Transmission intervals in Disease Modeling

Jonathan Dushoff, McMaster University

IBENS Minisymposium
Modeling Epidemics and Behaviour
May 2022, Paris

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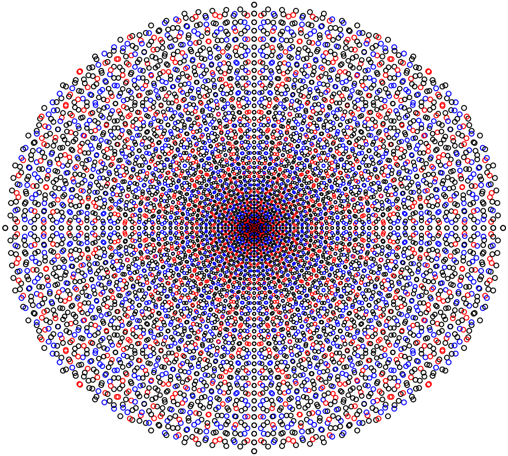
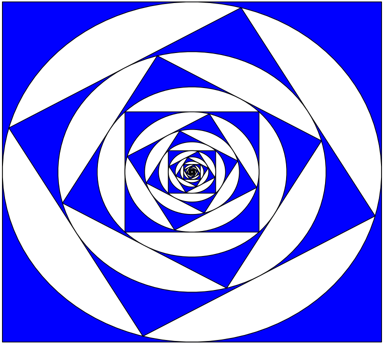
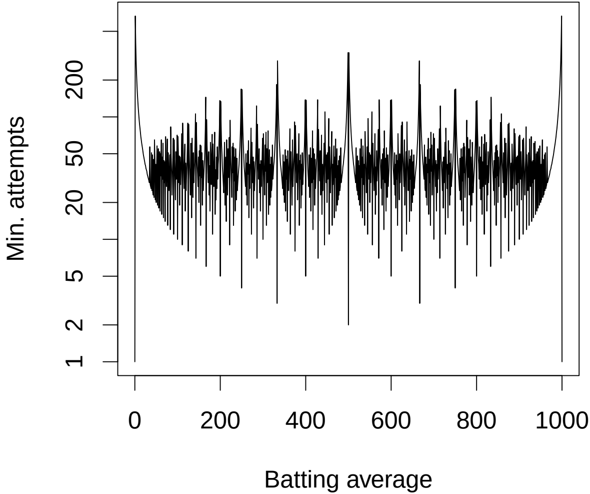


What is the pattern of Pythagorean triples of integers
$a^2 + b^2 = c^2$?



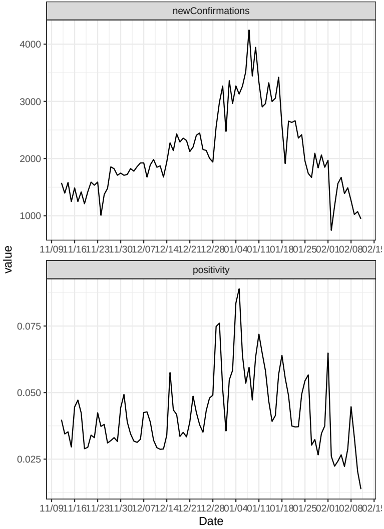
Divide a square and a circle each into two complementary subsets that are pairwise similar



How many at-bats does it take to get a given batting average?

Covid modeling questions

- How far and fast would it spread if unchecked?



https://wzmli.github.io/COVID19-Canada

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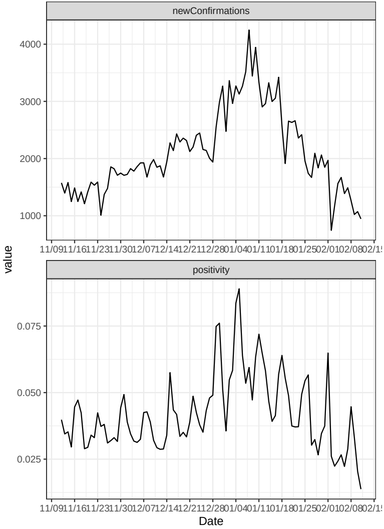
- How hard is it to eliminate?



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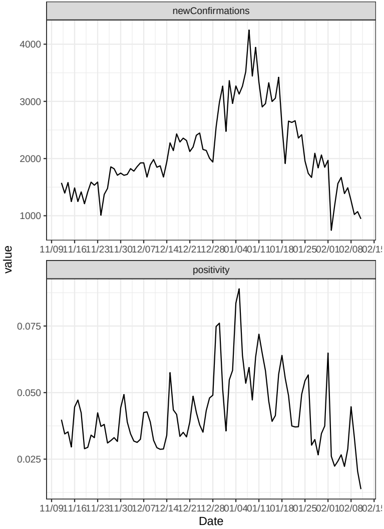

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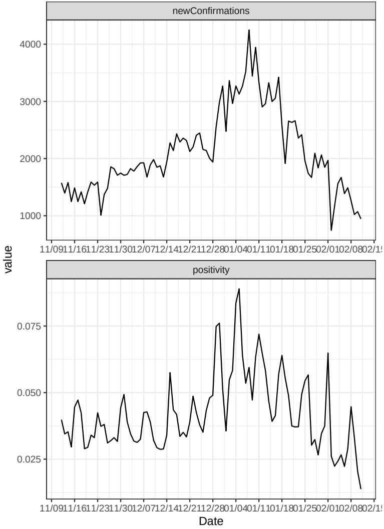

