Level 2 Stars, Workshop 4

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Nuclear Fusion

(a) Calculate the energy released from the fusion of three Helium nuclei to produce Carbon (i.e., the triple alpha process). Calculate the efficiency of the triple alpha process and compare to the efficiency of the proton-proton chain (0.7%).

(b) Describe the two main factors that determine the probability of a nuclear reaction occurring in the Sun.

(c) Derive an equation to calculate the temperature required for particles to surmount the Coulomb barrier to allow nuclear fusion to occur. Calculate the temperature required for nuclear fusion, assuming a fully ionized Hydrogen gas and a radius of $r=10^{-15}$ m (i.e., the range of the strong force).

(d) Why does nuclear fusion appear to be unfeasible on the basis of the classical approach taken above? Briefly explain what quantum effect needs to be considered to provide a better understanding of how nuclear fusion works, outlining the main principles behind this effect.

(e) At the core temperature of the Sun ($T_{c,Sun}=1.57 \times 10^7$ K), the proton-proton chain provides 98.5% of the luminosity while the CNO cycle provides just 1.5% of the luminosity. Assuming only that $\varepsilon_{pp} \propto T^4$ and $\varepsilon_{CNO} \propto T^{17}$, calculate the core temperature at which the proton-proton chain and CNO cycle provide an equal contribution to the luminosity.

(f) Following on from part (e), since the core temperature of main-sequence stars broadly scales as $\frac{T_c}{T_{c,Sun}} = \left(\frac{M}{M_{Sun}}\right)^{0.5}$, calculate the mass at which the CNO cycle provides 98.5% of the luminosity and the proton-proton chain provides just 1.5% of the luminosity.

[$m_H = 1.67 \times 10^{-27}$ kg; $e_c = 1.60 \times 10^{-19}$ C; $\varepsilon_o= 8.85 \times 10^{-12}$ F m^{-1}; $k = 1.38 \times 10^{-23}$ J K^{-1}; $\sigma_T = 6.65 \times 10^{-29}$ m^2; $h = 6.63 \times 10^{-34}$ J s; $M_{Sun}=1.99 \times 10^{30}$ kg; $c = 3.00 \times 10^8$ m s^{-1}; $m_p = 1.6726 \times 10^{-27}$ kg; $m_{He} = 6.6465 \times 10^{-27}$ kg; $m_C = 1.9926 \times 10^{-26}$ kg; $T_{c,Sun}=1.57 \times 10^7$ K]