Restricted Perceptions, Regime Switches and the Zero Lower Bound*

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Abstract

We analyze Markov-switching DSGE models under adaptive learning (AL) to study the interaction between expectations and the business cycle over the Zero Lower Bound (ZLB) episode. We assume that regime shifts by monetary policy are not directly observable by agents, instead they indirectly learn about regime switches and economic conditions based on past observations. A novel feature of this setup is the persistence amplification and accelerated learning around sharp regime switches. We use a variant of the well-known Kim & Nelson (1999) filter for the Bayesian estimation of MS-DSGE models under constant gain adaptive learning, which we apply to the Smets-Wouters (2007) model using U.S data. We find that (i) AL models typically outperform the regime-switching RE model, (ii) the switch to ZLB regime is usually followed by a sharp change in the learning parameters, suggesting a fast adoption of the new environment, (iii) the impulse responses in both RE and learning models change in the same direction with the switch to ZLB episode, but the magnitudes under learning models tend to be smaller. (iv) Counterfactual experiments suggest that learning dynamics prolonged the ZLB episode after the recent crisis.

JEL Classification: E37; E65; C11; C32 .

Keywords: Adaptive Learning; Markov-Switching; Bayesian Estimation of DSGE Models; Zero Lower Bound.

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1 Introduction

With the onset of the Global Financial Crisis in 2007-08 and the subsequent drop of interest rates to near-zero levels among leading central banks, there has been increased interest among policymakers and central bankers alike about the ZLB constraint on nominal interest rates. There is still ongoing debate about the precise impact of the zero lower bound constraint on the economy as a whole and in particular about its macroeconomic cost in terms of aggregate GDP levels. Monetary and fiscal policy recommendations of standard macroeconomic models are mixed: for instance there is no consensus on the propagation of a government spending shock and the size of a government spending multiplier during this period. As a consequence, while some researchers recommend a fiscal austerity program to jump-start the economy during the ZLB episode, others think it is best to adapt a fiscal consolidation strategy. A common approach in most macroeconomic models examining the ZLB episode is the assumption of Rational Expectations Equilibria (REE): agents are assumed to have perfect information about the underlying economic conditions along with all other cross-correlations of the relevant macroeconomic variables and form their expectations accordingly. In this paper, we contribute to the growing literature on the ZLB episode by relaxing the perfect information assumption, and instead estimating DSGE models under adaptive learning subject to the ZLB constraint.

In standard REE-DSGE models, the perfect foresight assumption about regime switches leads to short periods of anticipated ZLB episodes: the expected duration of this period is typically between three to five quarters in most DSGE models estimated on the U.S. economy, see e.g. Lindé et al. (2017) and Ji & Xiao (2016), while the ZLB episode between 2009 and 2016 lasted for twenty eight quarters. Another shortcoming of the standard REE models is the overestimation of the impact of forward guidance on the macroeconomy, usually known as the forward guidance puzzle Del Negro et al. (2012). These shortcomings call for a relaxation of REE-restrictions and introduce informational frictions into the models. A plausible and popular method to introduce such frictions is adaptive learning, which relaxes the assumption that agents have perfect knowledge about the underlying economic conditions. Instead, they have their own sub-models, possibly under- or over-parameterized, that may not coincide with the correct economic structure. Agents act as econometricians and update their models each period as new observations become available. There is a vast and growing literature on the empirical validation of adaptive learning in DSGE models as well as monetary and fiscal policy implications of adaptive learning, see Evans & Honkapohja (2012) for a textbook treatment and Woodford (2013) for a comprehensive review of the more recent work. Much of the earlier literature on adaptive learning focused on the learnability of Rational Expectations Equilibria and MSV-learning, focusing on small and temporary deviations from perfect foresight models. Milani (2007) and Eusepi & Preston (2011) are earlier examples of expectations-driven business cycles and how MSV-learning can improve the empirical properties of small-scale DSGE models, while Bullard & Mitra (2002) and Bullard & Eusepi (2014) examine monetary policy implications of this type of learning. In more recent work, Slobodyan & Wouters (2012a) and Slobodyan & Wouters (2012b) show that further deviations from perfect foresight models with the use of small forecasting rules can lead to further improvements in the fit of a medium-scale DSGE model. On a similar vein, Quaghebeur (2018) examines fiscal policy implications of a VAR-type adaptive learning rule and finds that government spending multipliers are larger under adaptive learning. Evans et al. (2008) and Evans & Honkapohja (2010) examine the implications of adaptive learning for fiscal policy.

There are various different approaches to modeling the ZLB constraint: Some researchers use a perfect foresight & endogenous duration approach, which allows for a joint determination of expectations and regime switches; see e.g. Maih (2015) or Lindé et al. (2017). Another approach which is more common in VAR-literature is to use a threshold-switching method, where the economy is assumed to be in the ZLB regime if interest rates fall below some pre-specified level, see e.g. Bonam et al. (2017). A final approach is to use a Markov-switching framework, where the presence of the ZLB regime is determined by its predictive density, see e.g. Binning & Maih (2016). Lindé et al. (2017) show that Markov-switching and endogenous duration approaches typically lead to similar results as long as the ZLB constraint is accounted for. In this paper, we use the Markov-switching (MS) approach to take into account the constraint. Aside from the ZLB episode, MS approach recently gained popularity in DSGE literature to model structural changes such as monetary policy switches or volatility breaks, see e.g. Sims & Zha (2006), Davig & Leeper (2007), Sims et al. (2008), Liu et al. (2011), Liu & Mumtaz (2011), Bianchi (2016), Bianchi & Ilut (2017) and Bianchi & Melosi (2017) for some of the recent work. Other related work includes Bullard & Duffy (2004) that studies learning about unanticipated structural change in productivity in an RBC framework, and Hollmayr & Matthes (2015) that studies consequences of fiscal policy shifts when agents have uncertainty about the switch.

While Markov-switching and adaptive learning have both been increasingly popular classes of time-varying DSGE models in recent years, there is surprisingly little work on DSGE models that combine both approaches. Closely related theoretical work includes Branch et al. (2013) that studies the properties of MSV-learning in Markov-switching models where agents are informed about regime switches but learn the remaining economic parameters; and Airaudo & Hajdini (2019) that studies equilibria in a Markov-switching framework where agents use an optimal AR(1) rule without accounting for regime switches. Empirical studies closely related to our work include Gust et al. (2018) that examines the ZLB episode and forward guidance in a Markov-switching setup under Bayesian learning, where agents are aware of regime switches but have to infer about the underlying regime of the current economy; and Lansing (2018) that analyzes the ZLB episode in a calibrated setup under adaptive learning where regime switches are unobserved. Our key difference from these empirical papers and one of our main contributions is to extend their framework to non-MSV and non-rational beliefs, and to estimate the resulting DSGE models during the ZLB episode. We then examine the consequences of deviating from the REE during this period, particularly how it might contribute to a prolonging of the crisis and how it might change implications of standard DSGE models about the potential impact of a government spending shock during this episode.

Our key assumption is that the underlying regime changes are unobserved to economic agents. Instead they use a constant gain econometric model, where they only indirectly become aware of regime changes if these switches have an observable and strong enough impact on their information set. To set the ideas, consider the following example: A central bank follows a simple Taylor rule that reacts to inflation in setting interest rates. This will only be known to economic agents to the extent that the central bank discloses its goal of inflation targeting, but the agents never know the exact reaction coefficient. Accordingly, the agents will not find out if the central bank suddenly and discreetly decides to change its reaction coefficient. Instead, the agents will slowly find out about this regime shift as long as it leads to observable consequences in the interest rate and the resulting inflation levels.

Farmer et al. (2009) and Farmer et al. (2011) explore the class of REE in Markov-switching models. Since we assume that regimes are never observed, an equilibrium concept in our framework can never coincide with a Rational Expectations Equilibrium. Instead, in this limited information environment, there are so-called Restricted Perceptions Equilibria (RPE) where the agents' misspecification of the economy becomes self-fulfilling and the system settles on a non-rational equilibrium. To start with, we compute these equilibria in a 1-dimensional setup with the Fisherian equation, where the agents' perceived law of motion (PLM) has the form of a Minimum State Variables (MSV) solution, except that the PLM does not take into account the possibility of regime-switches. We show that standard E-stability conditions apply to these equilibria, and therefore the systems will converge to the underlying equilibria under standard recursive algorithms such as constant-gain least-squares. Furthermore, the E-stability and convergence results continue to hold even if one of the underlying regimes is E-unstable as long as the remaining regimes are sufficiently E-stable. This is a simple extension of the long-run determinacy result of Davig & Leeper (2007), which they call the long-run Taylor principle. We therefore denote our result as the long-run *E-stability principle*. We then extend this idea further to higher dimensional systems, where the PLM can also deviate from the MSV-solution in the form of small VAR-type forecasting rules: this allows the information set of the agents to be smaller than the MSV-solution due to, for example, unobserved shocks or unaccounted cross-correlations. The underlying RPE are too complicated to compute either analytically or numerically in this more general setup, although the underlying systems can always be simulated to observe the system behaviour and E-stability.

Next we consider a variant of the Kim & Nelson (1999) filter to estimate our class of MS-DSGE models under adaptive learning, and we apply the filter to the Bayesian likelihood estimation of two standard DSGE models: The first one is the 3-equation NKPC model along the lines of Woodford (2013), which provides a good starting point to expose our main results. The second one is the more complex and empirically relevant Smets & Wouters (2007) model, which is popular among central bankers and policy makers as a benchmark for policy analysis. Our estimation results can be summarized as follows: The MS-AL models outperform the standard REE benchmark in all cases, and the also the regime-switching REE models in a majority of cases. Furthermore, we observe important differences in the impulse response and shock propagation structure of the models under consideration. The models have important implications for the impact of government spending shocks in particular: we find that, during normal times, government spending multipliers tend to be larger under adaptive learning compared to REE. Over the ZLB period with inactive monetary policy, the multipliers become larger for both REE and AL models compared with normal times. However, the proportional change for the REE model is substantially larger compared to all AL models. This suggests that the benchmark REE model may severely overestimate the effect of a fiscal expansion following the 2007-08 crisis.

The paper is organized as follows: Section 2 illustrates the main concepts in a simple framework with one-forward looking variable. Section 3 shows the computation and E-stability results of the two classes of Restricted Perceptions Equilibria in DSGE models. Section 4 provides the filter used in our estimations, while sections 5 and 6 discuss the estimations results in the 3-equation NKPC and SW models respectively. Finally Section 7 concludes.

2 Preliminaries

2.1 Fisher Equation and Long-run E-stability

Consider first a simple model of Fisherian inflation determination without regime switching:

$$\begin{cases} i_t = E_t \pi_{t+1} + r_t \\ r_t = \rho r_{t-1} + v_t \\ i_t = \alpha \pi_t, \end{cases} \tag{2.1}$$

where r_t is the exogenous AR(1) ex-ante real interest rate, i_t is the nominal interest rate, π_t is inflation, and v_t is an IID shock process. We assume that monetary policy follows a simple rule by adjusting nominal interest rate to inflation, denoted by α[1]. After eliminating nominal

[1] For the remainder, we assume that $Var(r_t) = 1$ to simplify the exposure.

interest rate i_t, the system can be re-written as follows:

$$\begin{cases} \pi_t = \frac{1}{\alpha}(E_t \pi_{t+1} + r_t), \\ r_t = \rho r_{t-1} + v_t. \\ \end{cases} \tag{2.2}$$

We use this small setup as our starting because, since it has been analyzed in Davig & Leeper (2007), which is one of the first studies on expectations in a regime switching setup; as well as in Airaudo & Hajdini (2019), which is the first study on small forecasting rules in a regime switching setup. The standard Minimum State Variable (MSV) solution takes the form of

$$\pi_t = dr_t. \tag{2.3}$$

In terms of adaptive learning terminology, (2.3) is known as the the *Perceived Law of Motion* (PLM). The Rational Expectations Equilibrium (REE) value of a is then pinned down by iterating the PLM forward to obtain the one-step ahead expectations, plugging the expectations back into the actual law of motion (2.2) and computing the associated fixed point, which yields $a = \frac{1}{\alpha - \rho}$. Hence the law of motion under REE is given by $\pi_t = \frac{1}{\alpha - \rho} r_t$. In this benchmark case, the equilibrium is said to be determinate if $\alpha > 1$, i.e. monetary policy is sufficiently aggressive.

Davig & Leeper (2007) consider scenarios where the interest rate coefficient α is subject to regime switches. Focusing on a two regime environment, assume that α changes stochastically between two regimes, $s_t = \{1, 2\}$ subject to the transition matrix:

$$Q = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix}$$

Then inflation dynamics are given as:

$$\begin{cases} \pi_t = \frac{1}{\alpha(s_t)}(E_t \pi_{t+1} + r_t) \\ r_t = \rho r_{t-1} + v_t. \end{cases} \tag{2.4}$$

Denoting by $\pi_{i,t} = \pi_t(s_t = i)$, we can rewrite the model in a multivariate form:

$$\begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \pi_{1,t} \\ \pi_{2,t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} E_t \pi_{1,t+1} \\ E_t \pi_{2,t+1} \end{bmatrix} + \begin{bmatrix} r_t \\ r_t \end{bmatrix}. \tag{2.5}$$

Since the ALM is regime-dependent, expectations are also regime-dependent in a REE framework. Denoting by a_i the regime-specific solutions, the corresponding regime-dependent 1-step ahead expectations are given by:

$$\begin{cases} E_t[\pi_{t+1}|s_t = 1] = (p_{11}d_1 + p_{12}d_2)\rho r_t, \\ E_t[\pi_{t+1}|s_t = 2] = (p_{21}d_1 + p_{22}d_2)\rho r_t. \end{cases}$$

In other words, agents hold two distinct laws of motion, know the transition matrix Q and form their expectations correctly after observing the current regime s_t. Davig & Leeper (2007) show that, in this setup, the equilibrium is determinate as long as the *long-run Taylor principle (LRTP)* is satisfied:

$$\alpha_1\alpha_2 > 1 - ((1 - \alpha_2)p_{11} + (1 - \alpha_1)p_{22}) \tag{2.6}$$

A key insight of this principle is that, the long-run dynamics of the model will be determinate even if one of the underlying regimes is indeterminate, provided there is at least one regime that is sufficiently determinate or the probability of entering into the indeterminate regime is sufficiently small. In what follows, we first relax the assumption of full information to replace it with that of learning, and then we extend the long-run determinacy insight into the concept of learnability, i.e. *E-stability* of equilibria.

Our main assumption in this paper is that agents do not directly observe or take into the regime shifts that occur in the economy when forming their expextations. Instead, they hold period-specific expectations that are updated each period as new observations become available. Therefore in our setup, regime switches are unknown to agents ex-ante, but only affect agents' expectations ex-post depending on their observable consequences. Before turning to learning, it is useful to first study the equilibrium properties of this setup. Hence assume that the economy evolves according to (2.4) with two regimes, where agents hold a regime-independent PLM and expectations as follows:

$$\pi_t = dr_t \Rightarrow E_t\pi_{t+1} = dE_t r_{t+1} = d\rho r_t. \tag{2.7}$$

The implied Actual Law of Motion (ALM) is then given by:

$$\begin{cases} \pi_t = \frac{1}{\alpha(s_t)}(d\rho + 1)r_t \\ r_t = \rho r_{t-1} + v_t. \end{cases} \tag{2.8}$$

The assumed form of PLM here does not nest the regime-dependent MSV solution. Therefore, any resulting notion of equilibrium under this scenario cannot coincide with the full-information REE. Instead, we consider a limited-information equilibrium associated with the above PLM. This type of equilibrium is commonly referred to as a Restricted Perceptions Equilibrium (RPE) in the adaptive learning literature (see e.g. (Evans & Honkapohja, 2012)), where the agents use a restricted (and therefore misspecified) information set, which becomes self-fulfilling at the underlying equilibrium[2].

Unlike a REE, one cannot use the method of undetermined coefficients as above to pin down the value of d associated with the RPE. Instead, following Hommes & Zhu (2014), we impose a moment consistency requirement on the model to pin down the value of d: the coefficient d determines the *perceived correlation* between inflation and real rate of interest in the PLM, i.e. $d = \frac{E[\pi_t r_t]}{E[r_t r_t]}$. In an RPE, the unconditional correlation $\frac{E[\pi_t r_t]}{E[r_t r_t]}$, implied by the ALM is equal to d. In other words, the agents' forecasting rule is consistent with the actual outcomes on average but it is misspecified along each regime. The associated unconditional moment in our example is given as:

$$\frac{E[\pi_t r_t]}{E[r_t r_t]} = E[\frac{1}{\alpha(s_t)} d\rho + \frac{1}{\alpha(s_t)}], \tag{2.9}$$

which involves involves the long-run distribution (i.e. ergodic distribution) of the Markov chain denoted by P. Given the transition matrix Q, this follows $P = [\frac{1-p_{22}}{2-p_{11}-p_{22}}, \frac{1-p_{11}}{2-p_{11}-p_{22}}]$[3]. Then the underlying RPE coefficient, which we denote as d^{RPE}, is given by[4]:

$$d^{RPE} = \frac{\alpha_1(1-p_{22}) + \alpha_2(1-p_{11})}{\alpha_1\alpha_2(2-p_{11}-p_{22}) - \rho\alpha_1(1-p_{22}) - \rho\alpha_2(1-p_{11})}. \tag{2.10}$$

Further note that, the regime-specific MSV solutions (i.e. the solution when the economy is always in regime i) are given by $d^i = \frac{1}{\alpha_i - \rho}$, $i \in \{1, 2\}$. Given these expressions, the underlying RPE boils down to a weighted average of the regime-specific equilibria, where the weights are determined by the long-run distribution of the regimes. Instead of the standard determinacy of Rational Expectations models, our main concept of interest in this case is E-stability[5]. E-stability governs whether the agents can learn the above fixed-point by starting from an arbitrary point a_0, and updating their beliefs about the coefficient each period using a recursive system as new observations become available. As shown in Evans & Honkapohja (2012), E-stability is governed by the mapping from agents' PLM to the implied ALM, defined as the T-map. In our example, the T-map is given by:

$$T : d \rightarrow T(d) = \frac{E[\pi_t r_t]}{E[r_t r_t]} = (d\rho + 1)\frac{\alpha_1(1-p_{22}) + \alpha_2(1-p_{11})}{\alpha_1\alpha_2(2-p_{11}p_{22})} \tag{2.11}$$

[2]Airaudo & Hajdini (2019) study RPE resulting from an AR(1) rule in this setup, i.e. not only the regimes are unobserved but the functional form of PLM is also misspecified.

[3]Note that ergodic distribution is obtained by solving $P'Q = P$.

[4]See Appendix A for details.

[5]Bullard & Eusepi (2014) shows that there is tight link between determinacy and E-stability of REE and in some special cases these conditions may even coincide.

The T-map is locally stable if the Jacobian matrix has roots with real parts less than one. When the local stability condition is satisfied, the equilibrium is said to be E-stable. In our case, the root and the associated E-stability condition are given as:

$$\frac{DT(d)}{D(d)} = \frac{\alpha_1(1-p_{22})+\alpha_2(1-p_{11})}{\alpha_1\alpha_2(2-p_{11}-p_{22})}\rho < 1, \tag{2.12}$$

which, after re-arranging, yields:

$$\alpha_1\alpha_2 > \frac{\alpha_1(1-p_{22})+\alpha_2(1-p_{11})}{2-p_{11}-p_{22}}. \tag{2.13}$$

This results in an E-stability criterion similar to that of *LRTP*. In order to obtain E-stability, a more aggressive monetary policy rule α_1 is needed whenever: (i) the average time spent in regime 1 (P_1) decreases, (ii) the average time spent in regime 2 (P_2) increases, or (iii) the monetary policy rule in regime 2 (α_2) becomes less aggressive. This suggests that it is possible to have E-stability despite having an E-unstable regime, as long as there is a sufficiently E-stable regime and the model does not spend too much time in the unstable regime on average. This is an intuitive extension of Davig & Leeper's insight on long-run determinacy to the learnability of equilibria, therefore we denote this as *the principle of long-run E-stability*.

2.2 Regime Switches and Constant Gain Learning

The RPE in (2.10) serves as a preliminary starting point to study the overall stability dynamics. However, our main point of interest in this paper is to study the transitory dynamics under adaptive learning when there is a monetary policy regime switch. In order to also consider learning dynamics about persistence, we first extend the model with a fraction ι_p of agents that have backward-looking expectations based on the previous period, while the remaining fraction $1-\iota_p$ form their expectations rationally as before. Accordingly, consider the following extension of the model:

$$\begin{cases} \pi_t = \frac{1}{\alpha(s_t)}(\tilde{E}_t\pi_{t+1} + \iota_p\pi_{t-1} + r_t), \\ \tilde{E}_t\pi_{t+1} = \iota_p\pi_{t-1} + (1-\iota_p)E_t\pi_{t+1}, \\ r_t = \rho r_{t-1} + v_t, \end{cases} \tag{2.14}$$

where $\tilde{E}_t$ denotes aggregate expectations operator and E_t refers to the Rational Expectations as before. Assuming again that agents' do not observe the regime switches, the associated PLM of the rational agents is given as[6]:

[6]We assume that rational agents take into account the presence of backward-looking agents when forming their expectations.

$$\pi_t = dr_t + b\pi_{t-1}, \tag{2.15}$$

where the T-map is given by:

$$\begin{pmatrix} d \\ b \end{pmatrix} \to \begin{pmatrix} E[(\pi_t - b(s_t)\pi_{t-1})r_t] \\ \frac{E[(\pi_t - d(s_t)r_t)\pi_{t-1}]}{E[\pi_t^2]}, \end{pmatrix} \tag{2.16}$$

with $b(s_t) = \frac{\iota_p}{\alpha(s_t)-(1-\iota_p)b}$ and $d(s_t) = \frac{(1-\iota_p)d\rho+1}{\alpha(s_t)-(1-\iota_p)b}$. With the addition of lagged inflation, the moments appearing in the above expression become analytically intractable, therefore the values a^{RPE} and b^{RPE} and the associated E-stability conditions are obtained numerically in the examples below[7].

