L2 Foundation of Physics 2B Optics 2019-20

Workshop O.W.2 Lens and Double slit

February 11, 2020

1. *Spherical waves and paraxial spherical waves*: Write an equation for the electric field of a spherical wave centred on the origin. [1 mark]

   Rewrite this equation in a plane a distance $z = f$ downstream in the **paraxial** regime, $z \gg \lambda$. Comment on the approximations used. [4 marks]

   A plano-convex lens with focal length f is placed in the $z = f$ plane. What is the form of the wave fronts downstream of the lens? [2 marks]

2. *Double slit*: In a Young's double-slit experiment, an aperture containing two narrow slits with positions $x' = \pm d/2$ is illuminated normally by uniform monochromatic light with amplitude E_0 and wavelength λ travelling along the z axis. In the far-field, $z \gg d$, the light field can be written as the sum of two cylindrical waves.

   (a) A cylindrical wave emitted from the origin may be written as

   $$E = \frac{E_0}{\sqrt{\mathrm{i}kr'}} \mathrm{e}^{\mathrm{i}kr'} ,$$

   where k is the magnitude of the wavevector and r' is the radial distance from the source. Comment on the choice of prefactor with respect to unis and energy conservation. [2 marks]

   (b) Write a general expression for the distance r' between an input point $(x', 0)$ and an observation point (x, z). [1 mark]

   (c) Use the paraxial condition, $|x - x'| < z$, to expand r' in terms of z, x, x', x^2, and x'^2. [1 mark]

   (d) Explain, briefly, why it is possible to neglect the x'^2 term for large z, but not the $x'x/z$ term. [3 marks]

   (e) Write an expression for the sum of two paraxial cylindrical waves arising in Young's double-slit experiment. Write your answer in terms of x, d, and x^2. You can neglect terms in d^2. [2 marks]

   (f) Identify terms in your expression that tell us whether the wavefronts in the far-field are planar or curved? What is the radius of curvature? [2 marks]

   (g) Calculate the intensity distribution as a function of x. Write your answer in terms of the wavelength. [2 marks]

   (h) If the wavelength is 0.50 μm, the spacing between the slits is $d = 0.50$ mm and the distance to the observation plane is $z = 1.00$ m, what is the spacing between the interference fringes? [2 marks]

   (i) Sketch, or describe, phasors diagrams corresponding to the positions: (i) $x = 0$, and (ii) any position where the intensity is one half of the maximum value. Why is it possible to have two different phasor diagrams for the same intensity? [5 marks]

For extra questions, see over.

3. *Extra questions*

(a) Explain, briefly, what is meant by the small-angle approximation, and why it is useful in optics. [2 marks]

(b) Estimate the fractional error (as a percentage) in using the small-angle approximation for case of light propagating at an angle $\theta = 30^\circ$ relative to the z-axis. [2 marks]

(c) *Angular frequency*: The angular frequency of a wave is denoted by the symbol ω, and is equal to 2π times the frequency. Explain, briefly, what angular refers to in this context. Is it related to a real angle? [2 marks]

(d) *Spatial frequency and the components of the wave vector*: Define spatial frequency and give a equation that relates the spatial frequency in a particular direction to the component of the wave vector in that direction. [2 marks]

(e) *Spatial frequencies everywhere*: Order the following in terms of increasing spatial frequency: (i) Bricks in a wall in a horizontal direction. (ii) Bricks in a wall in a vertical direction. (iii) The horizontal lines on a human forehead (furrows on a furrowed brow). (iv) Row of vines in a vineyard. (v) The teeth of a comb.

(f) *Spatial frequency calculation*: What is the spatial frequency (or wave number) and the magnitude of the wave vector for monochromatic light with a wavelength of 1.0 μm? What are the units in both cases?

(g) *Frequency to spatial frequency*: What is the spatial frequency of buses if there are two per hour and their average speed is 20 km/hour?

(h) *Plane wave properties*: How many independent parameters are required to specify the properties of a linearly polarised monochromatic plane wave (polarised along y) propagating along the z axis in vacuum at a particular instant in time, e.g., $t = 0$? [Hint: May be write the equation and note that the wave does not necessilarily need to have an antinode at the origin.] [4 marks]