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Assignment - 1

a) Test for consistency and solve :-

i) $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$

$\Rightarrow$ $AX = B$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$\Rightarrow$ $A:B \Rightarrow$

$$\begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 3 & 1 & -3 & : & 13 \\ 2 & 19 & -47 & : & 32 \end{bmatrix}$$

Applying Row operations :-

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 1 & 4 & -10 & : & 8 \\ 0 & 22 & -54 & : & 27 \end{bmatrix}$$

$R_3 \rightarrow \frac{1}{2} R_3$, $R_2 \rightarrow 2R_2 - R_1$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 11 & -27 & : & 11 \\ 0 & 11 & -27 & : & 13.5 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 11 & -27 & : & 11 \\ 0 & 0 & 0 & : & 13.5 \end{bmatrix} \rightarrow \begin{matrix} \rho(A) = 2 \\ \rho(A:B) = 3 \end{matrix}$$

$\Rightarrow$ $\rho(A) \neq \rho(A:B)$

~~Ans~~ So, the system of Eq^n is inconsistant.

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(ii) $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$

$$\Rightarrow A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Rightarrow$ $A : B$

$$\left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_1 + R_2$, $R_2 \to 2R_2 + R_1$

$$\Rightarrow \left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & -2 & -6 & 4 \end{array}\right]$$

$R_2 \to R_2 + 1.5 R_3$

$$\left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ 0 & 0 & -4 & 22 \\ 0 & -2 & -6 & 4 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ 0 & -2 & -6 & 4 \\ 0 & 0 & -4 & 22 \end{array}\right]$$

$\rho(A) = \rho(A:B) = 3$

Since, rank is equal then system of eq^n is consistent and have unique solution.

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(iii) $4x - y \cancel{+8z} = \cancel{8}\,12$, $\cancel{-x+2y+z}$ $-x+5y-2z=0$, $-2x+4z=-8$

$\Rightarrow$ $AX = B$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$A : B \Rightarrow$

$$\Rightarrow \left[\begin{array}{ccc:c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array}\right]$$

$R_3 \to 2R_3 + R_1$, $R_2 \to 4R_2 + R_1$

$$\left[\begin{array}{ccc:c} 4 & -1 & 0 & 12 \\ 0 & 19 & -8 & 12 \\ 0 & -1 & 8 & -4 \end{array}\right]$$

$R_3 \to R_3 + \frac{1}{19} R_2$

$$\left[\begin{array}{ccc:c} 4 & -1 & 0 & 12 \\ 0 & 19 & -8 & 12 \\ 0 & 0 & & -64/19 \end{array}\right]$$

$\cancel{8 - \frac{8}{19}}$

$\cancel{\frac{152-8}{19} = 144}$

$\cancel{-4 + \frac{12}{19}}$

$\cancel{\frac{-76+12}{19}}$

$\frac{-64}{19}$

$\rho(A) = \rho(A:B) = 3$

It is consistent and has unique solution.

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(6.) For what value of λ and μ the given system of eq^n. $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ has (i) no solution, (ii) a unique solution (iii) Infinit number of sol^n.

$[A:B]$

$\Rightarrow$
$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

i) ~~Fo~~ No sol^n.

$\lambda = 3$ & $\mu \neq 10$

ii) For unique sol^n.

$\lambda \neq 3$ & $\mu \neq 10$

iii) Infinite many sol^n.

$\lambda = 3$, $\mu = 10$

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(i) Find for what values of λ of the given eq^n.

$x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$

have a sol^n. and solve them completely in each cases.

=) $(A:B)$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & (\lambda^2-1-3(\lambda-1)) \end{array}\right]$$

For system having sol^n :-

$\lambda^2-1-3\lambda+3=0$

$\lambda^2-3\lambda+2=0$

$\lambda(\lambda-2)-1(\lambda-2)=0$

$(\lambda-2)(\lambda-1)=0$

$\lambda=2$, $\lambda=1$

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d.) Find the $soln$. of the system of eq^ns. $x+3y-2z=0$
$2x-y+4z=0$, $x-11y+14z=0$

e.)

