Lecture 5: The Michelson Interferometer

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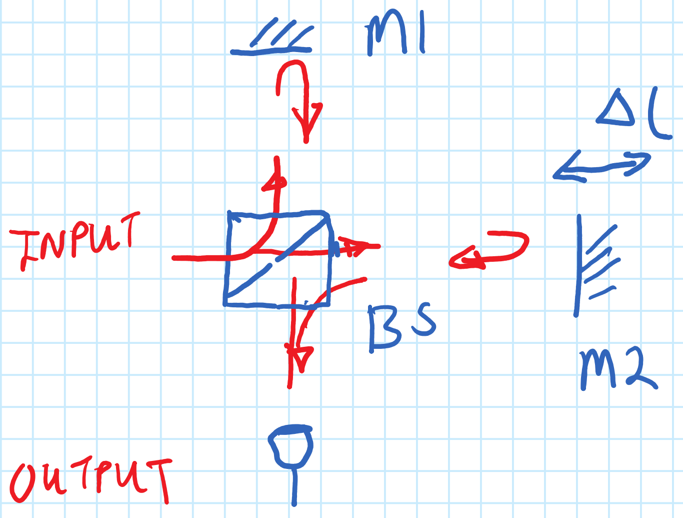
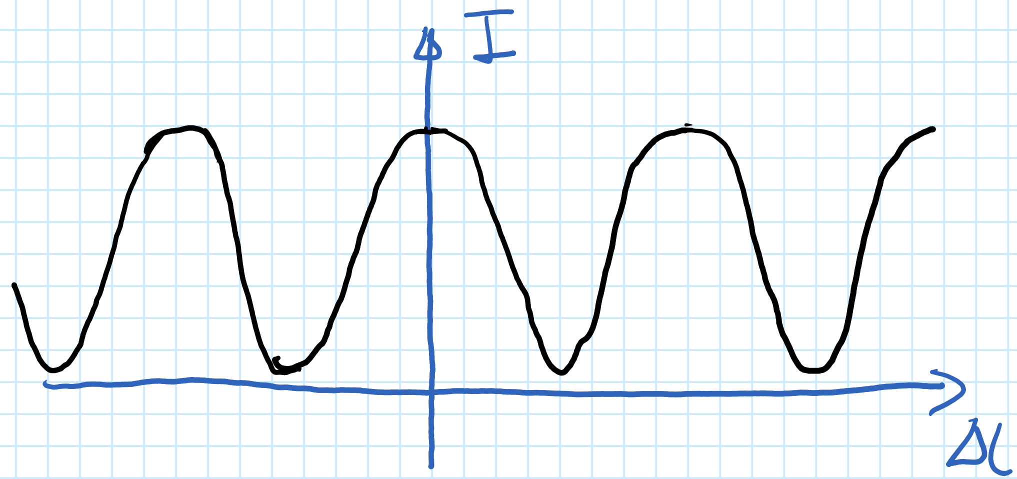
Optics f2f 3.12

LIGO is a type of MICHELSON INTERFEROMETER





DO AS A WORKED EXAMPLE Optics f2f Ex 3.11

(3.11) *Sensitivity of a gravitational wave detector*
A Michelson interferometer consists of a beamsplitter that divides an input with amplitude $\mathcal{E}_0$ into two equal amplitude 'arms' with lengths ℓ_1 and ℓ_2. A mirror retro-reflects each arms such that the two paths interfere at the beamsplitter.

Michelson Interferometer worked example

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(a) Write an expression for the output field after the two paths recombine at the beamsplitter. State any assumptions you make.

Let E_0 be input amplitude

Then amplitude in each arm is $\frac{E_0}{\sqrt{2}}$ (intensity $\frac{I_0}{2}$ 50:50 BS)

Propagation distance is $2L$ for each arm

phase is $ik2L$ "

$$\therefore \quad E = \frac{1}{\sqrt{2}} \cdot \frac{E_0}{\sqrt{2}} \left(e^{ik2L_1} + e^{ik2L_2} \right)$$

comes from 2nd pass through BS

ASSUMPTIONS

- plane waves in each arm
- neglect phase change on reflection @ BS

Worked example continued

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(b) Write an expression for the intensity at the output.

$$I \propto |E|^2 = EE^*$$

$$EE^* = \frac{1}{4}E_0^2\left(e^{i2kl_1} + e^{i2kl_2}\right)\left(e^{-i2kl_1} + e^{-2kl_2}\right)$$

$$= \frac{1}{4}E_0^2\left(2 + e^{i2k(l_1-l_2)} + e^{-i2k(l_1-l_2)}\right)$$

$$= \underbrace{\frac{I_0}{2}\left[1 + \cos\left(2k(l_1 - l_2\right)\right)}_{\text{fringes depend on arm length difference}}\Big]$$

Worked example continued

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(c) The path difference, $\ell_2 - \ell_1$, is chosen such that the intensity at the output is equal to one-half of its maximum possible value. Write an expression for $\ell_2 - \ell_1$ in terms of the wavelength, λ.

Max intensity is I_0 (when $\cos(\ ) = 1$)

$\therefore \frac{I_0}{2}$ when $\cos(2k(l_2 - l_1)) = 0$

$$2k(l_1 - l_2) = (2m+1)\frac{\pi}{2}$$

$$\therefore \quad l_1 - l_2 = \left(m + \frac{1}{2}\right)\frac{\lambda}{4}$$

Worked example continued

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(d) A gravitational wave arriving at a Michelson interferometer increases the length of one arm by $\Delta\ell$, and decreases the length of the other arm by $\Delta\ell$. Write an expression for the output intensity as a function of $\Delta\ell$, assuming that $\Delta\ell$ is small.

Let l_1 increase l_2 decrease THEN

$$I = I_0\left[1 + \cos 2k\left[(l_1 + \Delta l) - (l_2 - \Delta l)\right]\right] = I_0\left(1 + \cos\left[2k(l_1 - l_2) + 4k\Delta l\right]\right)$$

Use $\cos(A+B) = \cos A \cos B - \sin A \sin B$ (HINT)

$= \cos A - B \sin A$ for small B $\quad \Big| \begin{array}{l} \cos B = 1 \\ \sin B = B \end{array}$

$A = 2k(l_1 - l_2) = (2m+1)\pi/2$ (unperturbed phase difference)

$B = 4k\Delta l$ (gravity wave signal)

THEN $\sin A = \sin \pi/2 = 1$ (for $m = 0$ (or even))

$\cos A = 0$

$$\therefore I = \frac{I_0}{2}\left[1 - 4k\Delta l\right]$$

Worked example continued

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(e) If the power circulating in each arm is 0.8 MW and the minimal detectable signal is 1 μW, the wavelength is 0.5 μm and the length of each arm is 4 km, estimate the minimum strain, $\Delta\ell/\ell$, that can be detected in principle.

Power is intensity ($W m^{-2}$) $\times$ area

P IN EACH ARM $\propto I_0/2$ (I_0 is total intensity)

$$\therefore \frac{\Delta P}{P} = \frac{\Delta I}{I_0/2} = 4k\Delta l = \frac{8\pi \Delta l}{\lambda}$$

min sig

λ

$$\therefore \Delta l = \left(\frac{10^{-6}}{0.8\, 10^{6}}\right) \times \left(\frac{5\times10^{-7}}{24}\right) = 3\times10^{-20}$$

P

8π

$$\frac{\Delta l}{l} = \frac{3\times10^{-20}}{4\times10^{3}} \approx 10^{-23}$$

Next lecture

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(f) Give two reasons why Young's double-slit interferometer is less well suited to measure gravitational waves than a Michelson interferometer.