Thermodynamics and statistical mechanics in heavy-ion collisions

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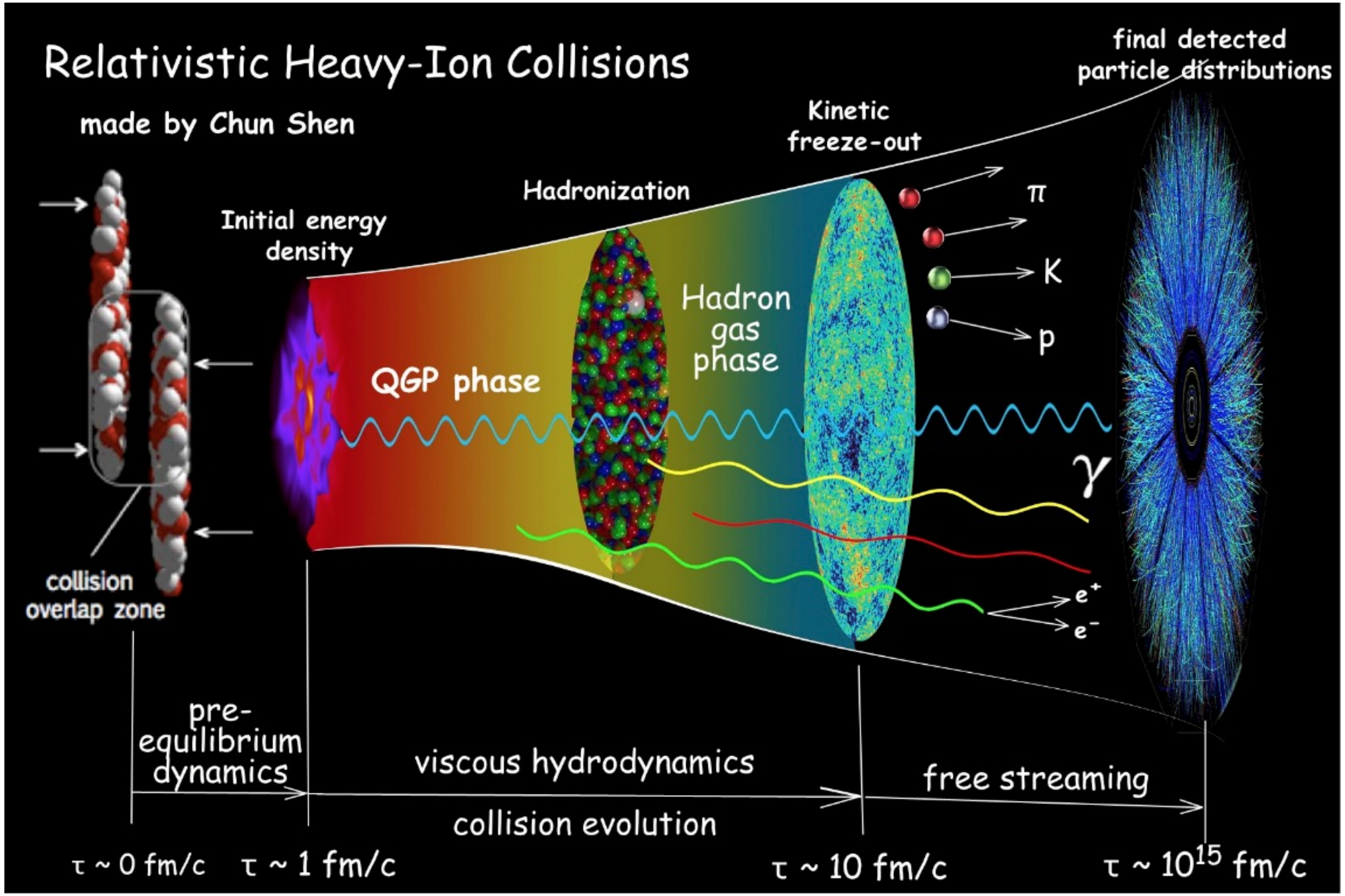
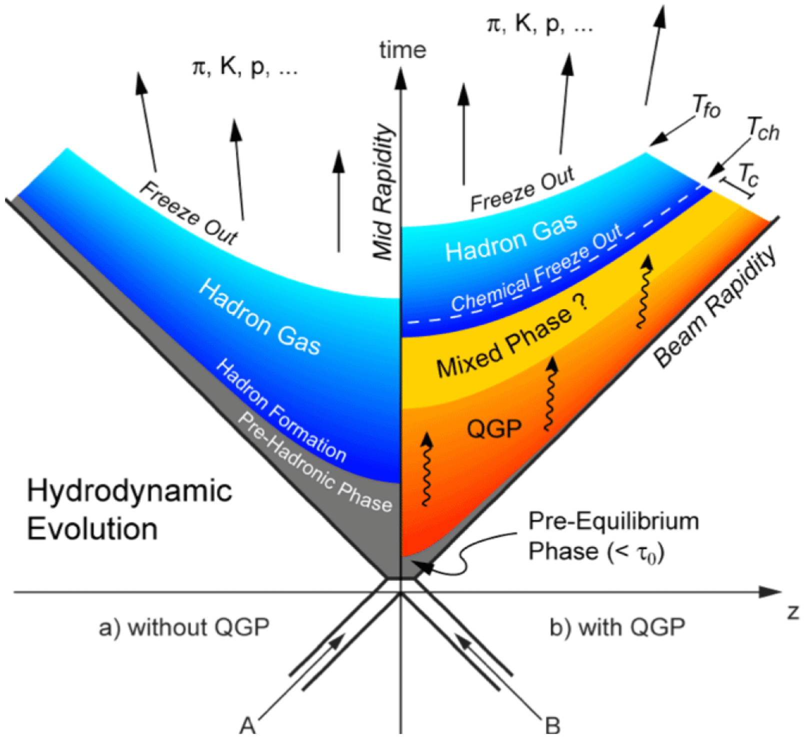
HEAVY-ION COLLISION

