Mathematical Methods in Physics

Workshop 8

8.1

The work done by a force $\mathbf{F}$ along a path C is $\int_C \mathbf{F} \cdot d\mathbf{r}$. Calculate the work done by the force $\mathbf{F} = 4xy\,\mathbf{i} - 8y\,\mathbf{j} + 2\mathbf{k}$ in the displacement:

a) Along the helix $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + 3t\mathbf{k}$ with $0 \leq t \leq 2\pi$.

b) Counterclockwise around the circle $x^2 + y^2 = 4$, $z = 0$.

c) Along the ellipse $x^2 + 4y^2 = 4$, $z = 0$ counterclockwise from $A = (0, -1, 0)$ to $B = (0, 1, 0)$.

[*Hint: In b) and c) use polar coordinates in order to parametrise the curve.*]

8.2

The curve

$$\mathbf{r}(u) = u\,\mathbf{i} + \cosh u\,\mathbf{j}$$

is called catenary.

a) Find the unit tangent vector $\hat{\mathbf{t}} = d\mathbf{r}/ds$.

b) Find its radius of curvature ρ, where $\mathbf{n} = \frac{\hat{\mathbf{n}}}{\rho} = \frac{d\hat{\mathbf{t}}}{ds}$.

[*Hint: Use the chain rule, for instance* $\hat{\mathbf{t}} = \frac{d\mathbf{r}}{du}\frac{du}{ds}$ *and remember that* $\cosh^2 u - \sinh^2 u = 1$.]

8.3

Calculate

$$I = \int_C (y^2 dx - x^2 dy)$$

where C is the parabolic arc $y^2 = x$ from $A = (1, -1)$ to $B = (4, 2)$.

8.4

Consider the vector field

$$\mathbf{a} = \left(-zx\,\mathbf{i} - zy\,\mathbf{j} + (x^2 + y^2)\,\mathbf{k}\right) r^{-3}$$

where r is the modulus of the position vector. Establish whether the field is conservative. If it is, find its potential.