Mathematical Methods II
Weekly problem set 6

(1) Consider the 2D wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right]$$

where k is a real consant. Show that it can be separated into two independent ODEs and form a general solution to the PDE.

(2) Solve the 1D heat equation for a long, thin metal rod of length L

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

using the boundary conditions for insulated end points, $u_x(0,t) = 0$, $u_x(L,t) = 0$, $t > 0$ and the initial condition $u(x,0) = x$ for $0 < x < L$.