

# Foundations of Physics 2B/3C 2015/2016

## Thermodynamics – Summer Examinations 2016

### Short Questions Vetted

- a) System property described by exact differentials, and  $\left(\frac{\partial^2 z}{\partial x \partial y}\right) = \left(\frac{\partial^2 z}{\partial y \partial x}\right)$ .

[2 marks – conceptual]

For the gas,  $dp = \left(\frac{\partial p}{\partial T}\right)_V dT + \left(\frac{\partial p}{\partial V}\right)_T dV$ , s

$$\left(\frac{\partial^2 p}{\partial V \partial T}\right) = -\frac{R}{(V-b)^2} \quad ; \quad \left(\frac{\partial^2 p}{\partial T \partial V}\right) = -\frac{R}{(V-b)^2}$$

Identical, hence  $p$  is a function of state.

For the magnet,  $dM = \left(\frac{\partial M}{\partial H}\right)_T dH + \left(\frac{\partial M}{\partial T}\right)_H dT$ ,

$$\left(\frac{\partial^2 M}{\partial T \partial H}\right) = -\frac{\gamma}{T^2} \quad ; \quad \left(\frac{\partial^2 M}{\partial H \partial T}\right) = -\frac{\gamma}{T^3}$$

Not identical, so  $M$  cannot be a function of state.

[2 marks – conceptual]

- b) Isothermal heat input at high temperature heat rejection at some lower temperature, joined by two adiabatic processes.

[1 mark - synoptic]

[1 mark – synoptic, curves need labelling]

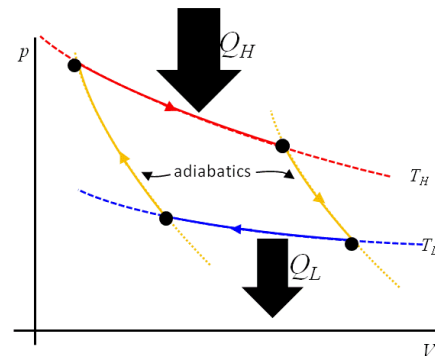
No engine operating between two heat reservoirs can be more efficient than a Carnot cycle.

[1 mark - conceptual]

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{293}{572} = 48.9 \%,$$

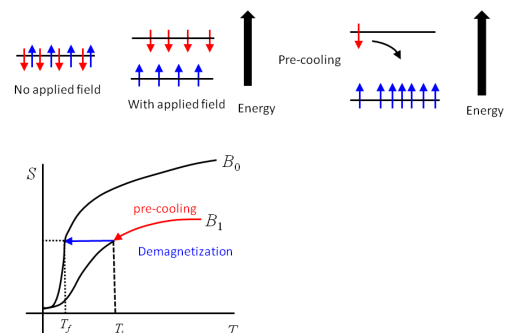
the maximum engine efficiency. Efficiency of 60.0 % is impossible.

[1 mark - conceptual]



- c) [Figure 2 marks – synoptic]

Spin-up more energetically favourable. Pre-cooling forces the spins into ordered spin-up state. As the field is adiabatically decreased, the spins return to their original, more disordered state, but overall entropy constant. Entropy of the atoms/molecules carrying the spins must decrease leading to a cooling.



[2 marks - synoptic]

- d) How much heat energy,  $\delta Q$  is required to raise the temperature of the substance by an amount  $dT$ ,  $C_\alpha = \left(\frac{\partial Q}{\partial T}\right)_\alpha$ .

**[1 mark - synoptic]**

$$TdS = \delta Q, \text{ so } C_\alpha = \left(\frac{\partial(TdS)}{\partial T}\right)_\alpha = T \left(\frac{\partial S}{\partial T}\right)_\alpha.$$

**[1 mark - conceptual]**

$$dU = \delta Q - pdV \text{ and } dV = 0, \text{ hence } dU = \delta Q \quad C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V.$$

**[2 marks - conceptual]**

# Foundations of Physics 2B/3C 2015/2016

## Thermodynamics – Summer Examinations 2016

### Long Question 1 – Vetted

a)  $I = T_0 \Delta S_U,$

[1 mark]

$$m^{Fe} c_p^{Fe} (T_E - 700) = -m^{Al} c_p^{Al} (T_E - 400)$$

$$T_E = \frac{700 m^{Fe} c_p^{Fe} + 400 m^{Al} c_p^{Al}}{m^{Fe} c_p^{Fe} + m^{Al} c_p^{Al}} = 519 \text{ K.}$$

