Introduction Statistical Methods- Session-2

Axioms of Probability, Probability basics, Mutually exclusive and Independent events

Probability Counting Principles for Outcomes in Experiments

Counting Rule 1: Multiplication Principle

- If there are k different outcomes for each of n trials, the total number of possible outcomes is given by k^n.

Example:

- A coin is tossed 5 times. Since each toss has 2 outcomes (Heads or Tails), the total number of outcomes is:

$$2^5 = 32 \text{ outcomes.}$$

Counting Rule 2: Generalized Multiplication Principle

- If there are k_1 outcomes on the first trial, k_2 outcomes on the second trial, ..., and k_n outcomes on the n-th trial, then the total number of possible outcomes is:

$$k_1 \cdot k_2 \cdot \ldots \cdot k_n$$

Example:

- A restaurant menu offers:
  - 3 appetizers,
  - 6 beverages,
  - 9 entrées, and
  - 5 desserts.
- The total number of meal combinations a diner can choose is:

$$3 \cdot 6 \cdot 9 \cdot 5 = 810 \text{ combinations.}$$

Counting Rule 3: Rule of Sum vs. Rule of Product

- The **Rule of Product** applies when one event **AND** another event occur.
- The **Rule of Sum** applies when one event **OR** another event occurs.

Rule of Product Example (AND):

- You are selecting:
  - 1 shirt from 4 options,
  - 1 pair of pants from 3 options.
- Since you are choosing both (shirt **AND** pants), the total number of outfits is:

$$4 \cdot 3 = 12 \text{ outfits.}$$

Rule of Sum Example (OR):

- You are deciding to travel by:
  - Train (3 possible routes),
  - **OR** Bus (5 possible routes).
- Since you can choose one mode of transportation (Train **OR** Bus), the total number of route options is:

$$3 + 5 = 8 \text{ route options.}$$

Key Rule of Thumb:

- Use **Rule of Product** for "AND" scenarios where multiple events or choices occur together.
- Use **Rule of Sum** for "OR" scenarios where you choose between distinct, mutually exclusive events.

Difference Between Permutations and Combinations

Permutations

- **Definition:** Arrangements of objects where the **order matters**.
- **Formula:**

$$P(n,r) = \frac{n!}{(n-r)!}$$

  where:

  - n is the total number of objects,
  - r is the number of objects to arrange,
  - $n!$ represents the factorial of n.

Example of Permutations:

- **Problem:** In a race with 5 participants, how many ways can the top 3 positions (Gold, Silver, Bronze) be arranged?
- **Solution:**
  - Here, the order matters because Gold, Silver, and Bronze represent different ranks.
  - Total ways:

$$P(5,3) = \frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60 \text{ ways.}$$

Combinations

- **Definition:** Selections of objects where the **order does not matter.**
- **Formula:**

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

where:

- n is the total number of objects,
- r is the number of objects to select.

Example of Combinations:

- **Problem:** From 5 friends, how many ways can you choose 3 to form a team?
- **Solution:**
  - Here, the order doesn't matter because the team composition is the same regardless of the arrangement of members.
  - Total ways:

$$C(5,3) = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10 \text{ ways.}$$

Key Differences

Aspect	Permutations	Combinations
Order	Matters	Does not matter
Formula	$P(n,r) = \frac{n!}{(n-r)!}$	$C(n,r) = \frac{n!}{r!(n-r)!}$
Use Case	Arranging people in a line, assigning roles	Forming teams, selecting items

Illustration:

- **Scenario:** You have 3 letters: A, B, C. You need to select 2.
  1. **Permutations (Order Matters):**
     - Possible arrangements: AB, BA, AC, CA, BC, CB.
     - Total: $P(3,2) = 6$.
  2. **Combinations (Order Does Not Matter):**
     - Possible selections: AB, AC, BC.
     - Total: $C(3,2) = 3$.

Choosing Marbles with Repetition Allowed

Understanding the Problem

- You have n distinct objects (e.g., 9 marbles of different colors).
- You are tasked to select r objects **with repetition allowed**:
  - This means you can choose the same object multiple times.
- Since repetition is allowed, for every selection, you have n choices.

General Formula

- The total number of arrangements (permutations) when selecting r objects from n distinct types with repetition allowed is given by:

$$n^r$$

  - n: Total number of distinct objects.
  - r: Number of objects to select.

Explanation with Examples

1. **Selecting 3 Marbles Out of 9 (Repetition Allowed)**
   - Each marble selection has $n = 9$ options (since colors are distinct).
   - You select $r = 3$ marbles.
   - For each selection, the choices are independent, and repetition is allowed.
   - Total permutations:
$$n^r = 9^3 = 9 \times 9 \times 9 = 729$$
   - **Interpretation:** You can choose combinations like (Red, Red, Blue), (Green, Yellow, Red), etc., allowing duplicates.
2. **Selecting 5 Marbles Out of 9 (Repetition Allowed)**
   - Again, $n = 9$ and $r = 5$.
   - Total permutations:
$$n^r = 9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59,049$$

Why This Works

- **Repetition:** Since the same object can be chosen multiple times, every choice for an object remains the same throughout the process.
- **Independent Choices:** Each selection is independent of the others, and there are n possibilities for each of the r positions.

Generalization

- If r objects are selected from n types, **with repetition allowed**, the total number of permutations is:

$$n^r$$

This formula accounts for all possible sequences of r choices where each choice has n options.

Applications

1. **Password Generation:** Generating passwords with n characters and r-length passwords.
2. **Lottery Tickets:** Choosing numbers or symbols where repetition is allowed.
3. **Combinations in Design:** Arranging items with the possibility of repeating choices (e.g., painting patterns).

Choosing Objects Without Repetition: Introduction to Factorials

Understanding the Problem

- You have n distinct objects (e.g., 9 marbles of different colors).
- You are tasked to choose r objects, but **without repetition:**
  - Once you select an object, it is no longer available for the next selection.
- The number of available choices decreases after each selection.

General Principle

- For the first selection, there are n choices.
- For the second selection, there are $n-1$ remaining choices.
- For the third selection, there are $n-2$ remaining choices.
- This process continues until r objects are selected.

The total number of arrangements (permutations) is:

$$P(n,r) = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$$

Example: Choosing 3 Marbles Out of 9 (Without Repetition)

1. The first selection has $n = 9$ choices.
2. The second selection has $n - 1 = 8$ choices.
3. The third selection has $n - 2 = 7$ choices.
4. Total permutations:

$$P(9,3) = 9 \times 8 \times 7 = 504$$

General Case: Choosing All n Objects Without Repetition

- If you select all n objects from a set of n, the number of permutations is:

$$P(n,n) = n \times (n-1) \times (n-2) \times \ldots \times 1$$

- This is represented by the **factorial function**, written as $n!$ ("n-factorial").
- **Factorial Definition:**

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 1$$

Example:

- For $n = 9$, the factorial is:

$$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362,880$$

General Formula for Permutations (No Repetition)

- If r objects are selected from n distinct objects, the total number of permutations is:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Explanation:

- The numerator $n!$ counts all possible arrangements of n objects.
- The denominator $(n-r)!$ removes arrangements of the $n-r$ objects not selected.

Example:

- Choosing 3 marbles from 9:

$$P(9,3) = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \ldots 1}{6 \times 5 \ldots 1} = 9 \times 8 \times 7 = 504$$

Key Points

1. **Factorial Function:** $n!$ is the product of all integers from n down to 1. It represents the total number of ways to arrange n distinct objects.
2. **Permutations Formula:**
   - $P(n,r)$ is used to calculate the number of arrangements of r objects selected from n distinct objects **without repetition.**
   - The formula is:

   $$P(n,r) = \frac{n!}{(n-r)!}$$

3. **Relevance of Factorials:** Factorials simplify calculations for arranging or selecting subsets of objects.

