University of Durham

EXAMINATION PAPER

May/June 2015 Examination code: PHYS2621WE01

STARS AND GALAXIES

SECTION A. Observational Techniques
SECTION B. Stars
SECTION C. Galactic Astronomy

Time allowed: 3 hours

Additional material provided: None

Materials permitted: None

Calculators permitted: Yes **Models permitted:** Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

Visiting students may use dictionaries: No

Instructions to candidates:

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **any three** of the four optional questions. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.

Information

A list of physical constants is provided on the next page.

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Electron mass:	$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^{2}\,kg^{-2}}$
Proton mass:	$m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}$ J s
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_{\mathrm{A}} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	AU $= 1.50 \times 10^{11}$ m
Parsec:	pc $= 3.09 \times 10^{16}$ m
Solar Mass:	$M_{\odot} = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_{\odot} = 3.84 \times 10^{26}$ W

SECTION A. OBSERVATIONAL TECHNIQUES
Question 1 is compulsory. Question 2 is optional.

1. (a) You have two instruments to put onto a telescope. At which focus would you recommend putting each of these instruments? State your reasons.
   (i) Wide field imaging camera to be used for surveys
   (ii) Very high resolution spectrograph to be used to search for Doppler shifts of bright stars caused by exoplanets.

   [4 marks]

   (b) Explain how the dark current, sky background and variations in the sensitivity of a CCD from pixel to pixel (flat fielding) are measured and corrected for in an astronomical image. [4 marks]

   (c) Use the condition for constructive interference to show that

$$\frac{d\lambda}{dx} = \frac{d\ cos\theta}{nf}$$

   where $d\lambda\,/\,dx$ is the wavelength per unit distance along the detector, θ is the angle which the spectrum makes with the grating face, n is the spectral order, d is the distance between the lines on the diffraction grating and f is the focal length of the telescope. [3 marks]

   A diffraction grating is used in second order with the spectrum observed at angle of 30° with respect to the grating face. If the camera used to record the spectrum has focal length of 150 mm, how many lines per millimeter does the grating require in order to achieve a linear dispersion of 1 nm / mm [1 mark]

Continued Next Page

2. (a) When starlight passes through the Earth's atmosphere, some of it is absorbed.

   (i) Show that the apparent magnitude of a star as observed from the Earth's surface ($m_{\rm observed}$) is related to the intrinsic magnitude at the top of the Earths atmosphere ($m_{\rm intrinsic}$) via

   $$m_{\rm intrinsic} = m_{\rm observed} - A(\lambda)\sec(z)$$

   where A(λ) is the wavelength dependent atmospheric absorption coefficient and z is the angle of the star from the zenith. [5 marks]

   (ii) The apparent V-band magnitude of a star at the top of Earth's atmosphere is $m_{\rm V} = 15.0$. If the star is observed from the ground at an angle of 30° from the zenith, what magnitude will be derived by a ground-based telescope? [The extinction coefficient in the V-band is A(λ) = 0.15 mag] [2 marks].

   (b) This 15$^{\rm th}$ magnitude star is orbited by a planet which is ten magnitudes fainter than the star. You want to image the planet in the V-band using the *Hubble Space Telescope*, which has a 2-meter diameter primary mirror. You choose a time for the observation when the planet is well separated from the star.

   (i) Calculate the spatial resolution of the *Hubble Space Telescope* in the V-band ($\lambda = 550$ nm) in arcseconds [1 mark]

   (ii) The *Hubble Space Telescope* has a throughput of 60%. The CCD has a quantum efficiency of 75% and the V-band filter has a bandpass of 100 nm. A magnitude zero star ($m_{\rm V} = 0$) has a flux density of $f = 3.92 \times 10^{-8}\,{\rm W\,m^{-2}\,\mu m^{-1}}$. Calculate the total power collected by the telescope from the planet [3 marks]

   (iii) The plate scale of the CCD onboard the *Hubble Space Telescope* is 0.03″ / pixel and the gain is 1. What is the photon detection rate of the planet per second per pixel by the CCD? [4 marks]

   (iv) This star–planet system is observed through the plane of the solar system and due to zodiacal light, the sky background has an effective magnitude of 19 magnitudes per square arc second. What integration time is required to ensure that the sky background noise is greater than the read noise? Assume that the read-noise is 5 electrons and that the dark current is negligible. [3 marks]

   (v) If the exposure time is adjusted so that the read noise is also negligible, what integration time is required to reach a *total* signal-to-noise ratio for the planet of 100? [2 marks]

SECTION B. STARS

Question 3 is compulsory. Questions 4 and 5 are optional.