Next we introduce adaptive learning into the system, where beliefs about a and b are updated each period as new observations become available, using a constant-gain least squares method à la Evans & Honkapohja (2012). Denoting by $\theta = [d, b]'$ and $y_t = [r_t, \pi_{t-1}]'$, the coefficients are updated using:

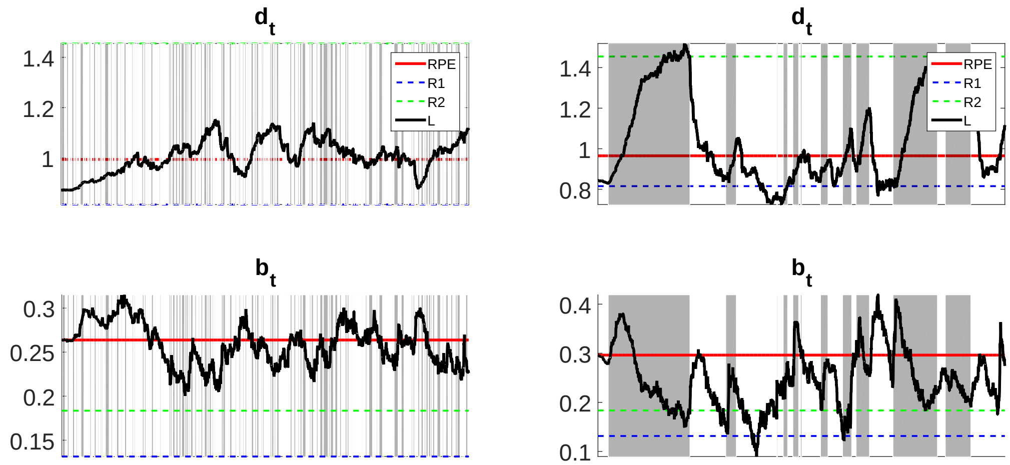
$$\begin{cases} R_t = R_{t-1} + \gamma(y_t^2 - R_{t-1}), \\ \theta_t = \theta_{t-1} + \gamma R_t^{-1} y_t(\pi_t - \theta_{t-1}y_t), \end{cases} \tag{2.17}$$

where γ denotes the gain value, i.e. the weight that agents put into the most recent observation. A costant gain implies geometric discounting of the past and allows agents to put more weight into recent observations, thereby allowing them to potentially detect the consequences of regime switches. We first illustrate the model dynamics for a parameterization where both regime-specific MSV-solutions, as well as the underlying RPE are E-stable. Figure 1 shows two simulations with different gain values and transition probabilities. Panel (a) shows an example with frequent regime switches, $p_{11} = p_{22} = 0.9$, and a small gain value of 0.005. In this case the learning coefficients oscillate around the RPE-consistent values, illustrating the stability of the system. An interesting feature of the RPE is that, while d^{RPE} is a weighted-average of the regime-specific equilibrium values, b^{RPE} is larger than the regime-specific values. This suggests that RPE is not necessarily a simple weighted average of the underlying regime-specific equilibria, and that regime-switching may induce persistence amplification in the system[8]. Panel (b) shows instead an example with more persistent regimes, $p_{11} = p_{22} = 0.99$, and a larger gain value of 0.01. It is readily seen that when the gain value is sufficiently large and the regime durations are long, the system converges towards the regime-specific values, i.e. agents

[7] See Appendix B for details on the implied moments.

[8] We do not plot the RPE as a function of the structural parameters for brevity, but our simulations show that persistence amplification arises whenever the persistence of the exogenous driving process ρ is sufficiently large.

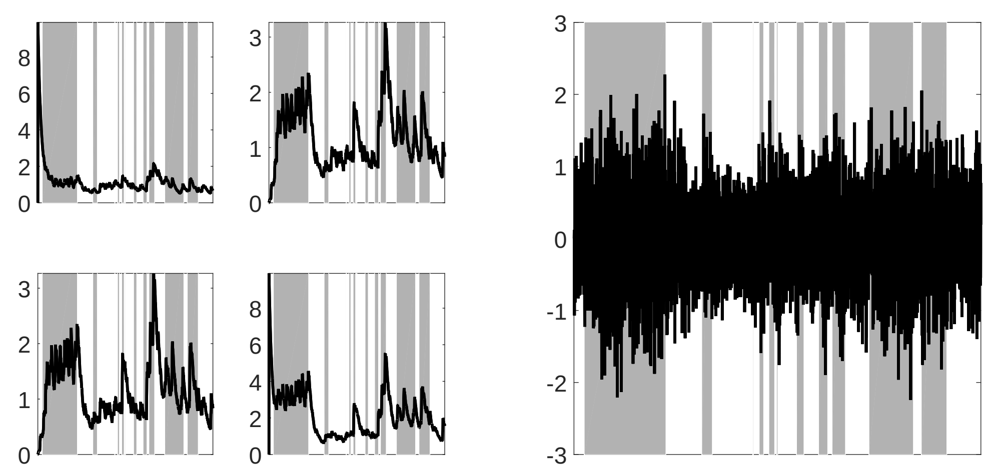
forget about the past regime switches. When the regime shift occurs, there are two possible outcomes: if the RPE and the new regime specific value are in the same direction, such as for d_t, then the learning process gradually moves towards the new direction. If the RPE and the new regime specific value are in different directions, such as for b_t in this example, then the learning process first jumps towards the RPE, then starts to gradually move towards the regime specific value. This figure illustrates that, under the right circumstances, regime switches may be characterized by periods of temporarily amplified persistence. More importantly, learning of the new regime can be very quick, especially when exiting a very long regime or entering into a new regime that has not been observed before. These results are in line with Hollmayr & Matthes (2015), where unacticipated structural change leads to a temporary period of fast learning and amplified volatility. In our framework, this phenomenon occurs as a temporary shift towards the RPE. These characteristics are particularly important from an empirical point of view, since the recent ZLB episode is similar to such a switch from a persistent regime to a new regime that was not experienced in the recent past. To examine further where the jumps in the learning coefficients are coming from, Figure 2 shows the perceived covariance matrix along with the agents' forecast errors for the second simulation with $p_{11} = p_{22} = 0.99$ and $\gamma = 0.01$. It is readily seen that regime switches are accompanied both by jumps in the forecast errors, as well as jumps in the perceived perceived variance of inflation and perceived covariance between inflation and real interest rates. As such, the accelerated learning and jumps in the coefficients over regime switches are driven not only by the increased forecast errors, but also due to perceived changes in the covariance structure of the system.



(a) $\gamma = 0.005, p_{11} = p_{22} = 0.9$.

(b) $\gamma = 0.01, p_{11} = p_{22} = 0.995$.

Figure 1: Learning coefficients along with the RPE-consistent and regime-specific values. The parameters $\iota_p = 0.25, \rho = 0.9, \alpha_1 = 1.5, \alpha_2 = 2$, are fixed in both simulations. Given the values of α_1 and α_2, both regime-specific equilibria and the RPE are E-stable.



(a) $\gamma = 0.005, p_{11} = p_{22} = 0.9$.

(b) $\gamma = 0.01, p_{11} = p_{22} = 0.995$.

Figure 2: Perceived covariance matrix and forecast errors from the simulation on the right in Figure 1 with $\gamma = 0.01, p_{11} = p_{22} = 0.995$. The left panel shows the perceived covariance matrix, i.e. $E[y_t y_t']$, while the right panel shows the forecast errors, i.e. $(\pi_t - \theta_{t-1} y_t)$.

2.3 Mean Dynamics

As a final step in this section, to motivate learning dynamics about the mean, we assume that nominal interest rates react to deviations of inflation from its non-zero target rate $\bar{\pi}$, i.e. $i_t - \bar{\pi} = \alpha(s_t)(\pi_t - \bar{\pi})$. After re-arranging, this can be re-written as:

$$\begin{cases} \pi_t = \frac{1}{\alpha(s_t)}((\alpha(s_t) - 1)\bar{\pi} + \tilde{E}_t\pi_{t+1} + r_t), \\ \tilde{E}_t\pi_{t+1} = \iota_p\pi_{t-1} + (1 - \iota_p)E_t\pi_{t+1}, \\ r_t = \rho r_{t-1} + v_t, \end{cases} \tag{2.18}$$

where the rational agents' PLM is given by:

$$\pi_t = a + b\pi_{t-1} + dr_t, \tag{2.19}$$

and the T-map:

$$\begin{pmatrix} a \\ b \\ d \end{pmatrix} \to \begin{pmatrix} E[(\pi_t - b(s_t)X_{t-1} - d(s_t)\epsilon_t)] \\ E[\frac{(\pi_t - a(s_t) - b(s_t)\pi_{t-1})r_t}{E[r_t^2]}] \\ \frac{E[(\pi_t - a(s_t)d(s_t)r_t)\pi_{t-1}]}{E[\pi_t^2]} \end{pmatrix}. \tag{2.20}$$

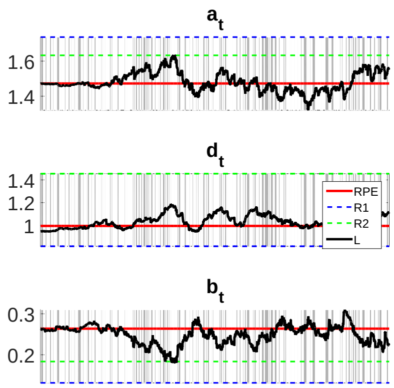
with $a(s_t) = \frac{(\alpha(s_t)-1)\bar{\pi}+(1-\iota_p)a}{\alpha(s_t)-(1-\iota_p)b}$, $b(s_t) = \frac{\iota_p}{\alpha(s_t)-(1-\iota_p)b}$ and $d(s_t) = \frac{(1-\iota_p)d\rho+1}{\alpha(s_t)-(1-\iota_p)b}$. Figure 3 illustrates two simulations with the same parameterization from before and $\bar{\pi} = 2$, where both regime-specific equilibria and the RPE are E-stable. Panel (a) again shows frequent regime switches with a small gain value, while Panel (b) shows infrequent switches with a larger gain. We observe that in this case, the RPE value for a is lower than both regime-specific values, which confirms our result from the previous section that the RPE is not necessarily given as a simple weighted average of the regimes. In this case, the lower value of a^{RPE} suggests that the perceived inflation target is lower under RPE than both regime specific values. While we observe oscillations near the RPE-value in the first simulation, the second one shows jumps towards the RPE value along with regime switches, followed by a gradual movement towards the regime-specific values as the regime persists.

A standard result in New Keynesian models is that the mean dynamics are not stable under learning with a passive interest rate rule. This has received considerable attention in the recent macro literature since the monetary policy is restricted to be passive during the ZLB episode, which leads to E-unstability for mean dynamics. Within our framework, using the notion of long-run E-stability such a regime leads to temporary episodes of instability and a return to stable dynamics once the regime switches back. Figure 4 illustrates a parameterization with such dynamics: monetary policy switches between $\alpha_1 = 2$ and $\alpha_2 = 0.75$[9] with transition

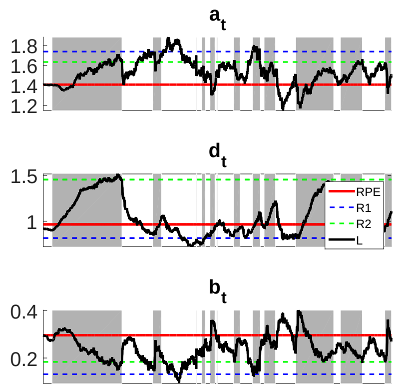
[9] The reaction coefficient of 0.9 is close to the stability region and still far from a passive policy rule; we use

probabilities $p_{11} = 0.99$ and $p_{22} = 0.95$. At these values, the first regime is E-unstable and the RPE is still stable. We shut off $\iota_p = 0$ and $\rho_r = 0$ such that the dynamics around b^{RPE} and d^{RPE} are still stable. It is readily seen that a_t still fluctuates around the RPE value, but now with short bursts of unstable periods. Further, one can still observe the same phenomenon of jumps not only in b_t, but also in a_t, which is visible in both simulations. It is important to note that the overall stability of the model is driven purely by the exogeneity of regime transition probabilities, and we consider extensions with endogenous probabilities in latter sections that can give rise to unstable dynamics.

In the next section, we extend our analysis to a general multivariate setup in order to approximate the ZLB episode as a regime-switch in New Keynesian models.



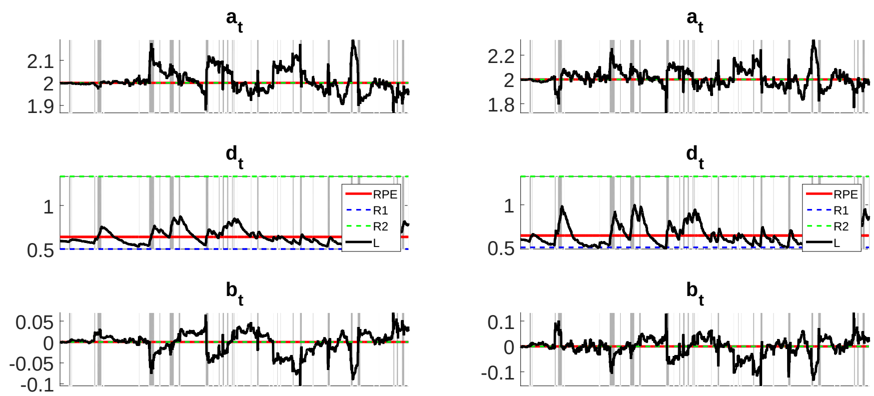
(a) $\gamma = 0.001, p_{11} = p_{22} = 0.99$.



(b) $\gamma = 0.005, p_{11} = p_{22} = 0.999$.

Figure 3: Learning coefficients along with the RPE-consistent and regime-specific values. The parameters $\bar{\pi} = 2, \iota_p = 0.25, \rho = 0.9, \alpha_1 = 1.5, \alpha_2 = 2$, are fixed in both simulations. Given the values of α_1 and α_2, both regime-specific equilibria and the RPE are stable.

this value since even lower values have a large impact in this 2-equation setup. Simulations with a passive rule are provided in latter sections in more realistic model setups.



(a) $\gamma = 0.005, p_{11} = 0.9, p_{22} = 0.95$. (b) $\gamma = 0.01, p_{11} = 0.99, p_{22} = 0.95$.

Figure 4: Learning coefficients along with the RPE-consistent and regime-specific values. The parameters $\mu = 2, \iota_p = 0, \rho = 0, \alpha_1 = 0.75, \alpha_2 = 2$, are fixed in both simulations. Given the values of α_1 and α_2, the mean dynamics in regime 1 are not E-stable but the RPE is E-stable.

3 General Setup and Estimation

Our simple example in the previous section with only one forward-looking variable serves as an introduction to the main concepts that we consider in this maper. In this section, we extend our notion of restricted perceptions to a general class of linear multivariate models. Consider the following data generating process:

$$\begin{cases} X_t = A(s_t) + B(s_t)X_{t-1} + C(s_t)E_tX_{t+1} + D(s_t)\epsilon_t \\ \epsilon_t = \rho\epsilon_{t-1} + \eta_t \end{cases} \tag{3.1}$$

where X_t denotes the state-variables that may depend on their lags, 1-step ahead expectations and the structural shocks ϵ_t, which itself follow a VAR(1) process. We assume that the corresponding matrices A, B, C, D and E contain the structural parameters of the model, some of which are subject to regime switches captured by s_t[10]. The corresponding regime-independent PLM of agents is given by:

$$\begin{cases} X_t = a + bX_{t-1} + d\epsilon_t, \\ E_tX_{t+1} = a + bX_t + (d\rho)\epsilon_t, \end{cases} \tag{3.2}$$

where we use a *period t dating* assumption, i.e. structural shocks and contemporaneous variables are observed at the time of forming expectations[11]. Further note that the above

[10] One may also consider regime switches in the shock processes, we omit these cases here.

[11] The alternative is to assume period t-1 dating, which uses information from the previous period, which

specification nests many benchmark PLMs as a special case, which will be discussed below. Plugging the expectations in (3.2) back into (3.1) yields the implied ALM:

$$X_t = A(s_t) + B(s_t)X_{t-1} + C(s_t)a + C(s_t)bX_t + (C(s_t)(d\rho) + D(s_t))\epsilon_t, \tag{3.3}$$

which can be re-written as

$$X_t = a(s_t) + b(s_t)X_{t-1} + d(s_t)\epsilon_t, \tag{3.4}$$

where $a(s_t) = (I - C(s_t)b)^{-1}(A(s_t) + C(s_t)a), b(s_t) = (I - C(s_t)b)^{-1}B(s_t)$ and $d(s_t) = (I - C(s_t)b)^{-1}(C(s_t)(d\rho) + D(s_t))$. In this case the T-map is given as:

$$\begin{pmatrix} a \\ b \\ d \end{pmatrix} \rightarrow \begin{pmatrix} E[X_t - b(s_t)X_{t-1} - d(s_t)\epsilon_t] \\ E[(X_t - a(s_t) - d(s_t)\epsilon_t)E[X_tX_t']^{-1}] \\ E[(X_t - a(s_t) - b(s_t)X_{t-1}]E[\epsilon_t\epsilon_t']^{-1} \end{pmatrix} \tag{3.5}$$

Appendix C provides the first and second moments that appear here for a general setup with m regimes. While equilibrium values for a^{RPE} and d^{RPE} are easily computed for a given matrix b^{RPE}, the matrix b^{RPE} is intractable, which means the corresponding E-stability conditions are also intractable. Therefore for the multivariate models considered in the remainder of this paper, we rely on simulations to check for the stability of the systems. Finally, to introduce adaptive learning, denote by $\Phi_t = [a, d, b]'$ and $Y_t = [X_{t-1}, \epsilon_t]'$, the coefficients are updated using least squares:

$$\begin{cases} R_t = R_{t-1} + \gamma(Y_t^2 - R_{t-1}), \\ \Phi_t = \Phi_{t-1} + \gamma R_t^{-1} y_t(X_t - \Phi_{t-1}Y_t). \end{cases} \tag{3.6}$$

With the introduction of adaptive learning into the the Markov-switching framework, we have a system characterized by two types of time variation, which can be written in the following compact state-space form:

$$\Big\{ S_t = \gamma_{2,\Phi_t}^{(s_t)} + \gamma_{1,\Phi_t}^{(s_t)} S_{t-1} + \gamma_{3,\Phi_t}^{(s_t)} \eta_t, \quad , \eta_t \sim N(0, \Sigma) \tag{3.7}$$

with $S_t = [X_t', \epsilon_t']'$ and $\gamma_{1,\Phi_t}^{(s_t)}, \gamma_{2,\Phi_t}^{(s_t)}$ and $\gamma_{3,\Phi_t}^{(s_t)}$ conformable matrices in terms of structural parameters, which depend on the assumption of the PLM. We next discuss the estimation of the general model in (3.7).

effectively leads to 2-step ahead forecasts in expectations.

3.1 Estimation

The benchmark algorithm for Markov-switching state-space models is the modified Kalman filter by Kim & Nelson (henceforth KN-filter): in a Markov-switching model with m regimes, a dataset of size T leads to m^T possible timelines, which quickly makes the standard Kalman filter intractable. The main idea in the KN-filter is to introduce a so-called *collapsing* technique to deal with this issue, which amounts to taking weighted averages of the state vector and covariance matrix at each iteration of the filter. This effectively reduces the number of timelines at each iteration by an order of m, thereby making the filter tractable again. The standard recommendation is to carry as many lags of the states as appears in the transition equation. Since we only consider we consider DSGE models that have a reduced-form VAR(1) representation in this paper, only a version of the filter with a single lag is presented here, although the same framework can be easily extended to any VAR(p) framework. Accordingly, if there are m different regimes in the model, we carry m different timelines in each period. Therefore there are m^2 different sets of variables in the forecasting and updating steps of each iteration. These are then collapsed at the end of each iteration to reduce to m sets of variables. An important question is how to introduce adaptive learning into this framework. We use an approach that is consistent with the theoretical framework of the previous section: the agents have a unique PLM based on observables, independent of the regime switches. We model this formally by collapsing the m different states further at each iteration to obtain the final states estimated by the filter, which are then used for the adaptive learning step. The unique learning coefficients are then used in each Kalman filter timeline of the next period's iteration[12]. Extending (3.7) with a set of measurement equations, the state-space representation of the model is given as follows:

$$\begin{cases} S_t = \gamma_{2,\Phi_t}^{(s_t)} + \gamma_{1,\Phi_t}^{(s_t)} S_{t-1} + \gamma_{3,\Phi_t}^{(s_t)} \eta_t, & ,\eta_t \sim N(0, \Sigma) \\ y_t = E + F S_t, \end{cases} \tag{3.8}$$

Given this representation, Figure 5 illustrates the KM-filter for the special case of two regimes, see Appendix B for the details. The filter as illustrated below yields the likelihood function, which is then combined with a set of prior distributions for Bayesian inference.

