Nuclear and Particle Physics
Workshop 1

1. For heavy, instable atomic nuclei α-decay is a comparatively common phenomenon, but proton emission is virtually non-existent. What is the reason for this observation?

a) Based on the semi-empirical mass formula, calculate the energy release for α- decay (the pairing term can be neglected $\delta = 0$)

$$E_\alpha = M(A, Z) - M(A-4, Z-2) - M(4, 2)\,.$$

b) Show that an approximate expression for E_α for $Z = A/2$ and $\frac{1}{A} \ll 1$ is given by

$$E_\alpha \approx -4a_V + \frac{8}{3}\frac{a_s}{A^{1/3}} + a_c\frac{5}{3}A^{2/3} + B(4, 2)$$

c) Use the numerical values $B(4, 2) = 28.3\,\text{MeV}$ and

$$\begin{aligned} a_V &= 15.84\,\text{MeV}\,, \quad a_s = 18.33\,\text{MeV}\,, \\ a_c &= 0.71\,\text{MeV}\,, \quad a_a = 92.80\,\text{MeV}\,. \end{aligned}$$

to show that $E_\alpha \geq 0$ for $A \gtrsim 93$. Therefore, only for heavy nuclei the energy release ispositive and α-decay is possible. Note that $Z = A/2$ is not a good approximation for heavy nuclei. The more realistic value derived in the homework exercise $Z = A/(2 + 0.015A^{2/3})$ yields $E_\alpha \geq 0$ for $A \gtrsim 150$.

d) Calculate the energy release for proton emission

$$E_P = M(A, Z) - M(A-1, Z-1) - M(1, 1)\,.$$

and show that in the same approximation as above, $Z = A/2$ and $\frac{1}{A} \ll 1$,

$$E_P = -a_V + \frac{2}{3}\frac{a_s}{A^{1/3}} + a_c\frac{11}{12}A^{2/3}\,,$$

which is always smaller than E_α for nuclei with $A \gtrsim 93$. Proton emission is typically energetically less favourable than α-decay for nuclei with $E_\alpha \geq 0$.

2. A Uranium nucleus ${}^{236}_{92}\mathrm{U}$ can break apart through spontaneous fission. Assume that it breaks in two roughly equal parts. Using that the electrostatic energy of a sphere with uniformly distributed charge Q is given by $E_{\mathrm{stat}} = \frac{3}{5}\frac{Q^2}{4\pi R}$ and the radius of both the Uranium atom and the fission products can be described by $R = 1.2 \cdot 10^{-13} A^{1/3}$, calculate the energy released in ${}^{236}_{92}\mathrm{U}$ fission.