The energy eigenfunctions of a particle in an infinite square well from $0 \leq x \leq L$ are

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right).$$

A particle in the well has an initial wave function which is **not** itself an energy eigenfunction,

$$\Psi(x, t=0) = A \sin^3\left(\frac{\pi x}{L}\right) \quad \text{where } A = \left(\frac{16}{5L}\right)^{1/2}$$

Look up any integrals you need at `http://www.wolframalpha.com/` .

i) $\Psi(x, 0)$ can be decomposed into a weighted sum of energy eigenfunctions so that

$$\Psi(x, 0) = \sum_n c_n \psi_n(x),$$

where $c_n = \int \psi_n^*(x)\Psi(x, 0)dx$. Use Wolfram alpha to do this integral and determine c_n for each of n=1,2,3 and 4. [5 marks]

ii) Use the trigonometric identity $\sin^3 x = \frac{1}{4}[3 \sin x - \sin(3x)]$ to show that

$$\Psi(x, 0) = (3\psi_1 - \psi_3)/\sqrt{10}.$$

Hence determine c_n analytically and check your answers above. [2 marks]

iii) What is the probability that a measurement of energy gives the value E_1? [1 mark]

iv) What is the expectation value of the energy $\langle H \rangle = \sum_n c_n^2 E_n$? Give your answer in terms of E_1. [1 mark]

v) Write down the fully time dependent wave function $\Psi(x, t)$ [1 mark]