[12]A natural alternative here is to apply the adaptive learning step distinctly to each collapsed state; one can then take a weighted average of these expectations to obtain the filtered expectations. Our results in the upcoming sections are not sensitive to such an alternative, but we only present the results under the first approach since it is more in the spirit of our theoretical framework.

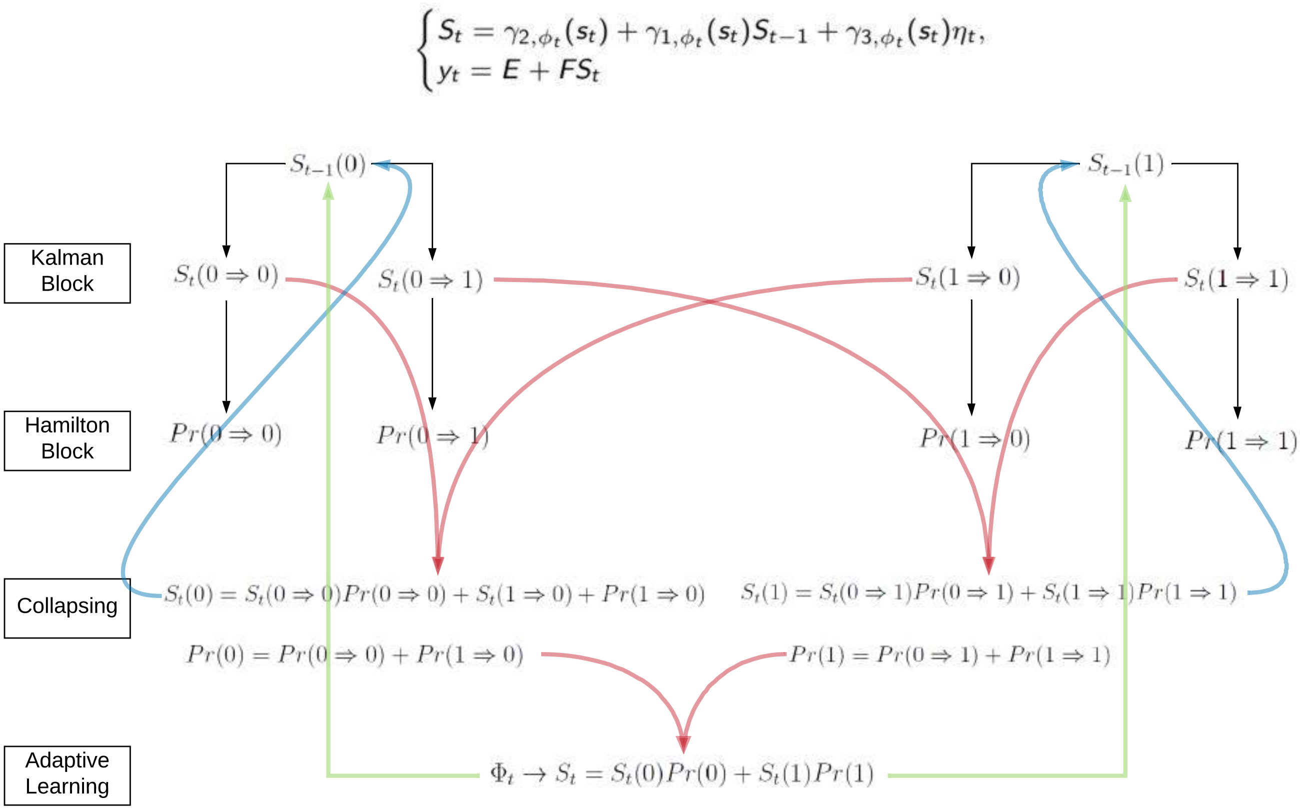


Figure 5: Illustration of the filter in a 2-regime model.

Initial Beliefs

A first practical issue in empirical studies on learning is where to initialize the beliefs. This has been shown to play a key role in driving the estimation results and model fit in previous studies, and various different approaches have been considered: Milani (2007) uses an estimation-based approach, where the initial beliefs are treated as free parameters and estimated jointly along with the other structural parameters of the model; Slobodyan & Wouters (2012b,2012a) consider REE-based and training-sample based approaches along with the estimation-based approach; while Berardi & Galimberti (2017c) proposes a smoothing-based approach. A common result in these studies is that the results are generally sensitive to initial beliefs, and the best-fitting approach depends on the specific model under consideration; see Berardi & Galimberti (2017a, 2017b) for a detailed overview on initial beliefs.

In this paper we follow the approach in Slobodyan & Wouters (2012b) with REE-based initial beliefs[13]. Specifically, we first estimate the benchmark REE model without regime switching over the pre-ZLB period. The relevant moments implied by the matrices γ_1 and γ_3 from this estimation are used as initial beliefs for the learning models[14]. This method has the advantage of not being computationally demanding since the initial beliefs are only computed once and remain fixed throughout the rest of the estimation.

Projection Facilities

A second issue with the estimation of adaptive learning models relates to retaining the stationarity of the model. A well-known issue with adaptive learning, particularly with constant gain recursive least squares learning, is that the stationarity of the underlying models are not guaranteed. Particularly when the PLM involves lagged state variables, the learning process may occasionally push the system into non-stationary and explosive regions, even if the underlying equilibrium is stable. Models subject to the zero lower bound constraint may be even more prone to encounter this problem, since typically an inactive monetary policy implies indeterminacy and E-unstability for the regime-specific dynamics.

A common method in the adaptive learning literature to deal with these potential instabilities is to impose a projection facility on the model, which forces the model dynamics to be stationary

[13]Alternative specifications with different initial beliefs methods can be found in the online appendix to this paper. Specifically, we consider: (i) a training-sample based approach where the filter at diffuse moments and the first 5-year period is discarded; (ii) an estimation-based approach where beliefs are estimated as free parameters in an auxiliary model, which are then used in the learning estimations; (iii) a filter-based approach, where, for each parameter draw, the beliefs are initialized at diffuse points and the filter is ran once, then the converged values of the beliefs coefficients are used as initial values for the second run of the filter. The results of the alternative estimations are provided in the Appendix, and while the relative fit of each model is sensitive to the initial beliefs, our main conclusions continue to hold in all specifications.

[14]The REE-implied intercepts are always zero, therefore the vector γ_2 is always initialized at the vector of zeros

by projecting the learning parameters into the stable region whenever instability is encountered. The simples approach to do this is to leave the parameters at their previous value if the learning update leads to non-stationarity, which is the method adopted in Slobodyan & Wouters (2012a). Specifically, we set up the projection facility as follows in our estimations: we stop updating the learning parameters each period if the update pushes the largest eigenvalue of the ergodic distribution of the model (3.7) outside the unit circle. In other words, we allow the regime-specific dynamics to be temporarily non-stationary as long as the underlying implied ergodic distribution remains stable. Importantly, this approach also allows the agents' PLM (3.2) to become temporarily explosive as long as the underlying ergodic distribution is stable. These choices reflect our desire to keep the projection facility as inactive as possible; and while they do not have an impact on our estimation results in general, but they indeed affect the frequency of periods with an active projection facility.

It is important to note that, since the projection facility imposes stability on the model, it essentially overrides the E-stability of the underlying system. In other words, the estimated models are stable regardless of whether E-stability holds. This is a necessary restriction since the T-map (3.5) for the general system is not tractable, i.e. we do not have a simple expression that verifies whether the model is E-stable or not for a given parameter draw. Therefore, to check the E-stability of the models, we resort to Monte Carlo (MC) simulations at the posterior mode, i.e. stability in the estimations is assumed ex-ante via projection facility, and it is verified ex-post at the point estimates via MC simulations.

Learning Rules

Our discussion up to this point is based on the information set consistent with the MSV solution, where the only source of model misspecification arises from unobserved regimes. However, in principle, any information set may be considered in agents' PLM. In this paper, we will focus our attention on three types of learning rules: (i) MSV-consistent rule as discussed before. (ii) a VAR-based rule, which assumes unobserved shocks but otherwise keeps the same set of state variables as in the MSV solution[15]. Similar learning rules have been applied in the learning literature, see e.g. *Exuberance Equilibria* (Bullard et al. (2008)) and *Limited Information Equilibria* (Chung & Xiao (2013)). (iii) a parsimonious AR(1) rule, which assumes a univariate process for each forward-looking variable[16]. This type of univariate forecasting rules have been applied in recent past to improve the empirical fit or match stylized facts of otherwise standard DSGE models, see e.g. Slobodyan & Wouters (2012b), Gaus & Gibbs (2018), Di Pace et al. (2016) andHommes et al. (2019). In the following sections, this framework is applied both

[15] In terms of (3.2), this assumes d is the zero matrix, but keeps the same b matrix as in the MSV solution.

[16] In terms of (3.2), this assumes d is the zero matrix and b is diagonal.

to a baseline 3-equation New Keynesian model, and to the Smets-Wouters (2007) model to investigate the ZLB episode for the U.S. economy.

4 Estimation of the Smets-Wouters Model

Priors and Measurement Equations

In this section we estimate a version of the Smets-Wouters (2007) model. The details of the model are omitted here for brevity; the readers are referred to Smets & Wouters for a close-up of the model. We have two small deviations from the benchmark model: first, we assume the price and wage mark-up shocks follow exogenous AR(1) processes, instead of the original ARMA(1,1) assumption[17]. Second, we shut off the flexible economy side of the model, which is used to obtain the potential output and the associated level of output gap. Instead, we follow Slobodyan & Wouters (2012a) and derive the output gap from the natural level of output, based on the underlying productivity process. This has the advantage of reducing the size of the model, thereby making its estimation computationally less demanding. The rest of the model, along with the prior distributions and measurement equations remain unchanged. Accordingly, the estimation is based on seven observables on the U.S. data over the period 1966:I-2016:IV as follows:

$$\begin{cases} d(log(y_t^{obs})) = \bar{\gamma} + (y_t - y_{t-1}) \\ d(log(c_t^{obs})) = \bar{\gamma} + (c_t - c_{t-1}) \\ d(log(inv_t^{obs})) = \bar{\gamma} + (inv_t - inv_{t-1}) \\ d(log(w_t^{obs})) = \bar{\gamma} + (w_t - w_{t-1}) \\ log(l_t^{obs}) = \bar{l} + l_t \\ (log(\pi_t^{obs})) = \bar{\pi} + \pi_t \\ (log(r_t^{obs})) = \bar{r} + r_t \end{cases}$$

where $d(log(y_t^{obs}))$, $d(log(c_t^{obs}))$, $d(log(inv_t^{obs}))$ and $d(log(w_t^{obs}))$ denote real output, consumption, investment and wage growths with the common growth rate $\bar{\gamma}$ respectively, while $log(l_t^{obs})$, $(log(\pi_t^{obs}))$ and $(log(r_t^{obs}))$ denote (normalized) hours worked, inflation rate and federal funds rate respectively. We use the same estimation sample as in the 3-equation NKPC with quarterly U.S. data covering the period from 1966:I to 2016:IV.

There are five additional parameters due to regime-switching and learning that are not

[17] This is due to the fact that, as shown in Slobodyan & Wouters (2012a), these shock processes are typically close to being white noise when expectations are based on small learning rules, in which case the AR(1) and MA(1) terms are close to being locally unidentified. Therefore we assume away the MA(1) terms in these shocks.

present in the benchmark REE model. In terms of the regime transition probabilities, we estimate the exit probabilities from the normal and ZLB regimes, denotes as $1 - p_{11}$ and $1 - p_{22}$ respectively. We use uniform distributions over $[0, 1]$ for these parameters to see how informative the data is about these two parameters. This differs from previous studies in the literature where typically tight priors have been used, see e.g. Lindé et al. (2017), Chen (2017) and Ji & Xiao (2016), all of which use tight Beta priors for the transition probabilities. For the learning gain parameter, we use a Gamma prior with mean 0.035 and standard deviation 0.03, which follows from Slobodyan & Wouters (2012b). This distribution permits a prior credible interval between over the range $[0, 0.1]$ for the gain, which is consistent with previous findings in empirical learning literature. For the standard deviation of monetary policy shocks of the ZLB regime, $\eta_{r_{ZLB}}$, we use a Gamma distribution with mean and standard deviation 0.01, and finally for the steady-state level of interest rates at the ZLB regime, we use a normal distribution with mean 0.05 and standard deviation 0.025.

The model in its current form features seven forward looking variables, namely the rental rate of capital rk_t, asset prices q_t, consumption c_t, investment I_t, labor l_t, inflation π_t and real wages w_t; along with seven AR(1) structural shocks, namely technology $\epsilon_{a,t}$, government spending $\epsilon_{g,t}$, risk premium $\epsilon_{b,t}$, investment-specific technology $\epsilon_{I,t}$, monetary policy $\epsilon_{r,t}$, and two mark-up shocks in prices $\epsilon_{p,t}$ and wages $\epsilon_{w,t}$ respectively. Further, there are seven state variables that appear with a lag in the model, namely, consumption c_t, investment I_t, output y_t, inflation π_t, real wage w_t, nominal interest rate r_t and capital k_t. This translates into a learning matrix of size 7x15 for the MSV model (intercept, lagged state variables and shocks), 7x8 for the VAR(1) model (intercept and lagged state variables) and 2x1 for each variable in the AR(1) model (intercept and own lagged variable). See the online appendix for further detail on the learning equations.

Posterior Estimates

Using the KM-filter as illustrated in 5, we first obtain the posterior mode using standard optimization algorithms. The resulting modes are used to initialize the MCMC to sample from the posterior distribution, for which we use a Random Walk Metropolis Hastings with an adaptive covariance matrix for the proposal density. We simulate two chains of length 250000 for each model under consideration, and the first 40 % of each chain is discarded as the transient period. The remaining 150000 draws are checked for convergence using standard tests of Geweke (1992) within the chain, Gelman et al. (1992) between the chains. See the Online Appendix for further details and the full distributions of parameters.

Tables 1 and 2 show the results for the learning models, as well as the MS-REE model and REE benchmark case. First comparing the marginal (log)-likelihoods of the models, we observe that the Markov-Switching REE model (REE-MS) yields a substantial improvement over the benchmark REE: based on the Modified Harmonic Mean (MHM) estimators of -1194 and -1145, we obtain a Bayes Factor of 21.54 in favor of the Markov-switching model. Next comparing the learning models with REE-MS[18], we observe that all three learning models yield an improvement over REE-MS, but to varying degrees: MSV-learning results in a small and negligible improvement, while the VAR(1) and AR(1) models result in a relatively large improvements. The corresponding Bayes Factors relative to the REE benchmark are 21.97, 35.87 and 31.09 for MSV, VAR(1) and AR(1) respectively, which translate into Bayes Factors of 0.43, 14.32 and 9.55 relative to the REE-MS model. This indicates that, while the time-variation due to expectations under MSV-learning does not generate a meaningful improvement in the model fit, the parsimonious VAR(1) and AR(1) models yield a further improvement over REE-MS. Among the three learning models, VAR(1) specification emerges as the preferred specification based on the model fit.

Next we discuss the estimated parameter values across all models. The general pattern is that, the parameters are remain fairly similar across REE and REE-MS models. While the differences between the former and MSV-learning model remains minimal, some differences emerge between the small forecasting models of VAR(1) and AR(1) models.

We start by discussing the estimated regime transition probabilities and gain parameters. The exit probability from the normal regime, $1 - p_{11}$, is similar across all four regime-switching models. The posterior mean for this parameter oscillates between 0.99 % and 1.14 %, which translates into an expected duration between 101 and 88 quarters. The posterior HPD intervals for this regime do not exclude each other across all models. However, the exit probability from the ZLB regime, $1 - p_{22}$, turns out quite different between REE-MS and the learning models. The REE model attaches a high probability to leaving the ZLB regime: at the posterior mean, the exit probability is nearly 30 %, implying a short expected duration of only 3.3 quarters. For the learning models, this number decreases to values between 3.8 % and 6.6 %, with implied expected durations between 15 and 26 quarters, much closer to the empirical duration of the ZLB for the U.S. economy. It is also important to note that the implied HPD bands under REE and all three learning models for this parameter are mutually exclusive: the highest upper bound of the 90 % HPD interval across the learning models is 15.3 % (VAR(1) model), whereas the lower bound for REE-MS model is at 23 %. This shows that the REE-MS model favors a low expected ZLB duration due to agents' expectations. But since the model equates subjective and objective expectations about leaving the ZLB regime, this creates a trade-off between generating a short expected duration on the agents' part, and the matching the empirical duration of the

[18]It is readily seen from Tables 1 and 2 that, while the Laplace and MHM estimators of marginal likelihood result in similar values for REE and REE-MS models, there is some discrepancy between these two estimators for the learning models. Therefore our discussion is based on the MHM estimator throughout the remainder of the paper.

ZLB episode. Over the duration of the ZLB period, this result suggests that the agents are constantly surprised since they expect to stay at the ZLB for only 3.3 quarters, but the empirical duration of the episode lasts much longer than that. The learning models, by breaking this tight link between subjective and objective expectations, allow the model to generate a more persistent and realistic ZLB regime.

The resulting filtered (one-sided) regime probabilities for some of the key periods are reported for all regime-switching models in Table 3. Despite the differences between estimated transition probabilities, the models generally agree on the timing of entry to and exit from the ZLB episode. All models attach a probability of 100 % to the normal regime until 2008Q3, while 2008Q4 emerges as the only period with some uncertainty: AR(1)- and VAR(1)-learning models assign a probability of 72 % for the ZLB regime, while this number increases to 92 % for the REE-MS and 99.9 % for the MSV-learning models. The economy unambiguously stays at the ZLB regime from 2008Q4 until 2015Q2 across all models, where the ZLB regime probability remains between 99.8 % and 100 % during this period. From 2015Q3 onwards, the economy switches back to the normal regime. While exiting the regime, we do not observe the same uncertainty as in entering and all models leave the regime sharply.

Among the learning models, the estimated gain parameter is smallest for the MSV-learning model with a mean of 0.0012, and highest for the VAR(1)-learning model with a mean of 0.0064. Compared to the VAR(1) model, the AR(1) model yields a slightly lower gain with a mean of 0.005. These values suggest that the MSV-learning model with the largest information set results in the slowest update of learning parameters, while the remaining two models generate comparable levels of updating speed. The implications for the time-variation in the learning parameters will be discussed further in the next subsection.

Next we turn to shock and persistence parameters that mainly affect the persistence and cross-correlation dynamics in the model. Some parameters that have similar effects on model dynamics are best discussed in groups. The first of these groups is habit persistence λ and risk premium shock persistence ρ_b: both of these generate persistence in consumption Euler and the asset pricing equations. We observe that, on the one hand, for REE and MSV-learning models habit persistence is lower with values of 0.75 and 0.76, compared to the other three models with values between 0.78 and 0.85. On the other hand, for the low habit models, the shock persistence is somewhat higher values of 0.45 and 0.42, compared with the other high habit models where the persistence varies between 0.25 and 0.34. Overall, all parameters all parameters are within the HPD bands of each other, suggesting similar consumption and asset pricing dynamics across all models.

Next considering wage dynamics, we discuss the wage stickiness ξ_w, wage indexation ι_w and wage mark-up shock persistence ρ_w: these parameters mainly affect the wage setting dynamics, and it is readily seen that all three parameters are estimated at similar values across REE,

REE-MS and MSV-learning models. In particular, we observe high degrees of stickiness varying between 0.93 and 0.94, as well as high degrees of indexation varying between 0.79 and 0.82. This is combined with a low shock persistence values in the interval $[0.07, 0.12]$. For the VAR(1) and AR(1) models, we observe similarly low levels of shock persistence with values of 0.14 and 0.1 respectively, but in these models we also obtain a lower wage stickiness with 0.82 and 0.76, combined with a lower wage indexation with 0.67 and 0.55 respectively. This suggests that the change in the information set from MSV-learning to VAR(1) and AR(1)-learning results in more persistence, which in turn yields smaller estimates for these friction parameters.

Looking at the Phillips curve and inflation dynamics with a focus on price stickiness ξ_p, price indexation ι_p and price mark-up shock persistence ρ_p, we observe a similar story compared to the wage dynamics. The parameter estimates are similar across REE, REE-MS and MSV-learning models with a price stickiness between 0.79 and 0.83, a price indexation between 0.1 and 0.11 and a shock persistence between 0.7 and 0.78. This suggests that the roles of shock persistence and indexation change for these models compared to the wage setting dynamics. For the VAR(1) and AR(1) models, the price stickiness comes out similar to the other models with 0.79 and 0.74, while the price indexation is somewhat higher with 0.27 and 0.29. However, this slightly larger indexation is offset by a substantially lower shock persistence with 0.08 and 0.05 respectively. Taken together, these parameters suggest that VAR(1) and AR(1) models generate more persistence internally, which reduces the reliance on the exogenous persistence parameters.

Among the remaining shocks, we observe a similar difference in the estimated investment shock persistence ρ_i, which is lower under VAR(1) and AR(1) models with 0.59 and 0.52, while this number increases to values between 0.76 and 0.81 in the remaining models. Similar to the inflation and wage dynamics, this suggests more internal persistence for the investment dynamics under VAR(1) and AR(1) models. Due to the differences in estimated persistence parameters, the standard deviations for risk premium and investment shocks, η_b and η_i, turn out higher under VAR(1) and AR(1) models compared to the others. These larger standard deviations make up for the lower persistence parameters in the two models, resulting in similar levels of volatility for the corresponding AR(1) shock processes. Finally, the government spending and productivity shocks are both similar across all models specifications in terms of persistence and standard deviations: the productivity shock persistence ρ_a varies between 0.94 and 0.98, while the government spending shock persistence varies between 0.98 and 0.99. Similarly, the impact of productivity on government spending, captured by ρ_{ga}, varies between 0.5 and 0.53 across all models.

