University of Durham

EXAMINATION PAPER

May/June 2014 Examination code: 042621/01

LEVEL 2 PHYSICS: STARS AND GALAXIES

SECTION A. Observational Techniques
SECTION B. Stars
SECTION C. Galactic Astronomy

Time allowed: 3 hours

Examination material provided: None

Calculators: The following types **only** may be used: Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **any three** of the four optional questions. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.

ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK

Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_{\mathrm{B}} = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron mass:	$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_{\mathrm{A}} = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\,\mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Astronomical Unit:	AU = 1.50×10^{11} m
Parsec:	pc = 3.09×10^{16} m
Solar Mass:	$M_{\odot} = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_{\odot} = 3.84 \times 10^{26}$ W

SECTION A. OBSERVATIONAL TECHNIQUES
Question 1 is compulsory. Question 2 is optional.

1. (a) Describe four effects of the Earth's atmosphere on electromagnetic radiation passing through it. [4 marks]

   (b) An astronomer uses a CCD camera with a 10 cm diameter lens to detect low surface brightness galaxies. What exposure time is needed to detect such galaxies at the same signal-noise as a 1 minute exposure on a telescope with a 2.5 m diameter mirror and equipped with the same CCD detector? You may assume that readout noise and dark current are always negligible and that the throughput is 100% for both cases. [4 marks]

   (c) The spectral resolving power, R, of a spectrograph is given by the following formula:

$$R = \frac{m\rho\lambda W}{\chi D_T},$$

   where m is the diffraction order, ρ is the ruling density of the grating, λ is the wavelength, W is the size of the grating, χ is the angular size of the target star and D_T is the size of the telescope. What ruling density (in lines/mm) would be required to resolve two emission lines with wavelengths differing by 0.1 nm at $\lambda = 590$ nm in the second order with $W = 10$ cm, $D_T = 4$ m and with the entrance slit adjusted to 1 arcsec seeing? [3 marks]

   In the case of 0.5 arcsec seeing what difference in emission line wavelength could be resolved? [1 mark]

2. A 10 m telescope is fitted with perfect adaptive optics which allows diffraction limited imaging.

   (a) Calculate the diffraction limit of this telescope in the V-band with a central wavelength of 550 nm. Give your answer in milli-arcseconds. [2 marks]

   (b) A V-band image of a star is recorded using this telescope fitted with a detector with pixels which are $10 \times 10\ \mu\mathrm{m}^2$ in size. The effective focal length of the telescope is 500 m. Calculate approximately how many pixels the image will cover. [4 marks]

   (c) Calculate the total (i.e. over all the pixels) photon detection rate for the image of a star of magnitude $V = 20.0$ taken with the telescope. You may assume that the atmosphere and telescope have 100% throughput and the CCD has 100% quantum efficiency. The bandwidth of the V-band is 90 nm and a zeroth magnitude star provides $3.92 \times 10^{-8}\ \mathrm{Wm}^{-2}\mu\mathrm{m}^{-1}$. [6 marks]

   (d) The observation is made when the moon is full, and therefore the background is very large, $B = 10^8$ photons s^{-1} pixel^{-1}. Calculate the exposure time necessary to give a signal-to-noise ratio of 5. [4 marks]

   (e) It is suggested that the error on the position of the star can be estimated by dividing the diffraction limit by the above signal-to-noise ratio. Provide a brief justification in the shot-noise limited case and then estimate this error in the case of the above observation. [4 marks]

SECTION B. STARS

Question 3 is compulsory. Questions 4 and 5 are optional.

3. (a) In an eclipsing binary system, the time taken for the light to drop from uneclipsed to fully eclipsed is 7 hours. The relative velocity of the smaller star with respect to the larger star is 80 km s^{-1}. What is the radius of the smaller star? [4 marks]

   (b) Write down the three steps of the main proton-proton chain (PP I) for fusion of hydrogen into helium in the Sun. Make sure that the nuclear conservation laws are obeyed. [4 marks]

   (c) Draw a cross section through the centre of a massive star (8 solar masses) undergoing carbon burning at the core, indicating any other fusion-burning regions in the star. [4 marks]

   (d) Estimate the mean free path of a photon at the core of the Sun, given that the density there is $\rho = 1.5 \times 10^5$ kg m^{-3} and the Rosseland mean opacity is $\kappa = 0.04$ m^2 kg^{-1}. What is the dominant form of opacity at the core of the Sun? [4 marks]

   (e) The majority of pulsating stars lie in the instability strip on the Hertzsprung-Russell diagram. What zone within stars facilitates stellar pulsations? Why don't hotter stars to the blue side of the instability strip pulsate? Why don't cooler stars to the red side of the instability strip pulsate?
   [4 marks]

   (f) List the forces preventing the collapse of the following objects: (i) a normal star, (ii) a white dwarf, (iii) a neutron star, and (iv) a black hole, respectively. [4 marks]

   (g) Calculate the period of rotation for a neutron star with a mass twice that of the Sun rotating at the point of breakup. Assume that the neutron star has a radius of 10 km. [4 marks]

4. (a) What is the Jeans mass? [2 marks]

(b) Show that the Jeans mass (M_J) for a spherically symmetric gas cloud satisfies the relation

$$M_J = \left(\frac{5k_BT}{G\mu m_p}\right)^{\frac{3}{2}} \left(\frac{3}{4\pi\rho}\right)^{\frac{1}{2}},$$

where T is the temperature of the cloud, μ is the (dimensionless) mean molecular mass, and ρ is the average density of the cloud. [5 marks]

