# EM Waves and Light

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· Key points :

F2F Ch. 1

EM Waves, Harmonic Solution
Principle of Superposition, Scalar Approx.

## EM WAVES and LIGHT

Maxwell's Equations for the EM field :

① $\underline{\nabla} \times \underline{E} = -\dfrac{\partial \underline{B}}{\partial t}$ ② $\underline{\nabla} \times \underline{B} = \mu_0 \varepsilon_0 \dfrac{\partial \underline{E}}{\partial t}$

③ $\underline{\nabla} \cdot \underline{E} = 0$ ④ $\underline{\nabla} \cdot \underline{B} = 0$

ASIDE:
see
FOUNDATIONS 2A
(EM, HAMPSHIRE)
NO CHARGES
$q, \underline{j} = 0$

NOTE ON NOTATION

Optics {2} $\mathcal{E} \longrightarrow \underline{E}$

- consistent with exams
- easier to write

# Electromagnetic Waves

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Combining equations ① and ③ gives (see L2L)

$$⑤ \quad \nabla^2 \underline{E} - \frac{1}{c^2}\frac{\partial^2 \underline{E}}{\partial t^2} = 0$$

WAVE EQUATION
(second order in space & time)

FOUNDATIONS ① (Dr Peach)

Properties of ⑤:

- It is linear (no terms like $E^2$ etc)
- Therefore PRINCIPLE OF SUPERPOSITION holds

# Principle of Superposition

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KEY CONCEPT: PRINCIPLE OF SUPERPOSITION

If $\underline{E}_1$ is a solution of (S)

AND $\underline{E}_2$ is a solution of (S)

then $\underline{E}_1 + \underline{E}_2$ is a solution of (S)

NEW SOLUTIONS CAN BE CONSTRUCTED BY ADDING TOGETHER KNOWN SOLUTIONS FOR SIMPLE CASES

# The Harmonic Solution

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A general solution of 5 has the form

$$\underline{E} = \underbrace{\underline{E_0} \cos(\underline{k} \cdot \underline{r} - \omega t + \phi_0)}_{\text{harmonic solution}}$$

(cf harmonic oscillator)

- well defined angular frequency $\omega$
- wave vector $\underline{k} = (k_x, k_y, k_z)$
- $k = |\underline{k}| = \frac{2\pi}{\lambda}$ where $\lambda$ is wavelength

$$c = \omega / k$$

- $\phi_0$ is a phase offset (more later)

# Harmonic solution

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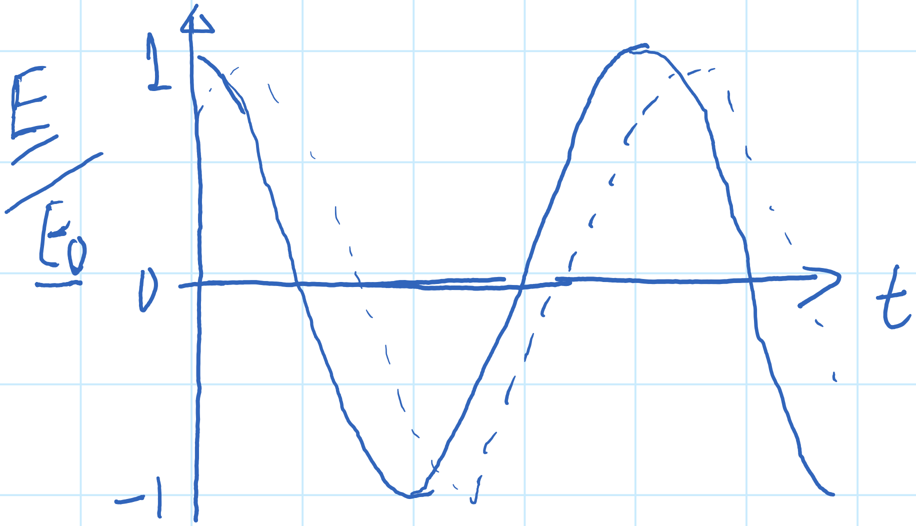


solid $\phi_0 = 0$

dashed $\phi_0 \neq 0$

at position $\underline{r} = 0$

Visible light $\lambda = 600\,\text{nm}$ $\Rightarrow$ Period $T = \frac{1}{f} = \frac{\lambda}{c} = \frac{6\,10^{-7}}{3\,10^{8}} = 2\,10^{-15}$

2 fs!

$\therefore$ usually measure time-averaged intensity $\boxed{I = \frac{1}{2} c \epsilon_0 E_0^2}$

J&L section 1.1

# Polarization and the Scalar Approximation

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$\underline{E}$ is a vector

Its direction is called the POLARIZATION (PART 3)

In parts 1 and 2 we ignore polarization and make the

$$\boxed{\text{SCALAR APPROXIMATION} \qquad \underline{E} \longrightarrow E}$$

VIEWPOINT: Assuming polarization is the same everywhere.

Breaks down for highly curved wavefronts

# Scalar approximation

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THEN: $$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$ scalar wave equation

SEE [2] section 1.12

and $$E = E_0 \cos(\underline{k} \cdot \underline{r} - \omega t + \phi_0)$$

NEXT: SPATIAL DEPENDENCE

Plane waves

Spherical waves

Paraxial Approximation