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c) Find for what values of λ the given eq^n.

$3x + y - \lambda z = 0$, $4x - 2y - 3z = 0$, $2\lambda x + 4y + \lambda z = 0$, may posses non-trivial sol^n, and solve them completely in each case.

$\Rightarrow$
$$A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$$

$$|A| = 3(-2\lambda + 12) - 1(4\lambda + 6\lambda) - \lambda(16 + 4\lambda) = 0$$
$$= -6\lambda + 36 - 10\lambda - 16\lambda - 4\lambda^2 = 0$$
$$= -32\lambda + 36 - 4\lambda^2 = 0$$
$$= 4\lambda^2 + 32\lambda - 36 = 0$$
$$\Rightarrow \lambda^2 + 8\lambda - 9 = 0$$
$$\Rightarrow \lambda^2 + 9\lambda - \lambda - 9 = 0$$
$$\Rightarrow \lambda(\lambda + 9) - 1(\lambda + 9) = 0$$
$$\Rightarrow (\lambda + 9)(\lambda - 1) = 0$$
$$\Rightarrow \lambda = -9, \quad \lambda = 1$$

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Assignment - 2

=> Are the following set of vectors linearly dependent or independent :

i) $[1\ 0\ 0]$, $[1\ 1\ 0]$, $[1\ 1\ 1]$

=> $V = C_1V_1 + C_2V_2 + C_3V_3 = 0$

=> $V = C_1[1\ 0\ 0] + C_2[1\ 1\ 0] + C_3[1\ 1\ 1] = 0$

$C_1 + C_2 + C_3 = 0$

$C_2 + C_3 = 0$

$C_3 = 0$

$$V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ => rank = 3 = n → trivial solⁿ.

=> Linearly dependent.

ii) $[7\ -3\ 11\ -6]$, $[-56\ 24\ -88\ 48]$

=> $V = C_1V_1 + C_2V_2 = 0$

$V = C_1[7\ -3\ 11\ -6] + C_2[-56\ 24\ -88\ 48]$

$7C_1 - 56C_2 = 0$

$-3C_1 + 24C_2 = 0$

$11C_2 - 82C_2 = 0$

$-6C_1 + 48C_2 = 0$

$$A = \begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -82 \\ -6 & 48 \end{bmatrix}$$

=> $R_1 \to R_1 + R_4$, $R_2 \to R_2 + 3R_4$, $R_3 \to R_3 - 11R_4$ =>

$$\begin{bmatrix} 7 & -56 \\ 0 & 0 \\ 0 & 0 \\ 1 & -8 \end{bmatrix}$$

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$R_1 \to R_1 - 7R_4$, $R_4 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & -8 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\rho(A) < 2 \to$ infinite solⁿ.

$\Rightarrow$ The vector is linearly dependent.

iii) $[-1 \quad 5 \quad 0]$, $[16 \quad 8 \quad -3]$, $[-64 \quad 59 \quad 9]$

2) $V = C_1[-1 \quad 5 \quad 0] + C_2[16 \quad 8 \quad -3] + C_3[-64 \quad 59 \quad 9]$

$-C_1 + 16C_2 - 64C_3 = 0$

$5C_1 + 8C_2 + 56C_3 = 0$

$-3C_2 + 9C_3 = 0$

$$V = \begin{vmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{vmatrix}$$

$|V| = -1[72 + 168) - 5(144 - 192)$

$-24 + 240 = 0$

$|V| = 0$; $\rho(V) < 3$

System has infinite solⁿ.

