



# Artificial & Computational Intelligence

**AIMLCZG557**

**Contributors & Designers of document content : Cluster Course  
Faculty Team**

**M2 : Problem Solving Agent using Search  
PSO & ACO**



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# Artificial and Computational Intelligence

## Disclaimer and Acknowledgement



- Few content for these slides may have been obtained from prescribed books and various other source on the Internet
- I hereby acknowledge all the contributors for their material and inputs and gratefully acknowledge people others who made their course materials freely available online.
- .I have provided source information wherever necessary
- This is not a full fledged reading materials. Students are requested to refer to the textbook w.r.t detailed content of the presentation deck that is expected to be shared over e-learning portal - taxilla.
- I have added and modified the content to suit the requirements of the class dynamics & live session's lecture delivery flow for presentation
- **Slide Source / Preparation / Review:**
- From BITS Pilani WILP: Prof.Raja vadhana, Prof. Indumathi, Prof.Sangeetha
- From BITS Oncampus & External : Mr.Santosh GSK

# Course Plan

- M1 Introduction to AI
- M2 Problem Solving Agent using Search
- M3 Game Playing
- M4 Knowledge Representation using Logics
- M5 Probabilistic Representation and Reasoning
- M6 Reasoning over time
- M7 Ethics in AI

## Optimization Problem

**Goal** : Navigate through a state space for a given problem such that an optimal solution can be found

**Objective** : Minimize or Maximize the objective evaluation function value

**Scope** : Local

**Objective Function** : Fitness Value evaluates the goodness of current solution

**Local Search** : Search in the state-space in the neighbourhood of current position until an optimal solution is found

### Single Instance Based

Hill Climbing

Simulated Annealing

Local Beam Search

Tabu Search

### Multiple Instance Based

Genetic Algorithm

Particle Swarm Optimization

Ant Colony Optimization

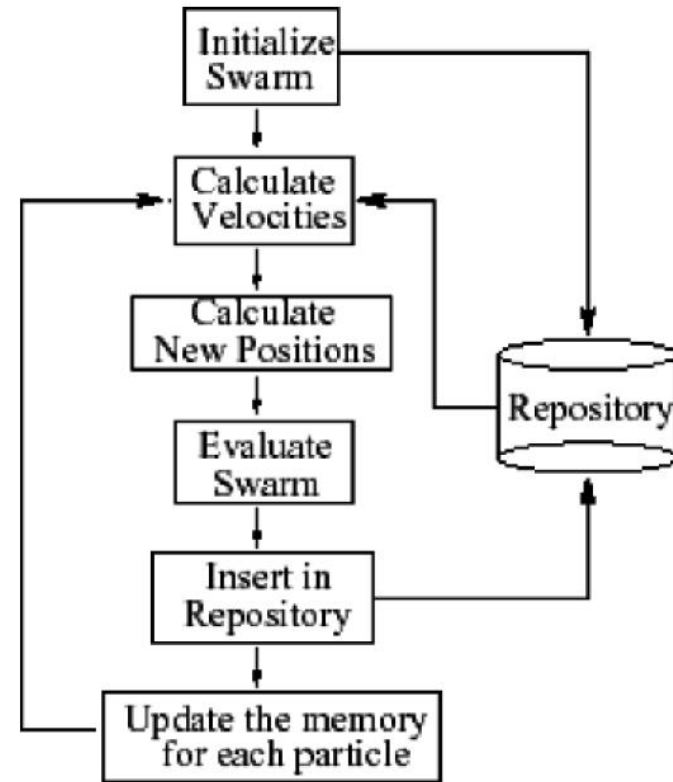
# Particle Swarm Optimization

# Particle Swarm Optimization

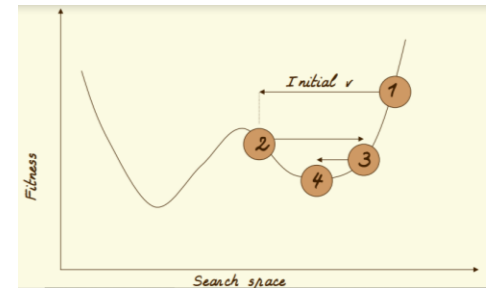
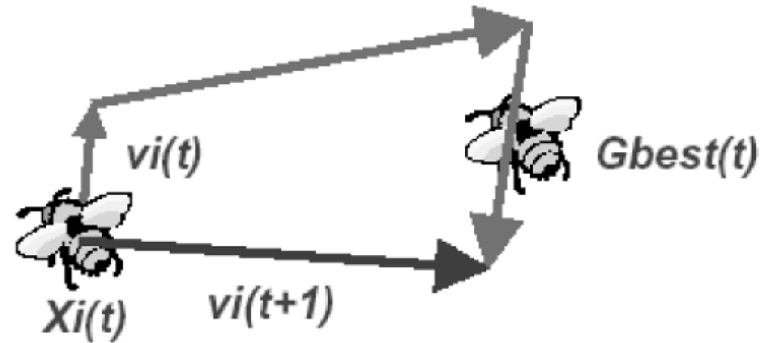
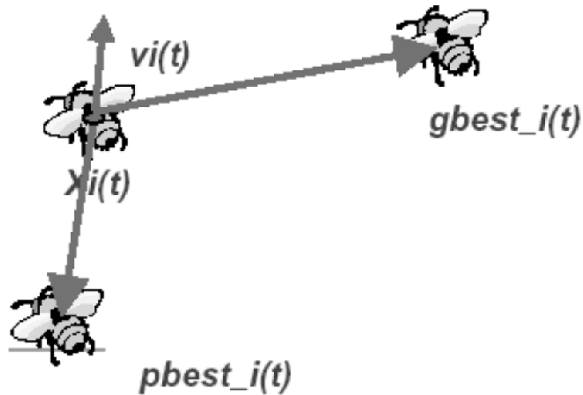
## Basic Flow of PSO

