

Mathematical modeling of infectious diseases

Insights and limitations

Jonathan Dushoff, McMaster University

2023

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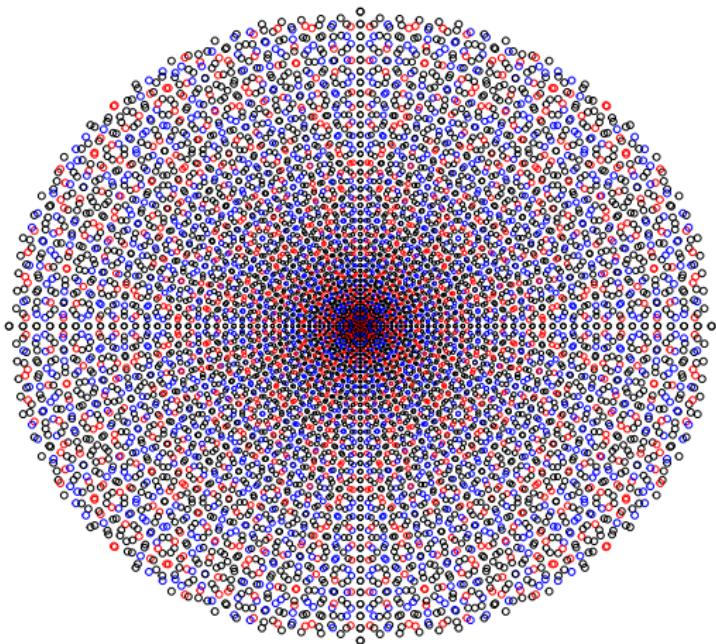
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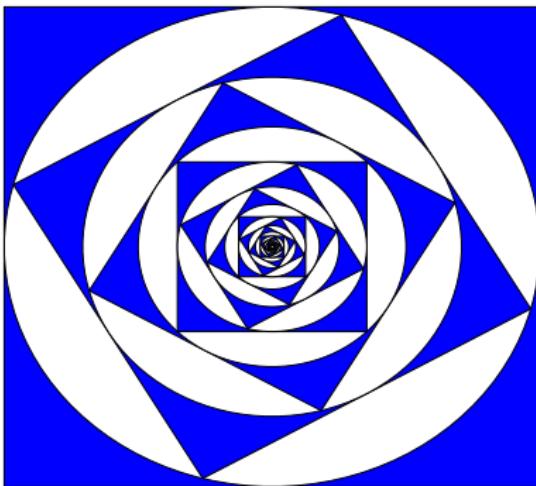






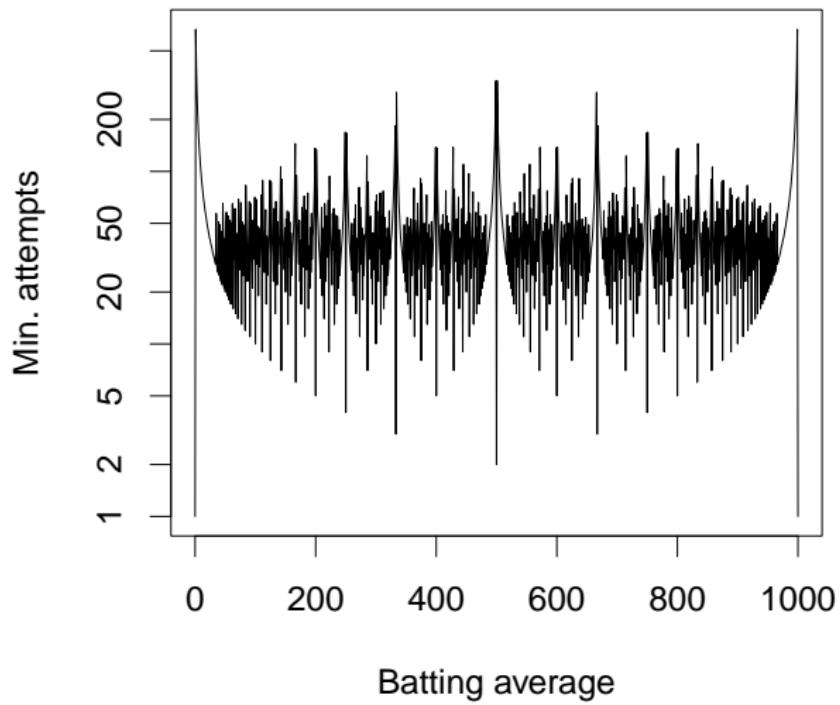


What is the pattern of Pythagorean triples of integers
 $a^2 + b^2 = c^2$?



Divide a square and a circle each into two complementary subsets
that are pairwise similar

Batting average (安打率)



Outline

What is dynamical modeling?

Modeling approaches

Transmission intervals

Intrinsic and realized intervals

Linking $r\mathcal{R}$

Serial-interval distributions

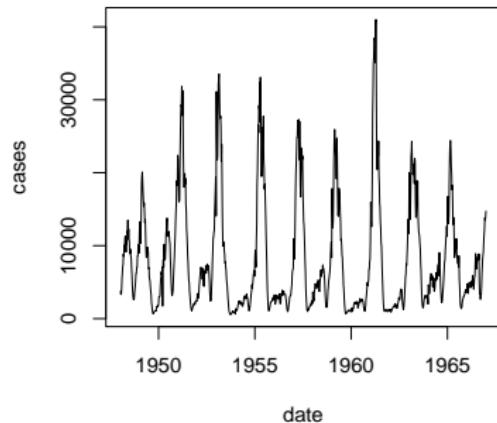
Summary

Dynamical modeling connects scales

Measles reports 麻疹病例報告



Measles reports from England and Wales



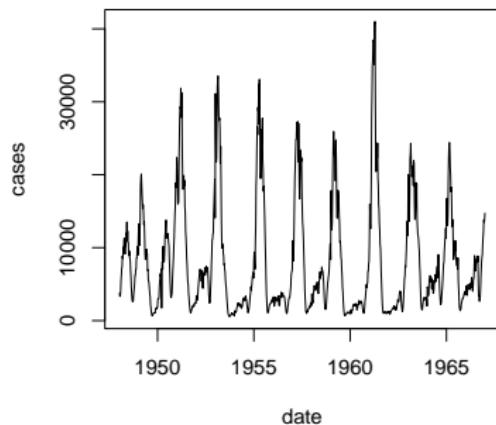
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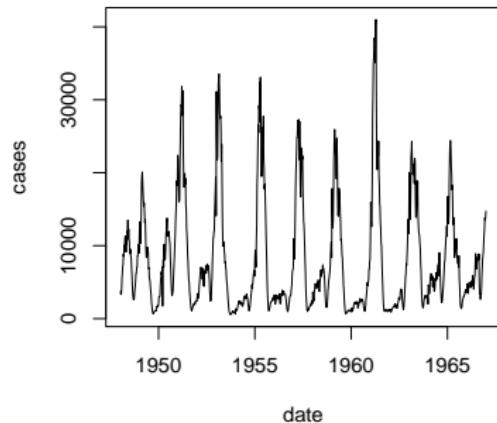
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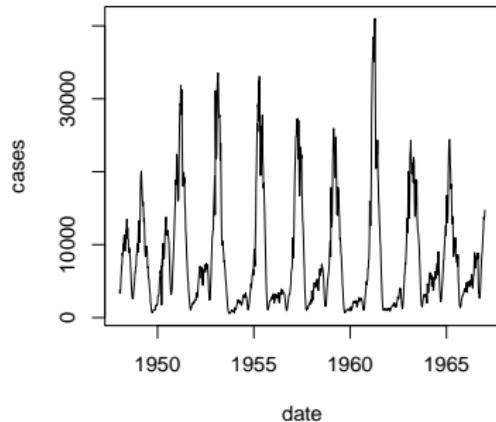
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- ▶ Calculate results (結果) over longer time periods

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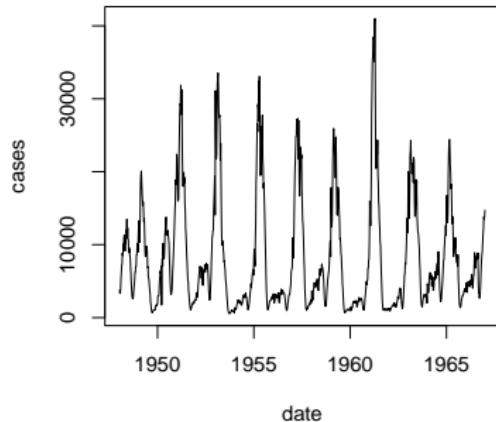
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Dynamical modeling bridges from individuals to populations

- We can learn about the result from the rules



Dynamical modeling bridges from individuals to populations

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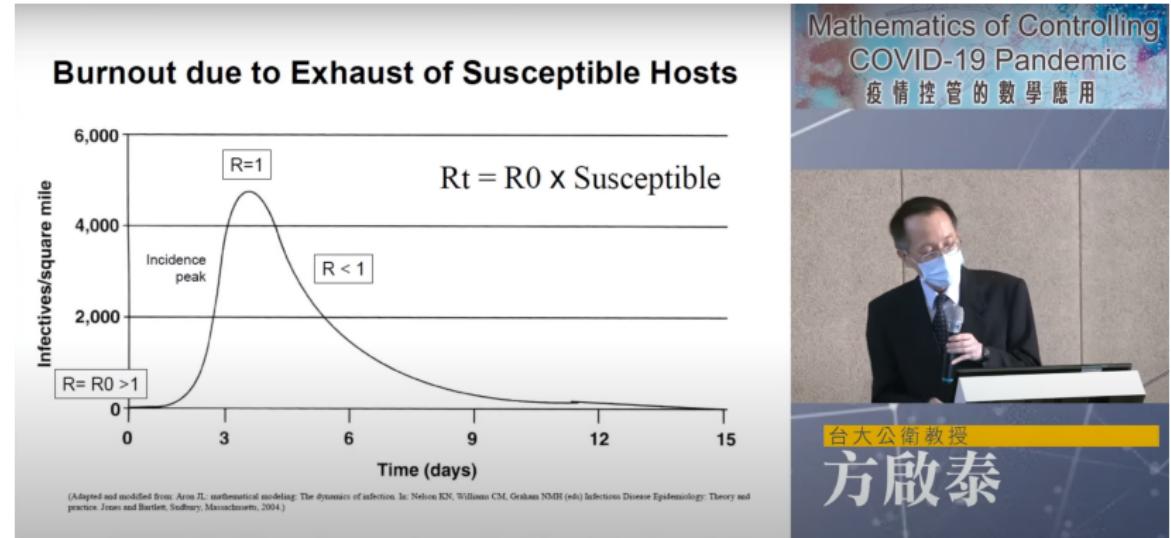
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How generation intervals shape the relationship between growth rates and reproductive numbers