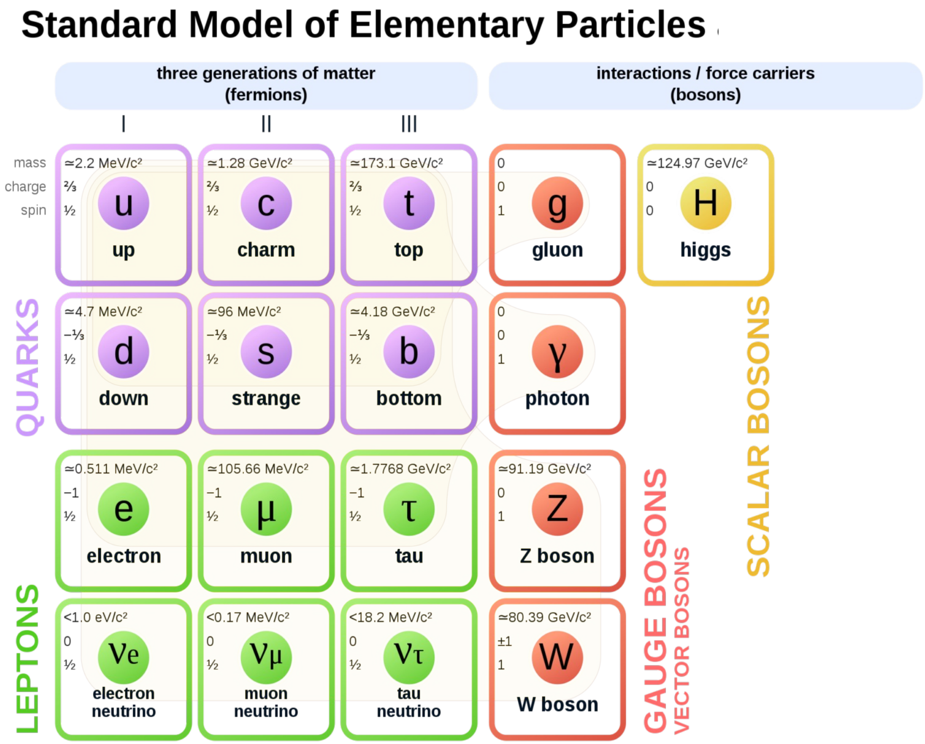


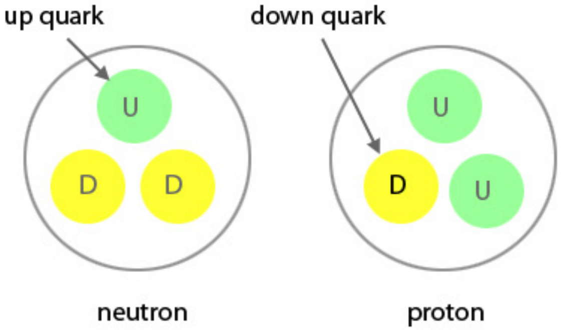


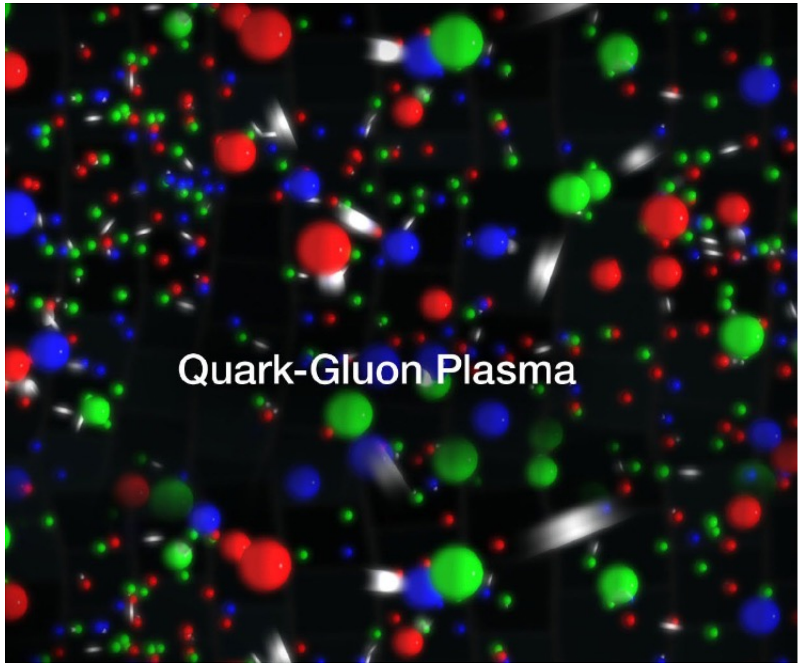
Schematic diagram

QUARK-GLUON PLASMA

- Quarks are the fundamental particles that make up protons and neutrons (and all mesons and baryons)
- Gluons are the gauge bosons of QCD
- Deconfined state of quarks and gluons → Quark-gluon plasma





