Epidemic strength and speed: rethinking metrics for infectious disease spread and control.

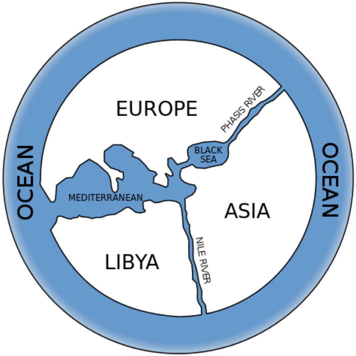
SMB 2017

Jonathan Dushoff
`http://lalashan.mcmaster.ca/DushoffLab`

Outline

Infectious diseases

Models



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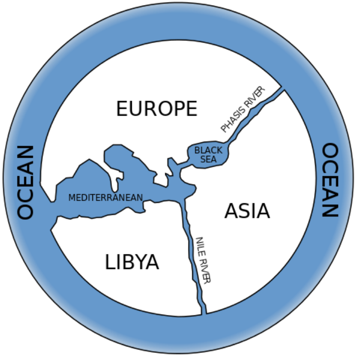

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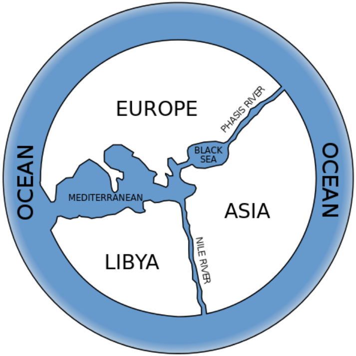

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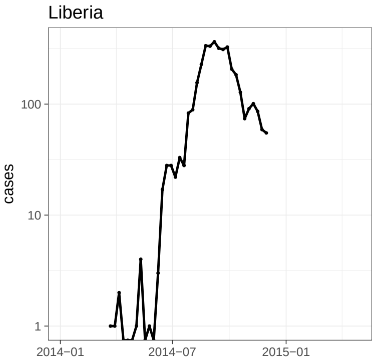
Ebola

Dynamic modeling connects scales



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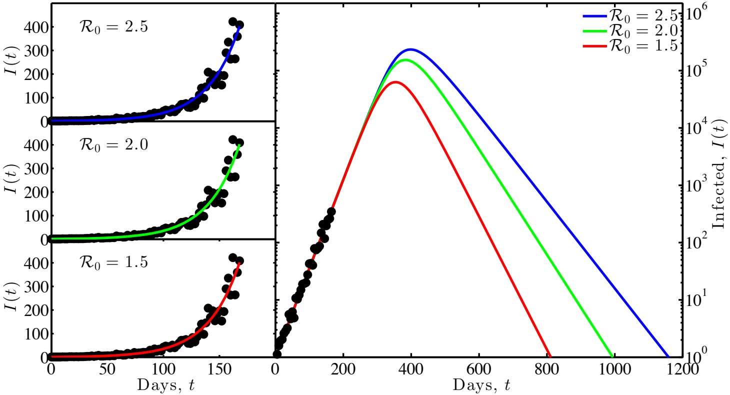

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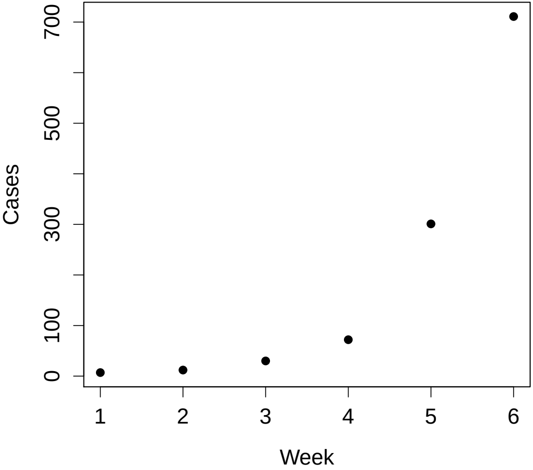