[1 mark]

$$\Delta S_{Fe} = \int_{700}^{T_E} \frac{m^{Fe} c_p^{Fe} dT}{T} = 10 \times 450 \ln \left( \frac{519}{700} \right) = -1.34 \text{ kJ K}^{-1} ;$$

$$\Delta S_{Al} 1.78 \text{ kJ K}^{-1} \Rightarrow \Delta S_U = 436 \text{ J K}^{-1}.$$

[2 marks]

$$I = 131 \text{ kJ.}$$

[1 mark]

b)  $S = k_B \ln \Omega$ , where  $\Omega = \frac{N!}{\prod_j n_j!}$

[1 mark]

$$\Omega = \frac{N_A!}{\left(\frac{10N_A}{16}\right)! \left(\frac{5N_A}{16}\right)! \left(\frac{N_A}{16}\right)!}$$

[1 mark]

$$S = k_B \ln \left[ \frac{N_A!}{\left(\frac{10N_A}{16}\right)! \left(\frac{5N_A}{16}\right)! \left(\frac{N_A}{16}\right)!} \right] = k_B \left[ \ln N_A! - \ln \left(\frac{10N_A}{16}\right)! - \ln \left(\frac{5N_A}{16}\right)! - \ln \left(\frac{N_A}{16}\right)! \right]$$

$$= k_B \left[ N_A \ln N_A - N_A - \frac{10N_A}{16} \ln \left(\frac{10N_A}{16}\right) + \frac{10N_A}{16} - \frac{5N_A}{16} \ln \left(\frac{5N_A}{16}\right) + \frac{5N_A}{16} - \frac{N_A}{16} \ln \left(\frac{N_A}{16}\right) + \frac{N_A}{16} \right]$$

$$= k_B \left[ N_A \ln N_A - \frac{10N_A}{16} \ln \left(\frac{10N_A}{16}\right) - \frac{5N_A}{16} \ln \left(\frac{5N_A}{16}\right) - \frac{N_A}{16} \ln \left(\frac{N_A}{16}\right) \right].$$

[2 marks]

$$S = k_B N_A \ln \left[ \frac{N_A}{\left(\frac{10N_A}{16}\right)^{10/16} \left(\frac{5N_A}{16}\right)^{5/16} \left(\frac{N_A}{16}\right)^{1/16}} \right] = 6.90 \text{ J K}^{-1}$$

[2 marks]

c) Reciprocity theorem says

$$\left( \frac{\partial T}{\partial V} \right)_S = - \left( \frac{\partial T}{\partial S} \right)_V \left( \frac{\partial S}{\partial V} \right)_T.$$

[1 mark]

$$C_V = T \left( \frac{\partial S}{\partial T} \right)_V \text{ and using Maxwell Relation on the second term.}$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{\left(\frac{\partial S}{\partial T}\right)_V} \left(\frac{\partial p}{\partial T}\right)_V = -\frac{T}{C_V} \left(\frac{\partial p}{\partial T}\right)_V.$$

[2 marks]

For an ideal gas,  $\left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V}$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{TR}{VC_V}.$$

[1 mark]

$$\int_{T_1}^{T_2} \frac{dT}{T} = -\frac{R}{C_V} \int_{V_1}^{V_2} \frac{dV}{V} \Rightarrow \ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{V_1}{V_2}\right)^{R/C_V}.$$

$\gamma = C_p/C_V$  and  $C_p - C_V = R$ , so  $\frac{R}{C_V} = \gamma - 1$ .

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow TV^{\gamma-1} = \text{Const} \Rightarrow pV^\gamma = \text{Constant}.$$

[2 marks]

Isotherm has,  $pV = RT_0$ , adiabatic has  $pV^\gamma = \text{Constant}$ .

$$\left(\frac{\partial p}{\partial V}\right)_{T_0} = -\frac{RT_0}{V^2} = -\frac{p}{V} ; \quad \left(\frac{\partial p}{\partial V}\right)_S = -\frac{\gamma \times \text{Constant}}{V^{\gamma+1}} = -\gamma \frac{p}{V}.$$

[2 marks]

$\gamma > 1$  so the adiabatic is always steeper, by a factor of  $\gamma$ ,  $\left(\frac{\partial p}{\partial V}\right)_S = +\gamma \left(\frac{\partial p}{\partial V}\right)_{T_0}$ ,

[1 mark]

## Foundations of Physics 2B/3C 2015/2016

### Thermodynamics – Summer Examinations 2016

#### Long Question 2 – Vetted

a)  $dG = dU - TdS - SdT - fdx - xdf$ , using,  $dU = TdS - fdx \Rightarrow dG = -SdT - xdf$ .