1. Initialize the swarm with random initializations
2. Evaluate fitness value for each of these individuals
3. Modify  $\mathbf{g}_{best}$ ,  $\mathbf{p}_{best}$ , and velocity
4. Move each particle to new particle
5. Goto step 2, and repeat until convergence

Particles velocities on each dimension are clamped to a max velocity  $v_{max}$



# Particle Swarm Optimization



Updating velocity vector:

$$v_i(t + 1) = \alpha v_i + c_1 \times rand \times (pbest(t) - x_i(t)) + c_2 \times rand \times (gbest(t) - x_i(t))$$

$$x_i(t + 1) = x_i(t) + v_i(t + 1)$$

$\alpha$  is inertia weight and controls exploration and exploitation  
 $c_1$  and  $c_2$  the cognition and social components respectively  
 rand is a random number generator

# Ant Colony Optimization



## Parameters used in ACO

General pseudo-code

**Procedure ACO**

**Schedule Activities**

Initialization

Construction

Update Pheromone

Daemon Actions {optional}

// local search, elitism

**End schedule activities**

**End ACO**

Parameter	Description
$N$	Total No of ants ; $N > 1$
$\tau_o$	Initial pheromone amount
$\tau_{ij}$	Amount of pheromone deposited while traversing from i to j
$\eta_{ij}$	Cost of link (i,j)
$\alpha$	Importance coefficient of pheromone intensity
$\beta$	Importance coefficient of route cost
$\rho$	evaporation co-efficient; $0 < \rho < 1$
$visit_k$	Visited nodes table of $k^{th}$ ant
$Q$	Importance - Constant value pertaining to pheromone trail
$f_k$	Route cost obtained by ant k

## Initialization

Place predefined number of ants on starting point

Set values for parameters  $\alpha, \beta, \rho$ .

Set  $\tau_0$  to 0.

## Construction

Compute the next node transition probability

$$NTP_{ij} = \frac{(\tau_{ij})^{\alpha} (\eta_{ij})^{\beta}}{\sum_{h \notin \text{visit}_k} (\tau_{ih})^{\alpha} (\eta_{ih})^{\beta}}$$

## Pheromone updation:

Pheromone reinforcement & pheromone evaporation

Direct impact on the exploitation (enhancing found food path) & exploration (discovering new path) of ant algorithms

$$\tau_{ij}^{new} = (1 - \rho)\tau_{ij}^{old} + \Delta\tau_{ij}^k$$

Amount of pheromone deposited on (i,j) by kth at that timestamp is given by

$$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{f_k} & \text{if } k^{th} \text{ ant passes } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

