QUANTUM MECHANICS 2 - WORKSHOP 7

A particle of mass M is trapped in a 3D isotropic harmonic potential $V(x,y,z) = m\omega^2(x^2 + y^2 + z^2)/2$ with unperturbed energy levels $E^0_{n_x,n_y,n_z} = (n_x + n_y + n_z + 3/2)\hbar\omega$. (The particle could be a single atom or ion — see, for example

http://www.physics.otago.ac.nz/nx/mikkel/single-atom.html,
http://www.physicscentral.com/buzz/blog/index.cfm?postid=7403268213516526572,
https://www.sciencedaily.com/releases/2010/10/101025090006.htm.)

Q1: The system is perturbed by a potential $H' = \lambda x^2 yz$ where λ is a constant. Use non-degenerate perturbation theory to calculate the energy shift of the ground state $E^1_{0,0,0} = \langle \psi^0_{0,0,0}|H'|\psi^0_{0,0,0}\rangle$ where $\psi^0_{0,0,0} = (a/\pi)^{1/4} e^{-ax^2/2} e^{-ay^2/2} e^{-az^2/2}$ for $a = M\omega/\hbar$.

Q2: The first excited state is triply degenerate with unperturbed wavefunctions

$$\begin{aligned}
\psi^0_1 \equiv \psi^0_{0,0,1} &= Aze^{-ax^2/2}e^{-ay^2/2}e^{-az^2/2} \\
\psi^0_2 \equiv \psi^0_{0,1,0} &= Aye^{-ax^2/2}e^{-ay^2/2}e^{-az^2/2} \\
\psi^0_3 \equiv \psi^0_{1,0,0} &= Axe^{-ax^2/2}e^{-ay^2/2}e^{-az^2/2}
\end{aligned}$$

where $A = (2a)^{1/2}(a/\pi)^{3/4}$. Evaluate the matrix elements $W_{jk} = \langle \psi^0_j|H'|\psi^0_k\rangle$ for the perturbation in Q1, where j,k take values $1,2,3$ denoting each wavefunction.

Q3: Solve the resulting matrix equation for all possible values of E^1

$$\begin{pmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = E^1 \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}.$$

Is there still degeneracy? Write down the three wavefunctions $\chi = \alpha\psi_1 + \beta\psi_2 + \gamma\psi_3$ corresponding to each possible energy.

Useful Integrals

$$\int_{-\infty}^{+\infty} ue^{-\alpha u^2} du = 0, \qquad \int_{-\infty}^{+\infty} u^2 e^{-\alpha u^2} du = \frac{1}{2}\left(\frac{\pi}{\alpha^3}\right)^{1/2}$$