Heterogeneous Expectations and Regime Switching *

(Link to Latest Version)

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Abstract

This paper analyzes the empirical relevance of heterogeneous expectations at the zero lower bound (ELB) in the canonical New Keynesian model. Agents can switch between an anchored Rational Expectations (RE) rule and an adaptive learning rule that may result in a de-anchoring of expectations. The structural change in monetary policy during ELB episodes, and the heterogeneity of private sector expectations are both captured in a unified framework of endogenous regime switching. An application to the U.S. economy over 1982Q1-2019Q4 shows that expectations are characterized as a mixture of RE and learning over the pre-GFC period, while a larger fraction of expectations remain anchored at the RE during the ELB period after 2008Q4. Counterfactual simulations show that a higher fraction of learning agents, as well as a higher intensity of learning can both generate deflationary spirals and prolonged periods of recession.

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1 Introduction

Following the Global Financial Crisis (GFC) of 2007-08, many leading central banks around the globe cut their nominal interest rates to near zero levels and encountered the effective lower bound (ELB) constraint on their rates, which generated an increased volume of research about the relevance and impact of this constraint on the economy. Before the effects of the GFC fully dissipated and the interest rates in advanced economies started rising in a return to the normalization of monetary policy, the global economy was hit by the pandemic-induced recession in the first half of 2020, pushing the nominal rates to near-zero levels again and showing that the lower bound constraint is here to stay for the foreseeable future.

In a large part of macroeconomic research using DSGE modeling, the standard assumption to analyze the effects of ELB and the impact of unconventional monetary policy over this period is Rational Expectations (RE). Within a variety of different modeling approaches, the RE assumption has been shown to have a number of shortcomings. This paper aims to address two such shortcomings associated with RE models. The first one relates to a specific form of modeling the ELB, which uses a regime switching approach to capture the monetary policy shift during the ELB period. One of the well known failures of RE models within this context is their inability to generate sufficiently long ELB episodes on par with the empirically observed ones. For example, Ji & Xiao (2016), Chen (2017) and Lindé et al. (2017) report expected ELB duration estimates for the U.S. economy ranging between 3 and 9 quarters over the post-GFC period, while the empirical duration was 28 quarters over the period 2008-2015. This makes the RE models within the regime switching framework unsuitable for policy analyses and for studying counterfactual scenarios.

The second shortcoming is about the relevance of central bank credibility and the bank's ability to anchor private sector expectations at the desired targets, which is closely related to unconventional monetary policy tools such as forward guidance communication, and the signaling channel of quantitative easing measures (Bernanke, 2017). Indeed, if the assumption of RE holds and the central bank is fully credible, then private sector expectations are always anchored at the desired equilibrium. However, it has been documented that under this assumption, central bank communications about future paths of policy rates have implausibly strong and stimulating effects on current macroeconomic outcomes, a phenomenon that has been described as the *Forward Guidance Puzzle* (Del Negro et al., 2012). In light of these shortcomings associated with RE models, there has been increased interest in limited information and bounded rationality models when analyzing the post-GFC period in New Keynesian models. This paper contributes to the growing literature on limited information and heterogeneous expectations by studying their empirical implications on macroeconomic outcomes during ELB regimes.

In this paper, I estimate the canonical 3-equation hybrid New Keynesian model with heterogeneous expectations, subject to the ELB constraint on nominal interest rates. When forming their expectations, agents are allowed to choose between an anchored *pseudo-rational* model and a de-anchored adaptive learning model, based on their past predictive performance. Both the expectational heterogeneity and the ELB constraint on nominal rates are captured in a unified framework of endogenous regime switching. During normal times when monetary policy follows the Taylor rule, the model reduces to the standard heterogeneous expectations setup with a switching mechanism along the lines of Brock & Hommes (1997). During ELB periods, the model features a different framework, where the central bank's desired interest rate is no longer observed or taken into account by all agents. In this case, rational agents form their expectations *as if* the central bank is not constrained by the ELB, and *as if* the desired interest rate affects macroeconomic outcomes. In other words, these agents continue forming their beliefs as though there is no structural change in monetary policy. This class of agents can be interpreted as anchoring their expectations at the targeted equilibrium, which proxies for the central bank's forward guidance communications, as well as the signaling channel of quantitative easing measures. In other words, they form their expectations under the assumption that unconventional policy measures substitute for the central bank's inability to lower the nominal interest rates further.

The adaptive learning agents instead ignore or do not observe the central bank's desired policy rate during the ELB regime. Instead, they use the observed variables, including the pegged nominal rates during ELB periods, and act like econometricians who update their beliefs every period to learn the new structural relations. As shown in Ozden & Wouters (2020), expectational dynamics under adaptive learning are not stable in New Keynesian models when interest rates are pegged.[1] As such, learning dynamics over ELB regimes leads to a de-anchoring of expectations, which may be gradual or fast depending on the speed and intensity of learning. The modeling framework generates a conditionally linear structure, which can be combined with the standard filtering algorithm in Markov-switching literature à la Kim & Nelson (1999) as a state space model with time-varying parameters, and estimated with Bayesian MCMC methods.

I estimate the model based on historical U.S. data over the period 1982Q1-2019Q4. The main contributions of the paper are threefold. First, during normal times before the GFC, expectations are characterized as a mixture of the two forecasting rules associated with RE and adaptive learning with nearly equal weights. Over the post-GFC period where the ELB constraint on nominal rates starts binding, the forecasting rule based on RE receives nearly twice as much weight as the adaptive learning rule. To the extent that adaptive learning leads to a de-anchoring of expectations, this result can be interpreted as a successful central bank communication that steers expectations in the right direction. In order to assess different aspects of the heterogeneous expectations regime switching model, I further provide estimation results for a a number of other models, namely the

[1] Expectational stability is henceforth denoted as E-stability, as is standard in the adaptive learning literature (Evans & Honkapohja, 2001).

benchmark REE without any regime switching, a REE model with exogenous regime switching in monetary policy, and a pure adaptive learning model with regime switching in monetary policy.

As a second contribution, I carry out a set of counterfactual experiments to show that keeping expectations anchored at the RE has an important stabilizing role. Since adaptive learning is inherently unstable in New Keynesian models when interest rates are pegged, having a large fraction of de-anchored expectations increases the likelihood of observing deflationary spirals and prolonged recessions. In my modeling approach, the degree of de-anchoring is captured through both the fraction of agents using adaptive learning, as well as how much weight these adaptive learning agents place on the most recent observations in their models, which is captured through the *constant gain* parameter. Third, I carry out a pseudo out-of-sample forecasting exercise with the model over the period 2020Q1-2024Q4 under the assumption of a set of pandemic-induced supply and demand shocks throughout the year of 2020. While the baseline projections with endogenous fractions generate a probability of recovery around 54% from the pandemic-induced ELB episode by the end of the projection period, this probability drops to 41% if a large proportion of expectations are based on adaptive learning. This shows the importance of keeping private sector expectations anchored through central bank communication tools.

Literature Review

The paper relates to the growing literatures on regime switching, adaptive learning and heterogeneous expectations. A large part of regime switching models in DSGE literature is centered around the RE framework, and particularly the theoretical properties and solution methods of such models.[2] More recently, a number of papers also focus on endogenous regime switching DSGE models.[3] A complication of RE models in this framework is that subjective expectations are equated to the objective expectations of the model, which leads to non-linearities when solving for model-consistent expectations. The advantage of adaptive learning models in this context is their conditionally linear structure, which can often be handled using standard filtering algorithms.

While there is ample research in regime switching models with rational agents, research in this class

[2] Examples include Farmer et al. (2009), Farmer et al. (2011) and Cho (2016) that study the theoretical properties and determinacy conditions associated with RE equilibria in Markov-switching models; Bianchi (2016) proposing new methods for measuring expectations and uncertainty in Markov-switching models; and Kulish & Pagan (2017) who propose solution and estimation methods for forward-looking models with structural changes under a variety of assumptions for agents' beliefs about those structural changes. Other empirical applications in regime-switching DSGE models include, among others, Sims & Zha (2006), Liu & Mumtaz (2011), Bianchi (2016) and Bianchi & Ilut (2017).

[3] See e.g. Barthélemy & Marx (2017) using perturbation methods to solve and estimate endogenous regime switching models; Chang et al. (2018) proposing an efficient filtering method to handle the estimation of state space models with endogenous switching parameters depending on latent autoregressive factors; and Benigno et al. (2020) considering an endogenous regime-switching framework to study financial crises.

of models with imperfect information/learning agents has been scarce. Examples include Branch et al. (2007) establishing theoretical properties of learning about both regime switches and structural relations; Airaudo & Hajdini (2019) studying a class of *Consistent Expectations Equilibria* in a regime-switching framework; and Ozden & Wouters (2020) considering the estimation of regime switching models under a class of adaptive learning rules. A closely related paper within this framework is Gust et al. (2018), who study the effectiveness of forward guidance in a model where agents are aware of regime switches, but do not know the transition probabilities and instead infer about them using a form of Bayesian learning.

Another closely related line of research is on heterogeneous expectations. Earlier work on heterogeneous expectations in New Keynesian models consider a variety of topics; e.g. Branch (2004) studies the empirical properties of heterogeneous expectations with survey data on inflation expectations; Branch & McGough (2009) analyze the micro-foundations of New Keynesian models with heterogeneous expectations; Anufriev et al. (2013) consider different interest rate rules and macroeconomic stability under heterogeneous expectations; Di Bartolomeo et al. (2016) study how heterogeneous expectations affect the design of optimal monetary policy in a New Keynesian model; and Cornea-Madeira et al. (2019) estimate the New Keynesian Phillips Curve with heterogeneous expectations.

More recently, there have been a number of papers that study the interactions between the ELB, unconventional monetary policy and heterogeneous expectations. The closest study to this paper along these lines is Busetti et al. (2017), where the authors study how prolonged periods of subdued price developments in the Eurozone may induce a de-anchoring of expectations. This is done in a heterogeneous expectations framework, similar to the one presented in this paper, where agents choose between anchored and de-anchored forecasting rules depending on their past performance. This paper can be seen as an extension of their modeling approach to estimate a heterogeneous expectations model in a unified framework that includes monetary policy switching. Other related papers include Andrade et al. (2019), who consider forward guidance in a heterogeneous expectations framework with optimistic and pessimistic agents; Hommes & Lustenhouwer (2019), who study the theoretical properties of a NK model with a ELB under heterogeneous expectations, with fundamentalists who believe in the target of the CB, and naive expectations who believe in a random walk; Goy et al. (2020), who analyze the effects of different types of forward guidance in a New Keynesian model with heterogeneous expectations and the ELB constraint; Lansing (2019) where a representative agent contemplates between a targeted equilibrium and a deflationary equilibrium, where a non-trivial probability on the deflationary equilibrium becomes partially self-fulfilling by lowering the averages of observed variables; and Arifovic et al. (2020) who study heterogeneous expectations through a novel mechanism called social learning, where the authors analyze the coordination and de-anchoring of expectations and how forward guidance may affect these results. This paper contributes to the literature by estimating a heterogeneous expectations model in a tractable way, which is done by re-formulating the standard heterogeneous expectations approach in a regime-switching environment.

Finally, the paper also relates to representative agent models studying the effects of ELB and unconventional monetary policy under imperfect information and adaptive learning. Examples include Evans et al. (2008), where the global dynamics of liquidity traps under adaptive learning are studied; Haberis et al. (2014), who analyze macroeconomic effects of transient interest rate pegs in an imperfect information model; Eusepi & Preston (2010), who consider central bank communication in a model where agents' expectations are not consistent with the central bank policy; Cole (2018), who studies the effectiveness of learning on forward guidance, where forward guidance is introduced into monetary policy with a sequence of shocks; and similarly Cole & Martínez-García (2020), who study the effectiveness of forward guidance in a New Keynesian model with imperfect central bank credibility. The present paper relates to this literature by allowing a fraction of agents to use adaptive learning rules through an evolutionary selection mechanism.

The paper is organized as follows. Section 2 presents the main concepts and heterogeneous expectations within the canonical 3-equation New Keynesian model. Section 3 presents the estimation results for the model, along with a discussion of three other REE and learning models as different points of comparison. Section 4 presents a number of counterfactual exercises to analyze the effects of heterogeneous and de-anchored expectations. Section 5 concludes.

2 Model Setup

2.1 Structural Equations and Rational Expectations

I consider the simple canonical version of the New Keynesian model as in Clarida et al. (1999). Similar setups have been considered in closely related papers of Busetti et al. (2017), Lansing (2019), and Goy et al. (2020). I first present the basic form of the model without any regime switching, given by the following structural equations:

$$\begin{cases} y_t = (1-\iota_y)E_t y_{t+1} + \iota_y y_{t-1} - \frac{1}{\tau}(r_t - E_t\pi_{t+1}) + u_{y,t}, \\ \pi_t = \beta((1-\iota_p)E_t\pi_{t+1} + \iota_p \pi_{t-1}) + \kappa y_t + u_{\pi,t}, \\ r_t = \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\delta y}(y_t - y_{t-1}) + \varepsilon_{r,t}, \end{cases} \tag{2.1}$$

where y_t, π_t and r_t denote the output gap, inflation and nominal interest rate respectively. The first equation represents the IS curve, where ι_y is the intrinsic level of inertia (or indexation) in output gap, and τ is the intertemporal elasticity of substitution for households. The second equation is the Phillips curve, with ι_p the price indexation and κ denoting the slope of the Phillips curve. The last equation is the monetary policy reaction function subject to the zero lower bound on nominal interest rates, with ρ_r the interest rate smoothing rate, ϕ_π inflation reaction, ϕ_y output gap reaction, and $\phi_{\Delta y}$ output gap growth reaction. The model is supplemented with 3 shocks. The demand shock $u_{y,t}$ and cost-push shock $u_{\pi,t}$ are AR(1) processes given by:

$$\begin{cases} u_{y,t} = \rho_y u_{y,t-1} + \varepsilon_{y,t}, \\ u_{\pi,t} = \rho_\pi u_{\pi,t-1} + \varepsilon_{\pi,t}, \end{cases} \tag{2.2}$$

while the monetary policy shock $\varepsilon_{r,t}$ is assumed to be an i.i.d. process. Before introducing the zero lower bound (ELB) constraint on the nominal rates and the regime switching setup, it is useful to start with the Rational Expectations Equilibrium (REE) of the model, associated with the Minimum State Variable (MSV) solution. The model can be written in the standard matrix form:

$$\begin{cases} AX_t = BX_{t-1} + CE_tX_{t+1} + D\varepsilon_t, \\ \varepsilon_t = \rho\varepsilon_{t-1} + \eta_t, \end{cases} \tag{2.3}$$

for conformable matrices A, B, C, D and ρ, with $X_t = [y_t, \pi_t, r_t]'$, $\varepsilon_t = [\varepsilon_{y,t}, \varepsilon_{\pi,t}, 0]'$, and $\eta_t = [\eta_{y,t}, \eta_{\pi,t}, \eta_{r,t}]'$. Under RE, the equilibrium solution takes the following form, along with the implied 1-step ahead expectations:

$$\begin{cases} X_t = bX_{t-1} + d\varepsilon_t, \\ E_tX_{t+1} = bX_t + d\rho\varepsilon_t. \end{cases} \tag{2.4}$$

Plugging the expectations back into the law of motion (2.3) yields:

$$(A - Cb)X_t = BX_{t-1} + (Cd\rho + D)\varepsilon_t. \tag{2.5}$$

The RE solution is then pinned down by the following fixed-point conditions:[4]

$$\begin{cases} b = (A - Cb)^{-1}B, \\ d = (A - Cb)^{-1}(Cd\rho + D). \end{cases} \tag{2.6}$$

[4] I make use of the toolkit introduced in Uhlig et al. (1995) to solve for the fixed-point conditions.

