Foundation of Physics 2B/3C Optics 2019-20

O.WP.5 Fresnel and Fraunhofer

March 5, 2020

1. *Cartesian separability* The Fresnel diffraction integral is

$$E^{(z)} = \frac{E_0}{i\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') e^{ikr_\mathrm{p}} \mathrm{d}x' \mathrm{d}y',$$

   where $r_\mathrm{p} = z + \left[(x - x')^2 + (y - y')^2\right] / (2z)$ and $k = 2\pi/\lambda$. For an aperture that is uniform and infinite in the y directionn, we can write $f(x', y') = f(x')$.

   (i) Show that the field at $y = 0$ is given by

$$E^{(z)} = \frac{E_0}{\sqrt{i\lambda z}} \int_{-\infty}^{\infty} f(x') e^{ik(x-x')^2/(2z)} \mathrm{d}x'. \text{ [4 marks]}$$

$$\text{Hint:} \qquad \int_{-\infty}^{\infty} e^{\pi (y')^2/(i\lambda z)} \mathrm{d}y' = \sqrt{i\lambda z}$$

   (ii) What is the field along the z axis if the aperture is also infinite and uniform along the x direction? How does your answer compare to the incident field? [4 marks]

2. *Other wavelengths* A typical wavelength for X-Ray crystallography is of the order of 1×10^{-10} m, and a typical separation of planes in a crystal is of the order of a few $\times 10^{-10}$ m. Show therefore that the relevant regime for X-Ray crystallography is Fraunhofer. [3 marks]