Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift

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Stochastic gradient descent (SGD)

- Minimize the expected loss over the training set:

$$\hat{\theta} = \arg\min_{\theta} \mathrm{E}_{x \sim D}[\ell(x, \theta)]$$

- Parameters update according to the gradient of mini-batches

$$\theta \leftarrow \theta - \frac{\alpha}{m} \sum_{i=1}^{m} \frac{\partial \ell(x_i, \theta)}{\partial \theta}$$

- Careful tuning of learning rates and initial parameters.

Internal covariate shift

- Covariate shift: Changes of input distribution to a learning system

$$\ell = F(x, \theta)$$

- Internal covariate shift: Extension to the deep network

$$\begin{aligned} \ell &= F_2(F_1(u, \theta_1), \theta_2) \\ &= F_2(x, \theta_2) \end{aligned}$$

Reducing internal covariate shift

- Whitening the inputs to each layer:

$$x \leftarrow \frac{x - \mathrm{E}[x]}{\sqrt{\mathrm{Var}[x]}}$$

  However, gradient descent does not take into account the normalization [Ioffe and Szegedy 2015].

- Mean and variance of an activation depend on model parameters

$$\frac{\partial\,\mathrm{E}[x]}{\partial\theta} \text{ and } \frac{\partial\,\mathrm{Var}[x]}{\partial\theta}$$

Batch normalizing transform

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\ldots m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_\mathcal{B} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

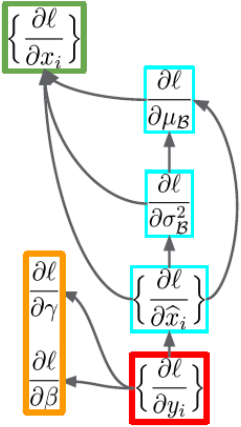
$$\sigma_\mathcal{B}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m} (x_i - \mu_\mathcal{B})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_\mathcal{B}}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Algorithm 1: Batch Normalizing Transform, applied to activation x over a mini-batch.

Backpropagation with batch normalization



(a)

$$
\begin{aligned}
\frac{\partial \ell}{\partial \widehat{x}_i} &= \frac{\partial \ell}{\partial y_i} \cdot \gamma \\
\frac{\partial \ell}{\partial \sigma_{\mathcal{B}}^2} &= \sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot (x_i - \mu_{\mathcal{B}}) \cdot \frac{-1}{2}(\sigma_{\mathcal{B}}^2 + \epsilon)^{-3/2} \\
\frac{\partial \ell}{\partial \mu_{\mathcal{B}}} &= \sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{-1}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \\
\frac{\partial \ell}{\partial x_i} &= \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{1}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} + \frac{\partial \ell}{\partial \sigma_{\mathcal{B}}^2} \cdot \frac{2(x_i - \mu_{\mathcal{B}})}{m} + \frac{\partial \ell}{\partial \mu_{\mathcal{B}}} \cdot \frac{1}{m} \\
\frac{\partial \ell}{\partial \gamma} &= \sum_{i=1}^{m} \frac{\partial \ell}{\partial y_i} \cdot \widehat{x}_i \\
\frac{\partial \ell}{\partial \beta} &= \sum_{i=1}^{m} \frac{\partial \ell}{\partial y_i}
\end{aligned}
$$

(b)

Training a batch-normalized network

Input: Network N with trainable parameters Θ;
subset of activations $\{x^{(k)}\}_{k=1}^{K}$

Output: Batch-normalized network for inference, $N_{\mathrm{BN}}^{\mathrm{inf}}$

$N_{\mathrm{BN}}^{\mathrm{tr}} \leftarrow N$ // Training BN network
for $k = 1 \ldots K$ **do**
  Add transformation $y^{(k)} = \mathrm{BN}_{\gamma^{(k)},\beta^{(k)}}(x^{(k)})$ to $N_{\mathrm{BN}}^{\mathrm{tr}}$ (Alg. 1)
  Modify each layer in $N_{\mathrm{BN}}^{\mathrm{tr}}$ with input $x^{(k)}$ to take $y^{(k)}$ instead
end for
Train $N_{\mathrm{BN}}^{\mathrm{tr}}$ to optimize the parameters $\Theta \cup \{\gamma^{(k)}, \beta^{(k)}\}_{k=1}^{K}$
$N_{\mathrm{BN}}^{\mathrm{inf}} \leftarrow N_{\mathrm{BN}}^{\mathrm{tr}}$ // Inference BN network with frozen // parameters
for $k = 1 \ldots K$ **do**
  // For clarity, $x \equiv x^{(k)}, \gamma \equiv \gamma^{(k)}, \mu_{\mathcal{B}} \equiv \mu_{\mathcal{B}}^{(k)}$, etc.
  Process multiple training mini-batches $\mathcal{B}$, each of size m, and average over them:

$$\mathrm{E}[x] \leftarrow \mathrm{E}_{\mathcal{B}}[\mu_{\mathcal{B}}]$$
$$\mathrm{Var}[x] \leftarrow \frac{m}{m-1}\mathrm{E}_{\mathcal{B}}[\sigma_{\mathcal{B}}^2]$$