2.2 Zero Lower Bound and Regime Switching

The main objective of the paper is to evaluate the effects of the ELB constraint on macroeconomic outcomes. Introducing the constraint on the interest rate rule leads to the following form:

$$r_t = max\{0, \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\delta y}(y_t - y_{t-1}) + \varepsilon_{r,t}\}, \tag{2.7}$$

which is an occasionally binding constraint (OCB) on the nominal rates. A popular method in the literature to approximate the non-linearity resulting from the OCB is to use a regime-switching approach, e.g. in Binning & Maih (2016), Chen (2017), and Lindé et al. (2017) to name a few. In this setup, monetary policy is subject to two different regimes: a Taylor rule regime where interest rates follow the intended reaction function when the ELB constraint does not bind, and a ELB regime where monetary policy becomes inactive when the reaction function becomes constrained by the lower bound. Under this approach, denoting by s_t the regime switching process, which can take on values $s_t = Z$ (ELB regime) and $s_t = T$ (Taylor rule regime), the monetary policy rule evolves according to:

$$\begin{cases} r_t(s_t = T) = \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\delta y}(y_t - y_{t-1}) + \varepsilon^T_{r,t}, \\ r_t(s_t = Z) = \varepsilon^Z_{r,t}. \end{cases} \tag{2.8}$$

During the ELB regime, monetary policy is assumed to be unable to follow its standard reaction function and therefore switches to the second function with pegged interest rates, i.e. $\rho_r = \phi_\pi = \phi_y = \phi_{\Delta y} = 0$, with some additional white noise $\varepsilon^Z_{r,t}$. The regime probabilities evolve according to a transition matrix Q, with constant transition probabilities for convenience, given as:

$$Q = \begin{bmatrix} q^T & 1-q^T \\ 1-q^Z & q^Z \end{bmatrix},$$

with q^T denoting the probability that period t with $s_t = T$ is followed by period $t+1$ with $s_{t+1} = T$. Likewise, q^Z denotes the probability that a ELB period with $s_t = Z$ is followed by a ELB period with $s_{t+1} = Z$. A time-varying version of these transition probabilities will be introduced in the next section. Under this approach, the model dynamics can be captured with the notation:

$$\begin{cases} A(s_t)X_t = B(s_t)X_{t-1} + C(s_t)X_{t+1} + D(s_t)\varepsilon_t, \\ \varepsilon_t = \rho\varepsilon_{t-1} + \eta_t, \end{cases} \tag{2.9}$$

with conformable regime-dependent matrices $A(s_t)$, $B(s_t)$, $C(s_t)$ and $D(s_t)$. The standard approach in Markov-switching DSGE literature, including the aforementioned studies, is to use the RE assumption to solve the above system. In the current framework, using RE boils down to the assumptions that agents are aware of (i) the current underlying regime s_t, and (ii) the transition matrix Q associated with the regimes. They form their expectations according to these two criteria, which leads to regime-dependent expectations in the following form:

$$\begin{cases} E_t[X_{t+1}|s_t = T] = q^T(b(s_{t+1} = T)X_t + d(s_{t+1} = T)\rho\varepsilon_t) + (1 - q^T)(b(s_{t+1} = Z)X_t + d(s_{t+1} = Z)\rho\varepsilon_t), \\ E_t[X_{t+1}|s_t = Z] = q^Z(b(s_{t+1} = Z)X_t + d(s_{t+1} = Z)\rho\varepsilon_t) + (1 - q^Z)(b(s_{t+1} = T)X_t + d(s_{t+1} = T)\rho\varepsilon_t). \end{cases} \tag{2.10}$$

In other words, Markov-switching REE (MS-REE) models equate agents' subjective expectations about regime switches to the objective expectations of the model. In this paper, I use the RISE toolbox (Maih, 2015) to handle the solution and estimation of the MS-REE system with exogenous regime switching.

A well-known result in the macroeconomic literature is that the RE solution in the baseline version of the model in (2.1) is determinate when the Taylor principle of $\phi_\pi > 1$ is satisfied, while the equilibrium becomes indeterminate with pegged interest rates. Davig & Leeper (2007) establish that in a regime switching environment with RE, the equilibrium determinacy can still hold even if one of the underlying regimes is indeterminate. They define this property as the *Long-run Taylor principle* (LRTP). The implications of this for the New Keynesian model with active and passive policy rules as described above is that, as long as the passive (indeterminate) periods are sufficiently short-lived relative to the active (determinate) periods, the model dynamics can still be characterized by a determinate equilibrium.

Even when the overall model dynamics remain determinate, the regime-specific indeterminacy of pegged interest rates leads to an important shortcoming in MS-REE models in the context of ELB with pegged interest rates. The regime-specific indeterminacy typically generates more volatility and adverse economic outcomes than intended, which become more severe as the expected duration of these regimes increases. As a consequence, the MS-REE models are unable to generate persistent ELB regimes in this context, e.g. Chen (2017) and Lindé et al. (2017) report expected duration estimates between 3-9 quarters for the U.S. economy, while the empirical duration between 2008-2015 was 28 quarters. This makes MS-REE models unsuitable for counterfactual simulations and policy analysis, which will be discussed further in Section 3. It is also important to note that having short expected durations in the model leads to an implicit form of non-rationality on the agents' part: expecting short periods of ELB and experiencing long durations leads them to be repeatedly surprised over these periods without ever revising their beliefs.

Ozden & Wouters (2020) show that breaking the tight link between subjective and objective expectations in MS-REE models, and instead replacing agents' expectations with adaptive learning leads to substantial improvements in terms of model-implied expected durations and fitting the data. However, adaptive learning also comes with a regime-specific E-unstability during ELB periods, which puts a downward pressure on economic variables. This leads to frequent deflationary spirals and crashes, which is also inconsistent with the historical experiences on the ELB. Therefore, the main goal of this paper is to introduce heterogeneous expectations into the previous framework, thereby reducing the impact of adaptive learning on the economy and improving the model's longer-term projection performance.

2.3 Endogenous Regime Switching and Heterogeneous Expectations

The standard REE, and the MS-REE models presented in the previous section serve as a benchmark for the model presented in this section. I make two important deviations from the aforementioned models. First, I relax the assumption of exogenous regime switching and introduce endogenous transition probabilities for monetary policy, which follows the approach in Ozden & Wouters (2020).[5] Second, I introduce heterogeneous expectations in the form of an endogenous regime switching process, which will be explained in further detail below.

Starting with the monetary policy function, the transition matrix is first replaced with the time-varying matrix:

$$Q_t = \begin{bmatrix} q_t^T & 1-q_t^T \\ 1-q_t^Z & q_t^Z \end{bmatrix},$$

where the probabilities q_t^T and q_t^Z depend on the central bank's desired policy rate at every period, which will be defined as the *shadow rate* henceforth. More formally, I assume that the shadow rate r_t^* follows:

$$\begin{cases} r_t^*(s_t = T) = \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\Delta y}(y_t - y_{t-1}), \\ r_t^*(s_t = Z) = \rho_r r_{t-1}^* + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\Delta y}(y_t - y_{t-1}). \end{cases} \tag{2.11}$$

This structure makes use of the following assumptions: r_t^* is the central bank's level of desired nominal interest rate *in the absence of monetary policy shocks and the ELB constraint*. During normal times with the Taylor rule, the shadow rate is smoothed over the observed nominal interest rate. Therefore during normal times, the only difference between these two rates is the presence of i.i.d. monetary policy shocks. During ELB periods with pegged nominal rates, the shadow rate is smoothed over itself, which allows for persistent deviations from the nominal rate beyond the i.i.d. monetary policy shocks. This captures the idea of keeping the interest rates *lower-for-longer*, where the central bank wants to keep the policy rate at near zero levels until the shadow rate, recovers back to non-negative levels.

[5]Endogenous regime switching models have also been considered within the context of RE models. Examples include Binning & Maih (2016), Barthélemy & Marx (2017) and Benigno et al. (2020), among others. I abstract from these in this paper.

Given the shadow rate r_t^*, the transition probabilities are determined according to:

$$q_t^T = \frac{\theta_1}{\theta_1 + exp(-\Phi_1(r_t^* + (\bar{r_T} - \bar{r_Z})))}, \quad q_t^{ELB} = \frac{\theta_2}{\theta_2 + exp(\Phi_2(r_t^* + (\bar{r_T} - \bar{r_Z})))}, \tag{2.12}$$

where $\bar{r_T}$ and $\bar{r_Z}$ are the steady-state levels of the nominal interest rate during normal and ELB regimes respectively. In this study, these steady-state values are taken to be the historical average rates over the normal and ELB periods respectively, and they are introduced into the measurement equations rather than the structural equations, which will be discussed further in Section 3.

Anchored Expectations

Given the endogenous monetary policy switching, expectations are formed according to two types of models: the first type is based on the RE solution of the baseline version of the model (2.1), where monetary policy is active and the equilibrium is unique. I assume this type of agents always use the RE equilibrium associated with (2.6) where the Taylor principle $\phi_\pi > 1$ is satisfied. In other words, they always form their expectations based on a determinate RE solution. During normal periods, this assumption boils down to the standard model solution associated with RE. During the ELB periods, expectations associated with this type take on a different interpretation where monetary policy is passive, but expectations evolve *as if* the central bank's desired interest rate, i.e. the shadow rate r_t^*, matters for the economy.

The assumption that agents always use the RE solution associated with active policy rule implicitly means that they know the shadow rate at any given period, even though the shadow rate is not directly observable during ELB periods. Therefore this assumption is interpreted as a successful *central bank communication* on the desired interest rate, which proxies for the impact of central bank's unconventional policy tools on expectations. I assume that forward guidance communications and quantitative easing measures allow the central bank to correctly signal the desired interest rate and anchor this class of agents' expectations on the targeted equilibrium. Put differently, the agents believe that unconventional monetary policy measures substitute for the ELB constraint on the nominal rates.

It is important to note that this expectation formation rule ignores the presence of the ELB constraint, as well as the presence of other agents in the economy that form their expectations differently. Therefore these expectations correspond to a form of pseudo-rationality only, i.e. what would happen if all expectations were rational, and if the monetary policy was not constrained by the ELB. Such behavior is usually referred to as *fundamentalist* rules in heterogeneous expectations studies.[6] In this paper, I refer to this type as *anchored* expectations.

[6] See e.g. Hommes & Lustenhouwer (2019) and Goy et al. (2020), where *fundamentalist* agents use the steady-state

Adaptive Learning

The second class of agents use a constant gain recursive least squares (RLS) learning rule based on the observable variables of output gap, inflation and nominal interest rates. Specifically, I assume that agents have the following regression model, along with the implied 1-step ahead expectations:

$$\begin{cases} X_t = \alpha_{t-1} + \beta_{t-1} X_{t-1} + \delta_t, \\ E_t X_{t+1}^L = \alpha_{t-1} + \beta_{t-1} X_t, \end{cases} \tag{2.13}$$

where α_{t-1} is a vector of perceived means, β_{t-1} is the perceived first-order correlation matrix, and δ_t is a vector of i.i.d. shock processes. The first equation in (2.13) is denoted as the agents' *Perceived Law of Motion* (PLM) henceforth. This particular VAR(1) form of learning has been frequently used in the learning literature, see e.g. Milani (2011) and Chung & Xiao (2013). It has the advantage of being close to the beliefs consistent with the MSV solution of the model. The only difference is that with the VAR(1) learning rule, the exogenous AR(1) shocks are not included in the regression, which keeps the PLM relatively small and more tractable. While the results in the paper are provided under this learning rule, I also provide a robustness check for the main results with a more parsimonious AR(1) PLM in Appendices B, C and D.[7]

It is important to note that in this paper, I use the assumption of *t-timing* on expectations, which means that agents are able to use period t information when forming their expectations. This corresponds to a joint determination of expectations and period t variables, which is also the standard assumption in REE and MS-REE models in general. Keeping the information structure in both learning and RE models is crucial for heterogeneous expectations in this context since agents evaluate the rules based on their forecasting performance, which is in turn affected by the information content.[8] Agents update the perceived parameters in their PLM after the endogenous variables are determined, hence these parameters appear with a lag in (2.13) in the form of α_{t-1} and β_{t-1}. Under constant gain RLS, they evolve according to:

$$\begin{cases} R_t = R_{t-1} + \gamma(\tilde{X}_{t-1}\tilde{X}_{t-1}' - R_{t-1}), \\ \Phi_t = \Phi_{t-1} + \gamma R_t^{-1} \tilde{X}_{t-1}(X_t - \Phi_{t-1}\tilde{X}_{t-1})', \end{cases} \tag{2.14}$$

values or long-run averages of the relevant endogenous variables when forming their expectations.

[7]Specifically, this alternative learning rule assumes a diagonal β_t matrix in the notation of (2.13), where agents ignore the cross-correlations between the variables. The results presented in the paper are robust to these two PLMs considered. Ozden & Wouters (2020) provide a more thorough comparison and assessment of different learning rules, including an MSV-learning rule with observed shocks.

[8]The alternative is to use the assumption of $t-1$ *dating* for both types of agents, which takes on a sequential structure where first expectations are formed using information from period $t-1$, and then period t variables are determined given the expectations.

where $\tilde{X}_{t-1} = [1, X'_{t-1}]'$, $\Phi_t = [\alpha_t, \beta_t]$ and R_t is the second moments matrix of perceived autocovariances. γ denotes the constant gain value, which determines the weight that agents place on the latest available observations.

A well-known result in the adaptive learning literature is that, akin to the determinacy condition in RE models, the learning dynamics are generally stable when the Taylor principle $\phi_\pi > 1$ is satisfied (Bullard & Mitra, 2002). In general, the stability of learning dynamics is defined as the *E-stability* principle (Evans & Honkapohja, 2001). During ELB periods where monetary policy is inactive, the E-stability principle breaks down and learning dynamics become unstable, which may give rise to deflationary spirals. Ozden & Wouters (2020) derive the long-run E-stability (LRES) principle, akin to the LRTP of Davig & Leeper (2007), which shows that as long as the ELB episodes are relatively short-lived compared to normal episodes, the overall model dynamics remain stable and deflationary spirals do not arise. Both LRTP and LRES conditions depend on the assumption of exogenous regime switches, whereas the model of interest in this paper is based on endogenous regime switching. Nevertheless, the general stability principles associated with the RE equilibria and adaptive learning serve as an intuitive starting point to analyze the endogenous switching model.

Aggregate Dynamics

Given the RE- and learning-based expectation formation rules, the fraction of agents using each rule evolves according to a fitness measure based on their 1-step ahead forecasting performance as in Busetti et al. (2017), Hommes & Lustenhouwer (2019), Lansing (2019) and Goy et al. (2020). Specifically, I assume the following fitness measures ζ_t^{RE} and ζ_t^L associated with each rule:[9]

$$\begin{cases} \zeta_t^{RE} = (1-\omega)FE^{RE} + \omega\zeta_{t-1}^{RE}, \\ \zeta_t^{L} = (1-\omega)FE^{L} + \omega\zeta_{t-1}^{L}, \end{cases} \tag{2.15}$$

where FE^{RE} and FE^L denote the sum of forecast errors for inflation and output gap under for the RE- and learning-based PLMs respectively. Given the fitness measures, agents' fractions are determined by:

$$n_t^{RE} = \frac{exp(\chi\zeta_t^{RE})}{exp(\chi\zeta_t^{RE}) + exp(\chi\zeta_t^{L})}, \quad n_t^{L} = \frac{exp(\chi\zeta_t^{L})}{exp(\chi\zeta_t^{RE}) + exp(\chi\zeta_t^{L})}, \tag{2.16}$$

where n_t^{RE} and n_t^L denote the fractions of agents associated with each type, and χ is an *intensity of choice* measure, common across both types, which determines the frequency of switching between

[9] The fitness measures follow the standard assumption in the heterogeneous expectations literature as in the aforementioned studies.

the rules.

In this paper, different from previous studies, I use the expectational heterogeneity as a regime switching mechanism. In previous studies, the fractions n_t^{RE} and n_t^L determine the aggregate expectations as a weighted average, whereas I instead interpret these fractions as the probability of realization for each regime. Accordingly, the law of motion (2.3) is determined separately under (2.4) and (2.13). Then I take a weighted average of each regime, based on the fractions n_{t-1}^{RE} and n_{t-1}^L, in order to obtain the realized regimes. The advantage of this approach is that it allows me to re-cast the heterogeneous expectations framework as a regime switching process, which can be estimated with the conditionally linear filter described in Ozden & Wouters (2020). Together with the monetary policy switching, the expectational switching gives rise to a 4-regime model, which are summarized and labeled as follows: (1) Taylor rule regime with learning (E-stable), (2) Taylor rule regime with RE equilibrium (determinate equilibrium), (3) ELB regime with learning (E-unstable and possibly de-anchored expectations), (4) ELB regime with RE equilibrium (anchored expectations at the determinate RE solution). Putting together all 4 regimes, the transition matrix is given by:

$$\begin{bmatrix} q_t^T n_{t-1}^L & q^T n_{t-1}^{RE} & (1-q_t^T)n_{t-1}^L & (1-q_t^T)n_{t-1}^{RE} \\ q_t^T n_{t-1}^L & q^T n_{t-1}^{RE} & (1-q_t^T)n_{t-1}^L & (1-q_t^T)n_{t-1}^{RE} \\ (1-q_t^{ELB})n_{t-1}^L & (1-q_t^{ELB})n_{t-1}^{RE} & q_t^{ELB} n_{t-1}^L & q_t^{ELB} n_{t-1}^{RE} \\ (1-q_t^{ELB})n_{t-1}^L & (1-q_t^{ELB})n_{t-1}^{RE} & q_t^{ELB} n_{t-1}^L & q_t^{ELB} n_{t-1}^{RE} \end{bmatrix},$$

where I make the assumption that the fraction of expectations enter into the transition matrix with a 1-period lag, similar to the learning parameters in agents' PLM. This leads to a sequential intra-period timeline as follows: (i) first the shadow rate is calculated, which determines the monetary policy regime, (ii) expectations and the endogenous variables are jointly determined for each possible regime transition, (iii) regimes probabilities are calculated based on period t monetary policy regime probabilities, and period $t-1$ fraction of agent types (iv) states are collapsed, the adaptive learning rule is updated, and period t fraction of expectations are realized based on their forecast errors.

