# ELECTROMAGNETISM

## Level 2 Physics problems – Foundations of physics 2

**Solution 3 Cycle 2 Version 1**

**Professor D P Hampshire – 2nd Year Physics Lecture Course**

*Information underlined or indicated by red text is required for marks to awarded. The mark scheme is a guide and solutions should not be considered to be unique. Marks are awarded for correct relevant Physics.*

**1.** Faraday's law;

$$\xi = -\frac{\partial \phi}{\partial t} \text{ where, } \phi = \int \underline{\boldsymbol{B}} \cdot \mathrm{d}\underline{\boldsymbol{S}} \qquad \text{1-1}$$

Alternatively,

$$\xi = -\int \frac{\partial \underline{\boldsymbol{B}}}{\partial t} \cdot \mathrm{d}\underline{\boldsymbol{S}} \qquad \text{1-2}$$

where $\xi$ is the electromotive force, $\phi$ is the magnetic flux and $t$ is the time.

**1 mark if equation 1-1 or 1-2 is written and terms correctly defined**

**[Qn 1: 1 mark total]**

**2.**

a) Use Faraday's law to calculate the electromotive force. First calculate the magnetic flux.

$$\phi = \mathrm{B_z(t)} \cdot \mathrm{S} = B(0)S\cos\omega t \qquad \text{2-1}$$

And using,

$$\xi = -\frac{\partial \phi}{\partial t} \qquad \text{2-2}$$

$$=> \xi = \omega B(0)S\sin(\omega t) \qquad \text{2-3}$$

**1 mark if equation 2-3 is correct.**

b) Coil is now rotating so perpendicular area is given by;

$$S_\perp = S\cos(\omega t)$$

Magnetic flux is;

$$\phi = \int \underline{\boldsymbol{B}} \cdot \mathrm{d}\underline{\boldsymbol{S}} = B(0)\cos(\omega t) \cdot S\cos(\omega t) = B(0)S\cos^2(\omega t)$$

Use $\cos 2x = 2\cos^2 x - 1$;

$$=> \xi = -\frac{\partial \phi}{\partial t} = B(0)S\omega\sin(2\omega t) \qquad \text{2-4}$$

**1 mark if equation 2-4 is correct.**
**[Qn 2: 2 marks total]**

**3.** The wings 'sweep' out flux of magnitude,

$$\phi = vBlt \qquad 3\text{-}1$$

Using Faraday's law,

$$\xi = -\frac{\partial\phi}{\partial t} = -vBl$$

Potential difference $= |e.m.f.| = vBl = 500.10^{-5} \cdot 2 \cdot \cos(60°)$
$= 5$ mV 3-2

**Or** alternatively by using the Lorentz equation,

$$\underline{\boldsymbol{F}} = q(\underline{\boldsymbol{v}} \times \underline{\boldsymbol{B}}) \qquad 3\text{-}3$$

An electric field will be set up between the wing tips of the plane.

$$eE = F = evB$$

$El =$ Potential difference $= |e.m.f.| = vBl = 500 \cdot 10^{-5} \cdot 2 \cdot \sin(30°)$
$= 5$ mV 3-4

**If Faraday's law is used, equation 3-1 must be stated. 1 mark if 3-2 is correct.**
**If Lorentz force equation is used, equation 3-3 must be stated. 1 mark if 3-4 is correct.**
**[Qn 3: 1 mark total]**

**4.** Using Ampère's law;

$$\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \mu_0 I \qquad 4\text{-}1$$

Path integral intercepts nL wires carrying current.

$$BL = \mu_0 InL$$

where $L$ is the length of the coil, $n$ the number of turns per unit length

$$n = 1/N$$

$$B = \mu_0 nI$$

$$\phi = BA = \mu_0 nIA \qquad 4\text{-}2$$

$$=> \phi = \mu_0 nIA = 4\pi \times 10^{-7} \left({}^{2500}\!/_{1}\right) 10\pi (10^{-2})^2$$

$$= 9.9 \times 10^{-6} \text{ Wb} \qquad 4\text{-}3$$

**1 mark if 4-3 is correct.**
**[Qn 4: 1 mark total]**

**5.** a) The perpendicular area swept out is,

$$S = v\cos\theta lt \qquad 5\text{-}1$$

$$|\xi| = \left|-\frac{\partial\phi}{\partial t}\right| = \cos\theta Bvl = IR$$

$$=> I = \frac{Blv\cos\theta}{R} \quad 5\text{-}2$$

Using Lorentz force;

$$\underline{\boldsymbol{F}} = q(\underline{\boldsymbol{v}} \times \underline{\boldsymbol{B}}) => F = qvB$$

$$F = BIl$$

Perpendicular force;

$$F_{mag} = F\cos\theta = BIl\cos\theta = \frac{B^2l^2v\cos^2\theta}{R} \quad 5\text{-}3$$

**1 mark for identifying correct area, 5-1.**
**1 mark for correct current, 5-2**
**1 mark for correct answer, 5-3**

b) The component of the gravitational force on the rod down the plane,

$$F_{grav} = mg\sin\theta \quad 5\text{-}4$$

Balancing the forces,

$$F_{grav} = F_{mag} \quad 5\text{-}5$$

$$mg\sin\theta = \frac{B^2l^2v_t\cos^2\theta}{R} \quad 5\text{-}6$$

$$=> v_t = \frac{mgR\sin\theta}{B^2l^2\cos^2\theta} \quad 5\text{-}7$$

**1 mark for balancing forces, 5-5**
**1 mark for correct answer, 5-7**

**[Qn 5: 5 marks total]**

**Total for all questions 10 marks**