In terms of the measurement equation parameters, we find that the estimated steady-state of inflation $\bar{\pi}$ is somewhat lower under VAR(1) and AR(1) models with 0.63 and 0.67 respectively, while it is between 0.73 and 0.76 among the remaining models. This is due to the perceived mean dynamics: for these two models, the perceived mean remains substantially above zero over the estimation period, compared to the MSV-learning model where the perceived mean varies very little prior to the crisis, and the no-learning models where the perceived mean remains fixed at zero. For the remaining two parameters, the common growth rate $\bar{\gamma}$ turns out similar across all models with values between 0.38 and 0.41, while the steady-state labor $\bar{l}$ yields large difference across all models accompanied by wide HPD intervals, suggesting a large uncertainty around the estimates for this parameter.

We do not observe substantial differences in the remaining parameters. In particular, the monetary policy parameters are similar across all models, with HPD intervals well within the range of each other. Inflation reaction ϕ_π ranges between 1.35 and 1.72 across all models, while these number are $[0.85, 0.89]$ for interest rate smoothing ρ, $[0.06, 0.12]$ for output gap reaction r_y, and $[0.14, 0.19]$ for output gap growth reaction r_{dy}. Similarly, for Frisch elasticity of labor supply $\frac{1}{\sigma_l}$, we find values between 0.38 and 0.55, while for elasticity of intertemporal substitution $\frac{1}{\sigma_c}$, these values turn out to be 0.77 and 0.93. The posterior mean for capital adjustment cost ϕ takes on values between 4.84 and 6.47 (with relatively large HPD bands, so that none of the estimated HPD bands are mutually exclusive), while the share of fixed cost in production ϕ_p oscillates between 1.53 and 1.64. Similarly, the capital utilization adjustment cost ψ remains at comparable levels across all models with values between 0.64 and 0.77, and the share of capital in production α ranges between 0.17 and 0.19. The household discount factor β, defined as $\beta = \frac{1}{1+\frac{\bar{\beta}}{100}}$ ranges between 0.997 and 0.998, given the estimated values of $\bar{\beta}$.

Para	Prior Dist	Mean	REE Mean	HPD 90 %		REE-MS Mean	HPD 90 %		MSV Mean	HPD 90 %		VAR(1) Mean	HPD 90 %		AR(1) Mean	HPD 90 %	
ϕ	Normal	4	5.37	3.54	7.14	6.47	4.67	8.41	5.43	3.56	7.39	5.43	3.71	7.17	4.84	2.99	6.82
σ_c	Normal	1.5	1.3	1.11	1.51	1.16	1	1.34	1.16	0.98	1.39	1.07	0.99	1.15	1.11	0.54	1.68
λ	Beta	0.7	0.75	0.66	0.83	0.82	0.76	0.88	0.76	0.68	0.85	0.85	0.8	0.9	0.78	0.68	0.89
ξ_w	Beta	0.5	0.93	0.91	0.95	0.94	0.92	0.95	0.93	0.9	0.95	0.82	0.76	0.89	0.76	0.69	0.83
σ_l	Normal	2	2.11	0.95	3.12	1.83	0.51	3.19	2.12	0.74	3.39	2.65	1.56	3.84	2.12	0.88	3.37
ξ_p	Beta	0.5	0.81	0.76	0.86	0.83	0.78	0.87	0.79	0.74	0.85	0.79	0.72	0.85	0.74	0.64	0.83
ι_w	Beta	0.5	0.81	0.7	0.93	0.79	0.65	0.93	0.82	0.69	0.94	0.67	0.44	0.87	0.55	0.31	0.79
ι_p	Beta	0.5	0.10	0.03	0.17	0.11	0.03	0.21	0.11	0.03	0.21	0.27	0.12	0.43	0.29	0.09	0.58
ψ	Beta	0.5	0.77	0.65	0.9	0.75	0.59	0.9	0.64	0.46	0.83	0.67	0.46	0.86	0.67	0.45	0.87
ϕ_p	Normal	1.25	1.53	1.41	1.66	1.64	1.48	1.79	1.54	1.41	1.67	1.57	1.43	1.71	1.55	1.41	1.7
r_π	Normal	1.25	1.49	1.24	1.75	1.47	1.17	1.82	1.72	1.41	2.04	1.35	1.04	1.68	1.61	1.23	1.98
ρ	Beta	0.75	0.85	0.82	0.89	0.87	0.83	0.91	0.86	0.82	0.89	0.87	0.83	0.92	0.89	0.84	0.94
r_y	Normal	0.125	0.06	0.03	0.10	0.07	0.03	0.12	0.12	0.06	0.18	0.11	0.04	0.18	0.12	0.05	0.19
r_{dy}	Normal	0.125	0.18	0.14	0.21	0.18	0.14	0.21	0.19	0.15	0.23	0.17	0.13	0.22	0.14	0.11	0.18
$\bar{\pi}$	Gamma	0.625	0.74	0.58	0.89	0.76	0.57	0.98	0.73	0.58	0.93	0.63	0.45	0.8	0.67	0.48	0.85
$\bar{\beta}$	Gamma	0.25	0.15	0.06	0.24	0.15	0.05	0.26	0.2	0.08	0.33	0.18	0.06	0.3	0.21	0.07	0.36
$\bar{l}$	Normal	0	-0.11	-1.71	1.55	0.42	-1.58	2.41	1.44	0.1	2.91	0.32	-1.87	2.37	1.3	-0.93	3.44
$\bar{\gamma}$	Normal	0.4	0.38	0.36	0.41	0.41	0.38	0.43	0.41	0.39	0.44	0.41	0.38	0.45	0.39	0.36	0.42
α	Normal	0.3	0.19	0.17	0.22	0.19	0.16	0.22	0.17	0.14	0.21	0.19	0.15	0.22	0.17	0.13	0.2
Mode			-1112.17			-1043.09			-1041.5			-1010.1			-1024.9		
Laplace			-1194.87			1145.02			-1136.75			-1115.38			-1126.85		
MHM			-1194.72			-1145.12			-1144.11			-1112.19			-1123.188		
Bayes F			1			21.54			21.97			35.87			31.09		

Table 1: Estimation period: 1966:I-2016:IV, exogenous switching models.

	Prior		REE			REE-MS			MSV			VAR(1)			AR(1)		
Para	Dist	Mean	Mean	HPD 90 %		Mean	HPD 90 %		Mean	HPD 90 %		Mean	HPD 90 %		Mean	HPD 90 %	
ρ_a	Beta	0.5	0.96	0.95	0.98	0.95	0.93	0.98	0.94	0.91	0.97	0.98	0.96	0.99	0.96	0.93	0.99
ρ_b	Beta	0.5	0.45	0.22	0.69	0.34	0.18	0.51	0.42	0.24	0.63	0.25	0.14	0.36	0.33	0.2	0.46
ρ_g	Beta	0.5	0.98	0.98	0.99	0.98	0.97	0.99	0.98	0.97	0.99	0.98	0.96	1	0.99	0.98	1
ρ_i	Beta	0.5	0.81	0.75	0.88	0.76	0.68	0.84	0.8	0.72	0.88	0.59	0.48	0.69	0.52	0.4	0.64
ρ_r	Beta	0.5	0.11	0.03	0.18	0.1	0.01	0.18	0.15	0.04	0.28	0.22	0.07	0.43	0.17	0.05	0.3
ρ_p	Beta	0.5	0.78	0.69	0.86	0.76	0.65	0.86	0.7	0.58	0.81	0.08	0.01	0.18	0.05	0.01	0.1
ρ_w	Beta	0.5	0.08	0.01	0.14	0.07	0.01	0.14	0.12	0.03	0.23	0.14	0.03	0.25	0.1	0.01	0.2
ρ_{ga}	Beta	0.5	0.51	0.39	0.63	0.5	0.34	0.66	0.52	0.37	0.67	0.53	0.38	0.69	0.51	0.35	0.67
η_a	Inv. Gamma	0.1	0.45	0.49	0.44	0.44	0.4	0.49	0.44	0.39	0.48	0.45	0.4	0.5	0.45	0.4	0.5
η_b	Inv. Gamma	0.1	0.19	0.25	0.21	0.22	0.17	0.27	0.2	0.15	0.25	0.31	0.28	0.34	0.3	0.26	0.34
η_g	Inv. Gamma	0.1	0.48	0.52	0.48	0.49	0.44	0.54	0.49	0.44	0.54	0.49	0.44	0.54	0.49	0.44	0.54
η_i	Inv. Gamma	0.1	0.36	0.41	0.35	0.35	0.29	0.42	0.34	0.28	0.4	0.8	0.72	0.88	0.79	0.71	0.87
η_{r_N}	Inv. Gamma	0.1	0.22	0.24	0.21	0.23	0.2	0.25	0.23	0.2	0.25	0.23	0.2	0.26	0.22	0.2	0.25
$\eta_{r_{ZLB}}$	Gamma	0.03				0.01	0.01	0.01	0.02	0.02	0.03	0.01	0.01	0.01	0.01	0.01	0.01
η_p	Inv. Gamma	0.1	0.06	0.08	0.06	0.07	0.04	0.09	0.08	0.06	0.11	0.1	0.07	0.13	0.08	0.04	0.12
η_w	Inv. Gamma	0.1	0.37	0.41	0.37	0.37	0.32	0.41	0.35	0.3	0.4	0.4	0.36	0.44	0.39	0.35	0.43
$gain$	Gamma	0.035							0.0012	0.0001	0.003	0.0064	0.003	0.01	0.005	0.001	0.011
$1-p_{11}$	Unif.	0.5				0.0099	0.0023	0.0192	0.0122	0.007	0.0276	0.0113	0.0001	0.0264	0.0114	0.0002	0.0274
$1-p_{22}$	Unif.	0.5				0.299	0.2271	0.3737	0.0384	0.0001	0.112	0.0648	0.0017	0.1531	0.0658	0.0026	0.1494
$\bar{r_{zlb}}$	Normal	0.05				0.03	0.03	0.04	0.04	0.03	0.05	0.03	0.03	0.04	0.03	0.03	0.04
Mode			-1112.17			-1043.09			-1041.5			-1010.1			-1024.9		
Laplace			-1194.87			1145.02			-1136.75			-1115.38			-1126.85		
MHM			-1194.72			-1145.12			-1144.11			-1112.19			-1123.188		
Bayes F			1			21.54			21.97			35.87			31.09		

Table 2: Estimation period: 1966:I-2016:IV, exogenous switching models.

Date	Model			
	AR(1)	VAR(1)	MSV	REE-MS
08Q3	0	0	0	0
08Q4	0.72	0.72	0.999	0.92
09Q1	0.99	0.99	1	0.995
15Q2	0.998	0.998	1	0.998
15Q3	0	0	0	0

Table 3: Estimated ZLB regime probabilities during some of the important periods.

Learning Parameters

Our simulation analysis in Section 2 suggests that sharp regime switches, combined with sufficiently large gain values lead to a jumping effect on some of the learning coefficients. In this section, we analyze the implied time-variation in our estimated learning models and check whether the jumping phenomenon arises in some of the learning parameters during the switch to the ZLB period. Among the seven forward-looking variables, we focus on three variables that are characterized by large drops during the crisis period and the subsequent switch to the ZLB regime, namely asset prices q_t, consumption c_t and investment I_t. The general pattern that we observe is that, the learning parameters are more sensitive and react more to the ZLB switch as the forecasting rule becomes smaller. Hence we observe the the largest changes in learning parameters with the AR(1) model, while the MSV model yields the smallest changes. Further, we observe that the learning process reacts to the crisis through different parameters depending on the forecasting rule, as will be discussed below.

Figure 6 shows the perceived mean and persistence parameters for the AR(1)-learning model, along with the corresponding 90 % HPD intervals[19]. For all three variables, we observe that the perceived means jump down immediately following the crisis, which is more pronounced for asset prices and investment compared to consumption. For the perceived persistence parameters, we observe sizeable upward jumps for asset prices and investment, while for consumption there is a smaller jump in the opposite direction. As such, the learning patterns for investment and asset prices, and to a smaller degree also consumption, show similarities to simulation exercises. The same results also hold labor l_t and rental rate of capital rk_t with upward jumps in perceived persistence and downward jumps in perceived mean, whereas for inflation and real wage these structural breaks do not arise since there are also no sharp changes in the data[20].

Figure 7 shows a selected subset of the learning parameters for the VAR(1) model[21]. As

[19]In order to compute the HPD intervals for the learning parameters, we use the final 20 % of the MCMC sample for each model, which is further thinned with a factor 0.2, yielding a sample size of 10000 for the parameters. We then re-run the filter over these parameter draws to to obtain the credible intervals for the learning parameters.

[20]The corresponding figures for these variables are omitted for brevity

[21]Similar to the AR(1) model, only a small portion of the learning parameters are displayed given the size of

mentioned above, the learning processes react differently depending on the forecasting rule, which already becomes visible by comparing the AR(1) and VAR(1) models. Looking at the perceived mean parameters, we observe that the drops during the crisis period are substantially smaller. However, looking at the second and third columns, we observe that the feedback from lagged interest rates and inflation jump. Particularly for lagged interest rate, the jumps are towards zero, suggesting a weakened impact from interest rates. Similarly the last column shows the feedback from lagged investment, which show the jumping pattern during the crisis period, although to a much smaller extent for the asset prices[22].

Finally, figure 8 shows some of the learning parameters for the MSV-learning model. In this case, we observe that the largest change arises in the perceived mean parameters, all of which start to rapidly decrease following the crisis. As such, these variables show a disproportionately large response following the crisis, compared to their pre-crisis fluctuation levels. This large post-crisis response also explains the small estimated gain in the MSV model. While a larger gain could generate more fluctuations pre-crisis and possibly improve the model fit, it would also make the post-crisis response substantially larger. The following two columns, similar to the VAR(1) model, show the feedback from lagged interest rates and inflation respectively. In this case we observe more gradual responses rather than jumps, particularly for interest rates, which is not surprising given the small gain value. Nevertheless, we observe a similar change in direction for interest rates where the parameters move towards zero, suggesting a smaller impact from interest rates on these variables. The last column shows the perceived correlation parameters between the given variables and government spending process, which is of particular interest for the MSV model since the other two models assume unobserved shocks. We see that the perceived correlation moves away from zero for all three variables, suggesting a larger impact from an increased government spending. This is in line with the perceived weakened interest rate response, since it makes monetary policy less likely to offset any changes in government spending.

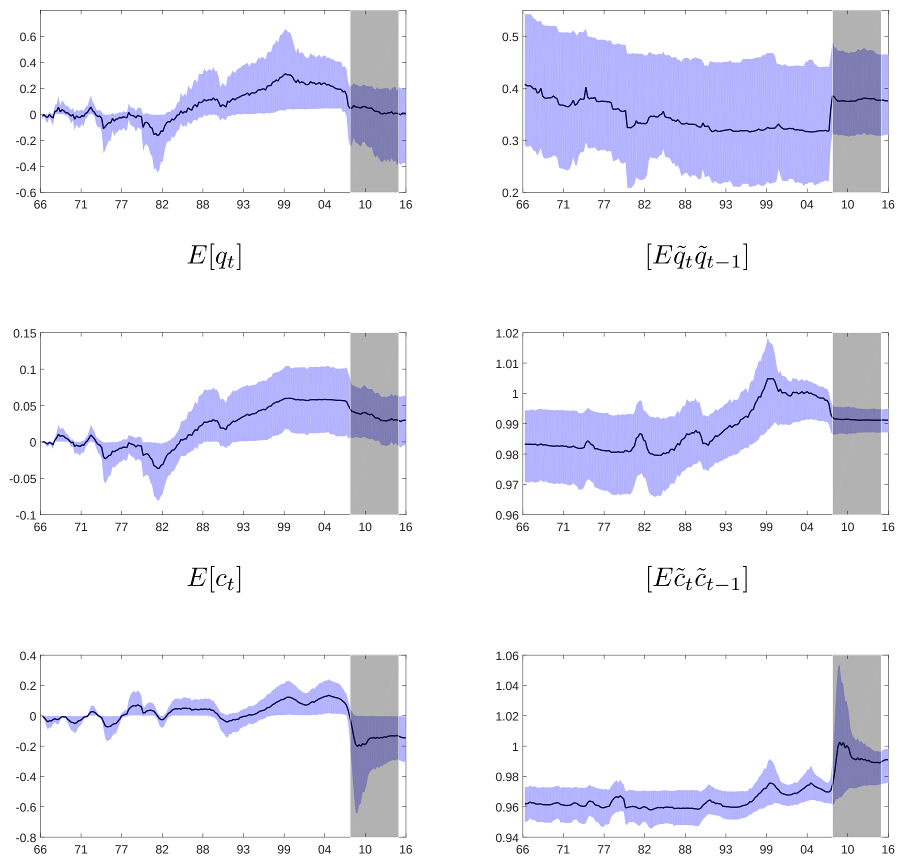
To see the effect of these changes in the perceived mean parameters on the model fit, Figure 9 plots the in-sample forecasts for the growth rates of output, consumption, investment and wages for the MS-REE and all three learning model[23]. A known issue with the REE models over the post-crisis period is the over-prediction of these growth rates: the sudden downward shift in the interest rates implies an increase in the growth rates in the model variables, whereas in fact the growth rates have been slightly lower than the pre-crisis historical averages for output, consumption and investment. As a consequence, the models tend to over-predict these variables

the learning parameter matrix.

[22]Note that we omit the learning parameters on own lagged variables with the exception of investment: the lagged asset price is not present in the learning matrix, and the response of consumption to lagged consumption does not show a meaningful change over the relevant period.

[23]i.e. these plots simply break down the model fit by period.

if no additional structural break is introduced into the model. It is readily seen in Figure 9d that this is indeed the case under MS-REE for the growth rates of output, consumption and investment. As opposed to this, Figure 9a-c shows that this over-prediction issue does not arise in the learning models. We interpret this downward shift in the learning models as a consequence of the time-variation in the learning parameters. Accordingly, the lower growth rates over the post-crisis period emerge as a simple consequence of a pessimistic wave reflected in the perceived mean parameters.



$E[q_t]$ $[E\tilde{q}_t\tilde{q}_{t-1}]$

$E[c_t]$ $[E\tilde{c}_t\tilde{c}_{t-1}]$

$E[I_t]$ $[E\tilde{I}_t\tilde{I}_{t-1}]$

Figure 6: Selected learning coefficients: AR(1) model. q_t, c_t and I_t denote asset prices, consumption and investment respectively, whereas $\tilde{q}_t$, $\tilde{c}_t$ and $\tilde{I}_t$ denote the corresponding demeaned variables.

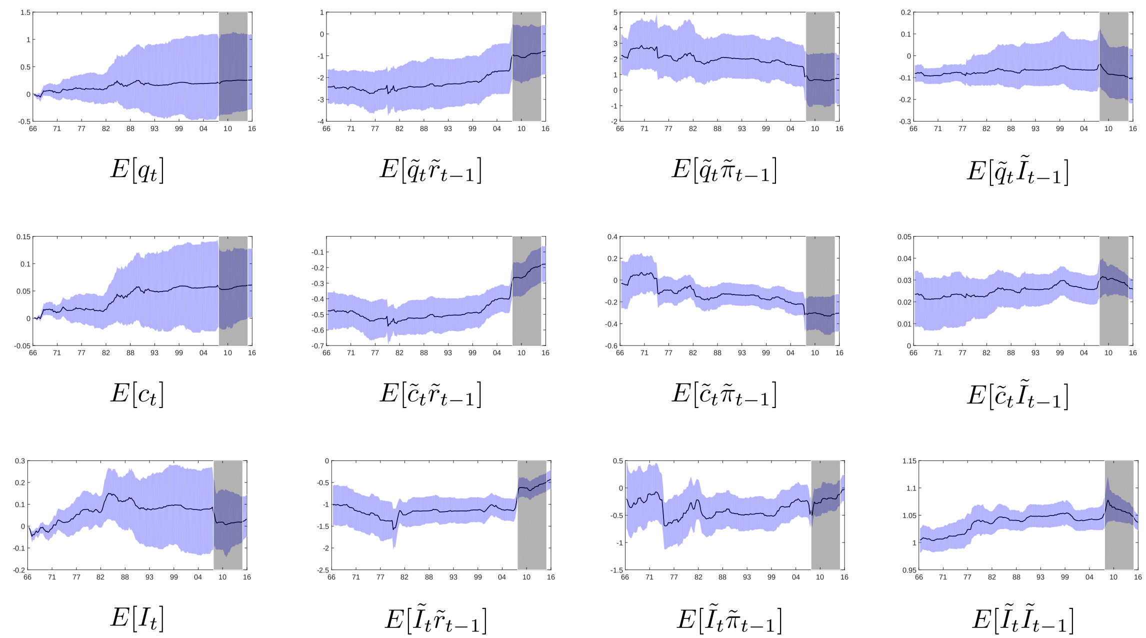


Figure 7: Selected learning coefficients: VAR(1) model. q_t, c_t, I_t, r_t and π_t denote asset prices, consumption, investment, nominal interest rates and inflation respectively, and the variables with a tilde denote their demeaned counterparts.

