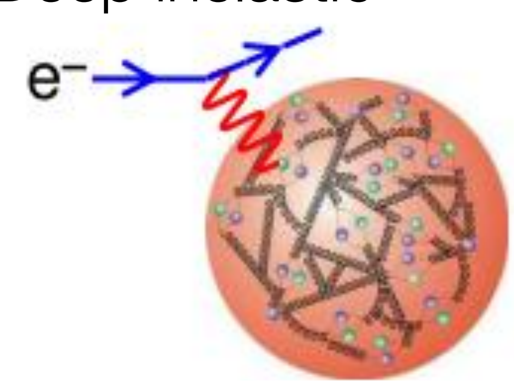
Rosenbluth



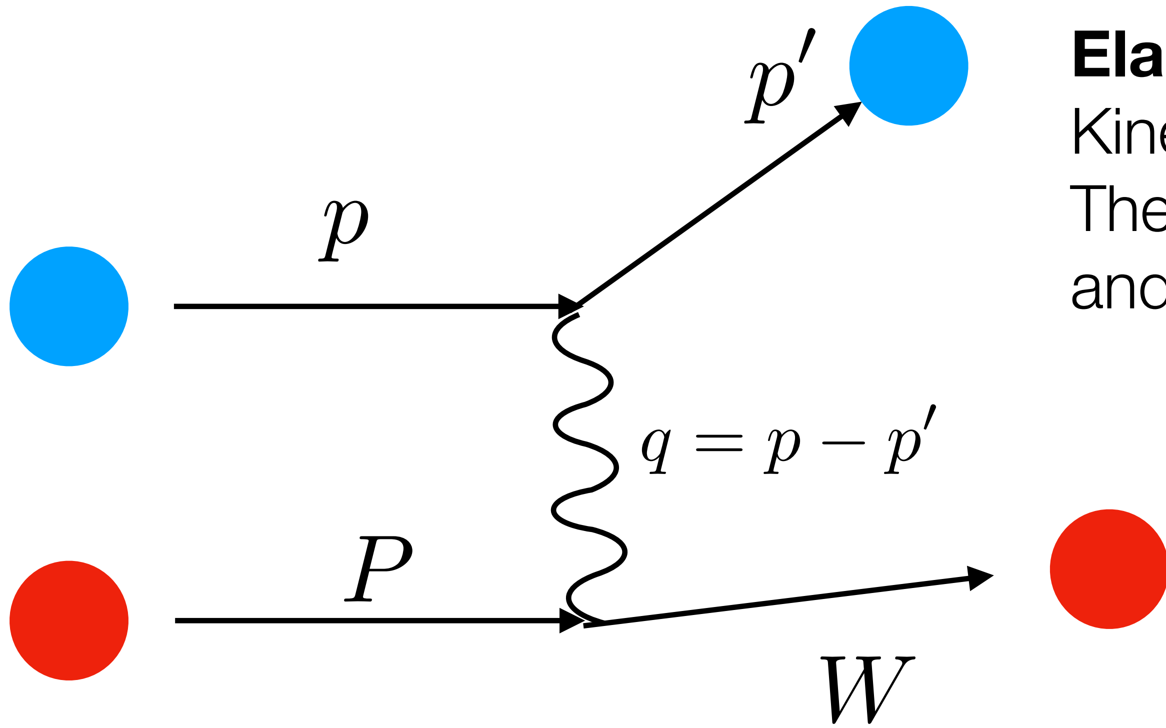
Inelastic



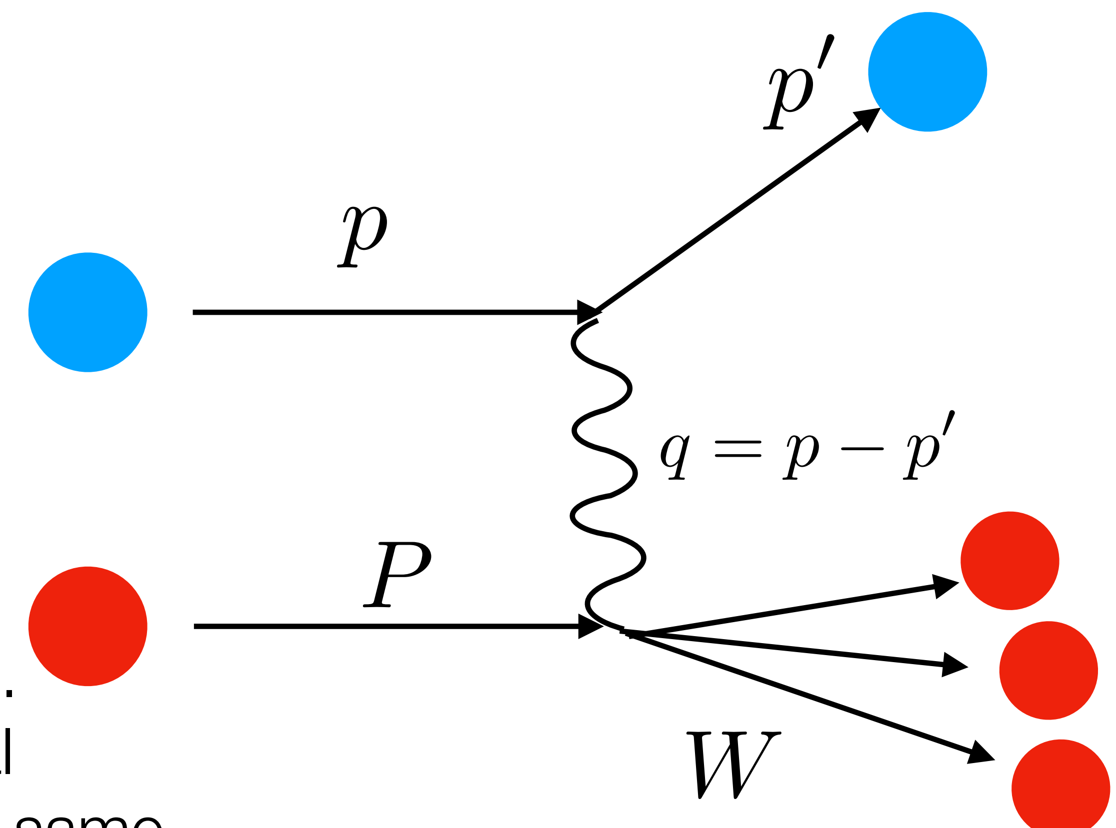
Deep inelastic



Scattering 7-8 QFT and inelastic scattering

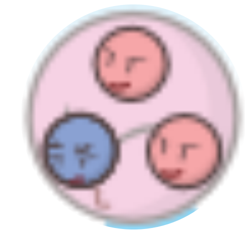


Elastic scattering:
Kinetic energy conserved.
The number and type of initial
and final state particles is the same.



Inlastic scattering:
Kinetic energy not conserved.
The number and type of initial
and final state particles is the same.

Scattering 7-8 QFT and inelastic scattering

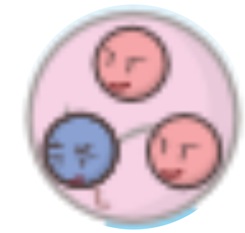


