Homework #2 *due October 25, Tuesday 2022, before recitation*

(1) Try to simplify the following expressions and justify your simplification.

(i) ***(0+1)*.1.(0+1) +(0+1) *.1.(0+1)***

(ii) ***(((0*.1*)+1)*(0+1)*)****

(iii) ***(L+M*)****

(iv) ***(L.M*)****

(2) Convert the *regular expression* $((0.0^{*}.(1.1))+\ 0.1)^{*}$ into an ***ε-NFA***

(3) Problems from the textbook

3.1.1 (b) and (c)

3.1.4 (b) and (c)

3.2.1 (c) and (d)

3.2.3

Homework 2 :

1- (i) $(0+1)^*.1.(0+1).1.(0+1) + (0+1)^*.1.(0+1)$

using A = lambda = λ

$= (0+1)^*.1.(0+1).\lambda + (0+1)^*.1.(0+1).\lambda$

using $A.b + A.c = A[b+c]$

$= (0+1)^*.1.(0+1)[\lambda+\lambda]$

$\rightarrow \lambda + \lambda = \lambda$

$\rightarrow$ using $A = A.\lambda$

$\boxed{(0+1)^*.1.(0+1)}$

(ii) $(((0^*.1^*)+1)^*(0+1)^*)^*$

→ The first portion expression can generate all kind of strings from $\{0,1\}$

→ The second portion of this expression can generate all kind of strings from $\{0,1\}$

→ Hence, the given regex refers to all the strings of 0,1.

→ Most simplified version is $(0+1)^*$, which represents the set of all the strings of 0,1.

(iii) $(L+M^*)^*$

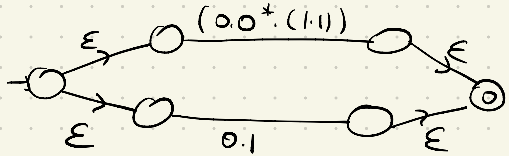
$M^* \Rightarrow$ zero or more occurences of M. Hence we can write it as

$(L+M^*)^* \Rightarrow (L+M)^*$. In other words, zero or more occurences of L or M.

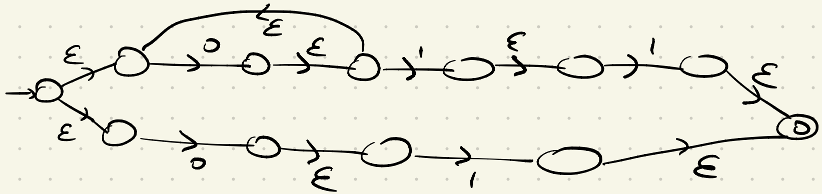
(iv) The answer will be same logic as (iii). Because, $(L.M^*)^*$ and $(L.M)^*$ are same. Hence, it can be simplified as $\underline{(L.M)^*}$

2- $((0.0^*.(1.1)) + 0.1)^*$ into an ε-NFA.

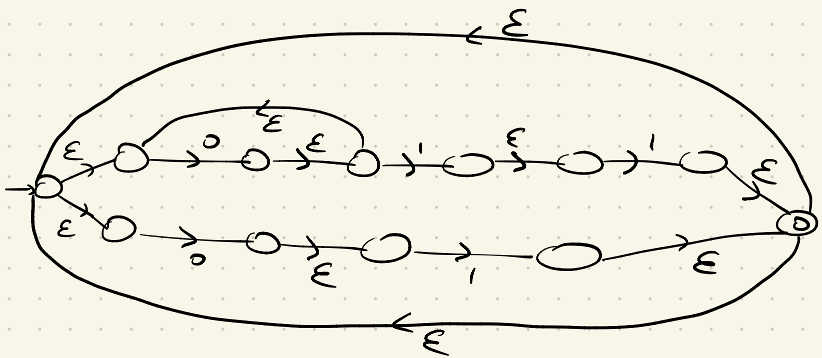
Step 1:



Step 2:



Step 3:



3.1.1

b) The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

$$(0+1)^* 1 (0+1)^9$$

c) The set of strings of 0's and 1's with at most one pair of consecutive 1's.

$$(0+10)^* (11+\epsilon) (0+10)^*$$

3.1.4

b) $(0^* 1^*)^* 000 (0+1)^*$

c) $(0+10)^* 1^*$

Answer: This is the language of strings in which there are no two consecutive 1's, except for possibly a string of 1's at the end.