J. Wallinga^{1,*} and M. Lipsitch²

¹Department of Infectious Disease Epidemiology, National Institute of Public Health and the Environment, PO Box 1, 3720 BA Bilthoven, The Netherlands

²Department of Epidemiology and Department of Immunology and Infectious Diseases, Harvard School of Public Health, 677 Huntington Avenue, Boston, MA 02115, USA

Mathematical models have become indispensable tools in predicting the course of emerging infectious diseases. A key variable in such models is the reproductive number R . For new emerging infectious diseases, the value of the reproductive number can only be inferred indirectly from the observed exponential epidemic growth rate r . Such inference is ambiguous as several different equations exist that relate the reproductive number to the growth rate, and it is unclear which of these equations might apply to a new infection. Here, we show that these different equations differ only with respect to their assumed shape of the generation interval distribution. Then, we show that the shape of the generation interval distribution, which estimates is appropriate for inferring the reproductive number from the observed growth rate. We show that by assuming all generation intervals to be equal to the mean, we obtain an upper bound to the range of possible values that the reproductive number may attain for a given growth rate. Furthermore, we show that by taking the generation interval distribution equal to the observed distribution, it is possible to obtain an empirical estimate of the reproductive number.

Keywords: basic reproduction ratio; epidemiology; influenza; Lotka–Euler equation; serial interval



双生伏士麻用六物及 Application of Mathematical Modelling in Global Infectious Disease Control



林先和教授

Outline

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Modeling approaches

Transmission intervals

Intrinsic and realized intervals

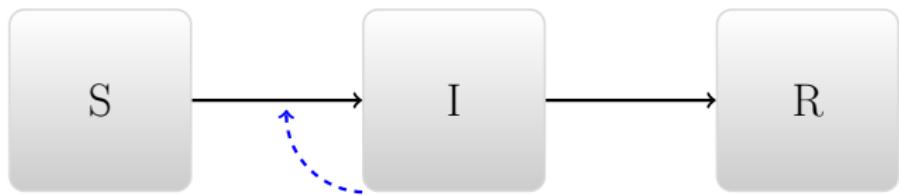
Linking $r\mathcal{R}$

Serial-interval distributions

Summary

Simple dynamical models use compartments

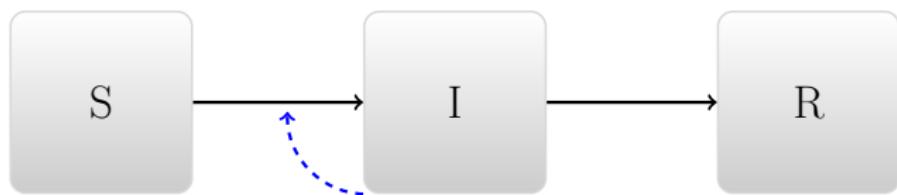
Divide people into categories:



- Susceptible → Infectious → Recovered

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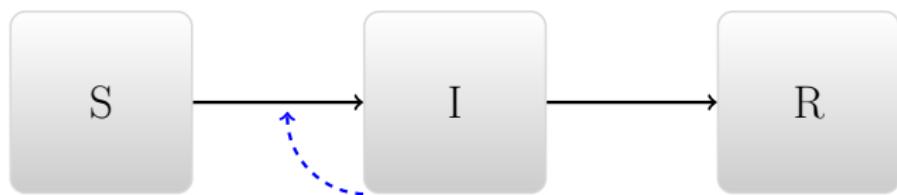
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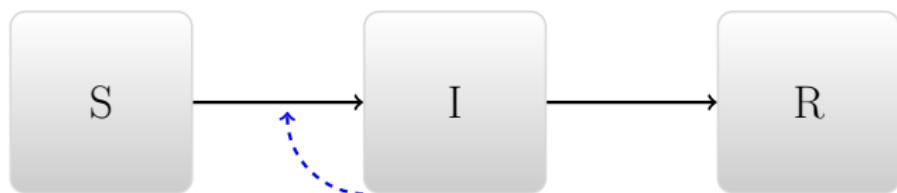
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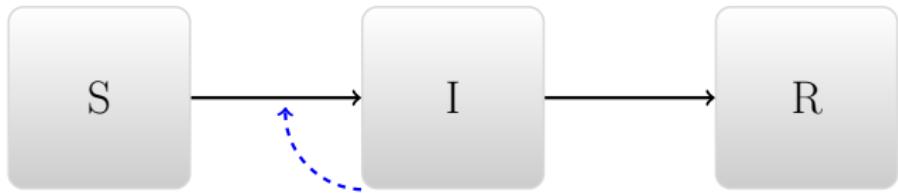
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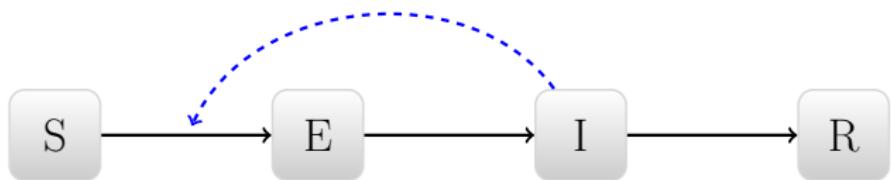


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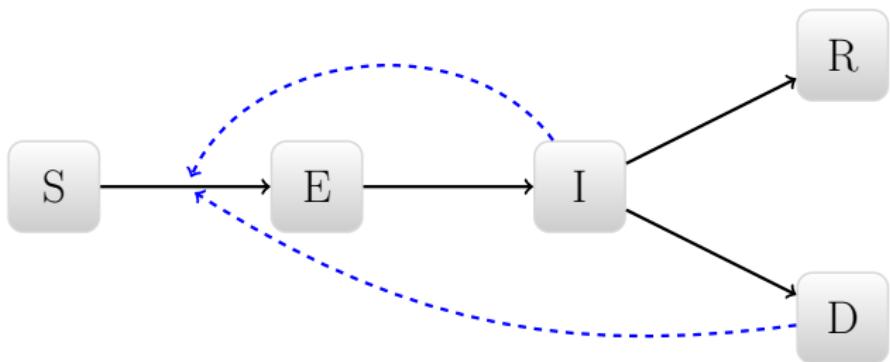


$$\begin{aligned}
 \frac{dS}{dt} &= \mu N - \beta \frac{SI}{N} - \mu S \\
 \frac{dI}{dt} &= \beta \frac{SI}{N} - \gamma I - \mu R \\
 \frac{dR}{dt} &= \gamma I - \mu R
 \end{aligned}$$

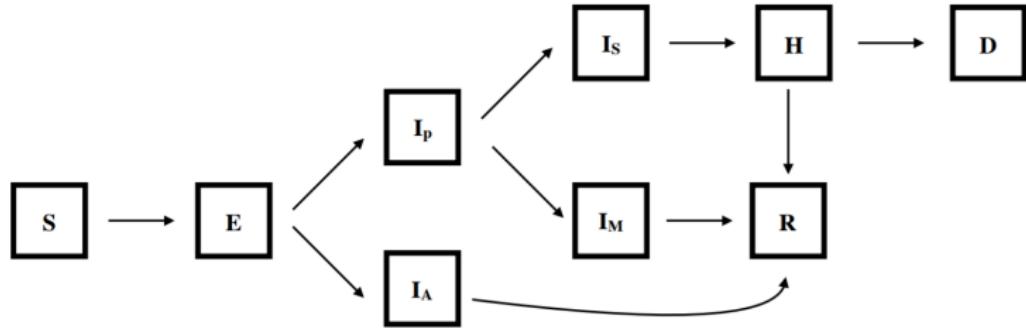
Delayed infectiousness



Ebola 伊波拉



COVID-19



S = susceptible

E = exposed

Ip = pre-symptomatic

Ia = asymptomatic

Is = symptomatic, severe case

Im = symptomatic, minor case

H = hospitalized

R = recovered

D = dead

Childs et al., <http://covid-measures.stanford.edu/>

Lessons

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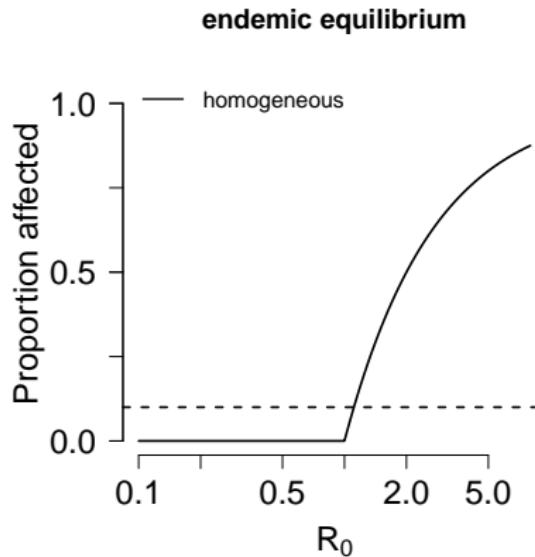
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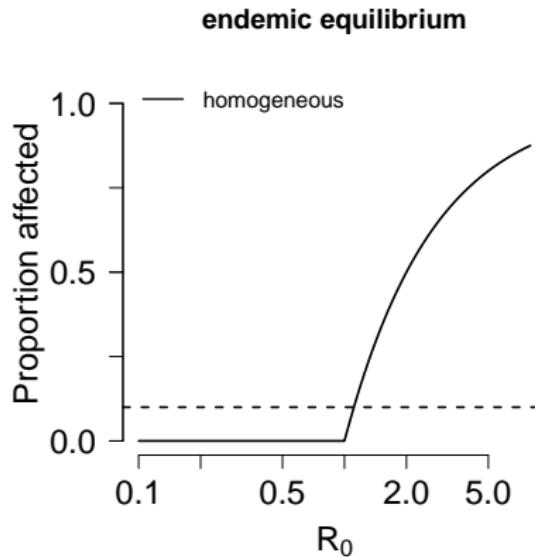
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Yellow fever in Panama



