(a) The following reasons for telescope size will be accepted:
collect more light
smaller diffraction limit (better angular resolution) [2 marks, bookwork]

The following reasons for mountain siting will be accepted:
clearer and / or drier skies
more stable atmosphere
away from light pollution (darker skies)
above clouds
less atmosphere (less turbulence) [2 marks for any combination, bookwork]

(b) The inverse square law links the flux, f, of a star at a distance, d, to the luminosity, L, it would have if it were at distance $D = 10\,\mathrm{pc}$:

$L\,/\,f = (d\,/D)^2$ [1 mark, bookwork]

The equation relating magnitudes and flux is:

$m_1 - m_2 = -2.5\log(f_1\,/\,f_2)$

If M corresponds to the magnitude at luminosity of L, and m corresponds to the magnitude at a flux of f, then the equation becomes:

$m - M = 2.5\log(L\,/\,f)$

$m - M = 2.5\log(d\,/D)^2$

$m - M = 5\log(d\,/\,10)$

$\underline{m - M = 5\log(d) - 5}$ [1 mark, bookwork]

For a parallax of $0.25''$, the distance is:

$d[\mathrm{pc}] = 1\,/\,\phi['']$

$d[\mathrm{pc}] = 1\,/\,0.25$

$\underline{d = 4\,\mathrm{pc}}$ [1 mark unseen]

The absolute magnitude is:

$M = m + 5 - 5\log(d)$

$M = 11.5 + 5 - 5\log(4)$

$\underline{M = 13.5}$ [1 mark unseen]

(c) The gain defines the number of photo-electrons detected by a pixel of the CCD which are converted to single ADU count. [1 mark bookwork]

or

$gain = \frac{\# photo-electrons/pixel}{\# counts/pixel\ (ADU)}$ [1 mark bookwork for this alternative]

In the photon-noise limited case:

$\mathrm{S} / \mathrm{N} = \sqrt{N_\gamma}$ [1 mark seen]

In this case:

$\mathrm{S} / \mathrm{N} = 10\,000 / 50 = \sqrt{N_\gamma}$

$\sqrt{N_\gamma} = 200$

$\underline{N_\gamma = 40\,000}$ [1 mark seen]

The conversion from photo-electrons to counts is:

$N_\gamma = g \times \mathrm{ADU}$

If 40 000 photo-electrons were detected and 10 000 ADU measured:

$g = 40\,000 / 10\,000$

$\underline{g = 4}$ [1 mark unseen]

(a) Advantages:
Low sky background
No atmospheric turbulence, so images are diffraction limited
No atmospheric absorption or refraction
No bad weather [2 marks for any sensible answers, bookwork]

Disadvantages:
Cost / Expense
Telescope / instruments can not be (easily) upgraded
Telescope must work first time
Size of the telescope is limited [2 mark for any sensible answers, bookwork]

(b) Telescope collecting area is proportional to D^2
At the diffraction limit, the resolution is $\theta \propto \lambda / D$

Hence the energy per unit area $\propto$ collecting area / resolution$^2 \propto D^4$ [2 marks bookwork]

(c) n = 1
$\rho = 200 \times 10^3$ lines m^{-1}
$\lambda = 9.01 \times 10^{-6}$ m
$W = 50 \times 10^{-3}$ m
$D_{\mathrm{T}} = 6.5$ m
$\chi = 1.22\,\lambda / \mathrm{D}$ [1 mark seen]

$\chi = 1.22 \times 9 \times 10^{-6} / 6.5$

$\chi = 1.69 \times 10^{-6}$ rads (0.35″) [1 mark unseen]

$\mathrm{R} = \frac{n\,\rho\,\lambda\,W}{\chi\,D_T}$

$\mathrm{R} = 1 \times (200 \times 10^3) \times (9.0 \times 10^{-6}) \times (50 \times 10^{-3}) / (1.69 \times 10^{-6} \times 6.5)$ [1 mark unseen]

$\underline{\mathrm{R} = 8192}$ [1 mark unseen]

(d) In the photon-noise limited case

S / N $= St / \sqrt{St}$

S / N $= \sqrt{St}$ [1 mark]

For a signal-to-noise limit of S / N = 100 and $t = 10$ hrs, the limiting signal (S) is:

$S = (\mathrm{S} / \mathrm{N})^2 \; / \; t$

$S = 100^2 / (10 \times 3600)$

$\underline{S = 0.278\ \mathrm{counts\,s^{-1}}}$ [1 mark unseen]

To calculate the limiting magnitude, we first need to calculate the photon rate from the 0-mag star ($\dot{N}_\star$). This can be calculated from the total power ($P_\star$) and photon energy using $\dot{N}_\star = \mathrm{P[J\,s^{-1}]\,/\,E[J]}$.