3 Estimation

Methodology, Data and Priors

This section discusses the estimation methodology, along with the dataset used in estimations and prior distributions for the estimated parameters. The regime switching model described in the previous section can be summarized as a time-varying recursive state-space system with the following structure:

$$S_t = \gamma_{1,\Phi_t}^{s_t} + \gamma_{2,\Phi_t}^{s_t} S_{t-1} + \gamma_{3,\Phi_t}^{s_t} \eta_t, \tag{3.1}$$

with $S_t = [X_t, \varepsilon_t]'$ and conformable matrices $\gamma_{1,\Phi_t}^{s_t}$, $\gamma_{2,\Phi_t}^{s_t}$ and $\gamma_{3,\Phi_t}^{s_t}$ with two layers of time-variation in the system matrices. The time-varying adaptive learning parameters are captured by Φ_t, while the regimes switches are captured by s_t. The filtering process and calculation of the likelihood function of the model is handled by the conditionally linear filter described Ozden & Wouters (2020), which is a straightforward extension of the standard Kim & Nelson (1999) filter (henceforth KN) used in Markov-switching state-space models. In a Markov-switching environment with m regimes, a sample size of T generates m^T distinct timelines associated with the model due to the history dependence of the Markov-switching structure. This number quickly becomes intractable as the sample size and the number of regimes grow. KN filter deals with this issue with a collapsing step, which amounts to taking a weighted average of the state vector and the covariance matrix at every iteration of the filter, effectively reducing the number of timelines from m^T to m^2. The adaptive learning step and updating of expectational fractions are applied on the collapsed variables, which feed back into the next iteration of the filter. This leads to a sequential and conditionally linear structure.[10]

For the estimation of the New Keynesian model, I use historical U.S. data on output gap, inflation and nominal interest rates over the post-Great Moderation period starting from 1982Q1 until 2019Q4. Further details and descriptions of the time series used in the estimation can be found in Appendix A. The measurement equations are straightforward and are related to the model variables as follows:

$$\begin{cases} y_t = \bar{y} + y_t^{obs}, \\ \pi_t = \bar{\pi} + \pi_t^{obs}, \\ r_t = \bar{r} + r_t^{obs}, \end{cases} \tag{3.2}$$

where the right-hand side variables are the historical data (observables), and the left-hand side variables are the model variables. The historical averages are denoted by $\bar{y}$, $\bar{\pi}$ and $\bar{r}$ respectively, which are included in the measurement equations rather than demeaning the data prior to the estimation. Following the approach in Gust et al. (2018), I assume there is a shift in the intercept of interest rates $\bar{r}$, which switches to a lower value during the ELB period. Different than monetary policy and expectational switching mechanisms in the model, I assume that this is a deterministic switch over the period 2008-2015, corresponding to the period where the U.S. Federal Reserve used forward guidance.[11] The regime specific values are subsequently denoted as $\bar{r_Z}$ during the ELB period between 2008-2015, and $\bar{r_T}$ during the Taylor rule regime. The output gap series is based on a quadratic de-trending of output over the sample period, following Cornea-Madeira et al. (2019).

All structural, learning and switching parameters are assigned prior distributions consistent with previous values used in the literature. The risk aversion parameter τ has a Gamma distribution with a mean 2 and standard deviation 0.5 as in An & Schorfheide (2007). The monetary policy reaction coefficients are all based on the Smets-Wouters (2007) model, henceforth denoted as SW07.

[10] See Appendix C in Ozden & Wouters (2020) for a more detailed discussion of the estimation algorithm.

[11] The same intercept shift is also assumed for the shadow rate over the same period.

Accordingly, inflation reaction ϕ_π is assigned a Gamma distribution with mean 1.5 and standard deviation 0.25; output gap reaction coefficients ϕ_y and $\phi_{\Delta y}$ are assigned Gamma distributions with mean 0.25 and standard deviation 0.1. The interest rate smoothing parameter ρ_r is assigned a Beta distribution with mean 0.75 and standard deviation 0.1. Similarly, shock parameters are based on the same model, where shock persistence parameters ρ_y and ρ_π are assigned a Beta distribution with mean 0.5 and standard deviation 0.2, and shock standard deviations are assigned Inverted Gamma Distributions with mean 0.1 and standard deviation 2. The standard deviation of the monetary policy shock over the ELB regime is an exception, which is instead assigned a Uniform distribution over the unit interval. For the slope of the Phillips curve κ, I use a relatively tight prior of a Beta distribution with mean 0.05 and standard deviation 0.025. This corresponds to a lower mean and standard deviation compared to previous studies, e.g. An & Schorfheide (2007) use a wider Beta distribution with mean 0.3 and standard deviation 0.15. Nevertheless, the prior used here includes parameter values consistent with most empirical studies as its credible interval. The indexation parameters ι_y and ι_π are assigned Beta distributions with mean 0.25 and standard deviation 0.1. The historical averages in the measurement equations are assigned Uniform distributions over the interval $[0, 2]$, except for the output gap mean which is fixed at 0 and is not included in the estimation. The average for interest rates during the ELB period, $\bar{r_Z}$, is assigned a more informative Normal prior with a mean of 0.1 and standard deviation 0.25 in order to restrict the range of parameter values over this period.

For the exogenous switching REE model, the transition probabilities $1-p_{11}$ and $1-p_{22}$ are assigned uniform priors over the unit interval, which differs from previous studies that assume tighter Beta distributions, e.g. Chen (2017) and Lindé et al. (2017). For the endogenous switching models, the parameters θ_1 and θ_2 in the monetary policy switching functions are fixed at 1 and are not included in the estimation; this is based on the analysis in Ozden & Wouters (2020) which shows that the data is not informative on both θ and Φ simultaneously. For the other two parameters on monetary policy switching, I assign Gamma distributions with mean 0.2 and standard deviation 0.1 on $\frac{\Phi_1}{1000}$ and $\frac{\Phi_2}{1000}$, which covers both gradual and abrupt transitions for monetary policy regime switching. The persistence of expectational switching, ω, is assigned the same distribution as the shock persistence parameters, i.e a Beta distribution with mean 0.5 and standard deviation 0.2. The intensity of choice χ is assigned a Gamma distribution with mean 5 and standard deviation 2, which is based on the findings of Cornea-Madeira et al. (2019) on inflation expectations. Finally, the constant gain parameter γ is assigned a Gamma distribution with mean 0.035 and standard deviation 0.015, which is based on Slobodyan & Wouters (2012) and Ozden & Wouters (2020), but assumes a tighter distribution compared to these two studies. The prior distributions for all estimated parameters are summarized in Table 1.

The initial transition probabilities for monetary policy switching are based on the unconditional moments for the normal and ELB periods in the sample. Accordingly, I use $q_1^T = 0.991$, which implies an expected duration of 107 quarters based on the initial period over 1982-2008, and $q_1^Z = 0.964$, which implies an expected duration of 28 quarters based on the ELB period over 2008-2015. For the expectational switching, I make no prior assumptions on the distributions and set n_1^{RE} and n_1^L both equal to 0.5. The results are not sensitive to the initial values as the data is informative about these probabilities over the entire sample.

The initial beliefs for adaptive learning are derived from the estimated RE model: I first estimate the baseline model in (2.1) under REE without regime switching. Then I retrieve the implied VAR(1) beliefs consistent with the estimated equilibrium. The initial values remain fixed at these values throughout the estimation. I use C. Sim's *csminwel* algorithm (1999) to obtain the posterior mode, which is used to initialize the MCMC algorithm using random-walk Metropolis-Hastings. I use 250000 parameter draws for all models under consideration. The first 40% of the draws are discarded as burn-in sample, and the convergence diagnostics for the remaining 60% are checked using Geweke's statistics (1992). Further details on posterior distributions and convergence diagnostics are provided in Appendix B.

Param.	Dist.	Prior Mean	Prior St. Dev.	Lower B.	Upper B.
$\bar{\pi}$	Uniform	0.5	0.29	0	∞
$\bar{r_T}$	Uniform	0.5	0.29	0	∞
κ	Beta	0.05	0.025	0	1
τ	Gamma	2	0.5	0	∞
ϕ_π	Gamma	1.5	0.25	1	∞
ϕ_y	Gamma	0.25	0.1	0	∞
$\phi_{\Delta y}$	Gamma	0.25	0.1	0	∞
ρ_y	Beta	0.5	0.2	0	1
ρ_π	Beta	0.5	0.2	0	1
ρ_r	Beta	0.5	0.2	0	1
η_y	Inv. Gamma	0.1	2	0	∞
η_π	Inv. Gamma	0.1	2	0	∞
η_r	Inv. Gamma	0.1	2	0	∞
ι_y	Beta	0.25	0.1	0	1
ι_π	Beta	0.25	0.1	0	1
$\bar{r_Z}$	Normal	0.1	0.25	0	∞
η_{r_Z}	Uniform	0.5	0.29	0	∞
$\frac{\Phi_1}{1000}$	Gamma	0.2	0.1	0	∞
$\frac{\Phi_2}{1000}$	Gamma	0.2	0.1	0	∞
γ	Gamma	0.035	0.015	0	1
ω	Beta	0.5	0.2	0	1
χ	Gamma	5	2	0	∞
$1 - p_{11}$	Uniform	0.5	0.29	0	1
$1 - p_{22}$	Uniform	0.5	0.29	0	1

Table 1. Prior distributions for the estimated parameters in the New Keynesian model.

Posterior Estimation Results

This section discusses the posterior estimation results for the heterogeneous switching model, along with three accompanying models to assess the impact of monetary policy and expectational switching mechanisms. In particular, I estimate the baseline REE model without any regime switching as described in Section (2.1), the exogenous MS-REE model with a switch in monetary policy reaction function as described in Section (2.2), and a 2-regime adaptive learning model with switching only in the monetary policy reaction function. This last model is a special case of the 4-regime heterogeneous expectations model, which assumes that all expectations evolve according to adaptive learning without the expectational switching mechanism. These four models are subsequently referred to as the baseline REE, the MS-REE, 2-regime and 4-regime adaptive learning models respectively. The estimation results for all four models are reported in Table 2, and the discussion below is mainly based on the posterior mean values.

I start with the parameters that are common across all models. The Slope of the Phillips curve κ is larger in the 2-regime VAR(1) model with a value of 0.01, compared with values ranging between $[0.002, 0.003]$ under the REE, MS-REE and 4-regime VAR(1) models. The larger slope under learning is consistent with previous studies in the literature, e.g. Milani (2007, 2011) and Slobodyan & Wouters (2012). The result that κ is smaller under the 4-regime learning model, compared the 2-regime learning is intuitive in this sense since the former model is closer to the REE benchmark. The risk aversion parameter τ is considerably lower in both 2-regime and 4-regime learning models with values of 1.44 and 1.53, and highest in the MS-REE model with 3.52 compared to the REE model with 2.43. The relatively high value in the MS-REE model is explained by the expectational feedback channel: when monetary policy becomes inactive, expectations directly account for this switch in the MS-REE model. As a result, the ex-ante real interest rate $r_t - E_t[\pi_{t+1}]$ has a larger feedback on output gap y_t in the IS equation. Therefore the higher risk aversion parameter in the MS-REE model has the effect of dampening this feedback channel, which is absent in the other model specifications since expectations either do not account for the ELB regime (in the baseline REE model), or only indirectly account for it (in the 2- and 4-regime learning models).

Next looking at the monetary policy parameters, these are typically consistent across all models with HPD intervals within the range of each other: the posterior means for ϕ_π range over the interval $[1.648, 1.959]$, whereas the output gap reaction ϕ_y and output gap growth reaction $\phi_{\Delta y}$ range over the intervals $[0.116, 0.203]$ and $[0.194, 0.221]$ respectively. The same argument also applies to interest rate smoothing ρ_r, which fluctuates between 0.89 and 0.936.

We observe some differences in the estimated indexation and shock persistence parameters: output gap shock persistence ρ_y is relatively lower under REE with 0.6, which ranges between 0.942 and 0.967 in the other models. The differences between the indexation parameter ι_y are smaller, the lowest value being 0.113 in the 2-regime learning, and highest value 0.322 in the 4-regime learning.

The picture reverses when we look at inflation dynamics: while the REE model has a highly persistence shock with a ρ_π of 0.965, the estimate ranges between 0.625 and 0.553 in other models. The indexation value ι_π again shows smaller differences, with values fluctuating between 0.213 and 0.344. These results suggest that the feedback from output gap shock persistence on inflation dynamics is stronger in the MS-REE and learning models, whereas the REE model needs a more persistent cost-push shock $\varepsilon_{\pi,t}$ to match the inflation persistence.

The intercept values in the measurement equations are higher under REE and MS-REE models, with 0.62 and 0.7 for $\bar{\pi}$ and 0.63 and 0.79 for $\bar{r_T}$ respectively. These values are 0.43 and 0.44 for $\bar{\pi}$ in learning models. Similarly, $\bar{r_T}$ is estimated at 0.182 and 0.27 in these two models. The smaller values are justified by the perceived mean dynamics fluctuating at above-zero values on average in the learning models, leading to lower values for the intercept terms in the measurement equations.

Among the remaining parameters, the exogenous regime transition probabilities in the MS-REE model are estimated at 0.207 for the ELB exit probability, and 0.037 for the normal regime probability. These values correspond to expected durations of roughly 4.83 quarters for the ELB regime, and 27.02 quarters for the Taylor rule regime. Particularly for the ELB regime, this leads to very short-lived ELB episodes compared to the empirical duration of 28 quarters, which makes the model unsuitable for policy analysis and counterfactual simulations over this period.

For the endogenous regime transition probabilities, the monetary policy switching parameters Φ_1 and Φ_2 are estimated at 177 and 147 under 2-regime learning, whereas they turn out to be 167 and 134 under 4-regime learning. In both models, these values suggest a faster transition from Taylor rule regime to ELB, compared to the transition from ELB to Taylor rule regime. Put differently, the policy switches from Taylor rule to ELB quickly when the shadow rate falls below the lower bound. However, once the policy is in the ELB regime, it may linger there for a while longer even after the shadow rate reaches non-negative values.

For the heterogeneous expectations and learning related parameters, we obtain similar values of 0.007 and 0.009 for the constant gain γ, which is in the range of values obtained in previous studies for the U.S., see e.g. Branch (2007), Milani (2007) and Slobodyan & Wouters (2012). For the memory parameter ω in expectational switching, I obtain a mean of 0.72 with a wide HPD interval covering values between $[0.39, 0.99]$, which suggests that the data is not very informative about this parameter. For the intensity of choice χ, I obtain a mean of 2.1, with an HPD interval of $[0.55, 3.89]$. This value is lower than the estimate in Cornea-Madeira et al. (2019).[12]

Overall, it is readily seen that relative to the REE model, the model fit is improved under MS-REE, as well as 2- and 4-regime learning models based on the posterior mode and Modified Harmonic

[12] This difference is explained by the lack of survey expectations as observables in my model. Including expectations data is likely to improve the inference on these parameters, which is left for future research.

Mean (MHM) estimators. The 4-regime learning model yields a somewhat worse model fit compared to MS-REE and 2-regime models, while the latter two models yield similar values. However, both the MS-REE and 2-regime learning models have important shortcomings: the MS-REE model, due to the exogenous nature of switching and given the estimated transition probabilities, can only generate short-lived ELB episodes of 4-6 quarters that are inconsistent with the empirically observed ELB durations. This makes the model unsuitable for studying the potential downside risks of ELB episodes. While the 2-regime learning model improves on this point by breaking the tight link of the MS-REE model between models' objective expectations and agents' subjective expectations, Ozden & Wouters (2020) show that a pure learning model typically leads to a fast de-anchoring of expectations, resulting in deflationary spirals with large probabilities. The 4-regime model, by incorporating heterogeneous expectations into the adaptive learning framework, allows for the possibility of more stable ELB episodes, as long as a sufficiently large fraction of expectations remain anchored at the REE. These points are further discussed with counterfactual simulations in Section 4 with applications to the GFC, as well as the pandemic-induced recession of 2020. I provide robustness checks on the estimation results discussed in this section in Appendix C, where the VAR(1) rule for the adaptive learning agents is instead replaced with a more parsimonious AR(1) rule.

For the remainder of this section, I focus on results under the 4-regime adaptive learning model. Figure 1 shows the filtered time series of the learning parameters α_t and β_t for output gap and inflation in agents' PLM over the estimation period. It is readily seen that the GFC and the subsequent switch to the ELB episode in 2008-2009 are associated with a substantial drop in the perceived means for both variables. Further, a clear pattern in some of the perceived β_t parameters is the upward or downward *jumps* when the economy switches to the ELB episode. In particular $\beta_{y,y}$ and $\beta_{\pi,y}$ jump down, while $\beta_{y,r}$, $\beta_{\pi,\pi}$ and $\beta_{\pi,r}$ jump up. This suggests a relatively fast adoption of the new environment by adaptive learning agents' PLM, where their regression model goes through large updates over a short period of a few quarters, after which the time variation in learning parameters resumes to a pattern of gradual changes as in the pre-crisis period. This quick updating during the monetary regime switch is consistent with the findings in Ozden & Wouters (2020), where such jumps have been investigated in more detail in simulation exercises in the context of long-run E-stability and Restricted Perceptions Equilibria (RPE).