Applications

1. **Seating Arrangements:** Arranging people in a line where each person occupies a unique spot.
2. **Password Generation (No Repetition):** Generating unique passwords from a set of characters.
3. **Sports Rankings:** Determining possible rankings of players or teams without ties.

Combination Locks: A Case of Permutations

Understanding the Problem

- A "combination lock" on a suitcase typically consists of a sequence of numbers that must be selected in the correct order to open the lock.
- Each number can range from **0 to 9**, giving **10 options per position.**
- For a 3-**number lock**, you must choose three numbers in a specific sequence.

Why the Order Matters

- If the correct sequence to open the lock is $3\ 5\ 7$, entering $7\ 3\ 5$ or $5\ 7\ 3$ will not open the lock.
- This means the **order of the numbers matters**; changing the sequence results in a completely different outcome.
- When the order matters, we are dealing with **permutations**, not combinations.

Calculation of Total Possible Permutations

1. Each of the three positions (slots) in the lock can be filled with any number from 0 to 9.
2. Repetition is **allowed** because a number can appear in multiple slots.
3. The total number of possible arrangements is:

   $$P(n,r) = n^r$$

   where:

   - $n = 10$ (total choices for each slot),
   - $r = 3$ (number of slots).

   Substituting the values:

   $$10^3 = 10 \times 10 \times 10 = 1,000 \text{ possible permutations.}$$

Why It Should Be Called a Permutation Lock

- The term "**combination**" suggests that the order **does not matter.** For example, in mathematics:
  - A combination of 3 numbers from 0 to 9 would imply choosing the numbers $3, 5,$ and 7 in any order (e.g., $3, 5, 7$ is equivalent to $7, 5, 3$).
  - However, in a lock, $3, 5, 7$ is not the same as $7, 5, 3$; only the exact sequence unlocks it.
- Since the sequence of numbers matters, **the mechanism relies on permutations, not combinations.**

Comparison of Permutations and Combinations

Aspect	Permutation Lock	Combination Lock
Order	Matters	Does not matter
Mathematical Concept	Permutations ($P(n, r) = n^r$)	Combinations ($C(n, r) = \frac{n!}{r!(n-r)!}$)
Example	$3, 5, 7 \neq 7, 5, 3$	$3, 5, 7 = 7, 5, 3$
Practical Application	Suitcase locks, padlocks	Lottery ticket selection, team formation

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Key Insights

1. **Naming Issue:** The term "combination lock" is a misnomer, as it relies on the principles of permutations.
2. **Correct Terminology:** These locks should technically be called "**permutation locks**" because the sequence of numbers is critical.
3. **Practical Importance:** Understanding this distinction is important in fields like cryptography, where order-dependent sequences play a crucial role.

Introduction to Probability and Random Experiments

Probability is the mathematical study of uncertainty and randomness. It deals with quantifying the likelihood of different outcomes in uncertain situations.

A **random experiment** is a process or action that produces an outcome which cannot be predicted with certainty in advance but has a well-defined set of possible outcomes.

Characteristics of a Random Experiment

1. **Uncertainty:** The outcome is not deterministic.
2. **Repeatability:** The experiment can be repeated under the same conditions.
3. **Sample Space (S):** The set of all possible outcomes of the experiment.

Examples of Random Experiments

1. Flip a Coin

- **Experiment:** Toss a coin.
- **Possible Outcomes (S):**

$$S = \{\text{Heads (H)}, \text{Tails (T)}\}$$

- **Randomness:** You cannot predict whether it will land heads up or tails up.
- **Real-World Application:** Coin tosses are often used in decision-making or determining the starting team in a sports match.

2. Walk to a Bus Stop

- **Experiment:** Measure the waiting time for the bus.
- **Possible Outcomes (S):** Any non-negative real number, representing the waiting time in minutes or seconds.

$$S = [0, \infty)$$

- **Randomness:** The arrival time of the bus depends on factors like traffic and scheduling, making it unpredictable.
- **Real-World Application:** Understanding bus arrival times helps in designing efficient public transportation systems.

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3. Give a Lecture

- **Experiment:** Count the number of students actively listening to the lecture.
- **Possible Outcomes (S):** A range of non-negative integers, e.g.,

$$S = \{0, 1, 2, 3, \ldots, n\}, \text{where } n \text{ is the total number of students.}$$

- **Randomness:** The number of attentive students may vary depending on factors like interest in the topic, time of day, or the lecturer's presentation style.
- **Real-World Application:** This can be used to assess audience engagement or improve teaching techniques.

4. Transmit One of a Collection of Waveforms Over a Channel

- **Experiment:** Transmit a waveform signal (e.g., a sine wave or square wave).
- **Possible Outcomes** (S):

$$S = \{\text{Waveform 1, Waveform 2, Waveform 3, ...}\}$$

- **Randomness:** The waveform that arrives at the receiver may differ due to noise, interference, or distortion in the channel.
- **Real-World Application:** This is a key concept in communication systems like radios or telecommunication networks.

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5. Identify the Received Waveform

- **Experiment:** The receiver attempts to identify the transmitted waveform from the distorted or noisy received signal.
- **Possible Outcomes** (S):

$$S = \{\text{Waveform 1, Waveform 2, ..., Incorrect Waveforms}\}$$

- **Randomness:** The receiver's identification depends on the accuracy of the transmission and decoding process.
- **Real-World Application:** This is critical in data transmission systems, where error correction ensures accurate communication.

Key Takeaways

1. A **random experiment** involves uncertainty in its outcome.
2. The **sample space** (S) represents all possible outcomes of the experiment.
3. These concepts are foundational in probability theory and have applications in fields like engineering, education, transportation, and communication systems.

Understanding Sample Space in Probability

The **sample space** (S) of a random experiment is the set of all possible outcomes that can result from the experiment. It represents the complete list of outcomes that could occur, ensuring no possibility is left out.

Characteristics of Sample Space

1. **Completeness:** S must include all possible outcomes of the experiment.
2. **Mutual Exclusivity:** Each outcome in S is distinct and does not overlap with any other.
3. **Exhaustiveness:** Every potential outcome is accounted for in S.

Examples of Sample Space

1. Throwing a Die

- **Experiment:** Throw a six-sided die and record the number that lands face up.
- **Possible Outcomes (S):**

$$S = \{1, 2, 3, 4, 5, 6\}$$

  - Each number corresponds to a distinct face of the die.
  - Example Outcome: If the die lands on "4," the observed outcome is 4.
- **Randomness:** The result of the throw is uncertain and depends on chance.
- **Real-World Application:** This sample space is used in board games and probability problems involving dice.

2. Testing an Integrated Circuit

- **Experiment:** Manufacture an integrated circuit and test it to determine whether it meets quality objectives.
- **Possible Outcomes (S):**

$$S = \{\mathrm{a}, \mathrm{r}\}$$

  - a: The circuit is **accepted** because it meets quality standards.
  - r: The circuit is **rejected** because it fails quality standards.
- **Randomness:** Whether the circuit passes or fails depends on factors like manufacturing defects, material quality, and process control.
- **Real-World Application:** This sample space is critical in quality control processes for electronics and manufacturing industries.

