Cosmology Part I: Workshop III

These questions illustrate different ways of measuring masses.

1. The sun orbits the centre of the galaxy at a distance of $r = 8\,\mathrm{kpc}$ with a speed of $220\,\mathrm{km\ s^{-1}}$. **Assuming spherical symmetry** (for simplicity), estimate the mass of the galaxy within the solar radius.

$[G = 6.67 \times 10^{-11}\mathrm{kg^{-1}m^3s^{-2}}.\ 1\mathrm{pc} = 3.09 \times 10^{16}\mathrm{m.}]$

The luminosity of the Milky Way within the solar radius is approximately $10^{10}\,\mathrm{L_\odot}$. Estimate the mass in these stars.

[Hint: The sun is a typical star, $M_\odot = 2 \times 10^{30}$ Kg.]

2. A galaxy cluster is composed of 100 galaxies within a radius of $R = 1\,\mathrm{Mpc}$, with rms (1-d) velocities of $1000\,\mathrm{km\ s^{-1}}$. Making some reasonable assumptions calculate the following:

i) The total mass of the galaxy cluster.

ii) The fraction of the cluster's mass which is in the form of stars, if each galaxy has a luminosity of $10^{11}\,\mathrm{L_\odot}$.

3. The XMM satellite observatory is used to make X-ray observations of the intracluster gas in a rich galaxy cluster. X-ray spectra reveal the gas to have a uniform temperature of $T = 1.6 \times 10^8$ K. Maps of the X-ray surface brightness are consistent with a spherically symmetric distribution of gas with

$$\rho_{\mathrm{gas}}(r) = \rho_0 \left(\frac{r}{1\ \mathrm{Mpc}}\right)^{-2},$$

where $\rho_0 = 1.43 \times 10^{13}\ \mathrm{M_\odot\ Mpc^{-3}}$.

i) Calculate the mass of gas within $r \leq 1$ Mpc.

ii) Use the equation of hydrostatic equillibrium to show that the total mass of the cluster within radius r is

$$M = 4\,\frac{r}{G}\,\frac{kT}{m_{\mathrm{p}}}$$

for $r \leq 1$ Mpc.

iii) Hence estimate the total mass within $r \leq 1$ Mpc and compare with your answer from part (i).

[Assume the intracluster gas to be ionized hydrogen, so that the average mass of a particle of the gas is $\frac{1}{2}m_p$. The Boltzman constant is $k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\ K^{-1}}$, the mass of a proton is $m_p = 1.67 \times 10^{-27}$ Kg.]