Cosmology: Problem Cos 1.3

(a) The disc of a spiral galaxy is observed to have a flat rotation curve in its outer parts with orbital speeds of ± 150 km s^{-1} relative to the centre. If this is due to a spherical dark matter halo, show that its mass density profile is of the form $\rho(r) \propto r^{-2}$.

(b) Calculate the local mass density of the halo of this galaxy at a radius of 50 kpc in units of $M_\odot$ pc^{-3}. If dark matter consists of particles with a mass of 180 GeV/c^2, calculate their space density at this radius in units of particles per cubic metre.

(c) A model Universe comprises a population of such galaxies with a co-moving space density of 0.02 Mpc^{-3}. Find the maximum radius out to which the dark matter haloes extend, in Mpc, if these galaxies contribute $\Omega_g = 0.15$ to the density paramater. Assume $H_0 = 75$ km s^{-1} Mpc^{-1}.

(d) A spherical cluster of galaxies contains pure ionized hydrogen with a gas density profile $\rho_{gas} \propto r^{-\beta}$ in hydrostatic equilibrium in a dark matter gravitational potential. If the gas temperature of 10^8 K is independent of radius and the total mass of the cluster (gas & dark matter combined) within 1 Mpc is 2×10^{14} $M_\odot$, calculate the index β. Repeat the calculation if the gas instead comprises ionized hydrogen mixed with 25% fully-ionized helium-4 (by mass).