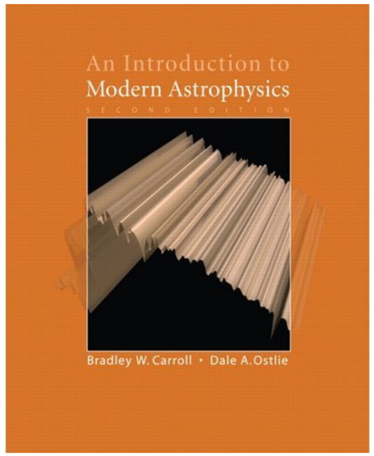
Lecture 12: Stellar structure –

Pulsating stars

Professor David Alexander
Ogden Centre West 119

Chapter 14 of Carroll and Ostlie



Aims of lecture

Key concept: physics of stellar pulsation

Aims:

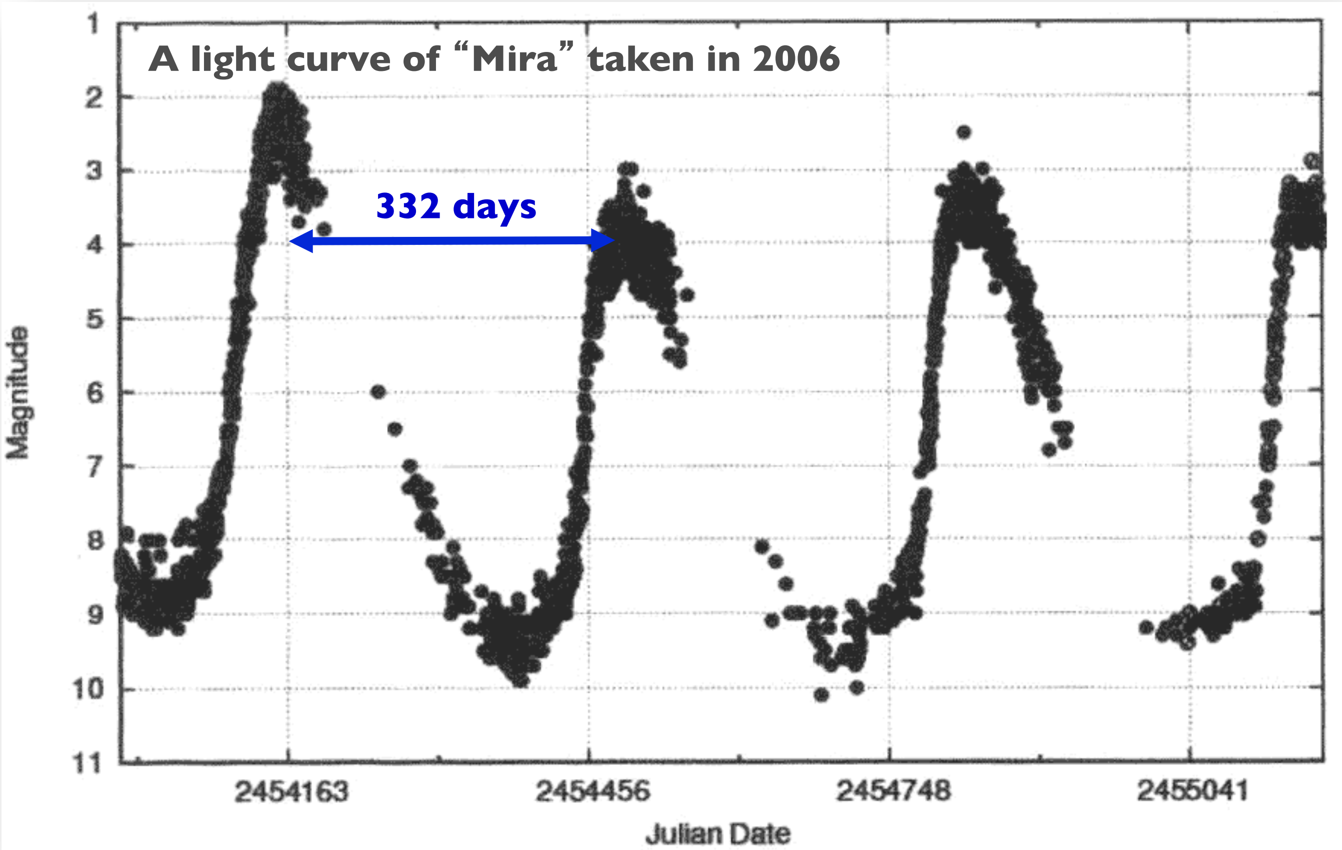
- Understand how the period-luminosity relationship for Cepheid variables can be used to determine distances in the Universe
- Be able to define the term instability strip and understand why the instability strip has a narrow temperature range
- Understand that sound waves and partial ionisation zones (opacity) power stellar pulsation
- Know and be able to show:

$$\Pi \approx \sqrt{\frac{3\pi}{2\gamma G\rho}}$$

Stellar pulsation period

Discovery of pulsating stars

The first variable star was discovered in 1595, by a Lutheran Pastor David Fabricius. He observed rhythmic variations in the brightness of a star called o Ceti (Omicron Ceti), which he called 'Mira' meaning 'wonderful'.