32.1 c) $R_{ij}^{(2)}$

$$R_{11}^{(2)} = R_{11}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{21}^{(1)}$$
$$= 1^* + 1^* 0 (\epsilon + 1^+ 0)^* 1^+ = (1+01)^*$$

$$R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} = R_{12}^{(1)} (R_{22}^{(1)})^*$$
$$= 1^* 0 (\epsilon + 1^+ 0)^* = (1+01)^* 0$$

$$R_{13}^{(2)} = R_{13}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{23}^{(1)} = \emptyset + 1^* 0 (\epsilon + 1^+ 0)^* 0 = (1+01)^* 00$$

$$R_{21}^{(2)} = R_{21}^{(1)} + R_{22}^{(1)} (R_{22}^{(1)})^* R_{21}^{(1)} = (R_{22}^{(1)})^* R_{21}^{(1)} = \epsilon + 1^+ 0) 1^+ = 1^+ (\epsilon + 01^+)$$

$$R_{22}^{(2)} = R_{22}^{(1)} + R_{22}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} = (R_{22}^{(1)})^+ = (\epsilon + 1^+ 0)^+ = (1^+ 0)^*$$

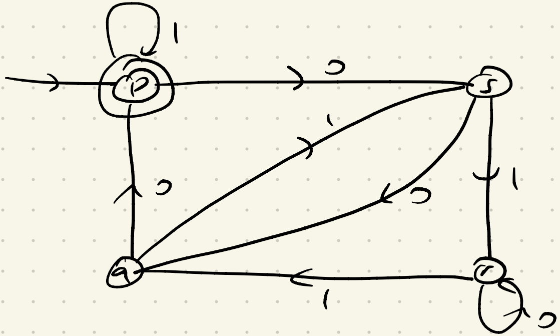
$$R_{23}^{(2)} = R_{23}^{(1)} + R_{22}^{(1)} (R_{22}^{(1)})^* R_{23}^{(1)} = (R_{22}^{(2)})^* R_{23}^{(1)} = (\epsilon + 1^+ 0)^* 0 = (1^+ 0)^* 0$$

$$R_{31}^{(2)} = R_{31}^{(1)} + R_{32}^{(1)} (R_{22}^{(1)})^* R_{21}^{(1)} = \emptyset + 1 (\epsilon + 1^+ 0)^* 1^+ = 1 (1^+ 0)^* 1^+$$

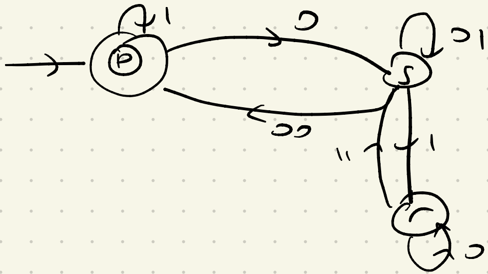
$$R_{32}^{(2)} = R_{32}^{(1)} + R_{32}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} = 1 + 1 (\epsilon + 1^+ 0)^+ = 1 (1^+ 0)^*$$

$$R_{33}^{(2)} = R_{33}^{(1)} + R_{32}^{(1)} (R_{22}^{(1)})^* R_{23}^{(1)} = (0 + \epsilon) + 1 (\epsilon + 1^+ 0)^* 0 = 0 + 1 (1^+ 0)^* 0 + \epsilon$$

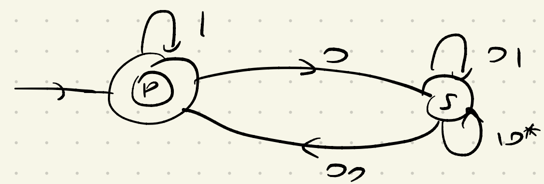
3.2.3



step 1: eliminate q:



step 2: eliminate r:



step 3: eliminate s

Regular expression: $(1+0(01+10^*11)^*00)^*$