ELECTROMAGNETISM
Level 2 Physics problems – Foundations of physics 2

Solution 5 Cycle 2 Version 1
Professor D P Hampshire – 2nd Year Physics Lecture Course

Information underlined or indicated by red text is required for marks to awarded. The mark scheme is a guide and solutions should not be considered to be unique. Marks are awarded for correct relevant Physics.

1. Zero marks unfortunately. 1-1

2. a) Only the current in the length of wire 2a contributes to the field at point P.

Using the Biot-Savart law;

$$dB = \frac{\mu_0 I \cos\theta dl}{4\pi(\mathrm{R}^2 + l^2)} \qquad \text{2-1}$$

$$\tan\theta = \frac{l}{R} \Rightarrow \frac{dl}{d\theta} = \mathrm{R}\sec^2\theta$$

Plugging in;

$$dB = \frac{\mu_0 I \cdot \cos\theta \cdot \sec^2\theta \cdot d\theta}{4\pi R(1+\tan^2\theta)} = \frac{\mu_0 I \cdot \cos\theta \cdot d\theta}{4\pi R} \qquad \text{2-2}$$

Hence for whole straight-line segment;

$$B_P = 2 \cdot \int_0^{\theta_{Max}} \frac{\mu_0 I \cdot \cos\theta \cdot d\theta}{4\pi R} = 2 \cdot \frac{\mu_0 I}{4\pi R}\sin(\theta_{Max}) = \frac{\mu_0 I}{2\pi R}\sin(\theta_{Max})$$

From the figure;

$$\sin(\theta_{Max}) = \frac{a}{\sqrt{a^2 + \mathrm{R}^2}} \qquad \text{2-3}$$

$$B_P = \frac{\mu_0 a I}{2\pi R\sqrt{a^2 + \mathrm{R}^2}} \qquad \text{2-4}$$

1 mark for correct result 2-4.

b) For an N-sided polygon:

$$\theta = \frac{\pi}{N} \qquad \text{2-5}$$

Each side of the polygon contributes B_P, therefore B_{Total} is:

$$B_P = \frac{\mu_0 a I}{2\pi R\sqrt{a^2 + \mathrm{R}^2}}$$

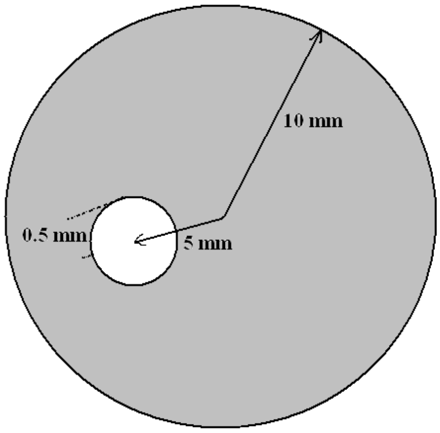
$$B_{\text{Total}} = NB_P = \frac{N\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{N}\right)$$

$$\text{As } N \to \infty,\ \sin\left(\frac{\pi}{N}\right) \to \frac{\pi}{N} \qquad \text{2-6}$$

Therefore, $B_{\text{Total}} \to \frac{\mu_0 I}{2R}$: the field at the centre of a current carrying loop. 2-7

1 mark for correct result 2-7. [Qn 2: 2 marks total]

3. Use superposition: add into the hole two identical wires with current density J = 10^5 Am^{-2} in opposite directions. One of the wires (wire A) serves to make the cable completely solid. Wire B has the current in the opposite sense. 3-1



a) At centre of the cable:
Wire A: zero net field, need to consider B field from wire A.

$$2\pi r B = \mu_0 I => B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(1 \times 10^5)(\pi)(5 \times 10^{-4})^2}{2\pi(5 \times 10^{-3})} = 3.14\ \mu\text{T} \quad 3\text{-}2$$

1 mark for correct result 3-2

b) At the centre of the hole:
Wire B: Zero net field, need to consider current enclose from wire A.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(1 \times 10^5)(\pi)(5 \times 10^{-3})^2}{2\pi(5 \times 10^{-3})} = 314\ \mu\text{T} \quad 3\text{-}3$$

