Homework #5 *due November 29 Tuesday before the recitation*

(1) Let ***M*** be the *PDA* defined by $Q = \{q, q_0, q_1, q_2\}$, $\Sigma = \{a,b\}$, $\Gamma = \{a\}$, $F := \{q, q_1\}$.

$\delta(q_0, a, Z_0) = \{(q, Z_0)\}$

$\delta(q, a, Z_0) = \{(q, aZ_0)\}$

$\delta(q, a, a) = \{(q, aa)\}$

$\delta(q, b, a) = \{(q_1, e)\}$

$\delta(q_1, b, a) = \{(q_1, e)\}$

$\delta(q_1, b, Z_0) = \{(q_2, e)\}$

a) Describe the language accepted by ***M***.

b) Trace all computations of the strings ***aab, abb, aba*** in ***M***.

c) Show that ***aaabb, aaab*** $\in L(M)$.

(2) Construct *PDA*s that accept each of the following languages.

a) $\{a^i b^j \mid 0 \le i \le j\}$

b) $\{a^i c^j b^i \mid i,j \ge 0\}$

d) $\{a^i b^j c^k \mid i+k = j\}$

e) $\{a^i b^j \mid 0 \le i \le j \le 2i\}$

f) $\{a^{i+j} b^i c^j \mid i,j > 0\}$

(3) $L = \{w \in \{a, b\}^* \mid$ *at least one prefix of **w** contains strictly more **b**'s than **a**'s.*$\}$.

For example, ***baa, abb, abbbaa*** are in ***L***, but ***aab, aabbab*** are not in ***L***.

a) Construct a PDA that accepts ***L*** by final state.

b) Construct a PDA that accepts ***L*** by empty stack.

(4) From the main text ***Exercises 6.2.6, 6.3.2, 6.3.4***

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a) In the given PDA, q_0 reads a single 'a' and moves to the final state q which means number of a's in any string must be a minimum of 1 at the start of the string.

→ q reads input symbol 'a' and pushes it to the stack when top of stack is either 'z_0' or 'a'. Transition from state q to q_1 on reading 'b', pops symbol from stack when top of stack has symbol 'a'. State q_1 has transition to itself on reading 'b', pops symbols from the stack contains 'z_0' which means that number of b's should be less than number of a's.

Therefore, $L = \{a \;\; i\, n \geq 1 \;;\; m \geq 0 \;;\; n \geq m\}$

b) For aab $\quad (q_0, aab, z_0) \vdash (q, ab, z_0) \vdash (q, b, z_0) \vdash (q_1, e, z_0) \Rightarrow$ Accepted

For abb $\quad (q_0, abb, z_0) \vdash (q, bb, z_0) \Rightarrow$ no transition possible from state q for input symbol 'b'.
⇓
Rejected

For aba $\quad (q_0, aba, z_0) \vdash (q, ba, z_0) \Rightarrow$ 4 Rejected

c) For aaabb,

$\delta(q_0, aaabb, z_0) \Rightarrow \delta(q, aabb, z_0) \Rightarrow \delta(q, abb, az_0) \Rightarrow \delta(q, bb, aaz_0) \Rightarrow \delta(q_1, b, az_0)$

$\Downarrow$

$\delta(q_1, e, z_0)$

Accepted !

aaabb $\in L(M)$.

For aaab,

$\delta(q_0, aaab, z_0) \Rightarrow \delta(q, aab, z_0), \delta(q, ab, az_0), \delta(q, b, aaz_0) \Rightarrow \delta(q_1, e, az_0)$

Accepted

aaab $\in L(M)$.

From main textbook

6.3.2 $S \to 0S1 \mid A$

$A \to 1A0 \mid S \mid \epsilon$

Let P have the single state {q}, input alphabet {0,1}, stack alphabet {0, 1, S, A}, transition function δ, start state {q}, and start symbol S.

We need to build all of the productions into our transition function, so we add the following transitions for our variables.

$\delta(q, \epsilon, S) = \{(q, 0S1), (q, A)\}$

$\delta(q, \epsilon, A) = \{(q, 1A0), (q, S)\}$

We also need to be able to match terminals against our input string, so we add a transition for each terminal

$\delta(q, 0, 0) = \{(q, \epsilon)\}$

$\delta(q, 1, 1) = \{(q, \epsilon)\}$

6.3.4

To apply the construction given in the proof of Theorem, we would first have to convert this machine to accept with an empty stack, rather than accepting when it is in state p. However, we note that this PDA will accept any string that starts with 0. We need to read a 0 when we see Z_0 on the stack, because there is no value given for $\delta(q, 1, Z_0)$. But after the first zero, we can just read 1's and 0's, staying in state q and pushing an extra X onto the stack for each 0. Then once we reach the end of the input, we can follow an ϵ transition to state p and accept.

Hence, we write a grammar that produces any string starting with 0.

$$S \to 0T$$
$$T \to 0T \mid 1T \mid \epsilon$$