$\to$ Linearly dependent.

iv) $[1 \quad -1 \quad 1]$, $[1 \quad 1 \quad -1]$, $[-1 \quad 1 \quad 1]$, $[0 \quad 1 \quad 0]$

2) $V = C_1[1 \quad -1 \quad 1] + C_2[1 \quad 1 \quad -1] + C_3[-1 \quad 1 \quad 1] + C_4[0 \quad 1 \quad 0]$

$C_1 + C_2 - C_3 = 0$

$-C_1 + C_2 + C_3 + C_4 = 0$

$C_1 - C_2 + C_3 = 0$

2) $\begin{vmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{vmatrix} \Rightarrow \begin{matrix} R_3 \to R_3 + R_2 \\ R_2 \to R_2 + R_1 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$

Rank $= 3 \neq n$ $\to$ Infinite solⁿ.

$\to$ Linearly dependent

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(v) $[2 \quad -4], [1 \quad 9], [3 \quad 5]$

$\Rightarrow$ $V_1 = C_1[2 \quad -4] + C_2[1 \quad 9] + C_3[3 \quad 5]$

$2C_1 + C_2 + 3C_3 = 0$

$-4C_1 + 9C_2 + 5C_3 = 0$

$$\begin{bmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{bmatrix} \Rightarrow R_2 \to R_2 + 2R_1 \Rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 11 & 11 \end{bmatrix}$$

No. of unknow $= 3$, Rank $\Rightarrow 2$, Rank $\neq n$

$\to$ Infinite solⁿ.

$\to$ Linearly dependent

(vi) $[3 \quad -2 \quad 0 \quad 4], [5 \quad 0 \quad 0 \quad 1], [-6 \quad 1 \quad 0 \quad 1], [2 \quad 0 \quad 0 \quad 3]$

$\Rightarrow$ $V = C_1[3 \quad -2 \quad 0 \quad 4] + C_2[5 \quad 0 \quad 0 \quad 1] + C_3[-6 \quad 1 \quad 0 \quad 1] + C_4[2 \quad 0 \quad 0 \quad 3]$

$\Rightarrow$ $3C_1 + 5C_2 - 6C_3 + 2C_4 = 0$

$-2C_1 + C_3 = 0$

$4C_1 + C_2 + C_3 + C_4 = 0$

$$V = \begin{bmatrix} 3 & -2 & 0 & 4 \\ 5 & 0 & 0 & 1 \\ -6 & 1 & 0 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix} \Rightarrow |V| = 0$$

$\cancel{3} \quad \cancel{5}$

Rank $< 4 \neq n$

$\to$ Infinite solⁿ. or non-trivial solⁿ.

$\to$ Linearly dependant.

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Assignment - 3

Find the Eigen Values and Eigen vectors of following matrices ?

1.) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} = A$

$\Rightarrow$ $A - \lambda I \Rightarrow$

$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & -1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix}$

$|A - \lambda I| = 0$

$\Rightarrow (-2-\lambda)[(-\lambda + \lambda^2) - 12] - 2[-2-\lambda-6] - 3[-4+1-\lambda] = 0$

$\Rightarrow 2\lambda - 2\lambda^2 + \lambda^2 - \lambda^3 + 24 + 12\lambda + 4\lambda + 12 + 9 + 3\lambda = 0$

$\Rightarrow \lambda^3 + \lambda^2 - 2\lambda - 45 = 0$

$\Rightarrow (\lambda - 5)(\lambda^2 + 6\lambda + 9) = 0$

$\Rightarrow (\lambda - 5)(\lambda^2 + 3\lambda + 3\lambda + 9) = 0$

$\Rightarrow (\lambda - 5)(\lambda + 3)(\lambda + 3) = 0$

$\Rightarrow \lambda = 5$ or $\lambda = -3$

For $\lambda = 5 \Rightarrow$

$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}$ $R_1 \leftrightarrow R_3 \Rightarrow$ $\begin{bmatrix} -1 & -2 & -5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix}$

$\Rightarrow$ $R_2 \to R_2 + 2R_1$, $R_3 \to R_3 - 7R_1$

$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix}$ $\Rightarrow R_3 \to R_3 + 2R_2 \Rightarrow$ $\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix}$

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$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = k$