Figure 2 shows the estimated fraction of agents with expectations anchored at the (determinate) RE equilibrium, together with the nominal interest rate and the estimated shadow rate over the sample period. It is readily seen that during normal times, prior to 2008 with the Taylor rule regime, the fraction fluctuates around 50%, indicating a roughly equal share of agents with anchored expectations and de-anchored learning agents. However, with the switch to the ELB regime, there is a large jump in the fraction of anchored expectations for a short period until 2010, which then goes back down to below 50% and gradually increases afterwards. This suggests that a de-anchoring

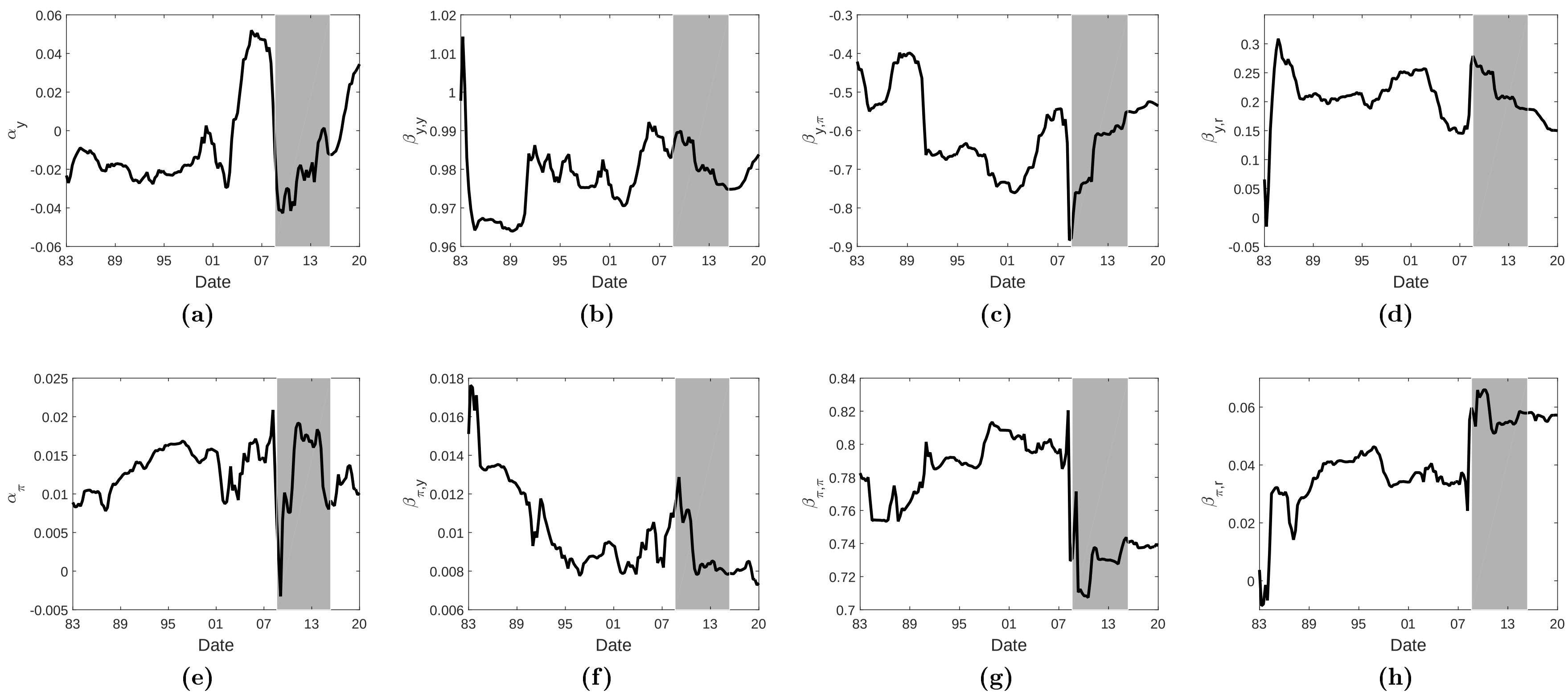
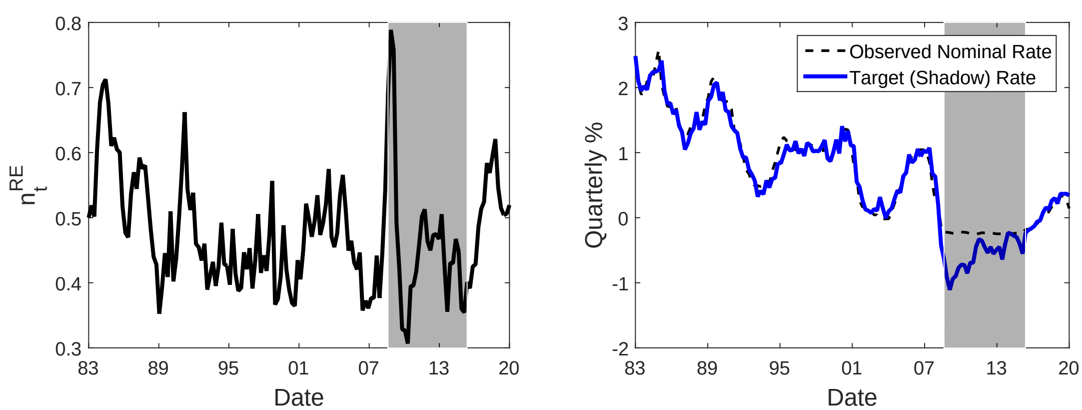


Figure 1. 4-regime learning model, α_t and β_t parameters in agents' PLM.

of expectations immediately following the crisis is not supported by the data. It is also important to note that the increase in the fraction of anchored expectations is accompanied by the jumps in adaptive learning agents' PLM: the crisis period creates a large impact on the learning agents' PLM through a large expectational error, leading to a quick de-anchoring for this type. In turn, the effect that this jump would have had on the aggregate variables is dampened by the reduction of agents using adaptive learning.

Next looking at the estimated shadow rate, we observe that during the pre-crisis period, it closely follows the nominal interest rate. As discussed in the previous section, this close relationship between the interest rates during the Taylor rule regime is by construction. During the initial phase of the ELB regime, a large disparity between the two rates arises once the crisis hits, where the shadow rate falls to roughly -1% on a quarterly basis. This is consistent with other studies in the literature, e.g. Kulish et al. (2014), where the authors report an annual rate of -4% for the lowest point of the shadow rate at the posterior mean. Importantly, the shadow rate gradually picks up after the initial crisis period and catches up with the nominal rates as the nominal rate starts rising during the last quarter of 2015 and first quarter of 2016, where the economy switches back to the Taylor rule regime.

The observed pattern in the shadow rate offers another interpretation for the estimated fraction of agents using adaptive learning: to the extent that learning with a passive Taylor rule is unstable, a larger difference between the shadow and nominal rates implies more instability in the learning dynamics. Therefore, the period with the largest difference between these two rates (i.e. the earlier crisis period) is associated with a smaller fraction of agents using adaptive learning, which serves as a dampening mechanism on the downward economic pressure of learning.



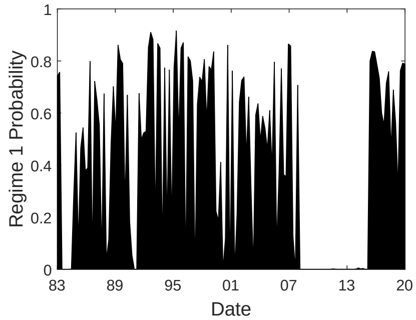
(a) (b)

Figure 2. (a) Fraction of agents using the Determinate RE solution. (b) Shadow rate and the nominal interest rate in the 4-regime learning model.

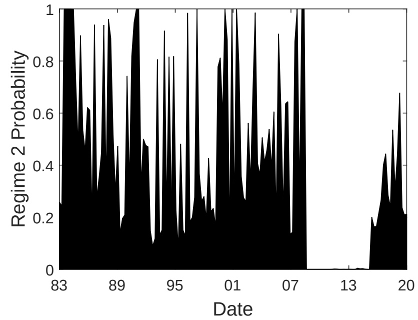
Figure 3 shows the estimated probabilities of each regime over the sample period. Not surprisingly, the first 2 regimes (learning + Taylor rule, and REE + Taylor rule respectively) are dominant over the earlier sample up to start of the GFC. During the post-GFC period, regime 4 (determinate REE + ELB) is more dominant over the earlier sample, while the probability of regime 3 (learning + ELB) somewhat increases after 2010. Once the economy switches back to the Taylor regime by the end of 2015, we observe that the fraction of learning agents is higher until the end of the sample. Looking at the average probabilities, also reported in Figure 3, we observe that the economy spends roughly the same amount of time in regimes 1 and 2 with around 40% over the entire sample period (corresponding to a 50% average during Taylor rule regimes). Regimes 3 and 4 are activated over the ELB period between 2008-2015, where the system spends nearly twice as much time in regime 4 compared to regime 3, with averages of 12% and 6.64% over the entire sample period. These values correspond to averages of 64.3% and 35.6% over the ELB regime.

The results based on the estimated regime probabilities suggest that, during the ELB period, the data supports a large fraction of expectations staying anchored, while the remaining one third of expectations are de-anchored. As I will show with counterfactual simulations in Section 4, a higher fraction of agents using adaptive learning puts a downward pressure on inflation and output gap, and sufficiently high fractions of adaptive learning can lead to deflationary spirals with ever-falling output gap and inflation. Accordingly, the high fraction of anchored expectations can be interpreted as the impact of unconventional monetary policy on expectations, to the extent that we evaluate such measures through their impact on expectations. In particular, by signaling a commitment to monetary easing through forward guidance, and through the signaling channel of quantitative easing, the central bank is able to keep a high fraction of expectations anchored during the crisis period, which in turn improves economic outcomes.

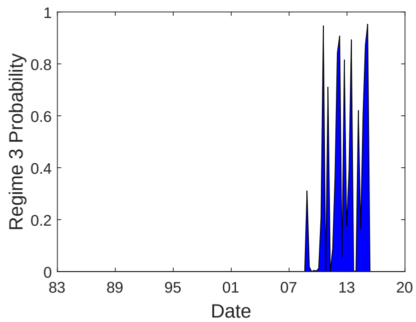
Before closing this section, I also briefly discuss some conditional and average impulse responses under 2- and 4-regime learning models. The aim of this exercise is to see the impact of heterogeneous expectations on impulse responses. As an example, Figure 4 shows the impulse response of output gap to a cost-push shock. The top two panels in the Figure show the conditional impulse responses of output over the period 2001Q1-2019Q4. The black and red lines at the beginning and end denote the IRFs under MS-REE model as a point of reference. The general pattern we observe in these two figures is that, when the system switches from the Taylor rule regime to the ELB regime, the direction of change in the impulse responses is the same in all three models. However, there is considerable time variation in the IRFs of learning models: they are characterized by a jump when the switch occurs, after which the IRFs change only gradually until the end of the ELB period. At that point the system switches back to the Taylor rule regime, leading to another jump in the IRFs. The overall change in the IRFs for learning models tends to be smaller than the MS-REE model, which is discussed extensively in Ozden & Wouters (2020) for a wide set of variables and shocks in the SW07 model. An additional observation here is that the regime-specific differences also tend to be smaller under the 4-regime learning model compared to the 2-regime model, since the fraction of anchored expectations keeps the IRFs more stable compared to the 2-regime model. The full set



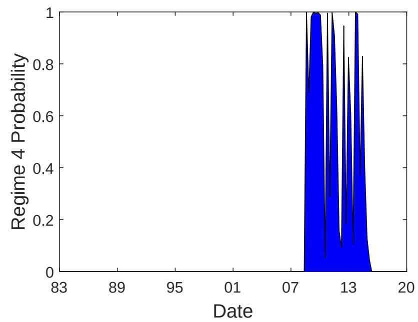
(a) Regime 1: Taylor Rule + Learning.



(b) Regime 2: Taylor Rule + (Determinate) REE.



(c) Regime 3: ELB + Learning.

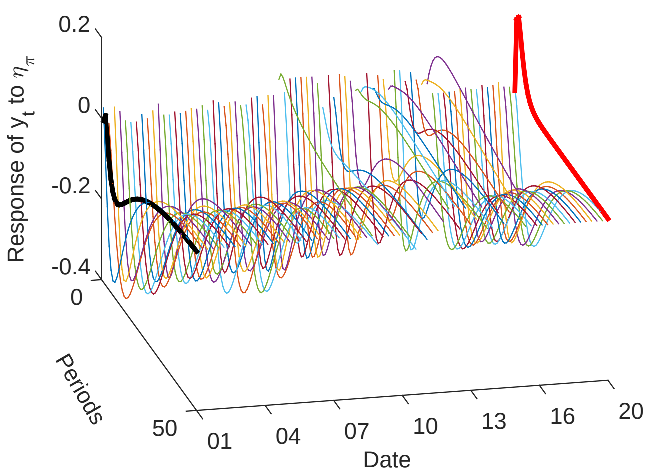


(d) Regime 4: ELB + (Determinate) REE.

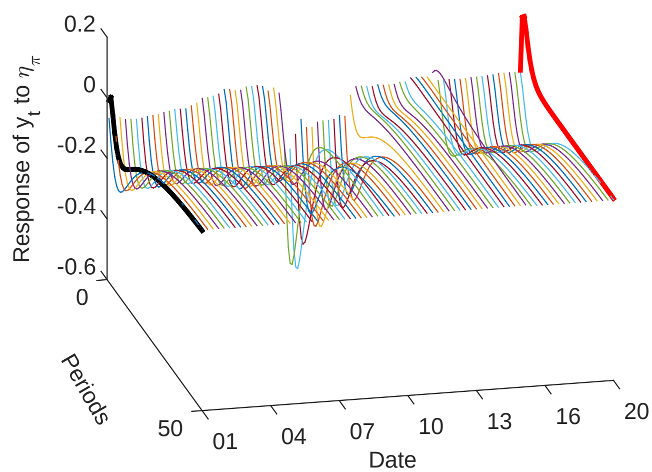
Regime Averages				
Expectations	Regime 1 Learning	Regime 2 REE	Regime 3 Learning	Regime 4 REE (Determinate Solution)
Monetary Policy	Taylor Rule	Taylor Rule	Passive	Passive
VAR(1) Learning				
Whole sample period	40.70%	40.60%	6.64%	12%
Taylor rule period	50%	50%	0%	0%
ELB period	0%	0%	35.6%	64.3%

Figure 3. 4-regime learning model: estimated regime probabilities, along with the average regime probabilities over the entire sample period, and during the Taylor rule and ELB periods.

of impulse response for both output gap and inflation to demand and cost-push shocks is omitted here, and can be found in Appendix D, along with the IRFs associated with the AR(1) learning model.



(a) 4-regime VAR(1), conditional IRFs.



(b) 2-regime VAR(1), conditional IRFs.

Figure 4. Conditional and average impulses responses of output gap y_t to a cost-push shock $\eta_{\pi,t}$ in the 4-regime learning model over the period 2001Q1-2019Q4.

switching	REE 1 regime -			MS-REE 2 regime (exo.)			VAR(1) 2 regime (endo.)			VAR(1) 4 regime (endo.)		
Param.	Mean	90 % HPD	Interval	Mean	90 % HPD	Interval	Mean	90 % HPD	Interval	Mean	90 % HPD	Interval
$\bar{\pi}$	0.624	0.473	0.768	0.704	0.519	0.882	0.43	0.26	0.601	0.448	0.364	0.537
$\bar{r}$	0.633	0.198	1.023	0.793	0.252	1.312	0.182	0.01	0.389	0.27	0.056	0.489
κ	0.003	0.001	0.005	0.002	0.001	0.004	0.01	0.002	0.019	0.003	0.001	0.006
τ	2.443	1.711	3.248	3.525	2.479	4.606	1.435	1.127	1.789	1.532	1.163	1.934
ϕ_π	1.771	1.311	2.25	1.721	1.276	2.221	1.648	1.149	2.156	1.959	1.512	2.413
ϕ_y	0.146	0.058	0.245	0.161	0.068	0.271	0.203	0.084	0.326	0.116	0.055	0.179
$\phi_{\Delta y}$	0.221	0.167	0.28	0.194	0.147	0.241	0.211	0.151	0.273	0.219	0.168	0.274
ρ_y	0.595	0.371	0.781	0.942	0.913	0.97	0.953	0.921	0.983	0.967	0.95	0.984
ρ_π	0.965	0.94	0.989	0.625	0.44	0.795	0.582	0.298	0.829	0.553	0.354	0.73
ρ_r	0.922	0.893	0.952	0.919	0.884	0.951	0.936	0.903	0.967	0.89	0.854	0.927
η_y	0.052	0.035	0.07	0.033	0.018	0.048	0.095	0.063	0.131	0.089	0.06	0.119
η_π	0.049	0.033	0.068	0.045	0.029	0.06	0.062	0.042	0.087	0.058	0.043	0.073
η_r	0.136	0.112	0.161	0.13	0.111	0.15	0.134	0.111	0.16	0.138	0.115	0.162
ι_y	0.274	0.199	0.348	0.243	0.169	0.317	0.113	0.059	0.174	0.322	0.244	0.401
ι_π	0.293	0.173	0.418	0.273	0.165	0.391	0.213	0.059	0.399	0.344	0.242	0.441
$\bar{r_2}$	-	-	-	0.032	0.029	0.036	0.032	0.028	0.036	0.032	0.029	0.036
η_{r_2}	-	-	-	0.01	0.007	0.013	0.01	0.007	0.013	0.01	0.007	0.013
$\frac{\Phi_1}{1000}$	-	-	-	-	-	-	0.177	0.067	0.312	0.167	0.065	0.309
$\frac{\Phi_2}{1000}$	-	-	-	-	-	-	0.147	0.045	0.271	0.134	0.047	0.246
γ	-	-	-	-	-	-	0.007	0.003	0.01	0.009	0.003	0.016
ω	-	-	-	-	-	-	-	-	-	0.721	0.386	0.993
χ	-	-	-	-	-	-	-	-	-	2.093	0.551	3.896
$1 - p_{22}$	-	-	-	0.207	0.127	0.304	-	-	-	-	-	-
$1 - p_{11}$	-	-	-	0.037	0.008	0.069	-	-	-	-	-	-
MHM		46.9			62.82			63.52			56.81	
Mode		89.07			122.09			126.01			122.99	

Table 2. Posterior distribution moments for the REE (no switching), MS-REE (2-regime exogenous switching), and learning models (2-regime endogenous switching, and 4-regime endogenous switching). The estimation period is from 1982Q1 to 2019Q4 for historical U.S. data. MHM refers to the Modified Harmonic Mean estimator.