[1 mark]

$G = G(T, f)$ , so  $dG = \left(\frac{\partial G}{\partial T}\right)_f dT + \left(\frac{\partial G}{\partial f}\right)_T df$  and Gibbs is a function of state.

[1 mark]

$$-S = \left(\frac{\partial G}{\partial T}\right)_f ; -x = \left(\frac{\partial G}{\partial f}\right)_T \Rightarrow \left(\frac{\partial(-S)}{\partial f}\right)_T = \left(\frac{\partial(-x)}{\partial T}\right)_f \Rightarrow \left(\frac{\partial S}{\partial f}\right)_T = \left(\frac{\partial x}{\partial T}\right)_f.$$

[2 marks]

$$dS = \left(\frac{\partial S}{\partial T}\right)_f dT + \left(\frac{\partial S}{\partial f}\right)_T df \Rightarrow TdS = T\left(\frac{\partial S}{\partial T}\right)_f dT + T\left(\frac{\partial S}{\partial f}\right)_T df.$$

[1 mark]

$C_f = T\left(\frac{\partial S}{\partial T}\right)_f$ , and using the Maxwell Relation to the second term,

$$TdS = C_f dT + T\left(\frac{\partial x}{\partial T}\right)_f df.$$

[2 marks]

Adiabat has  $dS = 0$ , so  $0 = C_f dT + T\alpha_f df$ .

[1 mark]

$$\int \frac{dT}{T} = -\frac{\alpha_f}{C_f} \int df \Rightarrow \ln T = -\frac{\alpha_f}{C_f} f + \text{Const} \Rightarrow T = A \exp\left(-\frac{\alpha_f f}{C_f}\right).$$

For the specific case of the elastic band,  $T = A \exp(kf)$ , with  $k$  a positive constant. Increasing the tension therefore exponentially increases the temperature.

[2 marks]

b)  $dG = dU - TdS - SdT + pdV + Vdp = -SdT + Vdp$ ,

[1 mark]

Phase change, the pressure is constant,  $\left(\frac{\partial G}{\partial T}\right)_p = -S$ .

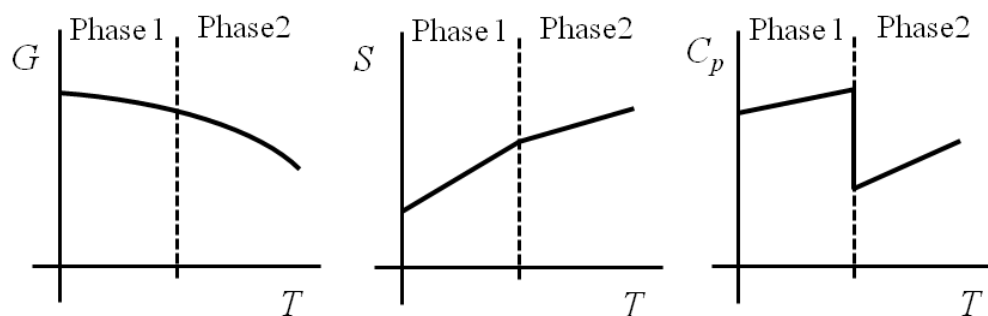
$$\left(\frac{\partial G_1}{\partial T}\right)_p = -S_1 \neq -S_2 = \left(\frac{\partial G_2}{\partial T}\right)_p.$$

[1 mark]

Third figure is the derivative of the second figure  $\left(\frac{\partial^2 G}{\partial T^2}\right)_p = -\left(\frac{\partial S}{\partial T}\right)_p = -\frac{C_p}{T}$  so the missing axis label is  $C_p$ .

[1 mark]

Second order change has a discontinuous second derivative



[2 marks –  $S$  and  $C_p$  are of opposite sign to  $G$ ]

Total derivative of the Gibbs function  $V = \left(\frac{\partial G}{\partial p}\right)_T$ .

[1 mark]

$$V = p_0 \frac{RT}{p} + A.$$

[1 mark]

$$p(V - A) = p_0 RT.$$

$A$  thus acts to reduce the volume of the gas, the gas molecules must have size.