$-8x_2 - 16x_3 = 0 \Rightarrow -8x_2 = 16x_3$

$x_2 = -2x_3 = -2k$

$x_1 + 2x_2 + 5x_3 = 0$

$x_1 + 2(-2k) + 5(k) = 0$

$x_1 = -k$

$$\begin{bmatrix} -k \\ -2k \\ k \end{bmatrix} \Rightarrow k \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

for $\lambda = -3$

$$\begin{bmatrix} -5 & 2 & -3 \\ 2 & -2 & -6 \\ -1 & -2 & -3 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \Rightarrow \begin{bmatrix} -1 & -2 & -3 \\ 2 & -2 & -6 \\ -5 & 2 & -3 \end{bmatrix}$$

$R_3 \to R_3 - 5R_1, \; R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} -1 & -2 & -3 \\ 0 & -6 & -k \\ 0 & 12 & 12 \end{bmatrix} \Rightarrow R_3 \to R_3 + 2R_2 \Rightarrow \begin{bmatrix} -1 & -2 & -3 \\ 0 & -6 & -12 \\ 0 & 0 & -12 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & -12 \\ 0 & 0 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_2 + 3x_3 = 0 \Rightarrow x_1 - 4k + 3k = 0 \Rightarrow x_1 = k$

$-6x_2 - 12x_3 = 0 \Rightarrow x_2 = -2x_3 \Rightarrow x_2 = -2k$

$-12x_3 = 0$

$$x_3 = k \Rightarrow \begin{bmatrix} k \\ -2k \\ k \end{bmatrix} \Rightarrow k \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

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Eigen value 5, -3 & corresponding Eigen vectors are $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

2.) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = A$

$\Rightarrow$ $A - \lambda I = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = 0$

$\Rightarrow (4-\lambda)(1-\lambda)^2 + 1\,(2(1-\lambda)) = 0$

$(4-\lambda)(1+\lambda^2-2\lambda)+(2-2\lambda)$

$4+4\lambda^2-8\lambda-\lambda-\lambda^3+2\lambda^2+2-2\lambda$

$\lambda^3+6\lambda^2-11\lambda+6=0$

$-\lambda^2(\lambda-1)+5\lambda(\lambda-1)-6(\lambda-1)=0$

$(\lambda-1)(\lambda^2-5\lambda+6)=0$

$(\lambda-1)(\lambda-2)(\lambda-3) = 0$

For $\lambda=1$

$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$3x_1+x_3=0 \rightarrow x_1=0$

$-2x_2=0 \rightarrow x_2=0$ $\Rightarrow$ $\begin{bmatrix} -1/3 \\ 0 \\ 1 \end{bmatrix}$

$-2x_3=0 \rightarrow x_3=0$

For $\lambda=2$

$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_3 \rightarrow R_3+R_1$, $R_2 \rightarrow R_2+R_1$ $\Rightarrow$ $\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$2x_1 + x_3 = 0 \Rightarrow x = -k/2$

$-x_2 + x_3 = 0$ $\Rightarrow k\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$

$x_2 = x_3 = k$

For $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1 \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1 + x_3 = 0$, $-2x_2 + 2x_3 = 0$, $x_3 = k$

$x_1 = -x_3 \Rightarrow$ ~~k = x = 1~~ $k\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

3.) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$\Rightarrow A - \lambda I \Rightarrow \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$ $\Rightarrow (5-\lambda)((-\lambda)(3-\lambda)) = 0$

$\Rightarrow (5-\lambda)(-3\lambda + \lambda^2) = 0$

$\Rightarrow -15\lambda + 5\lambda^2 + 3\lambda^2 - \lambda^3 = 0$

$\lambda^3 - 8\lambda^2 + 15\lambda = 0$

$(\lambda)(\lambda^2 - 8\lambda + 15) = 0$

$(\lambda)(\lambda^2 - 5\lambda - 3\lambda + 15) = 0$

$\lambda((\lambda-5)(\lambda-3)) = 0$

$\lambda = 0, 3, 5$

For $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \quad \begin{matrix} R_1 \leftrightarrow R_3 \\ R_2 \leftrightarrow R_3 \end{matrix} = \begin{bmatrix} -1 & 0 & 3 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + 5R_1 \Rightarrow \begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 15 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