We will discuss the cross section for inelastic scattering

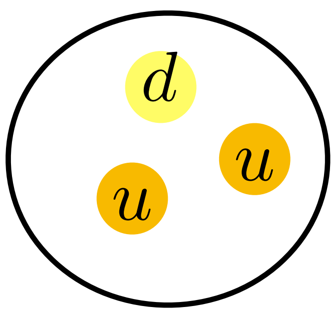
$$\boxed{\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + \frac{y^2}{2}\frac{2xF_1(x,Q^2)}{x}\right]}$$

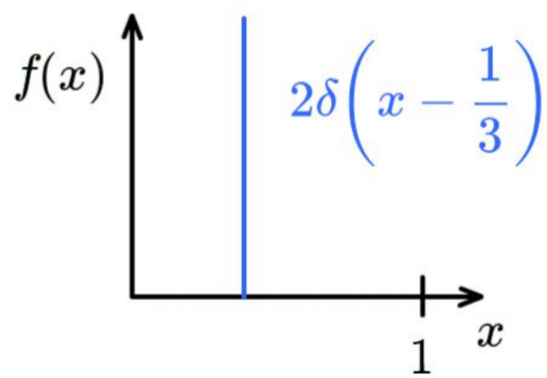
The form factors show a point-like substructure in the proton: partons

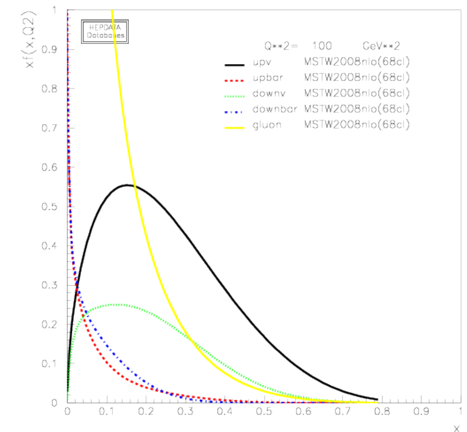
Scattering 7-8 QFT and inelastic scattering



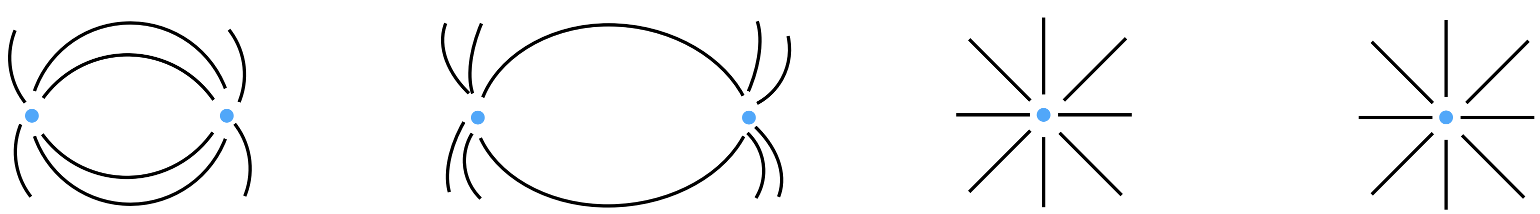
If there were no interactions, each parton would carry a fixed fraction x of the total momentum, but nuclei are bound states of the strong force.