[1 mark]

$$S = -\left(\frac{\partial G}{\partial T}\right)_p = -R \ln\left(\frac{p}{p_0}\right).$$

[1 mark]

$$S(4p) - S(p) = -R \ln\left(\frac{4p}{p_0}\right) + R \ln\left(\frac{p}{p_0}\right) = R \ln\left(\frac{1}{4}\right) = -R \ln(4)$$

Negative entropy change, compression decreases the gas volume so order is increased.

[1 mark]

**Examination Questions May/June 2016****Foundations of Physics 2B, Condensed Matter Physics, Q4 (5 parts):****SOLUTIONS**

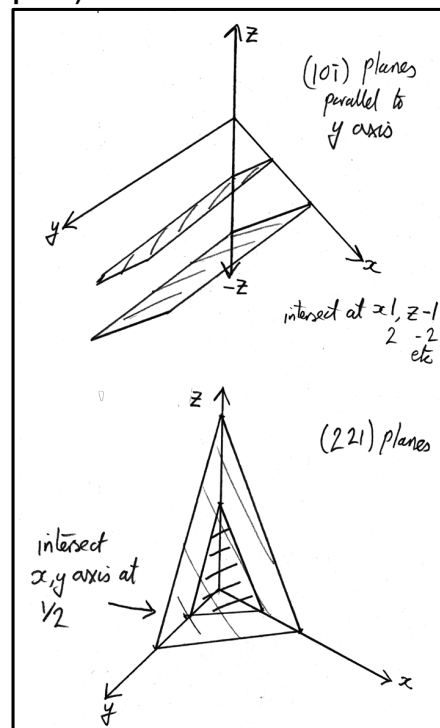
Synopsis

- a. Sketch shown in diagram. The spacing of each of the families of planes is given by:

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

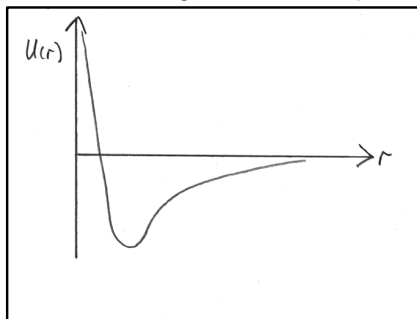
Problem

where  $a$  is the lattice constant (0.4 nm). Substituting in the  $(hkl)$  values gives: 0.28 nm and 0.18 nm  
2 marks for diagram and 1 mark for each answer.  
**[4 marks]**



Synopsis

- b. Sketch shows general form of potential (1 mark for correct shape of curve). The attractive component is due to electric dipoles (induced or permanent), the repulsive component is the quantum repulsion due to Pauli exclusion principle. Empirical relationship is Lennard Jones 6-12 potential written in general



form as  $U(r) = C \left[ -\left(\frac{\sigma}{r}\right)^6 + \left(\frac{\sigma}{r}\right)^{12} \right]$  **[4 marks]**

Conceptual

- c. The Drude model treats motion in electric field as drift. Assumes constant electric field produces constant acceleration, drift velocity increases linearly. After a certain time electron undergoes a collision and drift velocity drops to zero. Individual scattering times vary as it is a random process. The mean scattering time is used as a single parameter used to quantify the average time between

collisions, denoted by  $\tau$ . Drude conductivity is given by  $\sigma = \frac{ne^2\tau}{m_e}$ . **[4 marks]**

Synopsis

- d. The Bloch Theorem applies to electrons in a periodic potential (representing the crystal). Theorem indicates that wavefunction solution is composed of two parts: i) Bloch function which has periodicity of crystal lattice and ii) the wavefunction  $\psi(x)$ . This wavefunction  $\psi(x)$  is the same as the solution for free electrons and is a plane wave. The effect of the periodic potential is to modulate the intensity of the free electron plane waves through the Bloch function. (This modulation and mixing of free electron waves produces the nearly-free electron bandstructure with energy gaps. **[4 marks]**)