► Taiwan 登革熱

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Exponential growth

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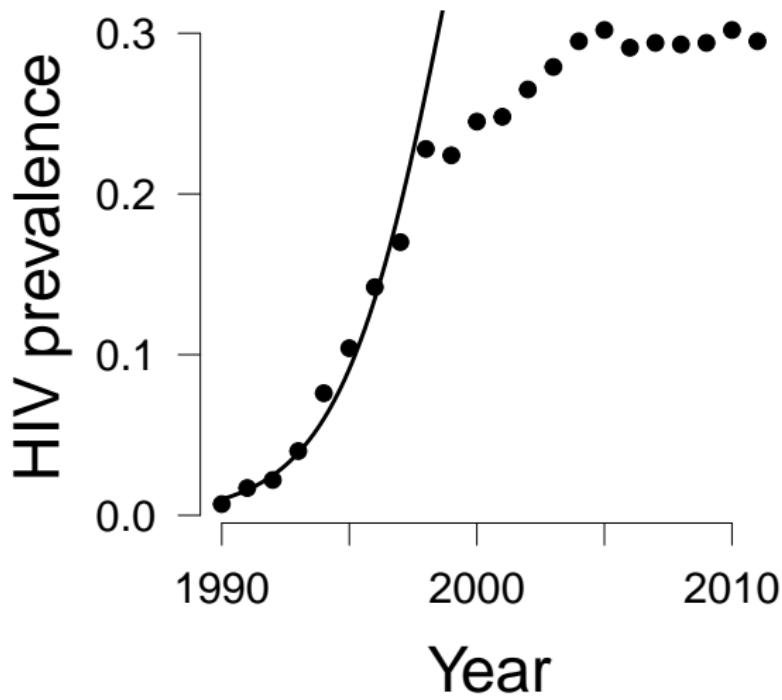
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little *r*

$$R_0 = 5.66$$



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Transmission intervals



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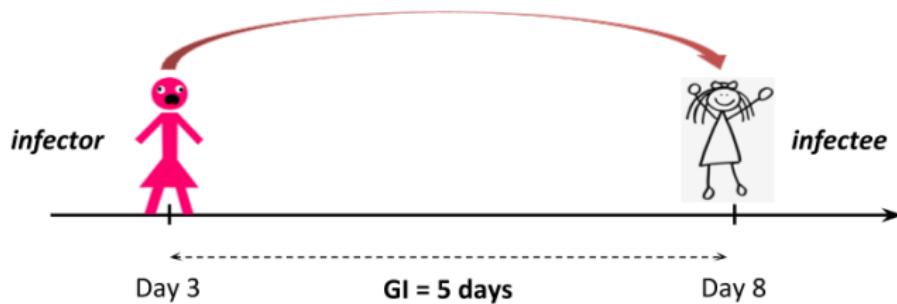
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How long is a disease generation?

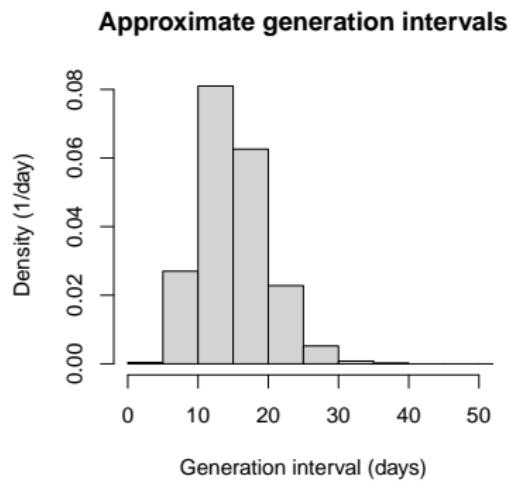
Definition

Generation Interval:

Interval between the time that an individual is infected by an infector and the time this infector was infected

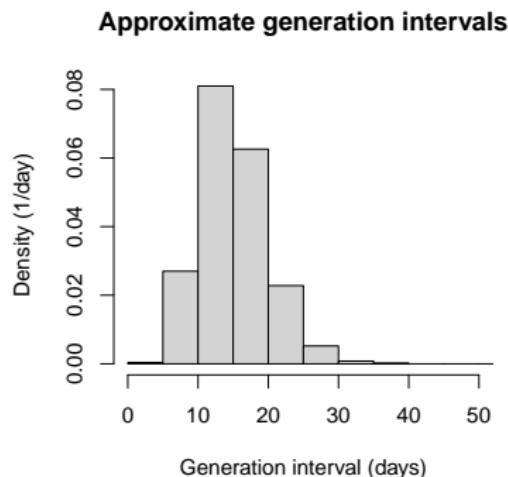


Generation-interval distributions



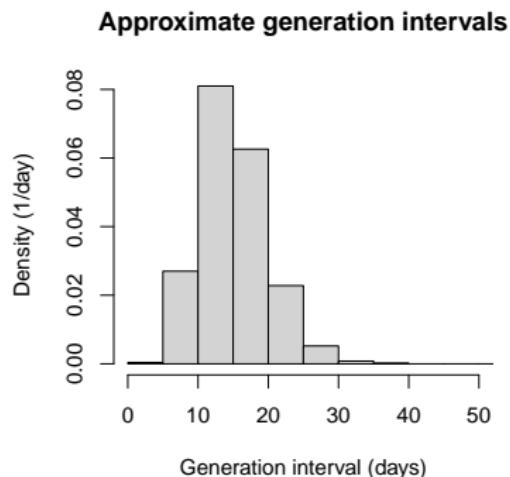
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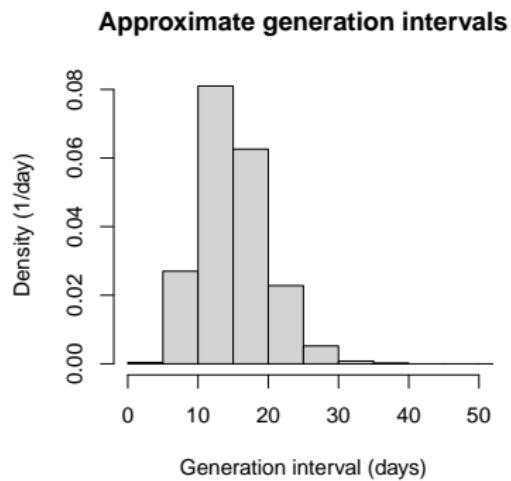
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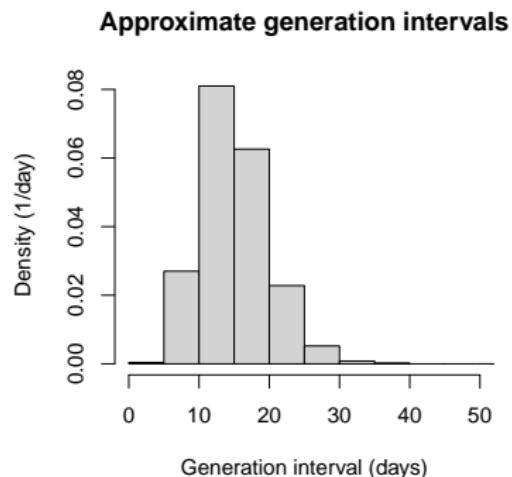
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Example: Post-death transmission and safe burial

- ▶ How much Ebola spread occurs before vs. after death



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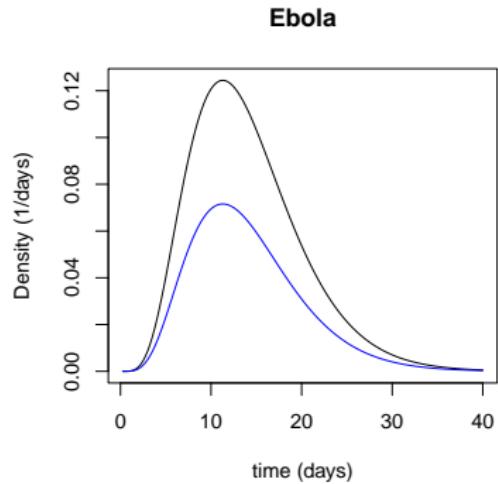
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Mechanistic perspective

