Foundations of Physics 2B/3C 2019-2020

Thermodynamics – Workshop 1 Problems

Week Commencing 14th October

1. Thermodynamic coordinates and system properties

The first two parts of this problem provide chance to practice using equations of state, and should really serve as revision of last year's material. If you are happy undertaking these manipulations, it would be much more beneficial to use the workshop time to attempt the harder problems: part (c) of this question, which looks at non-standard heat capacities, and those in section 2.

a) 8.0 moles of a diatomic gas are contained in a vessel having a volume of 5.0 litres at room temperature $(300\ \mathrm{K})$.
   i.) Determine the pressure of the gas inside the vessel.
   ii.) The gas undergoes an adiabatic expansion, such that the volume of gas quadruples. What is the new temperature of the gas?

(*Hint*: The adiabatic equation of state is $pV^{\gamma} = \mathrm{Constant}$, and a diatomic ideal gas has $C_p/C_V = 1.4$.)

b) $5.0\ \mathrm{kg}$ of Nitrogen gas (molecular mass $28\ \mathrm{g\ mol^{-1}}$) are compressed at a pressure of $2000\ \mathrm{kPa}$ and kept at a temperature of $90\ \mathrm{K}$. What is the volume of the vessel required to store this amount of gas?

c) Two materials have heat capacities at constant volume given by $C_1 = A$ and $C_2 = BT$, where A and B are constants of appropriate dimensions. If one material is initially at T_1 and the second is at T_2, determine the final equilibrium temperature, T_f of the blocks after they have been placed in thermal contact.

2. Differentials - mathematically and thermodynamically

The first part of the problem looks at differentials in their mathematical formalism. The following parts then apply the ideas to real thermodynamic situations. In particular, the last part explicitly links the idea of exact and inexact differentials to thermodynamic properties.

a) Consider the function $dz = 2xydx + (x^2 + 3y^2)dy$. Integrate this explicitly along the following two paths in the (x, y) plane:
   - $(x_1, y_1) \rightarrow (x_2, y_1) \rightarrow (x_2, y_2)$
   - $(x_1, y_1) \rightarrow (x_1, y_2) \rightarrow (x_2, y_2)$

   Is this function an exact differential? How else could you check?

b) A new form of matter has been discovered on Palace Green with equation of state $VC^2 = gT^3$, where C is the 'Cathedral function of state,' g is the 'Green constant' and other symbols have their usual meanings.

   Write down the expression for the total differential of the volume, dV. Hence show that $d(\ln V) = -2d(\ln C) + 3d(\ln T)$.

c) One mole of ideal gas has its state changed such that its pressure and temperature increase from (T_a, p_a) to (T_b, p_b). This increase can be undertaken in two different ways:

- a linear increase of pressure with temperature;
- an isothermal increase in the pressure, followed by an isobaric temperature increase.

i.) Sketch the two processes on a pT diagram and calculate the volume change which results along each path.

ii.) Evaluate the work done along each path.

iii.) Comment on your results, relating volume and work done to the idea of differentials.