Interference

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In this lecture we will learn how to add waves to describe the phenomenon of interference

- Basics: adding two plane waves
- Young's holes: adding two paxaxial spherical waves
- Michelson interferometer: testing gravity

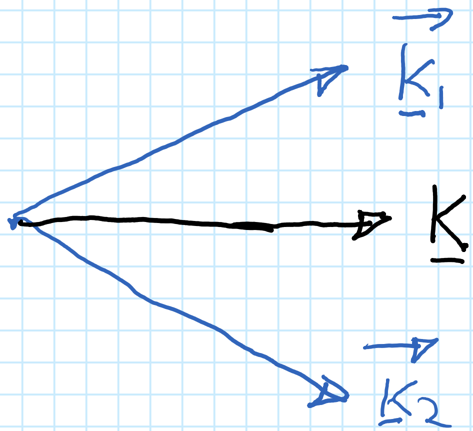
Key concept:

RELATIVE PHASE between waves $\xrightarrow{\text{MAPS ON TO}}$ INTENSITY PATTERN (eg spatially varying fringes)

INTENSITY PATTERN $\xrightarrow{\text{PROVIDES INFO ON}}$ RELATIVE PHASE (eg gravitational waves)

Example 1: Two plane waves at an angle

10 January 2020 15:01



$$\underline{k} = \frac{(\underline{k}_1 + \underline{k}_2)}{2} \quad \text{BISECTOR}$$

$$\Delta \underline{k} = \underline{k}_1 - \underline{k}_2 \quad \perp \text{ to BISECTOR}$$

Then field in the SCALAR APPROX. is:

$$E = \underbrace{E_1 e^{i(\underline{k}_1 \cdot \underline{r}_1 - \omega t)}}_{\text{WAVE } 1} + \underbrace{E_2 e^{i(\underline{k}_2 \cdot \underline{r}_2 - \omega t)}}_{\text{WAVE } 2} \qquad (1)$$

Now we measure $I \propto |E^2| = |EE^*|$ ← time average

$$\therefore I \propto E_1 E_1^* + E_2 E_2^* + \underbrace{E_1 E_2^* + E_2 E_1^*}_{\text{CROSS TERMS GIVE INTERFERENCE}}$$

Two plane waves continued

21 January 2020 14:32

Now let $|E_1|^2 = |E_2|^2 = |E_0|^2$ and $I_0 = \frac{1}{2}\varepsilon_0 c |E_0|^2$

$$I = I_0 + I_0 + \underbrace{I_0\left[e^{i\underline{k}_1\cdot\underline{r}}e^{-i\underline{k}_2\cdot\underline{r}} + e^{-i\underline{k}_1\cdot\underline{r}}e^{+i\underline{k}_2\cdot\underline{r}}\right]}_{\text{contains RELATIVE PHASE}}$$

$$= 2I_0 + I_0\left[e^{i(\underline{k}_1-\underline{k}_2)\cdot\underline{r}} + e^{-i(\underline{k}_1-\underline{k}_2)\cdot\underline{r}}\right]$$

$$= 2I_0 + 2I_0\cos(\Delta\underline{k}\cdot\underline{r}) \qquad \Delta\underline{k} = \underline{k}_1 - \underline{k}_2$$

$$= 4I_0\cos^2\left(\frac{\Delta\underline{k}\cdot\underline{r}}{2}\right) \quad \text{using } \cos^2\theta = \frac{1+\cos^2\theta}{2}$$

$\Delta\underline{k}$ contains RELATIVE PHASE DUE TO ANGLE

Another way

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Optics f2f sec. 3.3 uses another way