Pulsating stars: types

Astronomers have catalogued more than 40,000 pulsating stars of various classes

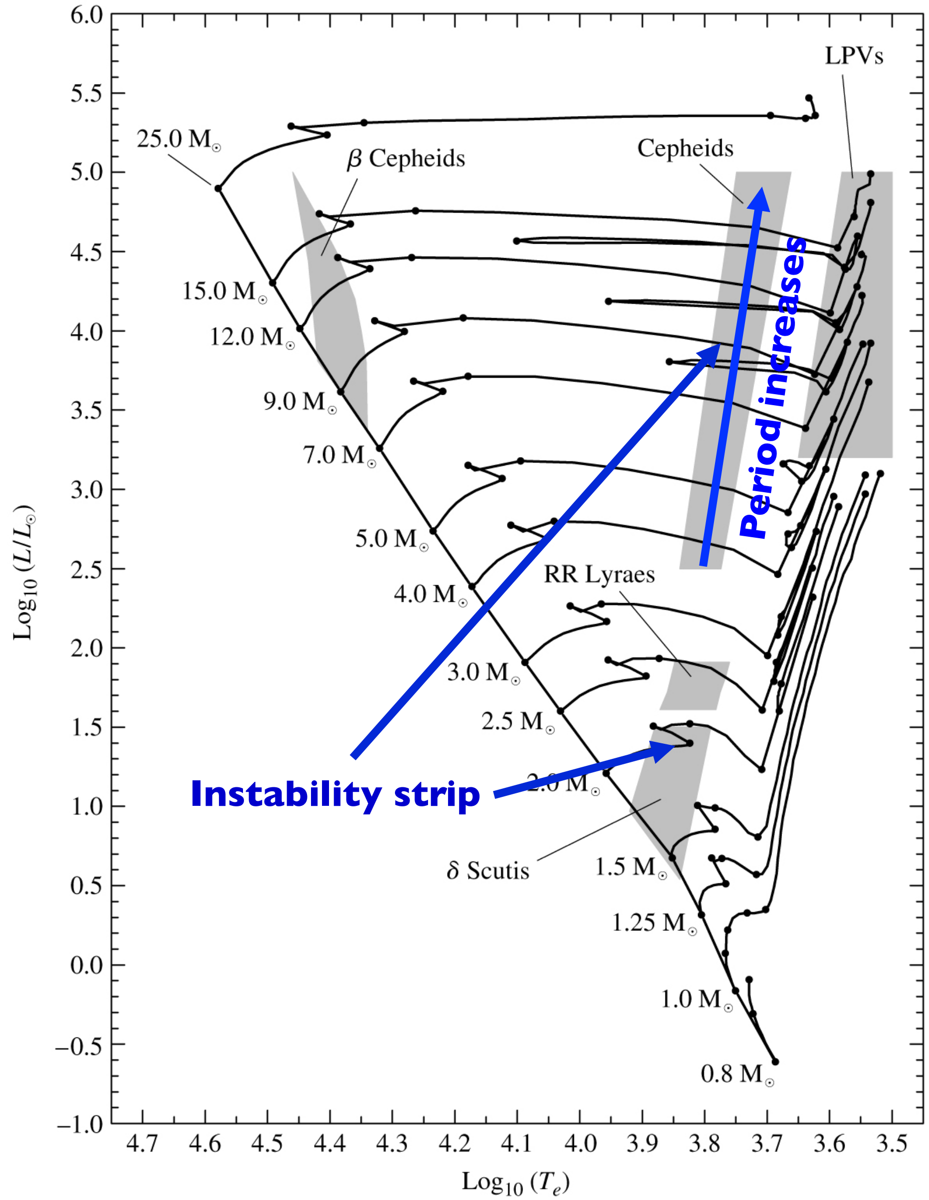
Type	Periods	Radial/ Non-Radial
Long-period variables	100-700 days	R
Classical Cepheids	1-50 days	R
W Virginis stars	2-45 days	R
RR Lyrae stars	1.5-24 hours	R
δ Scuti stars	1-3 hours	R,NR
β Cephei stars	3-7 hours	R,NR
ZZ Ceti stars	100-1000 secs	NR

Radial/non-radial refers to the mode of pulsation: either predominantly propagating through the star (radial) or propagating around the star (non radial).

We will focus on the radial mode: stars pulsating in and out

There are several million pulsating stars in the Galaxy but out of a population of several hundred billion so only ~10^{-4}-10^{-5} pulsate

Pulsating stars: instability strip on HR diagram



Pulsating stars are inherently unstable and the areas of the HR diagram on which they are found is known as the instability strip.