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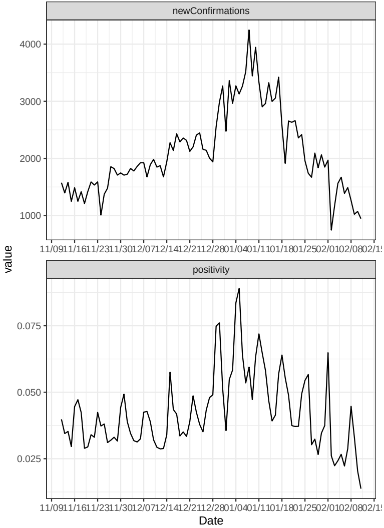

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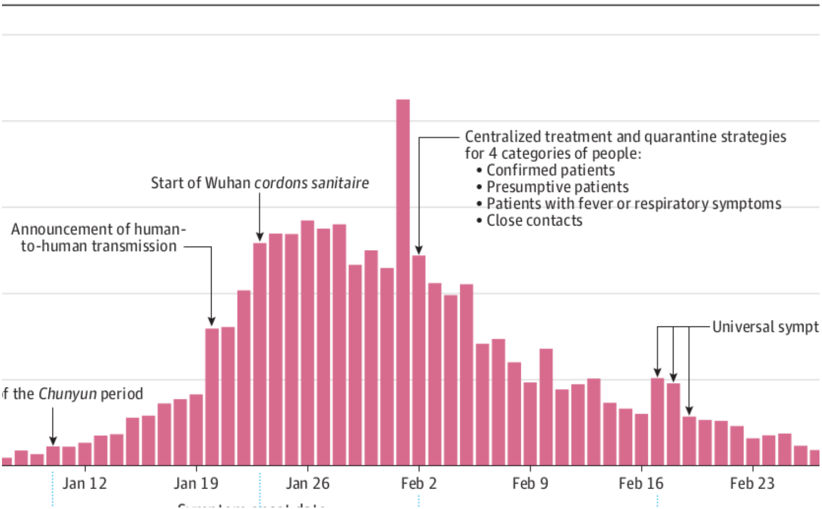
Wuhan control measures

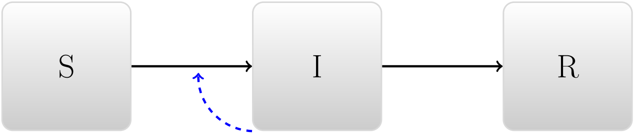


https://jamanetwork.com/journals/jama/fullarticle/2764658

Outline

Simple dynamical models use compartments

Divide people into categories:



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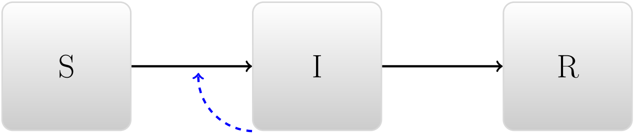

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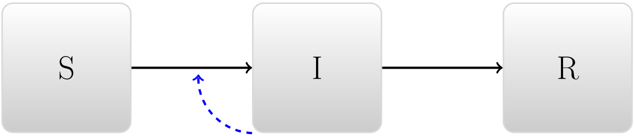

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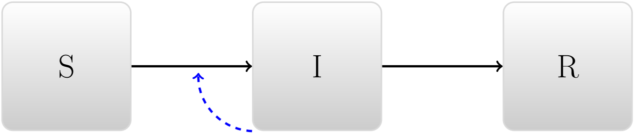

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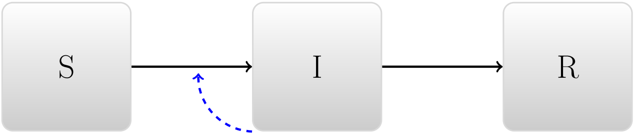

$$
\begin{aligned}
\frac{dS}{dt} &= \mu N - \beta \frac{SI}{N} - \mu S \\
\frac{dI}{dt} &= \beta \frac{SI}{N} - \gamma I - \mu R \\
\frac{dR}{dt} &= \gamma I - \mu R
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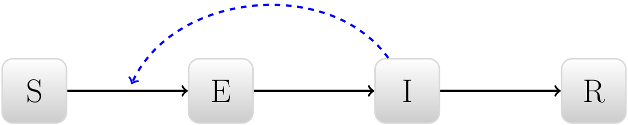
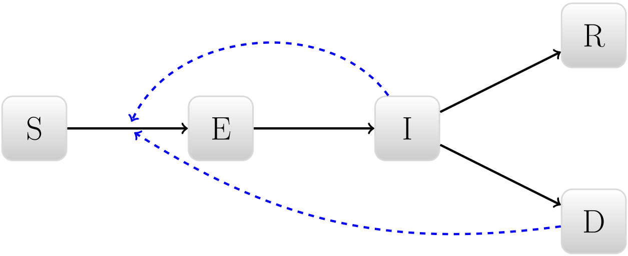
$$

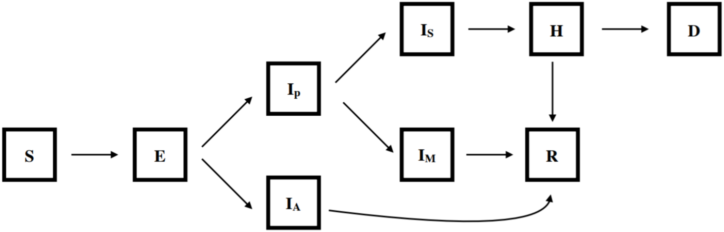
Delayed infectiousness



Ebola



Coronavirus



S = susceptible
E = exposed

I_P = pre-symptomatic
I_A = asymptomatic
I_S = symptomatic, severe case
I_M = symptomatic, minor case

