ELECTROMAGNETISM

Level 2 Physics problems – Foundations of physics 2

Question 2 Cycle 2 Version 1

Professor D P Hampshire – 2nd Year Physics Lecture Course

These problems are formatively self-assessed. Students who are good and brave, and showed the chutzpah to volunteer for the peer-marking pilot scheme will also mark one of their peer's scripts.

Reading Material (Please note that questions may not be exclusively associated with these chapters); Possible reading Feynman Lectures in Physics Chapters 1-3 (Vector calculus). Note - the most important issue this week is still for you to refine your own procedure for reading and understanding science textbooks and making notes that will aid you in revision.

1. If $\underline{\boldsymbol{A}} = 5yz\hat{\boldsymbol{\imath}} + 2y^2x\hat{\boldsymbol{\jmath}} - z\hat{\boldsymbol{k}}$ and $\underline{\boldsymbol{B}} = 3x\hat{\boldsymbol{\imath}} - (z^2 - x^2)\hat{\boldsymbol{k}}$, $f = xyz$ and $g = x^2z - y$, show that,

a) $\underline{\boldsymbol{\nabla}} \times (\underline{\boldsymbol{\nabla}} \times \underline{\boldsymbol{A}}) = \underline{\boldsymbol{\nabla}}(\underline{\boldsymbol{\nabla}} \cdot \underline{\boldsymbol{A}}) - \nabla^2\underline{\boldsymbol{A}}$ [1 mark]

b) $\underline{\boldsymbol{\nabla}}(fg) = f\underline{\boldsymbol{\nabla}}g + g\underline{\boldsymbol{\nabla}}f$ [1 mark]

c) $\underline{\boldsymbol{\nabla}} \times \underline{\boldsymbol{\nabla}}f = 0$ [1 mark]

d) And find $\underline{\boldsymbol{B}} \cdot (\underline{\boldsymbol{\nabla}} \times \underline{\boldsymbol{A}})$ [1 mark]

2. If $\underline{\boldsymbol{A}} = x\hat{\boldsymbol{\imath}} + 2yz\hat{\boldsymbol{\jmath}} + y^2\hat{\boldsymbol{k}}$,

a) calculate the line integral of the function from the origin to (1,1,1) by the path: (0,0,0) → (1,0,0) → (1,1,0) → (1,1,1) [1 mark]

b) Can you tell whether it is independent of path or not? [1 mark]

3. A soap bubble 20 cm in radius with a wall thickness of 3.3×10^{-6} cm is charged to a potential of 100 V. The bubble bursts and becomes a spherical drop. Estimate the potential of the drop. State the assumption you have used. [2 marks]

4. A conducting sphere of radius 10 cm is charged to 5 kV above Earth potential and isolated. Find the charge on the sphere, the $\underline{\boldsymbol{E}}$-field just outside it, and the $\underline{\boldsymbol{E}}$-field and potential 20 cm from its centre. [2 marks]