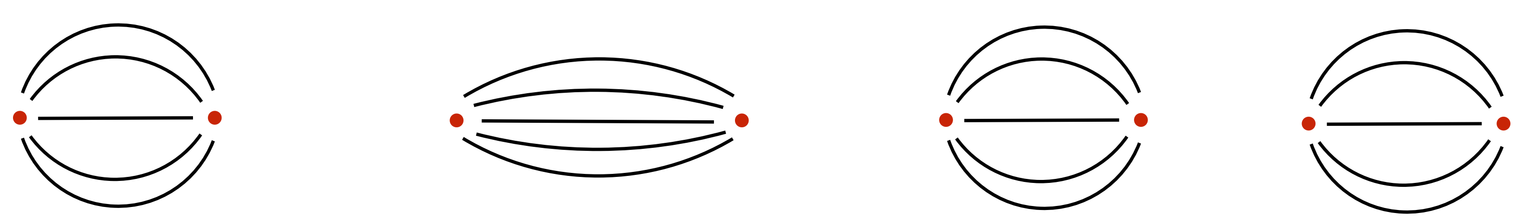




The mediators of the strong force are called gluons and the charge is called colour. Strongly bound objects can never be split.



QED



QCD

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Particle Physics 1 Feynman diagrams

Our best current knowledge of the fundamental constituents of nature

Fermions

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

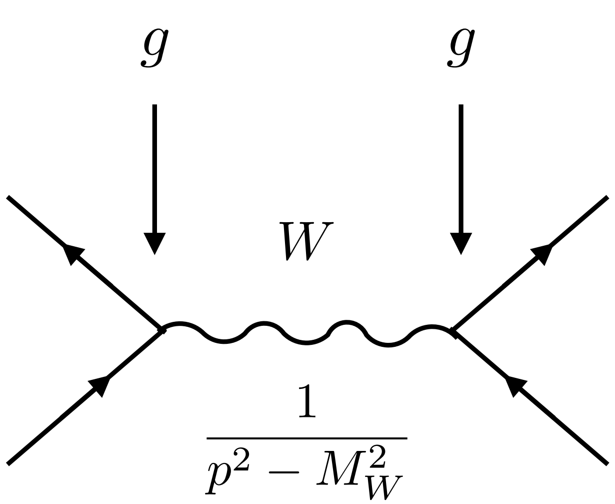
Bosons

electroweak	γ, Z, W^+, W^-
strong	g
Higgs	H

Particle Physics 1 Feynman diagrams

Our best current knowledge of the fundamental constituents of nature

And how to calculate fundamental cross sections with diagrams



$$\mathcal{M} \propto \quad g \, \frac{1}{p^2 - M_W^2} \, g$$

Fermions

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

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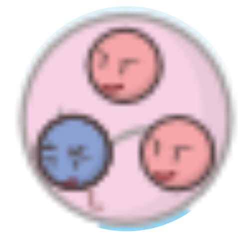
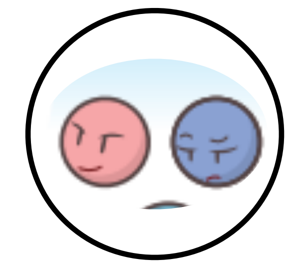
Particle Physics 2-3 Quarkonia and light mesons

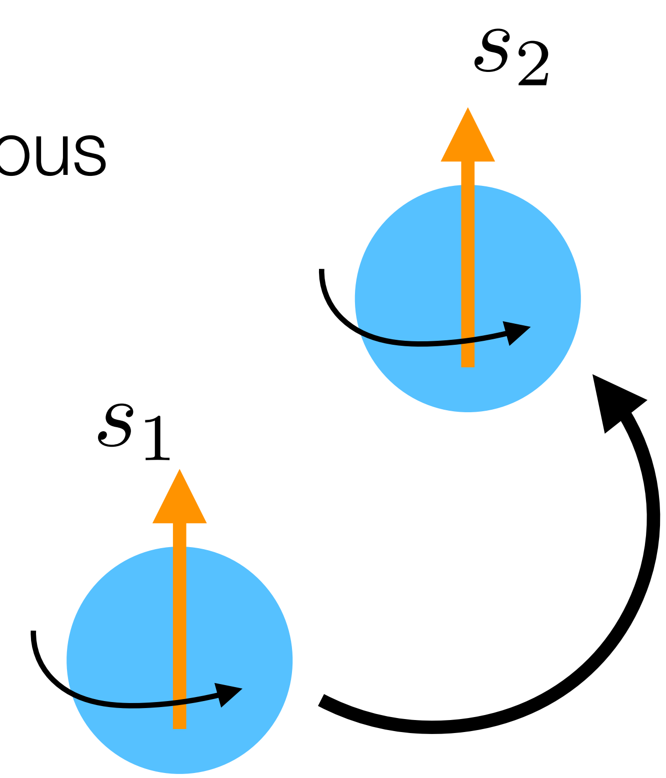
Partons are made of three quarks.



