Foundations of Physics 3A 2020/21 QM3 – Problem 2

Problem: Use the variational method to obtain the ground state energy approximately for the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + bx^4,$$

where m and b are two constants, using the function $\phi(\lambda, x) = \exp(-\lambda x^2)$ as a trial function (λ is a variational parameter, which we take to be real — i.e., zero imaginary part).

$$\left[\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}, \quad \int_{-\infty}^{\infty} x^2 e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}, \quad \int_{-\infty}^{\infty} x^4 e^{-ax^2}\,dx = \frac{3}{4}\sqrt{\frac{\pi}{a^5}}.\right]$$

Self-assessing your work: consider yourself to have been "successful" if you have got most things right or almost right, and "partially successful" if you made many errors or did not know what to do after you had obtained a result for $E[\phi]$.