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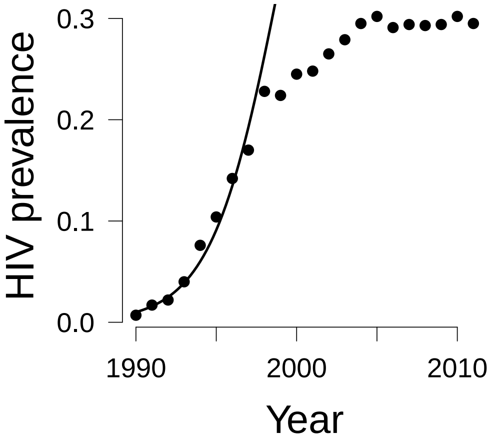

$C \approx 18\,\mathrm{month}$. Horrifyingly fast.

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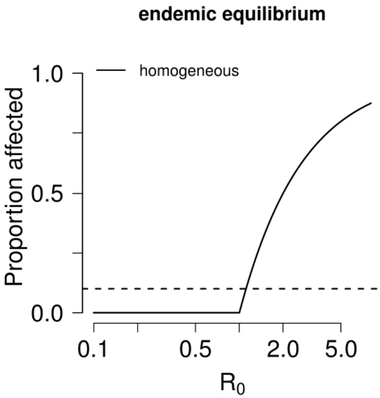

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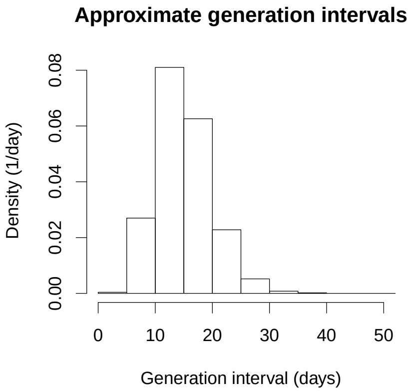

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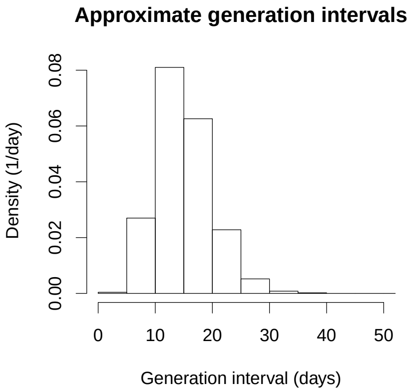

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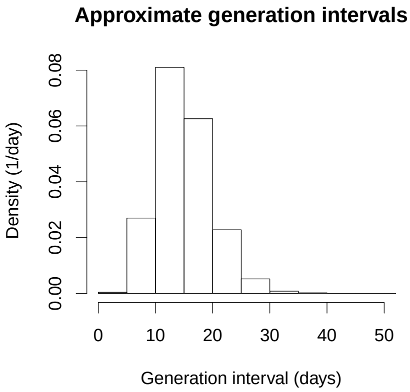

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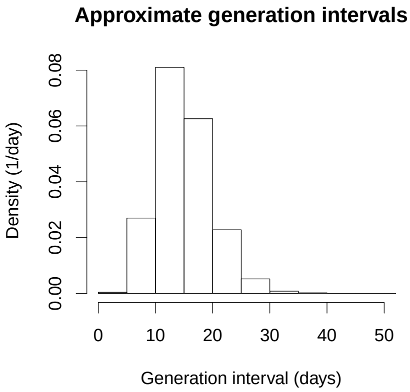

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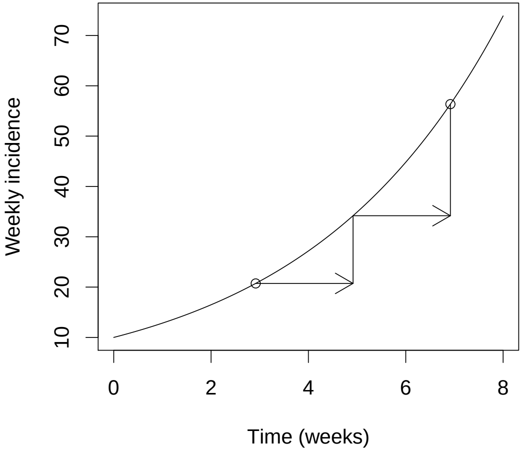