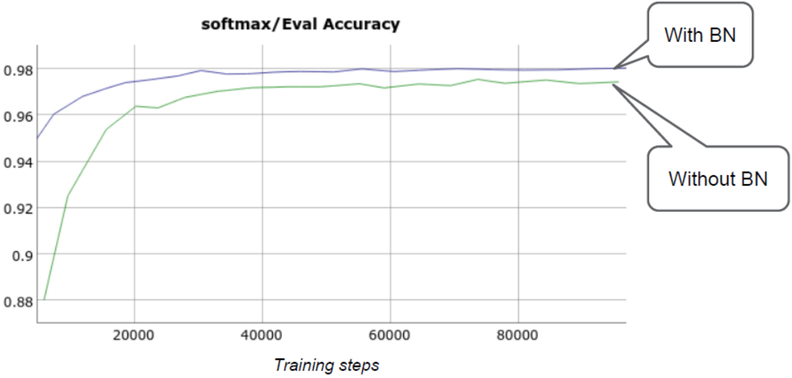
  In $N_{\mathrm{BN}}^{\mathrm{inf}}$, replace the transform $y = \mathrm{BN}_{\gamma,\beta}(x)$ with $y = \frac{\gamma}{\sqrt{\mathrm{Var}[x]+\epsilon}} \cdot x + \left(\beta - \frac{\gamma\,\mathrm{E}[x]}{\sqrt{\mathrm{Var}[x]+\epsilon}}\right)$
end for

Algorithm 2: Training a Batch-Normalized Network

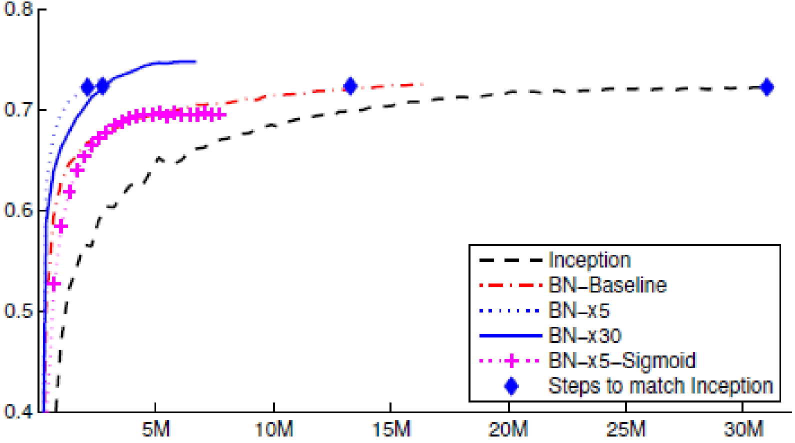
Experiments

- MNIST
  - 3 fully-connect hidden layers with 100 nodes in each layer.
  - Sigmoid activation function.
  - Mini-batch size to be 60.
- ImageNet
  - The Inception network [Szegedy et al. 2014].
  - SGD with momentum [Sutskever et al. 2013].
  - Mini-batch size to be 32.

Learning curve on MNIST [Ioffe 2015]



Learning curve on ImageNet with single networks



Classification results on ImageNet

Model	Resolution	Crops	Models	Top-1 error	Top-5 error
GoogLeNet ensemble	224	144	7	-	6.67%
Deep Image low-res	256	-	1	-	7.96%
Deep Image high-res	512	-	1	24.88	7.42%
Deep Image ensemble	up to 512	-	-	-	5.98%
MSRA multicrop	up to 480	-	-	-	5.71%
MSRA ensemble	up to 480	-	-	-	4.94%*
BN-Inception single crop	224	1	1	25.2%	7.82%
BN-Inception multicrop	224	144	1	21.99%	5.82%
BN-Inception ensemble	224	144	6	20.1%	**4.82%***

References I

- Sergey Ioffe. *Batch Normalization Presentation*. 2015.
- Sergey Ioffe and Christian Szegedy. "Batch normalization: Accelerating deep network training by reducing internal covariate shift." : *ICML*. 2015.
- Ilya Sutskever, James Martens, George Dahl, and Geoffrey Hinton. "On the importance of initialization and momentum in deep learning." : *ICML*. 2013.
- Christian Szegedy, Wei Liu, Yangqing Jia, Pierre Sermanet, Scott Reed, Dragomir Anguelov, Dumitru Erhan, Vincent Vanhoucke, and Andrew Rabinovich. "Going deeper with convolutions." *arXiv preprint arXiv:1409.4842* (2014).