知道了是規則

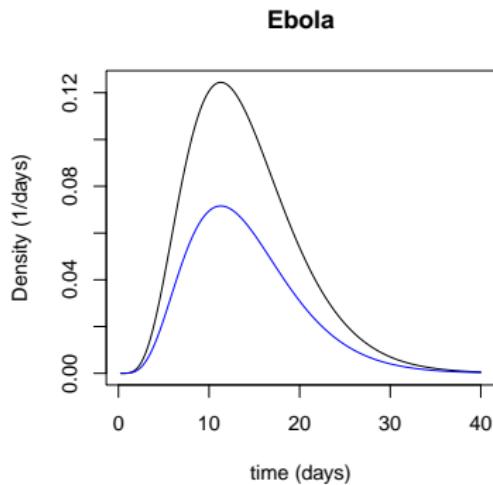
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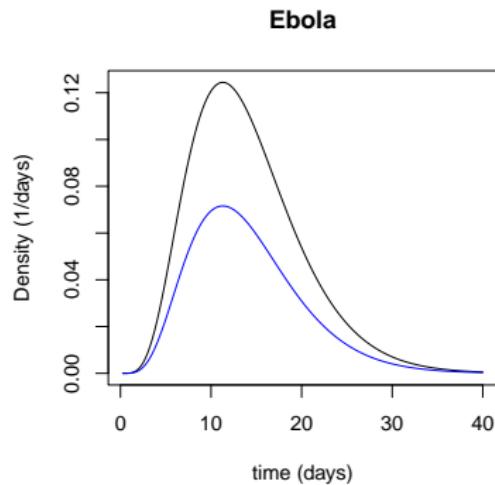
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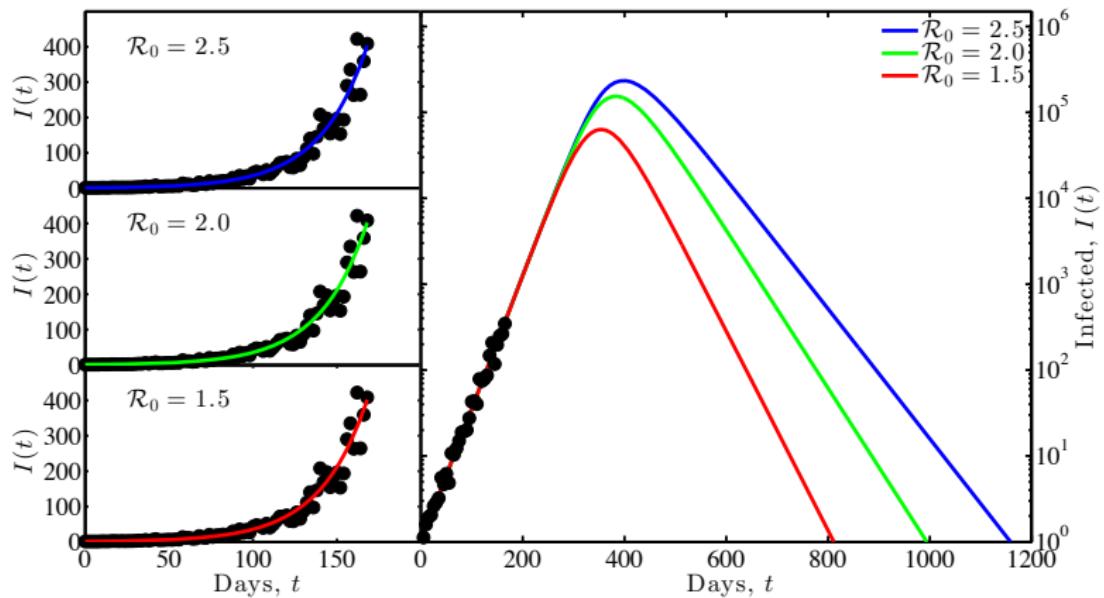
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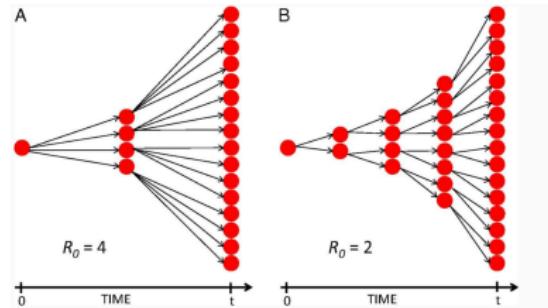
Ebola outbreak



Phenomenological perspective

知道了是結果

- ▶ Population-level speed r is observed

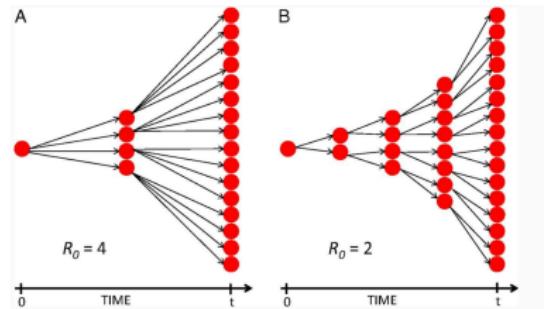


Powers et al.,
[https://www.pnas.org/
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Phenomenological perspective

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- ▶ Population-level speed r is observed
- ▶ Generation-interval distribution $g(\tau)$ can be estimated

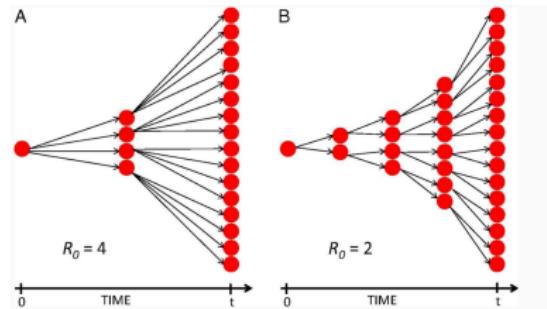


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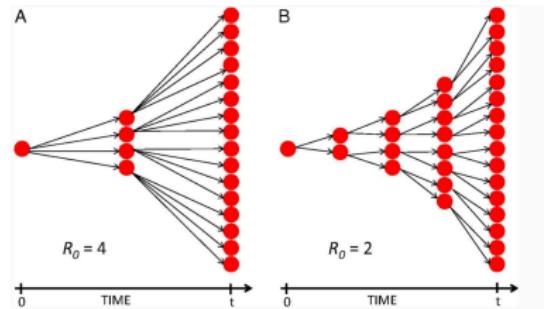


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- ▶ Quicker generations (low \bar{G})
 \implies lower \mathcal{R}



Powers et al.,
[https://www.pnas.org/
content/111/45/15867](https://www.pnas.org/content/111/45/15867)

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Outline

What is dynamical modeling?

Modeling approaches

Transmission intervals

Intrinsic and realized intervals

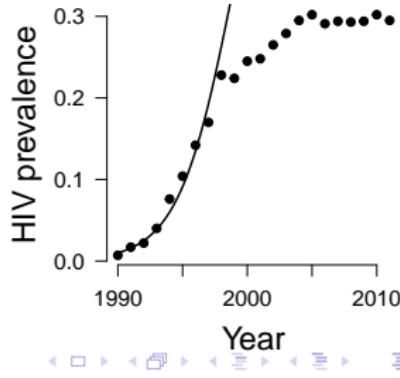
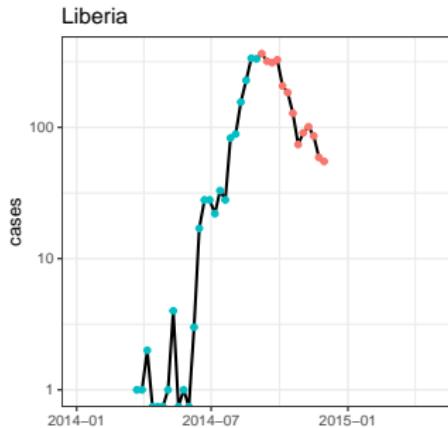
Linking $r\mathcal{R}$

