Statistical Physics: Workshop Problems 3

(1) (a) Find the entropy of rolling an octahedral dice and a dodecahedral dice.

(b) How does the phase space density of a microcanonical ensemble in equilibrium change with time?

(c) What are the units of each the micro-canonical, the canonical and the grand-canonical partition functions?

(d) Which quantities are constant in the micro-canonical ensemble, the canonical ensemble and the grand-canonical ensemble?

(2) A molecule has 3 non-degenerate vibrational modes with frequencies ω, 2ω and 3ω.

(a) Calculate the vibrational partition function of the molecule and hence obtain the probability of each of the modes being excited when the molecule is in contact with a heat bath at temperature T.

(b) Determine the high and low temperature limits of the probabilities and sketch a graph showing the probabilities as a function of temperature.

(c) What is the internal energy and free energy of the molecule at temperature T. What the difference is between internal energy and free energy?

(3) A system contains non-degenerate states with energies 0, ϵ, 2ϵ, 3ϵ,... . 3 particles are distributed amongst these states such that the internal energy, U, of the system is 3ϵ. By tabulating the number of microstates in the possible distributions work out what is the probability of finding the most likely distribution of particles in the states if the particles are (a) classical, (b) Fermions and (c) Bosons?

(4) A system of classical particles occupying single-particle levels is in thermal contact with a heat reservoir at a temperature T. The population distribution in the three lowest non-degenerate energy levels is given below. What is the mean temperature of the system?

State	Energy (meV)	Population
3	21.5	8.5%
2	12.9	23.0%
1	4.3	63.0%