H = hospitalized
R = recovered
D = dead

Childs et al., http://covid-measures.stanford.edu/

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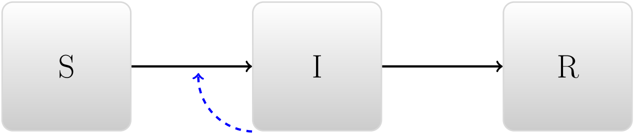
Cohort modeling



- Create ODEs to follow a *cohort* of people infected at the same time

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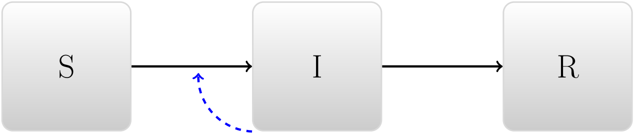

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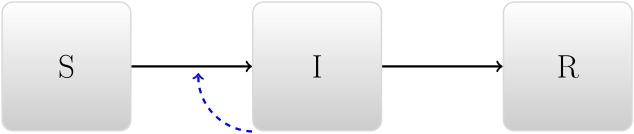

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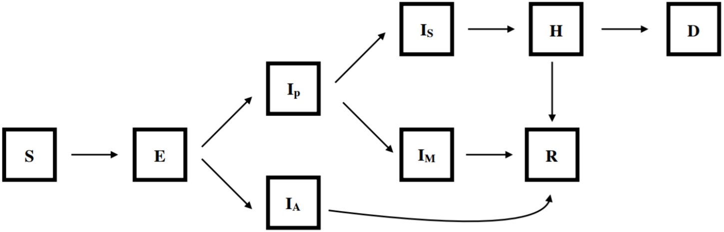

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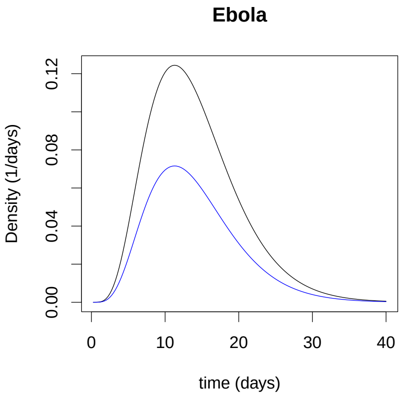

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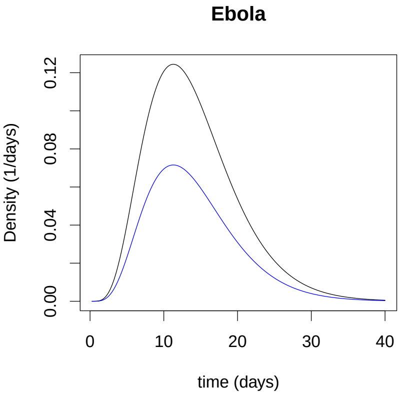

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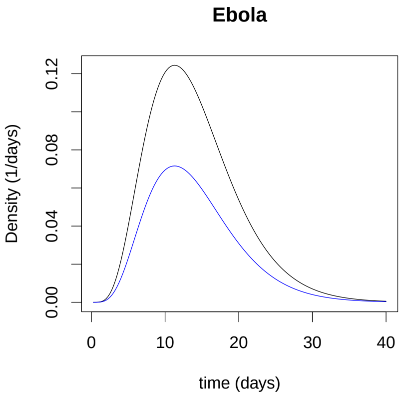

- Area is $\mathcal{R}$
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Outline

Transmission intervals

'*I cannot imagine how they will spend half of it.*'

- Sort of the poor relations of disease-modeling world

Transmission intervals

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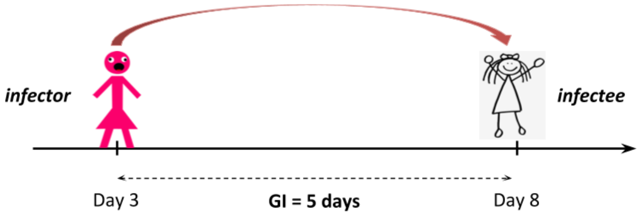
How long is a disease generation?

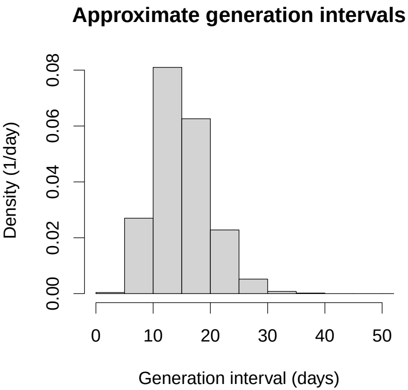
Definition

Generation Interval:

Interval between the time that an individual is infected by an infector and the time this infector was infected



Generation-interval distributions



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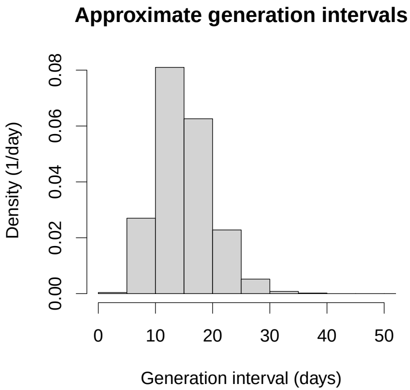

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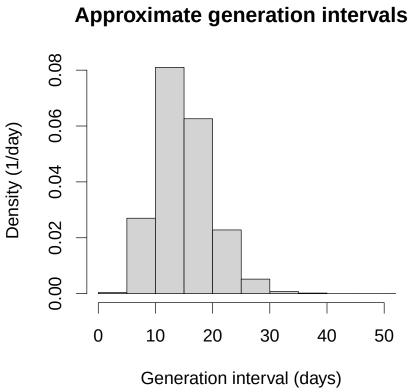

