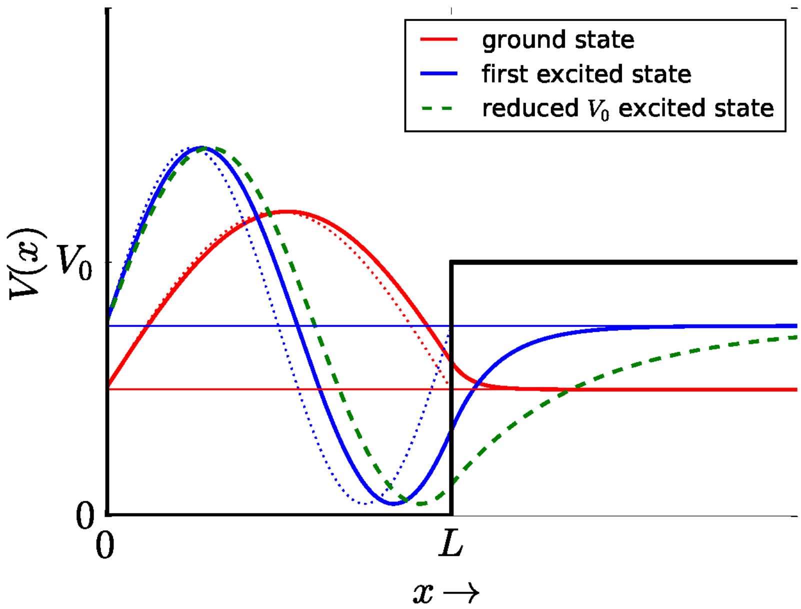
The sketch does not have to be as detailed as the following but should clearly show the following features:



i) Ground has one turning point and first excited state has two. [1 mark]

The wavefunctions go to zero at $x = 0$ and have a continuous gradient at $x = L$ [1 mark]

The peaks (and trough) are shifted to the right relative to those of the infinite square well [1 mark]

The exponential decay of the excited state is slower (longer length scale) than that of the ground state. [1 mark]

ii) For $0 < x < L$, $V = 0$ and so the Schrödinger equation is

$$\begin{aligned} \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} &= E\psi \\ \Rightarrow \frac{d^2\psi}{dx^2} &= \frac{-2mE}{\hbar^2}\psi = -k^2\psi \end{aligned}$$

where $k^2 = 2mE/\hbar^2$. This is the SHO equation and so its solution satisfying $\psi(0) = 0$ is $\psi(x) = A\sin kx$ [2 marks]

iii) Green dashed line on the figure above. The wavelength gets longer, but the second turning point must stay within $x < L$ [as otherwise the solution couldn't decay to zero and still have a continuous gradient at $x = L$.] [2 marks]

iv) The smallest allowable k is given by

$$kL = 3\pi/2$$

and at this point the state is marginally bound i.e.

$$V_0 = E = \frac{\hbar^2 k^2}{2m} = \frac{9\pi^2\hbar^2}{8mL^2}$$

[2 marks]