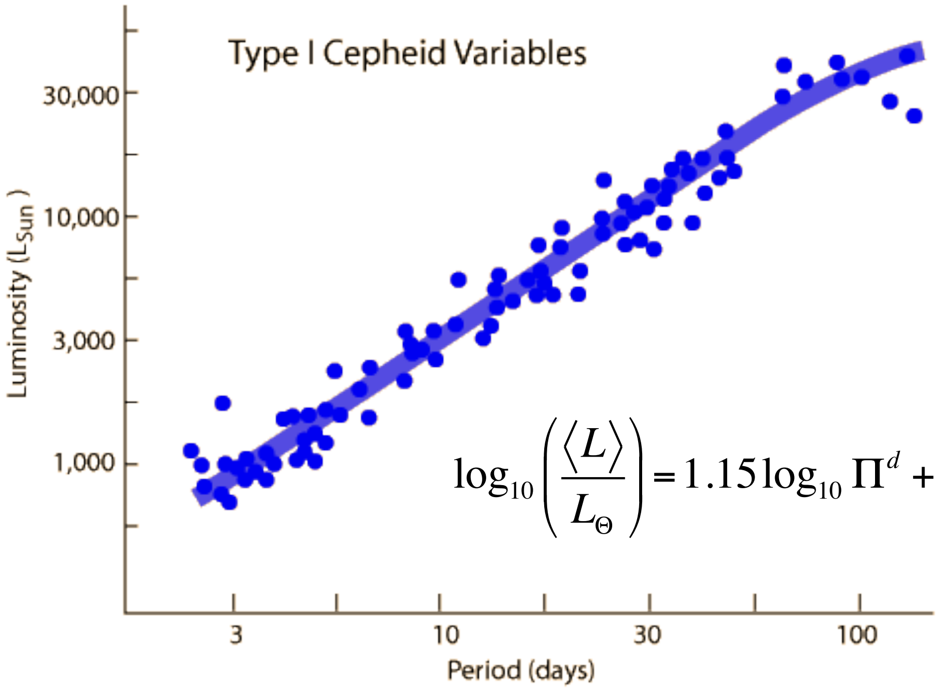
These areas are shown here (shaded regions), along with various types of pulsating stars.

Note the shaded regions are narrow: only ~600-1,000 K wide. We will explore why this is the case.

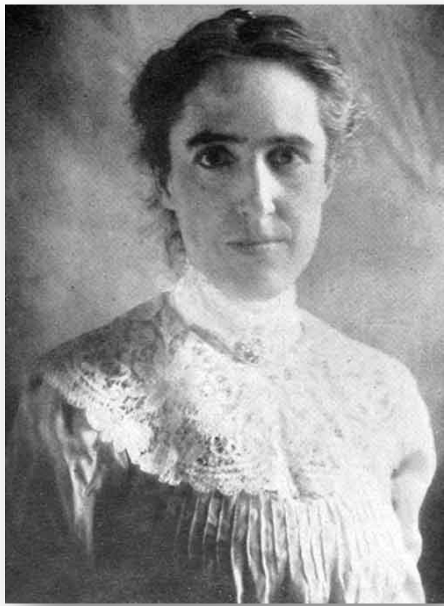
As can be seen, pulsating stars have a broad range of properties, from low-mass stars (such as delta Scutis and RR Lyraes) to high-mass stars (such as the famous classical Cepheid variables); see table on previous slide.

Cepheid variables: period-luminosity relationship

One of the most important classes of variable stars are Cepheid variables. In the early 1900s, Henrietta Leavitt was involved in the tedious task of identifying and cataloging variable stars, but discovered ~2400 classical Cepheids. She noticed that the brighter stars had longer periods than the faint ones, and plotting period vs luminosity astronomers found a strong correlation.



$$\log_{10}\left(\frac{\langle L\rangle}{L_\Theta}\right) = 1.15\log_{10}\Pi^d + 2.47$$