1 mark for correct result 3-3. [Qn 3: 2 marks total]

4. There are four parts to the rectangular path integral. We use Ampère's law:

$$\oint \underline{\boldsymbol{B}} \cdot \mathrm{d}\underline{\boldsymbol{l}} = \mu_0 \mathrm{I} \quad 4\text{-}1$$

Assume there is no fringe field:
The horizontal part [(b)→(c)] of the path integral between the pole pieces will contribute a (positive) non-zero term to the integral $\oint \underline{\boldsymbol{B}} \cdot \mathrm{d}\underline{\boldsymbol{l}}$.
The two vertical parts of the path integral [(a)→(b)] and [(c) →(d)] are perpendicular to the field, so do not contribute to the path integral. 4-2
The horizontal part of the path [(d)→(a)] far from the pole pieces is in zero field and also makes a zero contribution to the line integral in Ampère's law: $\oint \underline{\boldsymbol{B}} \cdot \mathrm{d}\underline{\boldsymbol{l}}$.
So, with no fringe field, the next value of $\oint \underline{\boldsymbol{B}} \cdot \mathrm{d}\underline{\boldsymbol{l}}$ is non-zero. However, there is no current enclosed by the curve, so $\mu_0 \mathrm{I}$ must equal zero. 4-3

Therefore, Ampère's law is not obeyed which is not allowed for this magnetostatic problem. We conclude there must be a fringing field. 4-4

Note that the fringe field contributes an equal (negative) amount to ensure the total path integral $\oint \underline{\boldsymbol{B}} \cdot \mathrm{d}\underline{\boldsymbol{l}} = 0$. The fringe field produces non-zero (negative) contributions along all three parts of the path [(c)→(d)→(a)] that were zero without the fringe field.

1 mark if statements 4-2, 4-3 are written explicitly if contradiction is shown with maths to reach conclusion 4-4. [Qn 4: 1 mark total]

5. a) At the centre of the sphere, the field produced by the surface charge is, Coulomb's law resolved along z-axis.

$$dE_z(\theta) = -\frac{\sigma \cdot A}{4\pi\epsilon_0 R^2}\cos(\theta) \tag{5-1}$$

$$= -\frac{[Pcos(\theta)] \cdot [Rd\theta \cdot 2\pi Rsin(\theta)]}{4\pi\epsilon_0 R^2}\cos(\theta) = -\frac{P\cos^2(\theta)\sin(\theta)\,d\theta}{2\epsilon_0} \tag{5-2}$$

$$=> E_z = -\frac{P}{2\epsilon_0}\int_0^\pi \cos^2(\theta)\sin(\theta)\,d\theta = -\frac{P}{2\epsilon_0}\left[-\frac{\cos^3(\theta)}{3}\right]_0^\pi \tag{5-3}$$

$$= -\frac{P}{3\epsilon_0} \tag{5-4}$$

So, using superposition,

$$E_{centre} = E_{applied} - \frac{P}{3\epsilon_0} \tag{5-5}$$

1 mark for the use of coulombs law to obtain 5-2. 1 mark for the setup of integrand 5-3. 1 mark for getting correct answer 5-4 and the use of superposition to yield 5-5.

b)

$$p = \alpha E_{applied} = \alpha(E + \frac{P}{3\epsilon_0}) \tag{5-6}$$

$$P = np$$

$$P = \epsilon_0(\epsilon_r - 1)E$$

By elimination of E and P,

$$\left(1 - \frac{n\alpha}{3\epsilon_0}\right) = \frac{n\alpha}{\epsilon_0(\epsilon_r - 1)} \tag{5-7}$$

Rearranging gives the Claussius-Mossotti result.

$$\alpha = \frac{3\varepsilon_0}{n}\left\{\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right\} \tag{5-8}$$

1 mark for writing the expression 5-6. 1 mark for obtaining 5-7 and rearranging to get answer 5-8. [Qn 5: 5 marks total]. Total for all questions 10 marks.