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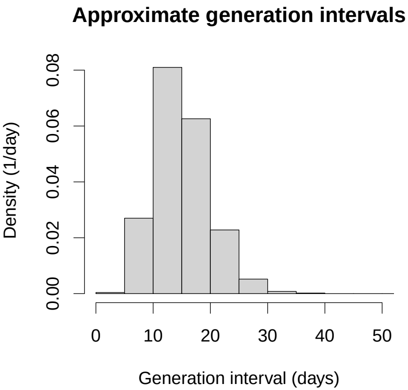

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Mechanistic perspective

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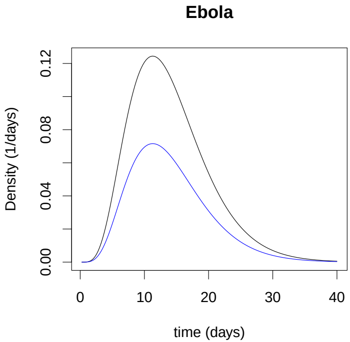

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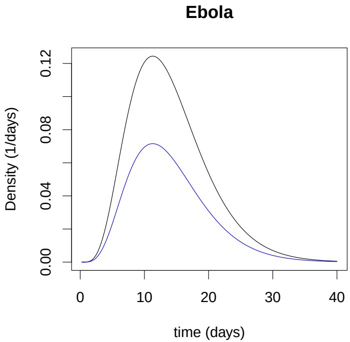

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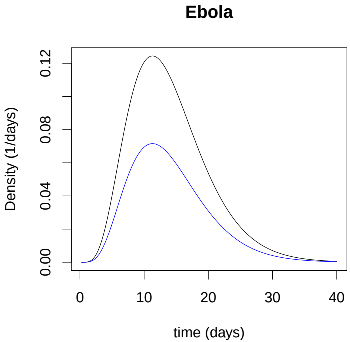
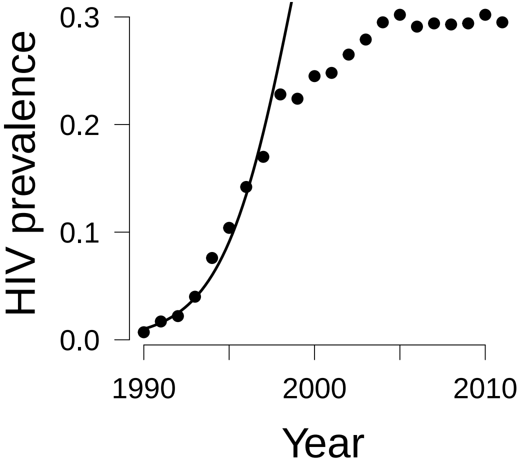
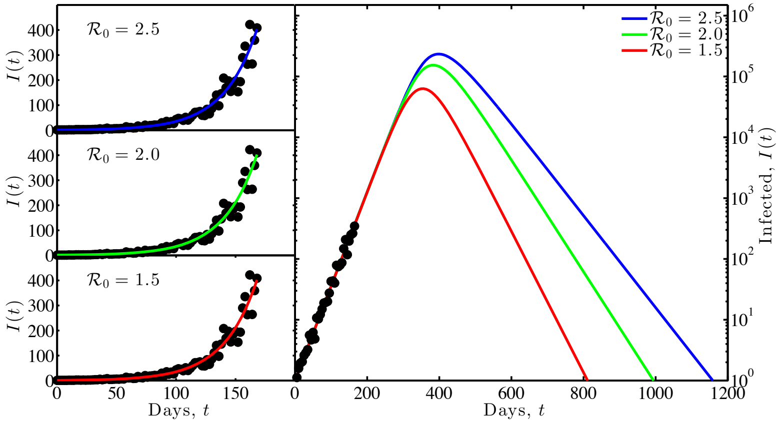


HIV in sub-Saharan Africa



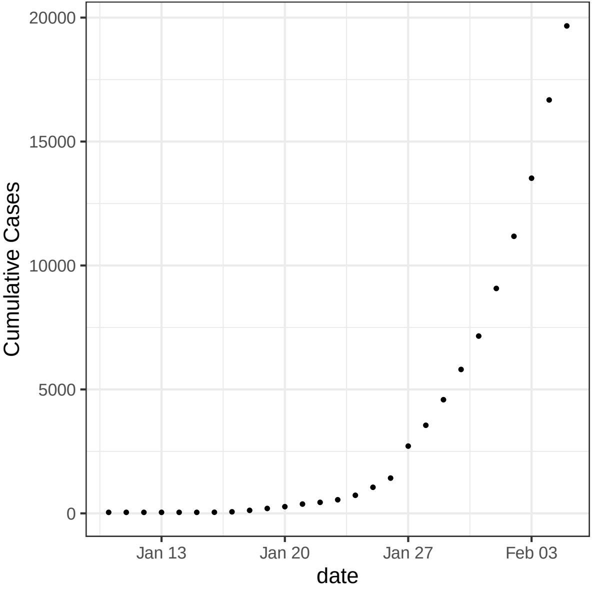
$C \approx 18\,\mathrm{month}$. Faster than expected.

Ebola outbreak



$C \approx 1\,\mathrm{month}$. Slower than expected.

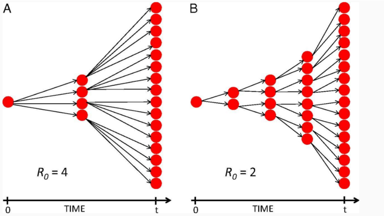
Coronavirus speed



$C \approx 5\,\mathrm{day}$. Coronavirus!