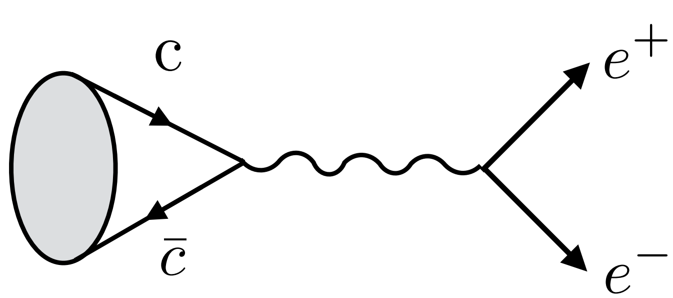
But there are also states made from quark-antiquark pairs, so called mesons



Mesons are two-particle bound states like the hydrogen atoms. We can understand much of the physics analogous to the hydrogen system. (Parity, Spin…)



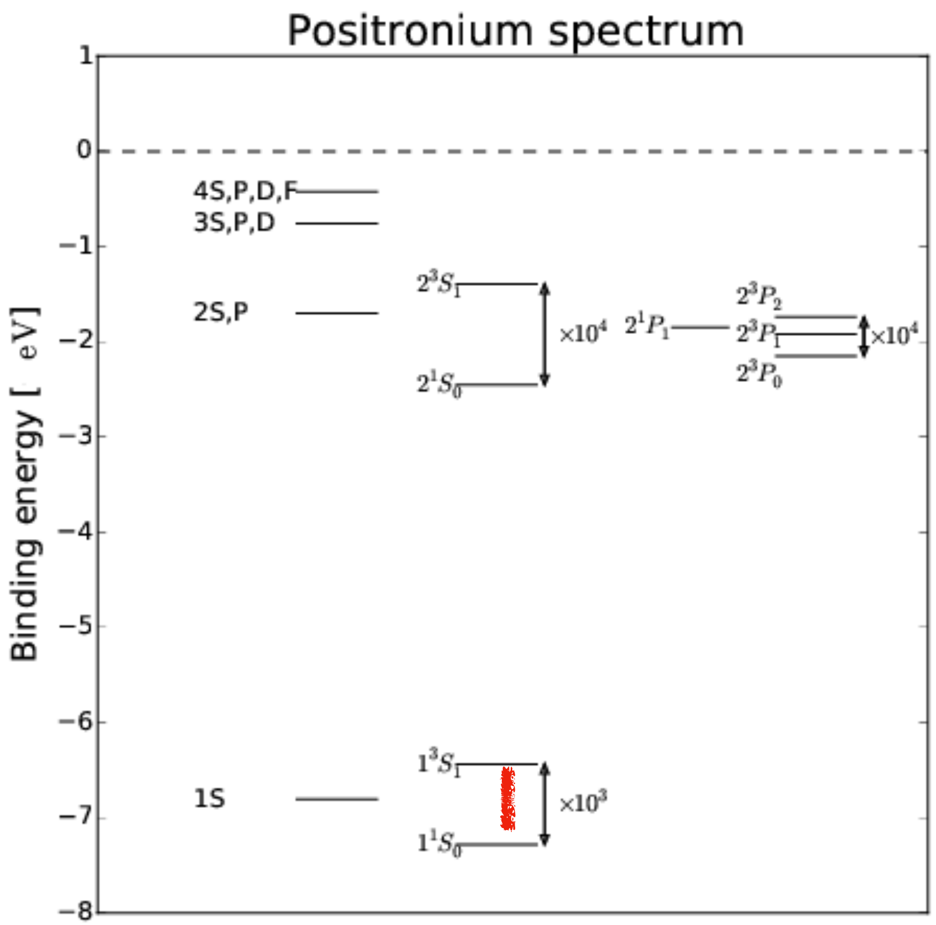
But mesons are unstable.