Detailed Explanation of the Two Examples

1. **Throwing a Die:**
   - The die has six faces, each numbered from 1 to 6.
   - The sample space S lists all the faces:

$$S = \{1, 2, 3, 4, 5, 6\}$$

   - Every outcome in S has an equal chance (for a fair die) of occurring, with a probability of $\frac{1}{6}$.
   - This is an example of a **discrete sample space**, as the outcomes are finite and distinct.
2. **Testing an Integrated Circuit:**
   - After testing, the circuit either meets or fails quality objectives.
   - The sample space is:

$$S = \{\text{accepted (a)}, \text{rejected (r)}\}$$

   - Each outcome is binary (either a or r), making this a **binary sample space**.

- Probabilities might be assigned based on prior data, e.g.:
$$P(\mathrm{a}) = 0.95,\ P(\mathrm{r}) = 0.05$$
indicating a 95% acceptance rate.

Types of Sample Spaces

1. **Discrete Sample Space:** Contains a finite or countable set of outcomes.
   - Example: Throwing a die ($S = \{1, 2, 3, 4, 5, 6\}$).
2. **Continuous Sample Space:** Contains an infinite number of outcomes, often associated with real numbers or intervals.
   - Example: Measuring the exact height of a person ($S = [0, \infty)$).

Key Takeaways

- The sample space S defines the scope of possible outcomes for a random experiment.
- Each experiment has a unique sample space that depends on its nature.
- Understanding S is fundamental to calculating probabilities and analyzing random processes.

Key Concepts in Probability

1. Event

An **event** is any subset of the sample space (S) of a random experiment. An event represents one or more outcomes of the experiment.

Example:

- **Experiment:** Roll a six-sided die.
- **Sample Space:** $S = \{1, 2, 3, 4, 5, 6\}$.
- **Event A:** Getting an even number.
  - $A = \{2, 4, 6\}$.

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2. Union of Two Events

The **union** of two events A and B, denoted as $A \cup B$, consists of all outcomes that are in A, in B, or in both.

Example:

- **Experiment:** Roll a six-sided die.
- **Event A:** Getting an even number ($A = \{2, 4, 6\}$).
- **Event B:** Getting a number greater than 3 ($B = \{4, 5, 6\}$).
- **Union $A \cup B$:**
  - $A \cup B = \{2, 4, 5, 6\}$.

3. Intersection of Two Events

The **intersection** of two events A and B, denoted as $A \cap B$, consists of all outcomes that are in both A and B.

Example:

- **Experiment:** Roll a six-sided die.
- **Event** A: Getting an even number ($A = \{2, 4, 6\}$).
- **Event** B: Getting a number greater than 3 ($B = \{4, 5, 6\}$).
- **Intersection** $A \cap B$:
  - $A \cap B = \{4, 6\}$.

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4. Mutually Exclusive Events

Two events are **mutually exclusive** if they cannot occur at the same time, i.e., $A \cap B = \emptyset$.

Example:

- **Experiment:** Roll a six-sided die.
- **Event** A: Getting an odd number ($A = \{1, 3, 5\}$).
- **Event** B: Getting an even number ($B = \{2, 4, 6\}$).
- A and B are mutually exclusive because $A \cap B = \emptyset$.

5. Definition of Probability

The **probability** of an event A, denoted $P(A)$, is a measure of the likelihood of the event occurring. It is defined as:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes in the sample space}}$$

Example:

- **Experiment:** Roll a six-sided die.
- **Event** A: Getting a 4.
  - $P(A) = \frac{1}{6}$.

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6. Empirical Approach

The **empirical approach** calculates probability based on observed data from repeated experiments:

$$P(A) = \frac{\text{Number of times event } A \text{ occurred}}{\text{Total number of trials}}$$

Example:

- Toss a coin 100 times, and "heads" appears 48 times.
- $P(\text{Heads}) = \frac{48}{100} = 0.48$.

7. Axiomatic Approach

The **axiomatic approach**, developed by Kolmogorov, defines probability using three axioms:

1. $0 \leq P(A) \leq 1$.
2. $P(S) = 1$ (the probability of the sample space is 1).
3. If A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B)$.

Example:

- **Experiment:** Roll a six-sided die.
- **Event A:** Getting a 2 ($P(A) = \frac{1}{6}$).
- **Event B:** Getting a 5 ($P(B) = \frac{1}{6}$).
- If A and B are mutually exclusive:

$$P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}.$$

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8. The Addition Rule

The **addition rule** is used to calculate the probability of the union of two events:

1. For **mutually exclusive events:**

$$P(A \cup B) = P(A) + P(B)$$

2. For **non-mutually exclusive events:**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example:

- **Experiment:** Roll a six-sided die.
- **Event A:** Getting an even number ($P(A) = \frac{3}{6}$).
- **Event B:** Getting a number greater than 3 ($P(B) = \frac{3}{6}$).
- **Intersection $A \cap B$:** Getting a 4 or 6 ($P(A \cap B) = \frac{2}{6}$).
- Using the addition rule:
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6}.$$

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9. Independent and Dependent Events

- **Independent Events:** The occurrence of one event does not affect the probability of the other.
$$P(A \cap B) = P(A) \cdot P(B)$$
  Example: Tossing two coins.
  - Event A: First coin shows heads.
  - Event B: Second coin shows tails.
- **Dependent Events:** The occurrence of one event affects the probability of the other.
$$P(A \cap B) = P(A) \cdot P(B|A)$$
  Example: Drawing two cards from a deck without replacement.
  - Event A: First card is an Ace.
  - Event B: Second card is an Ace.

10. Comparison Between Mutually Exclusive and Independent Events

Aspect	Mutually Exclusive Events	Independent Events
Definition	Events that cannot occur together.	Occurrence of one event does not affect the other.
Mathematical Relation	$P(A \cap B) = 0$	$P(A \cap B) = P(A) \cdot P(B)$
Example	Rolling a die: $A = \text{even}, B = \text{odd.}$	Tossing two coins: $A = \text{heads on coin 1}, B = \text{tails on coin 2.}$
Key Distinction	No overlap between events.	Events are independent of each other.

Key Takeaways

- **Events:** Represent outcomes or sets of outcomes.
- **Union and Intersection:** Useful for combining probabilities.
- **Mutually Exclusive vs. Independent Events:** A key distinction based on overlap and dependence.

EXAMPLE: 2

If two dice are thrown, what is the probability that the sum is

a) **Greater than 8**

b) **Less than 6**

c) **Neither 7 nor 11**

Experiment Setup

- Two dice are thrown.
- Each die has outcomes $\{1, 2, 3, 4, 5, 6\}$.
- Total possible outcomes for throwing two dice:

$$6 \times 6 = 36$$

- The **sample space** (S) contains all possible ordered pairs of the two dice:

$$S = \{(1,1), (1,2), \dots, (6,6)\}$$

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1. Probability That the Sum is Greater than 8

The sum of the two dice is greater than 8 when the possible sums are $9, 10, 11$, or 12.

Possible Pairs for Each Sum

- **Sum = 9:** $(3,6), (4,5), (5,4), (6,3)$ → 4 outcomes.
- **Sum = 10:** $(4,6), (5,5), (6,4)$ → 3 outcomes.
- **Sum = 11:** $(5,6), (6,5)$ → 2 outcomes.
- **Sum = 12:** $(6,6)$ → 1 outcome.

Total Favorable Outcomes:

$$4 + 3 + 2 + 1 = 10$$

Probability:

$$P(\text{Sum} > 8) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{10}{36} = \frac{5}{18}$$

2. Probability That the Sum is Less than 6

The sum of the two dice is less than 6 when the possible sums are $2, 3, 4$, or 5.

Possible Pairs for Each Sum

- Sum = 2: $(1, 1)$ → 1 outcome.
- Sum = 3: $(1, 2), (2, 1)$ → 2 outcomes.
- Sum = 4: $(1, 3), (2, 2), (3, 1)$ → 3 outcomes.
- Sum = 5: $(1, 4), (2, 3), (3, 2), (4, 1)$ → 4 outcomes.