Phenomenological perspective

- Population-level speed r is observed



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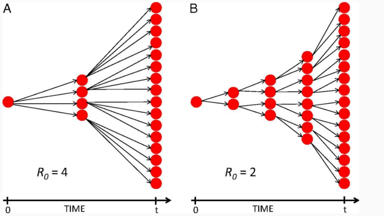

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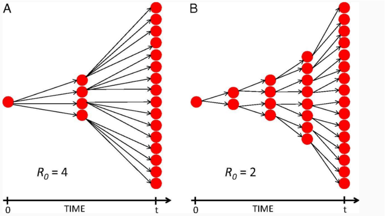

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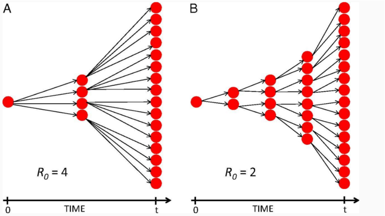

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How long until the bus comes?

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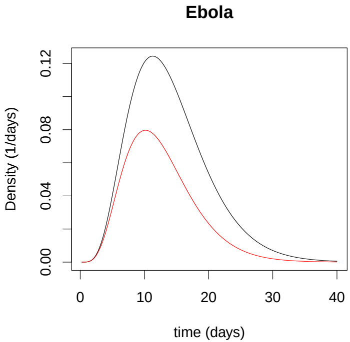

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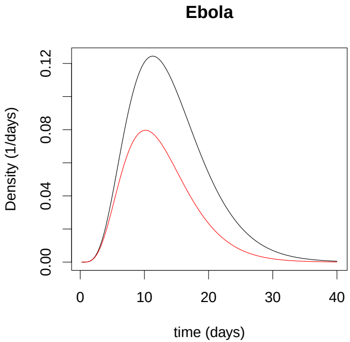

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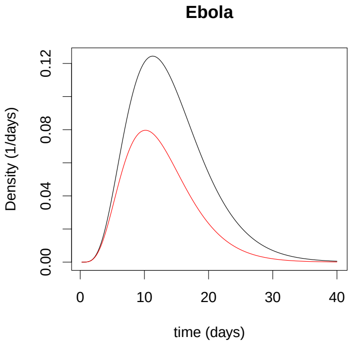

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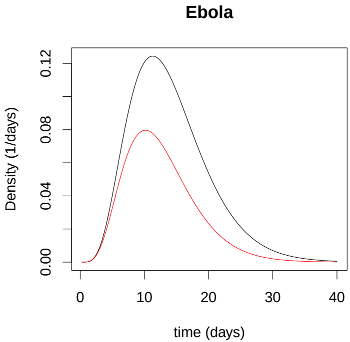

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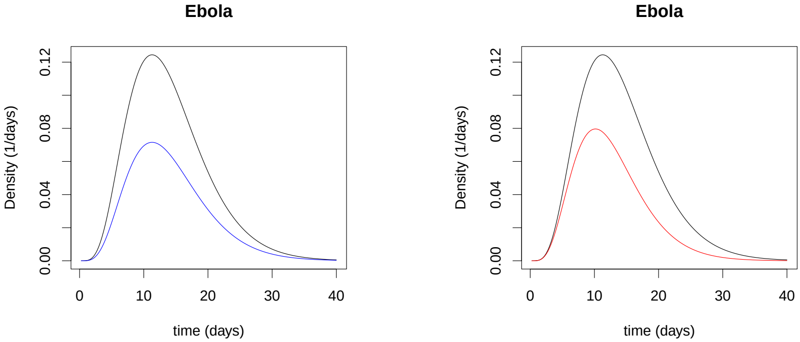

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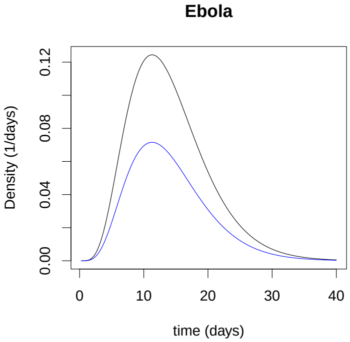

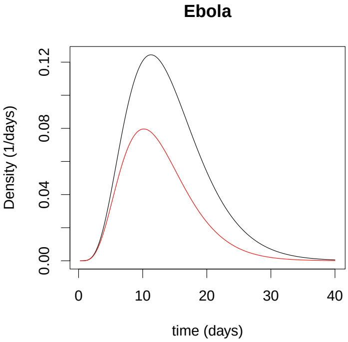

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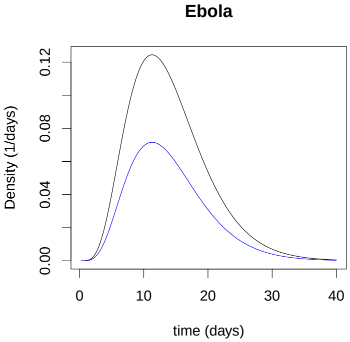

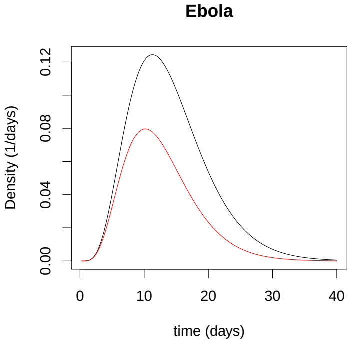

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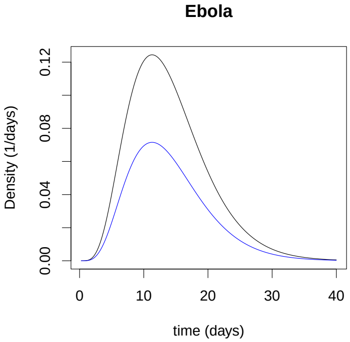

