**Camera Calibration Toolbox**

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1. **EMCCD imaging model**

Let be the number of electrons (ē) from the signal before the electron multiplying (EM) process. Let be the number of electrons generated by the dark current of the camera. Dark current is usually listed in the camera specifications in ‘ē/pixel/s’ and can be used to calculate when the exposure time is known. Both and are Poisson distributed. Let, be the read noise of the camera. The standard deviation of the read noise () is listed in camera specifications in ē. Read noise can be described by a normal distribution with zero mean and standard diviation. Let’s denote the EM process by a statistical function . Then we can write a formulation for the output signal (in ē) as,

We can then write the mean and variation equations for the signals from Eq1 as,

Here denotes the mean of the random variable . is the EM-gain of the camera and it is the average gain added by the EM process. The range of is listed in camera specs and the exact value is set during imaging.

Here quantifies the noise added by the EM process, , and is discussed in the next sub-section.

Above signals are given in ē. In practice the analog-to-digital (ADC) conversion process doesn’t map one ē to one analog-to-digital count (ADU); rather a certain number of electrons are mapped to a single ADU. This number if called the k-gain (denoted by here) with units ‘ē/ADU’. can be used to convert ē to ADUs or vise-versa. For a signal, in ē the corresponding signal in ADUs is,

In addition to the above, cameras offset the image by adding a fixed bias (denoted by [ADU] here). Knowing the exact values of Bias and k-gain is important to convert an experimental image in ADUs to electrons or vise-versa. For instance, using Eq4, we can write the Eq2 and 3 above in ADUs as,

1. **Electron Multiplication Process,**

The EM process in practice is noisy and is only the average value of the EM-gain. This noise is quantified by the excess noise factor (ENF), , defined Eq3. We can derive an expression for form Eq3 as.

Here is the variation of the signal that goes into the EM process, and is the variation of the signal that comes out of the EM process. If is 1 there is no noise added to by the EM process. Robins and Hadwen1 derived an expression for as,

Here, is the number of EM gain stages. EM is treated as Bernoulli process at each EM stage and is the probability of an electron multiplication event happening, i.e. the probability of a success event. The value of is usually small and is in the order of 1-2% [ref- Nuvu cam data sheet]. Considering all input electrons to a stage, the gain for each EM stage can be described by a Binomial distribution with a probability mass function,

Where the binomial coefficient is,

Here is the number of input electron to the gain stage and is the number of added electrons through the EM gain stage. The output electrons from the gain therefore equals to, . such gains stages are cascaded to get the final EM output signal .

<need more clarification on the mote-carlo sampling here>

1. **Image simulation procedure**

For the function we need to calculate the number of gain stages for a given EM-gain. For small , the average gain at each stage can be approximated as 1. Then for gain stages we can write,

First, A time series of images () were recorded using the Nuvu Hnu 512 EMCCD camera. Then the time series was averaged over to get . The dark current and exposure time was used to calculate . Then, using Eq5, the corresponding was calculated. This was then used as the ground truth image to simulate a synthetic image as below.

Then was divided by the EM-gain to get the noised image corresponding to the input image to the camera.

For comparison a random image from the time series was chosen (call it ). This image was converted to the scale of the input image as,

**Temp references**

1. <https://www.mirametrics.com/tech_note_ccdgain.php>
2. <https://www.princetoninstruments.com/learn/camera-fundamentals/emccds-the-basics>
3. <https://www.nuvucameras.com/products/hnu-512/>
4. <https://www.nuvucameras.com/wp-content/uploads/2020/02/nuvucameras_hnu512.pdf>
5. <http://slittlefair.staff.shef.ac.uk/teaching/phy217/lectures/instruments/L12/index.html>

**References**

(1) Robbins, M. S.; Hadwen, B. J. The noise performance of electron multiplying charge-coupled devices. *IEEE transactions on electron devices* **2003**, *50*, 1227-1232.