Summary of lecture 5 (16th January 2020)

Noise and Lock-in detection

Signal-to-noise Ratio:

$$\mathrm{SNR} = \frac{V_{signal}}{V_{noise}}; \ SNR_{dB} = 20\log_{10}\left(\frac{V_{signal}}{V_{noise}}\right)$$

Noise in analog systems:

Noise in analogue electronic systems is caused by perturbations in the rate that electrons flow in a conductor (i.e. in the number of electrons flowing past our measurement point per second). There are two fundamental types of mechanism which cause these perturbations; velocity fluctuations (i.e. thermal noise), and number fluctuations (i.e. electron distribution variation).

Johnson noise is due to the thermal motion of electrons in a conductor, it is white noise, i.e. the power distribution is uniform with respect to frequency. To reduce the effects of Johnson voltage noise in a system, we can reduce the resistance values used and limit the measurement frequency bandwidth to the minimum that is useful for our experiment.

Shot noise is caused by the fact that current is quantised (i.e. an electric current is composed of individual current 'pulses' as each electron passes our measurement point) and the fact that the distribution of the electrons within the conductor is stochastic, resulting in a variation in the number passing the measurement point per unit-time. Shot noise is also a white noise.

1/f noise is common across diverse physical systems. The 1/f dependency is believed to be characteristic of a distribution of 'lifetimes'. The assumption that we can improve our noise performance by averaging our readings over a longer time-period does not hold for 1/f noise; whilst it does limit the higher-frequency components, these are exactly balanced by an increase in low-frequency components. To reduce the effects of 1/f noise we can shield our cables and circuitry and use band-pass filters to limit the frequency spectrum to the region of interest.

Brown noise is characteristic of random-walk systems (i.e. Brownian motion). It is not common in electronic systems, but is recognisable by a $1/f^2$ power spectrum.

Analysis of the noise in a system can tell us about the underlying physics of why the noise arises; providing information on how to compensate or try to remove some of the noise from the signal, e.g. not using a cell-phone or microwave oven while conducting a sensitive experiment…

Phase-sensitive (lock-in) detection:

Phase-sensitive detection using a lock-in amplifier can be used to extract the amplitude of a modulated signal having very poor SNR. The basic principle of a lock-in amplifier is that, if we impose a modulation on the signal that we wish to observe, we can reject any component of the input that does not vary with that specific modulation frequency. Since noise in a system is typically uncorrelated to the signal, and is spread over a broad range of frequencies, we can achieve very good noise-rejection.
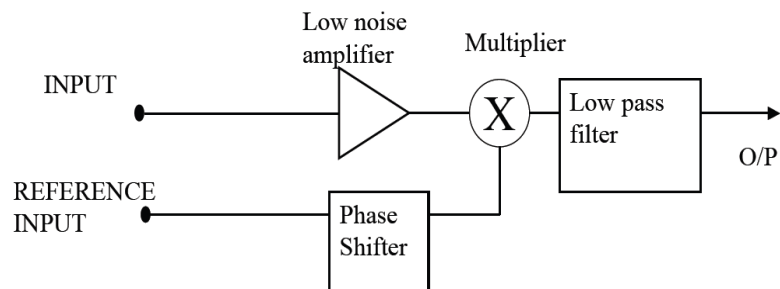
1. A low noise amplifier
2. A phase shifter
3. A multiplier
4. A low pass filter
5. An output display (voltmeter)

The lock-in amplifier has four primary components or subsystems; an amplifier, a phase-shifter, a multiplier and a low-pass filter. The lock-in-amplifier has two inputs; one, labelled 'input', is our modulated signal + noise; the other is our known 'reference' modulation signal, which we want the amplifier to 'lock' in to.

The mathematics behind the operation are found in the lecture notes, but we can describe the operating principle of lock-in detection in hand-waving terms. The multiplication process has the effect of shifting all the frequency components in our input (both modulated signal *and* noise) by an amount equal to the reference input frequency. Thus any component of the input which is very close to the frequency of the reference is shifted down to close to zero frequency; and passes through the low-pass filter. Components of the input at any other frequencies (i.e. the vast majority of the noise) are shifted to frequencies which are attenuated by the low-pass filter, and so are rejected. The output is maximised when the phase-shifter accounts for both any phase-difference between the input and reference signals, and any phase-shift induced in the input by the amplifier.

When setting up our lock-in-amplifier, we must define the gain of the amplifier stage. The key to this is to ensure that we don't suffer clipping at later points in the circuit as a result of our amplification (see op-amp lecture notes). Clipping results in the loss of information; since our signal is potentially small compared to the noise, we will very likely lose all signal information if the input is clipped by the amplifier.

We must also design the low-pass filter to have an appropriate corner frequency. The modulation frequency is typically ~100x the highest frequency component of the signal. The low-pass filter should have corner frequency such as to remove the high frequency modulation (and even higher frequency terms created by the multiplication), whilst passing all frequency components of the original signal.

One may determine the phase shift induced in the reference signal by the phase shifter by removing the phase-shifter from the circuit and measuring the difference between input and output signals across the phase-shifter using an oscilloscope. Any phase-shift induced by the amplifier at the reference frequency must also be considered.

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