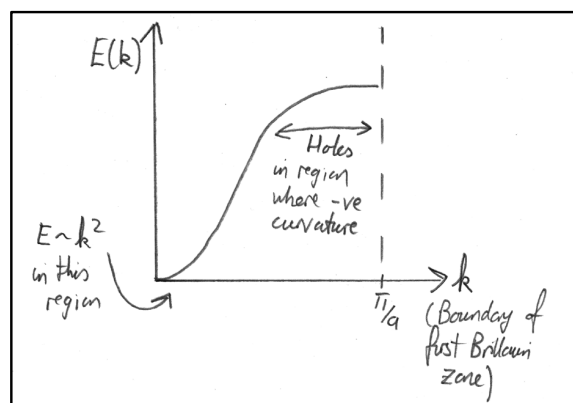
Conceptual

- e. Sketch shows general shape (1 mark for correct shape). The effective mass is determined from the inverse curvature of the  $E(k)$  curve using

$$m_e^* = \hbar^2 \left( \frac{d^2 E}{dk^2} \right)^{-1}$$

Effective mass is positive at

low  $k$  values increasing as  $k$  increases. It becomes negative at larger  $k$  as  $k$  approaches the first Brillouin zone boundary. It is in this region that holes occurs (corresponding to regions of negative effective mass). **[4 marks]**



**Examination Questions May/June 2016**  
**Foundations of Physics 2B, Condensed Matter Physics, Q5**  
**SOLUTION**

Problem

- a) The structure factor expression uses the coordinates of the unique atoms in the unit cell to determine the intensity of a scattered X-ray peak for each set of planes ( $h k l$ ) allowed by the Bragg Law. [1] The bcc lattice has two unique atoms at coordinates : (0 0 0) and ( $\frac{1}{2}\frac{1}{2}\frac{1}{2}$ ). [1] The structure factor expression is the sum over two atoms which becomes:

$$S_G(h k l) = f \left[ 1 + \exp \left[ -i\pi (h + k + l) \right] \right] \quad [1]$$

where  $f$  is the form factor for one atom. Evaluating the exponential means:

$$\mathbf{S = 0 \text{ when } h + k + l = \text{odd integer}}$$

$$\mathbf{S = 2f \text{ when } h + k + l = \text{even integer}} \quad [2]$$

These are the *structure factor rules* for the bcc lattice. They indicate which sets of Miller indices are missing from the X-ray diffraction pattern due to destructive interference within the unit cell. [1] [6 marks]

Problem

- b) Bragg Law is that  $\lambda = 2d \sin \theta$ . [1]

Lattice spacing for a cubic lattice is  $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{a}{\sqrt{N}}$  where  $N = h^2 + k^2 + l^2$  [1]

The structure factor rules specify which  $N$  values produce peaks.

Applying the structure factor rules means that the  $N$  values for the first five peaks are  $N = 2, 4, 6, 8$  and  $10$ . This corresponds to planes (110), (200), (211), (220) and (310). [1]

Rearranging relationships for  $\sin \theta$  gives  $\sin \theta = \frac{\lambda \sqrt{N}}{2a}$  [1]

Using this relationship gives:

Peak	$N$	$\sin \theta$	$\theta$	$2\theta$
1	2	0.2652	15.38	30.8
2	4	0.3750	22.02	44.0
3	6	0.4593	27.34	54.7
4	8	0.5303	32.03	64.1
5	10	0.5929	36.36	72.7

[2] for correct values of  $2\theta$

[6 marks]

Problem

- c) At temperature of 310 K structure will be fcc instead of bcc. *Assumption* is that the lattice constant will be the same as that at 300 K [1].

Must now use structure factor rules for fcc lattice. These are (OK just to assume these – given in lectures

$$\mathbf{S=0 \text{ when } hkl \text{ mixed parity}}$$

$$\mathbf{S=4f \text{ when } hkl \text{ all even or all odd}} \quad [2]$$

Applying the fcc rules means that the  $N$  values of the first five peaks are now 3, 4, 8, 11 and 12 [1]

This gives

Peak	$N$	$\sin \theta$	$\theta$	$2\theta$
1	3	0.3248	18.95	37.9
2	4	0.3750	22.02	44.0
3	8	0.5303	32.03	64.1
4	11	0.6219	38.45	76.9
5	12	0.6495	40.51	81.0

[2] for correct values of  $2\theta$

[6 marks]

Problem

- d) It is known that metals undergo thermal expansion. Heating the metal powder to 500 K will increase the lattice constant  $a$ . [1] This will result in the peaks occurring at smaller angles (in proportion to the reduction in lattice constant) [1]