Henrietta Leavitt: ~1910

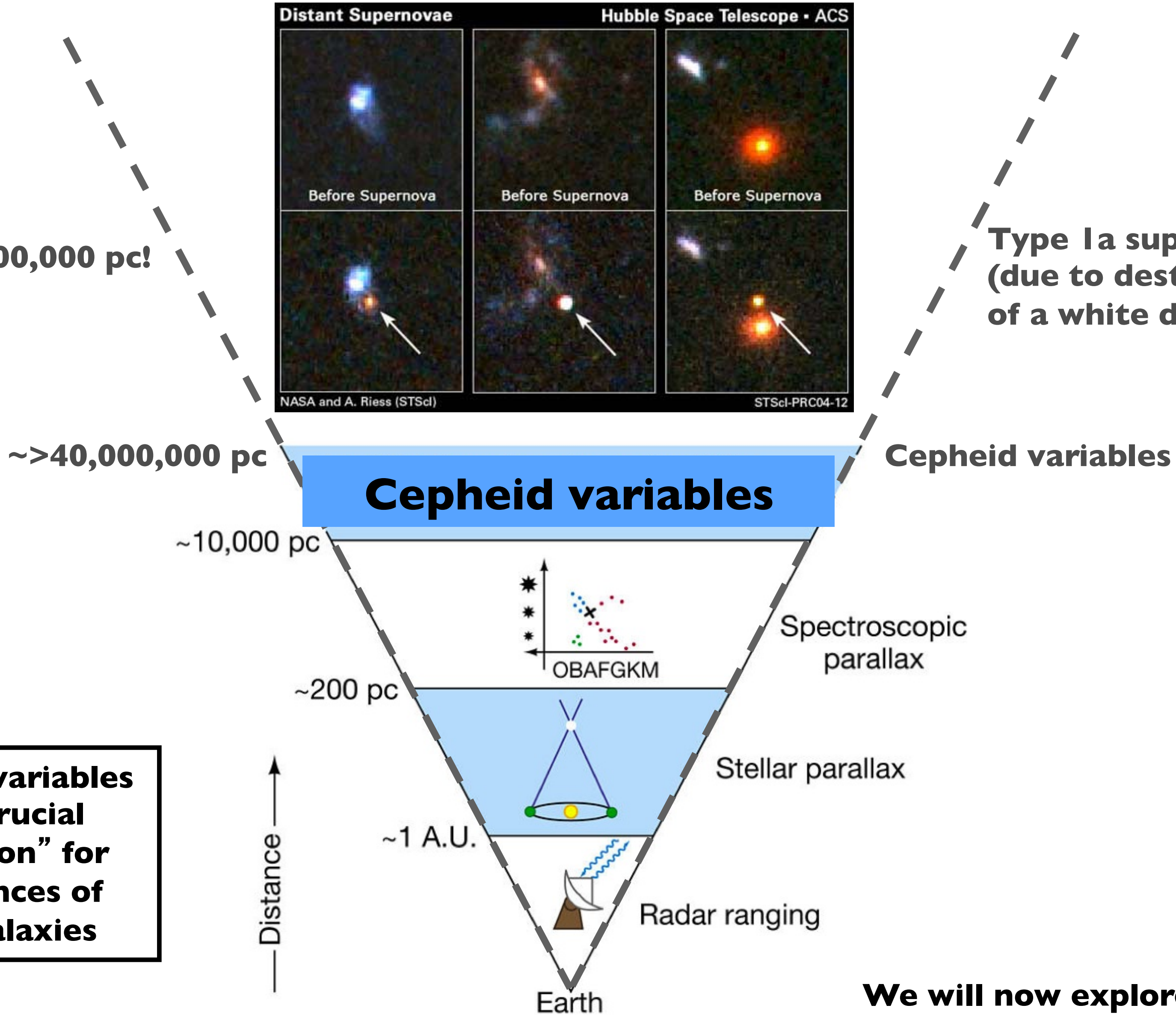
This equation can only be used for Cepheids (not other variable stars)

This tight period-luminosity relationship makes Cepheids “distance candles”: the distance to Cepheids in other galaxies can be calculated from the period and the flux of the Cepheid star.

Cepheid variables: standard candles

~4,000,000,000 pc!

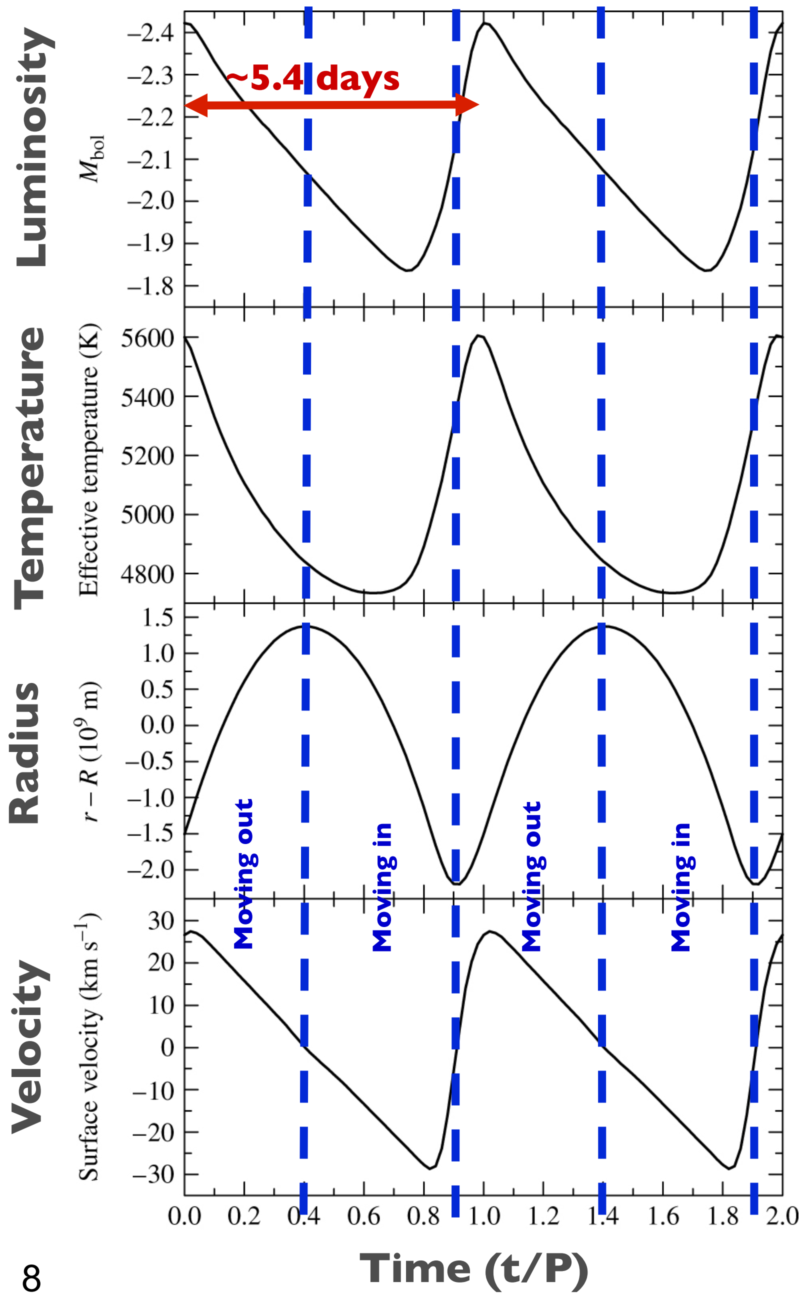
Type 1a supernovae
(due to destruction
of a white dwarf)



Cepheid variables provide crucial "calibration" for the distances of nearby galaxies

We will now explore what causes stellar pulsations...

