ELECTROMAGNETISM

Level 2 Physics problems – Foundations of physics 2

Question 5 Cycle 2 Version 1

Professor D P Hampshire – 2nd Year Physics Lecture Course

These problems are formatively self-assessed. Students who showed the chutzpah to volunteer for the peer-marking pilot scheme will also mark one of their peer's scripts.

Reading Material (Please note that questions may not be exclusively associated with these chapters); Possible reading Feynman Lectures in Physics Chapters 1-3 (Vector calculus). Note - the most important issue this week is still for you to refine your own procedure for reading and understanding science textbooks and making notes that will aid you in revision.

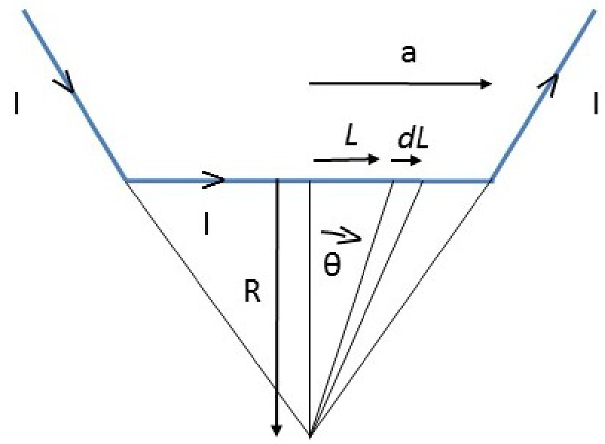
1. This week, I would like to recommend an electromagnetism activity appropriate when you are relaxing with family/friends. This can be fun in a bar or over lunch or perhaps when your friends start waxing lyrical about Shakespeare, Kant, football… [0 marks]
Start off by showing everyone a blank piece of paper. Then show them that you have nothing up your sleeves. Then write down Maxwell's four equations in differential form. Press on with aplomb to derive the velocity of light in a vacuum. Then take a few minutes to explain to the assembled audience why this is some of the most amazing and beautiful science humanity has ever completed. I can assure you that at the very least, the edges of their mouths will curl up. If something fun happens, let me know: d.p.hampshire@durham.ac.uk

2. The Biot-Savart law gives the steady field ($d\underline{\boldsymbol{B}}$) produced by a current element ($Id\underline{\boldsymbol{l}}$);

$$d\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{Id\underline{\boldsymbol{l}} \times \hat{\mathbf{r}}}{r^2}$$

a) Using the Biot-Savart law or otherwise, show that the magnetic field (B_P) at a point P, which is a distance R below the mid-point of a straight section of wire carrying current I as shown in the figure (below) and subtends an angle $2\theta_{Max}$, is given by; [1 mark]

$$B_P = \mu_0 I \sin\theta_{Max}/2\pi\text{R}$$



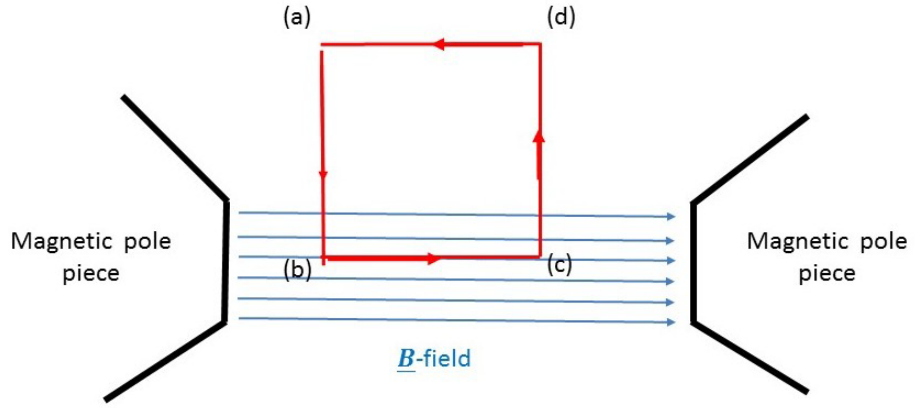
b) Use your result from part (a) to find the magnetic field at the centre of a polygon of N sides. Show that when N is very large, your result approaches that for a magnetic field at the centre of a circle. [1 mark]

3. A cable of circular cross-section and diameter 2 cm has a long cylindrical hole of diameter 1 mm drilled in it parallel to the cable axis. The distance between the axis of the hole and the cable is 5 mm. If the cable has a uniform steady current density of 10^5 Am^{-2} flowing in it, calculate the magnetic field;

a) At the centre of the cable. [1 mark]

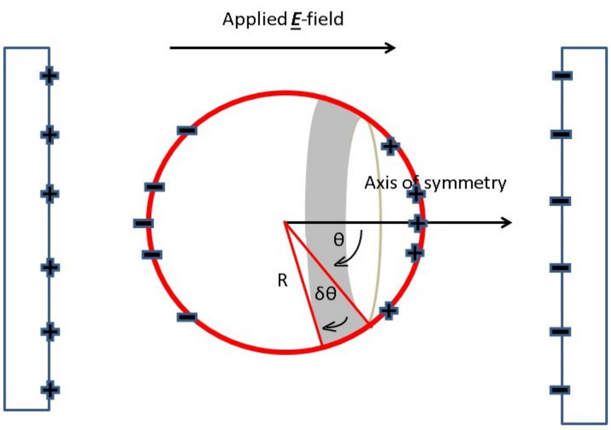
b) At the centre of the hole. [1 mark]

4. Show that a uniform field with no fringing field, such as that shown in the figure (below), is impossible because it violates Ampère's law. [1 mark]



[**Hint:** Do this by applying Ampère's law to the rectangular curve shown]

5. Consider a spherical dielectric with a uniform polarisation P in an applied field E_{applied} as shown in the figure (below).



The surface charge density, σ ,is given by,

$$\sigma = \underline{\boldsymbol{P}} \cdot \hat{\boldsymbol{n}} = P\cos\theta$$

a) Show that the net E-field, E_{net}, at the centre of the sphere (You may assume that the E-field inside the sphere is uniform) is given by, [3 marks]

$$E_{net} = E_{applied} - \frac{P}{3\varepsilon_o}$$

b) Use the definitions: The polarizability constant α, which for a single atom is defined using the electric dipole moment, p, where $p = \alpha E_{applied}$. The polarisation, P, where $P = Np$ where N is the number of dipoles per unit volume, and the relative dielectric constant ϵ_r where $P = \epsilon_0(\epsilon_r - 1)E$. Consider a lump of material where atoms interact and we can associate the space of each atom in the material to be a sphere. Using the result from a) or otherwise, derive the Clausius-Mossotti result; [2 marks]

$$\alpha = \frac{3\varepsilon_0}{n}\left\{\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right\}$$

[**Hint:** Take a look at Problem 4.38 in Griffiths]