Assignment 1 (DM)

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1 Solutions

1.1 Q1

Prove that there is no positive integer n such that n$^2 + n^3 = 100$.

Let n be any positive integer .

For n=5 , n$^2 + n^3 = 25 + 125 > 100$.

So n can be either 1 , 2 , 3 or 4 .

n=1 , n$^2 + n^3 = 2$.

n =2 , n$^2 + n^3 = 4 + 8 = 12$.

n = 3 , n$^2 + n^3 = 9 + 27 = 36$.

n= 4 , n$^2 + n^3 = 16 + 64 = 80$.

Hence there is no positive integer to satisfy the given relation .

1.2 Q2

Prove that n$^2 + 1 >= 2nwhen$

n is a positive integer with 1 ¡= n ¡= 4 .

Take n to be 1 , 2, 3 or 4 :

and let n$^2 + 1 = x$.

n=1 , x = 1+1 = 2^1

n=2 , x = 4 + 1 = 5 >2^2

n=3 , x = 9 + 1 = 10 >2^3

n=4 , x = 16 + 1 = 17 >2^4

Hence proved .

1.3 Q3

Find a compound proposition involving the propositional variables p, q, r, and s that is true when exactly three of these propositional variables are true and is false otherwise .

The various combinations of the p , q ,r and s to be True as the final truth value are :

p $\wedge q \wedge r \wedge \neg s$

$\neg p \wedge q \wedge r \wedge s$

p $\wedge \neg q \wedge r \wedge s$

p $\wedge q \wedge \neg r \wedge s$

Now if any of the above variation is true the complete proposition must be True so combine then with logical OR :

(p $\wedge q \wedge r \wedge \neg s) \vee (\neg p \wedge q \wedge r \wedge s) \vee (p \wedge \neg q \wedge r \wedge s) \vee (p \wedge q \wedge \neg r \wedge s)$.

1.4 Q4

Let P (x) and Q(x) be propositional functions. Show that $\exists x(P(x) \rightarrow Q(x))$

and $\forall x P(x) \rightarrow \exists x Q(x)$

always have the same truth value

1.5 Q5

Suppose that A and B are sets such that the power set of A is a subset of the power set of B. Does it follow that A is a subset of B.

No , it is not necessary .

Let A = {1 , 2 , 3 } and let power set of A = P(A) and take P(A) = set of natural number

and let B = { -3 , -5 , 2 } and let P(B) = set of all integers

here P(A) is a subset of P(B) but A is not a subset of B .

1.6 Q6

Let A and B be sets. Show that A $\subseteq B$

if and only if A $\cap B = A$.

We have to prove : 1) if A $\cap B = A then A \subseteq B$

2) and if A $\subseteq B then A \cap B = A$

1) Let x be an arbitrary element of A . since A $\cap B = A$

means x is in A $\cap B = A. By$

the definition of intersection ,

x is also in B and this will be true

for any x chosen from A hence A $\subseteq B$.

2) Let x be an arbitrary element of A . Since A$\subseteq B$,

x is also in B. Means x is in both A and B , hence also in A$\cap B$.

Let y be an arbitrary element of A$\cap B$.

By definition of intersection, y is in both A and B .

Since A$\subseteq B$,

every element of A is in B .

Hence y is also in A .