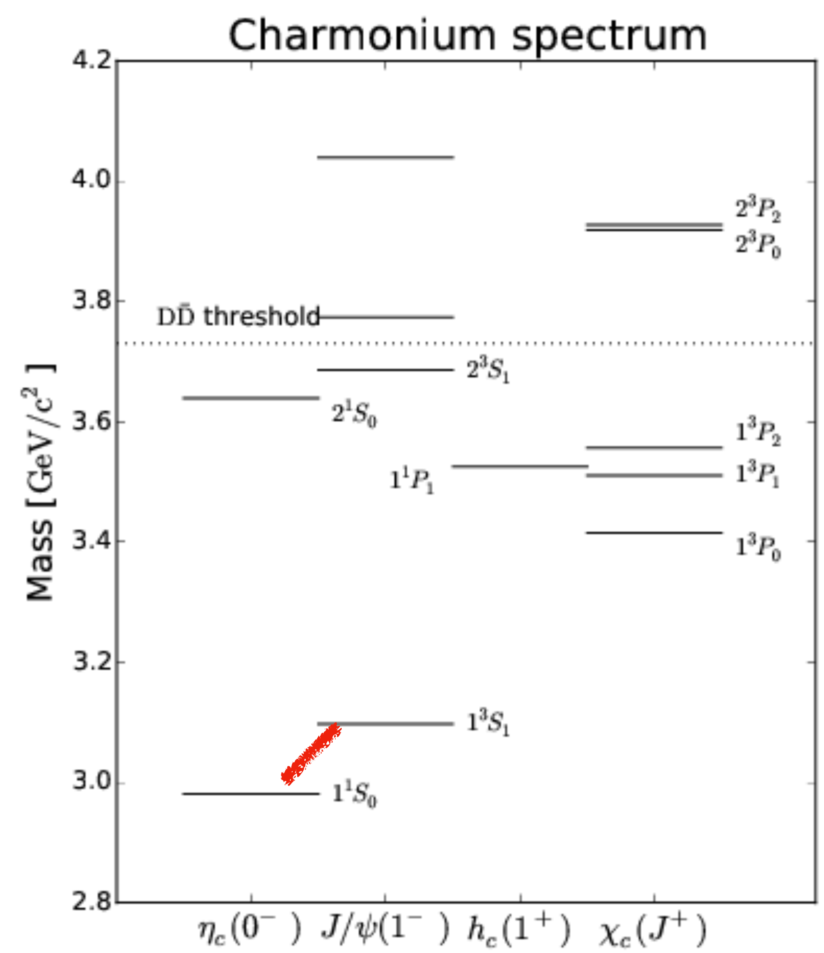


Particle Physics 2-3 Quarkonia and light mesons

They spectrum is very similar to positronium, but the energy splitting from spin-spin interactions is much larger

$$V_{ss}(e^+e^-) = \frac{8\pi}{3}\alpha\frac{\vec{s}_1\cdot\vec{s}_2}{m_e^2}\delta(\vec{x}) \qquad V_{ss}(q\bar{q}) = \frac{32\pi}{9}\alpha_s\frac{\vec{s}_q\cdot\vec{s}_{\bar{q}}}{m_q m_{\bar{q}}}\delta(\vec{x})$$





Particle Physics 2-3 Quarkonia and light mesons

The energy splitting can be calculated based on this potential

$$V_{ss}(q\bar{q}) = \frac{32\pi}{9}\alpha_s \frac{\vec{s}_q \cdot \vec{s}_{\bar{q}}}{m_q m_{\bar{q}}}\delta(\vec{x})$$

$$\langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{2S(S+1)-3}{4} = \begin{cases} -\frac{3}{4} & \text{for} \quad S=0 \\ +\frac{1}{4} & \text{for} \quad S=1 \end{cases}$$

$$\begin{aligned} E_{ss} &= \langle \psi | V_{ss} | \psi \rangle = \frac{8\pi\alpha_s}{9 m_q m_{\bar{q}}} \langle \psi | \delta(\vec{x}) | \psi \rangle \cdot \begin{cases} -3 & \text{for} \quad S=0 \\ 1 & \text{for} \quad S=1 \end{cases} \\ &= \frac{8\pi\alpha_s}{9 m_q m_{\bar{q}}} \left|\psi(0)\right|^2 \cdot \begin{cases} -3 & \text{for} \quad S=0 \\ 1 & \text{for} \quad S=1 \end{cases} \quad . \end{aligned}$$

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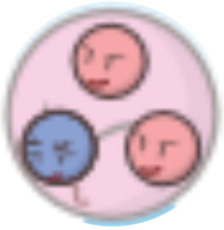
Particle Physics 4 Baryons



Baryons are strongly bound states made of three different quarks, such as protons (uud) and neutrons (ddu).