Total Favorable Outcomes:

$$1 + 2 + 3 + 4 = 10$$

Probability:

$$P(\text{Sum} < 6) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{10}{36} = \frac{5}{18}$$

3. Probability That the Sum is Neither 7 nor 11

Step 1: Calculate Favorable Outcomes for Sum = 7

- Sum = 7: $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$ → 6 outcomes.

Step 2: Calculate Favorable Outcomes for Sum = 11

- Sum = 11: $(5, 6), (6, 5)$ → 2 outcomes.

Step 3: Total Outcomes for 7 or 11

$$6 + 2 = 8$$

Step 4: Outcomes for "Neither 7 nor 11"

- Total outcomes = 36.
- Outcomes for "7 or 11" = 8.
- Outcomes for "neither 7 nor 11" = $36 - 8 = 28$.

Probability:

$$P(\text{Neither 7 nor 11}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{28}{36} = \frac{7}{9}$$

Example: 3

The probability that a student passes in statistics examination is 2/3 and the probability that he /she will not pass in mathematics examination is 5/9. The probability that he/she will pass in at least one of the examination is 4/5. Find the probability that he /she will pass in both the examinations

Step 1: Define the Events

- Let A represent the event that the student passes in the **Statistics** examination.
- Let B represent the event that the student passes in the **Mathematics** examination.

Step 2: Given Probabilities

- **Probability of passing Statistics** ($P(A)$) = $\frac{2}{3}$.
- **Probability of failing Mathematics** ($P(B^c)$) = $\frac{5}{9}$, where B^c represents the complement of event B (the event that the student does not pass Mathematics).

Therefore, the probability of passing Mathematics ($P(B)$) is:

$$P(B) = 1 - P(B^c) = 1 - \frac{5}{9} = \frac{4}{9}$$

- **Probability of passing at least one of the examinations** ($P(A \cup B)$) = $\frac{4}{5}$. This represents the event where the student passes either Statistics, Mathematics, or both.

Step 3: Use the Formula for Union of Two Events

The probability of passing at least one of the examinations is given by the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Where $P(A \cap B)$ is the probability that the student passes both examinations, which is what we need to find.

Step 4: Plug in the Known Values

We are given that $P(A \cup B) = \frac{4}{5}$, $P(A) = \frac{2}{3}$, and $P(B) = \frac{4}{9}$. Substituting these values into the formula:

$$\frac{4}{5} = \frac{2}{3} + \frac{4}{9} - P(A \cap B)$$

Step 5: Solve for $P(A \cap B)$

To solve for $P(A \cap B)$, we need to find a common denominator for the fractions on the right-hand side. The least common denominator between 3 and 9 is 9. So, we rewrite the fractions:

$$\frac{2}{3} = \frac{6}{9}$$

Thus, the equation becomes:

$$\frac{4}{5} = \frac{6}{9} + \frac{4}{9} - P(A \cap B)$$

$$\frac{4}{5} = \frac{10}{9} - P(A \cap B)$$

Now, subtract $\frac{10}{9}$ from both sides:

$$P(A \cap B) = \frac{10}{9} - \frac{4}{5}$$

Step 6: Find the Common Denominator

The least common denominator between 9 and 5 is 45. Let's rewrite the fractions:

$$\frac{10}{9} = \frac{50}{45}, \quad \frac{4}{5} = \frac{36}{45}$$

So,

$$P(A \cap B) = \frac{50}{45} - \frac{36}{45} = \frac{14}{45}$$

Final Answer:

The probability that the student will pass **both** the examinations is $P(A \cap B) = \frac{14}{45}$.

Summary

- The probability that the student passes Statistics ($P(A)$) is $\frac{2}{3}$.
- The probability that the student passes Mathematics ($P(B)$) is $\frac{4}{9}$.
- The probability that the student passes at least one of the two examinations is $\frac{4}{5}$.
- Using the formula for the union of events, we found that the probability that the student passes both exams is $\frac{14}{45}$.

Example: 4

Suppose that 75% of all investors invest in traditional annuities and 45% of them invest in the stock market. If 85% invest in the stock market and/or traditional annuities, what percentage invest in both?

Step 1: Define the Events

- Let A represent the event that an investor invests in **traditional annuities.**
- Let B represent the event that an investor invests in the **stock market.**

Given Information:

- $P(A) = 0.75$ (75% invest in traditional annuities).
- $P(B) = 0.45$ (45% invest in the stock market).
- $P(A \cup B) = 0.85$ (85% invest in either traditional annuities or the stock market, or both).

We need to find $P(A \cap B)$, which is the percentage of investors who invest in **both** traditional annuities and the stock market.

Step 2: Use the Formula for the Union of Two Events

The formula for the probability of the union of two events A and B is given by:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Where:

- $P(A \cup B)$ is the probability that an investor invests in either traditional annuities, the stock market, or both.
- $P(A)$ is the probability that an investor invests in traditional annuities.
- $P(B)$ is the probability that an investor invests in the stock market.
- $P(A \cap B)$ is the probability that an investor invests in both traditional annuities and the stock market (this is what we need to find).

Step 3: Substitute the Given Values into the Formula

We are given the following:

- $P(A) = 0.75$
- $P(B) = 0.45$
- $P(A \cup B) = 0.85$

Now, substitute these values into the formula:

$$0.85 = 0.75 + 0.45 - P(A \cap B)$$

Step 4: Solve for $P(A \cap B)$

Now, solve for $P(A \cap B)$:

$$0.85 = 1.20 - P(A \cap B)$$

$$P(A \cap B) = 1.20 - 0.85$$

$$P(A \cap B) = 0.35$$

Example: 5

Suppose the manufacturer's specifications for the length of a certain type of computer cable are 2000 ± 10 millimeters. In this industry, it is known that small cable is just as likely to be defective (not meeting specifications) as large cable. That is, the probability of randomly producing a cable with length exceeding 2010 millimeters is equal to the probability of producing a cable with length smaller than 1990 millimeters. The probability that the production procedure meets specifications is known to be 0.99.

(a) What is the probability that a cable selected randomly is too large?

(b) What is the probability that a randomly selected cable is larger than 1990 millimeters?

The Setup:

1. **Cable Specifications:** The manufacturer wants the cables to be **2000 millimeters** long, but they allow a variation of **±10 millimeters.** This means:
   - The acceptable cable length is between **1990 millimeters** (2000 - 10) and **2010 millimeters** (2000 + 10).
2. **Distribution of Cable Lengths:** The lengths of the cables follow a **normal distribution** (a bell curve). The majority of the cables will have a length close to 2000 millimeters, and fewer cables will be too short or too long.
3. **The Key Information:**
   - **99% of the cables** are within the range of 1990 millimeters to 2010 millimeters (i.e., meeting the specification).
   - **1% of the cables** are either **too short (less than 1990 mm)** or **too long (greater than 2010 mm).**

Now, we need to find two things:

1. **The probability that a cable is too long (greater than 2010 millimeters).**
2. **The probability that a cable is longer than 1990 millimeters.**

Part (a): Probability that the cable is too long (greater than 2010 millimeters)

Since we know that **99% of cables** are between 1990 mm and 2010 mm, that leaves **1%** of cables outside this range — either shorter than 1990 mm or longer than 2010 mm.

Because the situation is symmetrical (i.e., the same probability of being too long as being too short), we can divide this **1%** equally between the two ends. This means:

- **0.5%** of the cables will be **too short** (less than 1990 mm).
- **0.5%** of the cables will be **too long** (greater than 2010 mm).