4 Counterfactual Experiments

In this section, I consider a set of counterfactual simulations with the 4-regime learning model to assess the impact of de-anchored expectations on economic outcomes. The main focus is on how the fraction of learning agents n_t^L and the constant gain value γ affect output gap, inflation and the duration of ELB regimes. I first discuss a set of counterfactuals over the post-GFC period until the end of 2019. Then I discuss the model's projections over the post-2019 period under the assumption of a pair of adverse demand and cost-push shocks during 2020 to proxy for the pandemic-induced recession.

4.1 In-sample Counterfactuals

I: Estimated shocks, different (constant) fractions of agents

The first exercise is based on the period 2009Q4-2019Q4. The motivation for starting the counterfactual in 2009Q4 is that both inflation and output gap series attain their lowest values in 2009Q3, hence the counterfactuals start after the recession reaches its lowest point in this context. This exercise proceeds as follows: I first take the filtered shocks from the 4-regime learning model at the posterior mean. Then I use these shocks to re-simulate the economy with different fractions of learning agents n_t^L over the counterfactual period. I use a total of 1000 simulations, all using the same set of filtered shocks at the posterior mean, while learning agents' fraction n_t^L is varied over the interval $[0, 0.9]$.

The results of the exercise are shown in Figure 5. I focus on four variables of interest in particular, namely the output gap, inflation, the shadow rate and the probability of the Taylor rule regime.[13] The left-hand side panels show the period-specific variation in the variables as a function of learning agents' fraction n_t^L, where the solid black line denotes the baseline (estimated) scenario with endogenous fractions. The figure reveals a clear impact of the fraction n_t^L on the economic outcomes: as the fraction of agents using adaptive learning increases, output gap, inflation, and the shadow rate decrease on average over the counterfactual period. While the effects are negligible over the earlier periods, higher fractions of de-anchored expectations clearly generate a large downside risk over the post-2015 period. As a result, the probability of returning to the Taylor rule regime decreases as n_t^L increases and approaches its upper bound of 0.9. The overall effects are more clearly seen on the right-hand side panels, which show the averages over the counterfactual period as a function of the fraction n_t^L. A fixed value of n_t^L exceeding roughly 0.45 leads to counterfactual averages lower

[13] The Taylor rule regime probability is obtained by summing up the first two regime probabilities, i.e. Taylor rule learning and Taylor rule REE. Note that the probabilities of these two regimes are fixed, since the fractions of agents using each forecasting rule are fixed via n_t^L.

than the empirical ones. Around $n_t^L \approx 0.7$, the averages start to decrease exponentially, resulting in ever falling values of output gap, inflation and shadow rate. The last panel on the right-hand side shows the average probabilities of the Taylor rule regime across different sub-periods. When all expectations are anchored (i.e. $n_t^L = 0$), the average probability rises up to 50 % over 2009Q2-2019Q4, and remains around 12% over 2009Q2-2014Q4.[14] As we increase the fraction of adaptive learning agents, probabilities across all sub-subsamples decrease towards zero as n_t^L approaches 0.9. Overall, these results suggest that de-anchoring of expectations through a larger fraction of learning agents substantially increase the downside risk on the economy, making deflationary spirals more likely and increasing the probability of staying in the ELB regime.

II: Estimated shocks, different (constant) fractions and gain values

The second counterfactual exercise evaluates the impact of the constant gain parameter γ together with the fractions n_t^L, using the same setup as the previous exercise. Accordingly, I use the estimated (filtered) shocks at the posterior mean, with fixed constant gain γ and fraction values n_t^L over the intervals $[0, 0.01]$ and $[0, 0.9]$ respectively, with a grid of 100 points each.[15] For this exercise I only provide the averages over the sample period as a function of the gain γ and the fraction n_t^L. The results are shown for output gap, inflation and the shadow rate in Figure 6. The conclusions obtained in the first exercise continue to hold for sufficiently large values of the gain parameter. While the downside risk of a large fraction of de-anchored expectations is clearly visible for gain values exceeding 0.006, the sample averages become less sensitive to the fraction n_t^L as the gain value approaches 0. This result follows from the fact that the gain value controls the speed of learning. As the gain decreases, adaptive learning converges to a fixed belief, anchored expectations rule, albeit a different one than the underlying REE. At the other extreme with increasing values of the gain, the downside risk associated with larger n_t^L is compounded by larger values of the gain parameter, leading to the lowest averages in the region with high gain values and high learning fractions.

The two counterfactuals considered so far can be interpreted as an in-sample evaluation of agents' fractions and the speed of learning, where larger fractions and faster rates of learning both make it more likely to generate deflationary spirals and prolonged recessions. In what follows, I assess the model-implied projections over the post-2019 period.

[14] These results also imply that recovery earlier than 2014Q4 remains unlikely, since the average probability never exceeds 12% even with all expectations anchored.

[15] Note that the region for constant gain γ also includes the posterior mean value of 0.009.

4.2 Out-of-sample Projections

In this section I consider three more counterfactual exercises over the post-2019 period, which serve two purposes. First, they provide an assessment of the model's unconditional forecasts about the likelihood of the next ELB event in the absence of any other information. Second, with the introduction of a series of adverse shocks over 2020 to replicate the pandemic-induced recession, they show the model's assessment of how long the recession is likely to last and how different fractions of de-anchored expectations affect the probability of recovery and the return to the Taylor rule regime.

III: post-2019Q4, randomized shocks

The third exercise proceeds as follows: all parameters and variables are fixed at their posterior mean values up to 2019Q4, after which I simulate the economy 1000 times with randomized shocks for 20 quarters until 2024Q4. The simulation averages, along with the 10th and 90th percentiles of the Monte Carlo distributions for output gap, inflation, shadow rate and learning agents' fraction n_t^L are reported in Figures 7 and 8. In discussing the results, I divide the simulations into two categories: those that result in an average Taylor rule probability above 50% over the projection period 2020Q1-2024Q4, and those that result in an average Taylor rule probability below 50%. The simulations that end up in the first category turn out to be 68% of all simulations, while simulations with a Taylor rule probability below 50% make up the remaining 32%. In other words, in the absence of any other external effects, the model attaches a 32% probability to encountering another ELB episode over at least half of the projection period 2020Q1-2024Q4. Looking at the first category of simulations, it is readily seen that output gap remains stable around its pre-2020 levels while inflation picks up, and the nominal interest rate along with the shadow rate increases on average. Among these simulations, the nominal rate rarely falls into near-zero levels and therefore the Taylor rule probability stays stays close to 1 over the entire projection period.

Looking at the second category of simulations, output gap and inflation show a clear decreasing pattern on average, while nominal rates remain stuck at the ELB and the shadow rate falls into the negative territory. The average Taylor rule probability for these simulations quickly approaches zero during 2021, before slowly starting to rise after 2023. Taking both categories of simulations together, these results imply that the model has a bimodal prediction for the post-2019 period: the first case, with a probability of 68%, suggests the economy remains in the Taylor rule regime and the probability of hitting the ELB again before 2025 remains close to 0. In the second case, which has a probability of 32%, the economy hits the ELB again and enters into another recessionary period. Interestingly, the fractions of agents in the first set of simulations remain balanced around 50%, similar to the estimated fractions before the pre-GFC period. The second set of simulations is instead accompanied by an upward trend in the fraction of agents using adaptive learning. This result confirms the observation from the previous two counterfactuals that a larger fraction of de-anchored expectations are associated with worsening economic conditions.

IV: Post-2019Q4, "pandemic" shocks

The next exercise I consider relates to the pandemic-induced recession of 2020. I take the first set of simulations from the previous exercise where the Taylor-rule regime probability remains above 50%. Then I introduce a pair of large adverse demand and cost-push shocks over 2020Q1-2020Q4 into these simulations to proxy for the downfall associated with the recession. Specifically, this sequence of adverse shocks assume that the downturn already starts in the first quarter, that the main pandemic shock hits the economy in the second quarter and that the negative effects persist in the last two quarters of the year.[16] The aim of this exercise is to qualitatively assess how and when the economy recovers from the recession, rather than matching the exact magnitude of downfall in output gap and inflation.

The resulting time series for average shocks are shown in the first two panels of Figure 9. It is readily seen that the resulting downfall in the demand shock is comparable to the GFC period in magnitude, while the negative cost-push shock was absent during the GFC period. Given that the demand shock is a near unit root process with a persistence of 0.96 at the posterior mean, this sequence of adverse shocks resembles a permanent shock on the economy, while the cost-push shock recovers back to its baseline levels since its persistence is only 0.55. Under the assumption that the sequence of adverse shocks remain contained within the year of 2020, the model predicts a V-shaped pattern for inflation, which recovers to positive values by mid-2021. Output gap does not start recovering from the shock throughout the projection period, which again relates to the fact that the demand shock is a near unit root process.[17] Under this scenario, the nominal rate remains stuck at near-zero levels until the end of 2021, after which it slowly starts to rise again, which is when the shadow rate also catches up with the nominal rate.[18] Immediately following the large shock in 2020Q2, the fraction of agents using the de-anchored learning rule falls down to near-zero levels, which is qualitatively the same pattern as the estimated fractions during the GFC period. After this point, the fraction rises back up to nearly 70% and remains elevated until the end of the

[16] Namely, the shocks are of the the following magnitude: -1 st. dev. in 2020Q1, -6 st. dev. in 2020Q2, and -2 st. dev. in 2020Q3 and 2020Q4.

[17] In order to reduce the large impact of the demand shock on output gap, an alternative approach is to repeat the exercise with a less persistent demand shock. However, this has a small impact on the overall predictions of the model since monetary policy reaction output gap growth is larger than the reaction to output gap level. As such, persistently low levels of output gap does not prevent the interest rates from starting to rise during the projection period.

[18] Note that, even though the fall in both output gap and inflation is larger in this exercise compared to the GFC period, the fall in the shadow rate remains smaller compared to that period. This is due to the assumption that the period of 2008-2015 is accompanied by an intercept shift in the policy rate, whereas I do not make this assumption for the 2020-2024 period.

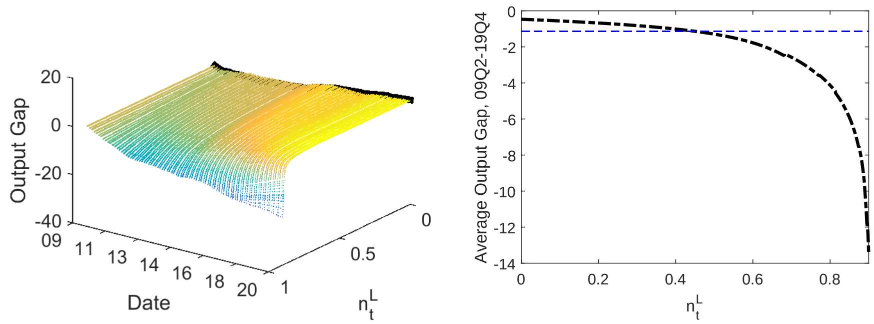
projection period. The Taylor rule probability remains close to zero throughout 2021 and slowly starts to rise again afterwards, but the probability of having escaped the ELB regime by 2024Q4 remains relatively low at around 54.1% under this benchmark scenario.

V: post-2019Q4, different fractions of expectations

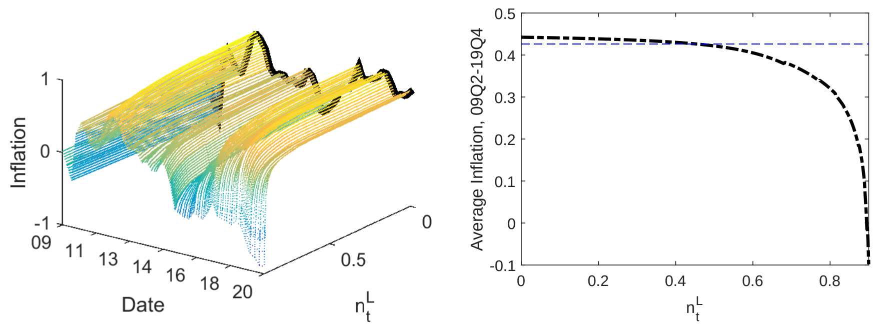
As a final exercise, Table 3 shows what happens under different fixed fractions over the 2020Q1-2024Q4 period, for which I use two end points of 10 % and 90 % de-anchored expectations. When only 10% of agents use adaptive learning, the probability of escaping the ELB episode by 2024Q4 decreases from 54.1% to 52%, whereas if 90% of agents use adaptive learning, the probability decreases to 41.5%. Accordingly, having an excessively large fraction of agents using adaptive learning decreases the probability of escaping the ELB episode by nearly 13 percentage points over this period.

	Degree of de-anchoring		
Date	Endogenous	fixed 10 %	fixed 90 %
20Q1	98.3%	98.3%	98.3%
20Q2	0.06%	0.06%	11.8%
21Q4	3.99%	04.3%	0.6%
22Q4	32.8%	33.7%	21.05%
23Q4	45.3%	44.6%	33.3%
24Q4	54.1%	52%	41.5

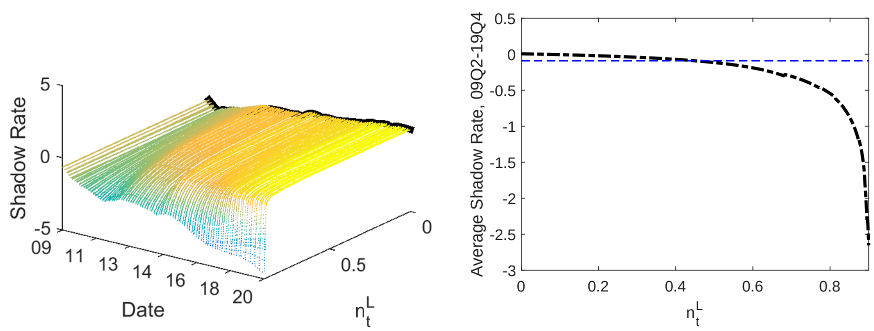
Table 3. Probabilities of returning back to the Taylor rule regime over different periods. Results from three exercises are reported: the benchmark scenario with endogenously determined fractions, the scenario where only 10% of agents use adaptive learning and the scenario where 90% of agents use adaptive learning.



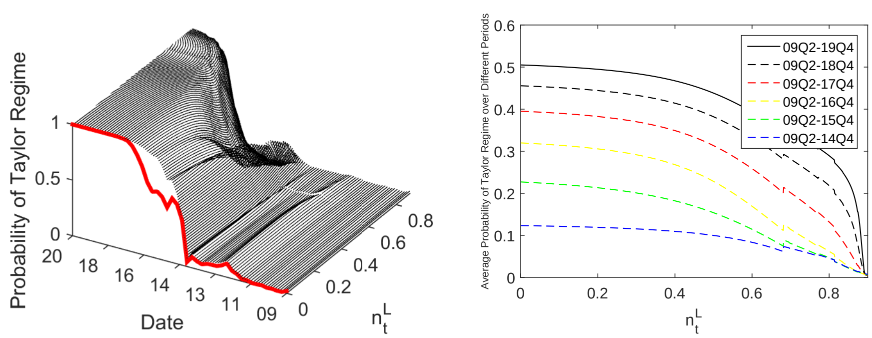
(a) Output gap



(b) Inflation

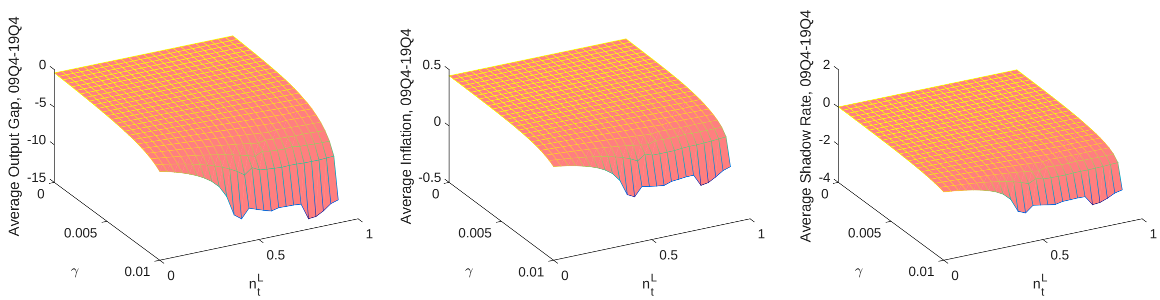


(c) Shadow Rate



(d) Taylor Rule Probability

Figure 5. Period-specific time paths (left) and averages (right) for key variables over the counterfactual simulation period from 2009Q4 to 2019Q4. The counterfactuals are based on the filtered shocks at the posterior mean, while the fraction of adaptive learning agents n_t^L is varied over $[0, 0.9]$ with a grid of 1000 points. In the left-hand side panels, the solid black line denotes the baseline (estimated) scenario with endogenous fractions. In the right-hand side panels, the black line denotes the counterfactual averages, while the blue line is the empirical average under the baseline scenario.



(a) Output Gap **(b)** Inflation **(c)** Shadow Rate

Figure 6. Averages for key variables over the counterfactual simulation period from 2009Q4 to 2019Q4. The counterfactuals are based on the filtered shocks at the posterior mean, while the fraction of adaptive learning agents n_t^L and the constant gain parameter γ in the adaptive learning rule are varied over intervals $[0, 0.9]$ and $[0, 0.01]$ respectively, with a grid of $20x20$ points.

