Level 2 Stars

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Problem Set S.1

(1) Estimate the effective temperature of a star with 10^{-3} times the luminosity of the Sun, and a radius equal to that of the Earth. At what wavelength does the emission from the star peak? [4 marks]

Solution

$$T_{eff} = \left(\frac{10^{-3} L_{sun}}{4\pi R_{earth}^2 \sigma} \right)^{1/4} = 10,700 K$$ [2 marks – 1 for equation 1 for numerical answer]

L = Luminosity (W), R = radius (m), σ = Stefan-Bolzmann constant (W m^{-2} K^{-4}) , Teff = effective temperature (K)

$\lambda_{max} = 3x10^{-3} / T$ [1 mark for equation]

λ_{max} = 270 nm [1 mark for numerical answer]

(2) Calculate the temperature where the ratio of electrons in the excited n=2 energy state to the ground state of Hydrogen is 1/100. What other physical process needs to be considered to calculate the strength of the Hydrogen absorption-line features in the optical spectrum of a star? [6 marks]

Solution

The equation needed to calculate the ratio of Hydrogen in different energy states is:

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

Re-arrange the equation so $N_b/N_a=1/100$

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT} = \frac{1}{100} \qquad \textit{therefore}$$

[2 marks – 1 for equation and 1 for equating this to the correct ratio of 1/100]

$$\frac{g_a}{100 g_b} = e^{-(E_b - E_a)/kT} \qquad \textit{and so}$$

$$\ln\left(\frac{g_a}{100 g_b}\right) = \frac{E_b - E_a}{kT} \qquad \textit{therefore}$$

[1 mark for the rearrangement]

$$T = \frac{E_b - E_a}{k \ln\left(100\left(g_b / g_a\right)\right)} = 19{,}800 K$$

[2 marks – 1 for the correct final equation and 1 for the answer]

Qualitative answer: ionisation is the other physical process that needs to be considered to calculate the strength of Hydrogen absorption-line features in the optical spectrum.

[1 mark]