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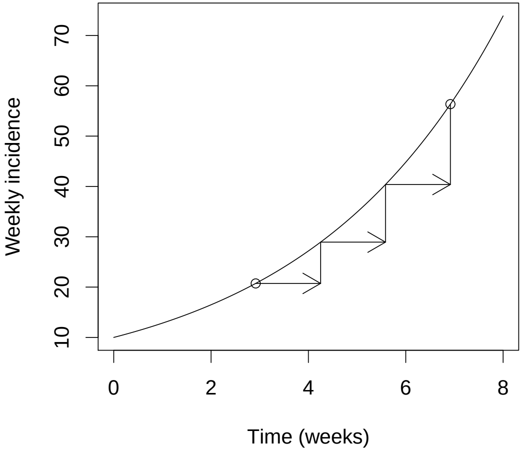

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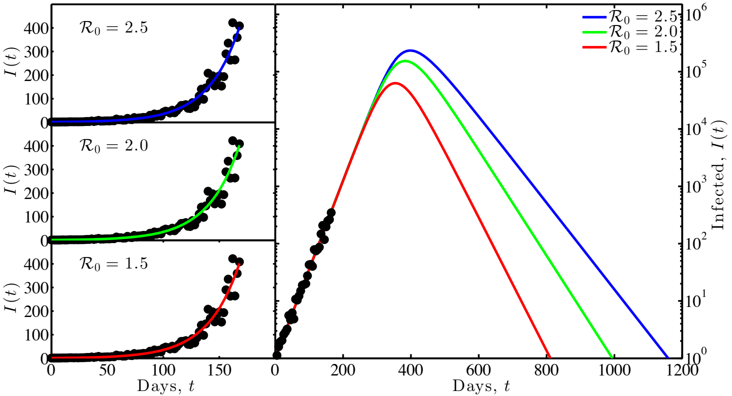

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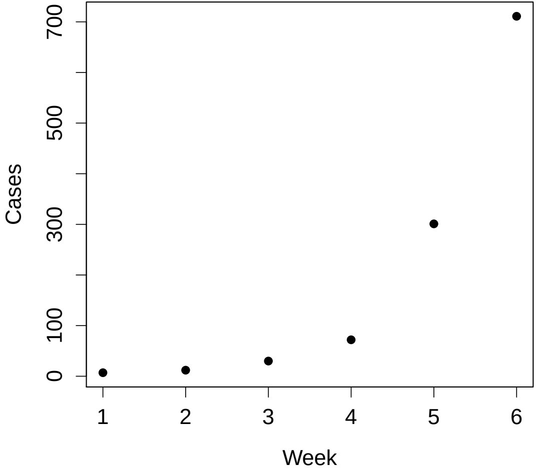

$C \approx 1\,\mathrm{week},\ G \approx 3\,\mathrm{day}$

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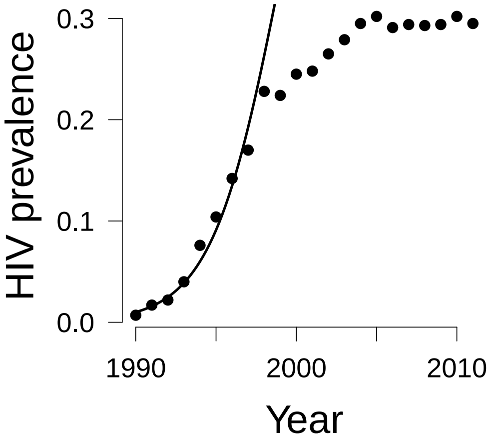