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Summary

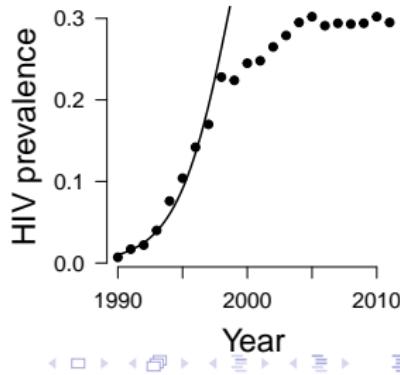
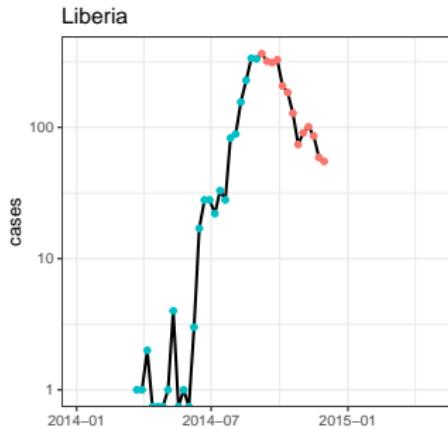
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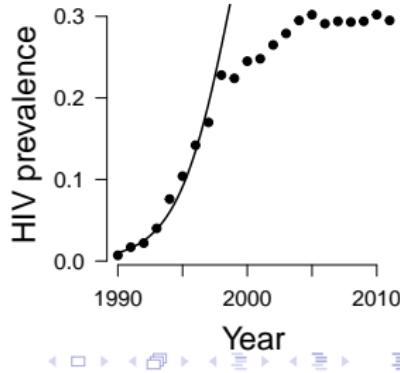
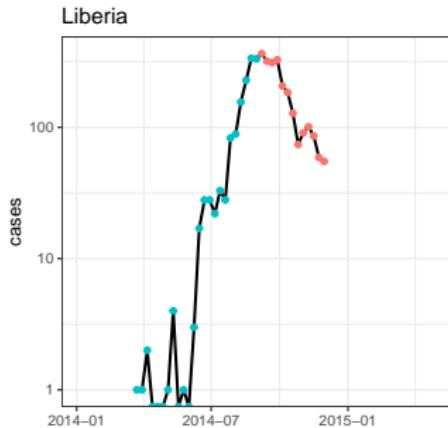
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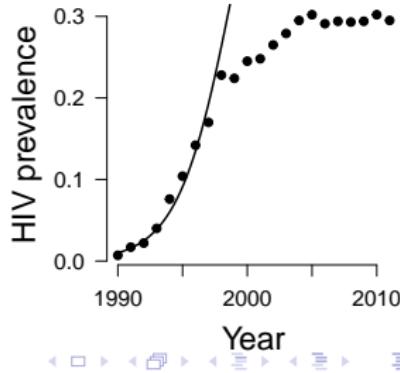
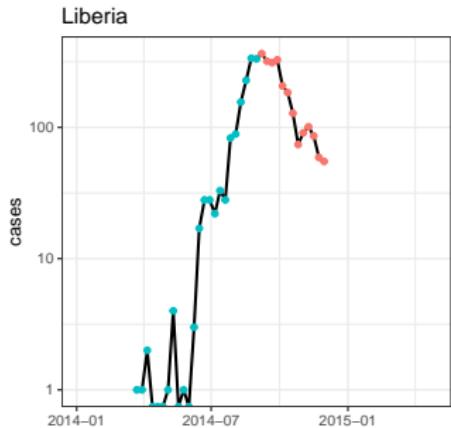
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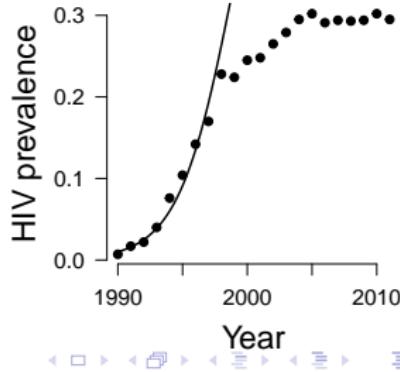
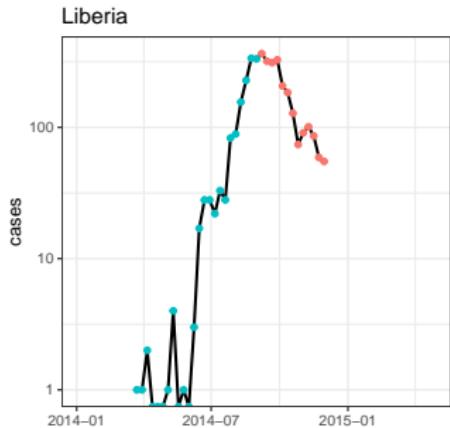
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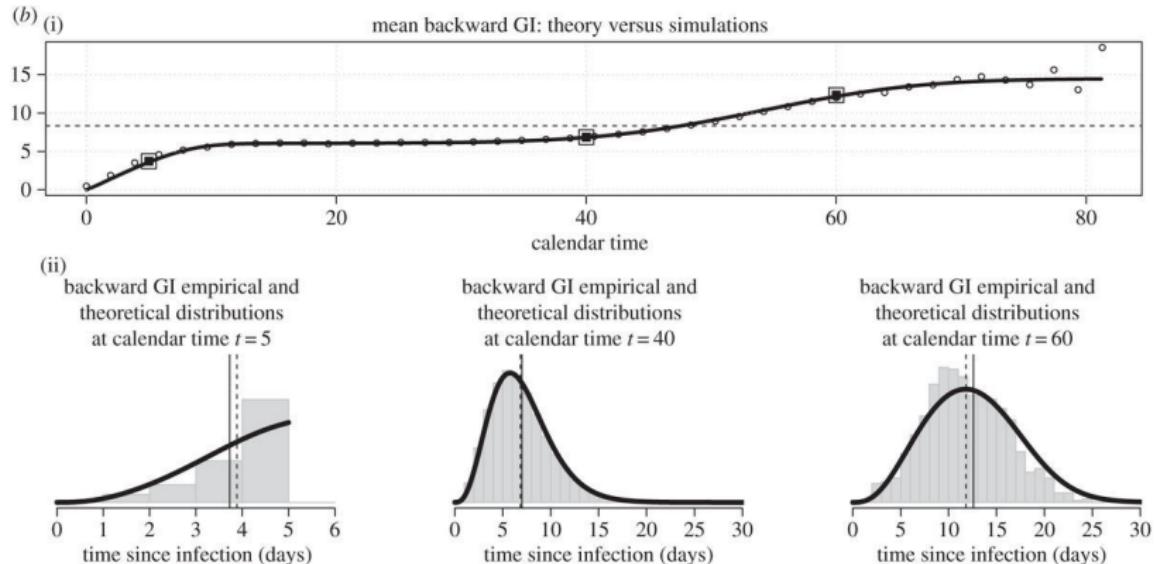
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Correcting backward intervals