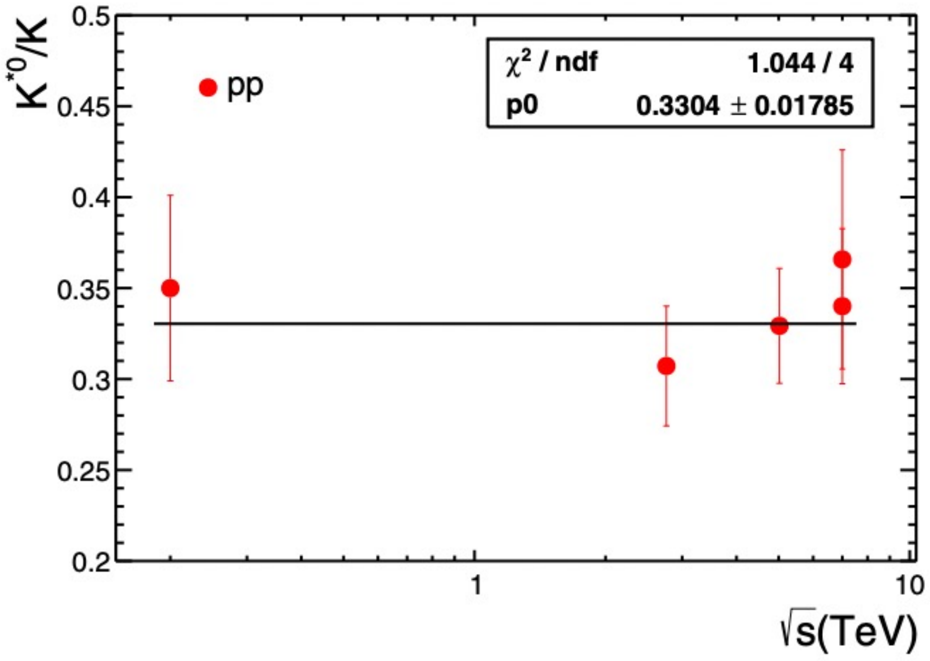


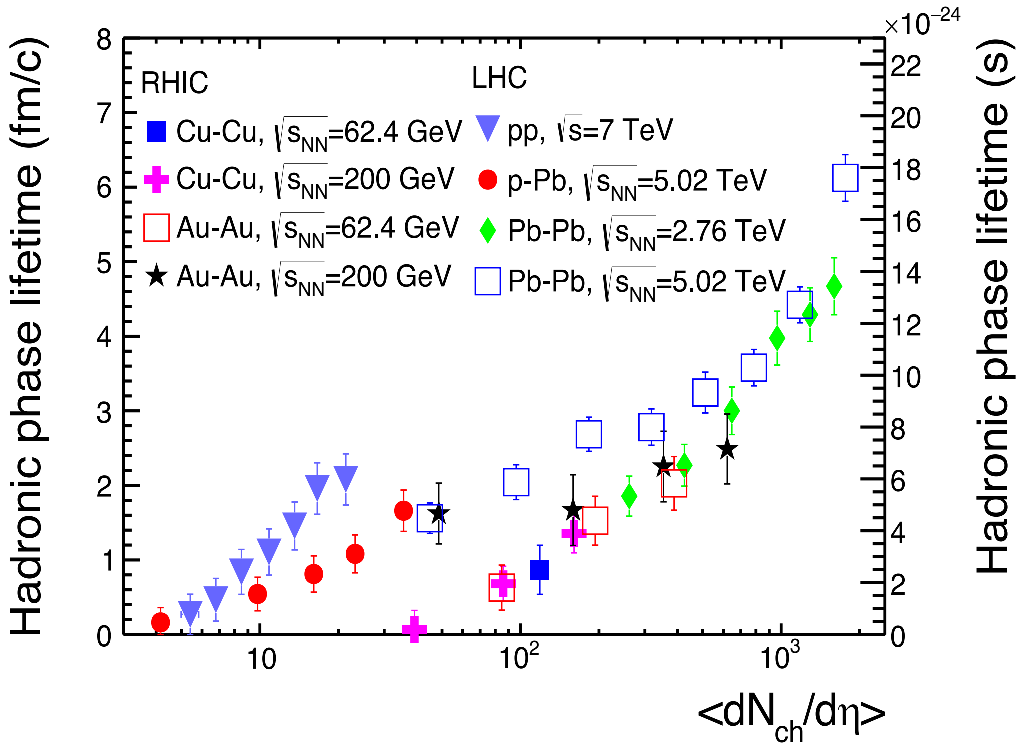
HADRONIC PHASE ?

- Time period between the chemical and kinetic freezeout
- Inspired from nuclear decay formula, $[K^{*0}/K]_{kinetic} = [K^{*0}/K]_{chemical} \times e^{-\Delta t/\tau}$