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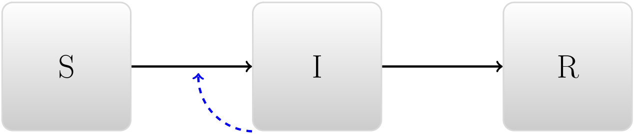

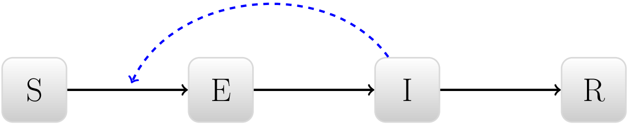

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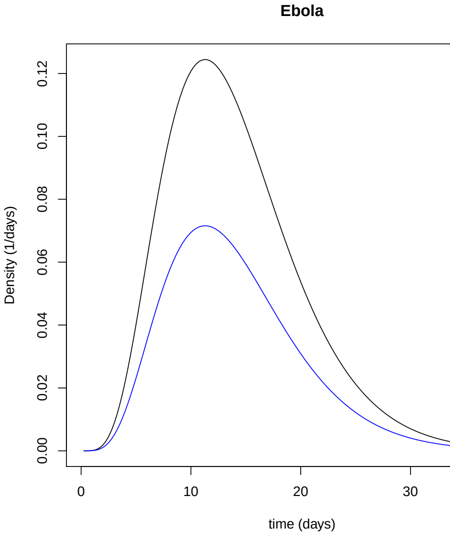

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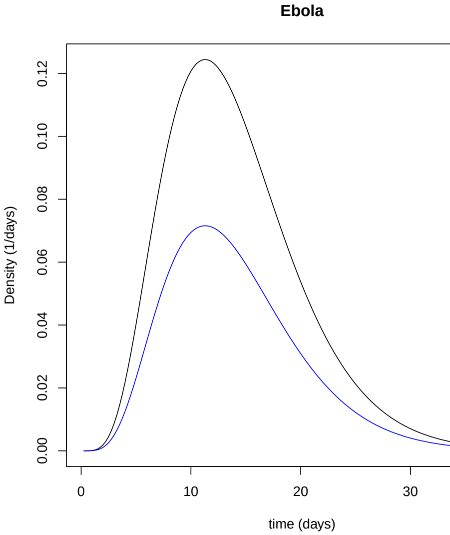

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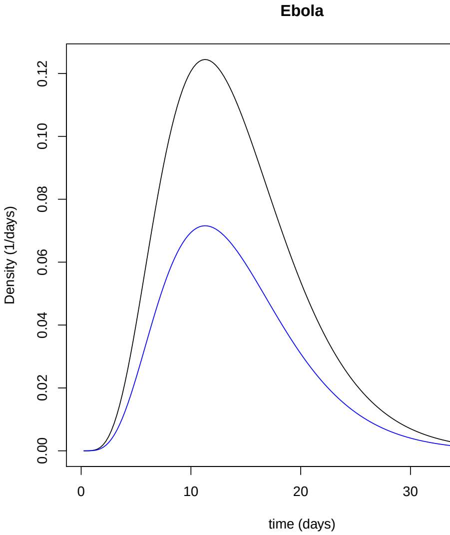

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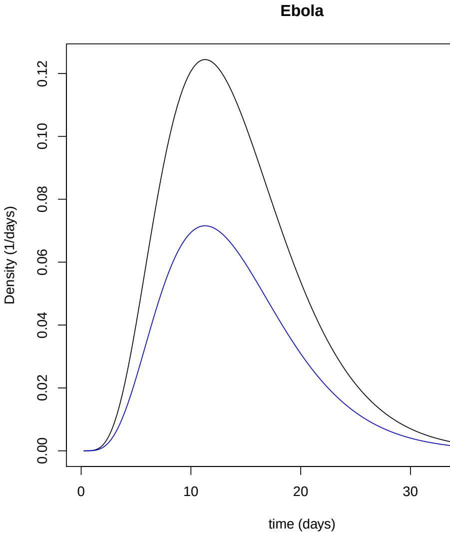

