Lecture 4: Times and Distances

4.1 Lookback time

[Liddle sec:5.2]

When we observe a distant galaxy at redshift z, we are observing photons which have travelled through space for a long time in order to reach us. They began their journey when the expansion factor of the Universe was $a = 1/(1+z)$. The length of time they have been travelling is therefore $t(a_0) - t(a)$. This time difference is called the **lookback time**.

Thus when we observe objects at redshift z we are observing the universe when it was younger than it is today.

This is the resolution of Olber's paradox. As we travel along any line of sight we are travelling back in time. Eventually we will come to an epoch before the first stars formed (and beyond that to the Big Bang itself) and hence not every line of sight will intersect a star. This is why it goes dark at night.

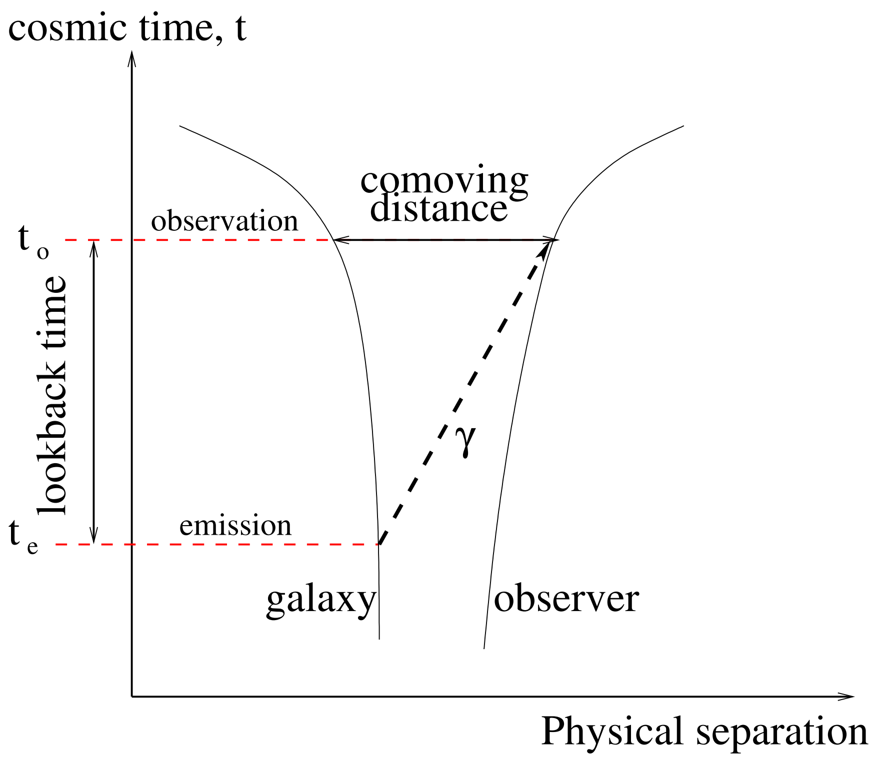

Figure 1: A schematic space time diagram for an expanding Universe, showing a photon being emitted by a galaxy at cosmic time t_e and detected by an observer at time t_o, defining the key concepts of *lookback time* and *comoving distance*.

4.2 The Distance redshift Relation

[Liddle sec:15.A.2.1, 15.A.2.2]

An obvious question is "how far away is a galaxy with redshift z". Think carefully about what you mean! For example do you want to know (a) how far did the photon travel to reach me? or (b) how far is the galaxy away from me now (ie. what is its co-moving distance)?

Consider a photon emitted at cosmic time t_e, as shown in Fig.1. At time t it will travel a physical distance $c\,dt$ in time dt. The expansion of the Universe will cause this interval of length to expand until at the present it will have a physical length of $(1/a(t))\,c\,dt$. In otherwords, the comoving distance the photon travels is

$$dx = \frac{c\,dt}{a(t)} = (1+z)\,c\,dt. \tag{4.1}$$

Note that we have converted the dependence on the expansion factor into a dependence on redshift.

In order to integrate this equation we need to express dt as a function of redshift. We can do this as follows: Using equations (3.7), (3.10) and (3.12) we can write the Friedmann equation in the form

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \frac{\rho(t)}{\rho_{\rm crit,0}} - \left(\frac{1}{a}\right)^2 H_0^2(\Omega_0 - 1). \tag{4.2}$$

We can convert this into an equation relating redshift and time using $a = 1/(1+z)$ and simplify further using $\rho(t) = \rho_0(1+z)^3 = \Omega_0 \rho_{\rm crit,0}(1+z)^3$.

