Condensed Matter Physics: Workshop 5 (5-9 March 2018)

Summary: In the May/June examination the 3-hour paper contains 15 short compulsory questions (divided among the three lecture courses: Thermodynamics, CMP and Optics). There will be five short questions on CMP. Short questions are worth 50% of the marks for the paper; each short question should be completed in around 6 minutes. The following are some short questions selected from previous year's examination papers covering the CMP course. For the workshop select those that interest you.

a. For a simple cubic lattice, sketch the planes with Miller indices (110) and (211). Include the x, y and z axes in your diagram. If the lattice constant a is 0.5 nm, determine the spacing between each of these two families of planes. **[4 marks]**

b. Calculate the fraction of space occupied by solid spheres in contact with each other packed in a face centred cubic lattice. **[4 marks]**

c. A metal comprises a single atom basis on each lattice point of a simple cubic lattice with a lattice constant of $a = 0.2$ nm. An X-ray scattering measurement is performed using X-rays of wavelength 0.1542 nm. Calculate the Bragg angles for scattering of X-rays from the (100) and (200) planes. Would you expect to see X-ray peaks from all possible planes in this crystal? **[4 marks]**

d. Describe the motion of atoms in a unit cell for transverse and longitudinal waves each having acoustic and optical modes of vibration. Illustrate your answer with a sketch of the phonon dispersion relation for the first Brillouin zone. **[4 marks]**

e. Silver has a free electron density of $n = 5.9 \times 10^{28}$ m^{-3} and an electrical conductivity of $\sigma = 6.2 \times 10^{7}$ Sm^{-1}. From the Drude model determine the mean time between collisions τ for electrons in this metal. Consider the motion of an individual electron under an applied electric field $\underline{E}$. Describe the behaviour of the velocity of this electron with time. **[4 marks]**

f. Explain the terms Fermi energy and Fermi surface in the context of the Sommerfeld free-electron model. What determines the value of the Fermi energy in a bulk 3D metal? **[4 marks]**

g. Describe van-der-Waals bonding. State how the bond energy is related to the bond length r. Give an example of an empirical function used to model the bond. **[4 marks]**

h. What assumptions form part of the Debye model of phonons in crystalline solids? The phonon dispersion relation for a simple cubic crystal is given by:

$$\omega(K) = \left(\frac{4C}{M}\right)^{1/2} \left|\sin\frac{1}{2}Ka\right|$$

where K is the phonon wavevector, a is the lattice constant, C is the interatomic force constant and M is the atomic mass. Obtain an expression for sound velocity in this system using the Debye model. **[4 marks]**

Corrected 05/03/18 Should be 10^{-8}

i. A metal with a single valence electron has a resistivity of 10^{-8} Ωm at T=300 K and an electron density of $1 \times 10^{28}\text{m}^{-3}$. Use the Drude model to calculate the mean free path for the electrons. State any assumptions you make. **[4 marks]**

j. Define the term Fermi energy. Copper has a free electron density of $8.4 \times 10^{28}\text{m}^{-3}$. Determine the value of the Fermi energy for Cu. Comment on your result in comparison to the thermal energy of electrons at room temperature. **[4 marks]**

k. Define the Hall coefficient. Aluminium has a Hall coefficient of $+1.02 \times 10^{-10}\text{m}^3\text{C}^{-1}$. Determine the density of charge carriers in Al. Explain why the Hall coefficient is positive and the implications of this for the free electron theory. **[4 marks]**