# Cosmology: Problem Cos.1.1

Cosmology is plagued by odd units as a result of history and the immense distances and timescales involved. These questions will help you get used to converting between the different units.

## Definitions

How are the following quantities defined?

(i) the critical density

(ii) the density parameter

## Peculiar Velocity and the Hubble flow

A group of galaxies has total mass $M = 6 \times 10^{13} \mathrm{M}_\odot$ and radius $r = 0.8\mathrm{Mpc}$. Galaxies in the group will orbit with typical velocities $v = \sqrt{(GM/r)}$. This velocity is refered to as the "peculiar velocity". For galaxies that are close to us this velocity makes it difficult to detect the Hubble expansion of the universe.

Estimate the peculiar velocity in units of $\mathrm{km\,s^{-1}}$.

Estimate the distance out to which the peculiar velocity exceeds the velocity of the Hubble expansion if $H_0 = 75\mathrm{km\,s^{-1}Mpc^{-1}}$?

## Lookback time in a flat universe

Starting from the general form of the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2}$$

show that for a critical density ($k = 0$) matter dominated universe:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 a^{-3}$$

The age of the Earth is estimated to be 4.6 Gyr. For this flat Universe, calculate the cosmic epoch, espressed as a redshift, at which the Earth formed. Assume $H_0 = 75\mathrm{km\,s^{-1}Mpc^{-1}}$.

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## The $kc^2$ Constant

Consider a universe which is "open", ie. the density parameter, $\Omega$, is less than unity. Starting from the general form, show that the Friedmann equation can be conveniently written

$$H(t)^2 = H(t)^2\Omega(t) - \frac{kc^2}{a^2}.$$

Assuming $H_0 = 75\mathrm{km\,s^{-1}Mpc^{-1}}$ and $\Omega_0 = 0.3$, determine the value of $kc^2$.