Physics of stellar pulsation: empirical evidence



These plots show the changing properties of δ Cephei, a typical classical Cepheid

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Top plot shows the change in luminosity (energy output). The change in temperature is a consequence of change in luminosity and radius; i.e., black body:

$$L = 4\pi R^2 \sigma T_e^4$$

The bottom two plots show what is physically going on (the "action": change radius and velocity): the star is effectively "breathing" in and out (pulsating) in accordance with the change in luminosity

Properties of δ Cephei:

M~5xSun; L~2000xSun; R~50xSun; d~270pc; spectral type: F5-G3!

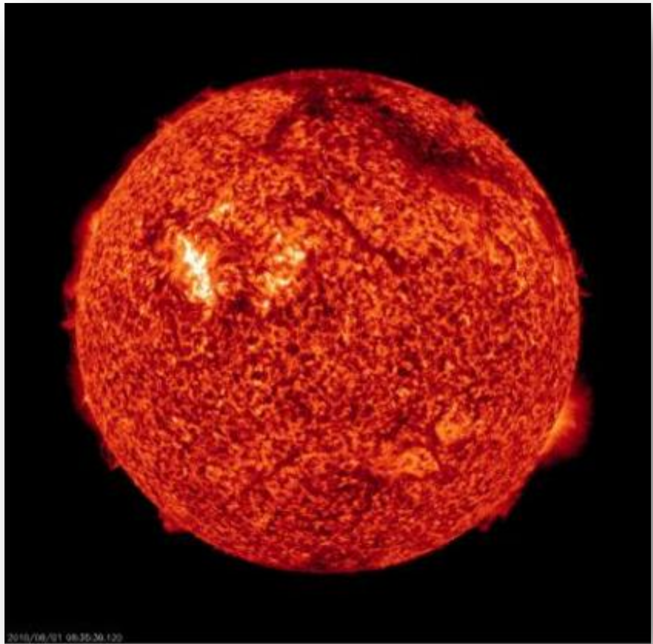
Physics of stellar pulsation: sound waves

The radial oscillations of a pulsating star are the result of sound waves resonating in the star's interior. A rough estimate of the pulsation period, Π, can be obtained by considering how long it would take a sound wave to cross the diameter of a hypothetical star of radius R and constant density ρ. The adiabatic sound speed is given by:

$$v_s = \sqrt{\frac{\gamma P}{\rho}}$$

where P=pressure and γ=ratio of specific heats.

Sounds waves move too quickly for significant heat transfer so an adiabatic approximation is good



These sound waves in stars are standing waves that are caused by the interplay between pressure and gravity.

In modelling the stellar pulsation we will estimate P(r) and use the sound-speed equation to determine the stellar pulsation period.

Physics of stellar pulsation: derivation

To find the pressure we can use the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2}$$

Let's assume that the star has constant density (not a very good assumption!). Then:

$$\frac{dP}{dr} = -\frac{G\left(\frac{4}{3}\pi r^3 \rho\right)\rho}{r^2}$$

$$\frac{dP}{dr} = -\frac{4}{3}\pi G \rho^2 r$$

Now integrate this using the boundary condition that P=0 at the surface of the star:

$$P(r) = \frac{2}{3}\pi G \rho^2 \left(R^2 - r^2\right)$$

This gives us a pressure as a function of r.

This derivation is essentially the same as the core-pressure derivation from lecture 4 but here we consider P(r) rather than the core pressure (Pc)

Physics of stellar pulsation: derivation

The pulsation period will be given roughly by the distance divided by the speed, or:

$$\Pi \approx 2\int_0^R \frac{dr}{v_s}$$

$$\Pi \approx 2\int_0^R \frac{dr}{\sqrt{\frac{2}{3}\gamma\pi G\rho\left(R^2 - r^2\right)}}$$

$$\Pi \approx 2 \times \sqrt{\frac{3}{2\gamma\pi G\rho}} \times \int_0^R \frac{dr}{\sqrt{\left(R^2 - r^2\right)}}$$

$$\Pi \approx 2 \times \sqrt{\frac{3}{2\gamma\pi G\rho}} \times \left[\sin^{-1}\left(\frac{r}{R}\right)\right]_0^R$$