First, calculate the total power collected from the 0-mag star:

$P_\star = \text{flux} \times \pi \times (D_T / 2)^2 \times \Delta\lambda \times \epsilon_{\text{ff}}$ [1 mark]

$P_\star = (3.90 \times 10^{-8}) \times \pi\,(6.5 / 2)^2 \times (1.0 \times 10^{-3}) \times 0.3$

$\underline{P_\star = 3.88 \times 10^{-10}\,\text{J s}^{-1}}$ [1 mark unseen]

$E = h\,c / \lambda$

$E = 6.63 \times 10^{-34} \times 3 \times 10^8 / (9.01 \times 10^{-6})$

$\underline{E = 2.21 \times 10^{-20}\,\text{J}}$ [1 mark unseen]

Photon arrival rate from 0^{th} magnitude star per spectral channel is therefore:

$\dot{N} = P_\star / E$

$\dot{N} = 3.88 \times 10^{-10} / 2.21 \times 10^{-20}$

$\underline{\dot{N} = 1.76 \times 10^{10}\,\text{s}^{-1}}$ [1 mark unseen]

The limiting magnitude is set by:

$m_0 - m_1 = -2.5\log(f_0 / f_1)$ [1 mark seen]

$m_1 = 2.5\log(1.76 \times 10^{10}\,\text{s}^{-1} / 0.278) + 0$

$\underline{m_1 = 27.0}$ [1 mark unseen]

(d) The resolution is given by: $R = \frac{n\,\rho\,\lambda\,W}{\chi\,D_T}$

For the same spectrograph, the product of $n\rho\lambda W$ does not change.

In the diffraction limited case, the product of $\chi\,D_T$ is also unchanged (since $\chi = 1.22\,\lambda / D_T$) [1 mark partly seen]

<u>Hence the resolution is unchanged</u> [1 mark unseen]

L2, Stars and Galaxies 2018 exam

David Alexander

June, DMA, Q3

7 short questions

a) Calculate the temperature of a white dwarf that has a radius of 6,000 km and emits 10% of the luminosity of the Sun. What physical process generates the pressure required to prevent a white dwarf from collapsing? [4 marks]

Solution

$L = 4\pi R^2 \sigma T_e^4$ *therefore*

$T_e = \sqrt[4]{\frac{L}{4\pi R^2 \sigma}}$ *so*

T=35,000 K

[1 mark for the equation, 1 mark for the manipulation, and 1 mark for the correct quantitative answer; Unseen]

A white dwarf is prevented from collapse by electron degeneracy pressure.
[1 mark; Seen]

b) What is opacity? Strong absorption lines are seen in the spectrum of a star. Briefly describe the physical process that causes absorption lines in the spectrum. [4 marks]