**Stopping criteria:** reaching predetermined number of iterations

**Problem:** Reaching pre-determined number of iterations before reaching destination leading to ant drop

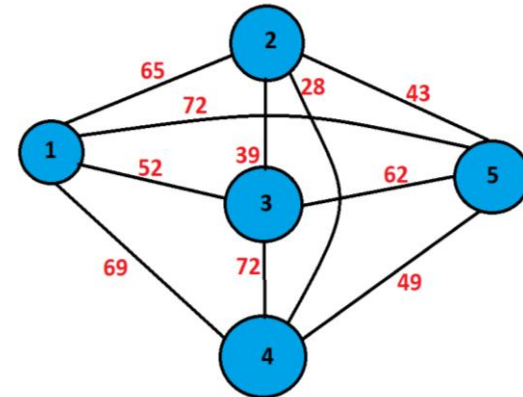
# Travelling Salesman Problem

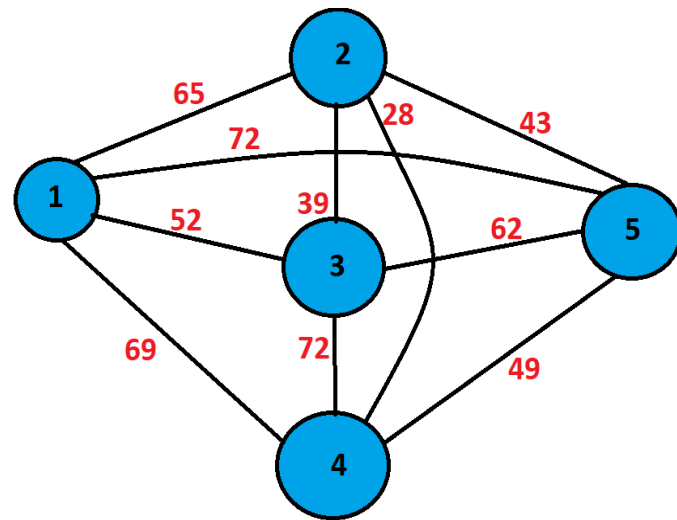


**Problem:** Given  $n$  cities, the goal is to find shortest path going through all cities and visiting each exactly once

- Consider a complete graph
- $d_{ij}$  is the route cost over  $(i,j)$   $\{f_k\}$

- Each ant builds its own tour from starting city
- Each ant chooses a town to go to with a probability
- Keep tabs on visit list of each ant
- When tour completed, lay pheromone on each edge visited
- Next city  $j$  after city  $i$  chosen according to probability rule





Initially No. of ants = No. of cities.  
 Start at 4.  $\alpha = 0.5$   $\beta = 0.75$  ( $0 \leq 1$ )  
 $Q = 100$   $\rho = 0.9$

$T = 0$

$$T_{12} = T_{21} = 0.54$$

$$T_{13} = T_{31} = 0.53$$

$$T_{14} = T_{41} = 0.35$$

$$T_{15} = T_{51} = 0.24$$

$$T_{23} = T_{32} = 0.53$$

$$T_{24} = T_{42} = 0.39$$

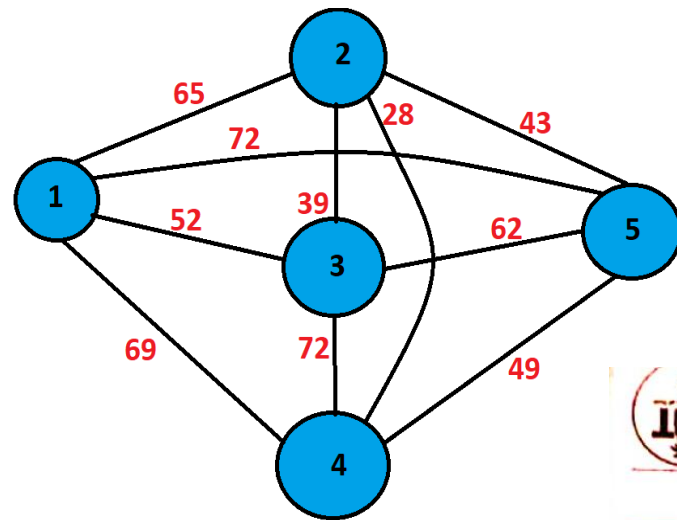
$$T_{25} = T_{52} = 0.18$$

$$T_{34} = T_{43} = 0.32$$

$$T_{35} = T_{53} = 0.90$$

$$T_{45} = T_{54} = 0.68$$

	1	2	3	4	5
1	0	65	52	69	72
2	65	0	39	28	43
3	52	39	0	72	62
4	69	28	72	0	49
5	72	43	62	49	0



Initially No. of ants = No. of cities.  
 start at 4.  $\alpha = 0.5$   $\beta = 0.75$  ( $0 \leq 1$ )  
 $Q = 100$   $\rho = 0.9$   ~~$\rho = 0.1$~~

$T = 0$

$$T_{12} = T_{21} = 0.54$$

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$$T_{34} = T_{43} = 0.32$$

$$T_{35} = T_{53} = 0.90$$

$$T_{45} = T_{54} = 0.68$$



Next Transition Probability

P

$$P_{41} = \frac{(T_{41})^\alpha (n_{41})^\beta}{(T_{41})^\alpha (n_{41})^\beta + (T_{42})^\alpha (n_{42})^\beta + (T_{43})^\alpha (n_{43})^\beta + (T_{45})^\alpha (n_{45})^\beta}$$

