SE sem IV / CBGS / May 2017 15/5/2017

COMP / IT – AM IV

QP Code : 541301

(3 Hours) **[Total Marks :80**

N.B. : (1) Questions no. **1** is compulsory.

(2) Attempt any **three** questions from Q. 2 to Q. 6

(3) Use of statistical table permitted.

(4) Figures to the **right** indicate **full** marks.

1. (a) Evaluate $\int_C (z-z^2)\,dz$, where C is the upper half of the circle $|z|=1$. **5**

(b) If $A=\begin{bmatrix}2 & 4\\ 0 & 3\end{bmatrix}$, then find the eigen values of $6A^{-1}+A^3+2I$ **5**

(c) State whether the following statement is true or false with reasoning : " The regression coefficients between 2x and 2y are the same as those between x and y." **5**

(d) Construct the dual of the following L.P.P. **5**

Maximise $Z=3x_1+17x_2+9x_3$

Subject to $x_1-x_2+x_3\geq 3$

$-3x_1+2x_3\leq 1$

$2x_1+x_2-5x_3=1$

$x_1, x_2, x_3\geq 0$

2. (a) Evaluate, $\int_C \frac{e^{2z}}{(z+1)^4}\,dz$, where C is the circle $|Z-1|=3$ **6**

(b) Show that the matrix $A=\begin{bmatrix}5 & -6 & -6\\ -1 & 4 & 2\\ 3 & -6 & -4\end{bmatrix}$ is derogatory. **6**

(c) A manufacturer knows from his experience that the resistance of resistors he produces is normal with $\mu=100$ ohms and standard deviation $\sigma=2$ ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms? **8**

3. (a) A discrete random variable has the probability distribution given below: **6**

x	−2	−1	0	1	2	3
p(x)	0.2	k	0.1	2k	0.1	2k

Find k, the mean and variance

[TURN OVER]

(b) Solve the following L.P.P. by simplex method 6

Maximise $Z=3x_1+2x_2$

Subject to $x_1+x_2 \leq 4$

$x_1-x_2 \leq 2$

$x_1, x_2 \geq 0$

(c) Expand $f(z)=\dfrac{z^2-1}{z^2+5z+6}$ around $z=0$, indicating region of convergence. 8

4. (a) Find the first two moments about the origin of Poisson distribution and hence find mean and variance. 6

(b) Calculate R and r from the following data : 6

x	12	17	22	27	32
y	113	119	117	115	121

(R - the rank correlation coefficient, r - correlation coefficient)

(c) Show that the matrix $A=\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. 8

Find the transforming matrix and the diagonal matrix.

5. (a) A tyre company claims that the lives of tyres have mean 42,000 kms with S.D of 4000 kms. A change in the production process is believed to result in better product. A test sample of 81 new tyres has a mean life of 42,500 kms. Test at 5% level of significance that the new product is significantly better than the old one. 6

(b) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{5+3\sin\theta}$ using Cauchy's residue theorem. 6

(c) Using the Kuhn-Tucker conditions solve the following N.L.P.P. 8

Minimise $Z=7x_1^2+5x_2^2-6x_1$

Subject to $x_1+2x_2 \leq 10$

$x_1+3x_2 \leq 9$

$x_1, x_2 \geq 0$

6. (a) 300 digits were chosen at random from a table of random numbers. The frequency of digits was as follows. **6**

Digit	0	1	2	3	4	5	6	7	8	9	Total
Frequency	28	29	33	31	26	35	32	30	31	25	300

Using χ^2-test examine the hypothesis that the digits were distributed in equal numbers in the table.

(b) Use the dual simple method to solve the following L.P.P. **6**

Minimise $Z = 6x_1 + x_2$

Subject to $2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$

(c) (i) Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches. **4**

(ii) A random variable X has the following probability distribution **4**

x	0	1	2	3
p(x)	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

Find M.G.F about the origin and hence first four raw moments.