$\tau \rightarrow$ lifetime of K^{*0}

$\Delta t \rightarrow$ hadronic phase lifetime

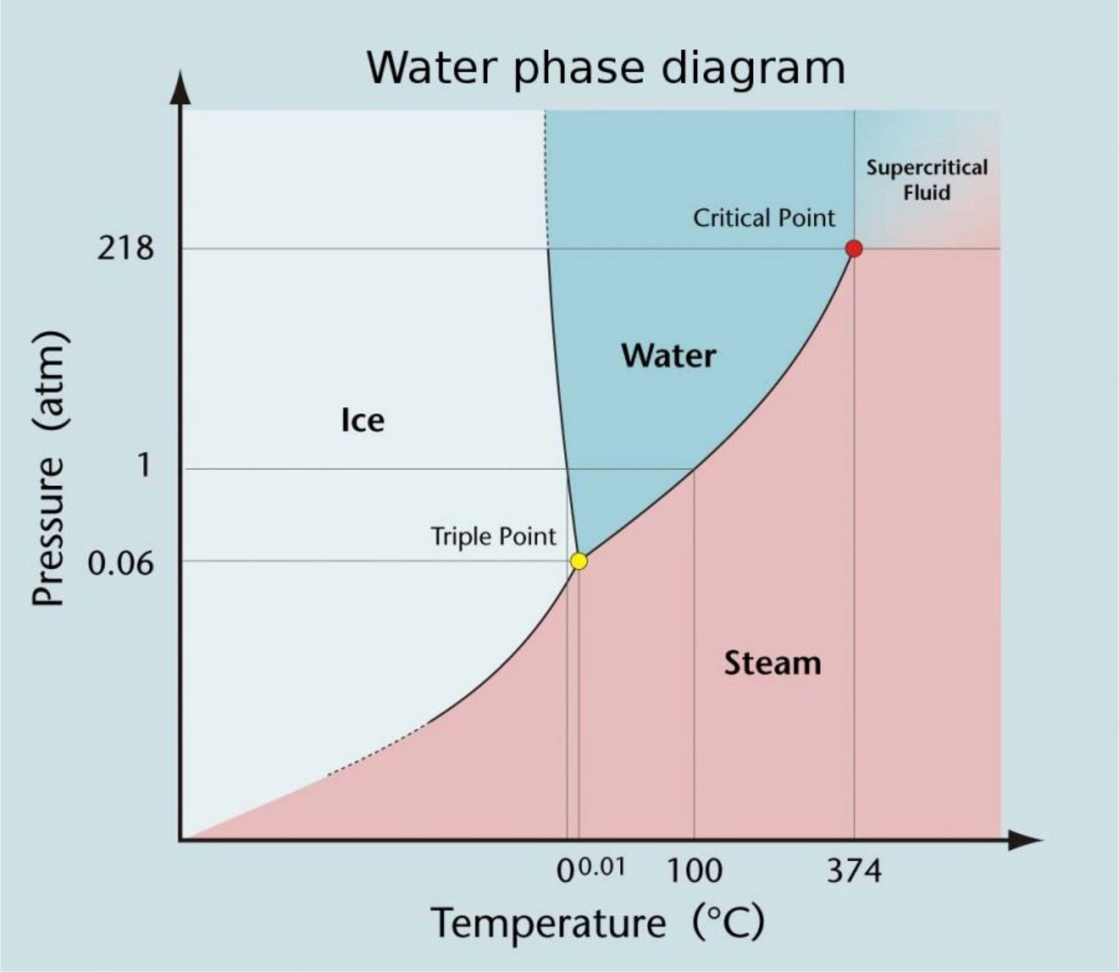


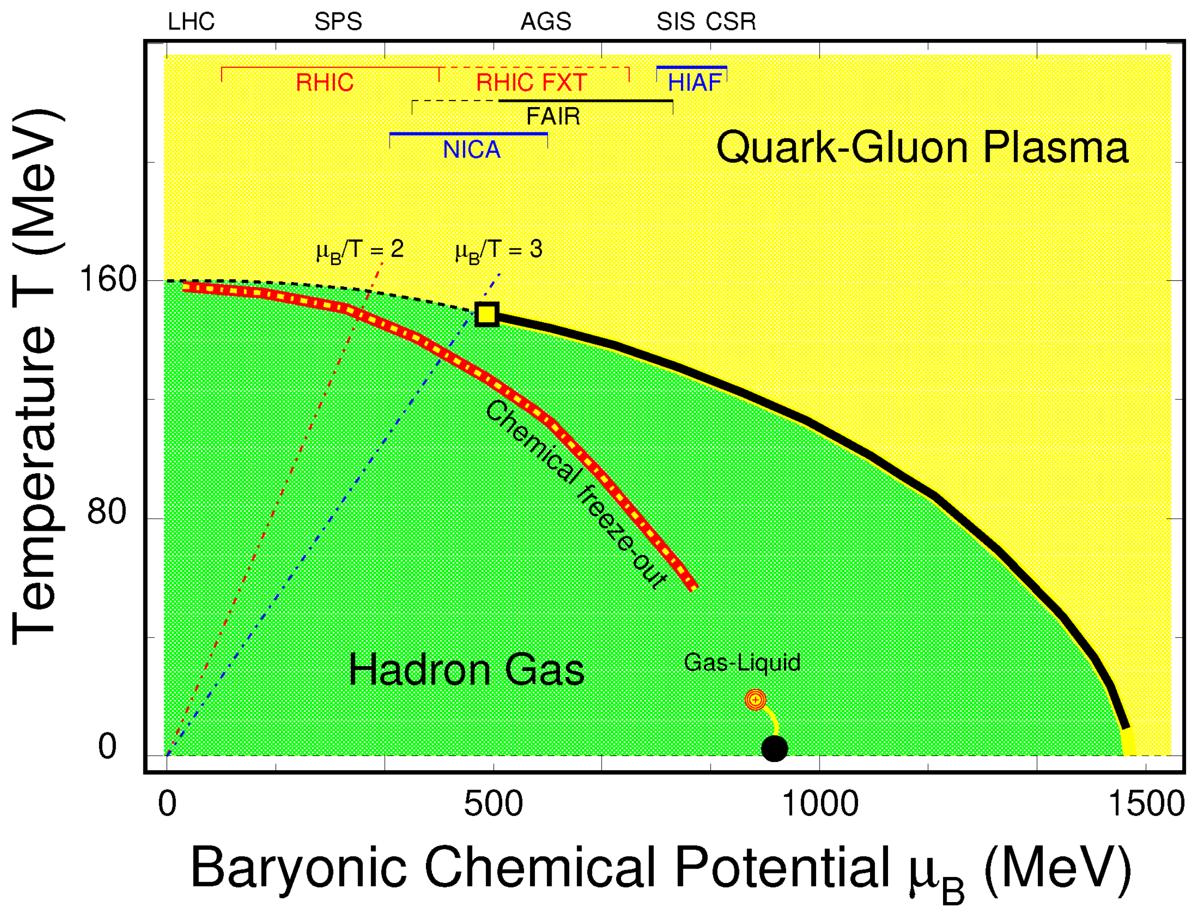


- Finite hadronic phase lifetime

D. Sahu, S. Tripathy, G. S. Pradhan and R. Sahoo, Phys. Rev. C 101, 014902 (2020)

PHASE DIAGRAM





HADRON GAS

- Heavy-ion collision leads to a multitude of particles in the final state → We apply Statistical mechanics

The hadron resonance gas model-

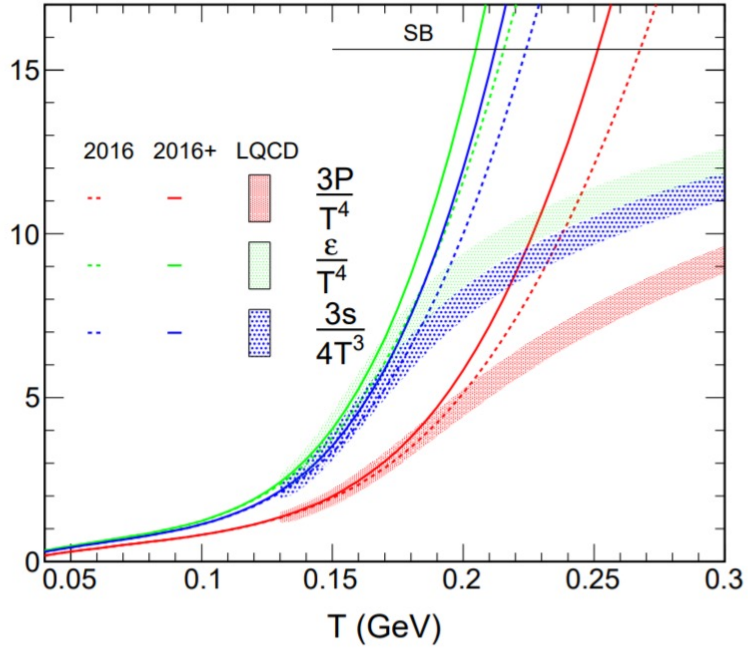
- The ideal HRG model is a non-interacting, multi-component gas of known hadrons and resonances
- The hadron resonance gas (HRG) model is very successful in describing physical observables from relativistic heavy-ion collisions at RHIC and LHC energies