Champredon and Dushoff, 2015. DOI:10.1098/rspb.2015.2026

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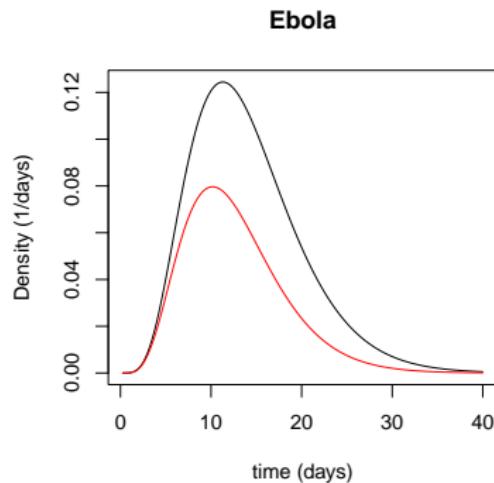
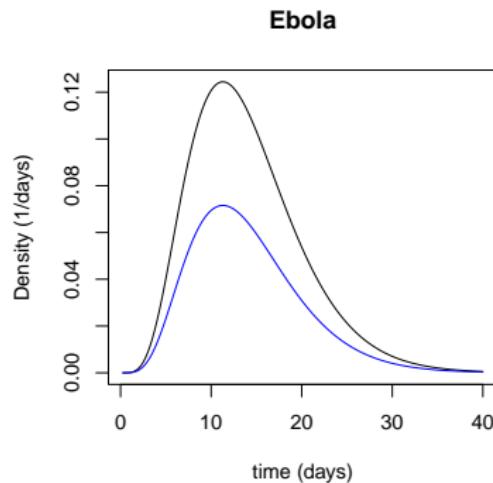
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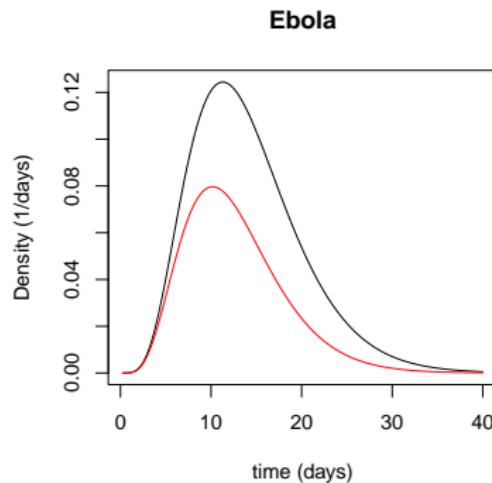
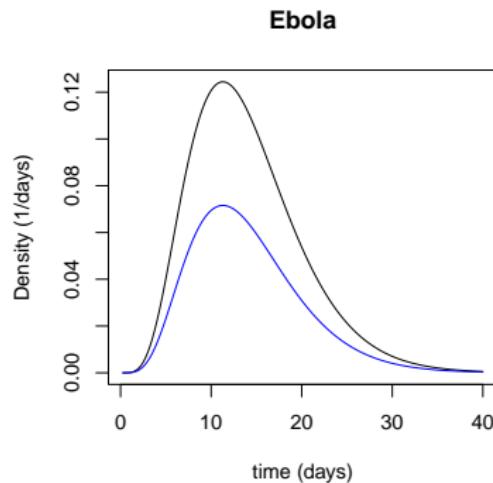
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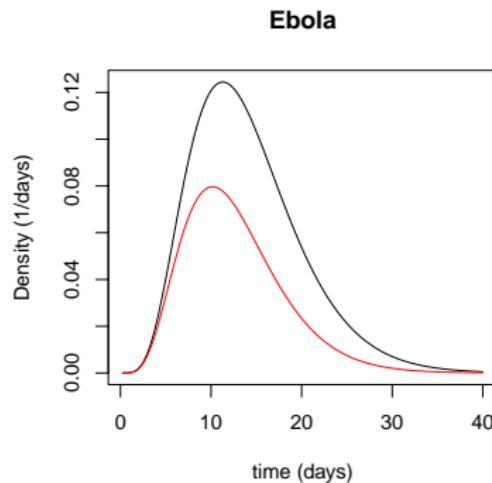
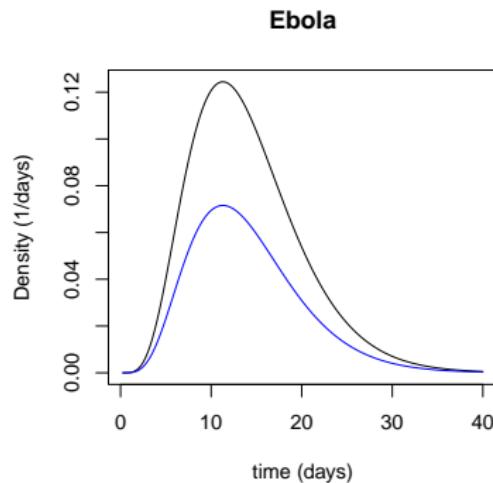
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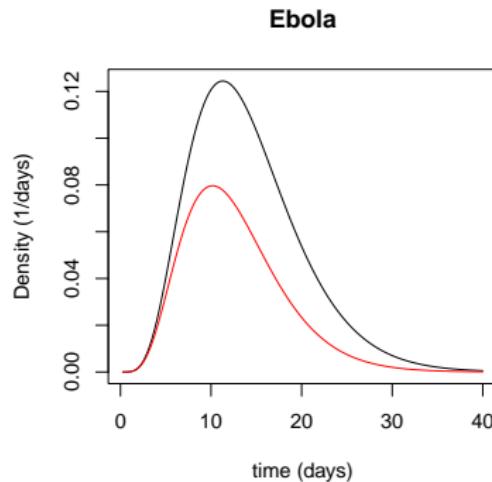
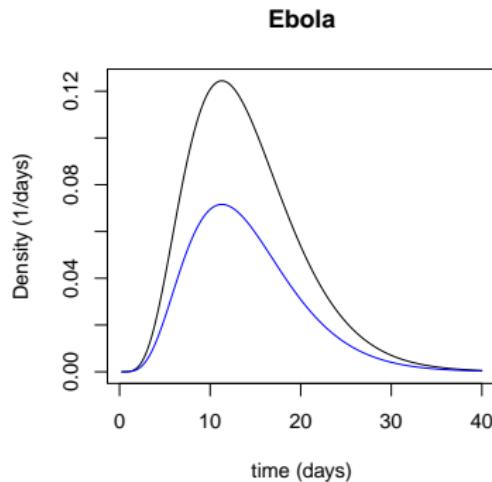
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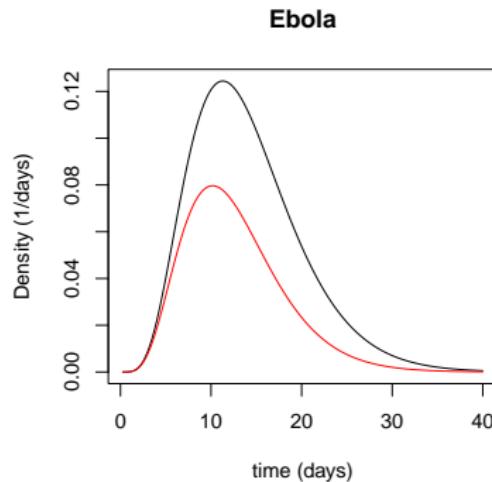
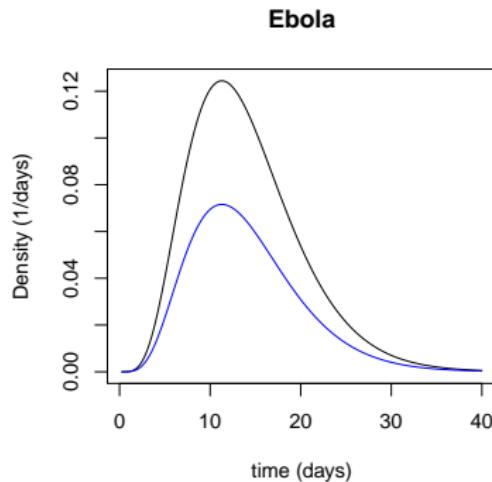
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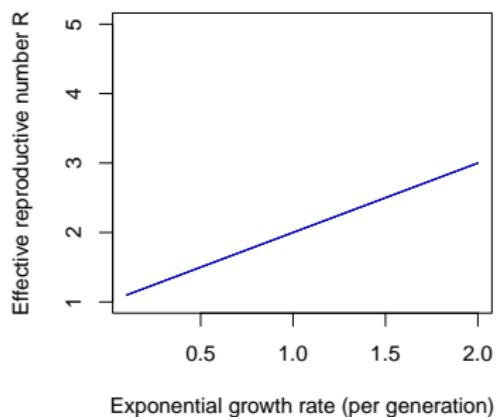
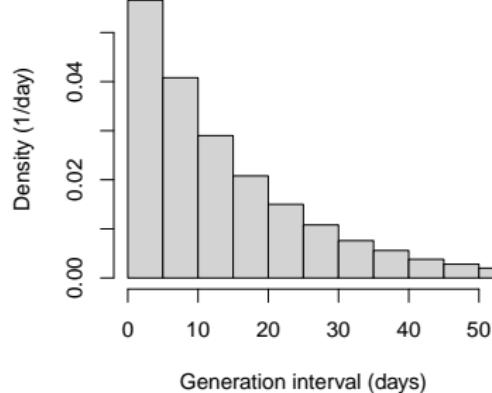
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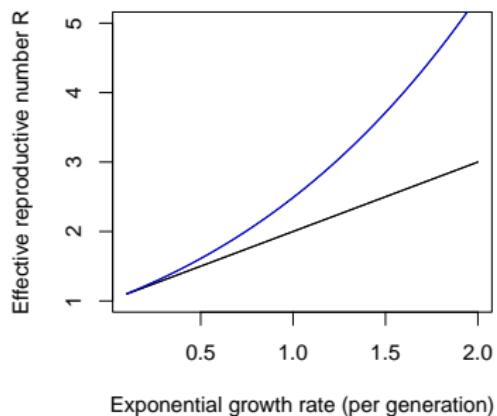
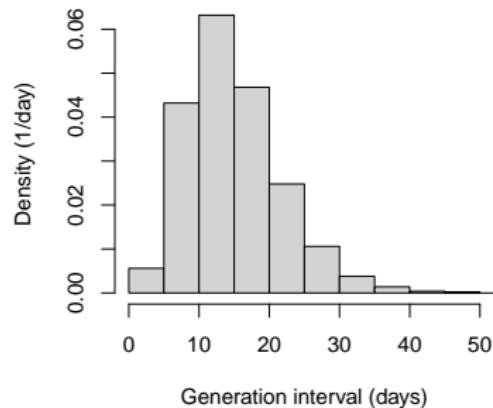
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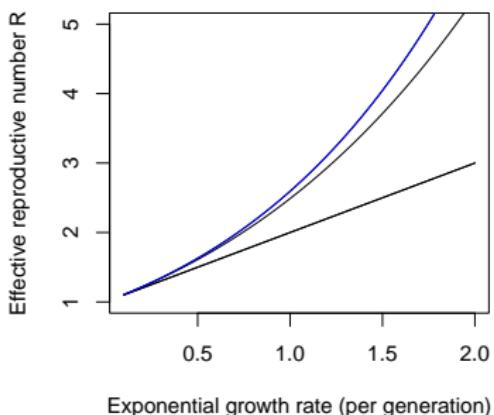
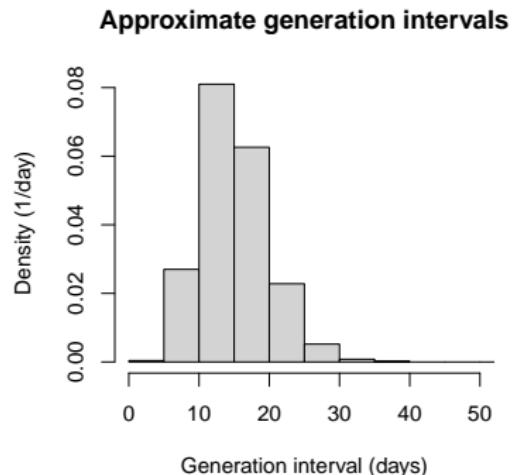


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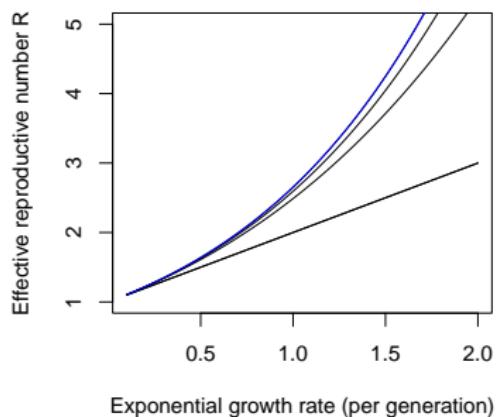
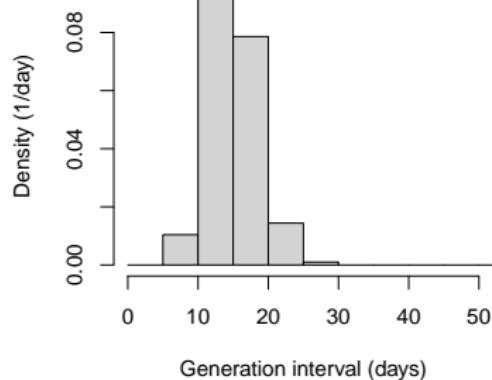


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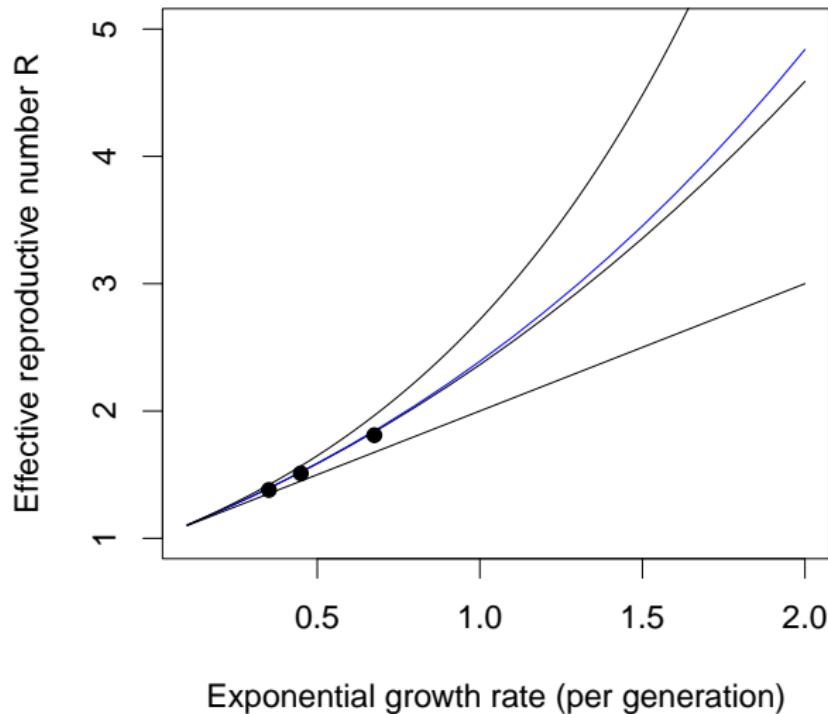


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Approximating the $r\mathcal{R}$ relationship