3. (a) What simple physical model and what physical parameter of the model determine the observed colour of a star? Which two additional physical processes do we need to consider to understand the observed spectral-line properties of a star? [4 marks]

   (b) The core of a star has the following properties: fully ionised hydrogen, a temperature of $T = 10^7$ K, and an average proton density of $n_p = 10^{32}$ m^{-3}. What is the dominant form of opacity (κ) in the core of the star? Calculate the mean free path of a photon in the core, assuming $\kappa = 0.040$ m^2 kg^{-1}. [4 marks]

   (c) The lifetime of the Sun on the main sequence is 10^{10} years. Assuming 26.2 MeV of energy is produced during each helium-fusing chain, what fraction of the mass of the Sun is converted to helium from hydrogen over the main-sequence lifetime of the Sun? [4 marks]

   [Hint: 1 eV is 1.60×10^{-19} J]

   (d) What determines the minimum possible mass of a main-sequence star? What limits the maximum possible mass of a main-sequence star? Give approximate values for the minimum and maximum masses of main-sequence stars. [4 marks]

   (e) Hayashi tracks trace the paths that protostars take on the Hertzsprung-Russell diagram before joining the main sequence for stars. Draw a Hertzsprung-Russell diagram highlighting the main sequence for stars and the Hayashi track of a low-mass star. [4 marks]

   (f) What are the dominant chemical elements in the cores of white dwarfs? Describe the physical process that generates the pressure that prevents white dwarfs from collapsing. [4 marks]

   (g) Sketch a semi-detached close-binary system, highlighting the principal components of the system. [4 marks]

4. (a) Show that the equation of hydrostatic equilibrium for a spherical mass distribution is given by

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2},$$

where M_r is the enclosed mass within radius r and ρ is the density at radius r. [6 marks]

(b) Using this, show that the pressure at the centre of a star with uniform density is given by

$$P_c = \frac{3}{8\pi}\frac{GM^2}{R^4},$$

where M is the stellar mass and R is the stellar radius. [6 marks]

(c) Given the derivation in part b, do you expect the central pressure estimated in this way to provide an accurate measurement of the true pressure at the centre of a star? Briefly justify your answer. [3 marks]

(d) Does particle or radiation pressure dominate at the centre of a star with temperature $T = 15 \times 10^6$ K, density $\rho = 1.5 \times 10^5$ kg m^{-3}, and mean-molecular mass $\mu = 0.7$? [5 marks]

[Hint: The radiation constant is $a = 7.57 \times 10^{-16}$ J m^{-3} K^{-4}]

  Continued Next Page

5.

(a) The Schwarzschild condition for convection is satisfied in different regions for different masses of stars. Draw cross sections through both a low mass ($1\ M_\odot$) and high mass ($8\ M_\odot$) main-sequence star to highlight the regions where convection is believed to occur in these stars. Why does convection occur in each of these regions? [4 marks]

(b) By considering the conditions required for an adiabatic mass element of material transported upwards in a star to be buoyant, show that the Schwarzschild criterion for convection to occur is $\frac{T}{P}\left(\frac{dP}{dT}\right) < \frac{\gamma}{\gamma-1}$, where P is the pressure, T is the temperature and γ is the ratio of specific heats. Recall that the adiabatic gas law equation is $P = K\rho^\gamma$, where K is a constant and ρ is the density, and that the condition for convection to occur (unstable against convection) is $\left(\frac{dP}{d\rho}\right)_{sur} > \left(\frac{dP}{d\rho}\right)_{ad}$, where the subscript ad refers to the conditions of the adiabatic mass element and sur refers to the conditions of the surrounding gas. [8 marks]