Since they are fermions the total wavefunction has to be antisymmetric

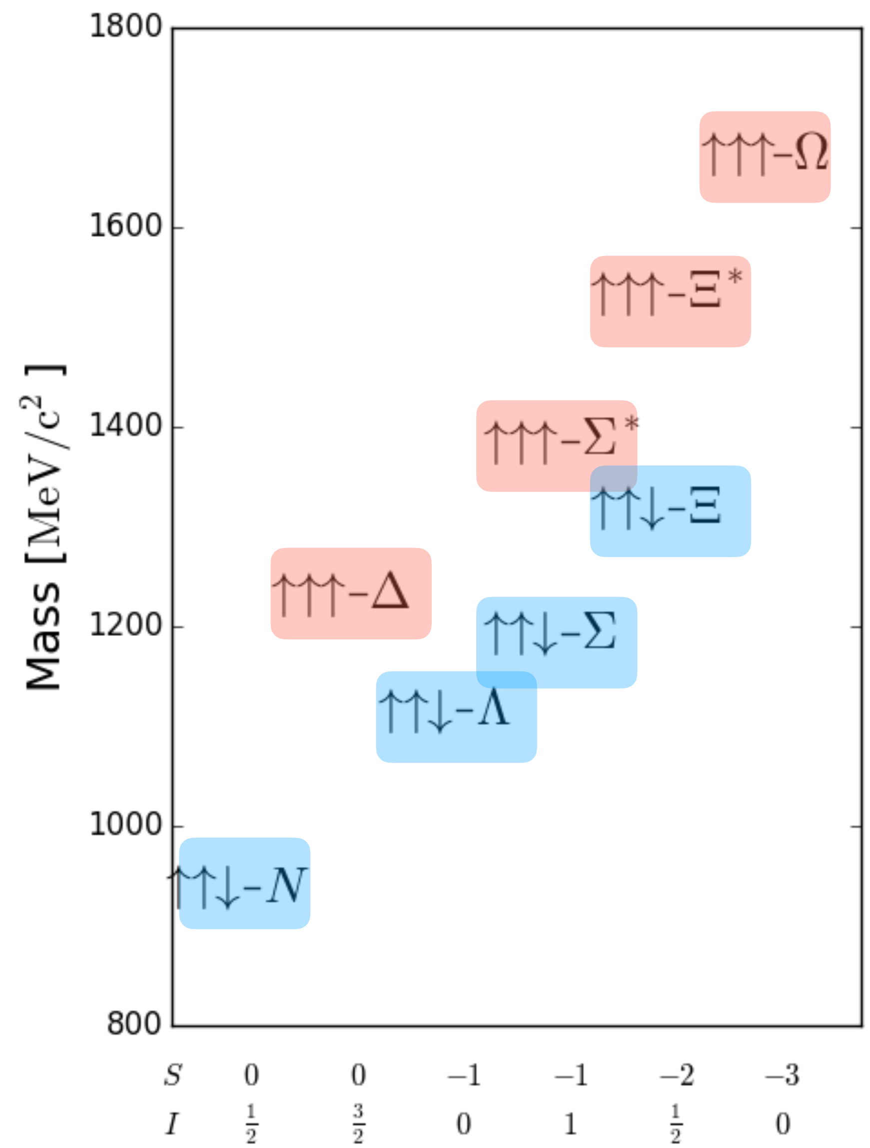
$$\psi_{\text{total}} = \psi_{\text{spatial}} \cdot \psi_{\text{flavour}} \cdot \psi_{\text{spin}} \cdot \psi_{\text{colour}}$$

Symmetry arguments help to understand the baryon spectrum.

Particle Physics 4 Baryons

Energy splitting can be calculated analogously to the calculation for mesons, but now one needs to consider 3 quark spins.

The energy splitting is larger if all spins are aligned.



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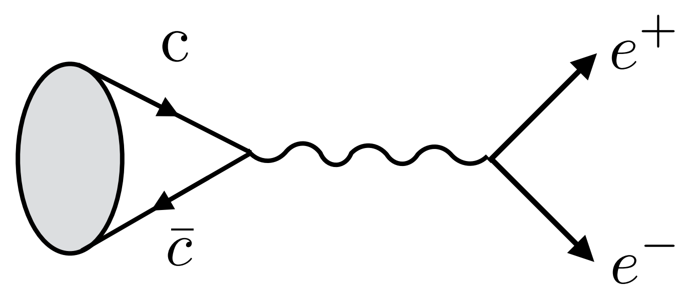
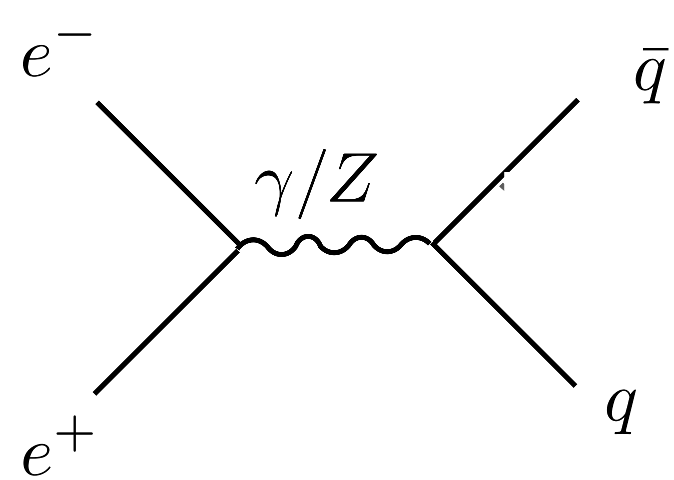
Particle Physics 5-6 e^+e^- collisions and weak interactions