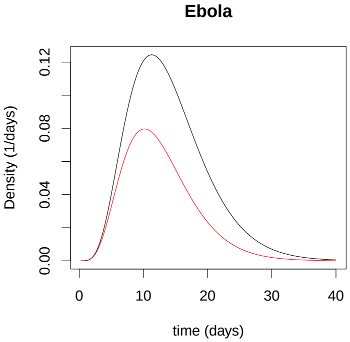

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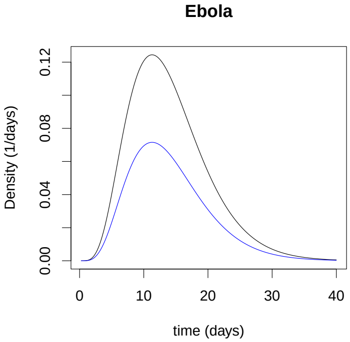

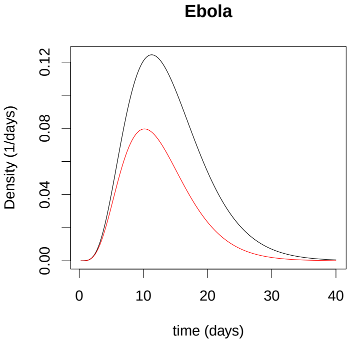

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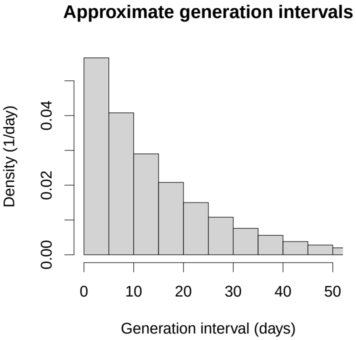

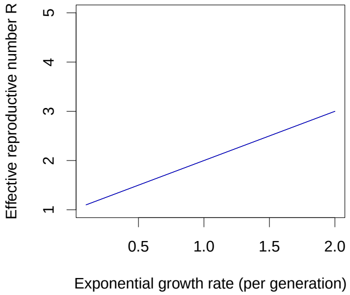

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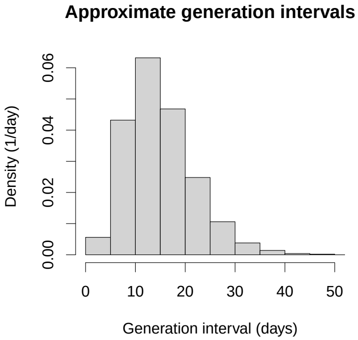

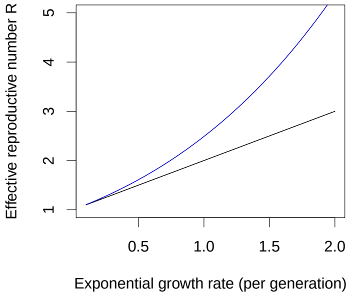

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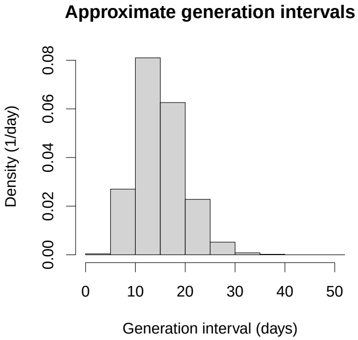

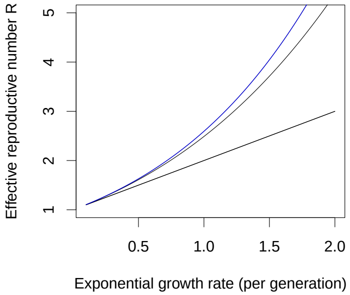

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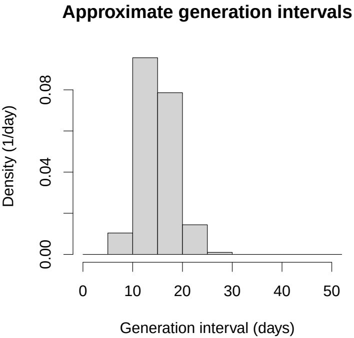
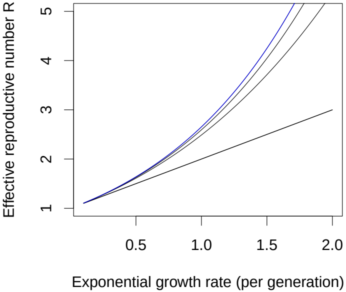
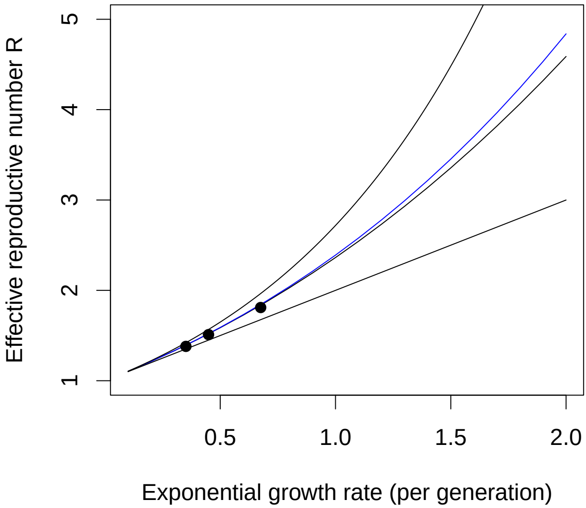