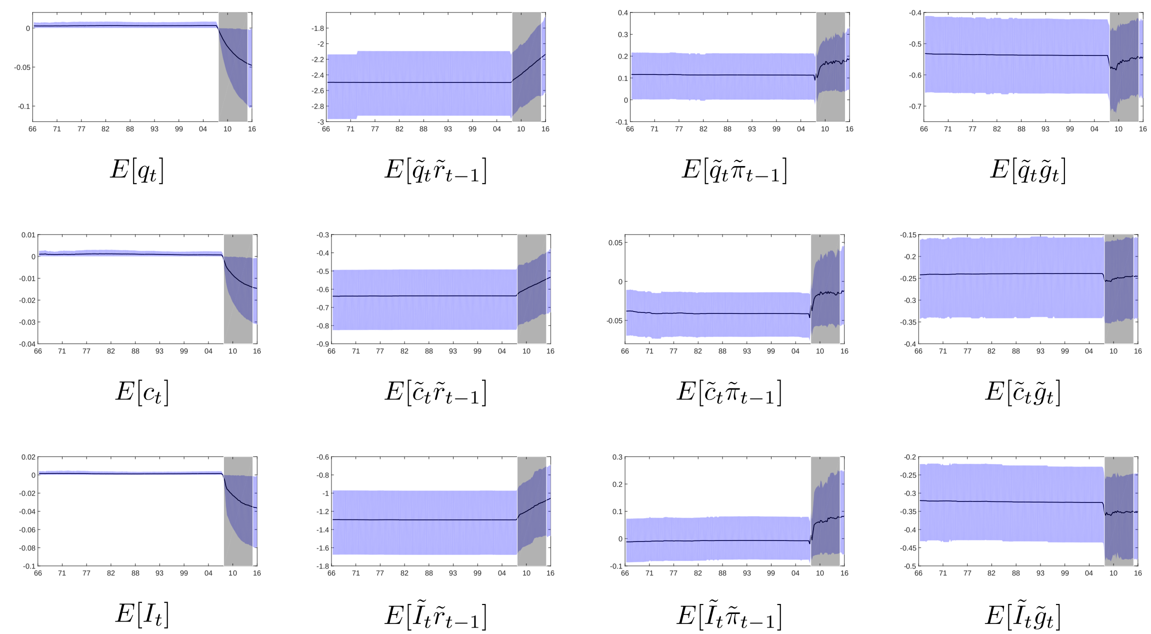
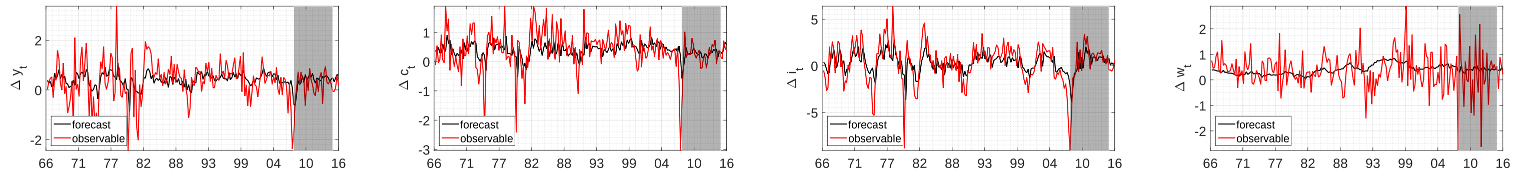
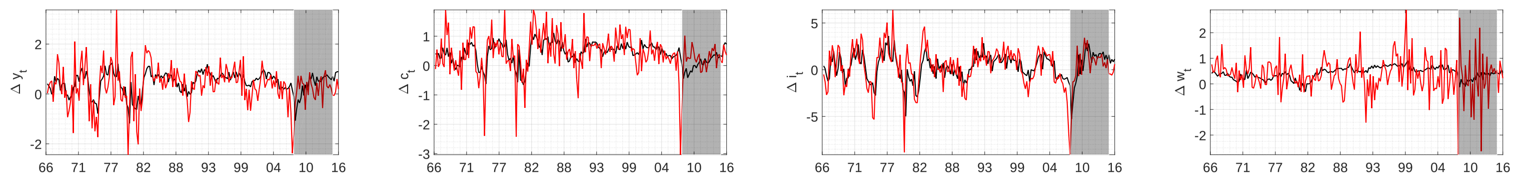


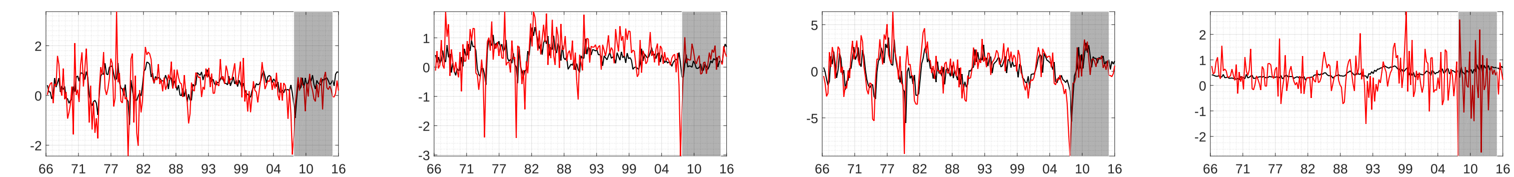
Figure 8: Selected learning coefficients: MSV model. q_t, c_t, I_t, r_t, π_t and g_t denote asset prices, consumption, investment, nominal interest rates, inflation and government spending process respectively, and the variables with a tilde denote their demeaned counterparts.



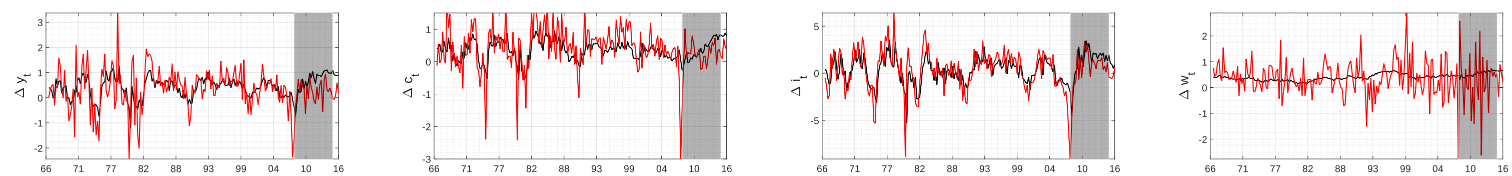
(a) AR(1)



(b) VAR(1)



(c) MSV



(d) REE-MS

Figure 9: In-sample forecasts (i.e. the forecasting step of the filter) for all observables: AR(1) and MS-REE models respectively.

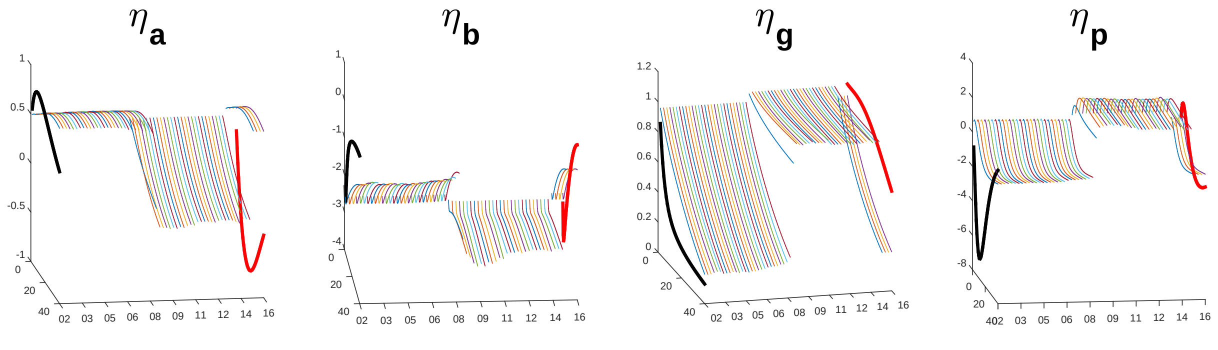
Impulse Responses

We next compare the impulse response functions (IRF) between learning and REE-MS models. Due to the time variation in expectations in learning models, the implied IRFs are period specific. Therefore for the learning models, we focus only on the last 15 years of the sample, i.e. over the period 2002-2016. The corresponding IRFs for output, consumption and investment are presented in Figures 10 through 12, where the black and red lines at the left and right sides denote the impulse responses for the REE-MS model under the normal and ZLB regimes respectively. We focus on four shocks, namely the productivity η_a, risk premium η_b, government spending η_g and price mark-up η_p.

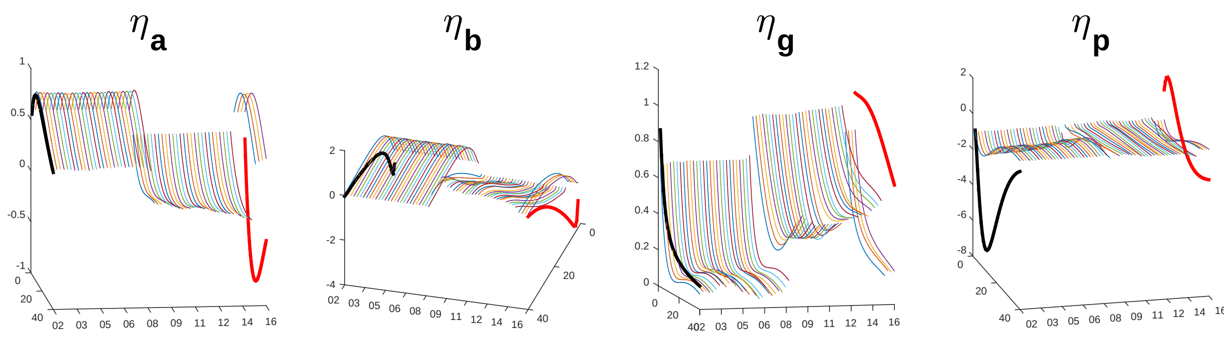
Starting with the responses of output in Figure 10, we observe that the IRFs are characterized by two jumps in 2008 and 2015, with the entry to and exit from the ZLB regime. The overall time variation before and after the crisis varies across models. For the AR(1) model in the first panel, the time variation for all shocks is fairly small compared to the jump in 2008. For the VAR(1) model, especially after the crisis period, we observe more time variation. Particularly for productivity and government spending shocks, the impulse responses gradually move in the direction of the REE-MS (i.e. the red line), until the regime switches back to the normal regime in 2015. For the MSV-learning model, we observe the gradual movement towards REE-MS impulse during the ZLB regime. Accordingly, the learning process manifests itself in these IRFs as a slow convergence towards the REE-MS model. As such, the learning and rational models generate different impulse responses at the beginning of the ZLB regime, where the difference slowly gets smaller as the system spends more time in the ZLB regime. Figures 11 and 12 show that a similar time variation also occurs for consumption and investment variables across all models.

A second important point related to IRFs is the difference between regime specific impulse responses for learning and REE-MS models. The general pattern in Figures 10 through 12 is that, the difference between regime-specific impulse responses under REE-MS models (i.e. the difference between black and red lines) is typically larger than the difference for learning models. To see this formally, we consider the following exercise: we choose five year periods during the normal regime before the crisis (2002:I-2006:IV) and during the ZLB regime after the crisis (2010:I-2014:IV). For the learning models, we then compute the median differences in the impulse responses between the two regimes, along with the minimum and maximum differences in the IRFs to serve as a pseudo confidence interval for these differences. Figure 13 plots these IRF differentials, along with the corresponding difference under REE-MS case. What becomes quickly evident is that, the differences in learning models are smaller than the differences in the REE model in a vast majority of cases: with the exception of the b-shock in VAR(1) and MSV models, the black line (learning model) and the associated pseudo confidence interval remains below the blue line (REE model). This result suggests that the REE-MS model may overestimate the impact of the ZLB regime on the propagation of shocks. An implication of these differences in impulse responses is on fiscal multipliers: a standard finding with the REE models is that, fiscal multipliers are substantially larger when the ZLB constraint is binding, compared to when it is not binding. Figure 14 shows the regime-specific cumulative fiscal multipliers for the REE-MS and all three learning models[24]. Over a 10-year period, the REE-MS model implies that the cumulative multiplier is up to 3.5 times larger in the ZLB regime, compared to the normal regime. For the VAR(1)- and MSV-learning models, the ratio remains similar to the REE-MS model up to 12 quarters, after which it remains below the REE-MS ratio over all horizons: for the VAR(1) model, the ratio reaches a maximum of 3, while for the MSV-learning model the maximum ratio is around 2.5. For the AR(1) model, the ratio is even smaller, with a maximum ratio staying below 2. These results imply that the impact of the ZLB constraint on fiscal multipliers is different under learning models: while the short-term effects are ambiguous, the multipliers are uniformly smaller under all learning models over longer horizons.

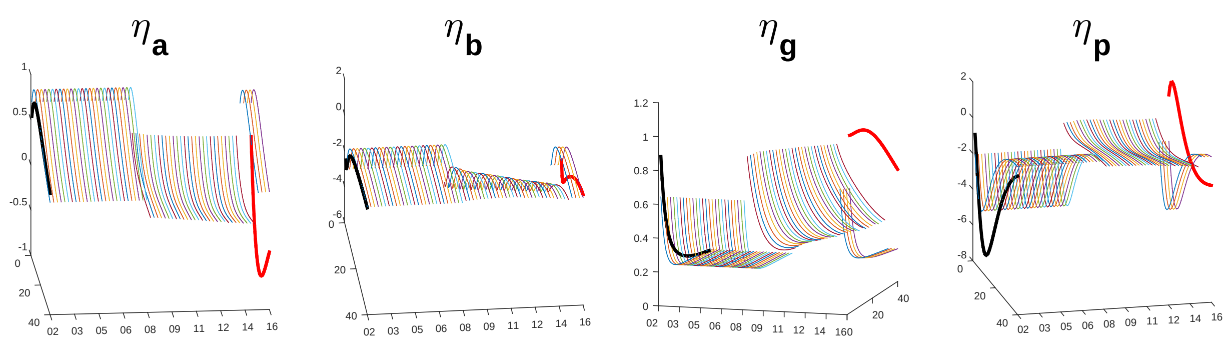
[24]The fiscal multiplier is computed as $FM = \frac{\sum_{i=1}^{N} \frac{\partial y_i}{\partial \eta_g}}{\sum_{i=1}^{N} \frac{\partial g_i}{\partial \eta_g}}$, i.e. the cumulative response of output to a one standard deviation shock, divided by the cumulative response of government spending process over the same period. In the figures we set $N = 40$, to consider multipliers up to 10 years.



(a) AR(1)

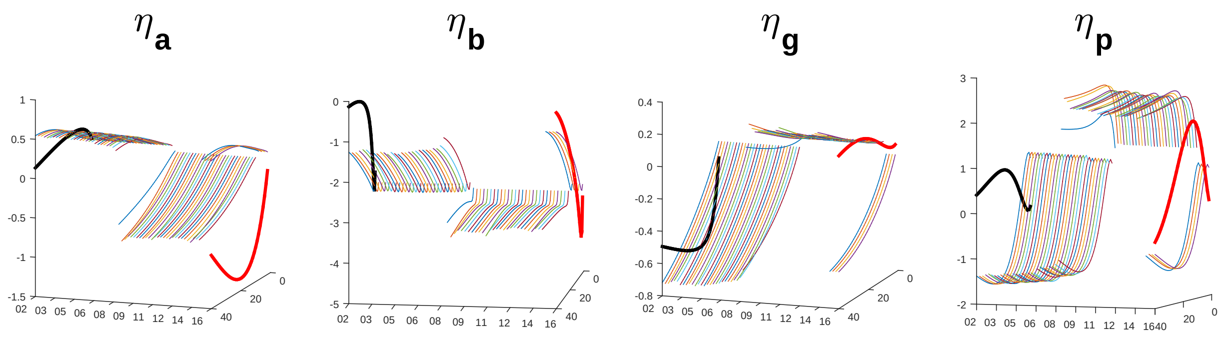


(b) VAR(1)

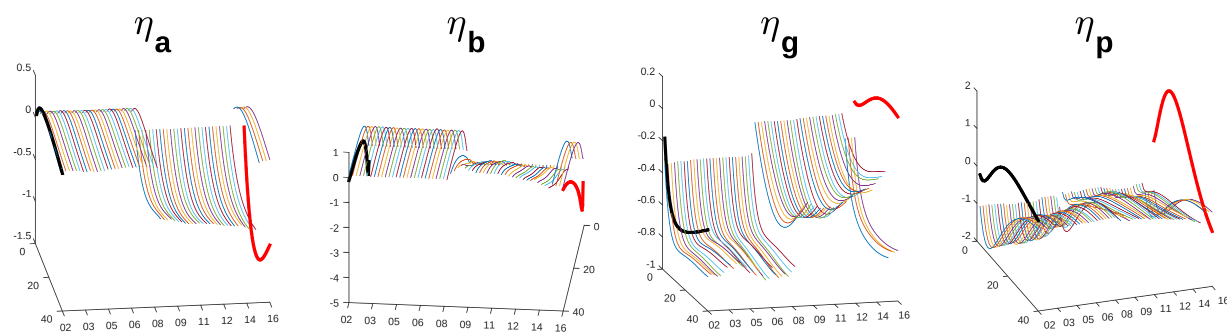


(c) MSV

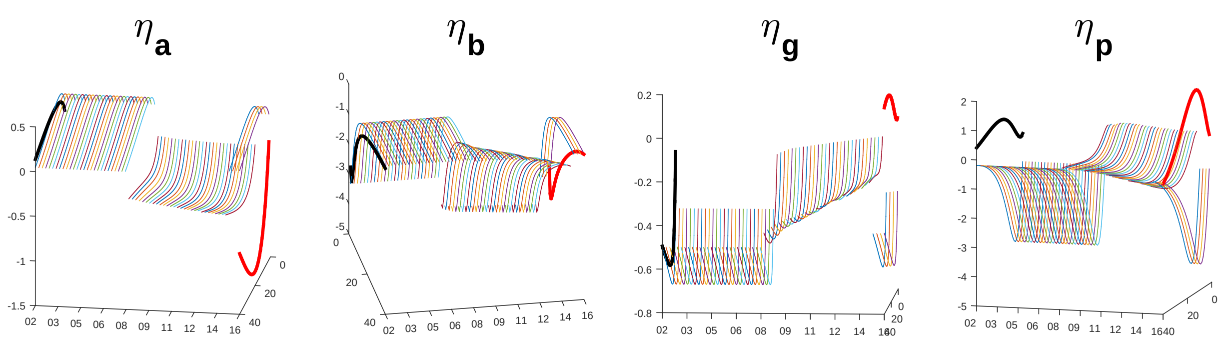
Figure 10: **Output:** Comparison of learning IRFs with REE IRFs. Each IRF shows a one standard deviation shock of η_a,η_b,η_g,η_p respectively.



(a) AR(1)

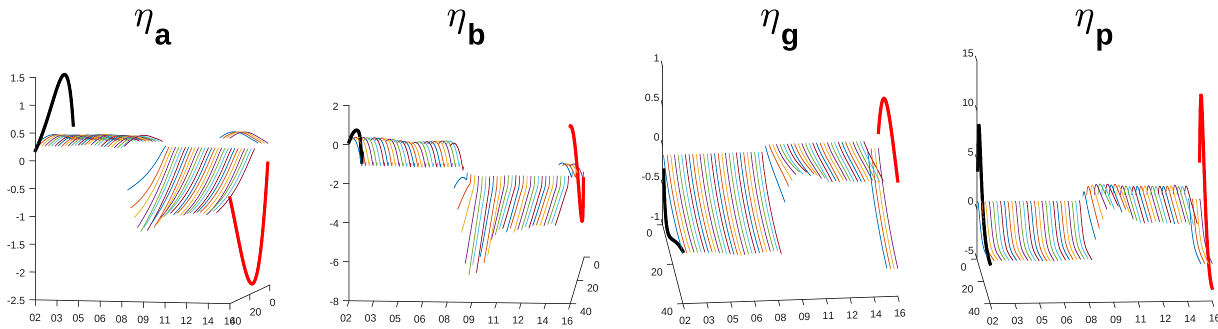


(b) VAR(1)

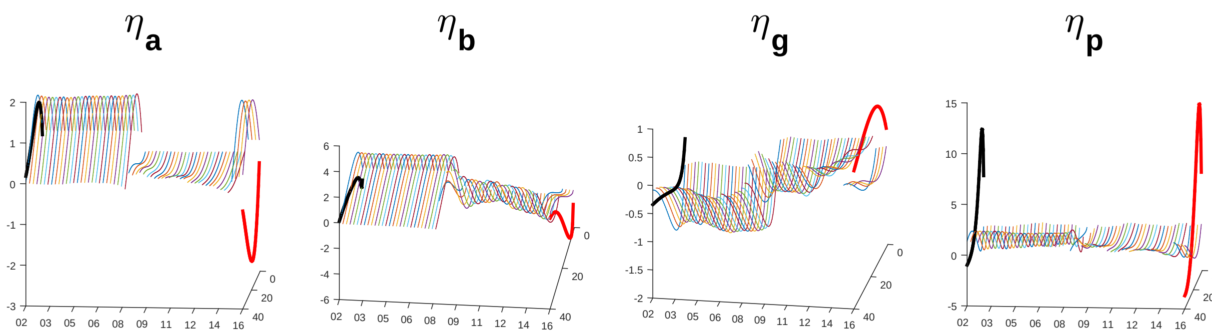


(c) MSV

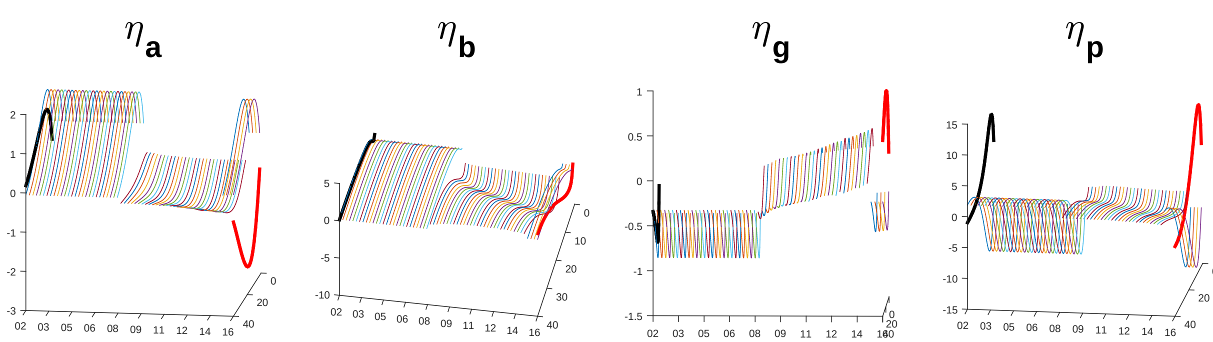
Figure 11: **Consumption:** Comparison of learning IRFs with REE IRFs. Each IRF shows a one standard deviation shock of η_a,η_b,η_g,η_p respectively.



