

## Webinar-2

1 Q Answer the following questions with justifications. (Mid Sem makeup(S2-23))

(A) Given the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix},$$

a professor asks two of his best students enrolled in Linear Algebra class to find the maximum value of  $\mathbf{x}^T \mathbf{A} \mathbf{x}$ , subject to the fact that  $\|\mathbf{x}\|^2 = 1$ , where  $\|\cdot\|$  is the Euclidean norm. Given that the students have not studied Calculus earlier, the first student says that this is impossible, whereas the second one is optimistic in estimating the value. Who is correct and why? Give adequate justifications.

HINT: Find a symmetric matrix  $\mathbf{B}$  such that  $\mathbf{x}^T \mathbf{A} \mathbf{x} = \mathbf{x}^T \mathbf{B} \mathbf{x}$   
(4 marks)

(B) Is  $\lambda = 4$  an eigenvalue of

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}?$$

If yes, find the corresponding eigenvector.

(3 Marks)

**NOTE:** 1 additional mark if you can do it **without explicitly** finding the eigenvalues and checking if 4 is one of the eigenvalues.

2 Q Answer the following questions with justifications. (Mid Sem regular(S2-23))

- (1) Given the characteristic equation of a matrix  $A$ , can we compute the characteristic equation of  $cA$  where  $c$  is a non-zero scalar without knowing the entries of  $A$ ? If so, show how to do it using detailed calculations. Otherwise explain why it is not possible. Clearly state all your assumptions

3 Q Answer the following (Mid Sem makeup(S2-23))

(A) Given that the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & a \\ 0 & 2 & 0 \\ a & 0 & b \end{bmatrix}, a, b \in \mathbb{R}$$

has 2 and -1 as its eigenvalues with algebraic multiplicity of 2 and 1 respectively. Further, the eigenvector corresponding to eigenvalue -1 is

$$\mathbf{e}_3 = \begin{bmatrix} 1 \\ 0 \\ -\sqrt{2} \end{bmatrix}$$

- (a) Find the value of  $b$  (1 mark)
- (b) Write all the eigenvalues and their geometric multiplicity (1 mark)
- (c) By observation, and using the properties of a symmetric matrix, find the other two eigenvectors ( $\mathbf{e}_1$  and  $\mathbf{e}_2$ ) of  $\mathbf{A}$ . (2 mark)
- (d) Write the spectral decomposition of the matrix  $\mathbf{A}$  (1 mark)
- (e) find the value of  $a$  (1 mark)

4 Q Answer the following for the given matrix: (Mid Sem regular(S2-23))

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(A) Obtain the left-singular vectors of  $\mathbf{A}$ .

(3 marks)

(B) Obtain the right-singular vectors of  $\mathbf{A}$ .

(3 marks)

(C) Obtain the singular value matrix  $\Sigma$ . What is the spectral norm of  $\mathbf{A}$ ?

(2 marks)