(c) Briefly describe the mixing-length theory of convection for stars. [2 marks]

(d) Calculate the radial extent (ℓ) of the convection cells at the surface of the Sun using the following equation $\ell = \alpha H_\mathrm{P}$, where the scale height (H_P) satisfies $\frac{1}{H_\mathrm{P}} = -\frac{1}{P}\frac{dP}{dr}$ and α is a scaling factor. In your calculation assume that the pressure at the surface of the Sun is $P = 10^{13}$ N m^{-2} and that the density is equal to the average density of the Sun. State a likely value of α and briefly justify the value you have given. [6 marks]

[Hint: the radius of the Sun $= 6.96 \times 10^8$ m]

SECTION C. GALACTIC ASTRONOMY

Question 6 is compulsory. Question 7 is optional.

6. (a) Describe Hubble's classification scheme for galaxies (Hubble Sequence). Briefly discuss two reasons why spiral galaxies are bluer than elliptical galaxies. [4 marks]

   (b) Which physical process produces the 21 cm line of neutral hydrogen? Star formation occurs in dense molecular clouds. Briefly discuss whether star formation can be studied using 21 cm radiation [4 marks]

   (c) Galactic disks are in differential rotation. Explain why this leads to the 'winding problem' of spiral arms. What is the resolution of this problem? [4 marks]

   (d) Use the viral theorem to derive the following relation between the mass (M), radius (R) and velocity dispersion of galaxies (σ) in a cluster of galaxies,

$$\sigma^2 = \frac{\mathrm{G}M}{R}\,.$$

   Calculate the mass of a cluster for which $\sigma = 800$ km s^{-1} and $R = 1.2$ Mpc. Express your answer in solar masses. [4 marks]

   (e) What is gravitational lensing? How did the MACHO project use gravitational lensing to study the nature of the dark matter in the halo of the Milky Way? What was the conclusion from the experiment? [4 marks]

7. (a) The Sun moves on a circular orbit with radius $R_\odot = 8$ kpc and speed $V_\odot = 220$ km s^{-1} around the centre of the Milky Way. Calculate the mass $M(< R_\odot)$ enclosed by the Sun's orbit in units of $M_\odot$. [4 marks]

   (b) The rotation curve of the Milky Way is (nearly) flat, $V_c(R) = V_\odot$, for $R > R_\odot$. Demonstrate that a spherical mass distribution with density distribution $\rho(R) \propto 1/R^2$ results in a flat rotation curve. [4 marks]

   (c) Why is it thought that most of the enclosed mass $M(< R_\odot)$ is due to invisible dark matter, rather than due to the stars we do see? [2 marks]

   (d) Assuming the mass $M(< R_\odot)$ is all composed of dark matter, calculate the density of dark matter $\rho(R = R_\odot)$ at the Sun's location assuming the same model as in part (b). The mean distance between stars at the Sun's location is 1 parsec. Estimate the ratio $\rho_\star/\rho$ of density in stars over density in dark matter at the Sun's location. Discuss whether this result is in contradiction with part (c). [4 marks]

   (e) What is the observational evidence that the Milky Way harbours a central supermassive black hole? Mass accretion makes the black hole shine with luminosity $L = 0.1\dot{M}c^2 = 10^2 L_\odot$. Calculate the mass accretion rate $\dot{M}$ in solar masses per year. [2 marks]

   (f) Assume that the black hole's radiation is all in the form of hydrogen ionising photons of energy $E = 2.2 \times 10^{-18}$ J. Calculate its Strömgren radius R_S in parsec, if it were embedded in a spherical cloud of hydrogen gas with number density $n = 10^2$ cm^{-3}. Use $\alpha = 3.1 \times 10^{-13}$ cm^3 s^{-1} for the recombination rate coefficient. [4 marks]

   [Hint: $R_S = \left(\frac{\dot{N}}{(4\pi/3)\alpha n^2}\right)^{1/3}$, where $\dot{N}$ is the ionising photon emission rate]