[Hint: Recall that the gravitational potential energy is $U = -\frac{3}{5}\frac{GM^2}{R}$]

(c) For a cloud of H_2 with a temperature of 50 K and an average density of 1.0×10^{-18} kg m^{-3}, calculate the minimum mass needed for it to collapse. [3 marks]

(d) Hayashi tracks trace the paths that protostars take on the Hertzsprung-Russell diagram before joining the main sequence. Draw a Hertzsprung-Russell diagram highlighting the main sequence and the Hayashi tracks for both a high-mass star and a low-mass star. [6 marks]

(e) The radiation from a protostar of radius 4.0×10^6 km is absorbed by a shell of dust with a radius 1.0×10^9 km. If all of the radiation from the protostar is re-emitted by the dust shell, and the shell is observed to have a temperature of 500 K, what is the effective temperature of the protostar? [4 marks]

[Hint: assume the radiation has a black body distribution.]

5. (a) What is a semi-detached close-binary system and how does it differ from other close-binary stellar systems? [3 marks]

(b) The approximate mass transfer rate in such a system can be written as

$$\dot{M} = \pi R d \rho \sqrt{\frac{3k_{\mathrm{B}}T}{m_{\mathrm{p}}}},$$

where ρ is the density and T is the temperature.

Make a simplified sketch of the geometry of the system and indicate what R and d in the above equation correspond to. [4 marks]

(c) For a system with $R = 1.0 \times 10^9$ m, $d = 1.0 \times 10^6$ m, $\rho = 2.0 \times 10^{-7}$ kg m^{-3}, and $T = 7500$ K, calculate the mass transfer rate of the gas stream. [1 mark]

(d) Calculate the mass-accretion luminosity assuming that the gas stream falls directly onto a neutron star with a mass of 3.0×10^{30} kg and a radius of 15 km. [3 marks]

(e) Calculate the mass-to-energy efficiency of the mass accretion. How does it compare to the mass-to-energy efficiency of nuclear fusion? [5 marks]

(f) What will happen to the gas stream if the orbital motion of the binary prevents the gas from falling directly on to the primary star? How will this change the mass-to-energy efficiency of the mass accretion? [4 marks]

SECTION C. GALACTIC ASTRONOMY

Question 6 is compulsory. Question 7 is optional.

6. (a) Describe two pieces of observational evidence that suggest that spiral galaxies are usually forming stars. Core-collapse supernovae are the final stage of the evolution of massive stars. Why do you not expect to find core-collapse supernovae in elliptical galaxies? [4 marks]

   (b) Which physical process generates the X-rays detected in clusters of galaxies? How do X-ray observations allow you to infer the mass of a cluster? Why does the inferred mass suggest the presence of dark matter? [4 marks]

   (c) Oort measured the radial (v_r) and tangential (v_t) speed of stars with respect to the Sun in terms of distance d, and galactic coordinate, l. How did he measure v_r and d? Why did these measurements suggest the presence of dark matter in the Milky Way? [4 marks]

   (d) An O-type star emits 1.0×10^{48} ionising photons per second. Compute the radius in parsec within which all HI gas in its surroundings is ionised after 1.0×10^{4} years for a hydrogen number density of 50 cm^{-3}. You may ignore hydrogen recombinations. The star also heats the surrounding gas. Given that the Jeans mass scales with temperature as $M_J \propto T^{3/2}$, will heating the gas tend to increase or decrease the typical mass of stars that form? Explain your reasoning. [4 marks]

   (e) What is gravitational lensing? How can gravitational lensing be used in the study of very distant galaxies? [4 marks]

  Continued Next Page

7. (a) Spiral galaxies have flat rotation curves, $V_c(r) = \text{constant}$, where r is the distance from the centre and V_c the circular velocity. Demonstrate that a spherical dark matter halo with density distribution $\rho_{\rm dm}(r) = V_c^2/(4\pi G r^2)$ yields a flat rotation curve. [4 marks]

   (b) The Milky Way's circular velocity is $V_c = 220$ km s^{-1}. Calculate the dark matter density, $\rho_{\rm dm}$, at the location of the Sun, $r = 8$ kpc, for the density distribution given in (a). Express your answer in $M_\odot \mathrm{pc}^{-3}$. [2 marks]

   (c) Most of the stars in the Milky Way are in its thin disc. The amount of light emitted from this disc per unit area, $I(R)$, drops exponentially with distance R from the centre, $I(R) = I_0\, e^{-R/R_h}$. Show that the total luminosity is $L = 2\pi\, I_0\, R_h^2$, and evaluate L for $R_h = 3.5$ kpc and $I_0 = 3.9 \times 10^8\, L_\odot\, \mathrm{kpc}^{-2}$. Express your answer in $L_\odot$. [4 marks]

   (d) The mass density in stars, $\rho_\star$, at the location of the Sun can be estimated by assuming all stars have mass $M = M_\odot$, and that the mean distance between stars is 1 pc. Use this to compute the ratio $\rho_\star/\rho_{\rm dm}$. Why is the rotation curve flat at the location of the Sun, even though the mass density in stars of the disc is much higher than the dark matter density of the halo at this location? [4 marks]

   (e) The rotation speed of spirals can be measured using the hydrogen 21 cm line. Which physical process produces this line? [2 marks]

   (f) Assume an exponential disc is observed face on from a distance D. Demonstrate that the flux observed per unit solid angle, the surface brightness, $dF/d\Omega$, is independent of D. [4 marks]

   [Hint: Recall that the solid angle $d\Omega$ subtended by a surface area dS at distance D is $d\Omega = dS/D^2$.]