

University of Durham

EXAMINATION PAPER

Examination session:

May/June

Year:

2018

Examination code:

PHYS3641-WE01

Title:

Advanced Physics 3

Time allowed:	3 hours		
Additional material provided:	None		
Materials permitted:	None		
Calculators permitted:	Yes	Models permitted:	Casio fx-83 GTPLUS or Casio fx-85 GTPLUS
Visiting students may use dictionaries:		No	

Instructions to candidates:

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper.
- Answer **one** other question from **each** section. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** the answers that are not to be marked.
- The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK.**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklets for Sections B and C, in order, inside your booklet for Section A, before they are collected by the invigilator.

Information

Section A: Soft Condensed Matter Physics

Section B: Optical Properties of Solids

Section C: Modern Atomic and Optical Physics

A list of physical constants is provided on the next page.

Revision:

Information

Elementary charge:	$e = 1.60 \times 10^{-19} \text{ C}$
Speed of light:	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
Boltzmann constant:	$k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Bohr magneton:	$\mu_B = 9.27 \times 10^{-24} \text{ J T}^{-1}$
Electron mass:	$m_e = 9.11 \times 10^{-31} \text{ kg}$
Gravitational constant:	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Proton mass:	$m_p = 1.67 \times 10^{-27} \text{ kg}$
Planck constant:	$h = 6.63 \times 10^{-34} \text{ J s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
Molar gas constant:	$R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
Avogadro's constant:	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Gravitational acceleration at Earth's surface:	$g = 9.81 \text{ m s}^{-2}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Astronomical Unit:	$\text{AU} = 1.50 \times 10^{11} \text{ m}$
Parsec:	$\text{pc} = 3.09 \times 10^{16} \text{ m}$
Solar Mass:	$M_\odot = 1.99 \times 10^{30} \text{ kg}$
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26} \text{ W}$

SECTION A: SOFT CONDENSED MATTER PHYSICSAnswer Question 1 and **either** Question 2 **or** Question 3.

1. (a) A material has a viscosity described by $\eta = \eta_0 + \alpha\dot{\epsilon}$, where $\eta_0 > 0$ is a constant and $\dot{\epsilon}$ is the shear rate imposed to the material. Plot the stress experienced by this material as a function of $\dot{\epsilon}$ for $\alpha < 0$, $\alpha = 0$, and $\alpha > 0$. Describe the characteristics of the material in each case. [4 marks]
- (b) A solution is composed of molecules able to form z permanent inter-molecular bonds over a given time period t , each with a probability p . We want to determine the conditions enabling a sol-gel transition. Assuming that $z = 3$, what is the minimum probability for the transition to eventually take place? Justify your answer. [4 marks]
- (c) The energy barrier for the nucleation of an ice cluster is G^* in water. The ice nucleation energy is given by $\Delta G_{nuc} = I - P$ where I is the interfacial energy of the ice nucleus with its surroundings and P the change in free energy associated with the water to ice phase transition. Determine by how much G^* would change if the ice were instead nucleating on the surface of a rock immersed in water. Assume that the rock's surface is flat, and that the ice nucleus makes a 90° contact angle with the rock in water. [4 marks]

$$\left[\begin{array}{l} \text{Hint: } \gamma_{LV} \cos \theta = \gamma_{SV} - \gamma_{SL} \quad (\text{Young}) \\ \text{where the symbols have their usual meanings} \end{array} \right]$$

- (d) A particle is placed at the surface of a liquid where it floats. At a given temperature T , the particle diffuses over a distance L during a time t . How would L change over the same time if the temperature were reduced by half? (ignore possible changes in the liquid viscosity). [4 marks]

$$\left[\begin{array}{l} \text{Hint: } D = \frac{k_B T}{6\pi} \quad (\text{Diffusion coefficient}) \\ \text{where the symbols have their usual meanings} \end{array} \right]$$

- (e) Alcohol and water are mixed together in equal proportion in a container at a temperature T . The solution is homogenous and the enthalpy of mixing is characterised by a parameter $\chi > 0$. By how much should the solution be cooled for demixing to take place? [4 marks]

$$\left[\begin{array}{l} \text{Hint: } \Delta \bar{G}_{mix} = \chi \phi(1 - \phi) + k_B T [\phi \ln \phi + (1 - \phi) \ln(1 - \phi)] \\ \text{where the symbols have their usual meanings} \end{array} \right]$$

2. A membrane, composed of an array of parallel fibres, separates two reservoirs. The membrane is placed horizontally so that a liquid (density ρ) placed in the top reservoir must cross the membrane to reach the bottom reservoir.