So, the probability that a randomly selected cable is **too long** is **0.5%** or **0.005**.

Part (b): Probability that the cable is longer than 1990 millimeters

Next, we want to find the probability that a cable is **longer than 1990 millimeters.**

- We already know that **0.5% of the cables** are shorter than 1990 mm.
- Therefore, the remaining percentage of cables must be **longer than 1990 mm.**

Since **100% of cables** are either shorter than or longer than 1990 mm, we can subtract the 0.5% that are shorter than 1990 mm from 100%:

- **100% - 0.5% = 99.5%** of cables are **longer than 1990 millimeters.**

Thus, the probability that a cable is **longer than 1990 millimeters** is **99.5%** or **0.995.**

Summary of Results:

1. **Probability that a cable is too long (greater than 2010 mm): 0.5%** or **0.005.**
2. **Probability that a cable is longer than 1990 mm: 99.5%** or **0.995.**

To determine whether two events A and B are independent, we need to check whether the following condition holds:

$$P(A \cap B) = P(A) \times P(B)$$

If the above equation is true, then the events are **independent**. Otherwise, they are **dependent**.

Step 1: Understand the Problem

We are throwing a die **twice**, which gives us two outcomes (let's call them X_1 and X_2) for the two rolls. Each roll of the die can produce outcomes from 1 to 6.

- **Event** A: One of the outcomes is 2, and the other outcome is **less than or equal to 2**. This means, for example, the pair of outcomes could be (2, 1), (1, 2), or (2, 2).
- **Event** B: One of the outcomes is 2, and the other outcome is **greater than or equal to 2**. This means, for example, the pair of outcomes could be (2, 2), (2, 3), (3, 2), etc.

Step 2: Find the Sample Space

When a die is thrown twice, the sample space contains all possible pairs of outcomes from the two rolls. The total number of outcomes is $6 \times 6 = 36$, because there are 6 possible outcomes for each die roll.

Step 3: Define the Events A and B

Let's list the outcomes for each event.

- **Event** A: One outcome is 2, and the other is less than or equal to 2. So, we need to find all pairs of numbers where one of them is 2, and the other is from the set $\{1, 2\}$.
  - Possible outcomes: (2, 1), (1, 2), and (2, 2).
  - Therefore, $A = \{(2, 1), (1, 2), (2, 2)\}$.
  - There are 3 outcomes in A, so $P(A) = \frac{3}{36} = \frac{1}{12}$.
- **Event** B: One outcome is 2, and the other is greater than or equal to 2. So, we need to find all pairs of numbers where one of them is 2, and the other is from the set $\{2, 3, 4, 5, 6\}$.
  - Possible outcomes: (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2).
  - Therefore, $B = \{(2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2)\}$.
  - There are 9 outcomes in B, so $P(B) = \frac{9}{36} = \frac{1}{4}$.

Step 4: Find the Intersection of A and B

The intersection $A \cap B$ consists of the outcomes that are in both A and B.

- From the lists of outcomes for A and B, the common outcomes are $(2, 2)$.
  - Therefore, $A \cap B = \{(2, 2)\}$.
  - There is 1 outcome in $A \cap B$, so $P(A \cap B) = \frac{1}{36}$.

Step 5: Check for Independence

Now we need to check whether $P(A \cap B) = P(A) \times P(B)$.

- $P(A) = \frac{1}{12}$
- $P(B) = \frac{1}{4}$
- $P(A \cap B) = \frac{1}{36}$

Now, check if $P(A \cap B) = P(A) \times P(B)$:

$$P(A) \times P(B) = \frac{1}{12} \times \frac{1}{4} = \frac{1}{48}$$

Since $P(A \cap B) = \frac{1}{36}$, and $P(A) \times P(B) = \frac{1}{48}$, we see that:

$$P(A \cap B) \neq P(A) \times P(B)$$

Conclusion:

Since $P(A \cap B) \neq P(A) \times P(B)$, the events A and B are **not independent**. Therefore, the two events are dependent.

Example: 9

A committee of 5 is chosen from a group of 8 men and 4 women. What is the probability that the group contains a majority of women?

Step 1: Total Number of Possible Committees

First, we need to find the total number of ways to form a committee of 5 members from the group of 8 men and 4 women (which is a total of 12 people).

The number of ways to choose 5 people out of 12 can be found using the **combination formula**:

$$C(n, k) = \frac{n!}{k!(n-k)!}$$

Where:

- n is the total number of people,
- k is the number of people to choose.

In this case, $n = 12$ (8 men + 4 women) and $k = 5$.

$$C(12, 5) = \frac{12!}{5!(12-5)!} = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$

So, the total number of possible committees is **792**.

Step 2: Committees with a Majority of Women

Now, we need to find how many of these committees contain a **majority of women.** A majority of women means that there must be **more women than men** in the committee. Since the committee has 5 members, the majority of women can only occur in the following cases:

- 3 women and 2 men
- 4 women and 1 man

We will calculate the number of ways to select these committees.

Case 1: 3 Women and 2 Men

We need to select 3 women from the 4 women, and 2 men from the 8 men.

- The number of ways to choose 3 women from 4 is:

$$C(4,3) = \frac{4!}{3!(4-3)!} = \frac{4}{1} = 4$$

- The number of ways to choose 2 men from 8 is:

$$C(8,2) = \frac{8!}{2!(8-2)!} = \frac{8 \times 7}{2 \times 1} = 28$$

So, the number of committees with 3 women and 2 men is:

$$C(4,3) \times C(8,2) = 4 \times 28 = 112$$

Step 3: Total Number of Committees with a Majority of Women

Now, we add up the results from the two cases:

$$\text{Total number of committees with a majority of women} = 112 + 8 = 120$$

Step 4: Probability of a Majority of Women

Finally, to find the probability that a randomly selected committee contains a majority of women, we divide the number of favorable outcomes (committees with a majority of women) by the total number of possible committees:

$$P(\text{Majority of Women}) = \frac{\text{Number of favorable committees}}{\text{Total number of committees}} = \frac{120}{792}$$

Simplifying the fraction:

$$P(\text{Majority of Women}) = \frac{120}{792} = \frac{5}{33}$$

Final Answer:

The probability that the committee contains a majority of women is $\frac{5}{33}$.

Practice problems:

Q1: A Survey conducted by a bank revealed that 40% of the accounts are savings accounts and 35% of the accounts are current accounts and the balance are loan accounts.

- **What is the probability that an account taken at random is a loan account?**
- **What is the probability that an account taken at random is NOT savings account?**
- **What is the probability that an account taken at random is NOT a current account?**
- **What is the probability that an account taken at random is a current account or a loan account?**

Given Data:

- 40% of the accounts are savings accounts ($P(\text{Savings}) = 0.40$).
- 35% of the accounts are current accounts ($P(\text{Current}) = 0.35$).
- The balance are loan accounts. Since the total probability is 1 (or 100%), the remaining probability corresponds to loan accounts.

