Level 2 Stars

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Problem Set S.3

(1) Estimate the temperature at the core of the star of pure Hydrogen using $P_c = \frac{3}{8\pi}\frac{GM^2}{R^4}$. You should assume a constant mass density and also assume that the contribution from radiation pressure is negligible. Comment on whether nuclear fusion of Hydrogen is possible assuming classical mechanics?

[6 marks]

Solution

Gas pressure is $P = nkT_c$ *which is* $P = \frac{M}{\frac{4}{3}\pi R^3}\frac{kT_c}{\mu m_H}$ *for constant density [1 mark]*

Equate both equations: $\frac{M}{\frac{4}{3}\pi R^3}\frac{kT_c}{\mu m_H} = \frac{3}{8\pi}\frac{GM^2}{R^4}$ *[1 mark]*

Rearranging gives: $T_c = \frac{1}{2}\frac{GM}{R}\frac{\mu m_H}{k}$ *[1 mark]*

Which gives: $T_c = 11{,}200{,}000K$ *[2 marks]*

Note: need to use μ=0.5 to get the correct answer as the Hydrogen is fully ionised so only get 1 mark if use a different value for μ.

Fusion is not possible assuming classical mechanics (it is only possible assuming quantum mechanics). [1 mark]

(2) Calculate the temperature at which radiation pressure exceeds the gas pressure, assuming a particle density of $n=10^{32}\ m^{-3}$?

[4 marks]

Solution

$P = nkT$ and $P = \frac{1}{3}aT^4$ where $a = \frac{4\sigma}{c} = 7.57 \times 10^{-16}\, Jm^{-3}K^{-4}$

so

$nkT = \frac{1}{3}aT^4$ therefore $T^3 = \frac{3nk}{a}$ and $T = \left(\frac{3nk}{a}\right)^{1/3}$

[3 marks]

$$T = \left(\frac{3 \times 10^{32} \times 1.38 \times 10^{-23}}{7.57 \times 10^{-16}}\right)^{1/3}$$

$T > 176{,}000{,}000 K$ *[1 mark]*