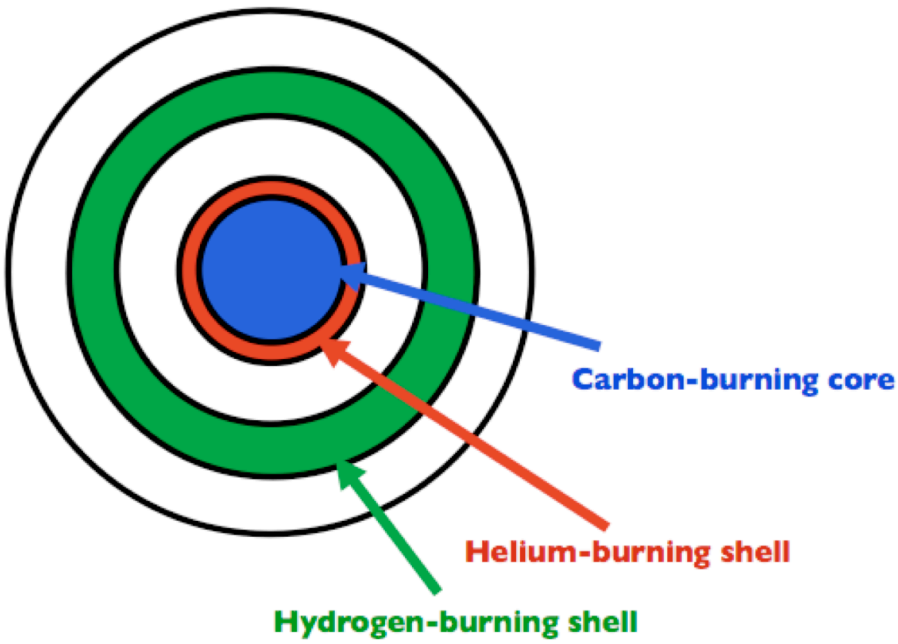
Solution

Opacity is the resistance to the flow of radiation from material. [2 marks; Seen]

The opacity process is called bound bound or excitation. This occurs when a photon has the exact energy required to excite a bound electron from one orbital to a higher orbital. [2 marks; Seen]

c) Draw a cross section of a star undergoing carbon burning at the core, indicating any other regions in the star where nuclear fusion is occurring. What is the origin of the carbon at the core of the star? [4 marks]

Solution



[1 mark for showing carbon-burning core, 1 mark for showing the helium and hydrogen burning shells, and 1 mark if also show the region in between helium and hydrogen burning shell where nuclear fusion isn't occurring; lose a mark if the radial order of the nuclear fusion is incorrect; Seen]

The carbon at the core was produced via helium burning in a previous nuclear-fusion phase. [1 mark; Seen]

d) Estimate the time it would take for a photon to escape from the Sun on a "random walk". Assume a mean-free path of 9.0×10^{-3} m, scattering with a 1.0×10^{-8} s delay each time a photon is scattered, and 7.0x10^8 m for the radius of the Sun. What is the dominant form of opacity in the core of the Sun?
[4 marks]

Solution

Number of steps are:

$$N = \left(\frac{d}{\ell}\right)^2 \qquad \textit{where d is the radius of the stellar core}$$

$$N = 6.1\times10^{21}\ \text{steps}$$

The equation for the total travel time of the photon is:

$$t = \frac{N\ell}{c} + \left(N\times10^{-8}\right) \qquad \textit{therefore}$$

$$t = 6.1\times10^{13}\,s$$

The dominant form of opacity at the core of the Sun is electron scattering.

[2 marks for the stages in the calculation, 1 mark for the correct quantitative answer, and 1 mark for the correct qualitative answer; Seen]

e) Briefly describe the physical process that leads to the production of neutrinos when the core of a massive star collapses. How is the production of neutrinos different in the Sun? [4 marks]

Solution

Neutrinos are produced during the stellar-core collapse because the protons and electrons are compressed strongly enough to form neutrons – this is called electron capture. To conserve the fermion number a neutrino is released. [2 marks; Seen]

The neutrinos from the Sun are produced in the nuclear fusion process of hydrogen into helium. However, the physical process (β decay) is the same as in the stellar-core collapse. [2 marks; Seen]

f) What determines the minimum possible mass of a main-sequence star? What limits the maximum possible mass of a main-sequence star? What is the name of the class of objects with masses just below the minimum possible mass of a main-sequence star? [4 marks]

Solution

The minimum mass is defined by the core temperature required for nuclear fusion to occur ($T \sim 4 \times 10^6$ K). [1 mark; Seen]

The maximum mass is defined as the point at which the star is unstable due to radiation pressure (which can be estimated from the Eddington limit). High-mass stars have strong radiation pressure which will drive away the outer regions of the star if it exceeds the gravitational pressure.
[2 marks; Seen]

Brown dwarfs have masses just below the minimum possible mass of a main-sequence star. [1 mark; Seen]

g) The rotation period of the Sun is approximately 28 days. Calculate the minimum rotation period of the Sun before it would break up. Assume a uniform density in your calculation. [4 marks]