Step 1: Calculate the probability of loan accounts

Since the total probability must sum to 1, we can calculate the probability of loan accounts:

$$P(\text{Loan}) = 1 - P(\text{Savings}) - P(\text{Current})$$

Substitute the values for savings and current accounts:

$$P(\text{Loan}) = 1 - 0.40 - 0.35 = 0.25$$

So, the probability that a randomly selected account is a loan account is:

$$P(\text{Loan}) = 0.25$$

Step 2: Probability that an account is NOT a savings account

The probability that an account is **not** a savings account is the complement of the probability that it is a savings account. The complement of an event is calculated by subtracting the probability of the event from 1:

$$P(\text{Not Savings}) = 1 - P(\text{Savings}) = 1 - 0.40 = 0.60$$

So, the probability that an account is **not** a savings account is:

$$P(\text{Not Savings}) = 0.60$$

Step 3: Probability that an account is NOT a current account

Similarly, the probability that an account is **not** a current account is the complement of the probability that it is a current account:

$$P(\text{Not Current}) = 1 - P(\text{Current}) = 1 - 0.35 = 0.65$$

So, the probability that an account is **not** a current account is:

$$P(\text{Not Current}) = 0.65$$

Step 4: Probability that an account is a current account or a loan account

The probability that an account is either a **current account** or a **loan account** is the union of these two events. Since a single account can only be one type (savings, current, or loan), the events "current" and "loan" are mutually exclusive. This means we can add the probabilities of these two events:

$$P(\text{Current or Loan}) = P(\text{Current}) + P(\text{Loan})$$

Substitute the known values:

$$P(\text{Current or Loan}) = 0.35 + 0.25 = 0.60$$

So, the probability that an account is either a current account or a loan account is:

$$P(\text{Current or Loan}) = 0.60$$

Summary of Results:

1. **Probability that an account is a loan account**: $P(\text{Loan}) = 0.25$
2. **Probability that an account is not a savings account**: $P(\text{Not Savings}) = 0.60$
3. **Probability that an account is not a current account**: $P(\text{Not Current}) = 0.65$
4. **Probability that an account is a current account or a loan account**: $P(\text{Current or Loan}) = 0.60$

Practice problems:

Q2. A speaks truth in 80% cases and B speaks in 60% cases. What percentage of cases are they likely to contradict each other in stating the same fact?

Given Data:

- A speaks the truth 80% of the time, so the probability that A speaks the truth ($P(A_{\text{True}})$) is:

$$P(A_{\text{True}}) = 0.80$$

- B speaks the truth 60% of the time, so the probability that B speaks the truth ($P(B_{\text{True}})$) is:

$$P(B_{\text{True}}) = 0.60$$

- Therefore, the probability that A lies is:

$$P(A_{\text{False}}) = 1 - P(A_{\text{True}}) = 1 - 0.80 = 0.20$$

- The probability that B lies is:

$$P(B_{\text{False}}) = 1 - P(B_{\text{True}}) = 1 - 0.60 = 0.40$$

Step 1: Identifying Contradictions

For A and B to contradict each other, one must speak the truth and the other must lie. There are two possibilities for this contradiction:

1. A speaks the truth and B lies.
2. A lies and B speaks the truth.

Step 2: Calculate the Probability of Each Contradiction Case

1. A speaks the truth and B lies:

The probability of this happening is the product of the probability that A speaks the truth and the probability that B lies:

$$P(\text{A True, B False}) = P(A_{\text{True}}) \times P(B_{\text{False}}) = 0.80 \times 0.40 = 0.32$$

2. A lies and B speaks the truth:

The probability of this happening is the product of the probability that A lies and the probability that B speaks the truth:

$$P(\text{A False, B True}) = P(A_{\text{False}}) \times P(B_{\text{True}}) = 0.20 \times 0.60 = 0.12$$

Step 3: Calculate the Total Probability of Contradiction

The total probability that A and B contradict each other is the sum of the probabilities of the two contradictory cases:

$$P(\text{Contradiction}) = P(\text{A True, B False}) + P(\text{A False, B True}) = 0.32 + 0.12 = 0.44$$

Step 4: Convert to Percentage

To express this as a percentage, multiply the probability by 100:

$$P(\text{Contradiction}) \times 100 = 0.44 \times 100 = 44\%$$

Final Answer:

The probability that A and B are likely to contradict each other is **44%**.

This means that in 44% of the cases, A and B will provide contradictory statements when stating the same fact.

Practice problems:

In a certain residential suburb, 60% of all households get Internet service from the local cable company, 80% get television service from that company, and 50% get both services from that company. If a household is randomly selected,

i) **What is the probability that it gets at least one of these two services from the company?**

ii) **ii) What is the probability that it gets exactly one of these services from the company?**

Given Data:

- **P(Internet):** The probability that a household gets Internet service from the cable company is 60% or 0.60.

$$P(\text{Internet}) = 0.60$$

- **P(Television):** The probability that a household gets television service from the cable company is 80% or 0.80.

$$P(\text{Television}) = 0.80$$

- **P(Internet ∩ Television):** The probability that a household gets both Internet and television services from the cable company is 50% or 0.50.

$$P(\text{Internet} \cap \text{Television}) = 0.50$$

Part (i): Probability that the household gets at least one of these two services

We want to calculate the probability that a randomly selected household gets at least one of these two services (either Internet, Television, or both). This is represented by the union of the two events:

$$P(\text{Internet} \cup \text{Television})$$

We can use the formula for the union of two events:

$$P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B})$$

Where:

- $P(\text{A})$ is the probability that the household gets Internet.
- $P(\text{B})$ is the probability that the household gets Television.
- $P(\text{A} \cap \text{B})$ is the probability that the household gets both services.

Substituting the given values:

$$P(\text{Internet} \cup \text{Television}) = P(\text{Internet}) + P(\text{Television}) - P(\text{Internet} \cap \text{Television})$$

$$P(\text{Internet} \cup \text{Television}) = 0.60 + 0.80 - 0.50$$

$$P(\text{Internet} \cup \text{Television}) = 1.40 - 0.50 = 0.90$$

So, the probability that a household gets at least one of the two services is:

$$P(\text{Internet} \cup \text{Television}) = 0.90$$

Answer for part (i): The probability that the household gets at least one of the services is **0.90**, or **90%**.

Part (ii): Probability that the household gets exactly one of these services

Now, we need to find the probability that the household gets exactly one of the services, which means it gets either Internet or Television, but not both.

We can break this down into two cases:

1. **The household gets only Internet**: This means it gets Internet but not Television. The probability of this is:

$$P(\text{Only Internet}) = P(\text{Internet}) - P(\text{Internet} \cap \text{Television})$$

Substituting the values:

$$P(\text{Only Internet}) = 0.60 - 0.50 = 0.10$$

2. **The household gets only Television**: This means it gets Television but not Internet. The probability of this is:

$$P(\text{Only Television}) = P(\text{Television}) - P(\text{Internet} \cap \text{Television})$$

Substituting the values:

$$P(\text{Only Television}) = 0.80 - 0.50 = 0.30$$

Now, to find the total probability of getting exactly one service, we sum the probabilities of getting only Internet and getting only Television:

$$P(\text{Exactly one service}) = P(\text{Only Internet}) + P(\text{Only Television})$$

$$P(\text{Exactly one service}) = 0.10 + 0.30 = 0.40$$

So, the probability that a household gets exactly one of the services is:

$$P(\text{Exactly one service}) = 0.40$$

Answer for part (ii): The probability that the household gets exactly one of the services is **0.40**, or **40%**.

Practice problems:

Q4: The next generation of miniaturised wireless capsules with active locomotion will require two miniature electric motors to manoeuvre each capsule. Suppose 10 motors have been fabricated but that, in spite of test performed on the individual motors 2 will not operate satisfactorily when placed into capsule, to fabricate a new capsule,2 motors will be randomly selected (that is, each pair of motors has the same chance of being selected) find the probability that

a) **Both motors will operate satisfactorily in the capsule.**

b) **One motor will operate satisfactorily and other will not.**

Problem Breakdown:

We are tasked with finding the probabilities for selecting two motors from a batch of 10, where 8 of the motors are satisfactory, and 2 are defective. The selection is random, so every pair of motors has an equal chance of being chosen.