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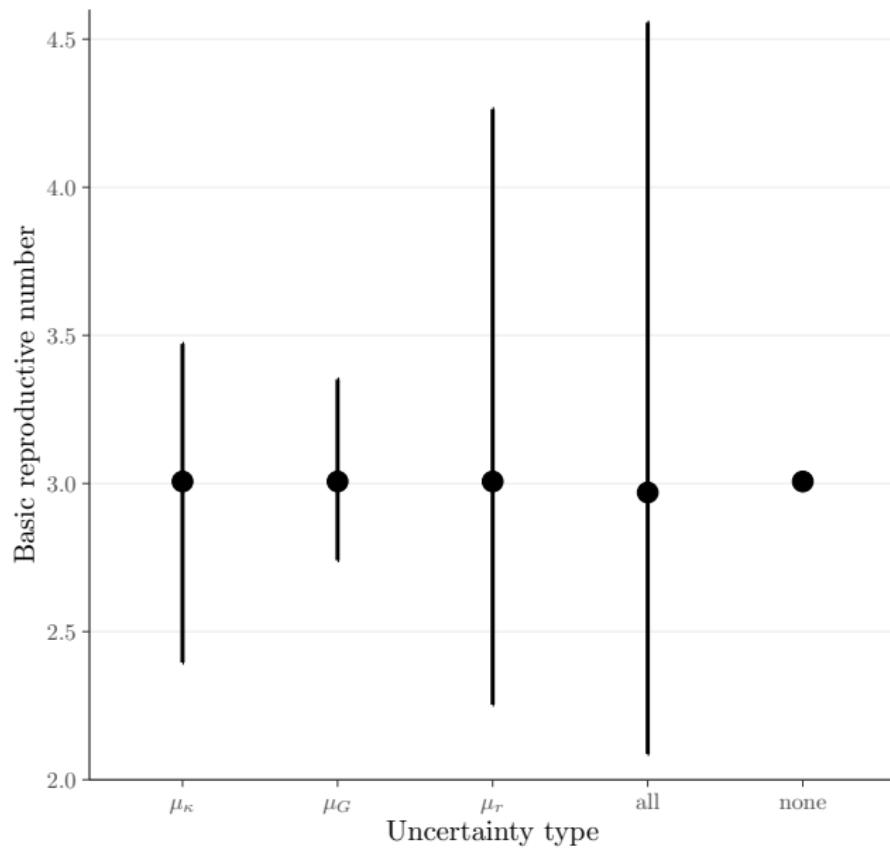
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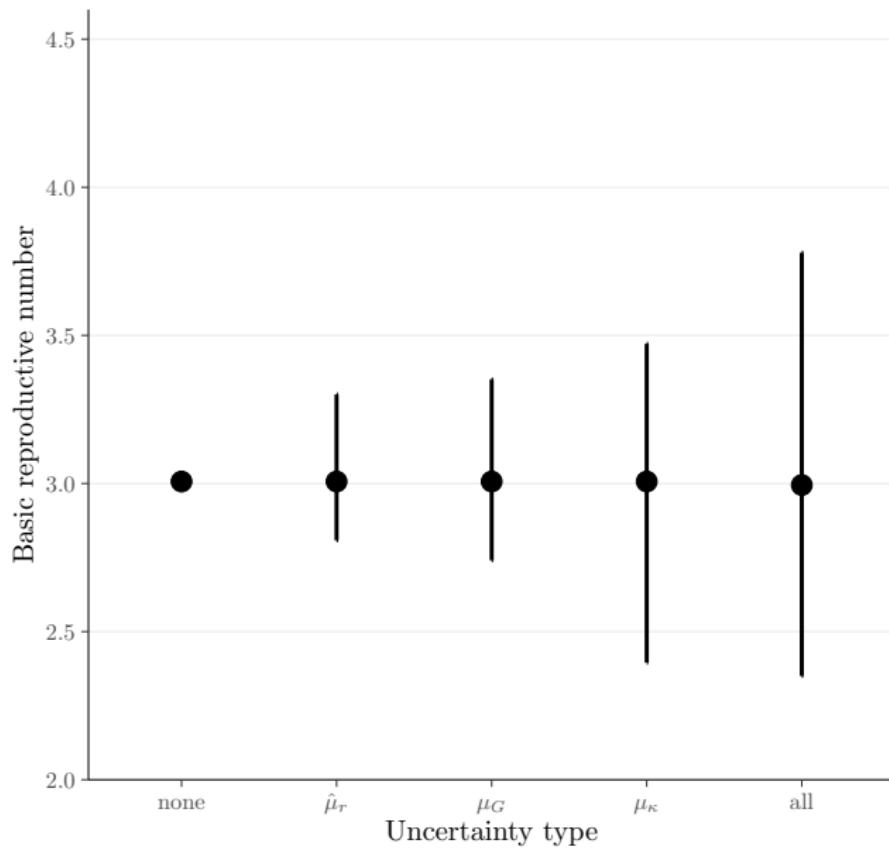
Propagating error

A. Baseline



Propagating error

B. Reduced uncertainty in r



Outline

What is dynamical modeling?

Modeling approaches

Transmission intervals

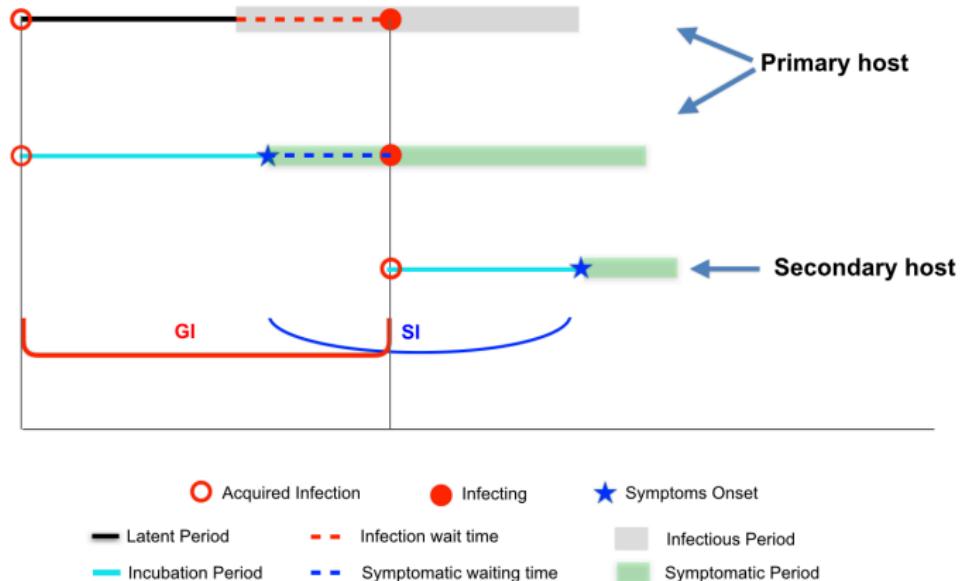
Intrinsic and realized intervals

Linking $r\mathcal{R}$

Serial-interval distributions

Summary

Serial-interval distributions



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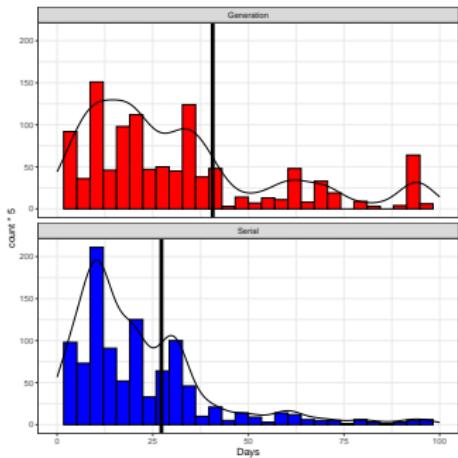
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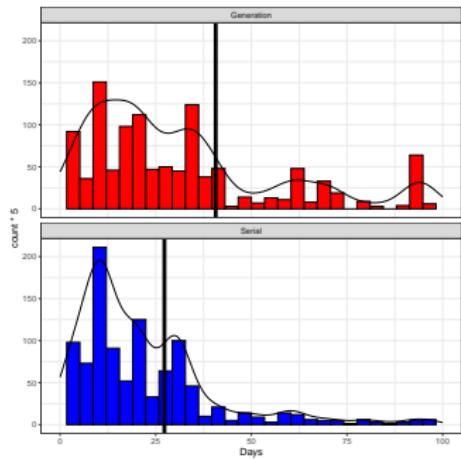
The serial-mean paradox

