Mathematical Methods in Physics

Weekly Problems 4

4.1

What are the Fourier coefficients of the function $f(x) = 1 + \sin(3x + \pi/5)$ seen as a periodic function of period 2π?
[*Hint: Expand the sine function using* $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$.]

4.2

Calculate the Fourier series of the function $f(x) = x^2$ periodically continued from the interval $[0, 2]$ to the entire real axis. Show explicitly your calculations for the integrals.

4.3

a) The Fourier series of the function $h(x) = \cos(x/2)$ for $-\pi \le x < \pi$ repeating itself with period 2π on the entire real axis is:

$$h(x) = \cos(x/2) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{r=1}^{\infty} \frac{(-1)^r}{(1 - 4r^2)} \cos(rx) \ . \tag{1}$$

b) By differentiating expression (1), calculate the Fourier series for the function $g(x) = \sin(x/2)$ for $-\pi \le x < \pi$ repeating itself with period 2π on the entire real axis.

c) Choosing an appropriate value for x, use the Fourier series of $h(x)$ to compute the sum

$$S = \sum_{r=1}^{\infty} \frac{(-1)^r}{1 - 4r^2} \ .$$