Step 1: Understand the Scenario

- Total motors = 10
- Satisfactory motors = 8
- Defective motors = 2
- Two motors are selected at random.

We are finding probabilities for two events:

1. **(a)** Both selected motors are satisfactory.
2. **(b)** One motor is satisfactory, and the other is defective.

Step 2: Total Possible Outcomes

The total number of ways to choose 2 motors from a set of 10 is calculated using combinations:

$$\binom{10}{2} = \frac{10 \times 9}{2} = 45$$

So, there are 45 possible ways to choose any 2 motors from the 10.

Part (a): Probability Both Motors Are Satisfactory

To have both motors satisfactory, we need to select 2 satisfactory motors out of the 8 satisfactory ones. The number of ways to do this is:

$$\binom{8}{2} = \frac{8 \times 7}{2} = 28$$

Thus, the probability that both selected motors are satisfactory is:

$$P(\text{Both satisfactory}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{\binom{8}{2}}{\binom{10}{2}}$$

Substitute the values:

$$P(\text{Both satisfactory}) = \frac{28}{45}$$

$$P(\text{Both satisfactory}) \approx 0.622 \text{ (or } 62.2\%\text{)}.$$

Part (b): Probability One Motor is Satisfactory and the Other is Defective

To have one satisfactory motor and one defective motor, we need:

- **1 satisfactory motor** selected from the 8 satisfactory motors: $\binom{8}{1} = 8$,
- **1 defective motor** selected from the 2 defective motors: $\binom{2}{1} = 2$.

The total number of favorable outcomes is:

$$\binom{8}{1} \times \binom{2}{1} = 8 \times 2 = 16$$

Thus, the probability that one motor is satisfactory, and the other is defective is:

$$P(\text{One satisfactory, one defective}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{16}{45}$$

$$P(\text{One satisfactory, one defective}) \approx 0.356 \text{ (or } 35.6\%\text{)}.$$

Practice problems:

Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

i) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

ii) What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

Problem Analysis

We are given:

- $P(\text{Coffee}) = 0.55$ (55% of adults regularly consume coffee).
- $P(\text{Soda}) = 0.45$ (45% of adults regularly consume soda).
- $P(\text{Coffee or Soda}) = 0.70$ (70% regularly consume at least one of the two products).

We aim to calculate:

1. $P(\text{Coffee and Soda})$: The probability that a randomly selected adult regularly consumes both coffee and soda.
2. $P(\text{Neither Coffee nor Soda})$: The probability that a randomly selected adult does not regularly consume at least one of the two products.

Part (i): Probability of Regularly Consuming Both Coffee and Soda

Using the formula for the union of two events:

$$P(\text{Coffee or Soda}) = P(\text{Coffee}) + P(\text{Soda}) - P(\text{Coffee and Soda}),$$

we can rearrange to solve for $P(\text{Coffee and Soda})$:

$$P(\text{Coffee and Soda}) = P(\text{Coffee}) + P(\text{Soda}) - P(\text{Coffee or Soda}).$$

Substitute the given probabilities:

$$P(\text{Coffee and Soda}) = 0.55 + 0.45 - 0.70.$$

Simplify:

$$P(\text{Coffee and Soda}) = 0.30.$$

Thus, the probability that a randomly selected adult regularly consumes both coffee and soda is **0.30 (or 30%)**.

Part (ii): Probability of Not Regularly Consuming At Least One of the Two Products

The probability that a randomly selected adult regularly consumes at least one of the two products is:

$$P(\text{Coffee or Soda}) = 0.70.$$

The complement of this event is the probability that a randomly selected adult **does not consume at least one of the two products**, which is:

$$P(\text{Neither Coffee nor Soda}) = 1 - P(\text{Coffee or Soda}).$$

Substitute the given probability:

$$P(\text{Neither Coffee nor Soda}) = 1 - 0.70.$$

Simplify:

$$P(\text{Neither Coffee nor Soda}) = 0.30.$$

Thus, the probability that a randomly selected adult does not regularly consume at least one of these two products is **0.30 (or 30%)**.

Practice problems:

Q6. Suppose a student is selected at random from 80 students where 30 are taking mathematics, 20 are taking chemistry and 10 are taking both. Find the probability 'p' that the student is taking Mathematics or chemistry?

Problem Analysis

We are tasked to find the probability that a randomly selected student is taking **Mathematics or Chemistry**. Let's define the events:

- M: Student is taking Mathematics.
- C: Student is taking Chemistry.

We know:

- $|M| = 30$: Number of students taking Mathematics.
- $|C| = 20$: Number of students taking Chemistry.
- $|M \cap C| = 10$: Number of students taking both Mathematics and Chemistry.
- Total number of students = 80.

The probability $P(M \cup C)$, i.e., the probability that a student is taking Mathematics or Chemistry, is calculated using the **union formula**:

$$P(M \cup C) = P(M) + P(C) - P(M \cap C).$$

Step 1: Calculate Individual Probabilities

The probability of a student taking Mathematics ($P(M)$) is:

$$P(M) = \frac{|M|}{\text{Total students}} = \frac{30}{80}.$$

Simplify:

$$P(M) = 0.375.$$

The probability of a student taking Chemistry ($P(C)$) is:

$$P(C) = \frac{|C|}{\text{Total students}} = \frac{20}{80}.$$

Simplify:

$$P(C) = 0.25.$$

The probability of a student taking both Mathematics and Chemistry ($P(M \cap C)$) is:

$$P(M \cap C) = \frac{|M \cap C|}{\text{Total students}} = \frac{10}{80}.$$

Simplify:

$$P(M \cap C) = 0.125.$$

Step 2: Apply the Union Formula

Substitute the values into the formula for $P(M \cup C)$:

$$P(M \cup C) = P(M) + P(C) - P(M \cap C).$$

$$P(M \cup C) = 0.375 + 0.25 - 0.125.$$

Simplify:

$$P(M \cup C) = 0.5.$$

Final Answer

The probability that a randomly selected student is taking **Mathematics or Chemistry** is:

$$P(M \cup C) = 0.5 \text{ (or } 50\%).$$

Practice problems:

Q 7: If A and B are events with P(A U B) = 7/8,

P(A ∩ B) = ¼ and P(A') = 5/8, find P(A), P(B) and P(A ∩ B')

We are tasked with finding:

1. $P(A)$ (the probability of event A),
2. $P(B)$ (the probability of event B),
3. $P(A \cap B')$ (the probability of A and B', where B' is the complement of B).

Given Information

- $P(A \cup B) = \frac{7}{8}$,
- $P(A \cap B) = \frac{1}{4}$,
- $P(A') = \frac{5}{8}$ (where A' is the complement of A).

Step 1: Find $P(A)$

The relationship between $P(A')$ and $P(A)$ is:

$$P(A') = 1 - P(A).$$

Substitute $P(A') = \frac{5}{8}$:

$$\frac{5}{8} = 1 - P(A).$$

Rearrange to solve for $P(A)$:

$$P(A) = 1 - \frac{5}{8} = \frac{3}{8}.$$

Thus, $P(A) = \frac{3}{8}$.