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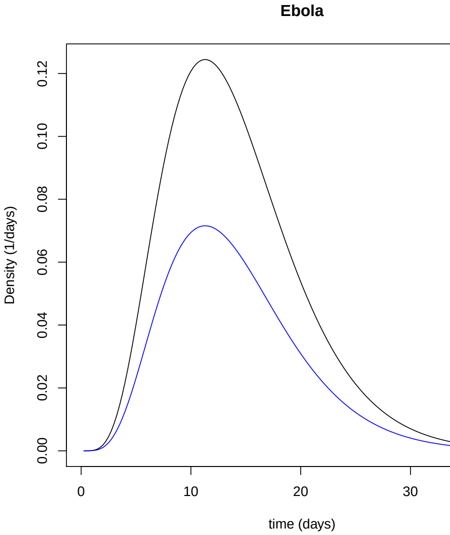

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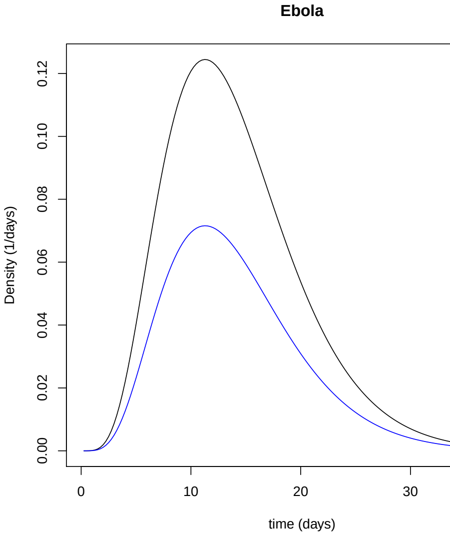

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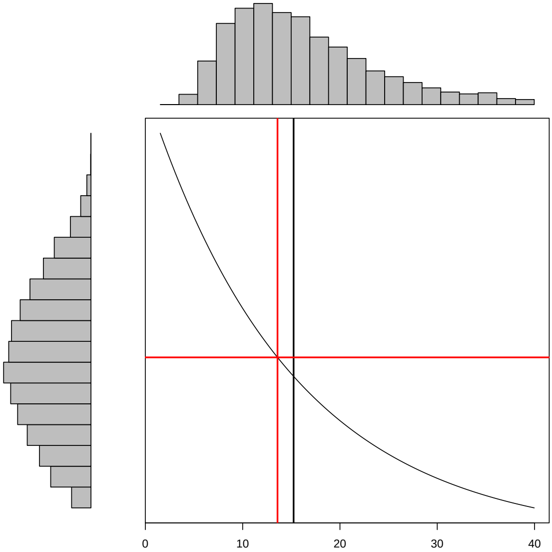
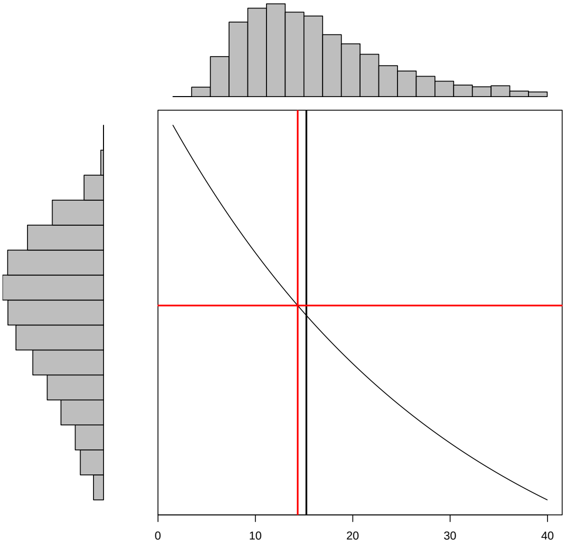
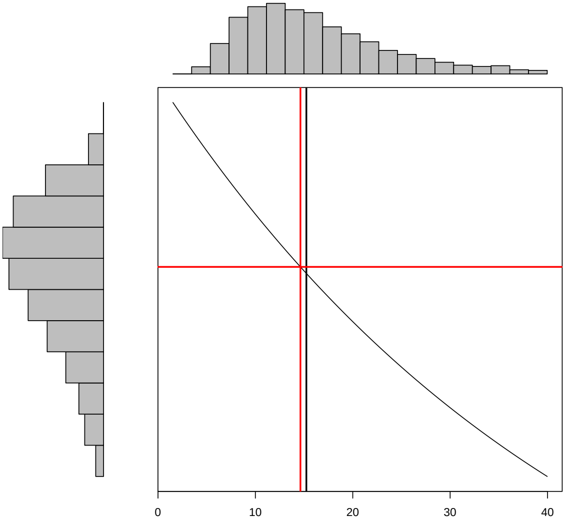


