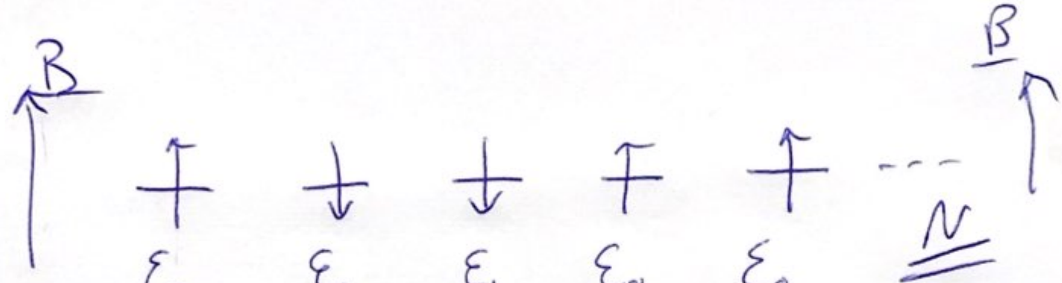
Example. Let's have N localised spins (spin-$\frac{1}{2}$ particles) in a magnetic field $\underline{B}$.

If a spin is parallel to the applied field, let



the energy be $-\mu B$, where μ is the magnetic moment, and the energy is $+\mu B$ if the spin is antiparallel. We have a 2 energy level system

$$\varepsilon_0 = -\mu B = -\varepsilon/2$$

$$\varepsilon_1 = +\mu B = +\varepsilon/2.$$

Recall that the partition function is

$$Z = \sum_i e^{-\beta \varepsilon_i}, \quad \text{where } \beta = 1/k_B T$$

Hence $Z = e^{\beta \varepsilon/2} + e^{-\beta \varepsilon/2} = e^{\beta\varepsilon/2}\left(1 + e^{-\beta\varepsilon}\right)$

Probability of finding state i occupied is Boltzmann factor for state i divided by Z, i.e. $P_i = e^{-\beta\varepsilon_i}/Z$.

$$P_0 = \frac{\cancel{e^{\beta\varepsilon/2}}}{\cancel{e^{\beta\varepsilon/2}}\left(1 + e^{-\beta\varepsilon}\right)} = \frac{1}{1 + e^{-\beta\varepsilon}}$$

$$\boxed{P_0(T) = \frac{1}{1+e^{-\varepsilon/k_BT}}} \quad \begin{matrix} \xrightarrow{\text{low } T} 1 \\ \xrightarrow[\text{high } T]{} 1/2 \end{matrix}$$

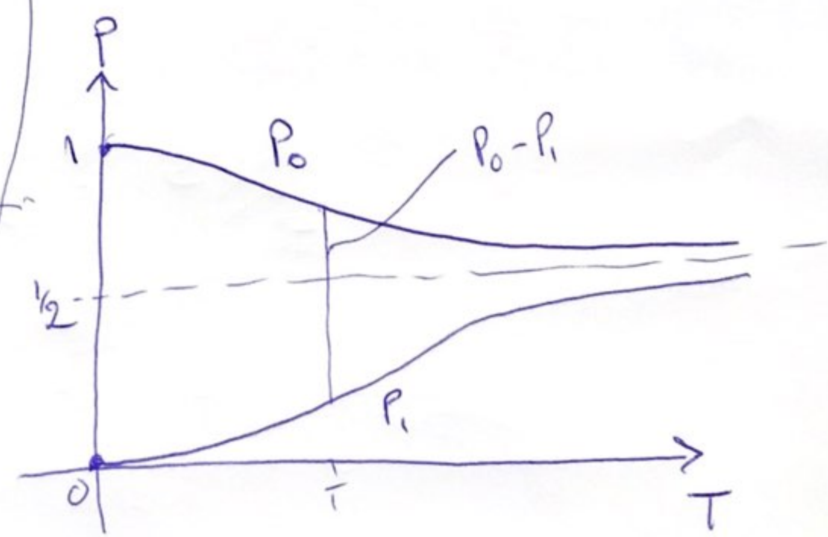
For state 1 we have

$$P_1 = \frac{e^{-\beta\varepsilon/2}}{e^{\beta\varepsilon/2}(1+e^{-\beta\varepsilon})} = \frac{e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$$

