L2 Foundation of Physics 2B Optics 2019-20

O.6 Young's interferometer: Summary

Learning outcomes:

1. To add two curved waves - **Young's two-hole experiment** [Optics $f2f$ Sec. 3.6].
2. To derive a similar result for **three slits** [Optics $f2f$ Sec. 3.8].

Key equations:

The **sum of two scalar spherical waves** originating at $(x', z) = (\pm d/2, 0)$—Young's two holes—is

$$E = E_\mathrm{s}\frac{\mathrm{e}^{\mathrm{i}kr_1}}{\mathrm{i}kr_1} + E_\mathrm{s}\frac{\mathrm{e}^{\mathrm{i}kr_2}}{\mathrm{i}kr_2} , \tag{1}$$

where E_s is the effective amplitude of the waves.

In the paraxial limit $d \ll z$ and $x < z$ we can write this as

$$E = \bar{E}_\mathrm{s}\mathrm{e}^{\mathrm{i}k(\bar{r}+d^2/8z)}\left(\mathrm{e}^{\mathrm{i}kdx/2z} + \mathrm{e}^{-\mathrm{i}kdx/2z}\right) , \tag{2}$$

where $\bar{E}_\mathrm{s} = E_\mathrm{s}/\mathrm{i}k\bar{r}$ and $\bar{r} = z + x^2/2z$. The first exponential terms is a **global phase** factor that disappears when we calculate intensity. However, the **relative phase** terms survive. The intensity is

$$I = 4\bar{I}_s\cos^2\left(\frac{\pi dx}{\lambda z}\right) . \tag{3}$$

For slits rather than circular apertures $\bar{E}_\mathrm{s} = E_\mathrm{s}/\sqrt{\mathrm{i}k\bar{r}}$. For **three slits** at $(x', z) = (0, 0)$ and at $(\pm d, 0)$ the sum of three paraxial cylindrical waves is

$$E = \bar{E}_\mathrm{s}\mathrm{e}^{\mathrm{i}(k\bar{r}-\omega t)}\left(\mathrm{e}^{\mathrm{i}kdx/z} + 1 + \mathrm{e}^{-\mathrm{i}kdx/z}\right) , \tag{4}$$

The first exponential terms is a **global phase** factor that disappears when we calculate intensity. However, the **relative phase** terms survive. The intensity is

$$I = \bar{I}_s\left[1 + 2\cos\left(\frac{2\pi dx}{\lambda z}\right)\right]^2 . \tag{5}$$

Outlook: In the next lecture, we shall look at going from 3 slits, to N slit (diffraction grating), before moving to infinitely many slits (Fresnel diffraction integral).