$C = 1/r = 30d$



$C = 1/r = 45d$



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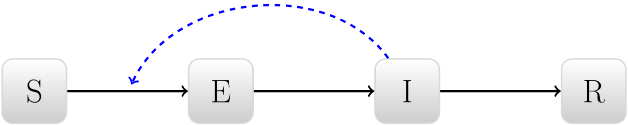
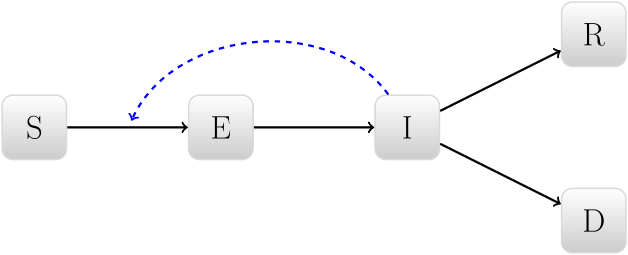
Standard disease model



Disease model including post-death transmission



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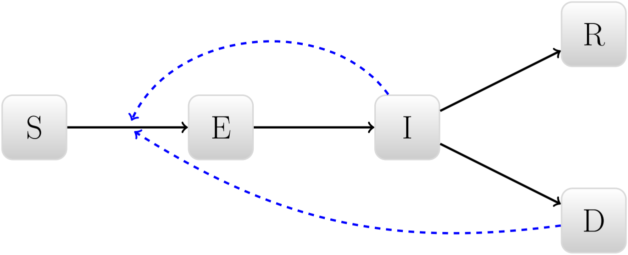
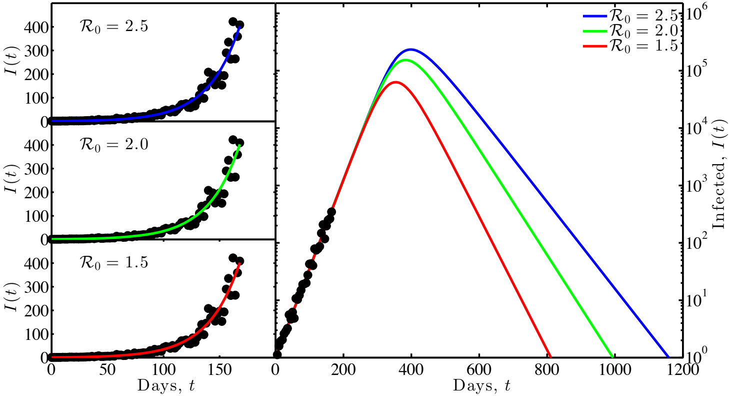


Scenarios



$\mathcal{R}_0 = 2.5$
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$I(t)$
400
300
200
100
0
0
50
100
150
Days, t
0
200
400
600
800
1000
1200
Days, t
10^0
10^1
10^2
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Infected, $I(t)$

