Cosmology Part I: Workshop II

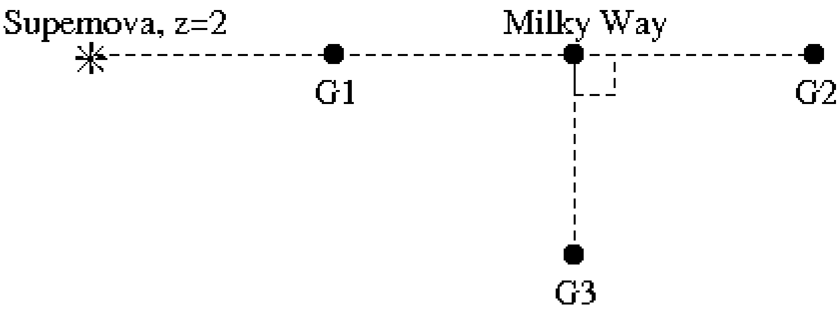
The following questions will test how well you have understood the key concepts underlying the theory of the expanding Universe.

1. Consider a matter-dominated flat Universe with Ω=1. At the current time, $t=t_0$, the expansion factor $a=a_0=1$. If an arbitrary galaxy emits light at arbitrary time t_1 which is subsequently received by a second galaxy at time t_2, show that the co-moving distance (i.e. that which would be physically measured if we halted the expansion at the present time t_0) between these two galaxies is given by:

$$X_{CM} = \frac{2c}{H_0}[a_2^{0.5} - a_1^{0.5}]$$

where a_2 and a_1 are the expansion factors at t_2 and t_1, respectively, and H_0 is the present-day Hubble parameter.

2. An astronomer observes a supernova explosion in a galaxy at redshift z=2. Galaxy G1 in the diagram below is located along the line of sight to the supernova but half-way in co-moving distance. Galaxies G2 and G3 are at the same co-moving distance from Earth as G1, but in the opposite and perpendicular directions, respectively. Calculate the observed redshifts of galaxies G1, G2 and G3 at the present time. [Assume the same cosmology as above].



3. Calculate the redshifts of the supernova as seen by observers in galaxies G1, G2 and G3.

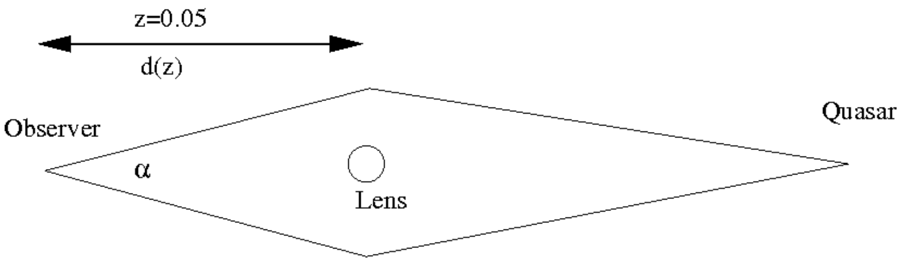
4. Calculate the elapsed cosmic time in Gyr between the observation of the supernova by observers G1 and G2 if H_0=70 km/s/Mpc.

5. [Optional] Show that all observers in the Universe, no matter how far they are away from the supernova, will eventually be able to see it.

The following question illustrates the use of gravitational lensing to measure H_0. You are given all the information you need in the question.

6. A galaxy at redshift z=0.05 acts as a gravitational lens and produces 2 images of a very distant quasar.

The diagram below illustrates the two paths that light takes from the source to the observer. Note that the 2 paths are not symmetric.



The difference in the length of the optical paths taken by the light-rays forming the 2 images is calculated to be $\Delta l = 0.42\ d \sin^2 \alpha$, where d is the co-moving distance to the lensing galaxy, and α is the angular separation of the 2 images.

In this case, α = 5 arcsec = 2.42×10^{-5} radians, and the time delay between receipt of the 2 images is Δt = 60 days = 5.18×10^{6} sec.

(a) Write down the relationship between co-moving distance of the galaxy, d, and the present-day Hubble constant (again assuming an $\Omega=1$ Universe). Note that z«1.

(b) By comparing the difference in light travel time with the difference in optical path length, estimate the value of H_0. [1 pc = 3.09×10^{16} m.]