Level 3 Planets & Cosmology

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Lecture 1: Overview

1.0 Our Neighbourhood and its Surroundings

The Sun and Solar System are not unusual among the 10^{11} stars that make up the Milky Way Galaxy. The Milky Way Galaxy is an average-sized spiral galaxy. [Incidentally, astronomers usually refer to the Milky Way as the Galaxy with an uppercase G].

The typical distance between bright galaxies is a few Mpc [1pc= 3.26 light years = 3.09×10^{16}m.]

The most distant observed galaxies are a few 1000 Mpc away.

Within this observable volume we estimate there are of order 10^9 bright galaxies.

1.1 Large Scale Isotropy and Homogeneity

[Liddle sec:2.3]

The distribution on the sky of very distant objects (e.g. faint galaxies, radio galaxies, quasars, or gamma-ray bursts) is very close to isotropic.

This is quite unlike what we see on smaller scales. For example, the stars in our Galaxy fall on a well defined rotating disk (the Galactic Plane).

Even on the scale of a few Mpc the distribution of galaxies is quite irregular. The Milky Way, along with our neighbouring Andromeda Galaxy (0.8 Mpc away) and a few dozen smaller galaxies form the so-called Local Group of galaxies. More massive still are clusters of galaxies, containing anywhere from several hundred to a thousand galaxies confined within a radius of just a few Mpc. Groups and clusters are gravitationally self-bound objects which, due to having an average matter density somewhat above the cosmic mean, have become detached from the general cosmic expansion of space (The Hubble Flow - section 1.3).

On larger scales still, groups and clusters form looser associations known as superclusters, which resemble a cosmic 'foam' with features described as 'filaments', 'nodes', 'walls' and 'sheets'. Together, these make up the so-called 'Large-scale Structure' of the Universe. Despite all this small-scale irregularity, once we move to scales larger than several tens to ~ 100 Mpc the distribution becomes homogeneous. Statistically-speaking, this means that the distribution of matter within any randomly-chosen box larger than a few 100 Mpc on the side looks much the same as in any other.

1.2 The Dominance of Gravity

In our current understanding of fundamental physics there are four fundamental forces of nature:
• the Strong nuclear force (Quantum Chromo Dynamics),
• the Weak nuclear force (mediated by massive W and Z particles),
• the Electromagnetic force (mediated by massless photons) and
• the Gravitational force.

The first two of these forces are short range. They are important in binding quarks into nucleons (neutrons and protons) and binding nucleons into atomic nulcei, but exert no significant forces between atoms let alone any larger scale.

The EM and gravitational forces are both inverse square laws and so, in principle, can exert forces over very large scales.

The EM force is much stronger than that due to gravity. E.g. consider the relative magnitude of EM and gravitational forces between and electron and proton in a hydrogen atom

$$F_{\rm EM} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$F_{\rm Gravity} = \frac{G m_e m_p}{r^2}$$

$$F_{\rm Gravity}/F_{\rm EM} = \frac{G m_e m_p 4\pi\epsilon_0}{e^2} = 4.4 \times 10^{-40}.$$

However EM forces do not dominate on large scales because electric charge comes in both positive and negative quanta. This results in bulk material being very accurately charge neutral and thus not producing EM forces on very large scales. In contrast all mass is positive and the gravitational attraction of very massive objects can be felt over vast distances.

Thus on scales of the solar system upwards gravity is the dominant force shaping the evolution of structure in the Universe.

1.3 Hubble Expansion and Big Bang

[Liddle sec:2.4]

All the galaxies in the universe are moving relative to one another.

Part of their motion is randomly orientated and caused by gravitational forces acting on galaxies, but superimposed on these random motions is a general expansion due to the expansion of space itself (a fundamentally General Relativistic effect).

Distant galaxies are receding away from us. The more distant ones are receding faster. Their general motion appears to fit a simple law

$$v_{\rm rec} = \mathrm{H}_0\, d$$

This is known as Hubble's law and H_0 is Hubble's "constant".

Modern day measurements indicate that $\mathrm{H}_0 \approx 75\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$.

If galaxies are now getting further apart then we can infer that in the past they were closer together. This observation gives rise to idea that the universe was once much smaller and denser and its expansion began in an initial **Big Bang**.

In these notes we shall use the suffix '0' (e.g. x_0) to mean the value of a quantity at the present time. In the next lecture we will show that H (as opposed to H_0) is not a true constant, and that it varies as a function of cosmic time. I will refer to $H(t)$ as the Hubble parameter.

1.4 The Evolution and Growth of Structure

If at some early time there exist slight variations in density from place to place, then gravity will cause overdense regions to contract and drawing additional material into these regions. Thus the amplitude of such density perturbations will grow with time. Over cosmic history this growth is enormous and only tiny density fluctuations are needed in the early universe to generate the structure we see today.

Background Material

This is ancillary material which we will draw on at various points. Some of it (particularly sections 0.0-0.3) should already be familiar to you, the rest will likely be covered in your other L3 physics modules.

0.0 Astronomical Units

Some formulae and results from various branches of physics which we will find useful in Cosmology.

Note that astronomers usually work in units of pc and yr. Here are the conversion factors you will need.