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$-x_1 + 5x_3 = 0$, $x_1 = 3x_3$

$15x_2 = 0$, $x_2 = 0$

$$k\begin{bmatrix}1\\0\\3\end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix}2 & 0 & 0\\0 & -3 & 0\\-1 & 0 & 0\end{bmatrix} \quad R_1 \leftrightarrow R_3 = \begin{bmatrix}-1 & 0 & 0\\0 & -3 & 0\\2 & 0 & 0\end{bmatrix}$$

$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix}-1 & 0 & 0\\0 & -3 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$-1x_1 = 0 \to x_1 = \cancel{0}\, k_1$

$-3x_2 = 0 \to x_2 = \cancel{0}\, k_2$, $x_3 = 0$ $\Rightarrow$ $\begin{bmatrix}k_1\\k_2\\0\end{bmatrix}$

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4.
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

$\Rightarrow (A - \lambda I) = 0 \Rightarrow \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$

$\Rightarrow (0-\lambda)(-6 - 3\lambda + 2\lambda + \lambda^2) = 0$

$6\lambda - 3\lambda^2 + 2\lambda^2 \cancel{+} - \lambda^3 = 0$

$\lambda^3 - \lambda^2 \cancel{+} - 6\lambda = 0$

$\lambda(\lambda^2 - \lambda - 6) = 0$

$\lambda(\lambda^2 - 3\lambda + 2\lambda - 6) = 0$

$\lambda(\lambda(\lambda - 3) + 2(\lambda - 3)) = 0$

$\lambda(\lambda - 3)(\lambda + 2) = 0$

$\lambda = 0, \lambda = 3, \lambda = -2.$

For $\lambda = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$, $R_3 \leftrightarrow R_2$

$$\begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_2 + 4x_3 = 0$

$x_2 = -4/3\, x_3$

$x_2 \to k$, $x_3 = -4/3\, k$, $x_1 = 0$

$$k \begin{bmatrix} 0 \\ 1 \\ -4/3 \end{bmatrix}$$

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For $\lambda = 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix}$$

$R_1 \rightarrow -1/3\, R_1$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $-3x_1 = 0$, $4x_2 = 0$, $-5x_3 = 0$

$x_1 = k_1$, $x_2 = k_2$, $x_3 = k_3$

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \rightarrow -\frac{1}{2} R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$x_1 = 0$, $x_2 + 4x_3 = 0$

$x_2 - 4x_3$

$x_3 \rightarrow k$, $x_2 = -4k$

$$k \begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix}$$

(5.) For the following matrix find one eigen value without calculation and justify your answer.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

2) $|A| = 0$

as $R_1 = R_2 = R_3$

As we know →

Product of Eigen value = det of vector

Since, det. is 0 so one of the eigen value will be 0.

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Assignment - 4

1.) Find the rank of the matrix A by reducing in Row reduced Echlon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

⇒ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 2R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$$

$R_{43} \to R_4 + R_3$, $\cancel{R_3 \leftrightarrow R}$. $R_4 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 4 & 5 & -1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 4 & 5 & -1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank (A) = 3 .

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2.) Let W be the vector space of all symmetric 2x2 matrices. $T: W \to P_2$ be linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$.

Find the rank and nullity of T.

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3.) Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find Eigen value & Eigen vectors of A^{-1} and $A+4I$.

=) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A : I$

$$\left[\begin{array}{cc:cc} 2 & -1 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 + R_1$

$$\left[\begin{array}{cc:cc} 2 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$R_2 \Leftrightarrow R_1$

$$\left[\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$\left[\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ 1 & -2 & 0 & -1 \end{array}\right]$$

$R_2 \to R_2 \times -1$

$$\left[\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ -1 & 2 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 + R_1$

$$\left[\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ 0 & 3 & 1 & 2 \end{array}\right]$$

$R_1 \to 3R_1 - R_2$

$$\left[\begin{array}{cc:cc} 3 & 0 & 2 & 1 \\ 0 & 3 & 1 & 2 \end{array}\right]$$

$R_1 \to \frac{1}{3} R_1$, $R_2 \to \frac{1}{3} R_2$

$$\left[\begin{array}{cc:cc} 1 & 0 & 2/3 & 1/2 \\ 0 & 1 & 1/3 & 2/3 \end{array}\right]$$

$A^{-1} = \begin{bmatrix} 2/3 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}$

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$\Rightarrow |A^{-1} - \lambda I| = 0$

$$\frac{1}{3}\left(\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\right) = \frac{1}{3}\left[(2-\lambda)^2 - 1\right] = 0$$

$$\Rightarrow \lambda^2 + 4 - 4\lambda - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda^2 - 3\lambda - \lambda + 3 = 0$$

$$\lambda(\lambda-3) - 1(\lambda-3) = 0$$

$$(\lambda-3)(\lambda-1) = 0$$

$$\lambda = 1, \lambda = 3$$

For $\lambda = 1$

$$\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

let $x_2 = k$

$$x_1 + x_2 = 0 \Rightarrow x_1 - x_2 = -k$$

$$\Rightarrow \begin{bmatrix} k \\ -k \end{bmatrix} \Rightarrow k\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad k=1 \Rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

For $\lambda = 3$

$$\frac{1}{3}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\frac{1}{3}\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x_1 + x_2 = 0, \quad x_2 = k \Rightarrow x_1 = x_2 = k$$

$$\begin{bmatrix} k \\ k \end{bmatrix} \Rightarrow k\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow k=1, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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$\Rightarrow \quad A + 4I \Rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

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4.) Solve by Gauss - Seidel Methode (Take three iteration)

$3x - 0.1y - 0.2z = 7.85$

$0.1 + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

with initial value $x(0) = 0,\ y(0) = 0,\ z(0) = 0$

$\Rightarrow$ $x_1 = \frac{7.85 + 0.1y + 0.2z}{3} \xrightarrow{y=0,\ z=0} x_1 = \frac{7.25}{3} = 2.616$

$y_1 = \frac{-19.3 - 0.1x + 0.3z}{7} \xrightarrow[z=0]{x=2.616} \frac{-19.3 - 0.1(2.616)}{7} = -2.794$

$z_1 = \frac{71.4 - 0.3x + 0.2y}{10} = \frac{71.4 - 0.3(2.616) + 0.2(-2.794)}{10} = 7.0056$

$x_2 = \frac{7.85 + 0.1y_1 + 0.2 \times z_1}{3} = \frac{7.85 + (0.1)(-2.4415) + 0.2(-2.794)}{3}$

$= 2.995$

$y_2 = \frac{-19.3 + 0.3(z) + 0.3z}{7} = \frac{-19.3 + 0.3(7.00563) + 0.1(2.2995)}{7} = \cancel{-2.44}$ -2.41418

$z_2 = \frac{71.4 - 0.3x_2 + 0.2y_2}{10} = \frac{71.4 - 0.3(2.9905) + 0.2(-2.41418)}{7} = 7.0020014$

$x_3 = \frac{7.85 + 0.1y_2 + 0.2z_2}{3} = \frac{7.85 + 0.1(-2.41418) + 0.2(7.0020014)}{3}$

$\Rightarrow 3.0029$

$y_3 = \frac{-19.3 + 0.3(z_2) + 0.1(x_3)}{7} = \frac{-19.3 + 0.3 \times (7.002) + 0.1 \times 3.0029}{7}$

$= -2.41415$

$z_3 = \frac{71.4 - 0.3(x_3) + 0.2(y_3)}{10} = \frac{71.4 - 0.3 \times (3.0029) + 0.2 \times (-2.41415)}{10}$

$= 7.00125$

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5.) Define consistent and inconsistent system of equations. Here solve the following system of equations if consistent $x+3y+2z=0$, $2x-y+3z=0$, $3x-5y+4z=0$, $x+17y+4z=0$.