Step 2: Find $P(B)$

Use the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Substitute the known values:

$$\frac{7}{8} = \frac{3}{8} + P(B) - \frac{1}{4}.$$

Convert $\frac{1}{4}$ to $\frac{2}{8}$ for consistency:

$$\frac{7}{8} = \frac{3}{8} + P(B) - \frac{2}{8}.$$

Simplify:

$$\frac{7}{8} = \frac{1}{8} + P(B).$$

Solve for $P(B)$:

$$P(B) = \frac{7}{8} - \frac{1}{8} = \frac{6}{8}.$$

Simplify:

$$P(B) = \frac{3}{4}.$$

Step 3: Find $P(A \cap B')$

The relationship between $P(A)$, $P(A \cap B)$, and $P(A \cap B')$ is:

$$P(A) = P(A \cap B) + P(A \cap B').$$

Substitute $P(A) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{4}$ (convert $\frac{1}{4}$ to $\frac{2}{8}$):

$$\frac{3}{8} = \frac{2}{8} + P(A \cap B').$$

Solve for $P(A \cap B')$:

$$P(A \cap B') = \frac{3}{8} - \frac{2}{8} = \frac{1}{8}.$$

Final Answers

1. $P(A) = \frac{3}{8}$,
2. $P(B) = \frac{3}{4}$,
3. $P(A \cap B') = \frac{1}{8}$.

Practice problems:

Q8. In a Sample space, events A and B are such that P(A)=P(B), $P(\bar{A} \cap \bar{B}) = P(A \cap B) = \frac{1}{6}$. *Find*

$$a)\ P(A),$$

$$b) P(\bar{A} \cup \bar{B})$$

$$c)\ P(\textit{exactly one of the events A or B})$$

We are tasked to calculate the following probabilities based on the given information:

1. $P(A)$,
2. $P(A' \cup B')$,
3. $P(\text{exactly one of the events A or B occurs})$.

Given Information

- $P(A) = P(B)$,
- $P(A^c \cap B^c) = \frac{1}{6}$,
- $P(A \cap B) = \frac{1}{6}$.

Let's proceed step by step:

Step 1: Find $P(A)$

We know the basic relationship for a sample space:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Substituting $P(A) = P(B)$ and $P(A \cap B) = \frac{1}{6}$, we have:

$$P(A \cup B) = 2P(A) - \frac{1}{6}.$$

Additionally, the complement of $A \cup B$ is $A^c \cap B^c$, and its probability is given as:

$$P(A^c \cap B^c) = \frac{1}{6}.$$

Since $P(A^c \cap B^c) + P(A \cup B) = 1$, we can write:

$$P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}.$$

Substitute $P(A \cup B) = \frac{5}{6}$ into the equation $P(A \cup B) = 2P(A) - \frac{1}{6}$:

$$\frac{5}{6} = (A) - \frac{1}{6}.$$

Add $\frac{1}{6}$ to both sides:

$$\frac{5}{6}+\frac{1}{6}=2P(A).$$

Simplify:

$$\frac{6}{6}=2P(A).$$

$$P(A)=\frac{1}{2}.$$

Thus, $P(A)=\frac{1}{2}$.

Step 2: Find $P(A'\cup B')$

Using the complement rule:

$$P(A'\cup B')=1-P(A\cap B).$$

Substitute $P(A\cap B)=\frac{1}{6}$:

$$P(A'\cup B')=1-\frac{1}{6}=\frac{5}{6}.$$

Thus, $P(A'\cup B')=\frac{5}{6}$.

Step 3: Find $P(\text{exactly one of A or B occurs})$

The probability of exactly one of A or B occurring is:

$$P(\text{exactly one of A or B})=P(A\cap B^c)+P(A^c\cap B).$$

Using the relationship:

$$P(A)=P(A\cap B)+P(A\cap B^c),$$

we can write:

$$P(A\cap B^c)=P(A)-P(A\cap B).$$

Substitute $P(A)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{6}$:

$$P(A\cap B^c)=\frac{1}{2}-\frac{1}{6}=\frac{3}{6}-\frac{1}{6}=\frac{2}{6}.$$

Similarly, for $P(A^c\cap B)$, use:

$$P(B)=P(A\cap B)+P(A^c\cap B),$$

$$P(A^c\cap B)=P(B)-P(A\cap B).$$

Substitute $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{6}$:

$$P(A^c\cap B)=\frac{1}{2}-\frac{1}{6}=\frac{2}{6}.$$

Add the two probabilities:

$$P(\text{exactly one of A or B})=P(A\cap B^c)+P(A^c\cap B)=\frac{2}{6}+\frac{2}{6}=\frac{4}{6}.$$

Simplify:

$$P(\text{exactly one of A or B})=\frac{2}{3}.$$

Practice problems:

Q9. An insurance company offers four different deductible levels—none, low, medium, and high—for its homeowner's policyholders and three different levels—low, medium, and high—for its automobile policyholders. The accompanying table gives proportions for the various categories of policyholders who have both types of insurance.

For example, the proportion of individuals with both low homeowner's deductible and low auto deductible is .06 (6% of all such individuals.

Auto	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.1

Suppose an individual having both types of policies is randomly selected.

a. What is the probability that the individual has a medium auto deductible and a high homeowner's deductible?

b. What is the probability that the individual has a low auto deductible? A low homeowner's deductible?

c. What is the probability that the individual is in the same category for both auto and homeowner's deductibles?

d. Based on your answer in part (c), what is the probability that the two categories are different?

e. What is the probability that the individual has at least one low deductible level?

f. Using the answer in part (e), what is the probability that neither deductible level is low?

Table Representation Recap:

Auto/Homeowner Deductible	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.10

(a) Probability that the individual has a medium auto deductible and a high homeowner's deductible:

From the table:

- The probability $P(\text{Auto} = \text{M and Homeowner} = \text{H}) = 0.10$.

Thus:

$$P(\text{Medium auto and High homeowner}) = 0.10$$

(b) Probability that the individual has:

1. **A low auto deductible:** To calculate this, sum up all the probabilities in the **"L" column:**

$$P(\text{Auto} = \text{L}) = 0.06 + 0.10 + 0.03 = 0.19$$

2. **A low homeowner's deductible:** To calculate this, sum up all the probabilities in the **"L" row:**

$$P(\text{Homeowner} = \text{L}) = 0.06 + 0.10 + 0.05 + 0.03 = 0.24$$

(c) Probability that the individual is in the same category for both auto and homeowner's deductibles:

Same categories mean:

- Auto = Homeowner = N
- Auto = Homeowner = L
- Auto = Homeowner = M
- Auto = Homeowner = H.

From the table:

$$P(\text{Same category}) = P(\text{N}, \text{N}) + P(\text{L}, \text{L}) + P(\text{M}, \text{M}) + P(\text{H}, \text{H}).$$

Substitute values:

$$P(\text{Same category}) = 0.04 + 0.10 + 0.20 + 0.10 = 0.44$$

(d) Probability that the two categories are different:

The complement of being in the same category is being in different categories:

$$P(\text{Different categories}) = 1 - P(\text{Same category}).$$

Substitute $P(\text{Same category}) = 0.44$:

$$P(\text{Different categories}) = 1 - 0.44 = 0.56$$

(e) Probability that the individual has at least one low deductible level:

At least one low deductible means:

$$P(\text{At least one L}) = 1 - P(\text{No L in either category}).$$

To calculate $P(\text{No L in either category})$, sum up probabilities where neither auto nor homeowner deductible is "L":

- $P(\text{No L}) = P(\text{N, N}) + P(\text{N, M}) + P(\text{N, H}) + P(\text{M, N}) + P(\text{M, M}) + P(\text{M, H}) + P(\text{H, N}) + P(\text{H, M}) + P(\text{H, H}).$

Substitute values:

$$P(\text{No L}) = 0.04 + 0.05 + 0.03 + 0.07 + 0.20 + 0.10 + 0.02 + 0.15 + 0.10 = 0.76.$$

Thus:

$$P(\text{At least one L}) = 1 - 0.76 = 0.24$$

(f) Probability that neither deductible level is low:

This is just $P(\text{No L})$, which we already calculated in part (e):

$$P(\text{Neither deductible is low}) = 0.76$$