$$\ln Z_i^{id} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2\, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$$p^{id} = \sum_i (\pm) \frac{g_i T}{2\pi^2} \int_0^\infty p^2\, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$$\varepsilon^{id} = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\, dp}{\exp[(E_i - \mu_i)/T] \pm 1} E_i$$

$$n^{id} = \sum_i \frac{g_i}{2\pi^2} \int_o^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T \pm 1}$$



Subhasis Samanta et al, J. Phys. G 46 065106 (2019)

VAN DER WAAL'S INTERACTION

- Disagreement between LQCD data and HRG model at high temperature
- Interaction with both attractive and repulsive parts has been introduced in the HRG model

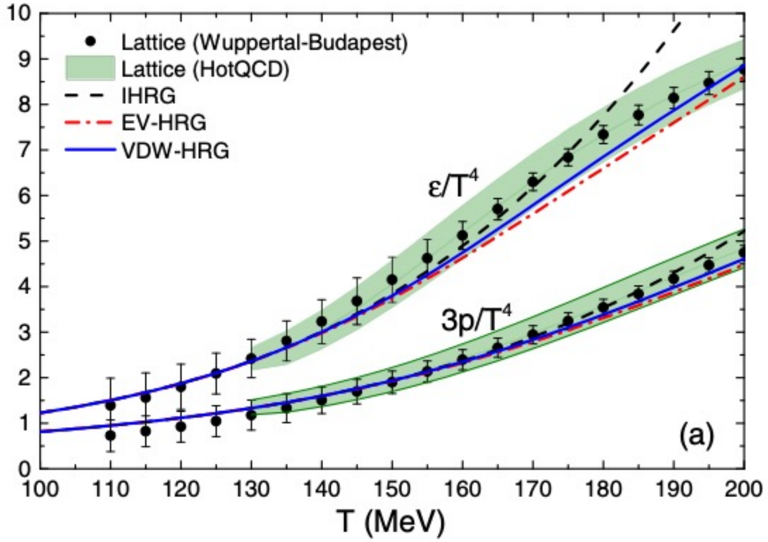
$$P(V,T,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2}$$

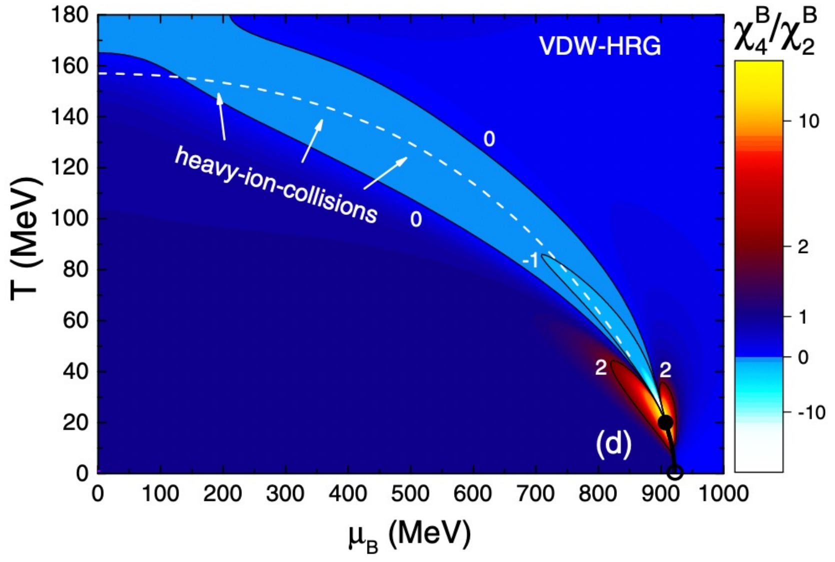
$$p(T,\mu) = p^{id}(T,\mu^*) - an^2$$

$$\mu^* = \mu - bp(T,\mu) - abn^2 + 2an$$

$$n \equiv n(T,\mu) \equiv \left(\frac{\partial p}{\partial \mu}\right)_T = \frac{n^{id}(T,\mu^*)}{1+bn^{id}(T,\mu^*)}$$

$$\varepsilon(T,\mu) = \frac{\varepsilon^{id}(T,\mu^*)}{1+bn^{id}(T,\mu^*)} - an^2$$





