L2 Foundation of Physics 2B Optics 2018-19

O.4 Two waves: Summary

Learning outcomes:

1. To discuss the concept of **interference** [Optics $f2f$ Chapter 3].
2. To add two planar waves [Optics $f2f$ Sec. 3.3].

When two or more waves intersect they interfere. **Interference** maps the **relative phase** between the two waves into intensity maxima and minima.

A (scalar) plane wave propagating with wavevector $\underline{k}_1$ is $\underline{E}_1 = E_1 \mathrm{e}^{\mathrm{i}(\underline{k}_1\cdot\underline{r}-\omega t)}$. The **sum of two plane waves** is

$$E = E_1 + E_2 = E_1 \mathrm{e}^{\mathrm{i}(\underline{k}_1\cdot\underline{r}-\omega t)} + E_2 \mathrm{e}^{\mathrm{i}(\underline{k}_2\cdot\underline{r}-\omega t)} \,. \tag{1}$$

Calculating the intensity $I \propto |E^2|$ gives cross terms like $E_1 \cdot E_2^\star$ that give rise to **interference fringes**. Another way to calculate the intensity (if we assume $E_1 = E_2 = E_0$) is to re-write the electric field as

$$E = 2\mathcal{E}_0 \mathrm{e}^{\mathrm{i}[\bar{\underline{k}}\cdot r-\omega t]} \cos\left[(\Delta\underline{k}\cdot\underline{r})/2\right] \,, \tag{2}$$

where $\bar{\underline{k}} = (\underline{k}_1 + \underline{k}_2)/2$ and $\Delta\underline{k} = \underline{k}_1 - \underline{k}_2$. The first exponential terms is a **global phase** factor that disappears when we calculate intensity. However, the **relative phase** terms survive. The intensity is

$$\mathcal{I} = 4\mathcal{I}_0 \cos^2\left[(\Delta\underline{k}\cdot\underline{r})/2\right] \tag{3}$$

If $\underline{k}_1$ and $\underline{k}_2$ are in the $x-z$ plane at an angle of $\theta = \pm\theta_0/2$ with respect to the z-axis then the spacing between the intensity maxima along x is

$$\Lambda = \frac{\lambda}{2\sin(\theta_0/2)} \,, \tag{4}$$

where the extra factor of 2 in the demonimator arises because for cosine squared, there is a maximum whenever the argument $2\pi x/[\sin(\theta_0/2)]$ equals an integer multiple of π.

For small θ, the spacing frequency of the interference fringes is proportional to angle, $1/\Lambda \approx \theta/\lambda$.

Outlook: In the next lecture, we shall look at an application of two-wave interference using the example of the **Michelson interferometer** [Optics $f2f$ Sec. 3.12].