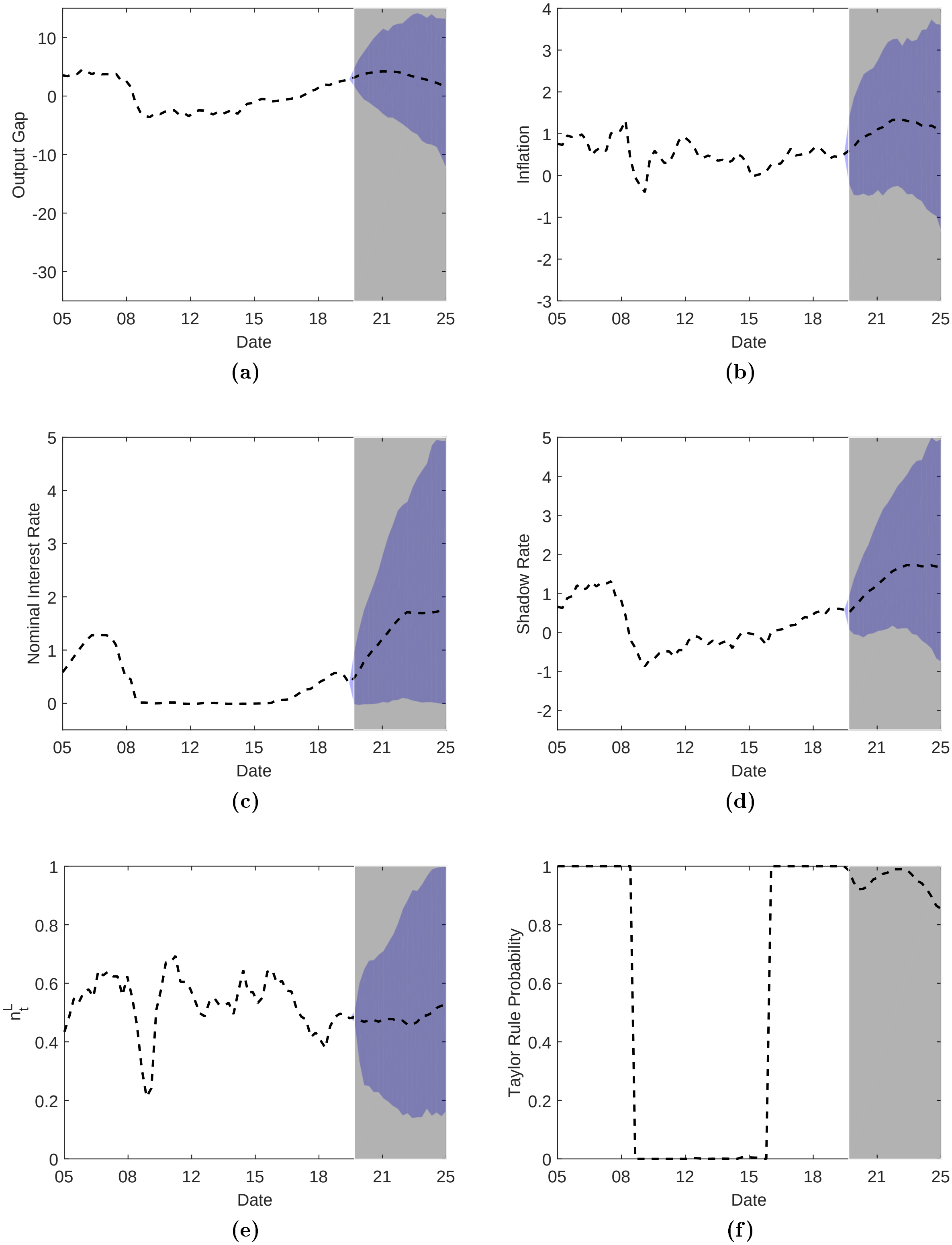


Figure 7. Counterfactual simulations with randomized shocks after 2019Q4 over a period of 20 quarters until 2024Q4. The panels show the Monte Carlo moments for key variables among simulations that yield an average Taylor rule probability of 50% or above, which make up 68% of all simulations. The dotted lines denote the simulation averages, whereas the blue regions correspond to 10% and 90% quantiles of the Monte Carlo distributions.

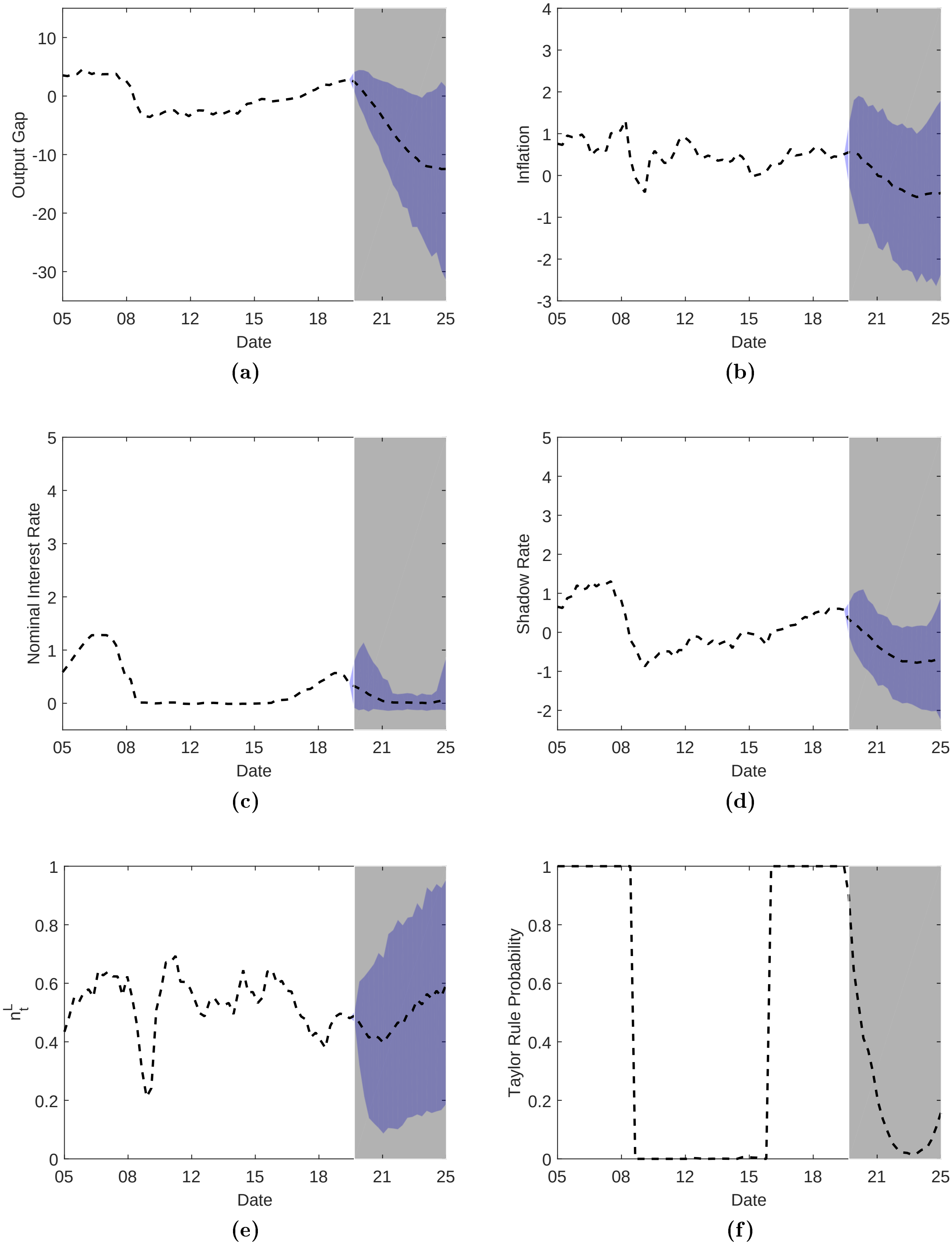
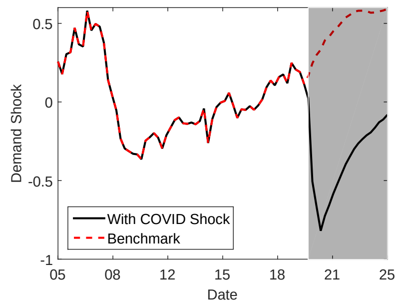
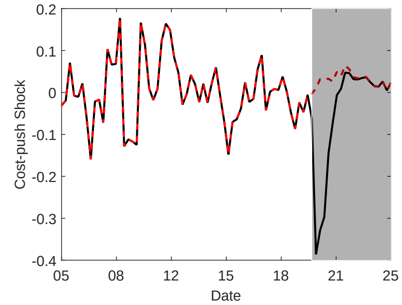


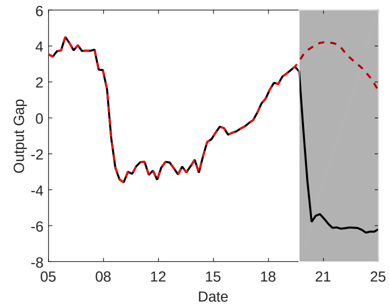
Figure 8. Counterfactual simulations with randomized shocks after 2019Q4 over a period of 20 quarters until 2024Q4. The panels show the Monte Carlo moments for key variables among simulations that yield an average Taylor rule probability below 50%, which makes up 32% of all simulations. The dotted lines denote the simulation averages, whereas the blue regions correspond to 10% and 90% quantiles of the Monte Carlo distributions.



(a)



(b)



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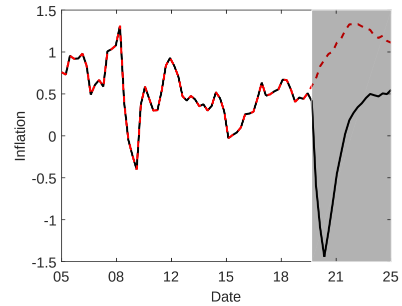
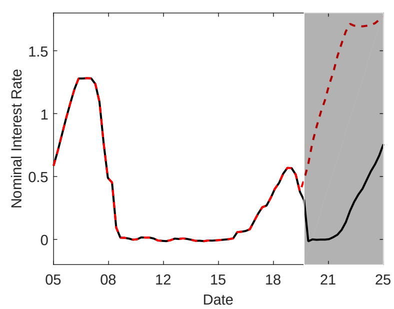
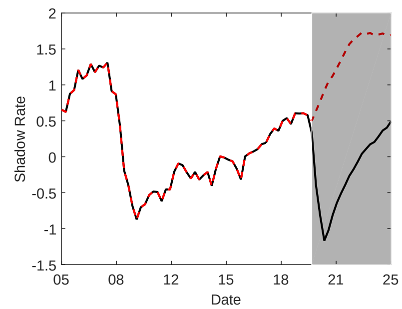


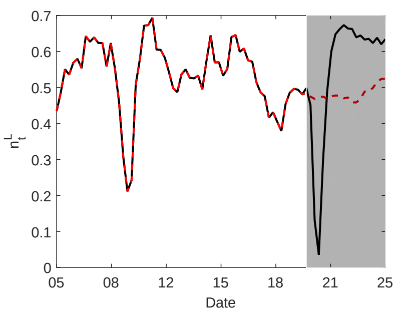
(d)



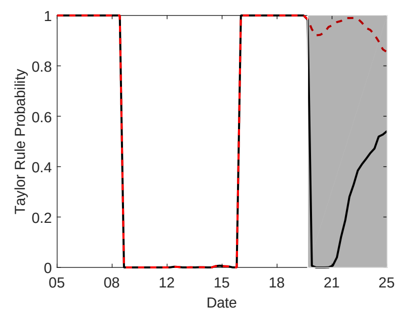
(e)



(f)



(g)



(h)

Figure 9. Counterfactual simulations with pandemic-shocks after 2019Q4 over a period of 20 quarters until 2024Q4. This exercise uses the simulations from the previous set of counterfactual simulations, where the average Taylor rule probability remains above 50% (i.e. 680 simulations in total). Using only these simulations ensures that the ELB regime would remain unlikely in the absence of other adverse shocks. The simulations are then re-run with the introduction of pandemic-shocks throughout 2020. The dotted red lines correspond to simulation averages without the pandemic-shocks, while the solid black lines are the averages with pandemic-shocks.

5 Conclusions

This paper proposes a heterogeneous expectations model based on the canonical New Keynesian model, with a monetary policy subject to the ELB constraint on the nominal interest rates. Both mechanisms are captured in a unified framework of endogenous regime switching. Several important lessons stand out. First and foremost, the results in the paper suggest that private sector expectations for the U.S. economy over the period 1982-2019 can be described as a mixture of anchored, pseudo-rational expectations and de-anchored expectations based on adaptive learning. This suggests that not accounting for the expectational heterogeneity in policy design, in particular unconventional monetary policy tools such as forward guidance and quantitative easing, may have unintended consequences. Second, the model shows that during the recent experience with ELB after the GFC, expectations remained mostly anchored, which can be interpreted as a successful central bank communication over this period. Third, counterfactual experiments show that more de-anchoring, either through a higher fraction of adaptive learning agents or a faster rate of learning by these agents, puts a downward pressure on economic variables by increasing the likelihood of deflationary spirals and prolonged recessions.

The paper also opens potential avenues of future research. The current framework only incorporates unconventional monetary policy through its expectational channel, and future studies should also model the direct effects of unconventional tools and in particular quantitative easing measures. Furthermore, in order to obtain more concrete policy recommendations, more research in heterogeneous expectations is needed in more realistic, medium-scale DSGE models on par with those used at central banks.

References

Airaudo, M. & Hajdini, I. (2019). Regime switching consistent expectations equilibria: General results with application to monetary policy.

An, S. & Schorfheide, F. (2007). Bayesian analysis of dsge models. *Econometric reviews*, 26(2-4), 113–172.

Andrade, P., Gaballo, G., Mengus, E., & Mojon, B. (2019). Forward guidance and heterogeneous beliefs. *American Economic Journal: Macroeconomics*, 11(3), 1–29.

Anufriev, M., Assenza, T., Hommes, C., & Massaro, D. (2013). Interest rate rules and macroeconomic stability under heterogeneous expectations. *Macroeconomic Dynamics*, 17(8), 1574–1604.

Arifovic, J., Grimaud, A., Salle, I., & Vermandel, G. (2020). *Social Learning and Monetary Policy at the Effective Lower Bound.* Technical report, Bank of Canada.

Barthélemy, J. & Marx, M. (2017). Solving endogenous regime switching models. *Journal of Economic Dynamics and Control*, 77, 1–25.

Benigno, G., Foerster, A., Otrok, C., & Rebucci, A. (2020). *Estimating Macroeconomic Models of Financial Crises: An Endogenous Regime-Switching Approach*. Technical report, National Bureau of Economic Research.

Bernanke, B. S. (2017). Monetary policy in a new era.

Bianchi, F. (2016). Methods for measuring expectations and uncertainty in markov-switching models. *Journal of Econometrics*, 190(1), 79–99.

Bianchi, F. & Ilut, C. (2017). Monetary/fiscal policy mix and agents' beliefs. *Review of economic Dynamics*, 26, 113–139.

Binning, A. & Maih, J. (2016). Implementing the zero lower bound in an estimated regime-switching dsge model.

Branch, W., Davig, T., & McGough, B. (2007). Expectational stability in regime-switching rational expectations models. *Available at SSRN 1073150*.

Branch, W. A. (2004). The theory of rationally heterogeneous expectations: evidence from survey data on inflation expectations. *The Economic Journal*, 114(497), 592–621.

Branch, W. A. (2007). Sticky information and model uncertainty in survey data on inflation expectations. *Journal of Economic Dynamics and Control*, 31(1), 245–276.

Branch, W. A. & McGough, B. (2009). A new keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 33(5), 1036–1051.

Brock, W. A. & Hommes, C. H. (1997). A rational route to randomness. *Econometrica: Journal of the Econometric Society*, (pp. 1059–1095).

Bullard, J. & Mitra, K. (2002). Learning about monetary policy rules. *Journal of monetary economics*, 49(6), 1105–1129.

Busetti, F., Delle Monache, D., Gerali, A., & Locarno, A. (2017). *Trust, but verify. De-anchoring of inflation expectations under learning and heterogeneity*. Technical report, ECB working paper.

Chang, Y., Maih, J., & Fei, T. (2018). State space models with endogenous regime switching.

Chen, H. (2017). The effects of the near-zero interest rate policy in a regime-switching dynamic stochastic general equilibrium model. *Journal of Monetary Economics*, 90, 176–192.

Cho, S. (2016). Sufficient conditions for determinacy in a class of markov-switching rational expectations models. *Review of Economic Dynamics*, 21, 182–200.

Chung, H. & Xiao, W. (2013). *Cognitive consistency, signal extraction, and macroeconomic persistence*. Technical report, mimeo, SUNY Binghamton.

Clarida, R., Gali, J., & Gertler, M. (1999). The science of monetary policy: a new keynesian perspective. *Journal of economic literature*, 37(4), 1661–1707.

Cole, S. J. (2018). The effectiveness of central bank forward guidance under inflation and price-level targeting. *Journal of Macroeconomics*, 55, 146–161.

Cole, S. J. & Martínez-García, E. (2020). (wp 2020-03) the effect of central bank credibility on forward guidance in an estimated new keynesian model.