Volodymyr Vovchenko, Phys. Rev. Lett. 118, 182301

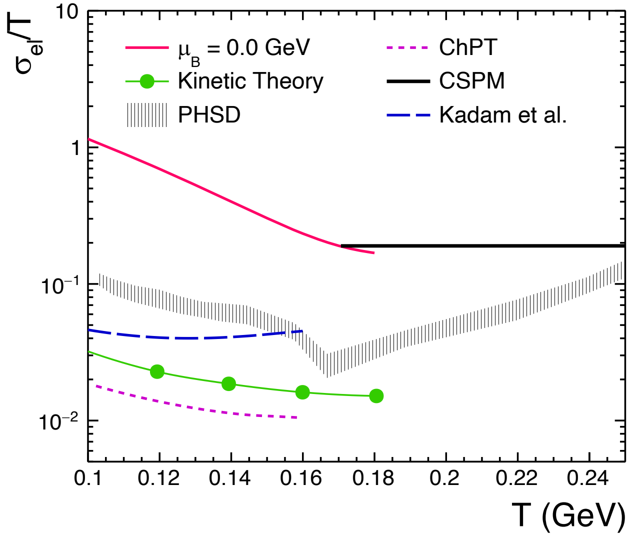
TRANSPORT PROPERTIES

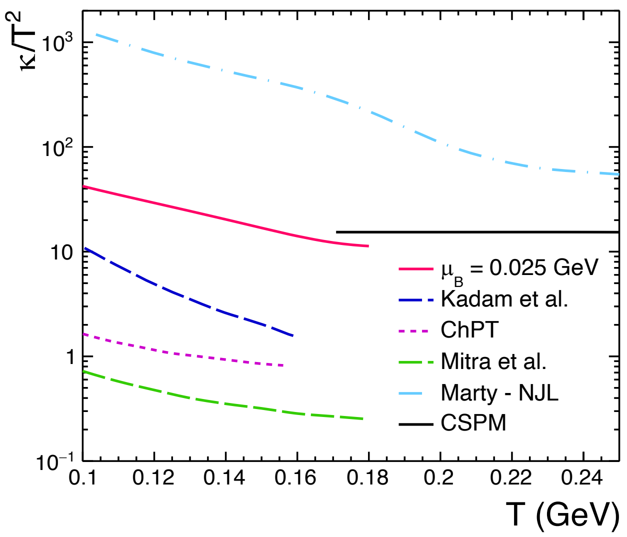
- Boltzmann transport equation: $$p^{\mu}\partial_{\mu}f_i(x,p) + q_i F^{\nu\rho}p_{\rho}\frac{\partial}{\partial p^{\nu}}f_i(x,p) = \mathcal{C}_i[f_i].$$
- The electrical conductivity is the response of a system to an externally applied field or due to the uneven distribution of charges
- Thermal conductivity describes the heat flow in interacting systems

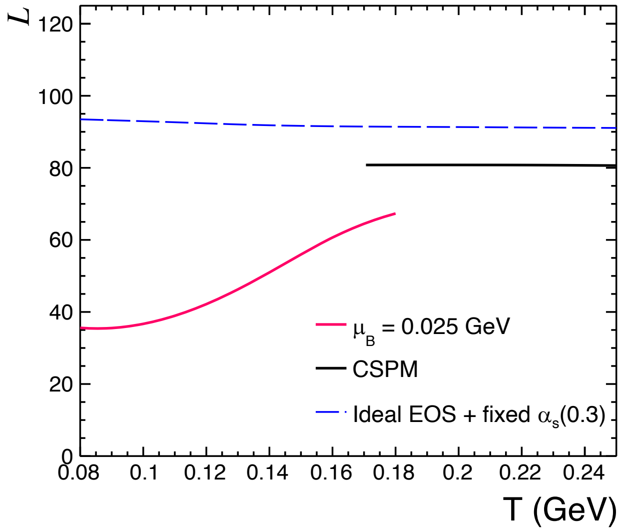
$$\sigma_{\rm el} = \frac{1}{3T}\sum_i g_i \tau_i q_i^2 \int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{E_i^2} \times f_i^0,$$

$$\kappa = \frac{1}{3T^2}\sum_i g_i \tau_i \int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{E_i^2}\left(E_i - \frac{t_i\omega}{n_{\rm net}}\right)^2 \times f_i^0,$$

Lorenz number



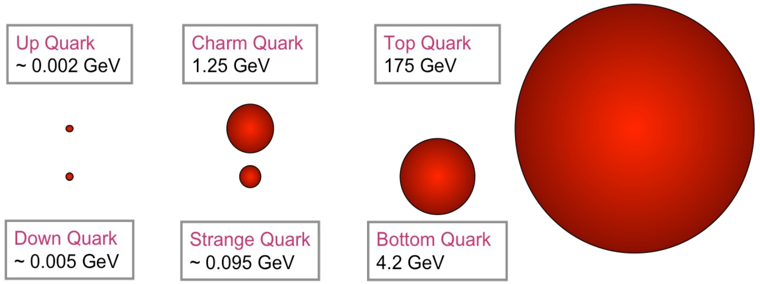




K. K. Pradhan, D. Sahu, R. Scaria and R. Sahoo, Phys. Rev. C 107, 014910 (2023)

DIFFUSION

- Heavy quarks produced early in the medium and acts as probe to study the medium
- After hadronization, the open charmed hadrons momenta get modified in hadronic medium
- Drag and diffusion of these hadrons describe the transport properties of hadron gas and helps to distinguish between the two phases



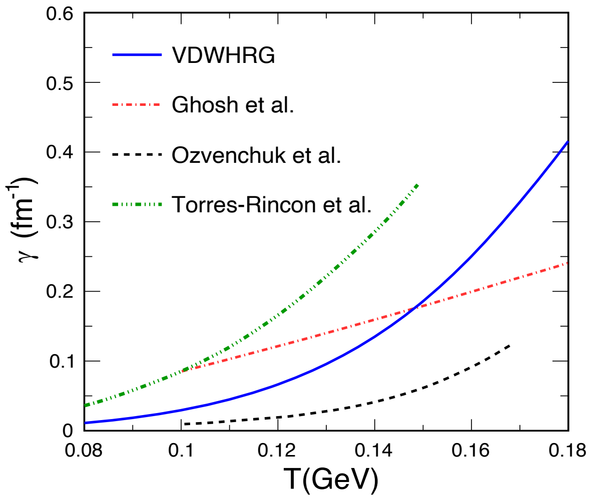
Fokker-Planck equation

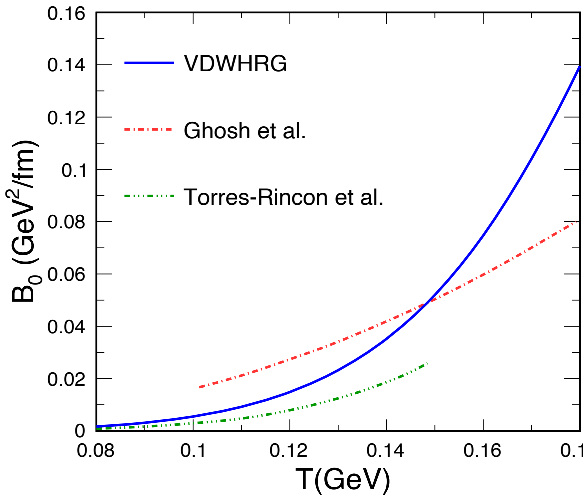
$$\frac{\partial f(t,p)}{\partial t} = \frac{\partial}{\partial p^i}\left[A^i(p)f(t,p) + \frac{\partial}{\partial p^j}\left(B^{ij}(p)f(t,p)\right)\right]$$