Just like meson can decay

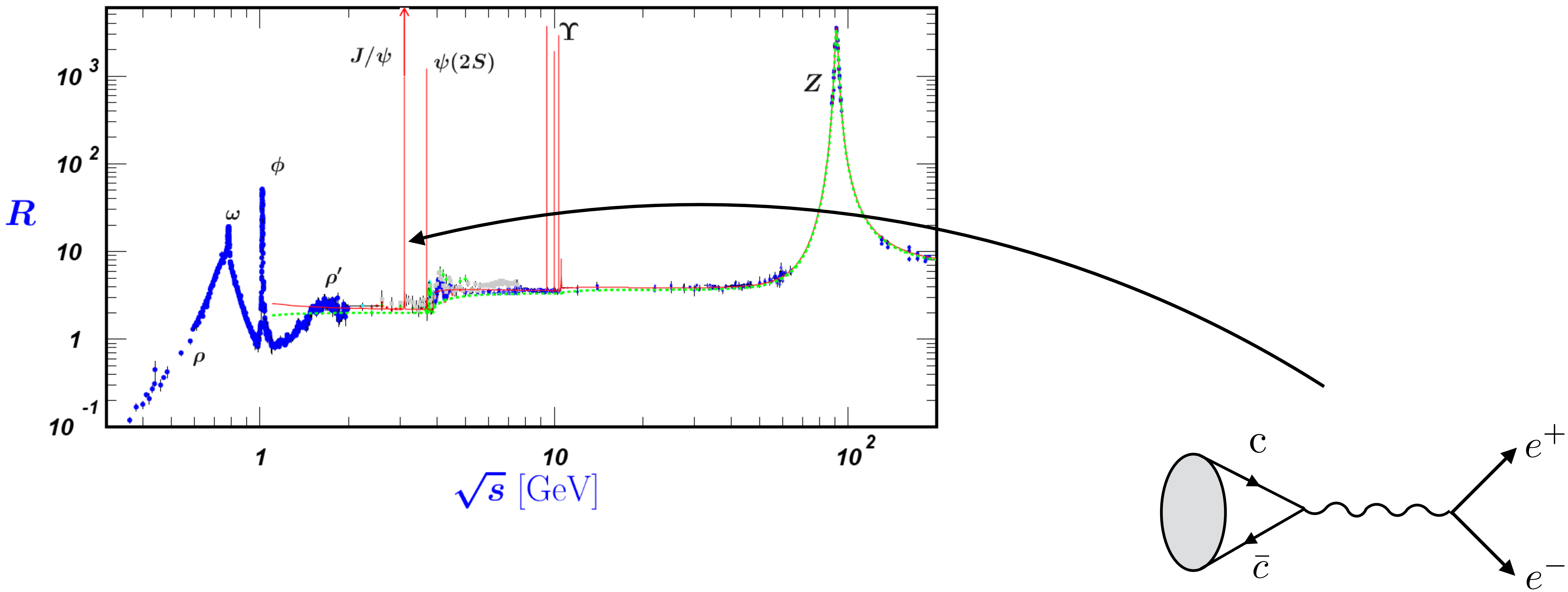


Pairs of fundamental particles can be produced in high-energy e^+e^- collisions.



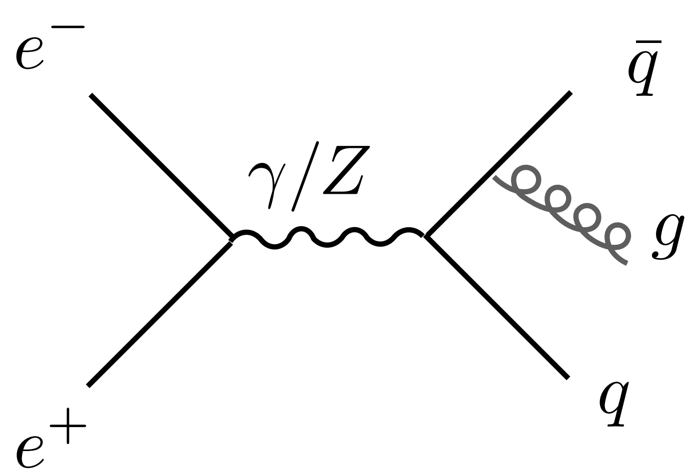
Particle Physics 5-6 e^+e^- collisions and weak interactions