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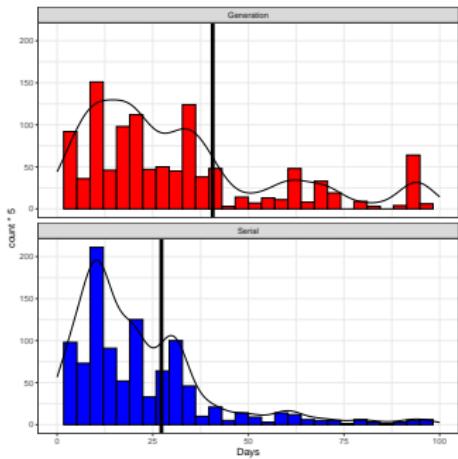
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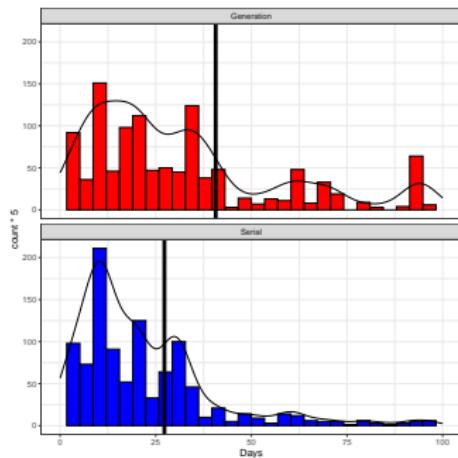
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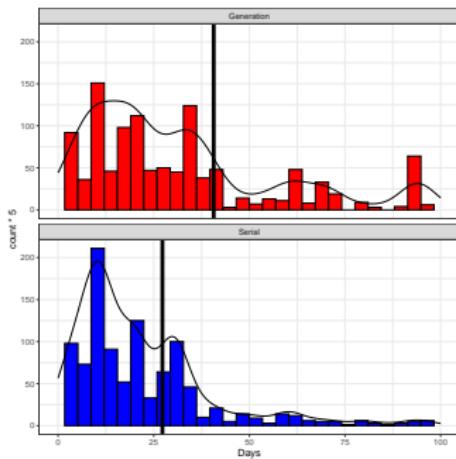
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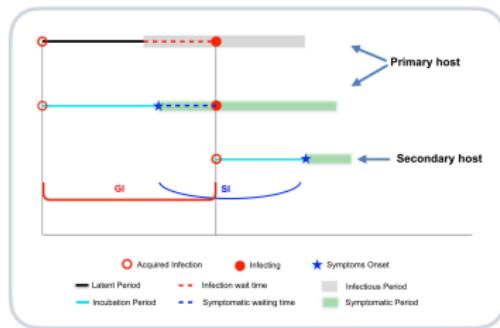
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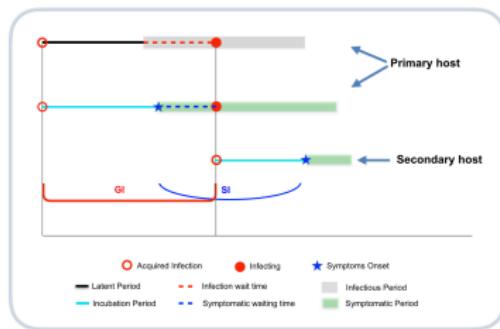
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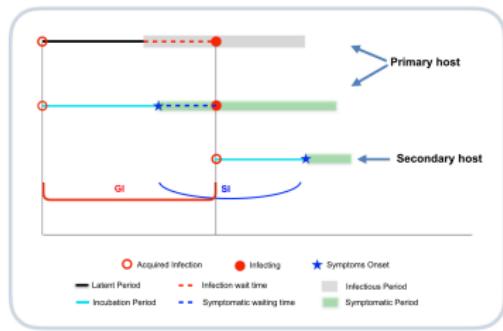
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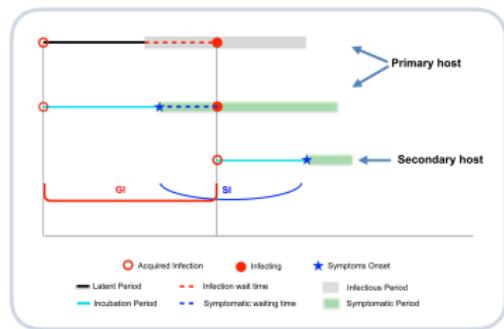
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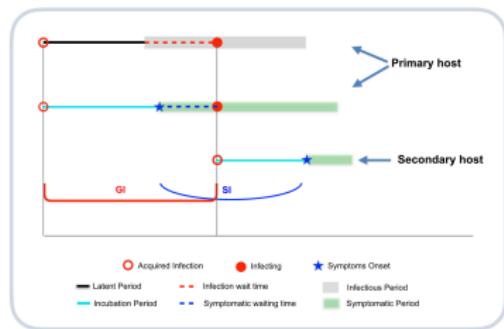
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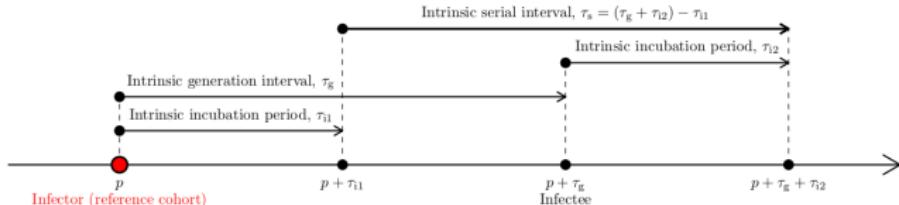
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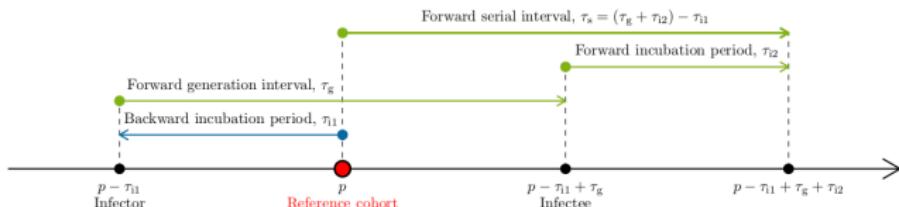
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The forward serial interval

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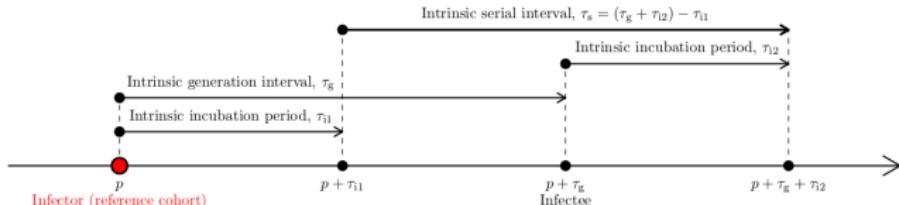
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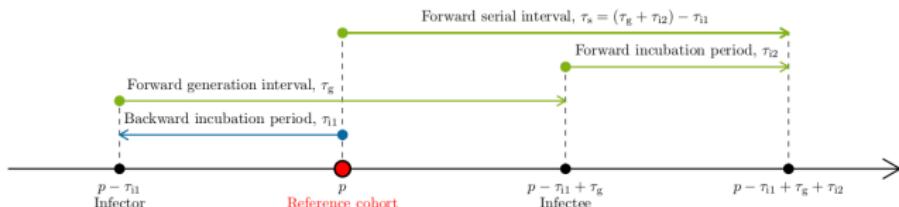
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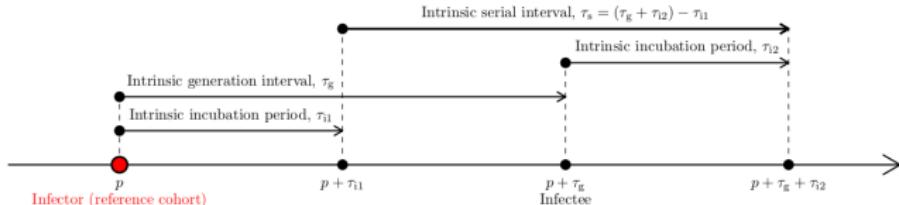
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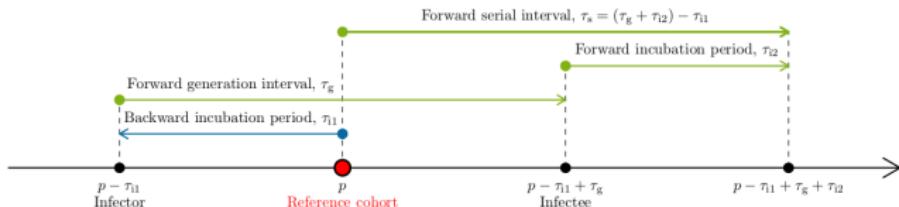
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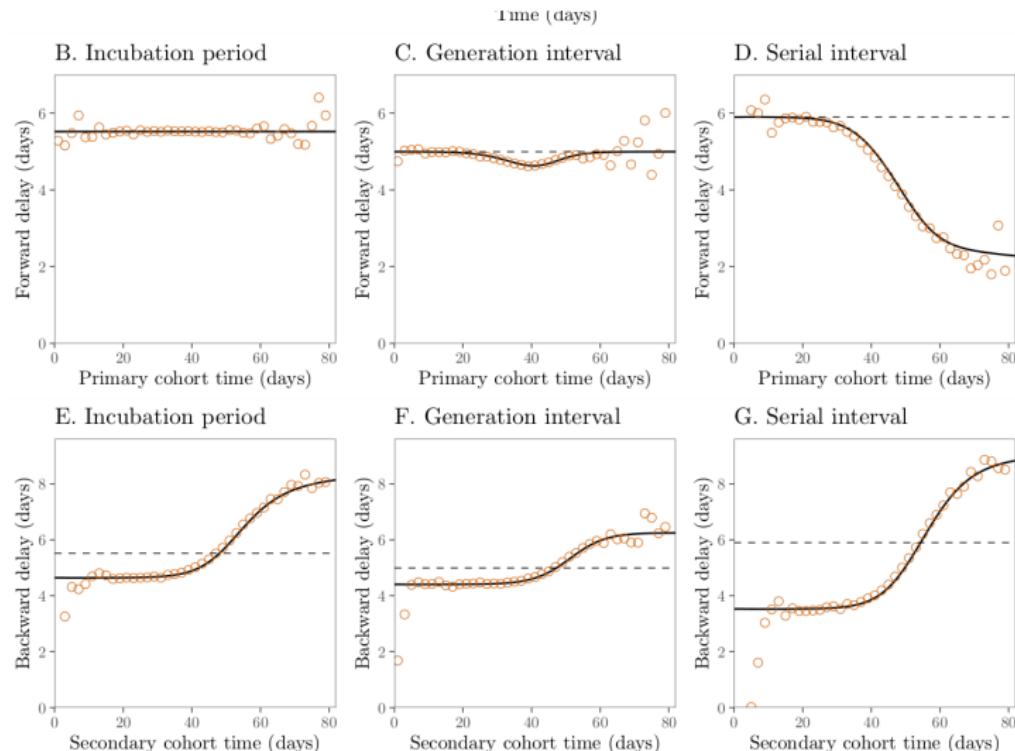


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Thanks

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