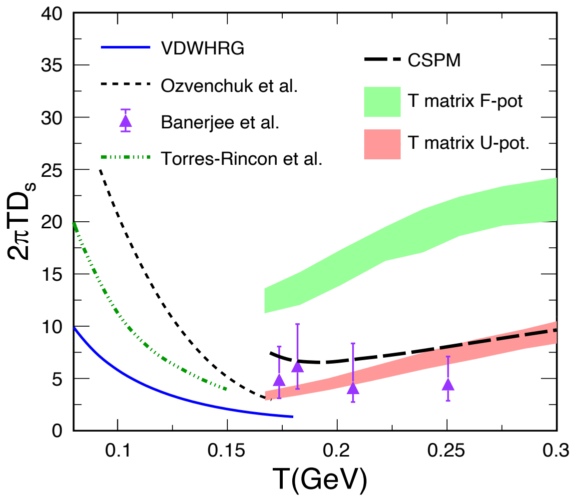
$$A_i = \gamma p_i$$

$$B_{ij} = B_0 P_{ij}^{\perp} + B_1 P_{ij}^{\parallel}$$

$$\mathrm{D_s} = \frac{T}{m_D \gamma}$$







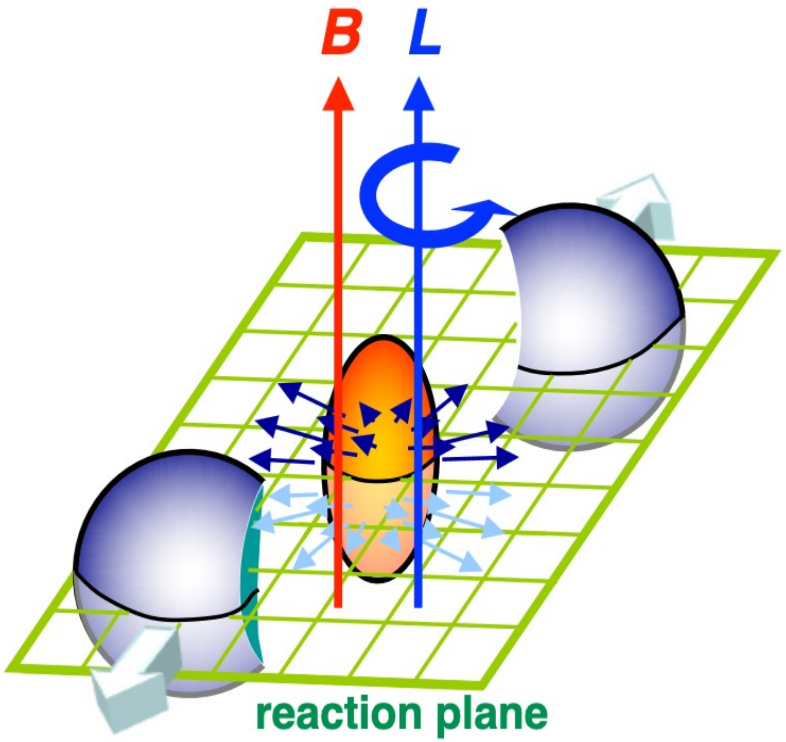
K. Goswami, K. K. Pradhan, D. Sahu and R. Sahoo, Phys. Rev. D108, 074011 (2023)

MAGNETIC FIELD AND ROTATION

- The non-central heavy-ion collision leads to production of strong transient magnetic field due to the motion of spectator protons

$eB \sim \mathrm{m}_\pi^2$, $\boxed{B \sim 10^{18}\text{Gauss}}$

$B \sim 100$ Gauss



- The peripheral heavy-ion collision also have a large initial orbital angular momentum L, can be written as

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$ $\boxed{\mathbf{L} \sim bA\sqrt{s_{NN}} \sim 10^6 \hbar}$

This leads to an angular velocity of $\omega \sim 10^{21} s^{-1}$

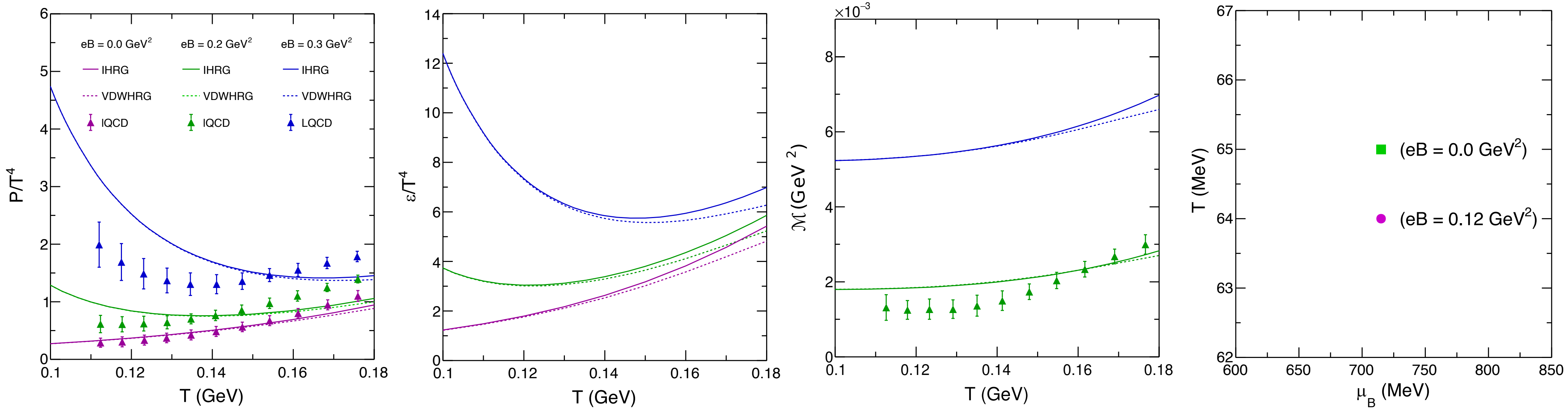
- The magnitude of magnetic field and rotation decays with the expansion of the medium