In this case we find

$$\left(\frac{1}{(1+z)}\right)^2 \left(\frac{dz}{dt}\right)^2 = (1+z)^3 H_0^2 \Omega_0 - (1+z)^2 H_0^2 (\Omega_0 - 1). \tag{4.3}$$

and hence

$$dt = \frac{-dz}{\mathrm{H}_0 (1+\Omega_0 z)^{1/2}(1+z)^2} \qquad (4.4).$$

Integrating this gives the **lookback time** to a galaxy at redshift z_1 as

$$\boxed{t_{LB} = \frac{1}{\mathrm{H}_0} \int_0^{z_1} \frac{dz}{(1+\Omega_0 z)^{1/2}(1+z)^2}} \tag{4.5}$$

Similarly, substituting (4.4) into (4.1) we find the **comoving distance** to a galaxy with observed redshift z_1 at the present day is

$$\boxed{x = \frac{c}{\mathrm{H}_0} \int_0^{z_1} \frac{dz}{(1+\Omega_0 z)^{1/2}(1+z)}} \tag{4.6}$$

For the special case of $\Omega = 1$ the integrand simplifies and the result of the integral is

$$x = \frac{2c}{\mathrm{H}_0}\left[1 - (1+z)^{-1/2}\right]. \tag{4.7}$$

Note that for $z \ll 1$ this reduces to

$$r \approx x \approx \frac{cz}{\mathrm{H}_0} = \frac{v}{\mathrm{H}_0}$$

compatible with Hubble's law (r is the physical distance).

Numerical integration of (4.5) for various values of Ω_0 is shown in Fig. 2, demonstrating that lookback times to a given redshift are longer in lower density (i.e. lower Ω_0) Universes. This is because Universes with lower Ω_0 decelerate more slowly than those with higher Ω_0 and hence need more cosmic time to expand from a given observed redshift (expansion factor) to the present day.

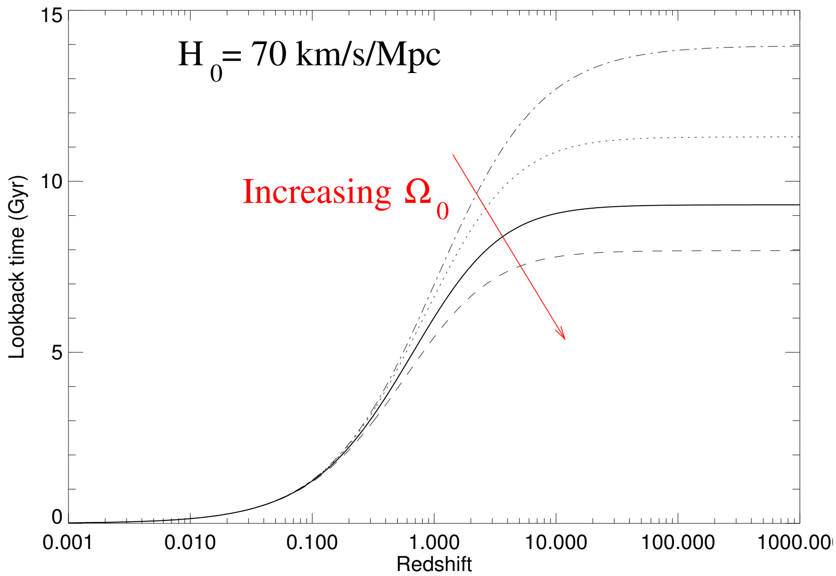

Figure 2: Lookback time versus redshift for Universes with, from top to bottom, $\Omega_0 = 0$, 0.3, 1 and 2.

4.3 Cosmological Time Dilation

Consider the emission of a pulse of light over a time interval Δt_e by a distant galaxy. If the galaxy is at redshift z, an observer will measure this pulse to a have a duration Δt_o where

$$\Delta t_o = \Delta t_e(1+z) \tag{4.8}.$$

In other words, the observed timescale is lengthened. This has been observationally verified in, for example, the decaying lightcurves of 'standard candle' supernovae. The result follows from equating the comoving distances of the emitting galaxy at the beginning and the end of the pulse, i.e.

$$\int_{t_e}^{t_o} \frac{c\,dt}{a(t)} = \int_{t_e+\Delta t_e}^{t_o+\Delta t_o} \frac{c\,dt.}{a(t)} \tag{4.9}$$

Examples

4.1 The spectrum of a distant galaxy is observed. Spectral lines are identified and the Lyman-α absorption line is measured to have a wavelength of 329 nm.

i) Compute the redshift of the galaxy.

ii) Compute the comoving distance between the galaxy and the observer.

iii) What is the comoving distance between the galaxy and the observer at the time the observed light was emitted by the galaxy?

iv) What is the lookback time to the galaxy?

v) What was the age of the universe when the observed light was emitted?

[Assume the Universe has the critical density and the Hubble Constant is 75 km s^{-1} Mpc^{-1}. The restframe wavelength of the Lyman-α spectral line is 121.8 nm]

4.2 Calculate the redshift and recession velocity of galaxies moving with the Hubble flow whose comoving distance from the Earth are:

i) 10 Mpc

ii) 300 Mpc

iii) 4000 Mpc

iv) 8000 Mpc

v) $>$ 8000 Mpc

[Assume the Universe has the critical density and the Hubble Constant is 75 km s^{-1} Mpc^{-1}.]

4.3 Calculate the fractional change in the Hubble parameter over the last 65 million years of cosmic history, assuming a Hubble Constant of 75 km s^{-1} Mpc^{-1} and $\Omega_0 = 0.3$.