Stars and Galaxies

Observational Techniques Homework Set 3 – Solutions

1) $m\rho\lambda = \sin(\alpha) + \sin(\beta)$

For $\lambda = 550\,\mathrm{nm}$, $\beta = -9.04°$

For $\lambda = 560\,\mathrm{nm}$, $\beta = -8.46°$

Hence $\Delta\beta = 0.6°$ [1 mark]

2) The designer can change the ruling density of grating (ρ), the size of the grating (W), or the order (n) at which the spectra are recorded. [2 marks]

In the diffraction limited case, the resolution will not change since the improvement in D_T is cancelled out by the improvement in spatial resoltion (i.e. $\chi_4 D_4 = \chi_8 D_8$). [1 mark]

3) We will treat the human eye in exactly the same way as a telescope and detector.

On a dark site with well adapted eyes, the average human eye can detect stars down to 6^{th} magnitude. To answer this question, we need to calculate how many photons this corresponds to per "observing" time for the eye.

First, calculate the flux density that this corresponds to using:

$m_0 - m_6 = -\,2.5\log(f_0\,/\,f_6)$

with $m_0 = 0$, $m_6 = 6$ and $f_0 = 3.9 \times 10^{-8}\,\mathrm{W\,m^{-2}\,\mu m^{-1}}$, this gives

$\underline{f_6 = 1.5 \times 10^{-10}\,\mathrm{W\,m^{-2}\,\mu m^{-1}}}$

Next, calculate the total power received at the front of the eye from this 6^{th} magnitude star:

$\mathrm{P} = f_6 \times \pi \times \mathrm{D}^2/4 \times \Delta\lambda$

$\mathrm{P} = 1.5 \times 10^{-10} \times \pi \times 0.009/4 \times 400 \times 10^{-9}$

$\underline{\mathrm{P} = 3.9 \times 10^{-15}\,\mathrm{W}}$

The photon energy at $\lambda = 550\,\mathrm{nm}$ is

$\mathrm{E} = hc\,/\,\lambda$

$\mathrm{E} = hc\,/\,550\mathrm{nm}$

$\mathrm{E} = 3.6 \times 10^{-19}\,\mathrm{J}$

Hence, the photon arrival rate at the front of the eye is

$\dot{n}_\gamma = \mathrm{P}\,/\,\mathrm{E}$

$\dot{n}_\gamma = 3.9 \times 10^{-15}\,/\,3.6 \times 10^{-19}$

$\underline{\dot{n}_\gamma = 10\,900\,\mathrm{s^{-1}}}$

But the efficiency of the eye is only 10%, so the number of photons that reach the retina is

$\dot{n}_\gamma = 0.1 \times 10900$

$\dot{n}_\gamma = 1090\,\mathrm{s}^{-1}$

Your eye refreshes at a rate of approximately 25 Hz (which is why a typical TV or screen refreshes at $\gtrsim 25\,\mathrm{Hz}$). Hence, the "exposure time" for the eye is $t = 1\,/\,25\,\mathrm{s}$. So, the number of photons from the faintest star that the human eye can detect is:

$\mathrm{n} = 0.04 \times 1090$

$\underline{n_\gamma = 44\,\mathrm{photons}}$

However, we do not yet know how spread out these photons are on your retina due to diffraction. To figure this out, we need to know how the rod (or "pixel") size compares to the diffraction limit.

Let's start by calculating the plate scale, $\mathrm{d}\theta\,/\,\mathrm{d}s$, at the retina, using the focal length, $f = 1.7\,\mathrm{cm}$:

$\mathrm{d}\theta\,/\,\mathrm{d}s = 1\,/\,f$

$\mathrm{d}\theta\,/\,\mathrm{d}s = 1\,/\,0.017$

$\mathrm{d}\theta\,/\,\mathrm{d}s = 1\,/\,0.017$ $(\times\, 206265/10^6$ to convert from radians / m to arcsec/μm)

$\underline{\mathrm{d}\theta\,/\,\mathrm{d}s = 12.1''\,/\,\mu\mathrm{m}}$

This means that each rod, which has a diameter of 2μm, observes an angle of $24.2''$.

How does this compare to the diffraction limit of the eye? At a wavelength of 550 nm, the diffraction limit of a lens with diameter 9 mm :

$\theta = 1.22\,\lambda\,/\,\mathrm{D}$

$\theta = 1.22 \times 550\,\mathrm{nm}\,/\,9\,\mathrm{mm}$ $(\times 206265$ to arcseconds)

$\underline{\theta = 15.4''}$

Hence, each rod covers an area greater than the diffraction limit, so all photons from a point source (e.g. a star) will fall on a single "pixel".

Therefore, the minimum number of photons that trigger a detection within the average human eye is approximately 44, and so the final answer is: No, the human eye can not detect a single photon (although, people with the "best" eyesight can detect stars that are $\sim$1-magnitude fainter, and can also be trained to speed up/slow down their refresh rate of the eye by $\sim 2\times$. The very best eyesight can "detect" ~ 8 photons).