Approximating the $r\mathcal{R}$ relationship



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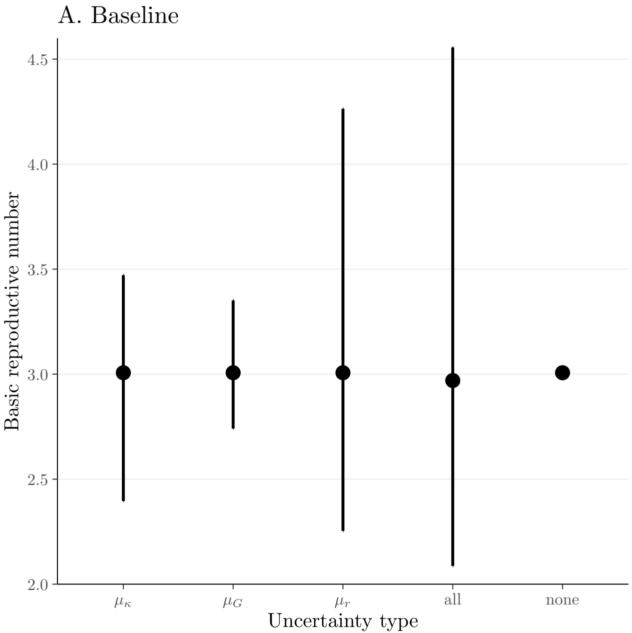

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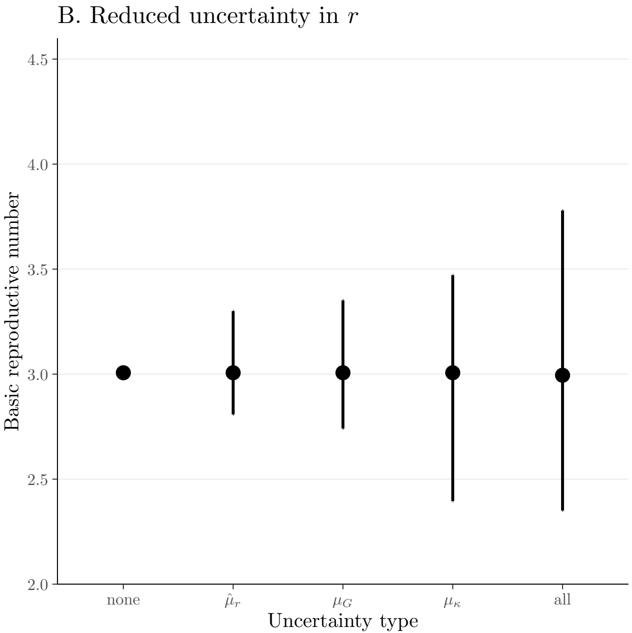


Outline

Modeling approaches

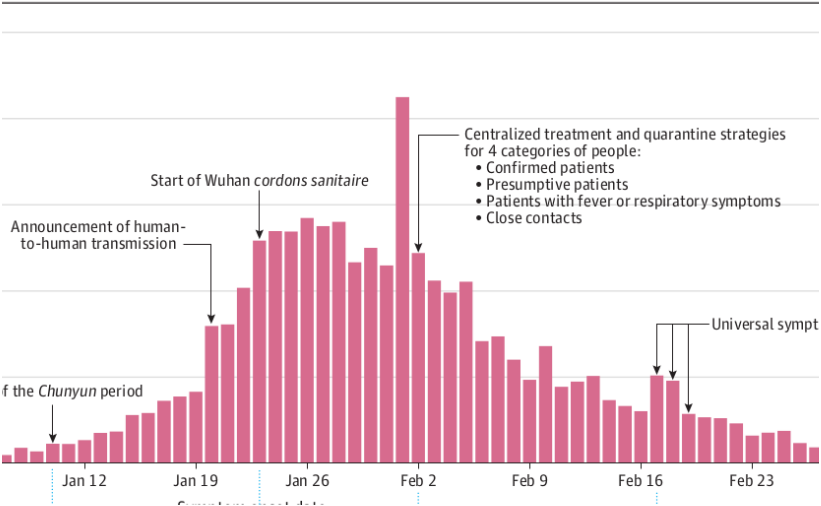
Transmission intervals

Linking $r\mathcal{R}$

Evaluating interventions

Summary

Wuhan control measures



https://jamanetwork.com/journals/jama/fullarticle/2764658

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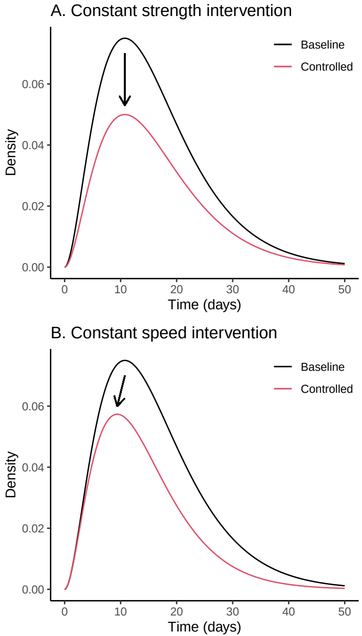