$$\boxed{P_1(T) = \frac{e^{-\varepsilon/k_BT}}{1+e^{-\varepsilon/k_BT}}}$$

$\swarrow$ low T $\quad \downarrow$ high T

0 $\quad$ 1/2

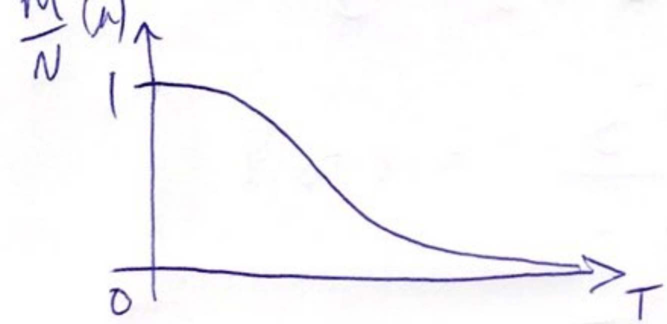


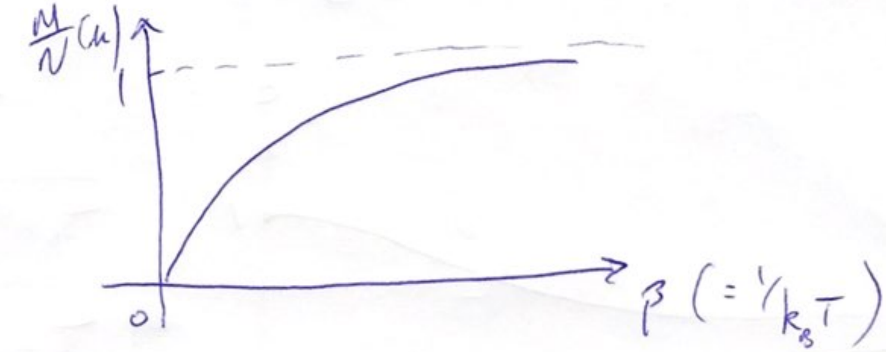
④

Magnetisation of system (M) per particle is the average magnetic moment

$$\frac{M}{N} = P_0 \mu + P_1(-\mu) = \mu (P_0 - P_1)$$

$$= \mu \frac{1 - e^{-\beta\varepsilon}}{1 + e^{-\beta\varepsilon}} = \mu \tanh\left(\beta\varepsilon/2\right)$$





Energy : $\ln(z) = \ln\left[e^{\beta\varepsilon/2}\left(1+e^{-\beta\varepsilon}\right)\right]$

$$= \beta\varepsilon/2 + \ln\left(1+e^{-\beta\varepsilon}\right)$$

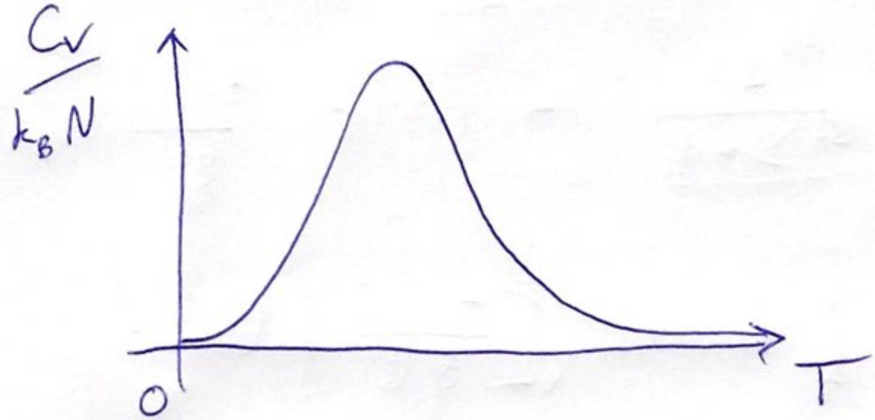
$$\frac{\partial \ln z}{\partial \beta} = \varepsilon/2 - \frac{\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$$

$$\mathcal{U} = -N\left[\frac{\partial \ln z}{\partial \beta}\right]_V = -\frac{N\varepsilon}{2} + N\varepsilon\,\frac{e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$$

Specific Heat : $C_v = \left[\partial \mathcal{U}/\partial T\right]_V = -k_B\beta^2\left[\partial \mathcal{U}/\partial \beta\right]$

$$= -Nk_B\beta^2\varepsilon\left[\frac{-\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}} - \frac{-\varepsilon e^{-2\beta\varepsilon}}{\left(1+e^{-\beta\varepsilon}\right)^2}\right]$$

$$\Rightarrow C_v = N k_B (\beta \varepsilon)^2 \frac{e^{-\beta\varepsilon}}{(1+e^{-\beta\varepsilon})^2}$$



Heat capacity per particle.

This peak is known as a Schottky ~~[illegible]~~ anomoly.