- (a) At first, only a drop of liquid is placed on the membrane. Assuming it forms a contact angle of 90° with the membrane, calculate the work of adhesion W of the droplet with the membrane as a function of the surface tension of the liquid γ_L , and the membrane-liquid interfacial energy γ_{ML} as a function of the membrane surface energy γ_M . [4 marks]

$$\left[\begin{array}{ll} \text{Hint: } W_{SL} = \gamma_L(1 + \cos \theta) & (\text{Young-Dupr  }) \\ \gamma_{12} = \gamma_1 + \gamma_2 - W_{12} & (\text{Dupr  }) \\ \text{where the symbols have their usual meaning} \end{array} \right]$$

- (b) We now fill up the top container until the whole membrane is just covered with liquid. Assume the membrane to be a single array of parallel cylinders (the fibres), each with a radius r and separated by a distance $2d$ from the adjacent fibre (surface to surface). Neglecting gravitational effects and assuming that d is much smaller than the capillary length of the liquid, make a schematic drawing of the interface between the liquid and the fibres composing the filter, and justify your drawing. [2 marks]
- (c) More liquid is added to the top container until it reaches a height h above the surface of the membrane. Taking into account gravity, calculate the pressure in the liquid P_L at the membrane, and show on a schematic drawing how the interface has changed. [2 marks]
- (d) Calculate the curvature of the liquid surface between two adjacent fibres as a result of the pressure in the liquid. [3 marks]

$$\left[\begin{array}{ll} \text{Hint: } P = \gamma_L \left(\frac{1}{R_1} + \frac{1}{R_2} \right) & (\text{Young-Laplace}) \\ \text{where the symbols have their usual meaning} \end{array} \right]$$

- (e) Calculate the maximum height h_{max} possible before the liquid starts flowing through the filter. [7 marks]
- (f) Qualitatively, how would adding detergent to the liquid affect h_{max} ? [2 marks]

3. The Gibbs free energy G of a polymer molecule dissolved into a solvent is given by:

$$G = k_B T v \left(1 - \frac{2\chi}{k_B T} \right) \frac{N^2}{2(r - r_0)^3} + \frac{3k_B T}{2a^2 N} (r - r_0)^2$$

where k_B is Boltzmann's constant, v is the volume of a monomer, a its length, N the numbers of monomers in a polymer, $\chi > 0$ the enthalpic factor describing the interfacial energy between a monomer and the solvent, and $r_0 = a\sqrt{N}$ the idealised end-to-end length of the polymer in the freely jointed chain model. Initially, the temperature T satisfies $k_B T > 2\chi$.

- Calculate the average end-to-end distance $\langle r \rangle$ of a single polymer molecule in the solvent, and its elasticity constant k when stretched by a small distance $d \ll \langle r \rangle$ around $\langle r \rangle$. [5 marks]
- The system is cooled down so that $T = 2\chi/k_B$. Show that the polymer is in the theta condition. Explain the significance of this particular temperature in terms of the balance of free energy contributions and equilibrium end-to-end distance. [3 marks]
- The system is further cooled to a temperature $T < 2\chi/k_B$ satisfying

$$T = \frac{2\chi}{k_B} \left(\frac{16\sqrt{N}}{16\sqrt{N} + 1} \right)$$

Compare the average end-to-end distance of a polymer with the diameter D of a sphere containing a collapsed polymer globule. You may assume that the average volume occupied by each monomer in a globule is $\rho = v \frac{\pi}{48} \sqrt{N}$, and that $v \approx a^3$. What can you conclude about the molecular arrangement of the polymer? [4 marks]

- We define ϕ as the relative concentration of polymers in the solution: $\phi = n_{\text{globule}}/n_{\text{tot}}$ with n_{globule} the number of globules in solution and $n_{\text{tot}} = n_{\text{globule}} + n_{\text{solvent}}$ the total number of molecules in the system. Assuming that $\gamma = \chi/2$ is the polymer-polymer interfacial energy and that n_{tot} is constant, determine by how much the free energy of the system would change if the polymers were to aggregate. You may assume that the aggregate is a sphere with no solvent in it. Is aggregation likely at all concentrations? [5 marks]
- Qualitatively, would you expect the result found in (d) to change if the entropy of mixing was also taken into account? Justify your answer. [3 marks]

SECTION B: OPTICAL PROPERTIES OF SOLIDSAnswer Question 4 and **either** Question 5 **or** Question 6.