Ydij

$$n_{41} = 0.014$$

$$n_{43} = 0.02$$

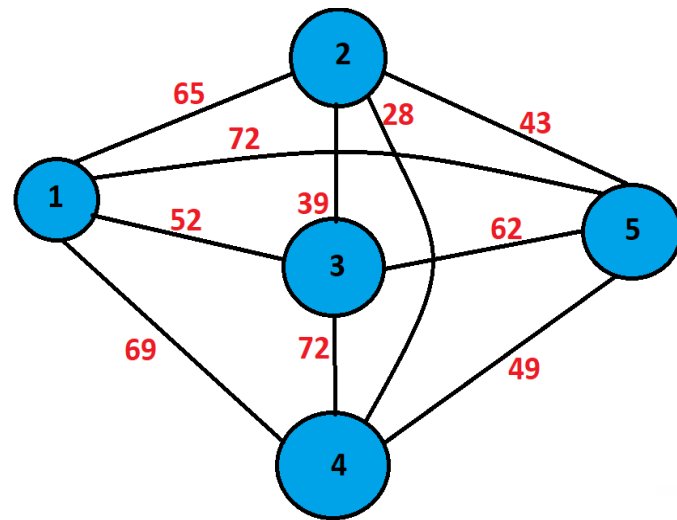
$$n_{43} = 0.014$$

$$n_{42} = 0.036$$

$$= \frac{(0.35)^{0.5} (0.014)^{0.75}}{(0.35)^{0.5} (0.014)^{0.75} + (0.399)^{0.5} (0.036)^{0.75} + (0.326)^{0.5} (0.014)^{0.75} + (0.683)^{0.5} (0.02)^{0.75}}$$

$$= \frac{0.024}{0.143} = 0.168$$





Initially No. of ants = No. of cities.  
 start at 4.  $\alpha = 0.5$   $\beta = 0.75$  ( $0 \leq 1$ )  
 $Q = 100$   $\rho = 0.9$   ~~$\rho = 0.1$~~

$T = 0$

$$T_{12} = T_{21} = 0.54$$

$$T_{13} = T_{31} = 0.53$$

$$T_{14} = T_{41} = 0.35$$

$$T_{15} = T_{51} = 0.24$$

$$T_{23} = T_{32} = 0.53$$

$$T_{24} = T_{42} = 0.39$$

$$T_{25} = T_{52} = 0.18$$

$$T_{34} = T_{43} = 0.32$$

$$T_{35} = T_{53} = 0.90$$

$$T_{45} = T_{54} = 0.68$$

$$= \frac{0.024}{0.143} = 0.168$$

$$P_{42} = \frac{0.053}{0.142} = \underline{0.364} \text{ max}$$

$$P_{43} = \frac{0.023}{0.143} = 0.160$$

$$P_{45} = 0.300$$

Since  $P_{42}$  is max, move from 4 to 2.

Initially No. of ants = No. of cities.  
 start at 4.  $\alpha = 0.5$   $\beta = 0.75$  ( $0 \leq 1$ )  
 $Q = 100$   $\rho = 0.9$   ~~$\rho = 0.1$~~

Pheromone updation ( $t=1$ )

$$\tau_{12} = 1 - \rho(\tau_{12}) + \Delta\tau_{12}$$

$$= 0.054$$

excepting for  
 $(4,2)$  &  $(2,4)$   
 $\Delta\tau_{ij}$  is zero for  
 all other  $(i,j)$

$$\tau_{13} = 0.053, \tau_{14} = 0.035,$$

$$\tau_{15} = 0.024, \tau_{23} = 0.05,$$

$$\tau_{24} = 0.039 + Q/d_{ij} = 0.039 + 100/28 = 3.610$$

$$\tau_{25} = 0.018, \tau_{34} = 0.032, \tau_{35} = 0.09$$

$$\tau_{45} = 0.068$$

Now ant is at 2.

$$P_{21} = 0.320$$

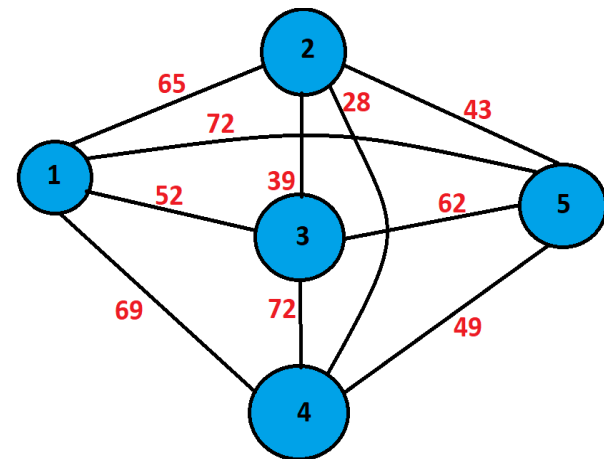
$$P_{23} = 0.429 \text{ Max}$$

$$P_{25} = 0.250$$

Now ant moves to 3!

$$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{f_k} & \text{if } k^{\text{th}} \text{ ant passes } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

$$\tau_{ij}^{\text{new}} = (1 - \rho)\tau_{ij}^{\text{old}} + \Delta\tau_{ij}^k$$