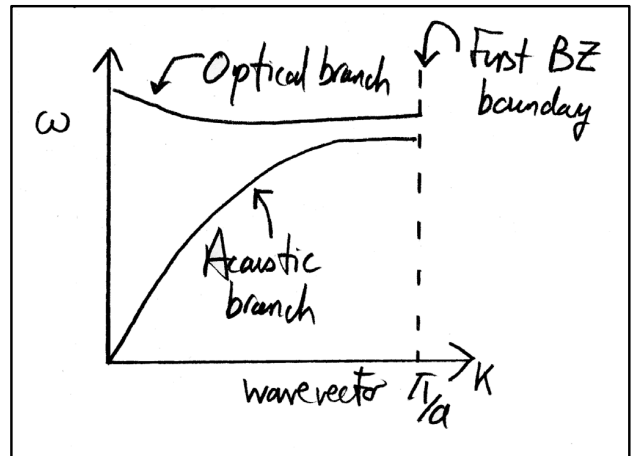
[2 marks]



**Examination Questions May/June 2016**  
**Foundations of Physics 2B, Condensed Matter Physics, Q6**  
**SOLUTION**

Synopsis

- a) Sketch shows two branches of the phonon dispersion curve. [2] marks for sketch and [1] for labels. The different curves are due to the different relative motion of the two atoms in the basis. Where the two atoms move in phase this is the *acoustic* mode, where the two atoms move out of phase this is the *optical* mode. [2] The acoustic branch is approximated using the *Debye* approximation having constant slope (or velocity) [1]. The *Einstein* approximation approximates the optical branch with a constant frequency or energy [1]



[7 marks]

Problem

- b) The group velocity is given by  $v_{\text{group}} = \frac{\partial \omega}{\partial K}$  [1]

Applying this to the dispersion relation gives  $v_{\text{group}} = \left( \frac{Ca^2}{M} \right)^{\frac{1}{2}} \cos \frac{Ka}{2}$  [1]

The group velocity of the acoustic phonon branch is equivalent to the velocity of sound when evaluated at the long wavelength limit defined by  $Ka \ll 1$  [1]

This gives  $v_{\text{group}} = \sqrt{\frac{C}{M}} a$ , giving  $v_{\text{group}} = 2664 \text{ ms}^{-1} \approx 2.7 \times 10^3 \text{ ms}^{-1}$  [1]

This value is substantially larger than the speed of sound in air (by a factor of ~8) and is typical of the speed of sound in metals. [1]

[5 marks]

Problem

- c) Sketch shows how group velocity varies across first Brillouin zone. [1] mark for sketch, [2] marks for appropriately labelled axes with values.

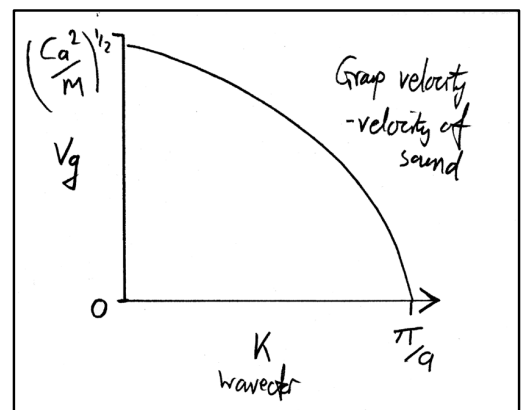
Value of velocity at  $K = \frac{\pi}{2a}$  is

$$v_{\text{group}} = \left( \frac{Ca^2}{M} \right)^{\frac{1}{2}} \cos \left( \frac{1}{2} a \frac{\pi}{2a} \right)$$

$$= \left( \frac{Ca^2}{M} \right)^{\frac{1}{2}} \cos \frac{\pi}{4} = \left( \frac{Ca^2}{2M} \right)^{\frac{1}{2}} \quad [1]$$

Substituting values gives  $1884 \text{ ms}^{-1}$  [1]

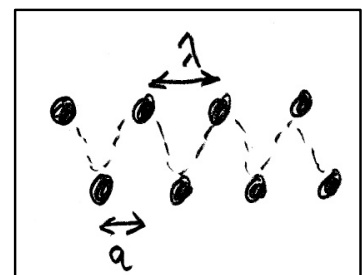
Velocity at Brillouin zone boundary is zero [1]



[6 marks]

Problem

- d) When the wavevector  $K$  is at the boundary of the first Brillouin zone this is equivalent to a wavelength of  $\lambda = 2a$ . Sketch (not required) shows situation. At this wavelength adjacent atoms are moving out of phase. This results in a standing wave that has zero group velocity. [2 marks]