Solution

Equate gravitational and centripetal accelerations at equator:

$$\omega_{\max}^2 R = \frac{GM}{R^2}$$

$\omega = 2\pi f$ *and* $P = \dfrac{1}{f}$ *and so* $\left(\dfrac{2\pi}{P_{\min}}\right)^2 = \dfrac{GM}{R^3}$

Therefore $P_{\min} = 2\pi\left(\dfrac{R^3}{GM}\right)^{1/2}$

For $R = 7.0\times10^8\,m$ *and* $M=M_{Sun}$*:* $P_{\min} = 1.0\times10^4\,s$ *or 0.12 days*

[2 marks for defining the equation and 2 marks for the correct numerical result; Unseen]

L2, Stars and Galaxies 2018 exam

David Alexander

June, DMA, Q4

Long question

a) What direct observational evidence provides strong support for nuclear fusion as the power source of the Sun? [2 marks]

Solution

The detection of neutrinos provides strong direct observational evidence that nuclear fusion occurs in the Sun. [Seen: 2 marks]

b) Write down the three steps of the dominant proton-proton chain for the fusion of hydrogen into helium in the Sun. Make sure that the nuclear reaction conservation laws are obeyed. [4 marks]

Solution

$$
\begin{aligned}
&{}^{1}\mathrm{H}_1 + {}^{1}\mathrm{H}_1 \Rightarrow {}^{2}\mathrm{H}_1 + e^{+} + \nu \\
&{}^{1}\mathrm{H}_1 + {}^{2}\mathrm{H}_1 \Rightarrow {}^{3}\mathrm{He}_2 + \gamma \\
&{}^{3}\mathrm{He}_2 + {}^{3}\mathrm{He}_2 \Rightarrow {}^{4}\mathrm{He}_2 + {}^{1}\mathrm{H}_1 + {}^{1}\mathrm{H}_1
\end{aligned}
$$

[2 marks for getting baryons right, 2 marks for leptons and hence conservations laws correct; Seen]

c) The Coulomb barrier energy is the energy that particles require to get within the range of strong interactions. Calculate the minimum temperature required for the particle kinetic energy to exceed the Coulomb barrier energy. Assume a pure helium gas and 1.0x10^{-15} m as the range of the strong nuclear force. [5 marks]

Solution

The particle kinetic energy is given by:

$$E_{particle} = \frac{3}{2} k T_c$$

[Seen: 1 mark]

The Coulomb barrier energy is given by:

$$E_{Coul} = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e_c^2}{r}$$

[Seen: 1 mark]

Therefore the temperature at which these are equal will be

$$T_c = \frac{1}{6\pi\varepsilon_0} \frac{Z_1 Z_2 e_c^2}{rk}$$

[Seen: 1 mark]

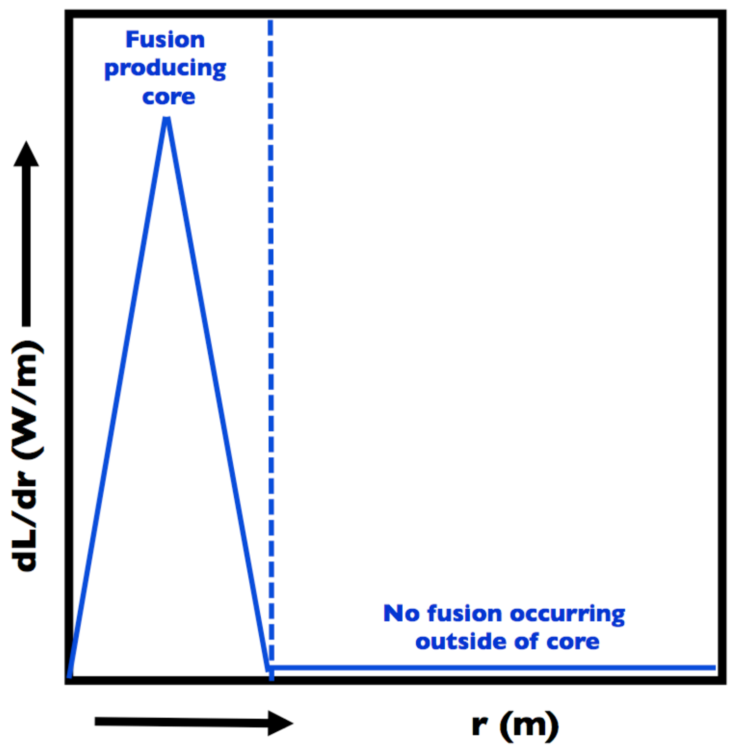
Putting in the radius for Z_1=Z_2=2 (for pure helium gas) gives a minimum temperature of:

$T > 4.5x10^{10}K$

[Unseen:2 marks]

d) Draw a plot to show how the energy production in the Sun depends on radius (dL/dr versus r). Highlight the region where nuclear fusion occurs. Why is there no significant nuclear fusion outside of this region? How will the dL/dr curve change when the Sun becomes a red-giant star? [9 marks]

Solution



[1 mark for giving the correct axes, 2 marks for indicating the fusion producing core in the central (~1/3) region of the star - to get both of the marks the student must show both the rise and the fall in the energy production (otherwise just 1 mark), and 1 mark for indicating no fusion occurring outside of the core (i.e., no significant energy production throughout the rest of the star); Seen]

Outside of the nuclear fusion region the temperature and density of the gas are too low for nuclear fusion to occur [Seen; 2 marks]

There will be three significant changes to the curve (1) there will be no fusion in the central regions (since all of the hydrogen will be converted into helium), (2) the peak in dL/dr will be shifted to a larger radius due to shell burning, and (3) the whole curve will rise due to the larger luminosity of the red-giant star. [Unseen; 3 marks]

L2, Stars and Galaxies 2018 exam

David Alexander

June, DMA, Q5

a) What is the Jeans mass and how is it applied in the formation of stars? [3 marks]

Solution

A gas cloud, or part of a gas cloud, with a mass greater than the Jeans mass will become unstable and begin to collapse. Above this mass, the cloud lacks sufficient gaseous pressure support to balance the force of gravity. It is therefore used to determine whether a gas cloud will collapse. [2 marks for a description of the Jeans mass and 1 mark to state how it is applied; Seen]

b) Show that the Jeans Mass (M_J) for a spherical gas cloud of uniform density satisfies the relation

$$M_J = \left(\frac{5kT}{G\mu m_p}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2},$$

where T is the temperature of the cloud, μ is the (dimensionless) mean molecular mass, and ρ is the average density of the cloud.

[4 marks]

[Recall that the gravitational potential energy is $U = -\frac{3}{5}\frac{GM^2}{R}$]

Solution

The collapse of a gas cloud to form a protostar is a consequence of competition between the gravitational potential energy (U) and the thermal energy (K).

$$U = -\frac{3}{5}\frac{GM^2}{R} \quad \textit{(as given)}$$
$$K = \frac{3}{2}NkT \ \textit{ where } \ N = \frac{M}{\mu m_p}$$

On the basis of the virial theorem the condition for collapse is $2K < |U|$*:*

$$\frac{3MkT}{\mu m_p} < \frac{3}{5}\frac{GM^2}{R}$$

Replace R assuming constant density:

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

Rearrange:

$$M_J = \left(\frac{5kT}{G\mu m_p}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2}$$

[2 marks for the correct definition for the collapse ($2K<|U|$), 1 mark for assuming constant density, and 1 mark for getting the basic steps right; Seen]

c) A spherical cloud of H_2 with uniform density has a radius of 1.0×10^{16} m, a mass of 4.2×10^{30} kg, and a temperature of 50 K. Determine whether this H_2 cloud is Jeans unstable. [4 marks]

Solution

The density of the cloud first needs to be calculated:

$$\rho = \frac{M}{V} = \frac{4.2 \times 10^{30}}{\left(\frac{4}{3}\pi\left(1.0 \times 10^{16}\right)^3\right)} = 10^{-18}\,kgm^{-3}$$

Now the Jeans mass can be calculated:

$$M_J = \left(\frac{5kT}{G2m_p}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2} = 9.42 \times 10^{32}\,kg$$

The Jeans mass is significantly in excess of that the cloud and therefore it will not collapse and so is not Jeans unstable.