(a) AR(1)

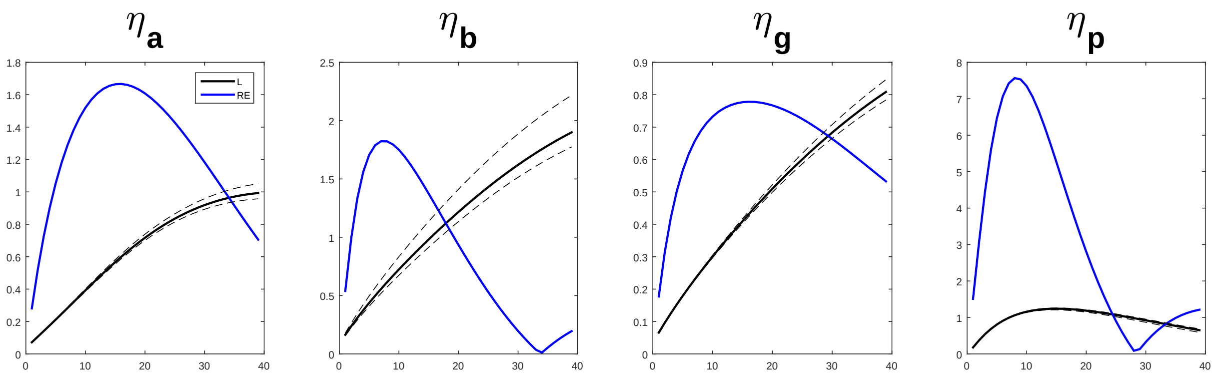


(b) VAR(1)

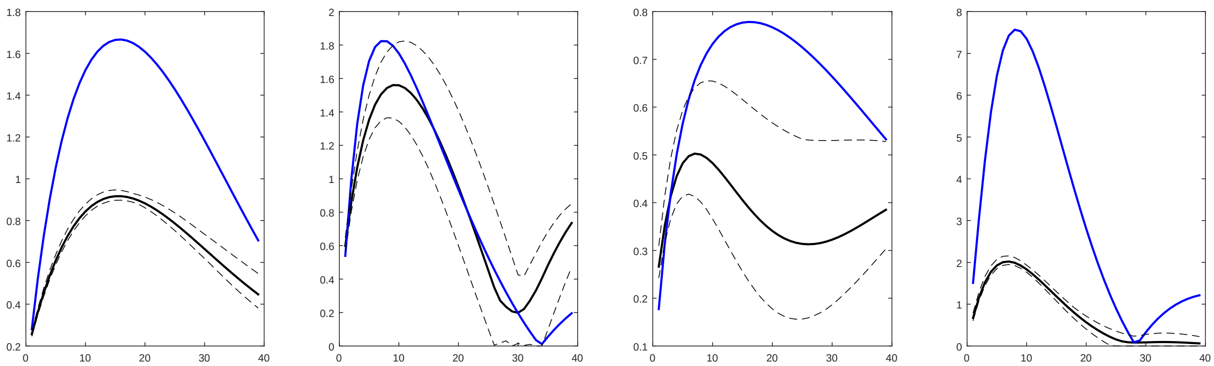


(c) MSV

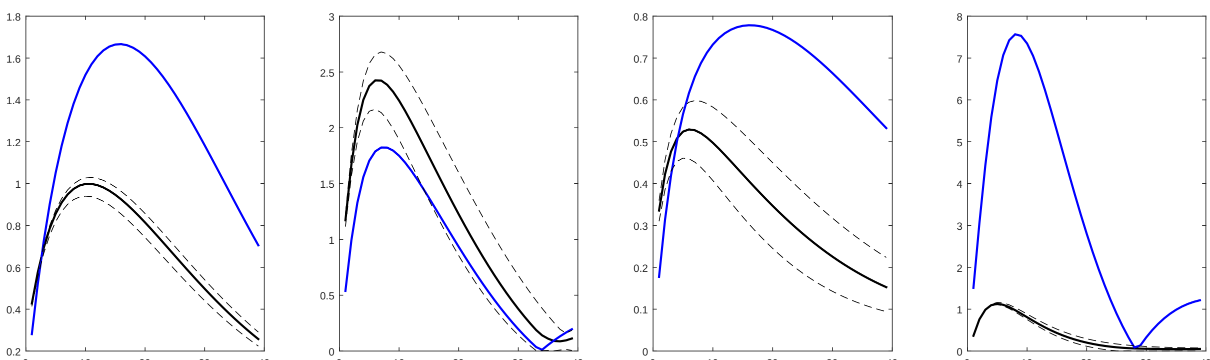
Figure 12: **Investment:** Comparison of learning IRFs with REE IRFs. Each IRF shows a one standard deviation shock of η_a,η_b,η_g,η_p respectively.



(a) AR(1)

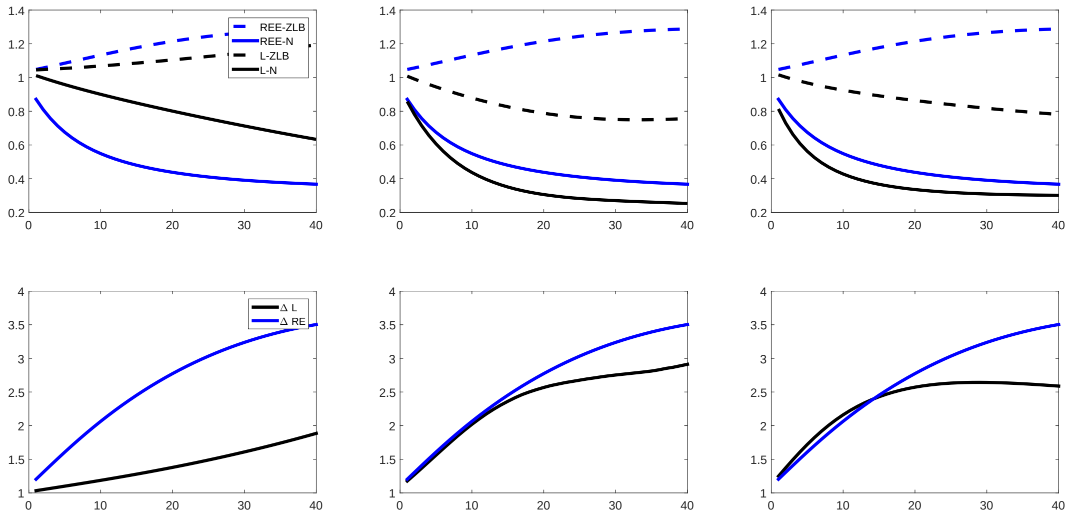


(b) VAR(1)



(c) MSV

Figure 13: **Output:** Impulse response differentials between normal and ZLB regimes.



(a) AR(1) (b) VAR(1) (c) MSV

Figure 14: Comparison of cumulative fiscal multiplier differences between REE and learning models over the normal and ZLB regimes.

4.1 Endogenous Switching Models

In this section, we consider an extension of the learning models with endogenous switching. While this will not lead to substantial changes in terms of our estimation results, having the endogenous transition probabilities will be an important stepping stone for the counterfactual simulations that we consider in the next section. Accordingly, we define the transition probabilities for the normal and ZLB regimes as follows:

$$p_{11}(t) = \frac{\theta}{\theta + exp(-\phi(r_t^* - \underline{r}))}, p_{22}(t) = \frac{\theta}{\theta + exp(\phi(r_t^* - \underline{r}))}$$

with $1 - p_{11}$ the exit probability from the normal regime, and $1 - p_{22}$ the exit probability from the ZLB regime. r_t^* denotes the shadow rate, and it is defined as follows:

$$r_t^* = \rho r_{t-1}^* + (1-\rho)(\phi_\pi \pi_t + \phi_y gap_t) + \phi_{dy} \Delta gap_t,$$

which means that it is the interest rate that would prevail in the absence of monetary policy shocks and the ZLB constraint. According to our transition functions, the probability of entering the ZLB regime increases as the interest rate approaches zero or falls below zero. In our estimations, we fix the first hyperparameter $\theta = 1$, and estimate the second hyperparameter ϕ with a Gamma prior with mean 0.2 and standard deviation 0.1. The second hyperparameter determines the shape of the transition function, where high values of ϕ sharpy increase the probability of being in the ZLB regime when interest rates get close to 0, and low values increase the probability more gradually.

Tables 4 and 5 show the estimation results for all three learning models. The VAR(1)- and MSV-learning models, the marginal likelihood is better compared to their exogenous switching counterparts, while for the AR(1) model there are no discernible differences. For the transition probability hyperparameters, the entry and exit parameters remain fairly close to each other. For the AR(1) model, the parameter on exiting the normal regime is lower at 0.1 compared to the ZLB parameter at 0.18, which suggests that the ZLB entry function is smoother than the ZLB exit function. Nevertheless, these two parameters have relatively large HPD bands covering both sharp and smooth cases of the transition functions, suggesting that the exact shape of the function is not important for the transition to and from the ZLB regime.

Table 6 shows the estimated regime probabilities, along with the estimated shadow rates: we observe that the entry and exit periods look similar to the exogenous switching models, where the all models enter into the ZLB regime from 2008Q4 onwards, and exit the regime from 2015Q3 onwards. In this case, different from the exogenous switching models, the ambiguity over the periods 2008Q4 disappears, where all models estimate a ZLB regime probability of 1. It is also interesting to note that in 2015Q3, the shadow rate exceeds zero 0 in AR(1) and VAR(1) models, while it is still slightly below 0 in the MSV-learning model. Nevertheless, all models estimate a ZLB probability of 0 over this period. Figure 15 shows the estimated shadow rate for the AR(1) model, along with the transition functions: it is readily seen that the transition functions are estimated very precisely, leading to tight HPD intervals for these functions. The implied time path of the shadow rate and transition functions for the other two models look similar, which are omitted here.

We use the endogenous switching models to run a set of counterfactual simulations in the next section.

Para	Prior Dist	Mean	MSV Mean	MSV HPD 90 %		VAR(1) Mean	VAR(1) HPD 90 %		AR(1) Mean	AR(1) HPD 90 %	
ϕ	Normal	4	5.26	3.39	7.23	5.03	3.41	6.76	4.7	2.93	6.53
σ_c	Normal	1.5	1.17	0.98	1.37	1.12	1.03	1.21	1.05	0.59	1.56
λ	Beta	0.7	0.75	0.67	0.84	0.82	0.76	0.88	0.78	0.67	0.88
ξ_w	Beta	0.5	0.93	0.9	0.95	0.84	0.76	0.91	0.76	0.68	0.82
σ_l	Normal	2	2.14	0.83	3.53	2.66	1.5	3.85	1.99	0.76	3.29
ξ_p	Beta	0.5	0.79	0.74	0.84	0.75	0.68	0.82	0.78	0.7	0.85
ι_w	Beta	0.5	0.81	0.68	0.94	0.7	0.49	0.91	0.51	0.28	0.75
ι_p	Beta	0.5	0.12	0.03	0.23	0.34	0.16	0.53	0.27	0.1	0.5
ψ	Beta	0.5	0.61	0.41	0.78	0.56	0.36	0.77	0.54	0.29	0.79
ϕ_p	Normal	1.25	1.55	1.41	1.69	1.57	1.43	1.71	1.54	1.4	1.68
r_π	Normal	1.25	1.81	1.51	2.1	1.48	1.09	1.88	1.63	1.19	2.07
ρ	Beta	0.75	0.86	0.82	0.89	0.86	0.82	0.91	0.95	0.93	0.97
r_y	Normal	0.125	0.12	0.08	0.17	0.14	0.09	0.18	0.06	0.01	0.1
r_{dy}	Normal	0.125	0.2	0.16	0.23	0.18	0.14	0.22	0.16	0.13	0.19
$\bar{\pi}$	Gamma	0.625	0.75	0.59	0.93	0.67	0.48	0.88	0.69	0.48	0.88
$\bar{\beta}$	Gamma	0.25	0.18	0.07	0.29	0.17	0.06	0.29	0.19	0.06	0.33
$\bar{l}$	Normal	0	1.91	0.63	3.12	1.88	0.12	3.56	0.58	-1.22	2.71
$\bar{\gamma}$	Normal	0.4	0.41	0.39	0.44	0.42	0.39	0.45	0.4	0.36	0.43
α	Normal	0.3	0.17	0.14	0.2	0.18	0.15	0.21	0.16	0.12	0.2
Mode			-1045.1			-1008.3			-1026.2		
Laplace			-1154.07			-1116.51			-1125.76		
MHM			-1134.25			-1108.17			-1123.32		
Bayes F.			26.26			37.59			31.01		

Table 4: Estimation period: 1966:I-2016:IV, endogenous switching models.

	Prior		MSV			VAR(1)			AR(1)		
Para	Dist	Mean	Mean	HPD 90 %		Mean	HPD 90 %		Mean	HPD 90 %	
ρ_a	Beta	0.5	0.94	0.91	0.97	0.97	0.95	0.98	0.96	0.93	0.99
ρ_b	Beta	0.5	0.43	0.24	0.63	0.26	0.14	0.37	0.33	0.2	0.46
ρ_g	Beta	0.5	0.98	0.97	0.99	0.99	0.98	1	0.98	0.96	1
ρ_i	Beta	0.5	0.8	0.72	0.87	0.61	0.5	0.72	0.5	0.38	0.63
ρ_r	Beta	0.5	0.12	0.03	0.22	0.23	0.08	0.38	0.11	0.02	0.2
ρ_p	Beta	0.5	0.7	0.57	0.82	0.07	0.01	0.16	0.05	0.01	0.11
ρ_w	Beta	0.5	0.12	0.02	0.22	0.15	0.04	0.27	0.1	0.01	0.2
ρ_{ga}	Beta	0.5	0.51	0.35	0.67	0.53	0.37	0.69	0.51	0.36	0.66
η_a	Inv. Gamma	0.1	0.44	0.39	0.49	0.45	0.4	0.5	0.45	0.4	0.51
η_b	Inv. Gamma	0.1	0.2	0.15	0.25	0.3	0.27	0.33	0.3	0.26	0.34
η_g	Inv. Gamma	0.1	0.49	0.44	0.54	0.49	0.45	0.55	0.48	0.44	0.53
η_i	Inv. Gamma	0.1	0.34	0.28	0.4	0.81	0.73	0.89	0.78	0.7	0.86
η_{r_N}	Inv. Gamma	0.1	0.23	0.2	0.26	0.23	0.21	0.26	0.22	0.2	0.25
$\eta_{r_{ZLB}}$	Gamma	0.03	0.03	0.02	0.03	0.02	0.02	0.03	0.01	0.01	0.01
η_p	Inv. Gamma	0.1	0.08	0.05	0.11	0.11	0.08	0.15	0.07	0.04	0.11
η_w	Inv. Gamma	0.1	0.35	0.3	0.4	0.4	0.36	0.44	0.38	0.34	0.42
$gain$	Gamma	0.035	0.0011	0.0001	0.0026	0.0047	0.0013	0.0084	0.0046	0.0001	0.0105
r_{zlb}^{-}	Normal	0.05	0.04	0.03	0.05	0.04	0.03	0.05	0.03	0.03	0.04
$\frac{\Phi_N}{1000}$	Gamma	0.2	0.17	0.03	0.38	0.19	0.03	0.39	0.1	0.01	0.25
$\frac{\Phi_{zlb}}{1000}$	Gamma	0.2	0.2	0.05	0.38	0.21	0.04	0.4	0.18	0.03	0.36
Mode			-1045.1			-1008.3			-1026.2		
Laplace			-1154.07			-1116.51			1125.76		
MHM			-1134.25			-1108.17			-1123.32		
Bayes F.			26.26			37.59			31.01		

Table 5: Estimation period: 1966:I-2016:IV, endogenous switching models.

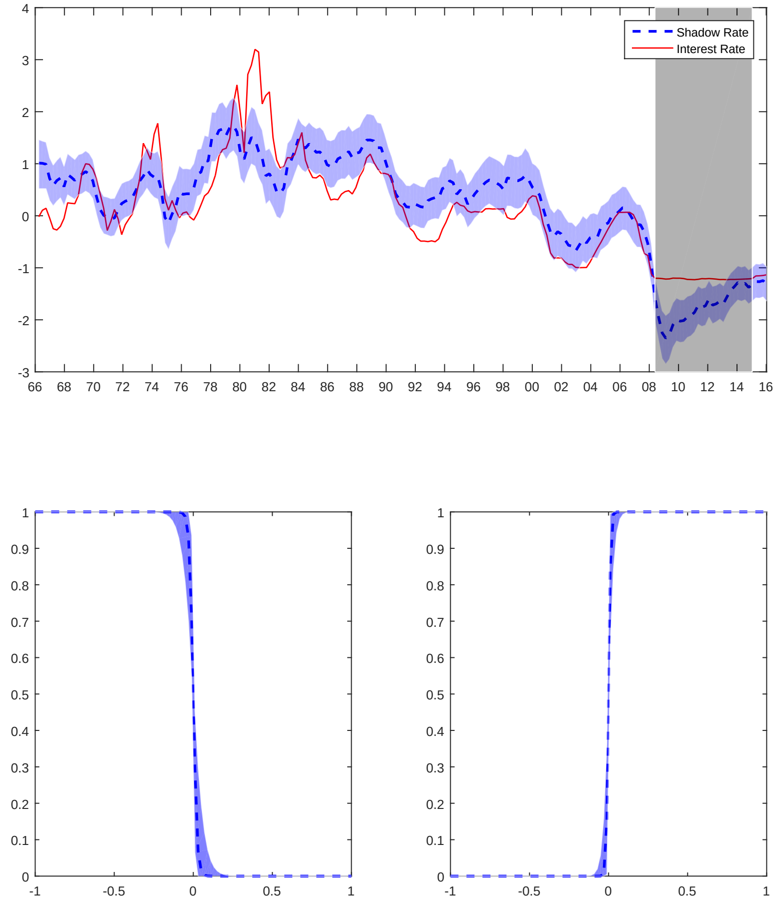


Figure 15: Shadow rate over the sample period for the AR(1) model, along with the transition probability functions.

Date	Model					
	AR(1)		VAR(1)		MSV	
	Regime prob.	Shadow rate	Regime prob.	Shadow rate	Regime prob.	Shadow rate
08Q3	0	0.27	0	0.5	0	0.32
08Q4	1	-0.37	1	-0.16	1	-0.34
09Q1	1	-0.74	1	-0.59	1	-0.78
15Q2	1	-0.05	1	-0.04	1	-0.08
15Q3	0	0.01	0	0.01	0	-0.04

Table 6: Estimated ZLB regime probabilities and shadow rates in endogenous switching models.

5 Counterfactual Simulations (Preliminary)

As outlined in the previous sections, the models uniformly agree that the empirical duration of the ZLB regime is 28 quarters over the period 2008Q3 to 2015Q2, after which the economy returns back to the normal regime. In this section we run a number of counterfactual exercises to see the effects of learning on macroeconomic outcomes. With a particular focus on inflation and output growth, we consider the following exercise using the estimated endogenous switching models: using the MCMC draws for each of the learning models, we first run the filter up to period 2008Q4, where the economy first enters the ZLB regime[25]. From 2008Q4 onwards, we simulate the economy for the following 32 quarters.

Figure 16 shows the resulting distributions of the ZLB duration for all three learning models. It is readily seen that this already reveals substantial differences among the learning models. For the AR(1) model, we obtain a bi-modal distribution for the ZLB duration, with one peak around 4 quarters with a frequency of around 5 %, and another peak at the boundary of 32 quarters with a frequency of around 17 %. As such, this model already attaches a high probability to still being in the ZLB regime after 8 years. For the VAR(1) model, where we observe a unimodal distribution peaking at 32 quarters with a probability close to 60 %. For this model, the majority of simulations are still in the ZLB regime at the end of the simulation. We observe a different pattern for the MSV model: while the distribution is still unimodal similar to the AR(1) model, we see the larger peak around 5 quarters with a frequency of 14 %, while the second peak at 32 quarters has a small probability of around 1.5 %. Accordingly, the first two learning models attach a substantially large probability to still being in the ZLB regime after 8 years, while the MSV model predicts an exit after only a year. A potential candidate for explaining this large difference between the learning models is the estimated gain value: recalling the estimated for the endogenous switching models, the gain parameters have similar posterior distributions with means of 0.0046 and 0.0047 for the VAR(1) and AR(1) models respectively, while this number decreases to 0.0011 for the MSV model. To see how the slower learning dynamics affect the ZLB frequency distribution, we consider two more experiments with a gain value of 0 (i.e. no learning dynamics), and a fairly large gain value of 0.01 (which roughly corresponds to the largest upper bound of the 90 % HPD interval for all learning models).