Equation (1) can be re-written

$$E = E_0 e^{i(\bar{\underline{k}}\cdot\underline{r} - \omega t)}\left(e^{i\Delta\underline{k}\cdot\underline{r}/2} + e^{-i\Delta\underline{k}\cdot\underline{r}/2}\right)$$

where $\bar{\underline{k}} = (\underline{k}_1 + \underline{k}_2)/2$

then $E = 2E_0 \underbrace{e^{i(\bar{\underline{k}}\cdot\underline{r} - \omega t)}}_{\text{GLOBAL PHASE does not affect fringe pattern}} \underbrace{\cos\left(\frac{\Delta\underline{k}\cdot\underline{r}}{2}\right)}_{\text{contains RELATIVE PHASE via } \Delta\underline{k}}$

MPAJ DEMO

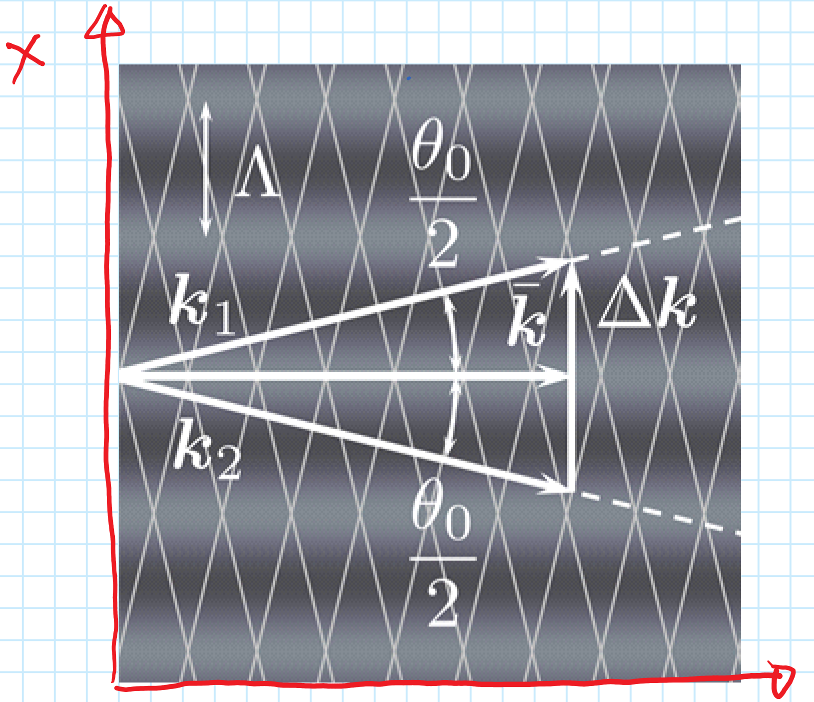
GLOBAL PHASE does not affect fringe pattern

contains RELATIVE PHASE via $\Delta\underline{k}$

THUS $I = 4 I_0 \cos^2\left(\frac{\Delta\underline{k}\cdot\underline{r}}{2}\right)$ as before

Two plane waves continued

10 January 2020 15:15



Optics f2f 3.4

Bright and dark fringes along $\Delta \underline{k}$

Now let

$\underline{k}_1$ & $\underline{k}_2$ lie in xz plane

and let them make angle

$\pm \theta_0/2$ with respect to z

Two plane waves continued

10 January 2020 15:42

THEN $\underline{K}$ is along z and $\Delta \underline{K}$ along x

ie $\Delta K = 2K \sin\left(\frac{\theta_0}{2}\right)$ (factor of 2 from the $\pm \theta_0/2$)

$$I = 4 I_0 \cos^2\left(K \sin\left(\frac{\theta_0}{2}\right) x\right)$$

and the fringe spacing Λ is

$K \sin\left(\frac{\theta_0}{2}\right) \Lambda = \pi$ (since $\cos^2$ is periodic in $n\pi$)

$$\therefore \Lambda = \frac{\pi}{K \sin\left(\frac{\theta_0}{2}\right)} = \frac{\cancel{\pi} \lambda}{2 \cancel{\pi} \sin\left(\frac{\theta_0}{2}\right)} \simeq \frac{\lambda}{\theta_0}$$

Small angle $\sin\frac{\theta_0}{2} \simeq \frac{\theta_0}{2}$

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Demos and show slides