MAGNETIC FIELD

- The Euler's thermodynamic equation get modifies as: $\varepsilon + P = sT + n\mu + \mathcal{M}(QB)$

$$P_{c,i}^{id}(T,\mu_i,B) = \pm\frac{Tg_i|Q_i|B}{2\pi^2}\sum_k\sum_{s_z}\int dp_z \ln\{1 \pm \exp[-(E_{c,i}^z - \mu_i)/T]\}$$

- The critical point shift towards low temperature in presence of magnetic field



B. Sahoo, K. K. Pradhan, D. Sahu and R. Sahoo, Phys. Rev. D 108, 074028 (2023)

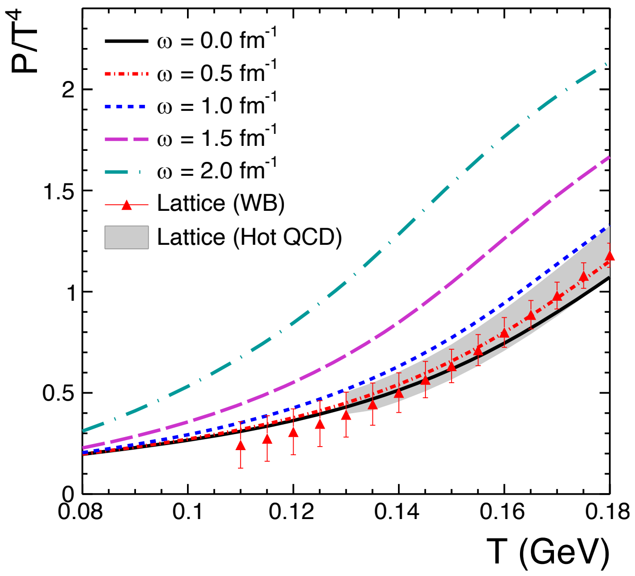
ROTATION

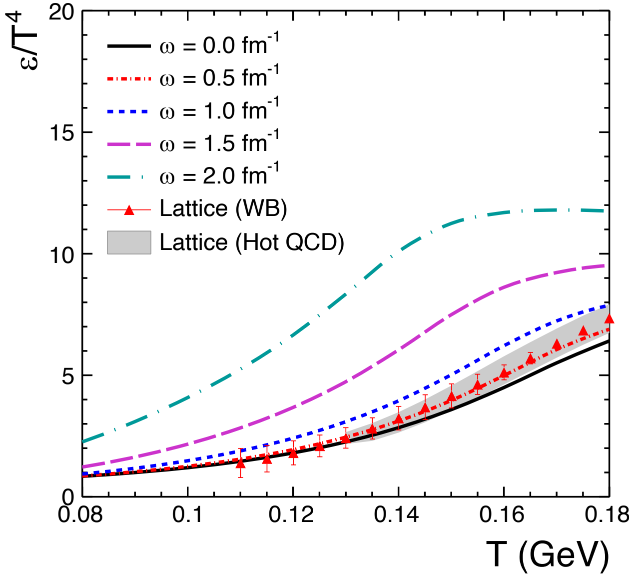
- The fundamental Euler's thermodynamic equation gets modified in presence of finite rotation adding a new Rotational Chemical Potential $$\varepsilon + P = sT + n\mu + W\omega$$

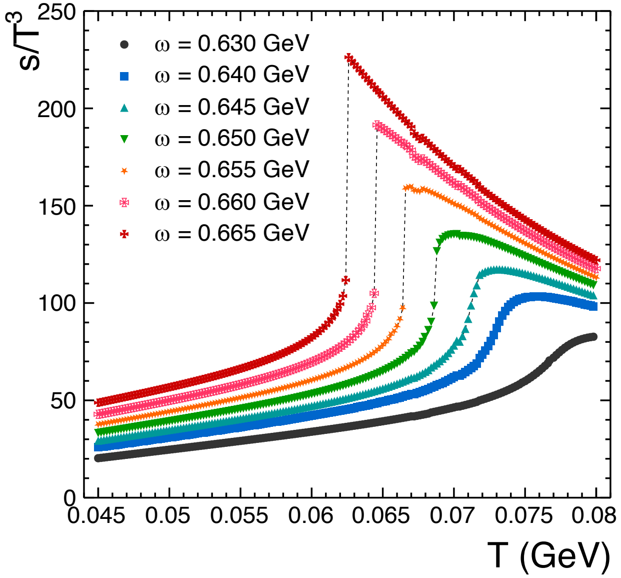
- The pressure for the rotational grand canonical ensemble is obtained as

$$P_i^{id}(T,\mu_i,\omega) = \pm\frac{g_i T}{2\pi^2}\int p^2 dp \ln\left\{1 \pm \exp\left[-\frac{E_i-\mu_i}{T}\right]\right\}\chi\left(\frac{\omega}{T}\right),$$

where $\chi\left(\frac{\omega}{T}\right) = \frac{\sinh\left(s+\frac{1}{2}\right)\frac{\omega}{T}}{\sinh\left(\frac{\omega}{2T}\right)}$







K. K. Pradhan, B. Sahoo, D. Sahu, and R. Sahoo, arXiv: 2304.05190

SUMMARY

- Finite hadronic phase in ultra-relativistic heavy-ion and hadronic collisions
- Importance of van der Waals type interaction in the hadron gas
- Estimation of various transport properties to understand the matter formed in heavy-ion collisions
- Modification of thermodynamic properties in the presence of magnetic field and rotation
- Modification in the liquid-gas critical point

What's next?

- Updated QCD phase diagram with rotation and magnetic field together with baryon chemical potential
- A model with both QCD critical point and the liquid-gas critical point to map the whole phase diagram

THANK YOU!!!