L2 Foundation of Physics 2B Optics 2019-20

Workshop O.W.3 Interference

February 27, 2020

1. *Lloyd's mirror*: The sum of two monochromatic waves with amplitude, $\bar{E}_0$, and wavelength, λ, is given by

$$E = \bar{E}_0(\mathrm{e}^{\mathrm{i}2\pi r_1/\lambda} + \mathrm{e}^{\mathrm{i}2\pi r_2/\lambda}) \ , \tag{1}$$

where r_1 are r_2 are the distances from point positions $(0, y_1, 0)$ and $(0, y_2, 0)$ to an observation point at $(0, y, z)$.

   (a) Are the wave fronts planar or curved? [1 mark]

   (b) Write an expression for r_1 in terms of the coordinates of the y_1, y, and z. [1 mark]

   (c) Re-write this expression using the **Fresnel** and **Fraunhofer** approximations (neglecting terms of higher order than y_1^2 and y_1, respectively). [4 marks]

   (d) A point source at $(0, d, 0)$ is a distance d above a flat mirror lying in the xz plane at $y = 0$. Use eqn (1) to derive an expression for the intensity along the y-axis at a horizontal distance z from the source, assuming that $z \gg d$. Assume that the mirror is perfectly reflecting and produces a π phase shift on reflection. [6 marks]

2. *Four holes*: An aperture containing 4 small holes located at points $(x' = \{-3d/2, -d/2, d/2, 3d/2\}, y' = 0)$ is placed in the $z = 0$ plane and illuminated using uniform monochromatic light with wavelength λ.

   (a) Derive an expression for the field at a point (x, z) in the far-field, $z \gg d$. State any approximations you make. [5 marks]

   [Hint: Follow the same derivation as we used for three slits.]

   (b) The intensity of light is proportional to the modulus-squared of the field amplitude. Write an expression for the intensity in terms of cosines. What is the maximum value? [3 marks]

   (c) How many positions of zero intensity are there between $x = 0$ and $x = [\lambda/(2d)]z$? Sketch the phasor diagrams or specify the phasor angles in each case. [4 marks]

3. *Double slit experiment with a green laser pointer: justification of paraxial approximations*: A spherical wave is written as $\mathcal{E} = \mathcal{E}_\mathrm{s}\mathrm{e}^{\mathrm{i}kr'}/(\mathrm{i}kr')$, where r' is the distance from the wave centre to an observer. Explain, why there is a factor of k in the denominator. [1 mark]

   In the Fraunhofer approximation, the distance r' between a point $(x', 0)$ in the input plane and a point (x, z) in the observation plane is given by $r' = \bar{r} - x'x/z$, where $\bar{r}$ is the distance between $(0, 0)$ and (x, z). Use this expression to substitute for r' and rewrite the spherical wave in terms of $\bar{r}$, x', x and z. [1 mark]

   Show that for $z \gg x'$ this can be written in the form

$$\mathcal{E} = \mathcal{E}_\mathrm{s}\frac{\mathrm{e}^{\mathrm{i}k\bar{r}}}{\mathrm{i}k\bar{r}}\,(1+\epsilon)\,\mathrm{e}^{\mathrm{i}\phi} \ .$$

Give expressions for ϵ and ϕ.

In a Young's double-slit experiment using a green laser pointer; the slit positions are at $x' = \pm 0.5$ mm and the distance to the screen is $z = 1.0$ m. Estimate the size of the phase term ϕ and the correction to the amplitude ϵ for a laser wavelength $\lambda = 0.5$ μm. As $\bar{r} = z + x^2/z$, we can write that $1/\bar{r} = 1/z$ to first order in x/z.

Use your answers to justify a further approximation in order to re-write the spherical wave in terms of x', x, and z only.