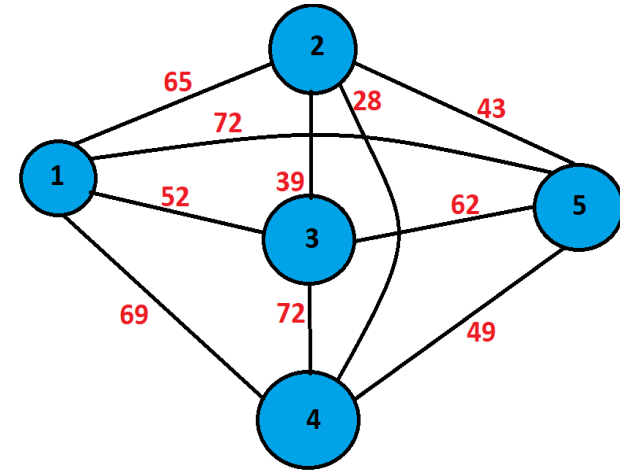
Pheromone updation ( $t_f = 2$ )

$$\tau_{12} = 0.005, \tau_{13} = 0.005, \tau_{14} = 0.003,$$

$$\tau_{15} = 0.002, \tau_{23} = 0.005 + 160/39 = 2.569$$

$$\tau_{24} = p(3.616) = 0.361, \tau_{25} = 0.001$$

$$\tau_{34} = 0.003, \tau_{35} = 0.009, \tau_{45} = 0.006$$



From 3, find  $P_{31}, P_{35}$

$$P_{31} = 0.500$$

$$P_{35} = \underline{0.5571} - \text{Max}$$

From 3, ant moves to 5.

Tabu list 4, 2, 3, 5

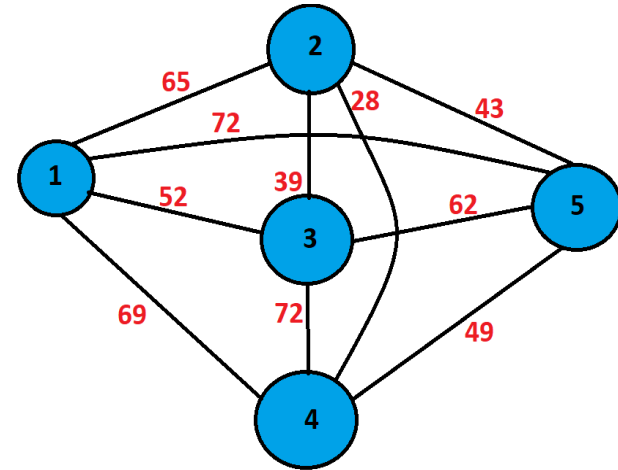
Pheromone updation  $t=3$ .

$$\tau_{12} = 0.0005, \tau_{13} = 0.0005, \tau_{14} = 0.0003,$$

$$\tau_{15} = 0.0002, \tau_{23} = 0.25, \tau_{24} = 0.03,$$

$$\tau_{25} = 0.0001, \tau_{34} = 0.0003, \tau_{35} = 0.0009 + 100\% \text{ evap}$$

$$\tau_{45} = 0.0006$$



From 5, move to 1 since it is the only non-visited city. Update pheromone.

$$\tau_{12} = 0.00005, \tau_{13} = 0.00005, \tau_{14} = 0.00003,$$

$$\tau_{15} = 1.388, \tau_{23} = 0.025$$

$$\tau_{24} = 0.003, \tau_{25} = 0.00001, \tau_{34} = 0.00003$$

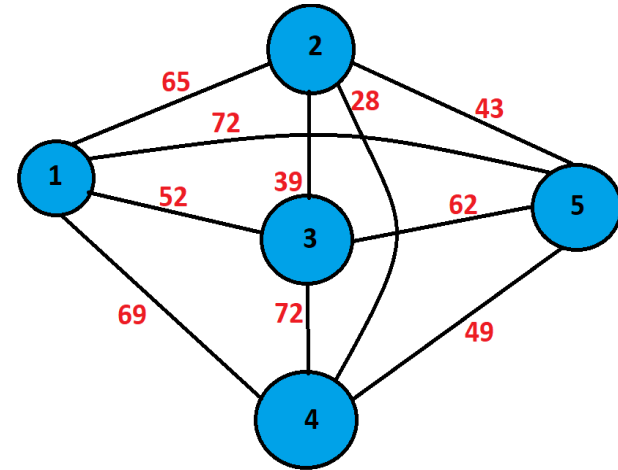
$$\tau_{35} = 0.16, \tau_{45} = 0.00006.$$

Now back to origin, since all the states are visited.

$$\begin{aligned} \text{update } \tau_{14} = \tau_{41} &= 0.00003 \times 100 / 69 \\ &= 1.449 \end{aligned}$$

Final route,

4-2-3-5-1-4.



## Previous Year Sample Question



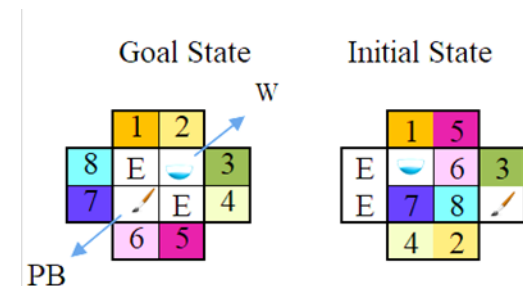
# Searching - Sample Question

Consider the below initial and goal state of the problem and answer the following questions. In a color palette board arrangement problem, there are 8 different colors, a water bowl (W), a paint brush (PB), and two empty cells (E) provided. The neighboring elements of an empty tile on the color palette board, such as colored tiles, the water bowl, and the paintbrush, can swap their positions with the empty tile. An agent should find a path i.e., series of swapping from the Initial state to achieve the Goal state.