4. (a) GaAs has a complex refractive index $\tilde{n} = 3.95 + 0.240i$ at a wavelength of 587.6 nm. What is the value of the reflection coefficient when light impinges on GaAs from vacuum at normal incidence? [4 marks]
- (b) Gold (Au) has a free electron density of $5.90 \times 10^{28} \text{ m}^{-3}$. Calculate the plasma frequency for gold. Sketch the form of the reflectivity of gold close to this frequency, based on the following assumptions: (i) no plasmon damping; (ii) no interband transitions. [4 marks]
- (c) Explain why spin-orbit coupling can allow radiative transitions between singlet and triplet states. [4 marks]
- (d) GaAs is a direct gap semiconductor with a band gap of 1.39 eV and may be assumed to have parabolic conduction and valence bands characterised by effective masses $0.072 m_e$ and $0.50 m_e$ respectively. What is the numerical value of the *joint density of states* (JDOS) at photon energy 1.89 eV? [4 marks]

$$\left[\begin{array}{l} \text{Hint: the expression for the JDOS can be obtained} \\ \text{from the free electron density of states below} \\ \\ D(E) = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} E^{1/2}. \end{array} \right]$$

- (e) Explain the term *exciton* as applied to the electronic excitation of a solid and briefly describe the difference between *Mott-Wannier* and *Frenkel* excitons. In which types of materials would these excitons be found? [4 marks]

5. The complex dielectric constant in a polar solid is frequency dependent and can be written in the form

$$\epsilon_r(\omega) = 1 + \chi + \frac{Nq^2}{\epsilon_0\mu} \frac{1}{(\Omega_{TO}^2 - \omega^2 - i\gamma\omega)},$$

where the symbols have their usual meaning.

- (a) Simplify the above equation to the following form

$$\epsilon_r(\omega) = \epsilon_\infty + (\epsilon_{st} - \epsilon_\infty) \frac{\Omega_{TO}^2}{(\Omega_{TO}^2 - \omega^2 - i\gamma\omega)}$$

and explain the significance of ϵ_{st} , ϵ_∞ , Ω_{TO} and γ . [5 marks]

- (b) From the expression given in (a) and by employing an appropriate assumption, derive the Lyddane-Sachs-Teller (LST) relationship

$$\frac{\Omega_{LO}}{\Omega_{TO}} = \left(\frac{\epsilon_{st}}{\epsilon_\infty} \right)^{1/2}$$

where the symbols have their usual meaning. [5 marks]

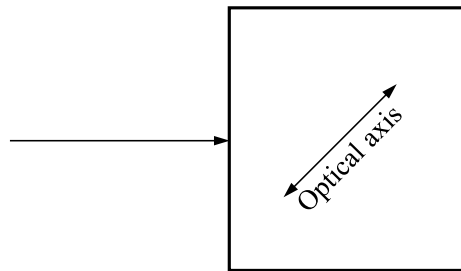
- (c) Find expressions for both the real ($\epsilon_1(\omega)$) and imaginary ($\epsilon_2(\omega)$) parts of the dielectric constant from the expression given in (a). [4 marks]
- (d) For NaCl, $\epsilon_{st} = 5.9$ and $\epsilon_\infty = 2.25$, and the TO phonon frequency is $\nu_{TO} = 4.9$ THz. The value of γ can be considered small but finite. Explain the form of $\epsilon_1(\nu)$ and indicate key values on a sketch. In particular, identify the Reststrahlen band frequency region and explain what happens to the reflectivity in this region. [6 marks]

6. Calcite has refractive indices for ordinary and extraordinary rays as $n_o = 1.6584$, $n_e = 1.4864$, respectively.

- (a) Consider an unpolarized monochromatic light beam normally incident on a birefringent crystal with the optical axis at an angle θ from the normal. Show that the angle between the refracted e-ray and the optical axis is given by the following expression. [7 marks]

$$\varphi = \arctan \left(\frac{n_o^2}{n_e^2} \tan \theta \right)$$

- (b) Consider an unpolarized monochromatic light beam normally incident on a calcite crystal with its optical axis at 45° from the surface normal (figure below). Calculate the angle difference between the o-ray and e-ray. [4 marks]



- (c) A calcite crystal plate with thickness of 0.04 mm is placed between two orthogonally aligned linear polarizers. The optical axis of the calcite crystal is in its surface plane and forming an angle γ with the polarization direction of the first polarizer ($\gamma \neq 0^\circ, 90^\circ$). Which wavelengths within the visible light range are completely blocked by this optical setup? [9 marks]

SECTION C: MODERN ATOMIC AND OPTICAL PHYSICSAnswer Question 7 and **either** Question 8 **or** Question 9.

7. (a) In the presence of a magnetic field B , the energy shifts for the components of the ground state of an alkali metal atom having angular momentum quantum number m_F along the field direction are

$$\Delta E = \pm \frac{A}{2} \mp \frac{A}{2} \sqrt{1 + \frac{8m_F\mu_B B}{A(2I+1)} + \frac{4\mu_B^2 B^2}{A^2}}.$$

A is the zero-field hyperfine splitting, I is the nuclear spin, μ_B is the Bohr magneton and B is the magnetic field. For what values of m_F is a linear Zeeman effect obtained? [4 marks]

- (b) Draw an energy-level diagram showing all the magnetic sub-levels in the ground state of an alkali atom with nuclear spin $I = 5/2$. [4 marks]
- (c) Ignoring spin, arrange the following atomic states in terms of increasing magnetic moment along the z-axis: $3d(m=2)$; $3p(m=0)$ and $4p(m=1)$. Justify your answer. [4 marks]
- (d) Orbitals of the hydrogen atom $\psi_{nlm}(r, \theta, \phi)$ are the product of an orbital angular momentum eigenfunction, $Y_{l,m}$, and a radial eigenfunction, $R_{n,l}$. Show graphically that the expectation value of the dipole moment operator $\hat{d} = -ez$ is zero for the orbital

$$\psi_{2p1}(r, \theta, \phi) = Y_{1,+1} R_{2,1} = \frac{1}{4\sqrt{2\pi}a_0^{3/2}} \frac{r}{a_0} \sin\theta e^{i\phi} e^{-r/2a_0}.$$

[4 marks]

- (e) Draw a schematic of an atomic beam clock using the following components: a feedback servo; an atomic beam; a microwave cavity; an oscillation counter; an atom detector and a microwave radiation source. Use your schematic to explain how such a clock can be used to measure time. [4 marks]

8. Transitions between the $1s$ and the $2p$ states of the hydrogen atom are electric-dipole allowed. You can ignore fine structure.

- (a) Explain why the $2p$ state of the hydrogen atom undergoes a spontaneous transition to the $1s$ state. [2 marks]
- (b) Explain the term *electric-dipole allowed*. [3 marks]
- (c) Selection rules for electric-dipole allowed transitions include $\Delta l = \pm 1$ and $\Delta m_l = 0, \pm 1$.
 - (i) Explain briefly why a transition with $\Delta l = 0$ is not electric-dipole allowed. [1 mark]
 - (ii) With the aid of an energy level diagram, indicate the electric-dipole allowed transitions between the $1s$ and $2p$ states of the hydrogen atom. Label each transition with linear and/or circular polarisations (with respect to a vertical quantisation axis) for which it is allowed. [4 marks]
- (d) During a transition between the $1s$ and $2p$ states of the hydrogen atom, the time-dependent wave function of the system at time $t = 0$ is

$$\psi(0) = \frac{1}{\sqrt{2}} (\psi_{1s} + \psi_{2p}).$$

- (i) Write down an expression for the wave function $\psi(t)$ at time t . [2 marks]
- (ii) The period of the phase evolution of the wavefunction is $T = 2\pi/\omega_0$, where ω_0 is the angular frequency of the transition. Derive an expression for the wave function $\psi(T_{1/4})$, where $T_{1/4}$ is one quarter of the period. [2 marks]
- (iii) Hence, derive expressions for the modulus squared of the wave function, $|\psi(t)|^2$ at $t = T_{1/4}$, $t = 2T_{1/4}$ and $t = 3T_{1/4}$. [6 marks]

9. The $3s^2S_{1/2} \rightarrow 3p^2P_{3/2}$ transition in ^{23}Na at an angular frequency of $\omega_0 = 2\pi \times 5.09 \times 10^{14}$ Hz (589 nm) is used for laser cooling. The lifetime of the $3p^2P_{3/2}$ state is 16.2 ns.

- Explain briefly why a resonant laser beam exerts a force on atoms. [4 marks]
- The maximum force experienced by an atom is $F_{\text{max}} = R_{\text{scatt}}\hbar k$, where k is the photon wavevector and R_{scatt} is the scattering rate. Calculate the value of the maximum force experienced by a ^{23}Na atom in a single, resonant laser beam. [3 marks]
- Calculate the detuning required to bring a laser beam into resonance with a ^{23}Na atom travelling at $v = 500$ m s $^{-1}$ towards the laser. [2 marks]
- In two counter-propagating laser beams, the force along the direction of the laser beams experienced by an atom due to each laser beam, F_+ and F_- , is given by

$$F_{\pm} = \pm \frac{\hbar k \Gamma}{2} \frac{I/I_{\text{sat}}}{1 + 2I/I_{\text{sat}} + 4(\delta \mp kv)^2/\Gamma^2},$$

where I/I_{sat} is the ratio of the laser intensity to the saturation intensity, δ is the detuning, k is the wavevector, v is the atom velocity and Γ is the linewidth. The total force an atom experiences in both laser beams is F_{molasses} .

- On the same diagram, sketch F_+ , F_- and F_{molasses} versus v . [3 marks]
- For small values of v , i.e. $|kv/\Gamma| < 1$, show that F_{molasses} is linearly proportional with respect to v . [8 marks]

$$\left[\begin{array}{l} \text{Hint: You may find the following relationship useful:} \\ 1/(A - Bx) - 1/(A + Bx) = 2Bx/(A^2 - B^2x^2) \end{array} \right]$$