=) $AX=0 \rightarrow$ Homogeneous eq^n $\rightarrow$ Always consistent.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$, $R_4 \rightarrow R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$, $R_4 \rightarrow R_4 + R_3$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

=) $\rho(A) = 2$

No. of variable $= 3$

$\rho(A) < n \rightarrow$ Infinite sol^n.

For sol^n. =)

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

=) $x+3y+2z=0$

$-7y-z=0$

$z=-7y$

$x+3y-7y=0$

$x+4y$ let $y=k$

$x=4k$, $z=7k$, $y=x$

$$\begin{bmatrix} 4k \\ k \\ 7k \end{bmatrix} \Rightarrow k \begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix}$$

Durgesh Shukla (03B)

(6) Determine whether the function $T: R\,P_2 \rightarrow P_2$ is linear transformation or not.

Where $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

=)

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(7.) Determine whether the set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis determine the dimension and the basis of the sub space spanned by S.

$\Rightarrow$ $$c_1\begin{pmatrix}1\\2\\3\end{pmatrix} + c_2\begin{pmatrix}3\\1\\0\end{pmatrix} + c_3\begin{pmatrix}-2\\1\\3\end{pmatrix} = \begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$$

$c_1, c_2, c_3 \rightarrow$ scalar & $(v_1, v_2, v_3) \in V$.

$$\begin{bmatrix}1 & 3 & -2\\2 & 1 & 1\\3 & 0 & 3\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}$$

$AX = B$

$[A : B] \Rightarrow$ $$\begin{bmatrix}1 & 3 & -2\\2 & 1 & 1\\3 & 0 & 3\end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix}1 & 3 & -2\\0 & -5 & 5\\0 & -9 & 9\end{bmatrix}$$

$R_3 \rightarrow R_3 - \frac{9}{5}R_2$

$$\begin{bmatrix}1 & 3 & -2\\0 & -5 & 5\\0 & 0 & 0\end{bmatrix}$$

$\rho \Rightarrow 2 < n$.

$\rightarrow$ Infinite sol^n.

$\rightarrow$ Linear dependent

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(8.) Using Jacobi's method (perfor 3 iteration), solve
$3x - 6y + 2z = 23$, $-4x + y - z = -15$, $x - 3y + 7z = 16$,
with initial values $x_0 = 1$, $y_0 = 1$, $z_0 = 1$.

⇒ $3x - 6y + 2z = 23$
$-4x + y - z = -15$
$x - 3y + 7z = 16$

$$x_1 = \frac{23 + 6y_0 - 2z_0}{3} = \frac{23 + 6\times1 - 2\times1}{3} = \frac{27}{3} = 9$$

$$y_1 = -15 + z_0 + 4x_0 = -15 + 1 + 4\times1 = -10$$

$$z_1 = \frac{16 - x_0 + 3y_0}{7} = \frac{16 - 1 + 3(1)}{7} = \frac{18}{7} = 2.57$$

$$x_2 = \frac{23 + 6y_1 - 2z_1}{3} = \frac{27 + 6\times-10 - 2\times2.57}{3} = -12.714$$

$$y_2 = -15 + z_1 + 4x_1 = -15 + 2.57 + 4\times9 = 23.5714$$

$$z_2 = \frac{16 - x_1 + 3y_1}{7} = \frac{16 - 9 + 3\times-10}{7} = -3.25$$

$$x_3 = \frac{23 + 6y_2 - 2z_2}{3} = \frac{23 + 6(23.5714) - 2(-3.25)}{3}$$
$$= 56.99$$

$$y_3 = -15 + z_2 + 4x_2 = -15 + (3.25) + 4(-12.714) = -69.14$$

$$z_3 = \frac{16 - x_2 + 3y_2}{7} = \frac{16 - (-12.714) + 3(23.5714)}{7}$$
$$\Rightarrow 14.20404$$