## Q.7 Modern Optics 2016

- (a) [Bookwork] Spatial freq. ( $\omega$ ) no. of waves per unit length [1]  
 Ang. spatial freq. ( $k$ ) phase change per unit length [1]  
 Phasor  $e^{i2\pi ur}$  [1]

- (b) [Bookwork] Spherical wave  $E = E_0 \frac{e^{ikr}}{kr}$  [1]  
 [2]

Paraxial spherical wave  $E = E_0 \frac{e^{ikz}}{kz} e^{ik\rho^2/2z}$  [2]

- (c) [Bookwork] (i)  $G(u) - H(u)$  [1]  
 (ii)  $G(u) e^{-i2\pi ud}$  [1]  
 (iii)  $G(u) * H(u)$  [1]  
 (iv)  $G(u) H(v) \leftarrow$  note different Fourier variable! [1]

- (d) [Unseen] Using  $w = w_0 \left(1 + \frac{z^2}{z_R^2}\right)^{1/2}$   
 $z_R = \pi w_0^2 / \lambda = \pi (10^{-3})^2 / 5 \times 10^{-7} = 6.3 \text{ m}$  [2]  
 $w = w_0 \left(1 + \frac{z^2}{z_R^2}\right)^{1/2} = 1.6 \text{ mm}$  [1]

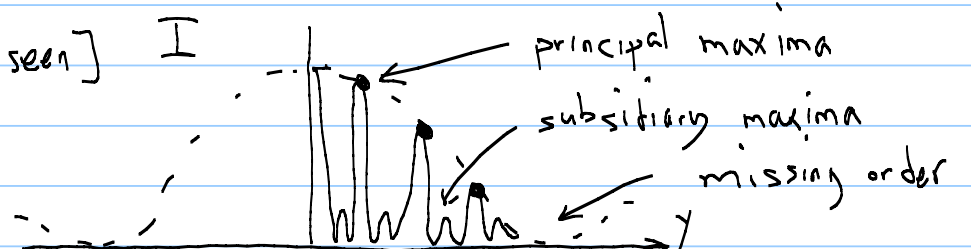
Far field  $z \gg z_R$  so this is NOT Par-field [1]

NB  $w = \frac{\lambda}{\pi w_0} z$  gives  $\frac{5 \times 10^{-7}}{3.1 \times 10^{-3}} 10 = 1.6 \text{ mm}$  as well!

- (e) [Unseen] The spacing between the fringes is  $\frac{f\lambda}{d}$  [2]

$$f\lambda/d = 0.15 \times 10^{-7} / 1 \times 10^{-3} = 5 \times 10^{-5} \text{ m} [1]$$

$\therefore$  visible on CCD camera. [1]

- (f) [Unseen]  [1]  
 [1]  
 [1]

4 order missing ratio of spacing  $d$  to width  $a$  is 4

$$\frac{E}{a} \neq \frac{d}{\lambda} [1]$$

## Q.8 Modern Optics

Plane wave:  $e^{i(k_x x + k_y y + k_z z)}$  planar wavefronts [1]

same everywhere [1]

Spherical waves  $e^{ikr}/r$  spherical wavefronts/define origin! [1]

Phasor  $e^{ikr}$  describes phase along a particular path [1]

$$\text{comb}_3\left(\frac{x}{2d}\right) - \text{comb}_2\left(\frac{x}{2d}\right) = \delta(x+2d) - \delta(x+d) + \delta(x) - \delta(x-d) + \delta(x-2d)$$

using  $\sum [\delta(x-d)] = e^{-ikd}$  [2]

$$F(k) = e^{i2kd} - e^{ikd} + 1 - e^{-ikd} + e^{-i2kd}$$
 [2]

where  $k_x = kx/z = 2\pi x/\lambda z$

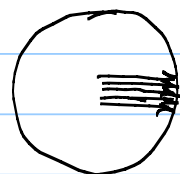
$$F(x) = e^{i4\pi xd/\lambda z} - e^{i2\pi xd/\lambda z} + 1 - e^{-i2\pi xd/\lambda z} + e^{-i4\pi xd/\lambda z}$$

$$x=0: 3 \text{ phasors } +1, 2 -1$$



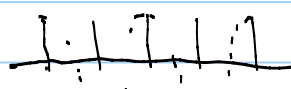
[2]