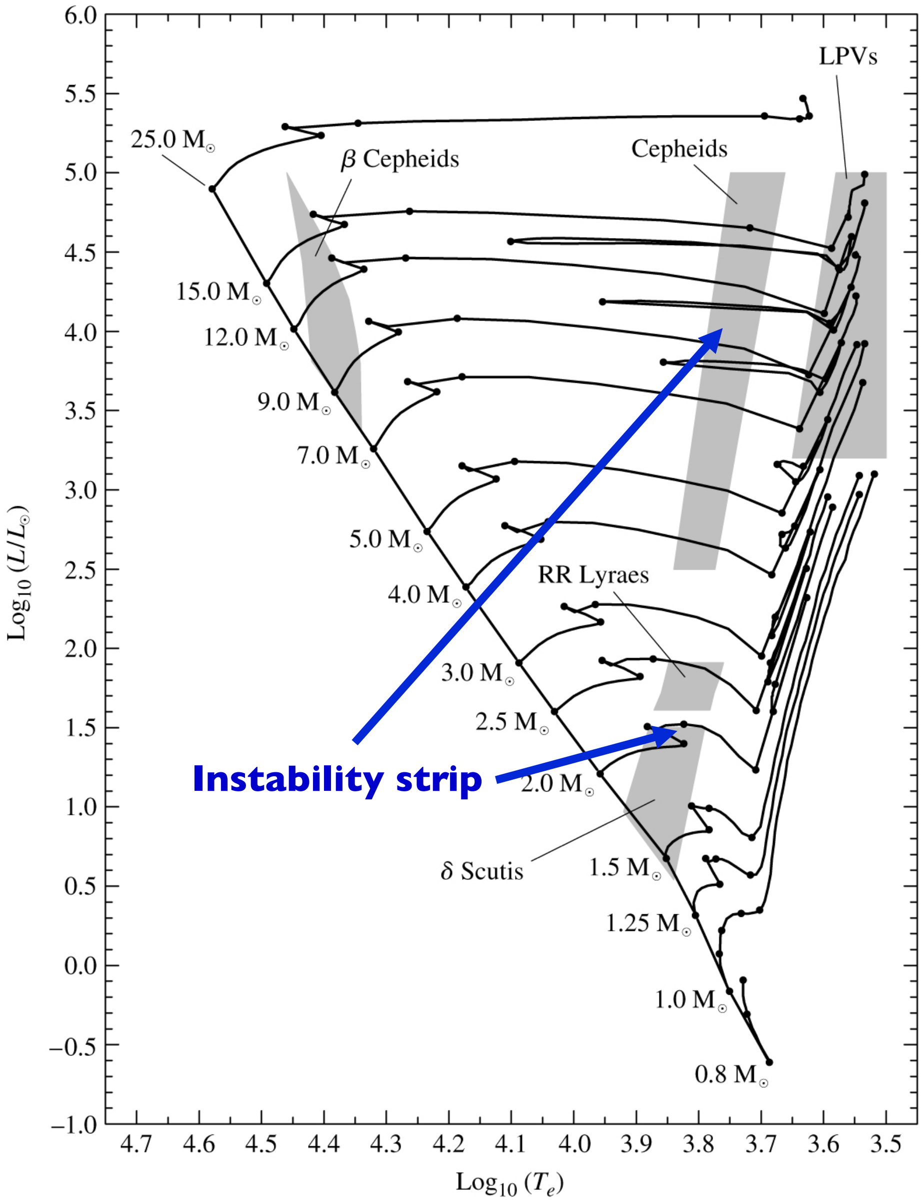
So:

$$\Pi \approx \sqrt{\frac{3\pi}{2\gamma G\rho}}$$

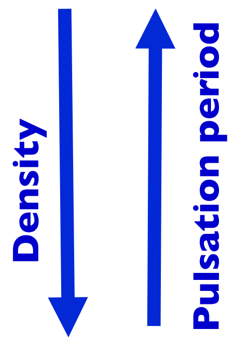
Equation 24

Pulsation period is inversely proportional to the square root of the density: Verify that this agrees with the period-luminosity relationship for Cepheids

Physics of stellar pulsation: instability strip



The last equation shows that the pulsation period is inversely proportional to the square root of the mean density. This explains why the pulsation period decreases as you move down the instability strip from the very tenuous supergiants to the very dense white dwarfs.



The pulsation period increases for more luminous objects, which are also less dense