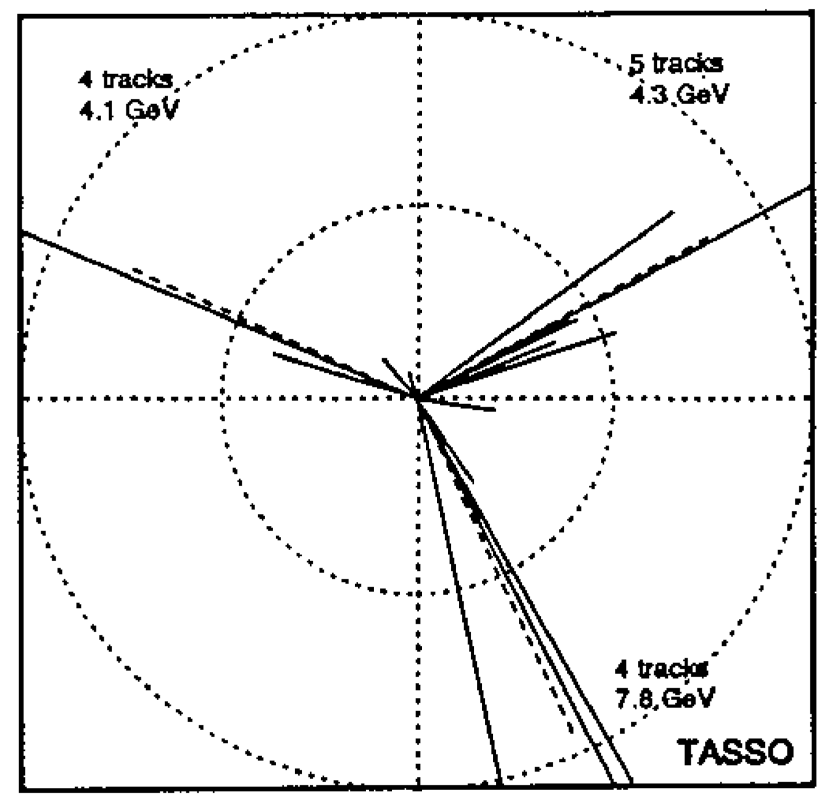
Resonances correspond to new particles



Particle Physics 5-6 e^+e^- collisions and weak interactions

e^+e^- collisions can also produce 3-jet events, which proof the existence of gluons

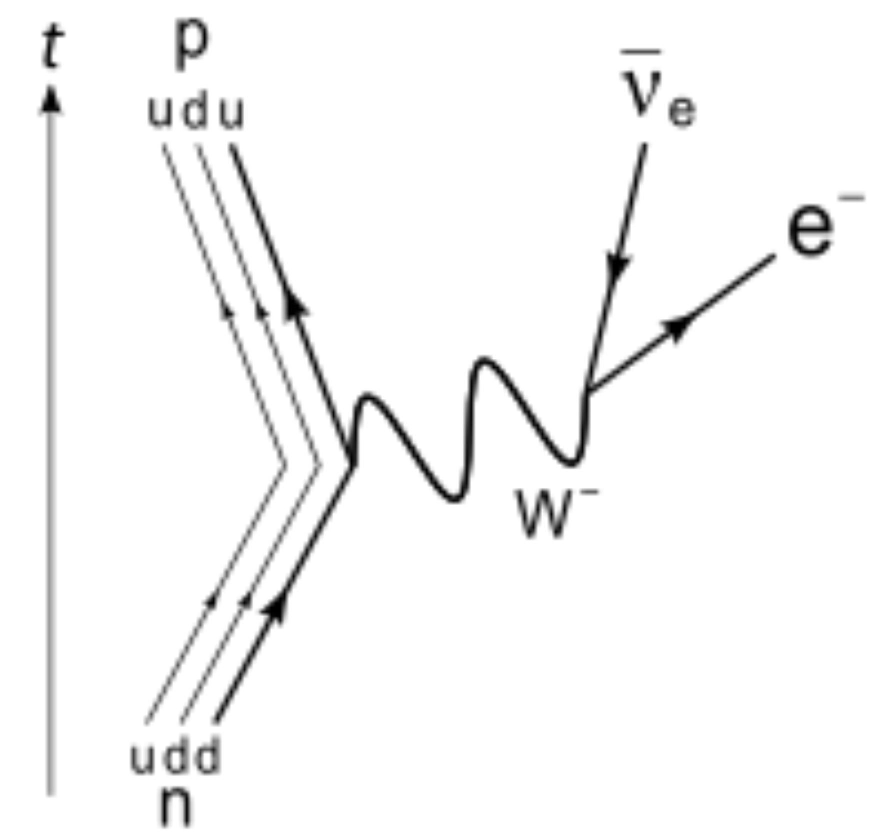




Particle Physics 5-6 e^+e^- collisions and weak interactions

We discuss the weak interaction, which fundamentally explains beta decay

interaction	carrier	mass
strong	gluon	0
electro-magnetic	photon	0
weak	W^+	80.4 GeV
	W^-	80.4 GeV
	Z^0	91.2 GeV



and the origin for different strength in weak interactions.

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