Cornea-Madeira, A., Hommes, C., & Massaro, D. (2019). Behavioral heterogeneity in us inflation dynamics. *Journal of Business & Economic Statistics*, 37(2), 288–300.

Davig, T. & Leeper, E. M. (2007). Generalizing the taylor principle. *American Economic Review*, 97(3), 607–635.

Del Negro, M., Giannoni, M. P., & Patterson, C. (2012). The forward guidance puzzle. *FRB of New York Staff Report*, (574).

Di Bartolomeo, G., Di Pietro, M., & Giannini, B. (2016). Optimal monetary policy in a new keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 73, 373–387.

Eusepi, S. & Preston, B. (2010). Central bank communication and expectations stabilization. *American Economic Journal: Macroeconomics*, 2(3), 235–71.

Evans, G. W., Guse, E., & Honkapohja, S. (2008). Liquidity traps, learning and stagnation. *European Economic Review*, 52(8), 1438–1463.

Evans, G. W. & Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*. Princeton University Press.

Farmer, R. E., Waggoner, D. F., & Zha, T. (2009). Understanding markov-switching rational expectations models. *Journal of Economic theory*, 144(5), 1849–1867.

Farmer, R. E., Waggoner, D. F., & Zha, T. (2011). Minimal state variable solutions to markov-switching rational expectations models. *Journal of Economic Dynamics and Control*, 35(12), 2150–2166.

Geweke, J., Bernardo, J. M., Berger, J. O., Dawid, A. P., & Smith, A. (1992). Bayesian statistics. *Bayesian statistics*.

Goy, G., Hommes, C., & Mavromatis, K. (2020). Forward guidance and the role of central bank credibility under heterogeneous beliefs. *Journal of Economic Behavior & Organization*.

Gust, C., Herbst, E., & Lopez-Salido, J. D. (2018). Forward guidance with bayesian learning and estimation.

Haberis, A., Harrison, R., & Waldron, M. (2014). Transitory interest-rate pegs under imperfect credibility.

Hommes, C. & Lustenhouwer, J. (2019). Inflation targeting and liquidity traps under endogenous credibility. *Journal of Monetary Economics*, 107, 48–62.

Ji, Y. & Xiao, W. (2016). Government spending multipliers and the zero lower bound. *Journal of Macroeconomics*, 48, 87–100.

Kim, C.-J. & Nelson, C. R. (1999). Has the us economy become more stable? a bayesian approach based on a markov-switching model of the business cycle. *Review of Economics and Statistics*, 81(4), 608–616.

Kulish, M., Morley, J., Robinson, T., et al. (2014). Estimating dsge models with forward guidance. *Discussion Papers*.

Kulish, M. & Pagan, A. (2017). Estimation and solution of models with expectations and structural changes. *Journal of Applied Econometrics*, 32(2), 255–274.

Lansing, K. J. (2019). Endogenous forecast switching near the zero lower bound. *Journal of Monetary Economics*.

Lindé, J., Maih, J., & Wouters, R. (2017). *Estimation of operational macromodels at the zero lower bound*. Technical report, Mimeo.

Liu, P. & Mumtaz, H. (2011). Evolving macroeconomic dynamics in a small open economy: An estimated markov switching dsge model for the uk. *Journal of Money, Credit and Banking*, 43(7), 1443–1474.

Maih, J. (2015). Efficient perturbation methods for solving regime-switching dsge models.

Milani, F. (2007). Expectations, learning and macroeconomic persistence. *Journal of monetary Economics*, 54(7), 2065–2082.

Milani, F. (2011). Expectation shocks and learning as drivers of the business cycle. *The Economic Journal*, 121(552), 379–401.

Ozden, T. & Wouters, R. (2020). Restricted perceptions, regime switches and the zero lower bound.

Sims, C. et al. (1999). Matlab optimization software. *QM&RBC Codes*.

Sims, C. A. & Zha, T. (2006). Were there regime switches in us monetary policy? *American Economic Review*, 96(1), 54–81.

Slobodyan, S. & Wouters, R. (2012). Learning in an estimated medium-scale dsge model. *Journal of Economic Dynamics and control*, 36(1), 26–46.

Smets, F. & Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, 97(3), 586–606.

Uhlig, H. et al. (1995). A toolkit for analyzing nonlinear dynamic stochastic models easily.

Woodford, M. (2011). *Interest and prices: Foundations of a theory of monetary policy*. Princeton University Press.

Appendix

A Data Descriptions

This section describes the quarterly time series used in the estimations. The dataset spans from 1982Q1 to 2019Q4 and the time series are available at:

- Real Gross Domestic Product (FRED mnemonic: GDPC1), denoted as GDP_t:
  https://fred.stlouisfed.org/series/GDPC1.
- Consumer Price Index for All Urban Consumers (FRED mnemonic: CPIAUCSL), denoted as P_t:
  https://fred.stlouisfed.org/series/CPIAUCSL.
- Effective Federal Funds Rate (FRED mnemonic: FEDFUNDS), denoted as R_t:
  https://fred.stlouisfed.org/series/FEDFUNDS.

Given GDP_t, P_t and R_t, the estimations are based on the following variables:

- Output Gap y_t is based on a second-order de-trending of $log(GDP_t)$ over the estimation sample.
- Inflation $\pi_t = \frac{P_t}{P_{t-1}}$.
- Nominal interest rate $r_t = R_t$.

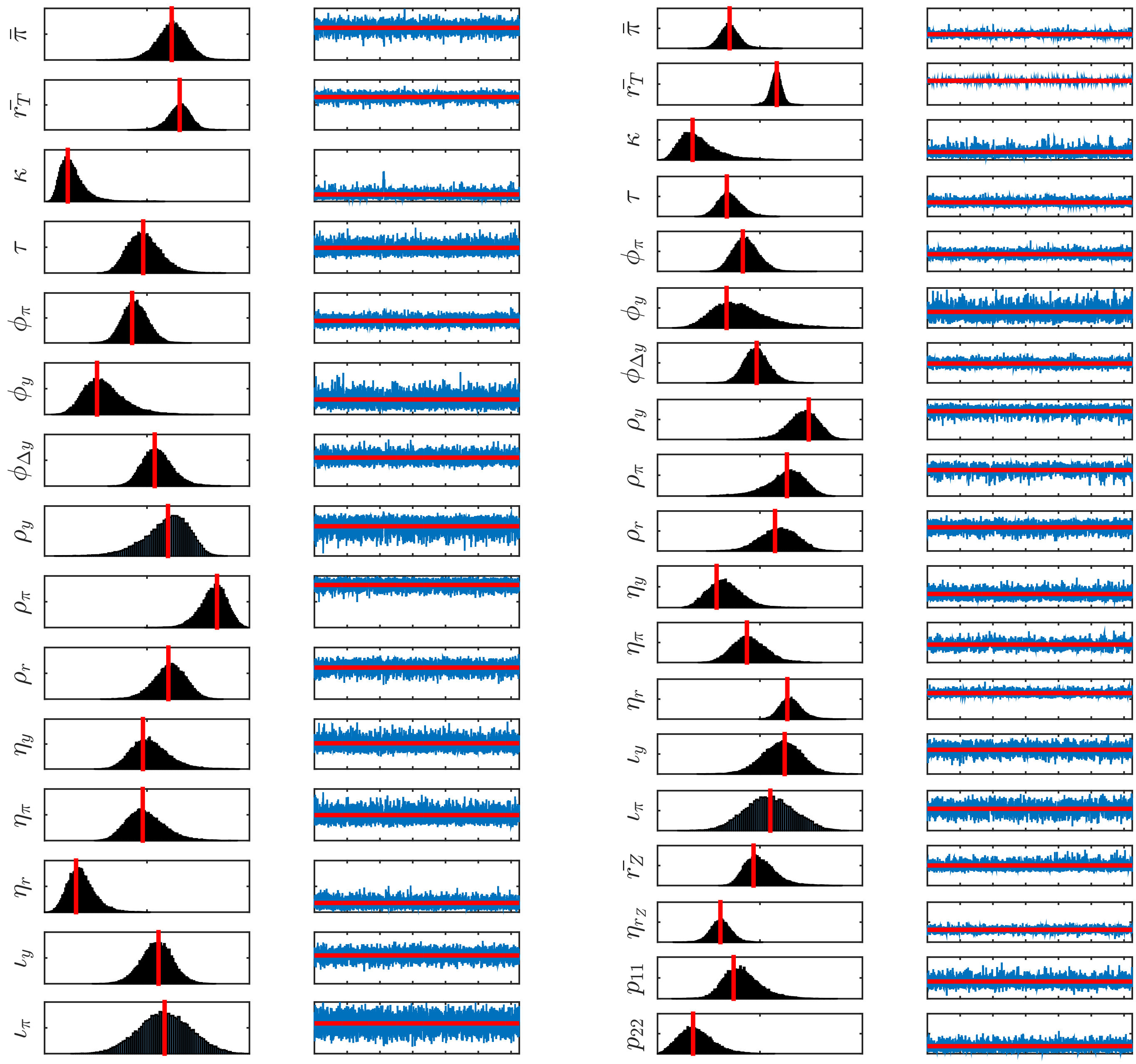
B Posterior Distributions, Diagnostic Checks

This section presents the posterior parameter distributions, trace plots and the results of Geweke's diagnostic tests (1992) for the convergence of MCMCs. Figures 10, 11 and 12 show the posterior distributions and trace plots of the parameter estimates for all models, namely the REE, MS-REE, 2- and 4-regime VAR(1) learning models that are presented in Section 3, as well as the 2- and 4-regime AR(1) learning models presented in Appendices C and D as a robustness check.

Table 4 shows the results of Geweke's convergence diagnostic tests (1992). The table reports the p-values of the tests with different tapering steps of 4%, 8% and 15% respectively, which accounts for potential autocorrelation in the Markov chains. Each model is estimated using a single chain of 250000 draws initialized at the posterior mode, of which the first 100000 or 40% of the draws are discarded as burn-in sample. For the remaining 150000 draws, the test compares the means of the first 20% and the last 50% of the posterior draws. The resulting p-values for all 6 models are well above the 5% significance level with all three tapering steps, indicating a failure to reject the null hypothesis that the distributions are different at the beginning and end of the chain. This suggests that all chains have converged.

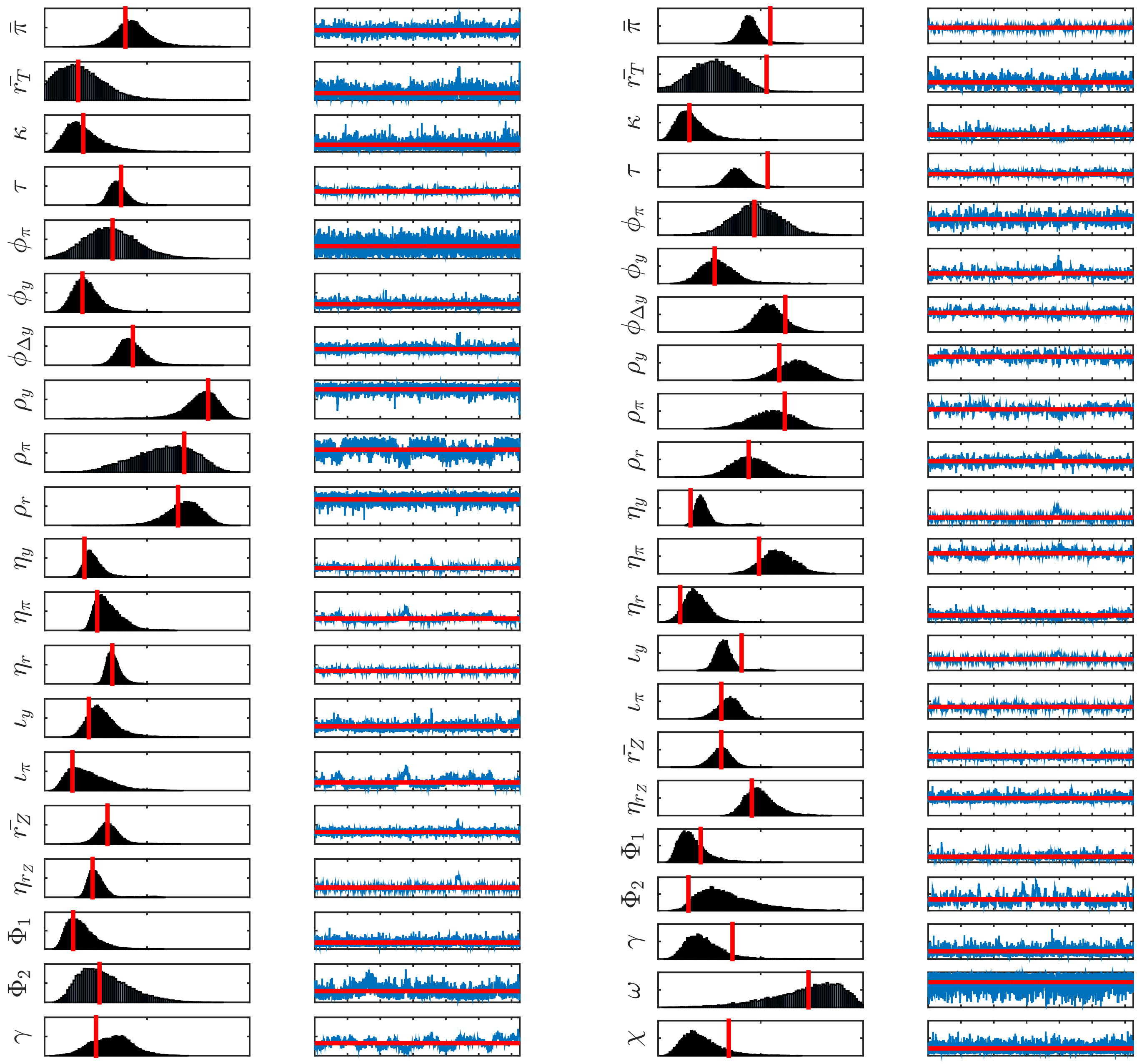
Tapering Step	Test Result REE 1-regime	RISE 2-regime	VAR(1) 2-regime	VAR(1) 4-regime	AR(1) 2-regime	AR(1) 4-regime
4%	39%	79%	18%	52%	88%	50%
8%	35%	75%	23%	57%	89%	50%
15%	32%	71%	26%	56%	89%	47%

Table 4. Geweke's Convergence Diagnostics (1992). The table reports the p-values of the tests with different tapering steps (4%, 8% and 15% respectively). The test compares the means of the first 20% and last 50% of the posterior draws after discarding the burn-in sample.



(a) REE **(b)** RISE

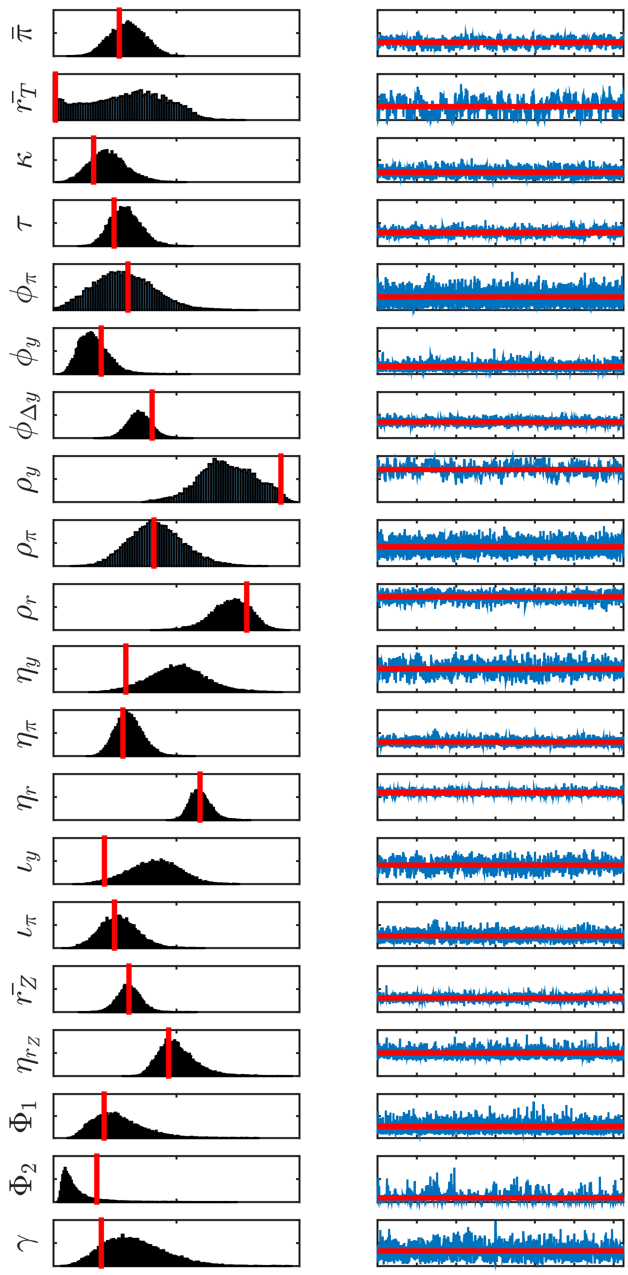
Figure 10. Rational Expectations models. For both models, the left-hand panel shows the posterior distributions, while the right-hand panel shows the trace plots.