[2 marks for calculating the Jeans mass using the correct mean molecular mass (μ=2), and 2 marks for the correct qualitative answer; Unseen]

d) The radiation from a protostar of radius 4.0×10^6 km is absorbed by a thin shell of dust at a radius 1.0×10^9 km, centred on the protostar. If 10% of the radiation from the protostar is re-emitted by the dust shell, and the shell is observed to have a temperature of 500 K, what is the effective temperature of the protostar? [4 marks]

Solution

Just 10% of the protostar luminosity is absorbed by the shell:

$$L_* = L_{cloud} \times 10$$

so:

$$4\pi R_*^2 \sigma T_*^4 = (4\pi R_{cloud}^2 \sigma T_{cloud}^4) \times 10$$

i.e.:

$$T_*^4 = \frac{(R_{cloud}^2 T_{cloud}^4) \times 10}{R_*^2}$$

$$T_* = \left(\frac{\left(10^{12}\right)^2 \times 500^4 \times 10}{\left(4 \times 10^9\right)^2} \right)^{1/4}$$

$$T_* = 14000\ K$$

[3 marks for the method and 1 mark for getting the correct numerical result; Unseen]

e) A protostar core has the following properties: fully ionized hydrogen with a pressure of P=6.0x10^{16} N m^{-2} and a density of ρ=1.0x10^{5} kg m^{-3}. Calculate the temperature of the gas at the core of the protostar. Briefly comment on whether this temperature is high enough for nuclear fusion via the proton-proton chain. [5 marks]

Solution

The temperature is calculated using the ideal gas law:

$$P = nkT = \frac{\rho kT}{\mu m_H} \qquad so \qquad T = P\frac{\mu m_H}{\rho k} = \frac{6.0\times10^{16}\times0.5\times1.67\times10^{-27}}{100000\times1.38\times10^{-23}} = 3.6\times10^{7}\,K$$

The temperature at the core is comparable to that at the centre of the Sun and hence nuclear fusion via the proton-proton chain can occur (note the proton-proton chain requires T>4x10^{6} K).

[1 mark for the method, 2 marks for correct quantitative answer (which requires assuming the correct mean-molecular mass) and 2 marks for getting the correct qualitative result; Unseen]

Answers to short questions

(a) Spirals contain young stars because they undergo star formation.[1 mark, bookwork]
Some of these young stars may be massive stars, and those are short-lived. These are hot and therefore blue. [1 mark, unseen]
Therefore spiral galaxies are bluer than elliptical galaxies and so they occupy region "2". Conversely, elliptical galaxies do not harbour massive blue stars, and hence are red and occupy region "1". [2 marks, unseen]
Simply stating the correct answer without explanation yields no marks.

(b) The three components are bulge, disc, and stellar halo. [3 marks, bookwork]
The Sun is part of the disc. [1 mark, bookwork]

(c) Thermal bremsstrahlung is produced by hot electrons being deflected when passing a nucleus. [1 mark, bookwork]
The gas therefore has to be very hot so this process occurs in the phase with $T \sim 10^7$ K. [1 mark, unseen]
Molecules are easily destroyed by collisions so can only occur at low temperature [1 mark, bookwork]
Therefore they are in the gas with $T \sim 10^2$ K. [1 mark, unseen]
Note that although these processes are discussed in the lectures, students need to connect how this depends on the temperature of the gas so that part of the question is unseen. Simply stating the correct answer yields no marks.

(d) Several are described in the lecture on AGN. 4 marks for 3 correct answers, 3 marks for 2 correct answers, 2 marks for 1 correct answer (max of 4), bookwork. Correct answers include (*i*) optically luminous central point source, (*ii*) strong radio emission, possibly from radio-lobes, (*iii*) strong emission of X-ray or γ-ray radiation, (*iv*) emergence of a relativistic jet, but there are other correct answers [4 marks, bookwork]

(e) Gravitational lensing is the deflection of light [1 mark, bookwork] by the bending of space-time due to the presence of a massive object [1 mark, bookwork]

The MACHO collaboration imaged stars in the LMC to try to detect increases in luminosity due to stars being lensed by an intervening massive compact halo object (*i.e.* by a MACHO). [1 mark, bookwork]
No lensing due to dark matter was detected, meaning it is not in the form of massive compact halo objects. [1 mark, bookwork]