Note: To understand the definition of neighbors, in the Goal state below colored tile Light Blue (8), Dark Orange (1) and water bowl (W), Paint brush (PB) are neighbors to one of the empty tile. Each transition will add a uniform path cost = 10 if the empty tiles are neither in the same row nor in the same column w.r.t resultant state. If they are in same column or in the same row the path cost must be cost = 5. When swapping the water bowl with an empty tile, add an additional cost of 3 for that resultant state.

H1: Number of Misplaced colored Tiles, water bowl and paint brush w.r.t to the goal state.

H2: Sum of Manhattan distances of all the misplaced tiles w.r.t to the goal state

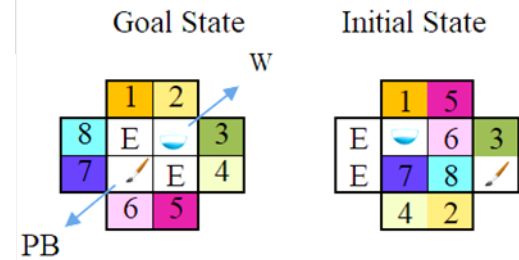
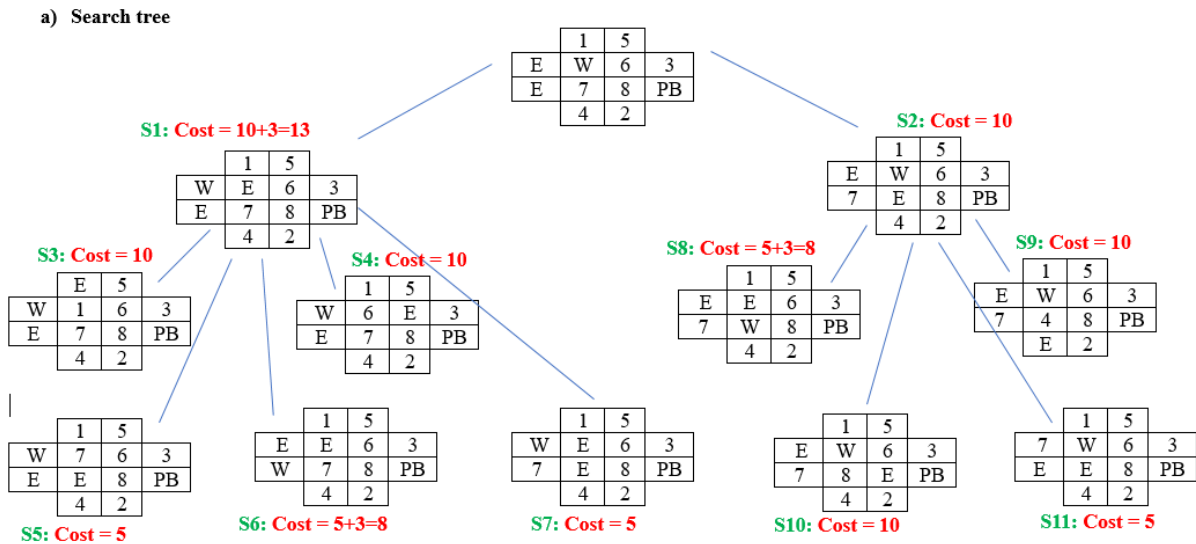


# Searching - Sample Question

Each transition will add a uniform path cost = 10 if the empty tiles are neither in the same row nor in the same column w.r.t resultant state. If they are in same column or in the same row the path cost must be cost = 5. When swapping the water bowl with an empty tile, add an additional cost of 3 for that resultant state.

