Mathematical Methods in Physics

Weekly Problems 1. Solution

1.1

a) $a_{11} + a_{22} + a_{33} = 8, \quad a_{12}a_{13} + a_{22}a_{23} + a_{32}a_{33} = 3, \quad a_{11}a_{21} + a_{21}a_{22} + a_{31}a_{23} = -6,$

$\boxed{\text{2 mark}}$

b) $a_{1j}\delta_{1j} = a_{11} = 1, \quad a_{12}\delta_{ii} = -3, \quad a_{1i}a_{2k}\delta_{ik} = a_{1i}a_{2i} = a_{11}a_{21} + a_{12}a_{22} + a_{13}a_{23} = -5.$

$\boxed{\text{2 mark}}$

1.2

a) When the ball bounces off the cushion along the x axis the x-component of its velocity is conserved, while the y-component is reversed. When the ball bounces off a cushion along the y-axis the y-component of the velocity vector is conserved but the x-component is reversed. So you should get:

$$\mathbf{a} \sim (a, -1)\ , \quad \mathbf{b} \sim (a, 1)\ , \quad \mathbf{c} \sim (-a, 1)\ , \quad \mathbf{d} \sim (-a, -1)\ .$$

$\boxed{\text{1 mark}}$

b) The point $(1, b)$ is on the line $\mathbf{b}$ such that the following relation holds

$$(1, b) = (a, 0) + t(a, 1).$$

By eliminating t from the two resulting equations you get $b = (1 - a)/a$. A similar calculation can be performed for the lines $\mathbf{c}$ and $\mathbf{d}$, that is

$$(c, 2) = (1, b) + t'(-a, 1)$$

gives $c = 1 - 2a + ab$ and

$$(0, 0) = (c, 2) + t''(-a, -1)$$

gives $c = 2a$. Putting everything together you should get $a = 2/5$. $\boxed{\text{3 marks}}$

1.3

A vector perpendicular to the plane is $\mathbf{n} = (-2, 3, 2)$. The component of $\mathbf{v}$ parallel to $\mathbf{n}$, and therefore perpendicular to the surface, is given by the orthogonal projection of $\mathbf{v}$ onto $\mathbf{n}$:

$$\mathbf{v}_\perp = \left(\frac{\mathbf{n}\cdot\mathbf{v}}{|\mathbf{n}|}\right)\frac{\mathbf{n}}{|\mathbf{n}|} = \frac{7}{17}\,(2, -3, -2)$$

$\boxed{\text{1 mark}}$

Hence:

$$\mathbf{v}_\parallel = \mathbf{v} - \mathbf{v}_\perp = \frac{1}{17}\,(3, -30, 48)$$

$\boxed{\text{1 mark}}$