# Relativistic Electrodynamics, Workshop 8

## Transformation of the ELectromagnetic Fields

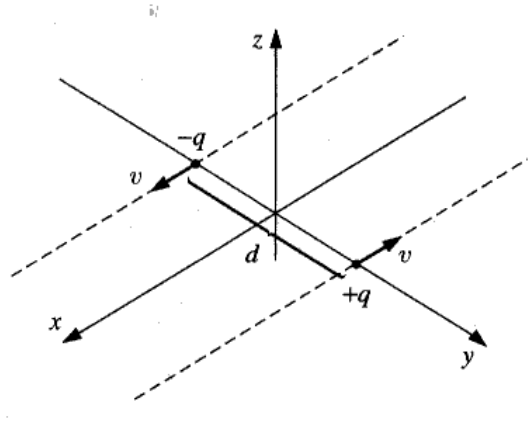
The electric and magnetic fields of a point charge moving with constant velocity (i.e. no acceleration) in terms of the *present* position (i.e. specifically not the retarded position) of the point charge is given by eq. 10.68 in Griffiths:

$$\underline{E}(\underline{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{1-v^2/c^2}{(1-v^2/c^2\sin^2\theta)^{3/2}}\frac{\hat{\underline{R}}_p}{R_p^2} \tag{1}$$

$$\underline{B}(\underline{r},t) = \frac{1}{c^2}(\underline{v}\times\underline{E}(\underline{r},t)) \tag{2}$$

where $\underline{R}_p$ is now the vector from the *present* position of the particle to $\underline{r}$, the position of the observer. $\theta$ is the angle between $\underline{R}_p$ and $\underline{v}$.

Two charges $\pm q$ are on parallel trajectories a distance $d$ apart in the $x-y$-plane. The charge $+q$ travels on a trajectory parallel to the $x$-axis, in the negative direction, and with a constant speed $v$. It crosses the $y$-axis at $d/2$. The charge $-q$ travels on a parallel trajectory, through $y=-d/2$, and with opposite velocity. We are interested in the force on $+q$ due to $-q$ at the the time $t_0$ where they cross their point of minimum separation $d$:




**1.** In the frame described above, calculate the $\underline{E}$-field and the $\underline{B}$-field at the position of $+q$ due to $-q$, at the time $t_0$.

**2.** Calculate the force $\underline{F}$ on $+q$ due to $-q$, at the time $t_0$.

**3.** Calculate the value at $+q$ for $\frac{1}{2}F^{\mu\nu}F_{\mu\nu}$, where $F^{\mu\nu}$ is the electro-magnetic field strength tensor for the fields generated by $-q$.

We now want to analyse the situation from the inertial frame $S'$, where $+q$ is at rest.

**4.** Calculate the velocity of $-q$ in this frame.

**5.** Find the Lorentz-transformed fields $\underline{E}'$ and $\underline{B}'$ in $S'$.

**6.** Calculate the force on $q+$ by $q-$ in $S'$.

**7.** Find the value of $\underline{E}'^2 - c^2\underline{B}'^2$.