SE-SEM-III CBGS Nov-2016 COMP/IT 25/11/16

Sub:- AM-III

Q.P. Code : 540701

(3 Hours) **[Total Marks : 80]**

NB: 1) Question No. 1 is compulsory.
2) Attempt any three of the remaining.
3) Figures to the right indicate full marks.

1. a) Find the Laplace transform of $te^{3t}\sin 4t$. **05**
b) Find half-range cosine series for $f(x)=e^x$, $0<x<1$. **05**
c) Is $f(z)=\frac{z}{\bar{z}}$ analytic? **05**
d) Prove that $\nabla\times(\bar{a}\times\nabla\log r)=2\frac{\bar{a}\cdot\bar{r}}{r^2}$, where $\bar{a}$ is a constant vector. **05**

2. a) Find the Z-transform of $\frac{1}{(z-5)^2}$ if $|z|<5$. **06**
b) If $V=3x^2y+6xy-y^3$, show that V is harmonic & find the corresponding analytic function. **06**
c) Obtain Fourier series for the function **08**

$$f(x)=\begin{cases}1+\frac{2x}{\pi}, & -\pi\le x\le 0\\ 1-\frac{2x}{\pi}, & 0\le x\le\pi\end{cases}$$

hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots$

3. a) Find $L^{-1}\left[\frac{(s+2)^2}{(s^2+4s+8)^2}\right]$ using convolution theorem. **06**
b) Show that the set of functions **06**
$1, \sin\left(\frac{\pi x}{L}\right), \cos\left(\frac{\pi x}{L}\right), \sin\left(\frac{2\pi x}{L}\right), \cos\left(\frac{2\pi x}{L}\right), \ldots\ldots$
Form an orthogonal set in (-L,L) and construct an orthonormal set.
c) Verify Green's theorem for $\int_C (xy-xy^2)\,dx+(ye^x+y^2)\,dy$ **08**
Where C is the closed curve bounded by $y^2=x$ & $x^2=y$.

4. a) Find Laplace transform of $f(x)=K\frac{t}{T}$ for $0<t<T$ & $f(t)=f(t+T)$. **06**
b) Show that the vector, $\bar{F}=(x^2-yz)\mathbf{i}+(y^2-zx)\mathbf{j}+(z^2-xy)\mathbf{k}$ is irrotational and hence, find ϕ such that $\bar{F}=\nabla\phi$. **06**
c) Find Fourier series for f(x) in $(0, 2\pi)$, **08**

$$f(x)=\begin{cases}x, & 0\le x\le\pi\\ 2\pi-x, & \pi\le x\le 2\pi\end{cases}$$

hence deduce that

$\frac{\pi^2}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots$

5. a) Use Gauss's Divergence theorem to evaluate **06**
$\int_S \bar{F}\cdot\hat{n}\,ds$ where $\bar{F}=2x\mathbf{i}+xy\mathbf{j}+zk$ over the region bounded by the cylinder $x^2+y^2=4$, $z=0$, $z=6$.
b) Find inverse Z – transform of $f(x)=\frac{z}{(z-1)(z-2)}$, $|z|>2$ **06**

TURN OVER

c) (i) Find $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$ 08

(ii) Find $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$

6. a) Solve $(D^2+3D+2)\,y = 2(t^2+t+1)$ with $y(0) = 2$ & $y'(0) = 0$. 06

b) Find the bilinear transformation which maps the points 0, i, -2i of z-plane onto the points -4i, ∞, 0 respectively of w-plane. Also obtain fixed points of the transformation. 06

c) Find Fourier sine integral of 08

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

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