$1\text{pc} = 3.26 \text{ light years} = 3.09 \times 10^{16}\text{m}$

$1\text{yr} = 3.16 \times 10^{7}\text{s}$

0.1 Newtonian Gravity

$$F_{\text{Gravity}} = \frac{Gm_1 m_2}{r^2}$$

0.2 Relativistic Doppler Shift

If a source of radiation is moving away from an observer with velocity v then a photon which in the frame of the source has wavelength λ will be redshifted and observed to have a longer wavelength $\lambda + \Delta\lambda$ (frequency $\nu - \Delta\nu$) where

$$\Delta\lambda/\lambda = \Delta\nu/\nu = \sqrt{\frac{(1+v/c)}{(1-v/c)}} - 1$$

which for $v \ll c$ reduces to

$$\boxed{\Delta\lambda/\lambda = \Delta\nu/\nu = v/c.}$$

Note that for distant cosmological objects, we also need to consider the propogation time for light, and how the recession velocity changes during this time. For this reason, you should be wary of using this formula to convert a redshift into a recession velocity. In particular, the redshift of a source can greatly exceed 1.

0.3 Particles of the Standard Model

There are 3 generations of quarks and leptons.

Quarks			**Leptons**		
+2/3	-1/3	Charge	0	- 1	
		Mass [GeV/c^2]			Mass [MeV/c^2]
u	d	0.35	ν_e	e	0.511
c	s	1.8/0.5	ν_μ	μ	105.7
t	b	175.0/4.5	ν_τ	τ	1784

1) They are ALL spin-1/2 fermions.

2) Quarks have fractional charge, but we only observe particles with integral charge. E.g. mesons with one quark and one antiquark and baryons with three quarks.

3) There are also anti-particles for each of these particles. e.g. e^+ (positron) and $\bar{\nu}_e$ (anti-electron neutrino).

In addition there are the boson particles that mediate the each of the four forces:

• The photon γ which mediates the EM force.

• The $W^\pm$ and Z^0 which mediate the Weak nuclear force.

• The gluons which, through QCD, give rise to the Strong nuclear force.

• There is no fully consistent quantum theory of gravity. Though if there were the massless spin-2 particle responsible would be called the graviton.

NOTE: Only the $W^\pm$ and Z^0 are massive ($M_{W^\pm} = 80.6\,\mathrm{GeV}/c^2$ and $M_{Z^0} = 91.2\,\mathrm{GeV}/c^2$).

Finally, in the Standard Model, there is the Higgs Boson with a mass $\sim$ 125GeV and which gives rise to the masses of the $W^\pm$, Z^0 and quarks.

0.4 Black-Body Radiation (Ultra-relativistic Thermal Bosons)

A thermal distribution of boson particles has a Bose-Einstein energy distribution.

$$N(p)dp = \frac{1}{\exp(E/kT) - 1}\, g\, \frac{4\pi p^2}{h^3} dp,$$

where g counts the number discrete polarization or spin states of the particles. For massless bosons such as photons with $E = pc = h\nu$ this results in the **black-body** energy spectrum

$$\epsilon(\nu)d\nu = N(p)\, E\, dp = \frac{g}{2}\frac{8\pi h\nu^3}{c^3}\, \frac{d\nu}{\exp(h\nu/kT) - 1}.$$

Integrating over all photon energies this gives a total energy density

$$\boxed{\epsilon_{\mathrm{phot}} = \frac{\pi^2 k^4}{15\hbar^3 c^3} T^4 = \frac{g}{2}\frac{4\sigma T^4}{c},}$$

where σ is Stefan's constant and $g = 2$ as photons have 2 polarization states. Integrating over the distribution yields the mean photon energy $\boxed{E_{\mathrm{mean}} = 2.70kT \approx 3kT}$.

0.5 Ultra-relativistic Thermal Fermions

A thermal distribution of fermion particles has a Fermi-Dirac energy distribution. For massless neutrinos, or for other fermions such as electrons and positrons at ultra-relativistic energies ($mc^2 \ll kT$, $E \approx pc$) the energy spectrum is

$$\epsilon(E)dE = \frac{g}{2}\, \frac{8\pi E^3}{h^3 c^3}\, \frac{dE}{\exp(E/kT) + 1}.$$

Integrating over all energies

$$\boxed{\epsilon_{\mathrm{rel\ fermions}} = \frac{7}{8}\frac{g}{2}\frac{4\sigma T^4}{c},}$$

where σ is Stefan's constant and g counts the number of discrete spin states. I.e. $\epsilon_{\mathrm{rel\ fermions}} = 7/8\, \epsilon_{\mathrm{phot}}$. The mean particle energy is $\boxed{E_{\mathrm{mean}} = (7/6)\, 2.70kT \approx 3kT}$.

0.6 Non-relativistic Particles

In this limit $kT \ll mc^2$, $E \approx mc^2 + 1/2mv^2$ and $p \approx mv$. Since $\exp(E/kT) \gg 1$ the ± 1 in the BE of FD distributions can be neglected and number of particles with velocity v is

$$N(v)dv \propto v^2 \exp(-mv^2/2kT)\, dv.$$

This is the normal Boltzmann distribution. Using this we find the familiar result that the mean particle energy is $E = mc^2 + 3/2\, kT$.

Examples

Example questions are provided at the end of each set of lecture notes. Where possible, we will work through them in the lecture. A file of fully-worked solutions is in any case available on DUO.

1.1 If $\mathrm{H}_0 = 70\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$, express $1/\mathrm{H}_0$ in units of time (seconds and Gyr).

1.2 Given a matter density of $\rho_0 = 3.8 \times 10^{10}\ \mathrm{M_\odot\ Mpc^{-3}}$, express ρ_0 in protons per cubic metre.