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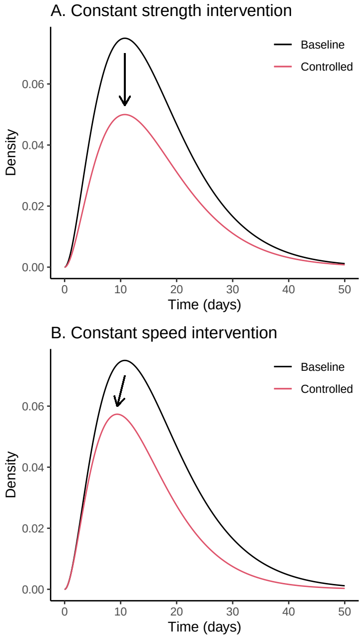

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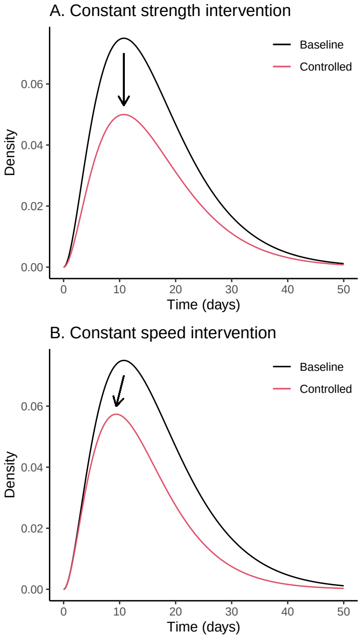

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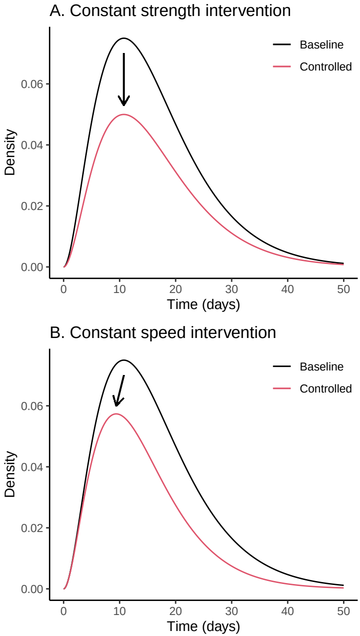

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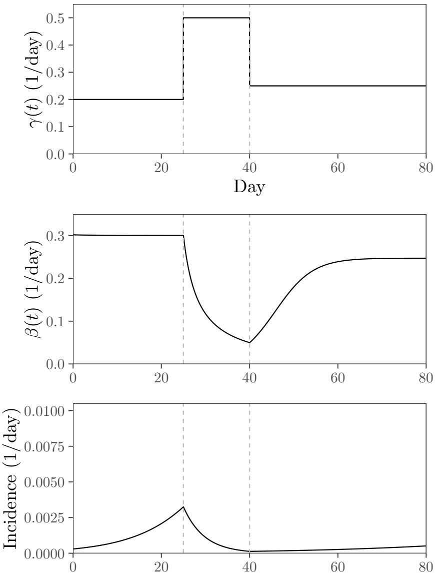

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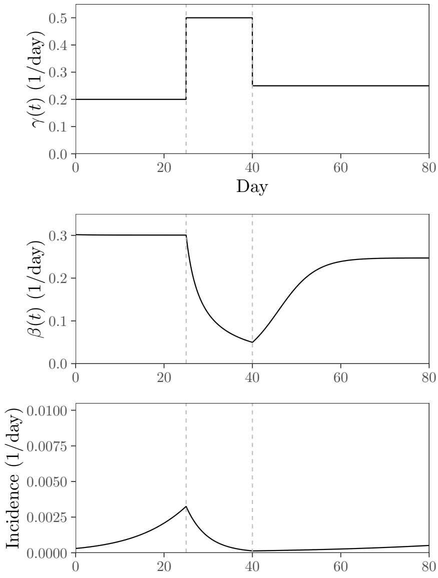

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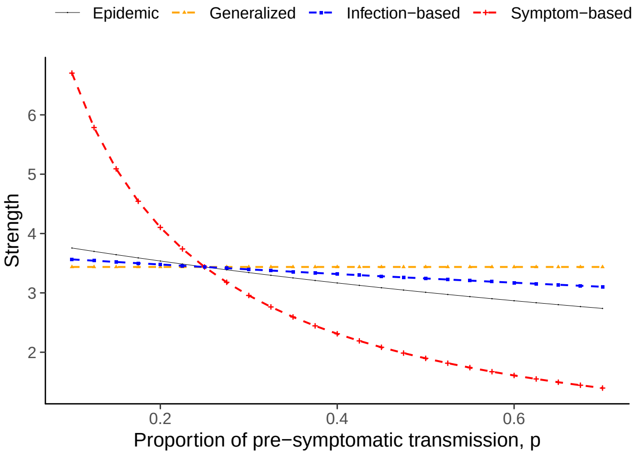

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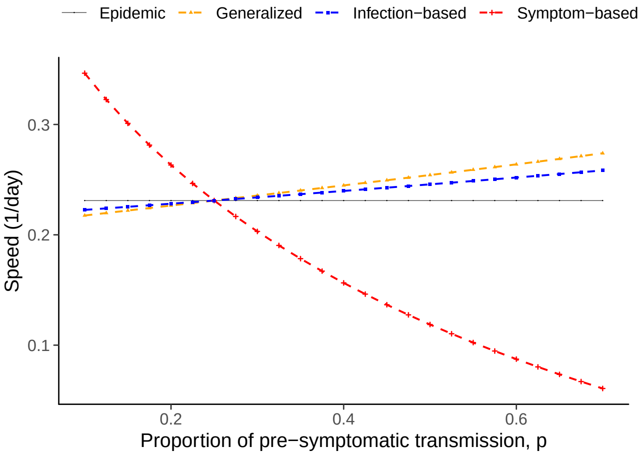

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