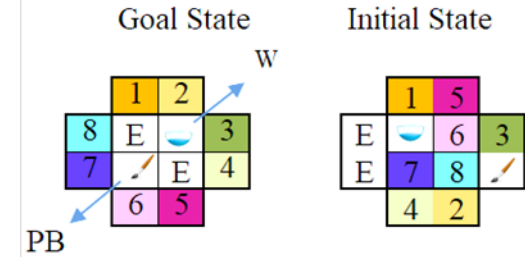
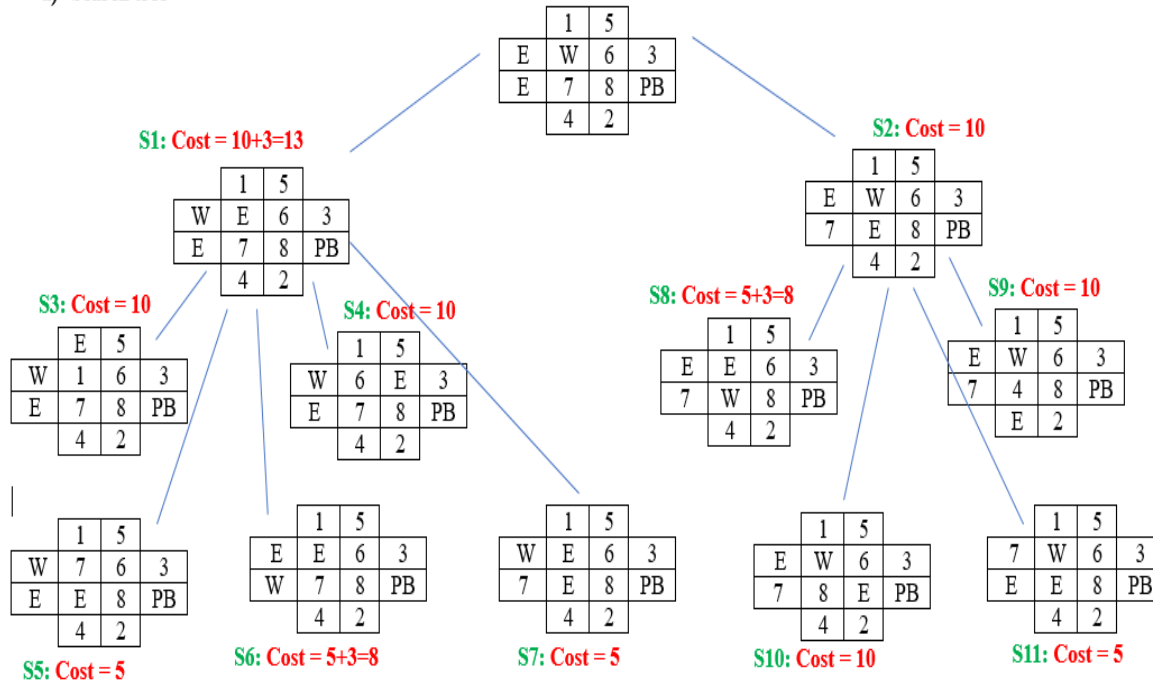
H1: Number of Misplaced colored Tiles, water bowl and paint brush w.r.t to the goal state.

H2: Sum of Manhattan distances of all the misplaced tiles w.r.t to the goal state



# Searching - Sample Question

a) Search tree



To find H1 & H2

H1 = Number of misplaced tiles, WB and PB with respect to Goal state

H2 = Sum of Manhattan distances of all the misplaced tiles with respect to the Goal state

**Goal State:**

	1	2	
8	E	W	3
7	PB	E	4
	6	5	

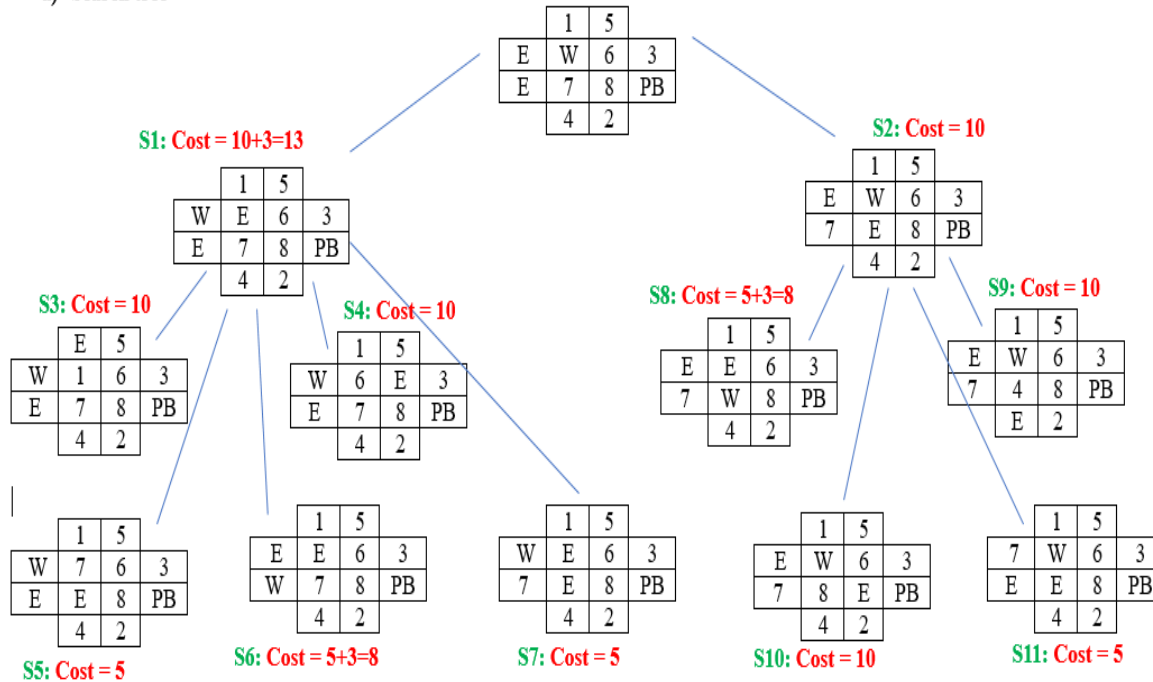
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Node	H1	H2	H(n) = h2(n) + g(n)
S1	8	3+2+3+1+3+2+3+3=20	33
S2	7	3+1+3+3+2+3+3=18	28
S3	9	21	31
S4	8	19	29