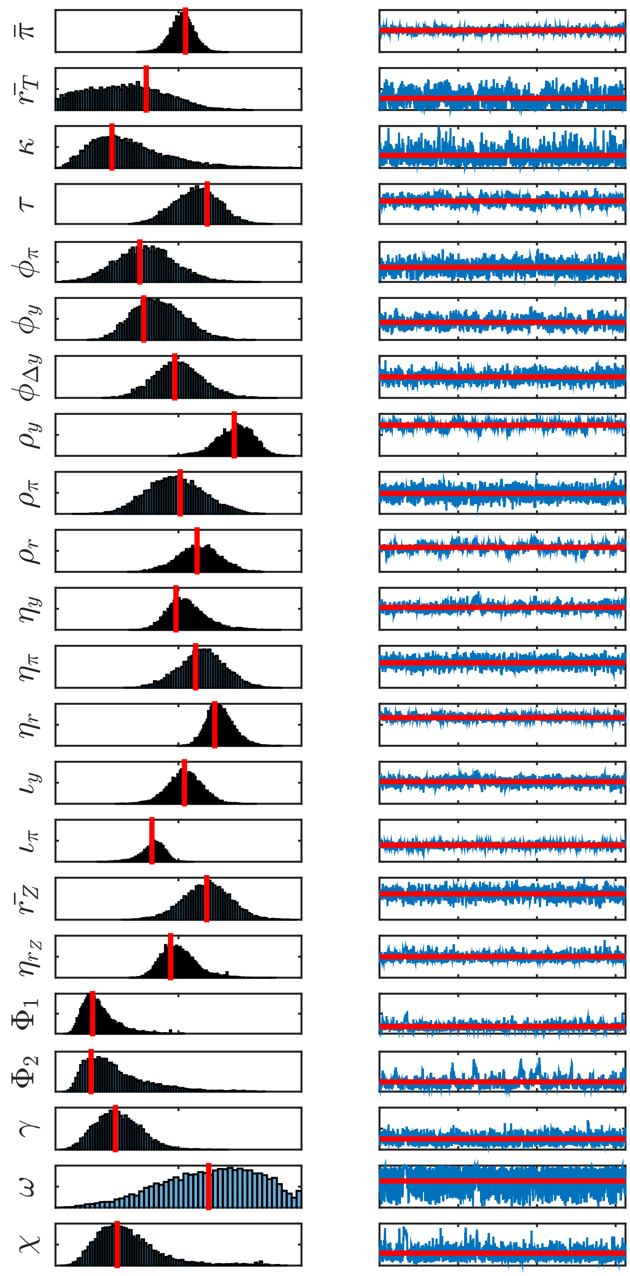
(a) 2-regime

(b) 4-regime

Figure 11. VAR(1) learning models. For both models, the left-hand panel shows the posterior distributions, while the right-hand panel shows the trace plots.



(a) 2-regime



(b) 4-regime

Figure 12. AR(1) learning models. For both models, the left-hand panel shows the posterior distributions, while the right-hand panel shows the trace plots.

C Estimation Results with AR(1) learning rule

This section presents the alternative estimation results, where I replace the VAR(1) forecasting rule in adaptive learning agents' PLM with a more parsimonious AR(1) rule. The parameter estimates under the 2- and 4-regime learning models are generally consistent with those presented in Section 3. A notable difference arises in the estimated gain parameter, which has a larger posterior mean for both 2- and 4-regime AR(1) learning models, compared to their VAR(1) counterparts. Aside from this, the memory parameter in expectational switching, ω, has a slightly lower mean but the HPD band is fairly wide similar to the results in Section 3, suggesting that the data is not informative about this parameter. Similarly, the intensity of choice parameter χ has a lower mean compared to the VAR(1) learning model, but the posterior intervals for the parameter in both models are well within the range of each other. A notable difference in this section, compared to the results in Section 3, is the deterioration in the model fit for both 2- and 4-regime learning models. In this case, the learning models yield a substantially worse likelihood compared to the MS-REE model, suggesting that the AR(1) learning rule does not fit the data as well as the VAR(1) rule in this context. Nevertheless, both learning models still provide a better fit compared to the baseline REE model without regime switching.

The estimated regime probabilities, agents' fractions, time-variation in PLM parameters and the shadow rate, as well as the counterfactual simulation results are consistent for the AR(1) learning model with the results presented in Sections 3 and 4. Therefore the full set of figures and tables are omitted here for brevity, and I only discuss some of the key results. Figure 13 shows the filtered PLM parameters in α_t and β_t for output gap and inflation. Given the univariate learning rule, there are 2 learning parameters for each variable. The intercept parameters in α_t are characterized by large downward shifts during the GFC period, while the parameter in β_t are characterized by "jumps" during the same period. These results are consistent with those discussed in Section 3.

Table 6 presents the results of the last simulation exercise in Section 4 with the AR(1) rule, which shows the probabilities of returning back to the Taylor rule regime over different periods until 2024Q4. It is readily seen that the results are qualitatively similar to the VAR(1) learning model, where a higher fraction of adaptive learning agents is associated with a lower probability of leaving the ELB regime. In this case, the benchmark scenario yields a probability of 58.2% for returning to the Taylor regime by the end of 2024Q4, compared to 54.1% under the VAR(1) learning model. When 90% of agents use adaptive learning, this probability decreases to 50.1%, compared to the 41.5% under the VAR(1) learning model. As such, the impact of agents' fractions is smaller when the AR(1) learning rule is used, but both models predict a downward shift in the probability of leaving the ELB with a higher fraction of learning agents.

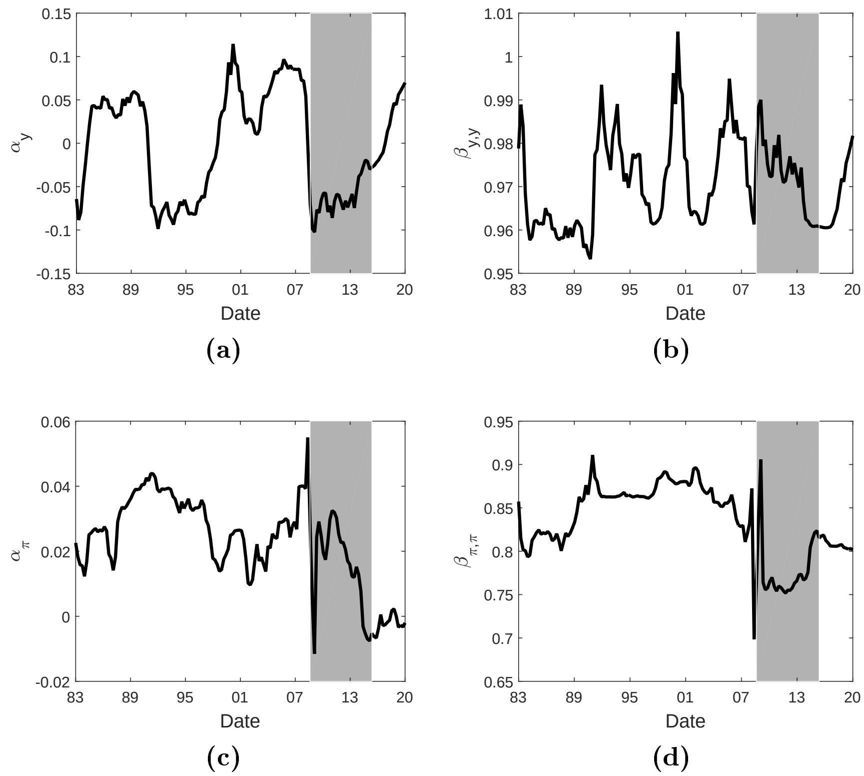


Figure 13. 4-regime model with AR(1) learning, α_t and β_t parameters in agents' PLM.

switching	REE 1 regime -			MS-REE 2 regime (exo.)			AR(1) 2 regime (endo.)			AR(1) 4 regime (endo.)		
Param.	Mean	90 % HPD	Interval	Mean	90 % HPD	Interval	Mean	90 % HPD	Interval	Mean	90 % HPD	Interval
$\bar{\pi}$	0.624	0.473	0.768	0.704	0.519	0.882	0.599	0.329	0.857	0.516	0.428	0.609
$\bar{r}$	0.633	0.198	1.023	0.793	0.252	1.312	0.588	0.001	1.077	0.28	0.005	0.543
κ	0.003	0.001	0.005	0.002	0.001	0.004	0.009	0.003	0.015	0.001	0.0002	0.003
τ	2.443	1.711	3.248	3.525	2.479	4.606	2.924	1.747	4.056	2.147	1.809	2.495
ϕ_π	1.771	1.311	2.25	1.721	1.276	2.221	1.572	1.106	2.032	1.732	1.296	2.2
ϕ_y	0.146	0.058	0.245	0.161	0.068	0.271	0.164	0.056	0.275	0.209	0.117	0.308
$\phi_{\Delta y}$	0.221	0.167	0.28	0.194	0.147	0.241	0.14	0.103	0.177	0.2	0.162	0.237
ρ_y	0.595	0.371	0.781	0.942	0.913	0.97	0.708	0.507	0.926	0.946	0.918	0.973
ρ_π	0.965	0.94	0.989	0.625	0.44	0.795	0.419	0.218	0.641	0.487	0.277	0.717
ρ_r	0.922	0.893	0.952	0.919	0.884	0.951	0.944	0.913	0.973	0.916	0.884	0.949
η_y	0.052	0.035	0.07	0.033	0.018	0.048	0.252	0.141	0.365	0.107	0.079	0.138
η_π	0.049	0.033	0.068	0.045	0.029	0.06	0.061	0.041	0.082	0.06	0.04	0.077
η_r	0.136	0.112	0.161	0.13	0.111	0.15	0.119	0.104	0.136	0.134	0.113	0.154
ι_y	0.274	0.199	0.348	0.243	0.169	0.317	0.41	0.205	0.599	0.41	0.358	0.463
ι_π	0.293	0.173	0.418	0.273	0.165	0.391	0.266	0.119	0.41	0.396	0.3	0.483
$\bar{r_2}$	-	-	-	0.032	0.029	0.036	0.032	0.029	0.036	0.032	0.029	0.036
η_{r_2}	-	-	-	0.01	0.007	0.013	0.01	0.007	0.013	0.01	0.007	0.013
$\frac{\Phi_1}{1000}$	-	-	-	-	-	-	0.263	0.09	0.458	0.176	0.074	0.316
$\frac{\Phi_2}{1000}$	-	-	-	-	-	-	0.093	0.02	0.237	0.122	0.032	0.255
γ	-	-	-	-	-	-	0.013	0.004	0.023	0.025	0.008	0.042
ω	-	-	-	-	-	-	-	-	-	0.633	0.285	0.998
χ	-	-	-	-	-	-	-	-	-	1.476	0.378	2.821
$1-p_{22}$	-	-	-	0.207	0.127	0.304	-	-	-	-	-	-
$1-p_{11}$	-	-	-	0.037	0.008	0.069	-	-	-	-	-	-
MHM		46.9			62.82				53.15			50.15
Mode		89.07			122.09				107.42			110.53

Table 5. Posterior distribution moments for the REE (no switching), MS-REE (2-regime exogenous switching), and learning models (2-regime endogenous switching, and 4-regime endogenous switching). The estimation period is from 1982Q1 to 2019Q4 for historical U.S. data. MHM refers to the Modified Harmonic Mean estimator.

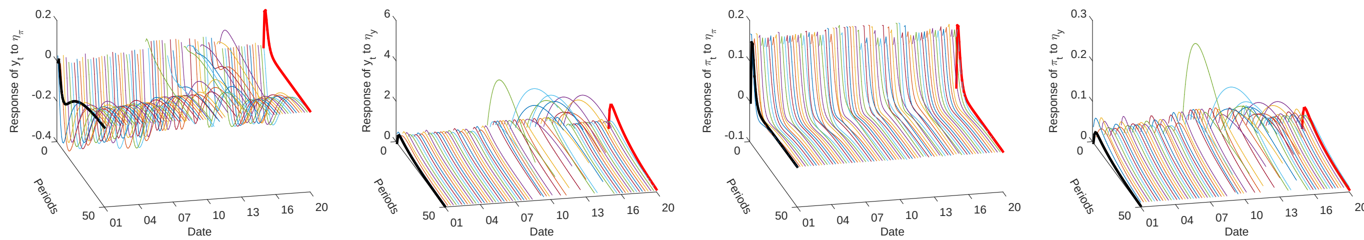
	AR(1) Degree of De-anchoring		
Date	Endogenous	fixed 10 %	fixed 90 %
20Q1	99.3%	99.3%	99.3%
20Q2		0.03%	11.8%
21Q4	2.56%	3.03%	0.07%
22Q4	31.9%	34.5%	24.2%
23Q4	46.9%	48.4%	41.5%
24Q4	58.2%	57.8%	50.1%

Table 6. Probabilities of returning back to the Taylor rule regime over different periods. Results from three exercises are reported: the benchmark scenario with endogenously determined fractions, the scenario where only 10% of agents use adaptive learning and the scenario where 90% of agents use adaptive learning.

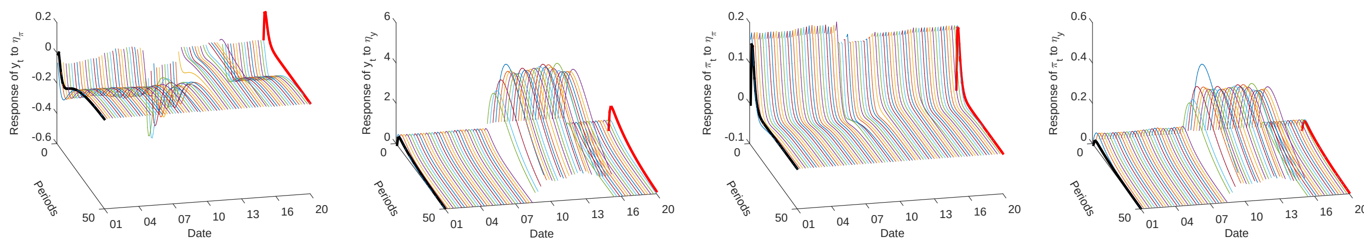
D Full Set of Impulse Responses for all Learning Models

This section presents the conditional impulse responses of output gap and inflation to both demand and cost-push shocks for all VAR(1) and AR(1)-learning models over the period 2001Q1-2019Q4. The IRFs are presented in Figure 14. Each panel also includes the regime-specific IRFs under the Taylor rule and ELB regimes for the MS-REE model as the solid black and red lines at the beginning and end of the panels respectively.

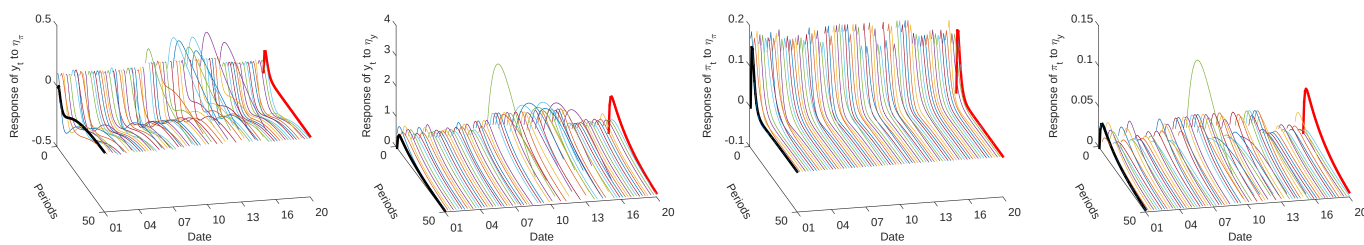
Similar to the discussion in Section 3, there are two main takeaways from the IRFs. The first is that, when the system switches from the Taylor rule regime to the ELB regime, the direction of change in the IRFs for all learning models is the same as the MS-REE model. A notable exception is the response of inflation π_t to a cost-push shock η_π, which remains fairly stable across both regimes and in all time periods. The remaining IRFs in the learning models are characterized by gradual movements over the sample period, and two jumps with with the entry to and exit from the ELB regime. For these IRFs, the time-variation in the 2-regime models with only adaptive learning agents is generally larger compared to the 4-regime models with heterogeneous expectations. This is due to the stabilizing effects of anchored expectations, which smooths the pattern in the IRFs. These results are similar for both VAR(1)- and AR(1)-learning models.



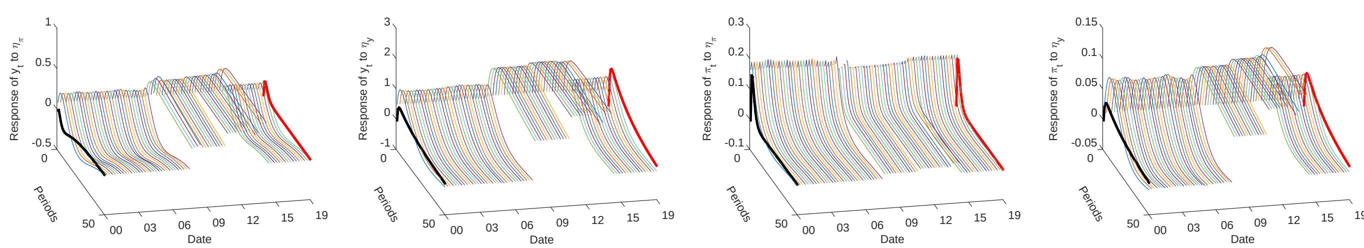
(a) 4-regime VAR(1), conditional IRFs



(b) 2-regime VAR(1), conditional IRFs



(c) 4-regime VAR(1), conditional IRFs



(d) 2-regime VAR(1), conditional IRFs

Figure 14. Conditional impulse responses for VAR(1) learning models.