Lecture 2: The Large Scale Properties of the Universe

2.1 The Cosmological Principle

The **Cosmological Principle** asserts that on sufficiently large scales the Universe is both homogeneous and isotropic.

Homogeneity: The property that the Universe looks the same at all points in space.

Isotropy: The property that the Universe looks the same in all directions.

N.B. If the universe is isotropic for all observers then it is necessarily homogeneous.

The cosmological principle was introduced by Einstein without any observational justification. At the time it wasn't even known that there were galaxies outside the Milky Way.

The reason for introducing the principle was that Einstein wanted to build a mathematical model of the Universe as a whole and to make a start it was necessary to simplify the problem. Now we have good empirical evidence of large scale homogeneity, but back then it wasn't so obvious.

2.2 Olber's Paradox

Olber (1826) reasoned that if the universe is infinite then any line of sight will eventually intersect the surface of a star like the sun. Thus he reasoned that all lines of sight should be as bright as the surface of the sun. So why does it go dark at night?

Assume the universe is:
1. Infinite (in space and time)
2. Static (unchanging)
3. Homogeneous
4. Euclidean

Consider a source of Luminosity L at a distance r from us.

The flux we will receive from it is

$$\boxed{f = \frac{L}{4\pi r^2}.}$$

Now consider a uniform distribution of n such objects per unit volume and compute the total flux we will experience.

$$f_{\rm tot} = \int_0^\infty \frac{L}{4\pi r^2}\; n\; 4\pi r^2 dr = nL\int_0^\infty dr$$

which is infinite. Olber was right!

Escape Clauses?

1. Lord Kelvin, suggested dust in the universe could absorb light making the flux fall faster than $1/r^2$. However, eventually the dust would be warmed and come into equilibrium re-radiating as much energy as it absorbs, although at longer wavelengths.

2. The recession of distant galaxies redshifts radiation from distant stars. However this still leaves the flux integrated over all wavelengths infinite.

3. The universe is not infinite either in space or instead in time.

This is the modern view in the Big Bang model in which the universe is infinite in space, but has a finite age. Moreover, stars themselves only have a finite lifetime and as they "burn" Hydrogen there can not be infinite generations of stars.

2.3 The Hubble Flow

[Liddle sec:4.1/5.1]

What non-static isotropic homogeneous universes are possible?

By homogeneity

$$\mathbf{v}(\mathbf{r}) - \mathbf{v}(\mathbf{a}) = \mathbf{v}(\mathbf{r} - \mathbf{a})$$

Hence $\mathbf{v}$ and $\mathbf{r}$ must satisfy a linear relation of the form

$$v_i = \Sigma_j A_{ij} r_j.$$

The matrix A_{ij} can be decomposed into symmetric and anti-symmetric parts

$$A_{ij} = A_{ij}^A + A_{ij}^S.$$

A_{ij}^A corresponds to a rotation and so can be transformed away by choosing coordinates rotating with the universe (i.e. non-rotating coordinates).

Then A_{ij}^S can be diagonalised

$$\mathbf{A}^S = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}$$

and hence $v_1 = \alpha r_1 \quad v_2 = \beta r_2 \quad v_3 = \gamma r_3$. But by isotropy $\alpha = \beta = \gamma = H(t)$.

Hence

$$\boxed{\mathbf{v} = H(t)\mathbf{r}.}$$

The recession velocity can be measured via the Doppler shift of spectral lines

$$\boxed{\Delta\lambda/\lambda = z \approx v/c \quad \text{if } v \ll c.}$$

In the mid-1920s Edwin Hubble measured the distances and redshifts of set of nearby galaxies and found them to follow this law – **the Hubble Law**.

2.4 Comoving Coordinates and Redshift

[Liddle sec:4.2/5.2]

In an expanding universe the physical separation of objects increases with time. It is convenient to define a dimensionless expansion factor a which grows with time. We define a such that the physical separation of objects moving with the Hubble flow is given by

$$r(t) = \left(\frac{a(t)}{a_0}\right) r_0 \tag{2.1}$$

where r_0 is their separation at time t_0 when the expansion factor $a_0 = a(t_0)$. A useful conventional is to take t_0 to be the present age of the universe and define the present value of the scale factor to be $a_0 \equiv 1$.

Taking the time-derivative we find $\dot{r} = (\dot{a}/a)\, r$ which means we can relate the time dependent Hubble parameter to the time derivative of the scale factor:

$$\boxed{H(t) = \dot{a}/a} \tag{2.2}$$

Applying this equation to the present time, we find $H_0 = \dot{a}_0$ since we define $a_0 \equiv 1$. We will come back to the time dependence of H in the next lecture.

It is often convenient to use **comoving** co-ordinates defined as

$$\mathbf{x} \equiv \mathbf{r}/a$$

as these then remain constant as points expand with the Hubble flow.

As the universe expands, the mass within a given co-moving radius stays the same. Thus the density of the universe is given by:

$$\boxed{\rho(t) = \rho_0/a^3} \tag{2.2}$$

where ρ_0 is the present-day density of the universe and $a_0 = 1$. This assumes that the energy density of the universe is dominated by matter, and does not include the contribution from photons (see Lecture 8) or "Dark Energy" (see Lecture 9). Recent estimates of the matter density of the universe suggest

$$\rho_0 \approx 2.5 \times 10^{-27} \mathrm{kg\,m^{-3}} \approx 3.8 \times 10^{10}\, \mathrm{M_\odot\, Mpc^{-3}} \approx 1.5 m_\mathrm{p}\, \mathrm{m^{-3}}.$$

The expansion of the universe also affects photons and thus the wavelength of a photon observed today that was emitted when the universe had a scale factor a is given by

$$\boxed{\lambda_\mathrm{observed}/\lambda_\mathrm{emitted} = \frac{1}{a}}$$

But the wavelength shift defines the redshift z through $\lambda_{\rm observed}/\lambda_{\rm emitted} = 1+z$ and hence redshift and expansion factor are related quite simply by

$$\boxed{1 + z \equiv 1/a.}$$

Its often convenient to label an event in the history of the universe by either: the time since beginning of the universe (t), the expansion factor (a) relative to the present-day, or even the redshift (z) we would measure for a photon emitted during the event. Note that

$$H(t) \equiv \left(\frac{\dot{a}}{a}\right) = \frac{-1}{1+z}\frac{dz}{dt}.$$

Examples

2.1 A civilization on a galaxy at redshift $z = 3$ uses a radio dish to communicate with a neighbouring galaxy. Their signal takes 1 million years to reach the other galaxy. Estimate the following two quantities, stating any assumptions you make:

i) The physical separation of the two galaxies at $z = 3$.

ii) The physical distance between these two galaxies today.

2.2 A certain type of galaxies has a present-day comoving volume density of 10^{-5} Mpc^{-3}.

a) What is the average separation of the galaxies today?

b) If the comoving density of these galaxies increases with redshift at a rate of $(1+z)^2$, how far apart were these galaxies at $z = 2$ in (i) comoving distance; (ii) proper distance?