

Mathematical Methods II

Weekly problem set 7

- (a) Find the general solution of the first-order partial differential equation (PDE)

$$x \frac{\partial u}{\partial x} + \frac{1}{2} \frac{\partial u}{\partial y} = 0,$$

by searching for a solution in the form $u(x, y) = f(p)$ where $p = p(x, y)$ is a certain function of x and y which you need to determine, and $f(p)$ is an arbitrary function of its argument.

- (b) Now solve a more complicated PDE

$$x \frac{\partial u}{\partial x} + \frac{1}{2} \frac{\partial u}{\partial y} + 5u = 0,$$

by looking for a solution in the form $u(x, y) = h(x, y) \cdot f(p(x, y))$ where $h(x, y)$ is any particular solution of this equation, and $f(p(x, y))$ is the general solution of the equation in part (a).

You will need to seek a valid particular solution for $h(x, y)$. Check it is valid by substituting it into the PDE.

- (c) Impose the boundary condition $u(x, y) = 3/x^2$ on the line $y = 0$ and hence derive the solution of the boundary value problem for the equation in part (b).