Table ?? shows the ZLB duration probabilities for several cases, along with the counterfactual averages for inflation and output growth. First looking at the moments for the benchmark case with estimated gains, we observe that the resulting counterfactual moments are in line with the ZLB duration frequencies: for the VAR(1) model that predicts the longest durations,

[25]Similar to the previous section, we use a total of 10000 MCMC draws for this exercise. The parameter draws are taken from the last 20 % of the MCMC simulation where we use a thinning factor of 0.2, corresponding to 10000 draws.

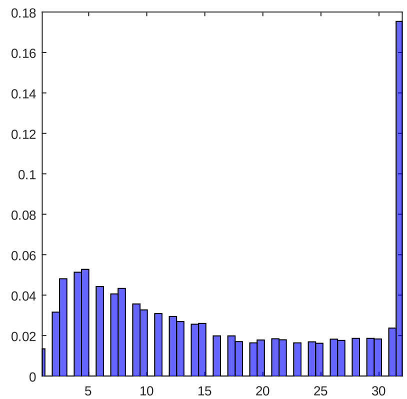
both the average inflation and output growth values are below 0 and substantially lower than the empirical averages over the simulation period. For the AR(1) model, the average inflation level is comparable and close 0, while the average output growth is higher than the VAR(1) case and in fact very close to the empirical average 0.19. For the MSV model, both inflation and output growth are well above the empirical averages, which is not surprising since the model also predicts very short-lived ZLB regimes.

Next we look at the corresponding moments and ZLB durations when we shut off the learning dynamics: it is readily seen that, for all models, the probability of long-lived ZLB regimes decreases in the absence of learning. For the AR(1) model, ZLB durations of longer than 5 and 7 years fall down to 26 % and 13 % respectively, from the benchmark levels of 38 % and 24 %. For the VAR(1) model, we observe a more substantial drop to 54 % and 24 % respectively, from the benchmark levels of 76 % and 65 %. For the MSV model, these probabilities are already very low at 3 % and 1 %, therefore shutting off the learning dynamics only marginally reduces the probabilities to 1 % and 0 % respectively. In line with the decreased ZLB durations, the counterfactual averages for inflation and output growth increase compared to the benchmark case in all three models.

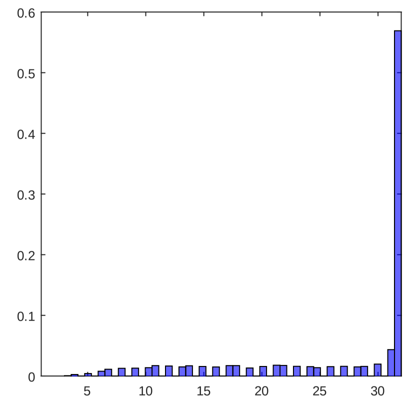
The last exercise we consider is fixing the gain value at a fairly large value of 0.01. As expected, the frequency of long-lived ZLB episodes increase in all models in this case. In particular, for the AR(1) model, ZLB durations exceeding 5 and 7 years increases to 45 % and 30 % respectively, from the benchmark levels of 38 % and 24 %. Similarly for the VAR(1) model, these values increase up to 81 % and 72 %, compared to the benchmark levels of 76 % and 65 %. We observe the largest increase for the MSV model, where the probabilities increase to 49 % and 44 % from benchmark values of 3 % and 1 %. This exercise reveals that the low ZLB durations in the MSV learning model is indeed mostly accounted for by the low estimated gain value. Similar to the previous case, the counterfactual averages for inflation and output growth and in line with the increased frequency of long-lived ZLB regimes. In particular, we observe a negative average output growth for the models, all of which are lower compared to the benchmark case. The changes for inflation averages are somewhat ambiguous, since they increase for the VAR(1) model, decrease for the MSV model and remains unchanged for the AR(1) model.

There are several takeaways from the counterfactual exercise. First and foremost, we see that the presence of learning dynamics unambiguously increase the frequency of long-lived ZLB episodes, which tend to result in lower inflation and output growth in most cases. This offers two potential interpretations for the 2008-2015 period through the lense of our models. The first one is that expectations were well anchored during this period and learning was limited, since our analysis suggests that inflation and output growth would have been lower in the presence of strong learning dynamics. The second is that, while learning dynamics created a downward pressure on the economy, there were other channels at play that offset the adverse effects of learning. In particular, unconventional monetary policy tools such as forward guidance and quantitative easing might have had such an effect, which we leave to future research.

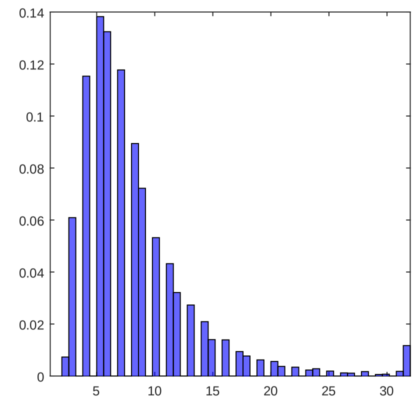
Variable	Empirical Moments	CF Moments		
		AR(1)	VAR(1)	MSV
Inflation	0.37	0	-0.09	0.46
Output Growth	0.19	0.2	-0.88	0.62
ZLB Duration Prob.				
<5 years		0.62	0.24	0.97
>5 years		0.38	0.76	0.03
>7 years		0.24	0.65	0.01
No Learning				
		AR(1)	VAR(1)	MSV
Inflation	0.37	0.16	-0.03	0.47
Output Growth	0.19	0.39	0.22	0.65
ZLB Duration Prob.				
<5 years		0.74	0.46	0.99
>5 years		0.26	0.54	0.01
>7 years		0.13	0.24	0
Fixed gain = 0.01				
		AR(1)	VAR(1)	MSV
Inflation	0.37	0	0.21	0.15
Output Growth	0.19	-0.13	-1.7	-1.12
ZLB Duration Prob.				
<5 years		0.55	0.19	0.51
>5 years		0.45	0.81	0.49
>7 years		0.3	0.72	0.44



(a) AR(1)



(b) VAR(1)



(c) MSV

Figure 16: Distributions of ZLB duration after the crisis for the learning models.

6 Conclusions

The literature on macroeconomics made great strides in the estimation of both Markov-switching and adaptive learning models in recent years, which are two alternative ways of introducing time-variation into DSGE models. Although there are numerous examples of both classes of models that have been successfully taken to the data, there is surprisingly little work on Markov-switching DSGE models under adaptive learning. In this paper, we provided a first attempt to estimate this class of DSGE models by combining these two approaches under the same roof. The resulting framework has an intuitive interpretation where the agents' do not know the details of a complex non-linear economy, but use simple linear rules to form their expectations about the future state of the economy. Our simulations show that, using simple adaptive rules allows the agents' to take into account the structural changes in the economy, albeit in an indirect manner. Our estimation results indicate that the two approaches can be complementary: while imposing the Markov-switching structure on the standard Smets-Wouters (2007) model improves the empirical fit, the introduction of learning can lead to further improvements under the right learning rules. Our results also have important implications in terms of the recently experienced ZLB period. We find that IRFs under REE and adaptive learning models typically move in the same direction once the economy switches to the ZLB regime. However, the magnitudes of the changes tend to be smaller under adaptive learning models, which suggest that standard REE models might overestimate the propagation of shocks over the ZLB period. Based on our counterfactual results, we also find that strong learning effects over the ZLB period typically yield low inflation and output growth rates, which suggest that other effects, such as unconventional policy tools, might have had an offsetting effect on learning dynamics. We leave a more detailed exploration of adaptive learning and unconventional monetary policy to future research.

Appendix

A RPE and T-map

Special case with 2 regimes, no lagged variables

Note that in the special case with $\iota_p = 0$, the model can be written as a generic 1-dimensional Markov-switching model of the form:

$$\begin{cases} \pi_t = d(s_t)r_t + u_t \\ r_t = \rho r_{t-1} + v_t \end{cases}$$

where $d(s_t) = \frac{d\rho - 1}{\alpha(s_t)}$. The moments necessary for the T-map are given as follows:

$$E[\pi_t r_t] = P_1 E[\pi_t r_t | S_t = 1] + P_2 E[\pi_t r_t | S_t = 2]$$

$$E[\pi_t r_t | S_t = 1] = E[d(s_t) r_t^2 | S_t = 1, S_{t-1} = 1] p_{11} + E[d(s_t) r_t^2 | S_t = 1, S_{t-1} = 2](1 - p_{22}) \frac{P_2}{P_1}$$

$$= d_1 p_{11} + d_1 (1 - p_{22}) \frac{P_2}{P_1}$$

Similarly, we have

$$E[\pi_t r_t | S_t = 1] = d_2 p_{22} + d_2 (1 - p_{11}) \frac{P_1}{P_2}$$

which yields

$$E[\pi_t r_t] = P_1 (d_1 p_{11} + d_1 (1 - p_{22}) \frac{P_2}{P_1}) + P_2 (d_2 p_{22} + d_2 (1 - p_{11}) \frac{P_1}{P_2})$$

Plugging in the steady-state probabilities P_1 and P_2, the T-map is given as follows:

$$d \to T(d) = \frac{\alpha_1 (1 - p_{22}) + \alpha_2 (1 - p_{11})}{\alpha_1 \alpha_2 (2 - p_{11} - p_{22})} (d\rho + 1)$$

with the E-stability condition

$$DT_a = \frac{\alpha_1 (1 - p_{22}) + \alpha_2 (1 - p_{11})}{\alpha_1 \alpha_2 (2 - p_{11} - p_{22})} \rho < 1$$

Re-arranging the expression above yields the LRES condition presented in Section 1. Further note that the regime-specific T-maps, and the associated regime-specific E-stability conditions are given by:

$$d \to \frac{d\rho + 1}{\alpha_i}$$

$$DT_d = \frac{\rho}{\alpha_i} < 1$$

which implies that regime specific E-stability of all regime-specific equilibria is a sufficient, but not necessary condition for LRES.

1-dimensional case with m regimes

Note that the Fisherian model considered in Section 2 can be written as a generic 1-dimensional Markov-switching model of the form:

$$\begin{cases} \pi_t = d(s_t)r_t + b(s_t)\pi_{t-1} \\ r_t = \rho r_{t-1} + v_t \end{cases}$$

where $b(s_t) = \frac{\iota_p}{\alpha(s_t)-(1-\iota_p)b}$ and $d(s_t) = \frac{(1-\iota_p)d\rho+1}{\alpha(s_t)-(1-\iota_p)b}$. In this Appendix we consider the general case with m regimes, with transition matrix given by:

$$Q = \begin{bmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \cdots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{bmatrix}$$

The 2-regime setup of Section is the special case with $m = 2$. We omit the first moment $E[\pi_t]$, which is trivially given as zero. Using this, we compute the second moments starting with the conditional variance. We have:

$$E[\pi_t^2] = \sum_{i=1}^{m} P_i E[\pi_t^2|S_t = i]$$

$$E[\pi_t^2|S_t = i] \sum_{i=1}^{m} E[\pi_t^2|S_t = i, S_{t-1} = j] p_{ji} \frac{P_j}{P_i}$$

where P_i denotes the i^{th} element of the steady-state vector of the Markov chain.

$$= \sum_{j=1}^{m} E[d(s_t)^2 r_t^2 + b(s_t)^2 \pi_{t-1}^2 + u_t^2 + 2b(s_t)d(s_t) r_t \pi_{t-1}|S_t = i, S_{t-1} = j] p_{ji} \frac{P_j}{P_i}$$

$$= \sum_{j=1}^{m} E[d_i^2 \sigma_r^2 + b_i^2 \pi_{t-1}^2 + \sigma_r^2 + 2b_1 d_i r_t \pi_{t-1}|S_{t-1} = j] p_{ji} \frac{P_j}{P_i}$$

Note that this last expression implies m equations in m unknowns for the conditional variances, *given* the conditional covariances $E[\pi_t r_t|S_t = j]$. Using this, the unconditional variance is given by:

$$E[\pi_t^2] = \sum_{i=1}^{m} P_i \sum_{j=1}^{m} (d_i^2 \sigma_r^2 + b_i^2 E[\pi_{t-1}^2|S_t = j] + \sigma_r^2 + 2b_1 d_i r_t E[\pi_{t-1}|S_{t-1} = j]) p_{ji} \frac{P_j}{P_i}$$

Next we move onto the covariance term $E[\pi_t r_t]$:

$$E[\pi_t r_t] = \sum_{i=1}^{m} P_i E[\pi_t r_t|S_t = i]$$

$$E[\pi_t r_t|S_t = i] = \sum_{j=1}^{m} E[\pi_t r_t|S_t = i, S_{t-1} = j] p_{ji} \frac{P_j}{P_i}$$

$$= \sum_{j=1}^{m} E[b(s_t)\pi_{t-1} r_t + d(s_t) r_t^2|S_t = i, S_{t-1} = j] p_{ji} \frac{P_j}{P_i}$$

$$\sum_{i=1}^{m} (b_i \rho E[\pi_t r_t|S_t = j] + d_i \sigma_r^2) p_{ji} \frac{P_j}{P_i}$$

Note that again, the last expression implies m equations in m unknowns for the conditional covariances. Using this, the unconditional covariance is given by:

$$E[\pi_t r_t] = \sum_{i=1} m P_i \sum_{j=1}^{m} (b_i \rho E[\pi_t r_t|S_t = j] + d_i \sigma_r^2) p_{ji} \frac{P_j}{P_i}$$

Next we compute the first-order autocovariance:

$$E[\pi_t \pi_{t-1}] = \sum_{i=1}^{m} P_i E[\pi_t \pi_{t-1}|S_t = i]$$

$$E[\pi_t \pi_{t-1}|S_t = i] = \sum_{j=1}^{m} E[b(s_t)\pi_{t-1}^2 + d(s_t)\pi_{t-1} r_t|S_t = 1, S_{t-1} = j] p_{ji} \frac{P_j}{P_i}$$

$$= \sum_{j=1}^{m} (b_i E[\pi_t^2|S_t = j] + d_i \rho E[\pi_t r_t|S_t = j]) p_{ji} \frac{P_j}{P_i}$$

Given the conditional covariance and conditional variance terms, the above expression yields the conditional autocovariances. Hence the unconditional autocovariance is given as:

$$E[\pi_t\pi_{t-1}] = \sum_{i=1}^{m} P_i \sum_{j=1}^{m} (b_i E[\pi_t^2|S_t = j] + d_i \rho E[\pi_t r_t|S_t = j]) p_{ji} \frac{P_j}{P_i}$$

Finally note that:

$$E[d(s_t)\pi_{t-1}r_t] = \sum_{i=1}^{m} P_i \sum_{j=1}^{m} d_i \rho E[\pi_t r_t|S_t = j] p_{ji} \frac{P_j}{P_i}$$

and

$$E[b(s_t)\pi_{t-1}r_t] = \sum_{i=1}^{m} P_i \sum_{j=1}^{m} b_i \rho E[\pi_t r_t|S_t = j] p_{ji} \frac{P_j}{P_i}$$

Recalling the T-map $\begin{pmatrix} d \\ b \end{pmatrix} \to T(d,b) = \begin{pmatrix} E[(\pi_t - b\pi_{t-1})r_t] \\ \frac{E[(\pi_t - dr_t)\pi_{t-1}]}{E[\pi_t^2]} \end{pmatrix}$, the above conditions fully pin down $T(d,b)$. It is generally not possible to obtain analytical expressions for this mapping, and therefore the RPE values d^{RPE} amd b^{RPE}. Therefore our results in Section 1 are computed numerically for given values of parameters.

N dimensional case with m regimes

Note that, after plugging in the PLM into ALM, the model considered in Section can be re-written as a generic Markov-switching model of the form:

$$\begin{cases} X_t = a(s_t) + b(s_t)X_{t-1} + d(s_t)\epsilon_t \\ \epsilon_t = \rho\epsilon_{t-1} + \eta_t \end{cases}$$

where $a(s_t) = A(s_t) + C(s_t)(a + ba)$, $b(s_t) = B(s_t) + C(s_t)b^2$ and $d(s_t) = C(s_t)(bd + d\rho) + D(s_t)$. We need the first and second moments of this system in order to compute the the resulting T-map for the RPE. Starting with the first moment, we have:

$$E[X_t] = \sum_{i=1}^{m} P_i E[X_t|S_t = i]$$

$$E[X_t|S_t = i] = \sum_{j=1}^{m} [a_i + b_i X_{t-1} + d_i \epsilon_t|S_{t-1} = j] p_{ji} \frac{P_j}{P_i}$$

$$= \sum_{j=1}^{m} (a_i + b_i E[X_t|S_t = j]) p_{ji} \frac{P_j}{P_i}$$

The expression above implies m equations in m unknowns for the conditions means. Using this yields:

$$E[X_t] = \sum_{i=1}^{m} P_i \sum_{j=1}^{m} (a_i + b_i E[X_t|S_t = j]) p_{ji} \frac{P_j}{P_i}$$

Moving onto the second moments and starting with the covariance term, we have:

$$E[X_t \epsilon_t'] = \sum_{i=1}^{m} P_i E[X_t \epsilon_t' | S_t = i]$$

$$E[X_t \epsilon_t' | S_t = i] E[) a(s_t) + b(s_t) X_{t-1} + d(s_t)\epsilon_t)\epsilon_t' | S_t = i]$$

$$= \sum_{j=1}^{m} E[(a_i + b_i X_{t-1} + d_i \epsilon_t)\epsilon_t' | S_{t-1} = j] p_{ji} \frac{P_j}{P_i}$$

$$= \sum_{j=1}^{m} (b_i \rho E[X_t \epsilon_t' | S_t = j] + d_i \Sigma_\epsilon) p_{ji} \frac{P_j}{P_i}$$

The last expression again implies m equations in m unknowns for the conditional covariances. The unconditional covariance is then given by:

$$E[X_t \epsilon_t'] = \sum_{i=1}^{m} P_i \sum_{j=1}^{m} (b_i \rho E[X_t \epsilon_t' | S_t = j] + d_i \Sigma_\epsilon) p_{ji} \frac{P_j}{P_i}$$

Next we compute:

$$E[X_t X_t'] = \sum_{i=1}^{m} P_i E[X_t X_t' | S_t = i]$$

$$E[X_t X_t' | S_t = i] = E[a(s_t)a(s_t)' + 2a(s_t)X_{t-1}' b(s_t)' + 2a(s_t)\epsilon_t d(s_t)' +$$

$$b(s_t) X_{t-1} X_{t-1}' b(s_t)' + 2b(s_t) X_{t-1} \epsilon_t d(s_t)' + d(s_t)\epsilon_t \epsilon_t' d(s_t)' | S_t = i]$$

$$= \sum_{j=1}^{m} E[a_i a_i' 2a_i X_{t-1}' b_i' + 2a_i \epsilon_t' d_i' + b_i X_{t-1} X_{t-1}' b_i' + 2b_i X_{t-1} \epsilon_t' d_i' + d_i \epsilon_t \epsilon_t' d_i' | S_t = j] p_{ji} \frac{P_j}{P_i}$$

Given the conditional means and covariances, the last expressions implies m equations in m unknowns for the conditional moments $E[X_t X_t' | S_t = i]$. The unconditional moment is then given by:

$$E[X_t X_t'] = \sum_{i=1}^{m} P_i \sum_{j=1}^{m} (a_i a_i' + 2a_i E[X_t' | S_t = j] b_i' + b_i E[X_t X_t' | S_t = j] b_i' + 2b_i E[X_t \epsilon_t' | S_t = j] \rho' d_i' + d_i \Sigma_\epsilon d_i') p_{ji} \frac{P_j}{P_i}$$

Finally we compute the autocovariance term:

$$E[X_t X_{t-1}'] = \sum_{i=1}^{m} P_i E[X_t X_{t-1}' | S_t = i]$$

$$E[X_t X_{t-1}' | S_t = i] = E[a_i X_{t-1}' + b_i X_{t-1} X_{t-1}' + d_i \rho \epsilon_{t-1} X_{t-1}' | S_t = i] =$$

$$\sum_{j=1}^{m} (a_i E[X_t | S_t = j] + b_i E[X_t X_t' | S_t = j] + d_i \rho E[\epsilon_t X_t' | S_t = j]) p_{ji} \frac{P_j}{P_i}$$

The last expression is pinned by the conditional first and second moments computed above. The unconditional autocovariance is then given as:

$$E[X_t X_t'] = \sum_{i=1}^{m} \sum_{j=1}^{m} (a_i E[X_t | S_t = j] + b_i E[X_t X_t' | S_t = j] + d_i \rho E[\epsilon_t X_t' | S_t = j]) p_{ji} \frac{P_j}{P_i}$$

Recall that the T-map is given by:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \to \begin{pmatrix} E[X_t - bX_{t-1} - d\epsilon_t] \\ E[(X_t - a - d\epsilon_t) X_{t-1}') E[X_t X_t']^{-1}] \\ E[(X_t - a - bX_{t-1}) \epsilon_t'] E[\epsilon_t \epsilon_t']^{-1} \end{pmatrix}$$

Hence, given the first and second moments computed above, the T-maps for a, b and c are pinned down. Similar to 1-dimensional case, it is generally not possible to find analytical expressions for these matrices. Further note that, the T-map for $b \rightarrow T(a,b,c)$ involves a 4^{th} order matrix polynomial of dimension N. This means there can be up to $\binom{4N}{N}$ for b. To our knowledge, there is no straightforward and general method to compute the full set of solutions to this problem. In this paper, we do not compute these fixed-points and rely on Monte Carlo simulations when necessary.