Answers to long question:

(a) Using spherical coordinates, $M(R) = \int_0^R \rho(r)\, 4\pi r^2\, dr = 4\pi\rho_0 R_0^3 \int_0^{R/R_0} dr = (4\pi\rho_0 R_0^3)(R/R_0)$. [2 marks, unseen]

(b) Rotation curve is circular velocity as a function of radius, $V_c(R)$.[2 marks, bookwork]
In a spherically symmetric mass distribution, the gravitational force per unit mass F at distance R is due to the enclosed mass, therefore $F = GM(R)/R^2 = (4\pi\, G\rho_0 R_0^2)/R$. [1 mark, unseen]
The magnitude of the radial acceleration on a circular orbit with radius R is $a = V_c^2/R$, with V_c the circular speed. [1 mark, unseen]
Equating these yield $V_c^2 = (4\pi\, G\rho_0 R_0^2)$, which is independent of R and the sketch looks like Fig. 1. 1 mark for equation, 1 mark for sketch

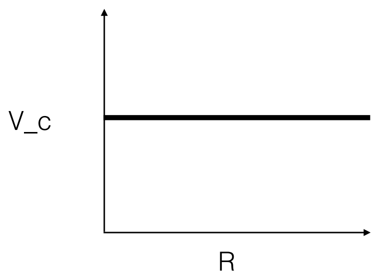


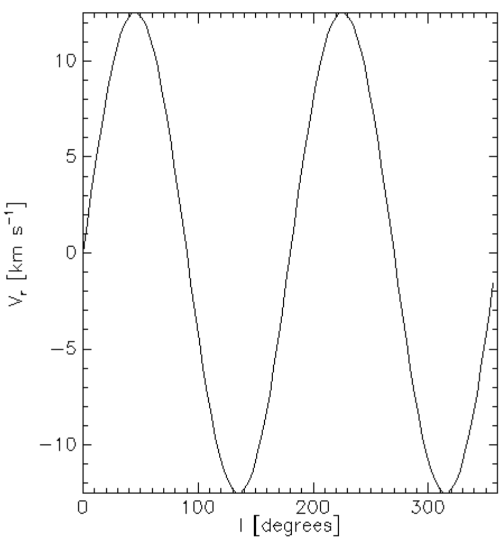
Figure 1: Circular speed as function of radius.

with labelled axes and indicating that V_c is constant.[2 marks, unseen]

(c) Inserting numerical values yields $\rho_0 = V_c^2/(4\pi\,\mathrm{G}R_0^2) = 0.01\ \mathrm{M}_\odot\ \mathrm{pc}^{-3}$.
[2 marks, unseen]
Lose 1 mark if result is not in $\mathrm{M}_\odot\ \mathrm{pc}^{-3}$, and one mark if not correctly rounded.

(d) Assuming the mean mass per stars is 1 $\mathrm{M}_\odot$, the mass density of stars is $\rho_\star = 1\ M_\odot \mathrm{pc}^{-3}$. [1 mark, unseen]
The ratio $\rho_\star/\rho_0 = 100$. [1 mark, unseen]
Because we know the rotation curve is dominated by dark matter, it is at first surprising that the mass density in the disc is dominated by stars. [2 marks]
This is however not a contradiction, because the stellar mass is (mostly) in a thin **disc** whereas the dark matter is approximately **spherically distributed**. Therefore the **enclosed mass**, which sets V_c is dominated by dark matter, not by stars. [2 marks, unseen]

(e) Using V_c is a constant and hence $\mathrm{d}V_c/\mathrm{d}R = 0$, yields $A = V_c/(2R_0) =$ 12.5km s^{-1} kpc^{-1} [1 mark, unseen]
This yields $V_r = 12.5\mathrm{km\ s}^{-1}\sin(2l)$ [1 mark, unseen]
The sketch looks like Fig. , full marks requires sinusoidal variation with $V_r = 0$ for $l = 0$, period 180°, amplitude 12.5 km s^{-1}, and labelled axes.
[2 marks, unseen]



10
5
0
−5
−10
V_r [km s^{-1}]
0
100
200
300
l [degrees]