Level 2 Stars

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Problem Set S.7

a) Estimate the lifetime of a 12 $M_\odot$ star on the main sequence, given that nuclear burning converts about 0.7% of the matter involved into energy. The star has a luminosity of 10^4 $L_\odot$ and is ~70% hydrogen by mass. [3 marks]

b) At the end of its life, the stellar core has a mass of 1.5 $M_\odot$. When it collapses what will the end product be? Give a reason for your answer. [2 marks]

c) Assuming that all of the gravitational potential energy liberated in the collapse is released in the form of neutrinos, and that the energy of a typical neutrino is 6 MeV, how many neutrinos are produced in total from the collapse? State what final radius you assumed. [3 marks]

d) Why are a large number of neutrinos produced during the collapse of the stellar core in massive stars? [2 marks]

[$M_\odot = 1.99 \times 10^{30}$ kg; $L_\odot = 3.84 \times 10^{26}$ W; 1 pc = 3.09×10^{16} m; 1 MeV=1.602×10^{-13} J; 1 AU = 1.50×10^{11} m; c = 3.00×10^8 m s^{-1}; G = 6.67×10^{-11} N m^2 kg^{-2}]

Solution

a) The main-sequence lifetime can be defined as:

$$t_{ms} = \frac{X\varepsilon Mc^2}{L} \qquad \textit{[1 mark]}$$

where X is the fraction of the mass in the star that will be used in H—H fusion, e is the mass-to-light efficiency conversion (based on the binding energy), M is the mass of the star, and L is the luminosity of the star.

Using this equation the main-sequence lifetime of this massive star is:

$$t_{ms} = \frac{0.7 \times 0.007 \times \left(12 \times 1.99 \times 10^{30}\right) \times \left(3.0 \times 10^{8}\right)^2}{3.84 \times 10^{26} \times 10^4}$$

This gives t_{ms}=8.7x10^7 years. *[2 marks]*

b) *The core will collapse to form a Neutron star. Cores with mass less than ~2.2-2.9 $M_\odot$ can support themselves by neutron degeneracy pressure. [2 marks]*

c) *The potential energy is liberated in the gravitational collapse) gives:*

$$E \sim -\frac{3}{10}\frac{GM^2}{R}$$

Likely radii for the collapsed core (a neutron star) are 10-30 km. Note: since the post-collapse radius is so much smaller than the pre-collapse radius we only need to consider the final (post-collapse) radius of 10-30 km.
[1 mark for a radius in this range]

Therefore:

$E \approx (0.86 - 2.57) \times 10^{46} J$ *for a 10-30 km radius [1 mark if answer in given range]*

The number of neutrinos produced is:

$N_{neutrino} = \dfrac{E}{(6 \times 1.6 \times 10^{-13})} = (0.90 - 2.68) \times 10^{58}!$ *[1 mark if answer in given range]*

d) *Neutrinos are produced during the stellar-core collapse because the protons and electrons are compressed strongly enough to form neutrons – this is called electron capture. To maintain the fermion number a neutrino is released. [2 marks]*