Further note that the regime-specific T-maps are given by:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \rightarrow \begin{pmatrix} A_i + C_i(a + ba) \\ B_i + C_i b^2 \\ C_i(bd + d\rho) + D_i \end{pmatrix}$$

These simply correspond to the standard MSV solutions given the regime-specific matrices. Computing the fixed-points yield the regime-specific equilibria as follows:

$$\begin{cases} a^{R_i} = (I - C_i - C_i b^{R_i})^{-1} A_i \\ vec(D^{R_i}) = (I - (I \otimes (C_i b^{R_i})))vec(d) + (\rho \otimes C_i)vec(d) + vec(D_i) \end{cases}$$

which yields the regime-specific values for a^{R_i} and d^{R_i} respectively, for a given matrix b^{R_i}. The second-order polynomial for b^{R_i} can be solved using standard toolboxes such as Adjemian et al. (2011) and Uhlig et al. (1995), which then completely pins down the regime-specific MSV. Denoting $\theta = (a,b,d)'$, the associated Jacobian for E-stability condition is given by:

$$\frac{DT}{D\theta} = \begin{bmatrix} C_i + C_i b & vec^{-1}_{n,n}(a' \otimes C_i) & 0 \\ 0 & 2C_i b & 0 \\ 0 & vec^{-1}_{n,n}(d' \otimes C_i) & C_i b + vec^{-1}_{n,n}(\rho' \otimes C_i) \end{bmatrix}$$

where $vec^{-1}_{n,n}$ denotes the matricization of a vector to an (n,n) matrix.

B Kim-Nelson Filter with Adaptive Learning

This section describes the KN-filter used in our estimations. The filter nests the standard Kalman filter for unobserved state variables with the Hamilton filter for unobserved regime probabilities. These two filters are followed by an approximation step via collapsing, which reduces the number of states from m^2 to m in order to keep the algorithm tractable. We extend the filter with an adaptive learning step, which takes a weighted average of the (Kalman) filtered states based on the (Hamilton) filtered regime probabilities. We assume that the resulting states are observable to the model's agents, who update their beliefs with linear-quadratic constant gain least squares using this latest available data. This leaves the Kalman and Hamilton filter blocks intact, since the model is conditionally linear at every period, given the previous period's adaptive learning update.

The endogenous regime-switching model replaces the transition probability matrix $Q(i,j)$ with a time-varying matrix $Q_t(i,j)$. This matrix is updated every period after the Kalman filter block given the shadow rate, which in turn is calculated based on the inflation and output gap variables contained in matrices $S_{t|t}^{(i,j)}$.

Table 7: **KN-filter for Markov-Switching DSGE Models under Adaptive Learning**

$$\begin{cases} S_t = \gamma_{2,\Phi_t}^{(s_t)} + \gamma_{1,\Phi_t}^{(s_t)} S_{t-1} + \gamma_{3,\Phi_t}^{(s_t)} \epsilon_t, & , \epsilon_t \sim N(0,\Sigma) \\ y_t = E + F S_t \end{cases}$$

0) Initial States :

$\tilde{S}_{0|0}^i, \tilde{P}_{0|0}^i, Pr[S_0 = i|\Phi_0], \Phi_0$ given.

1) Kalman Filter Block with the standard measurement and transition equations:

For $t = 1 : N$

For $\{S_{t-1} = i, S_t = j\}$

$$\begin{cases} S_{t|t-1}^{(i,j)} = \gamma_1^{(j)} S_{t-1|t-1}^{(i)} + \gamma_2^{(j)} \\ P_{t|t-1}^{(i,j)} = \gamma_1^{(j)} P_{t-1|t-1}^{(i)} \gamma_1^{(j)} + \gamma_3^{(j)} \Sigma^{(j)} (\gamma_3^{(j)})' \\ v_{t|t-1}^{(i,j)} = (y_t - F^{(j)} S_{t|t-1}^{(i,j)}) \\ Fe^{(i,j)} = F^{(j)} P_{t|t-1}^{(i,j)} F^{(j)} \\ S_{t|t}^{(i,j)} = S_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} (F^{(j)})' (Fe^{(i,j)})^{-1} v^{(i,j)} \\ P_{t|t}^{(i,j)} = P_{t|t-1}^{(i,j)} (F^{(j)})' (Fe^{(i,j)})^{-1} F^{(j)} P_{t|t-1}^{(i,j)} \end{cases}$$

2) Hamilton Block for transition probabilities:

Denote: $Pr[S_{t-1} = i, S_t = j|\Phi_{t-1}] = pp_{t|t-1}^{i,j}$, $f(y_t|\Phi_{t-1})$ the marginal likelihood,

$Pr[S_{t-1} = i, S_t = j|\Phi_t] = pp_{t|t}^{i,j}$ and $Pr[S_t = j|\Phi_t] = pp_{t|t}^{\tilde{j}}$.

$$\begin{cases} pp_{t|t-1}^{(i,j)} = Q(i,j) pp_{t-1|t-1}^{(i)} \\ f(y_t|\Phi_{t-1}) = \sum_{j=1}^M \sum_{i=1}^M f(y_t|S_{t-1} = i, S_t = j, \Phi_{t-1}) pp_{t|t-1}^{(i,j)} \\ pp_{t|t}^{(i,j)} = \frac{f(y_t|S_{t-1}=i, S_t=j, \Phi_{t-1}) pp_{t|t-1}^{(i,j)}}{f(y_t|\Phi_{t-1})} \\ p_{t|t}^j = \sum_i^M pp_{t|t-1}^{(i,j)} \end{cases}$$

3) Collapsing to reduce the number of states from m^2 to m:

$$\begin{cases} S_{t|t}^{(i)} = \frac{\sum_{i=1}^M pp_{t|t}^{(i,j)} S_{t|t}^{(i,j)}}{p_{t|t}^{(j)}} \\ P_{t|t}^{(i)} = \frac{\sum_{i=1}^M pp_{tt}^{(i,j)} (P_{t|t}^{(i,j)} + (S_{t|t}^{(j)} - S_{t|t}^{(i,j)})(S_{t|t}^{(j)} - S_{t|t}^{(i,j)})')}{p_{t|t}^{(j)}} \end{cases}$$

4) Update expectations based on filtered states:

Updating Expectations based on Filtered States:

$$\begin{cases} \tilde{S}_{t|t} = \sum_{j=1}^M p_{t|t}^{(j)} S_{t|t}^{(j)} \\ \Phi_t = \Phi_{t-1} + \gamma R_t^{-1} \tilde{S}_{t-1|t-1} (\tilde{S}_{t|t} - \Phi_{t-1}^T \tilde{S}_{t-1|t-1})^T \\ R_t^{-1} = R_{t-1} + \gamma (\tilde{S}_{t-1|t-1} \tilde{S}_{t-1|t-1}^T - R_{t-1}) \end{cases}$$

C Monte Carlo Simulations and Distributions of Learning Parameters

This section investigates the E-stability of the 3-equation and Smets-Wouters models that have been estimated in the previous sections. Recall from Section 3 that the underlying E-stability conditions for a general system of the form (3.4) is not available in closed form. Therefore, while the projection facility ensures that the models remain stable throughout the estimation, this does not guarantee the stability of the resulting systems under learning.

A well known result in the literature is that, when monetary policy is inactive, New Keynesian models are generally indeterminate and the mean dynamics are E-unstable under standard parameterizations. This result may also apply to the model specifications considered in this paper. As a first step, we illustrate this for the 3-equation model in a simplified framework to build intuition. Accordingly, consider the 3-equation model under MSV-learning, where we shut off all three parameters related to lagged endogenous variables, i.e. $\rho_r = 0, \iota_x = 0, \iota_\pi = 0$ to reduce the T-map in (3.5) to an analytically tractable form. We use the following parameterization for the remaining parameters: $\kappa = 0.02, \tau = 1, \beta = 0.99, \rho_y = 0.25, \rho_\pi = 0.25$. First consider the regime-specific T-maps and the associated Jacobian matrix:

$$\begin{pmatrix} a \\ b \end{pmatrix} \to T(a,d) = \begin{pmatrix} A_i + C_i a \\ C_i d\rho + D_i \end{pmatrix}, \frac{\partial T(a,d)}{\partial [a,d]} = \begin{pmatrix} C_i & 0 \\ 0 & (\rho C_i)' \end{pmatrix}, \tag{C.1}$$

where the underlying Rational Expectations Equilibrium is given by $a = (I - C_i)^{-1} A_i$ and $vec(d) = (I - \rho' \otimes C_i)^{-1} vec(D_i)$. At the given parameterization with an inactive Taylor rule, the largest root relating to the mean dynamics (i.e. the largest eigenvalue of C_i) is larger than 1, hence the model is E-unstable[26]. If we instead consider the Long-run E-stability with an active and a passive monetary policy rule, then we get the T-map:

$$\begin{pmatrix} a \\ b \end{pmatrix} \to T(a,d) = \begin{pmatrix} P_1(A_1 + C_1 a) + P_2(A_2 + C_2 a) \\ P_1(C_2 d\rho + D_2) + P_2(C_2 d\rho + D_2) \end{pmatrix}, \frac{\partial T(a,d)}{\partial [a,d]} = \begin{pmatrix} P_1 C_1 + P_2 C_2 & 0 \\ 0 & P_1(\rho C_1)' + P_2(\rho C_2)' \end{pmatrix}, \tag{C.2}$$

where the RPE is given by

$$\begin{cases} a = (I - P_1 C_1 - P_2 C_2)^{-1}(P_1 A_1 + P_2 A_2), \\ d = (I - \rho' \otimes (P_1 C_1) - \rho' \otimes (P_2 C_2))^{-1}(vec(P_1 D_1) + vec(P_2 D_2)). \end{cases} \tag{C.3}$$

[26] In this case the eigenvalues of C_i are given as $\lambda_{1,2} = \frac{\tau}{\tau + \phi_x + \kappa\phi_\pi}(\frac{-p}{2} \pm \sqrt{\frac{p^2}{4} - q})$, with $p = -(1 + \frac{\kappa + \beta\tau + \beta\phi_x}{\tau})$ and $q = \frac{\beta\tau + \beta\phi_x + \kappa\beta\phi_\pi}{\tau}$.

Hence, in the absence of lagged variables, the equilibrium and the associated stability conditions are simply weighted averages of the underlying regime-specific equilibria. Considering a standard Taylor rule with $\phi_\pi = 1.5$ and $\phi_x = 0.125$, the long-run E-stability is satisfied as long as $P_2 < \frac{1}{3}$, i.e. as long as the system does not spend an excessive amount of time in the passive regime. We conjecture that this result also applies to the estimated model specifications with lagged variables, i.e. the model dynamics are E-unstable during the ZLB regime, but the system as a whole may still be stable as long as it the duration of the passive regime is sufficiently short.

To investigate the stability of the estimated model specifications under learning, we use the following Monte Carlo experiment: we simulate all models at the estimated posterior mode 100 times, each of length 2000, where each simulation is initialized at the REE-based initial values (i.e. the same beliefs used in the estimation) and discard the first half of each simulation as the burn-in period. We collect the distributions of the learning parmaeters and the frequency of the projection facility activity over the remaining periods, which we use as an indicator of E-stability for the underlying system. Generally, the projection facility will be binding either when the underlying equilibrium dynamics are close to the non-stationary region, or when the equilibrium dynamics are E-unstable; a Monte Carlo experiment is unable to distinguish between these two. Nevertheless, it provides us with an overall picture of how much the system depends on the projection facility to remain in the stationary region.

Table 8 shows the results on the projection facility for all models under consideration. For a majority of cases, the frequency ranges between $[1.3\%, 6.3\%]$, i.e. the facility does not bind very often. This suggests that these models remain within the stationary region for over 90 % of the periods, including the ZLB regimes with inactive monetary policy. There are two exceptions with a higher frequency: SW model with AR(1), where the projection facility binds 14.7% of the time, and the NKPC model with the VAR(1) rule, where the facility binds 32.1% of the time. Particularly in the latter case, this high frequency is caused by the high estimated parameter value of 0.03. We provide one more experiment with this model with a lower gain value of 0.01, where the projection facility frequency indeed decreases to 19.2%. Overall, we conclude that all model specifications are able to generate stable dynamics most of the time, where the best performing model relies on the projection facility only 1.3 % of the time, whereas the worst one relies on the projection facility 32.1 % of the time). The underlying distributions of the learning parameters for all models specifications can be found in the Appendix. With the exception of a few parameters[27], the distributions are typically unimodal, suggesting stationary

[27]E.g. inflation persistence has a seemingly bimodal distribution both in the NKPC and SW models, with one mode around 0.6 and another one close to unit root. This could be an indicator or two stable equilibria, or simply a result of non-stationary dynamics that cluster around unit root due to the projection facility. We do not investigate this further since the simulations are only meant to investigate the overall stability of the models.

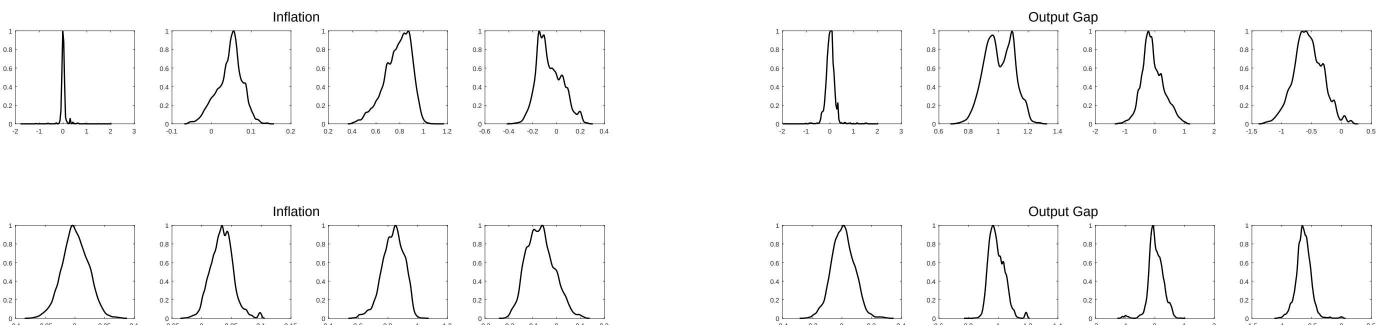
dynamics around a unique equilibrium.

	NKPC	SW
AR(1)	1.5 %	14.7 %
VAR(1)	32.1 %	6.3 %
VAR(1) (gain= 0.01)	19.2 %	
MSV	5.1 %	1.3 %

Table 8: Frequency of projection facility activity in all learning model. The results are based in 100 simulations each of length 2000, where the first half is discarded as burn-in sample in each case.

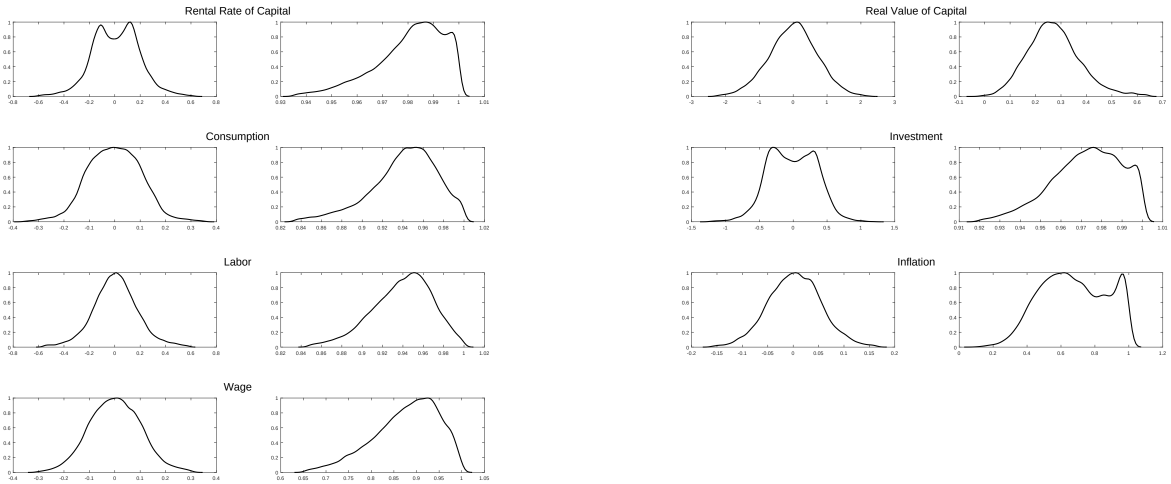
NKPC, AR(1)

NKPC, VAR(1) second row with $gain = 0.01$

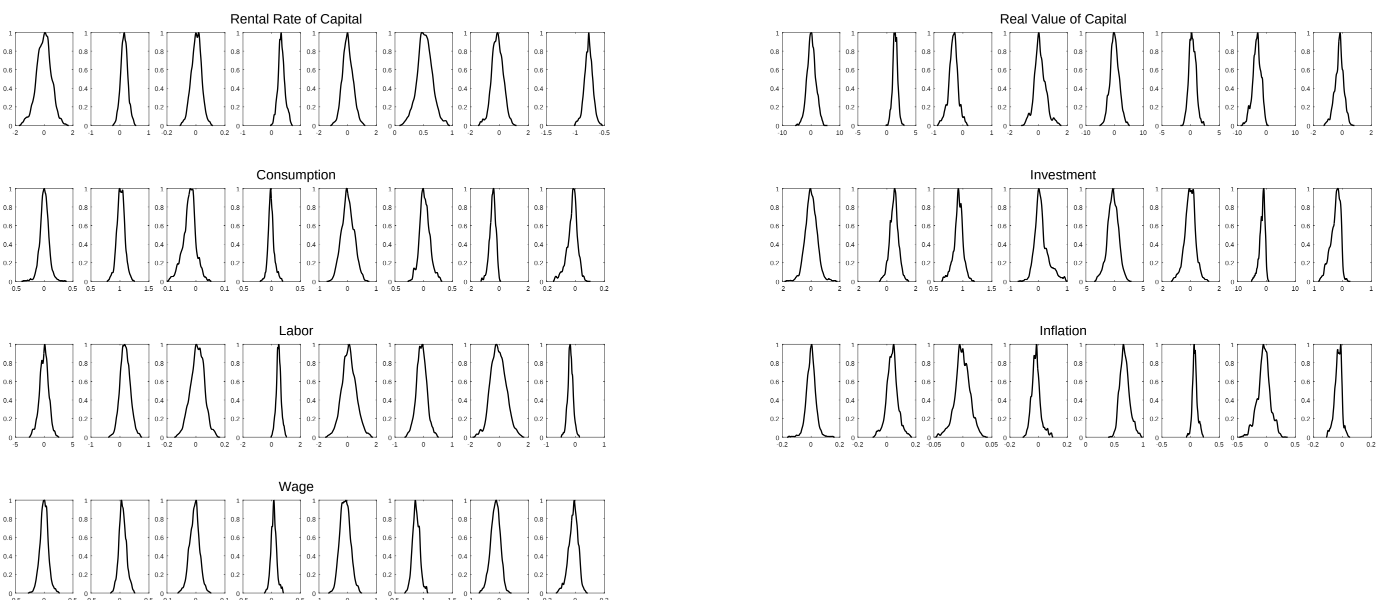


NKPC, MSV

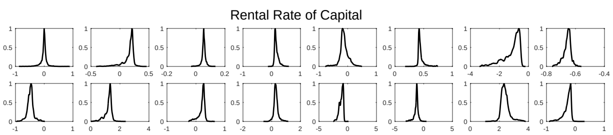
SW, AR(1)

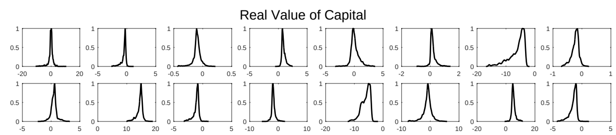


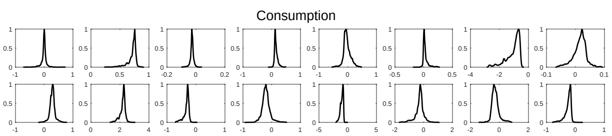
SW, VAR(1)

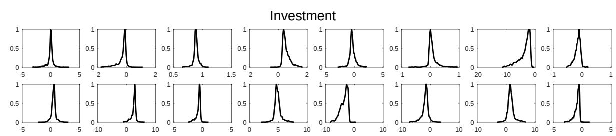


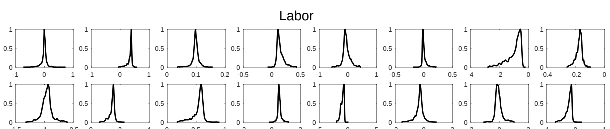
SW, MSV

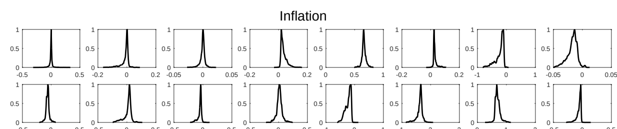

Rental Rate of Capital

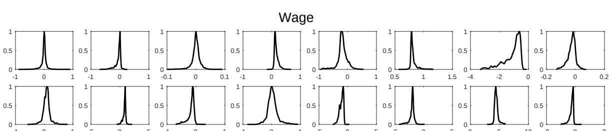

Real Value of Capital


Consumption


Investment


Labor


Inflation


Wage

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