# Searching - Sample Question



a) Search tree



To find H1 & H2

H1 = Number of misplaced tiles, WB and PB with respect to Goal state

H2 = Sum of Manhattan distances of all the misplaced tiles with respect to the Goal state

Goal State:

1	2		
8	E	W	3
7	PB	E	4
6	5		

+

Node	H1	H2	H(n) = h2(n) + g(n)
S1	8	3+2+3+1+3+2+3+3=20	33
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S3	9	21	31
S4	8	19	29

## Note:

Instruct the students to practice all the algorithms listed below. Emphasize that they should use the tabulated trace recommended by the faculty in class to show the step by step working of algorithm till first 4 iterations or the 3 levels of search tree generation whichever is achieved the latest in respective algorithms.

- Breadth first search
- Depth First Search
- Uninform Cost search
- Iterative Deepening Depth First search
- Greedy Best First Search
- A\* search
- Iterative Deepening A\*
- Recursive Best First Search





# Genetic Algorithm Sample Question

You have been asked to solve below linear equation problem with multiple variables:

$$2a + 7b - 5c + d = 0$$

where a, b, c, and d are integers in the range [-20,20] (equation can have more than one solution)

You have decided to solve this question using genetic algorithm. Show all the steps involved in solving this using Genetic Algorithm. Number each of your steps, and provide appropriate title to those steps. (No need to get the final solution, just the first iteration of each step involved in the algorithm)

Most likely representation of chromosome could be a list with 4 integers with each integer representing variables a, b, c or d. For example [3,2,3,1] would imply a=3, b=2, c=3, d=1.

Generate random population (list of lists)

Fitness function - use a fitness function (most likely  $f(x) = 2a + 7b - 5c + d$ ) and calculate the fitness of each individual

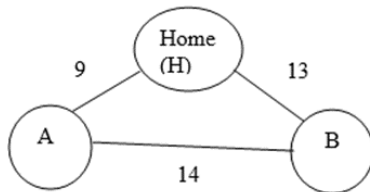
Apply some selection criteria based on fitness value (could be roulette or the best fit etc.) to come up with initial population

Apply crossover and mutation (how students chose to incorporate crossover and mutation may differ but it should be consistent)

# ACO - Sample Question



- a) Abhay is on a tour. He starts from home and visits two cities and has to reach back home. The cost of each route between cities and between his home and cities is given below in the form of graph. Determine the shortest path through Ant Colony Optimization. Use the following information regarding the various parameters for ACO.



Pheromone matrix is as given below:

	<i>H</i>	<i>A</i>	<i>B</i>
<i>H</i>	0	0.15	0.26
<i>A</i>	0.15	0	0.48
<i>B</i>	0.26	0.48	0

Rate of evaporation = 0.1;  $Q = 90$ ; The relative importance of pheromone is 0.3 and the relative importance of distance is 0.4;

# MID SEM Syllabus



Session #	List of Topic Title (from content structure in Part A)	References
1	(1) What is Artificial Intelligence: Acting Humanly, Thinking humanly, Thinking rationally, Acting Rationally (2) Foundations of AI (3) Brief Overview of Modern AI & Application Domains.	T1: 1.1 T1: 1.2, 1.4
2	(1) Intelligent Agents: Notion of Agents and Environments, Rationality (2) Nature of Environments, Structure of Agents	T1: Chapter 2
3	Problem Solving Agent: (1) Problem Formulation - Examples (2) Uninformed Search Algorithms: Uniform cost Search, Depth Limited Search, Iterative Deepening Search. (3) Notion of Heuristics (4) Informed Search Algorithms : Greedy best first search, A* search	T1: Chapter 3.1-3.4, 3.5.1, 3.5.2
4	Problem Solving Agent using Informed Search: (1) Optimality of A* Heuristic Functions: (1) Heuristic Accuracy & Algorithm performance (2) Admissible heuristics from relaxed problems, pattern databases. & Experience	T1: Chapter 3.5.2, 3.6
5	Local Search Algorithms & Optimization Problems (1) Hill Climbing Search (2) Simulated Annealing. (3) Local Beam Search	T1: Chapter 4.1
6	Local Search Algorithms & Optimization Problems (1) Genetic Algorithm	T1: Chapter 4.1, Research papers & web resources
7	Local Search Algorithms & Optimization Problems (1) Ant Colony Optimization (2) Particle Swarm Optimization	Research papers & web resources

**Required Reading:** AIMA - Chapter # 4.1, #4.2, #5.1

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials