Foundation of Physics 2B/3C Optics 2019-20

O.WP.5 Fresnel and Fraunhofer

March 5, 2020

1. *Cartesian separability*

   (i) The field at $y = 0$ is

   $$E^{(z)} = \frac{E_0}{i\lambda z}\mathrm{e}^{ikz}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x')e^{ik(x-x')^2/2z}e^{iky'^2/2z}\mathrm{d}x'\mathrm{d}y'^{[1]},$$

   We can separate the x' and y' integrals

   $$E^{(z)} = \frac{E_0}{i\lambda z}\mathrm{e}^{ikz}\int_{-\infty}^{\infty} f(x')e^{ik(x-x')^2/2z}\mathrm{d}x'\int_{-\infty}^{\infty} e^{iky'^2/2z}\mathrm{d}y'^{[1]},$$

   and using $k = 2\pi/\lambda$ we can write $iky'^2/2z = -\pi/(i\lambda z)$ and then use the hint [1] to get

   $$E^{(z)} = \frac{E_0}{\sqrt{i\lambda z}}\mathrm{e}^{ikz}\int_{-\infty}^{\infty} f(x')e^{ik(x-x')^2/2z}\mathrm{d}x'.^{[1]}$$

   (ii) If the field is also uniform along x we can carry out the integral over x' in the same way as we did for y' above[1] , giving another factor of $\sqrt{i\lambda z}$[1] . Therefore

   $$E^{(z)} = E_0\mathrm{e}^{ikz\,[1]},$$

   which is the same as the incident field no aperture, but multiplied by the expected phase factor for propagation over a distance z in the z-direction[1] .

2. *Cartesian separability*

   Basic answer:

   So far we have considered the far-field limit of Fraunhofer diffraction to be $z \gg \rho'$[1] , which for slit(s) we write as $z \gg d$ where d is the slit separation. Here d is the distance between the atomic planes[1] , which is far smaller than z, the distance from the crystal to the plane (film, camera) where the diffraction pattern is observed (typically a few cm)[1] .

   Better answer using Lecture 9: The applicability of the far field limit can be quantified using the Rayleigh distance[1]

   $$d_\mathrm{R} = \frac{a^2}{\lambda} \approx \frac{(5\times 10^{-10})^2}{10^{-10}} \approx 2.5\times 10^{-9}\ \mathrm{m}^{[1]}.$$

   Thus $z \gg d_\mathrm{r}$ and the Fraunhofer regime applies[1] .