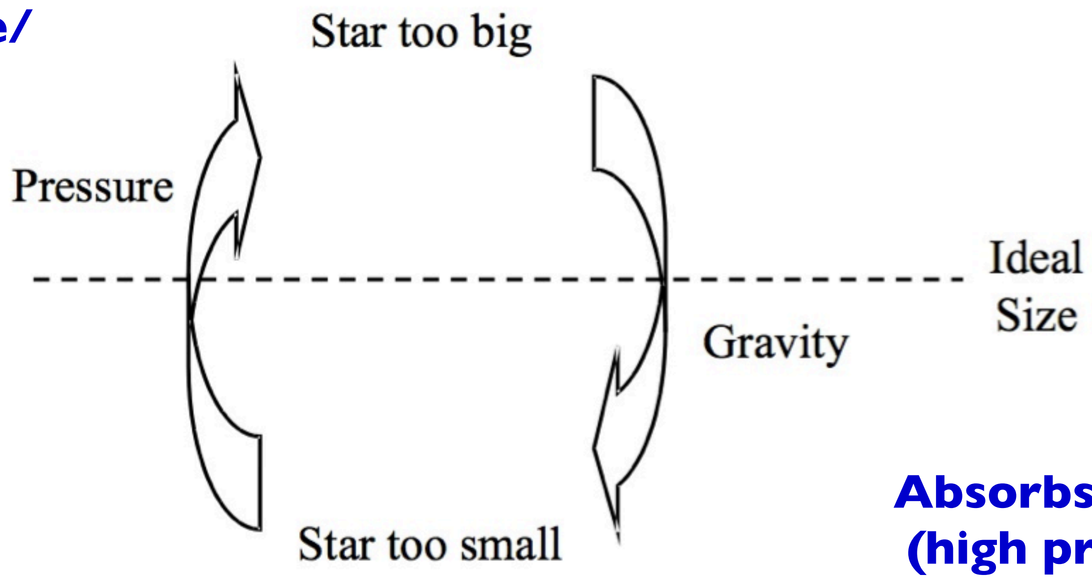
You can show more generally from Equation 24 and the luminosity equation that we would indeed expect to see a relationship between luminosity and pulsation period: <u>we explore this in a workshop</u>

We have shown:

(1) Stellar pulsations are due to the star moving in and out (oscillations)
(2) Pulsation period is due to sound waves moving through the star (standing waves) and is inversely proportional to the square root of the density
(3) This gives rise to a period-luminosity relationship

All stars can potentially power standing waves but why don't all stars oscillate? The answer is that some stars can "dam up" and release sufficient energy to allow the oscillations to continue while others cannot – the key is a "valve" mechanism, which let some stars operate like a huge "piston"

Releases energy (low pressure/ density)



Absorbs energy (high pressure/ density)

Valve mechanism: partial ionisation zones

Valve mechanism – opacity: if a layer becomes opaque upon compression it can "dam up" energy flowing towards the surface and push the layers upwards. As the expanding layer becomes transparent the trapped heat will escape and the layer will fall back to begin next cycle.

For this to work opacity must increase with compression, which means that opacity needs to increase as both ρ and T increases.

Consider how opacity changes with temperature (lecture 9)

Partial ionisation zones: opacity increases with temperature from excitation (bound bound) to ionisation (bound free) – partial ionisation zones are regions where only a fraction of the gas is ionised and hence the opacity is high.

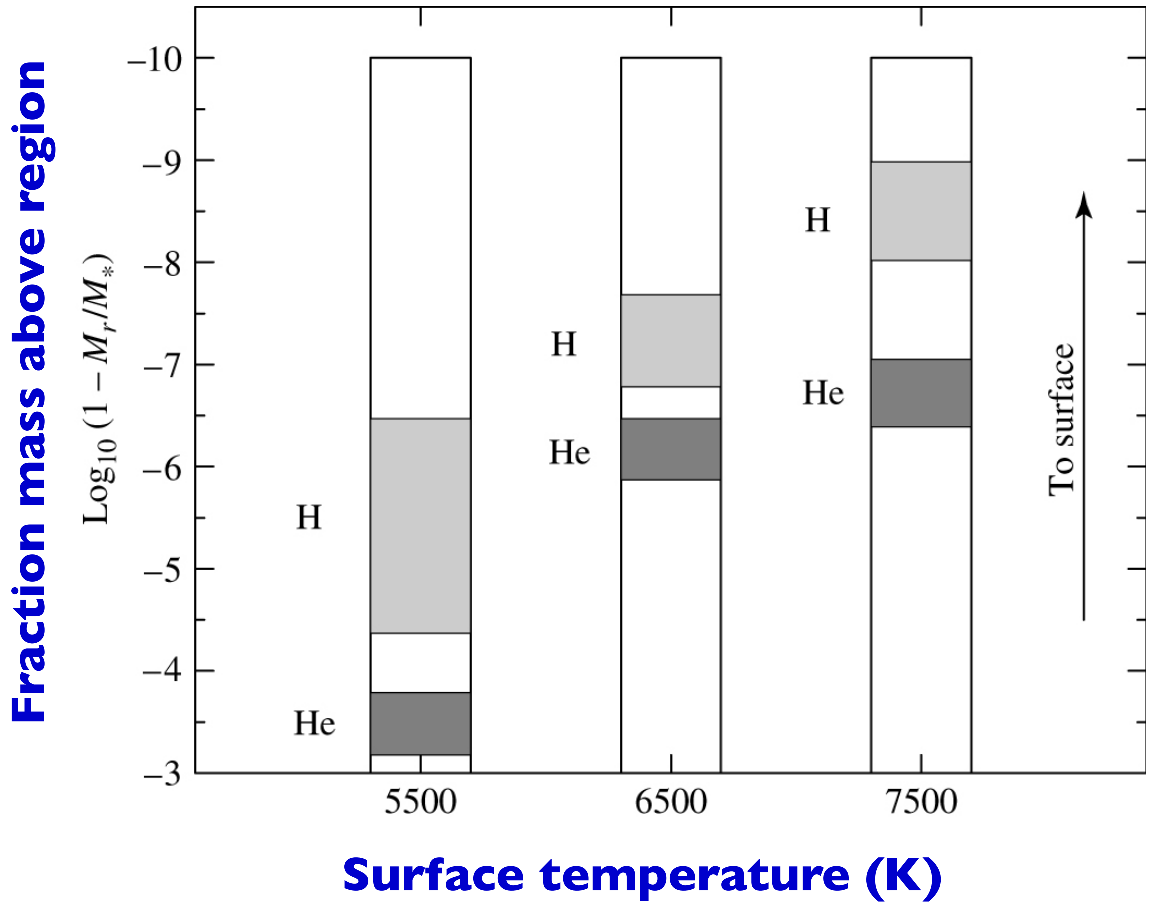
Part of the work done during the compression produces further ionisation rather than significantly raising the temperature. Then during expansion the temperature decreases slowly because ions recombine and release energy (luminosity increase).

