Lecture 0 - Some motivating problems

November 20, 2024

(1) Why does a linear system of the form $Ax = b$ where A is a $m \times n$ matrix and b is $m \times 1$ have either no solution, one solution or an infinite number of solutions? Why not two solutions for example?

(2) Consider a 10×5 matrix A in a linear system $Ax = b$. There are more equations than variables. Let us say we pick the first 5 equations and solve $A_1x = b_1$ and then the second set of 5 equations to solve $A_2x = b_2$? Is this a viable way of solving the original system? Will the different linear systems have similar solutions?

(3) Given a matrix of order 5×5 having a non-zero determinant, count the number of additions, multiplications and divisions required to convert from REF to RREF.

(4) Given two consistent systems $Ax_1 = b_1$ and $Ax_2 = b_2$, does it mean that $Ax = b_1 + b_2$ is consistent? Give reasons.

(5) Construct two non-zero matrices A and B of size 3×3 such that $AB = 0$ whereas BA is non-zero.

(6) Prove that if a matrix S can be written as A^TA, then S is positive definite. What can be said about the diagonal entries of S?

(7) Write a recursive function to determine the determinant of a square matrix of size $n \times n$. How does the complexity of this code compare to the numbers of steps needed in Gaussian elimination to compute a determinant?