$$x = \lambda z/2d: \text{ all phasors are parallel!}$$



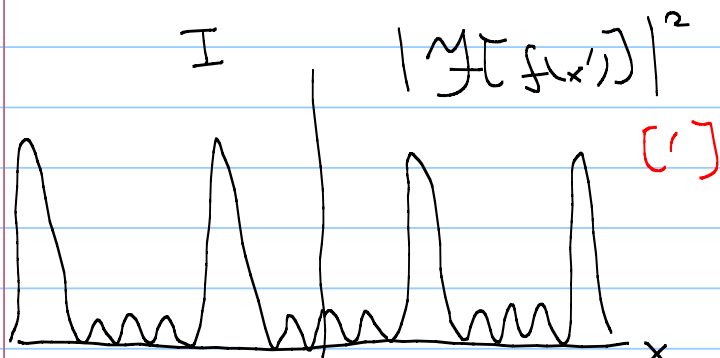
[2]

$$f(x) = \text{comb}_5\left(\frac{x}{d}\right) e^{i2\pi x/d}$$



[2]

$$FT = \sum \left[ \text{comb}_5\left(\frac{x}{d}\right) \right] * \delta\left(n - \frac{1}{d}\right) \text{ displaced comb}_5 \text{ pattern}$$
 [2]



[1]

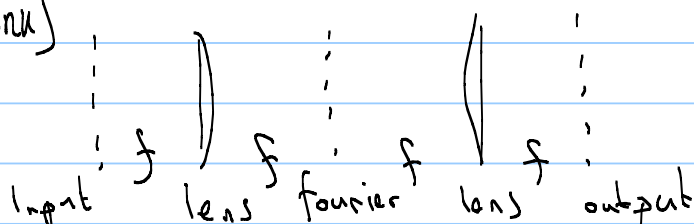


[1]

Displaced [1] 3 subsidiary maxima in both cases. [1]

## Q.9 Modern Optics 2016

[BOOKWORK]



[4]

[UNSEEN]

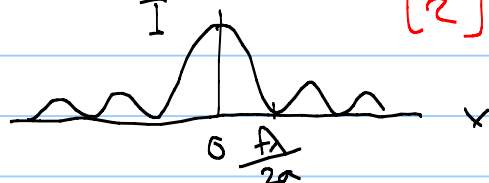
$$f(x) = \text{rect}(x/2a)$$

[1]

$$\begin{aligned} \text{In Fourier plane } E &= \frac{E_0}{\sqrt{i\lambda f}} \mathcal{F}[f(x')] = \frac{E_0}{\sqrt{i\lambda f}} 2a \text{sinc}(\pi u a) \\ &= \frac{E_0}{\sqrt{i\lambda f}} 2a \text{sinc}\left(\pi \frac{ax}{f\lambda}\right) \end{aligned}$$

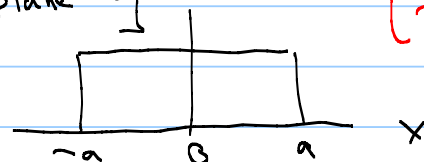
[1]

Fourier plane



[2]

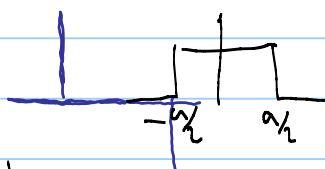
Output plane



[2]

$$\text{Modified field is } \text{rect}\left(\frac{x}{a}\right) * [\delta(x + \frac{a}{2}) - \delta(x - \frac{a}{2})]$$

[2]



Using convolution theorem

[1]

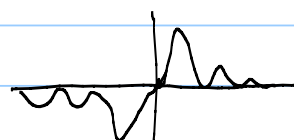
$$\begin{aligned} \text{Fourier plane } E &= \frac{E_0}{\sqrt{i\lambda f}} a \text{sinc}(\pi u a) \sin(\pi u a) \\ &= \frac{E_0}{\sqrt{i\lambda f}} a \text{sinc}\left(\pi \frac{ax}{f\lambda}\right) \sin\left(\frac{\pi ax}{f\lambda}\right) \end{aligned}$$

[2]

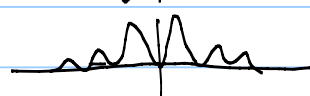
[1]

before filter

$$\frac{\sin^2}{x}$$



after filter



[1]

pattern is symmetric but with 0 on axis

∴ output must have equal + and -

CENTRAL ORDINATE THEOREM!

[1]

hence the 2nd filter does not restore the even parity of the field.