All stars will have partial ionisation zones: why don't all pulsate?

Depth of the partial ionisation zones and the location of the convection zones.

Valve mechanism: partial ionisation zone depth

The key is the depth of the partial ionisations zones (T~(1-4)x10^4 K; highlighted below): too close to the surface and there is too little material to drive pulsations. Too deep and convection occurs in the outer regions (lecture 10) and dampens out the pulsations.



In a hot star (>7,500K) the H and He partial ionisation zones are near the surface - the density is quite low and there is not enough mass to drive oscillations effectively.

In a cooler star (6,500K) the H and He partial ionisation zones are deeper in the star and there is more mass to push around.

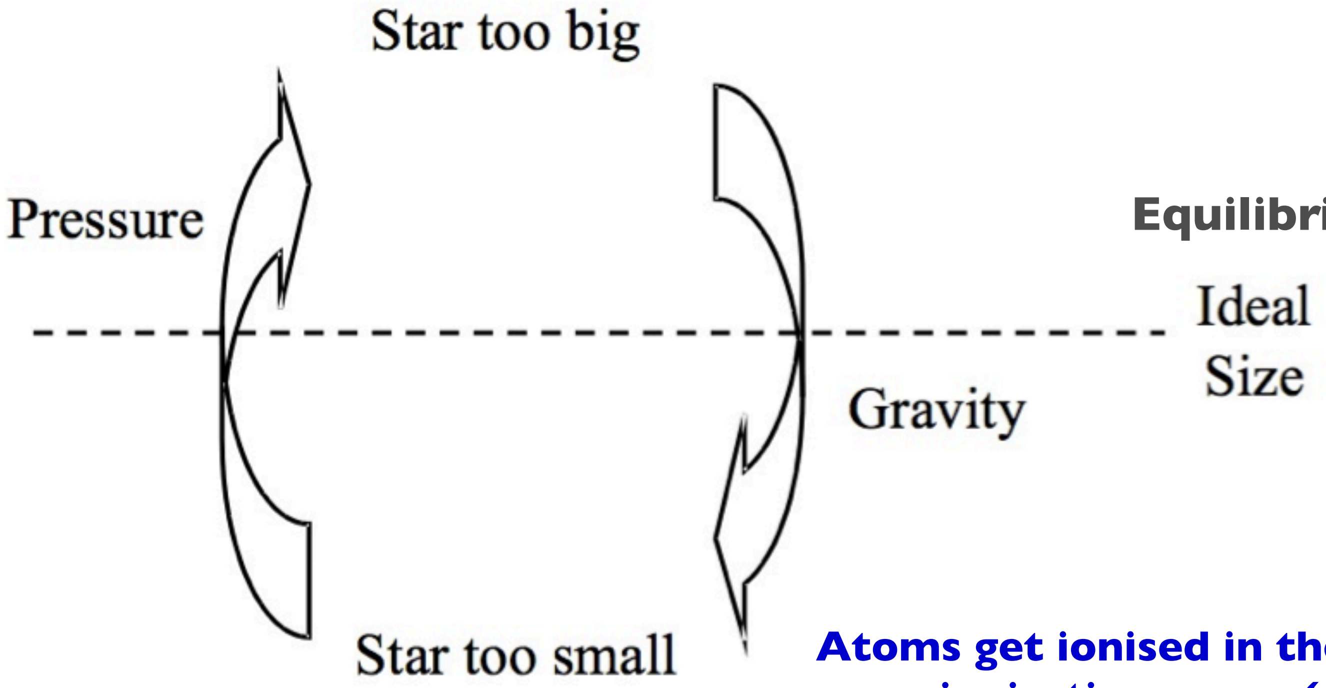
So why no pulsations in the coolest stars (<5,500K) where the partial ionisation zones will be deepest? Convection occurs and dampens out the pulsation (e.g., consider the Sun).

H partial ionisation zone: T~(1.0-1.5)x10^4 K. He partial ionisation zone: T~4x10^4 K.

Physical model: partial ionisation zone “pistons”

Pulsations driven by standing waves but “powered” by partial ionisation zones

Atoms recombine in the partial ionisation zones: luminosity increases which powers the expansion (radius increases)



Equilibrium

Atoms get ionised in the partial ionisation zones (opacity): luminosity decreases as star is compressed (radius decreases)

In summary: all stars have the potential to pulsate but either the lack of significant material above the partial ionisation zone or convection dampens the pulsations