Foundations of Physics 2B/3C 2019-2020

Thermodynamics – Workshop 6 Problems

Week Commencing 18th November

1. Thermodynamics of Elastic Bands

It is possible to look at the thermodynamics of systems other than ideal gases. Here, the term for the work changes to something appropriate for the particular case (e.g. $\delta W = +f dx$ for a material that is stretched by an amount dx when placed under a force, f), and any instances of pV in the potentials (Enthalpy and Gibbs) are replaced by an appropriate term with appropriate sign, having units of energy, so -fx here. To solve you then use all the techniques we have already learned, but with some different variables. In this problem we will look at some of the thermodynamic properties of an elastic band.

a) By considering the *Helmholtz free energy* of an elastic system, of length x, which when placed under tension df, has work $\delta W = f dx$ done on it. Derive the following *Maxwell Relation* using this information

$$\left(\frac{\partial f}{\partial T}\right)_x = -\left(\frac{\partial S}{\partial x}\right)_T.$$

b) For a stretched rubber band, experimental observations show that the tension in the band is proportional to its temperature when the band's length is held constant. Determine how the entropy of an elastic band changes when it is isothermally stretched and contracted, commenting on your results in terms of the order in the rubber band.

[Hint: first consider what partial derivative the first sentence of the question is referring to.]

c) Show that the internal energy of an elastic band only depends on its temperature, $U = U(T)$, which means that the heat capacity at constant length can be expressed as $C_x = \left({\partial U}/{\partial T}\right)_x$ or $dU = C_x dT$. (This is analogous to being able to express the constant volume heat capacity of an ideal gas in terms of the internal energy). Using this result, show that if the band's length is increased adiabatically, the temperature must rise.