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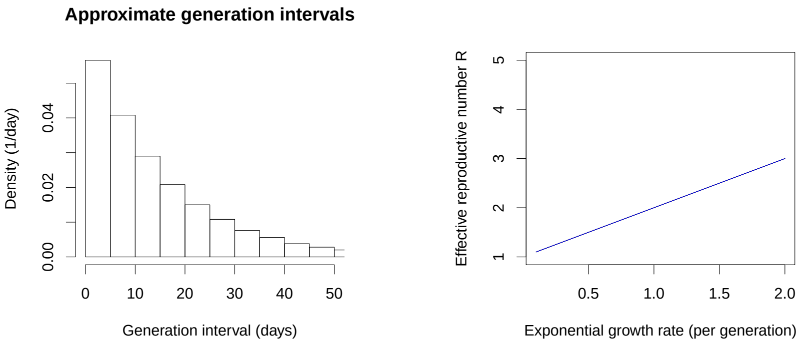

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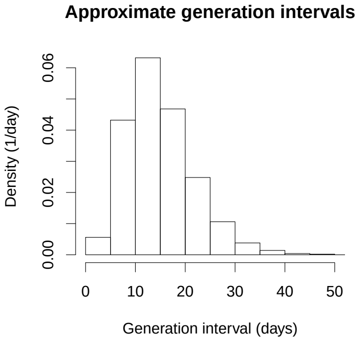

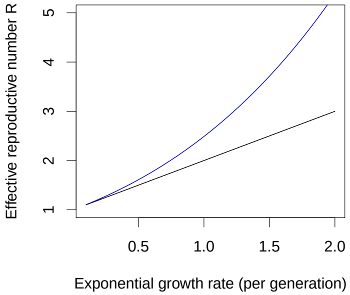

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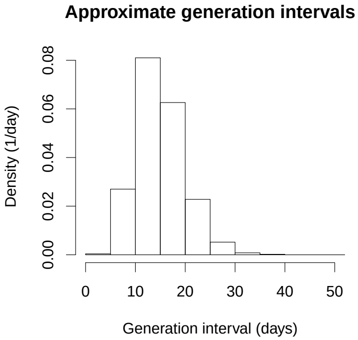

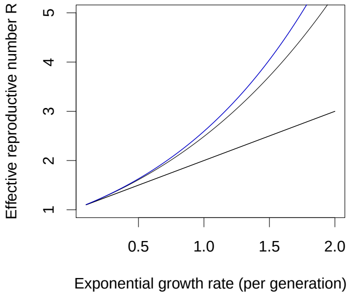

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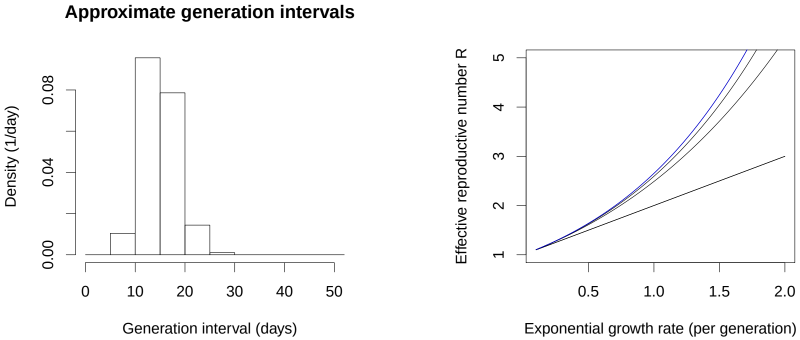

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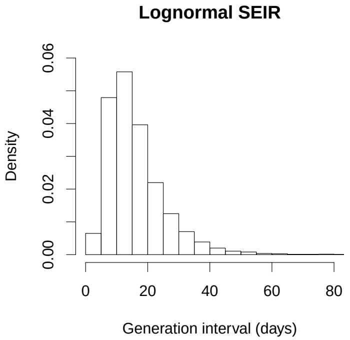

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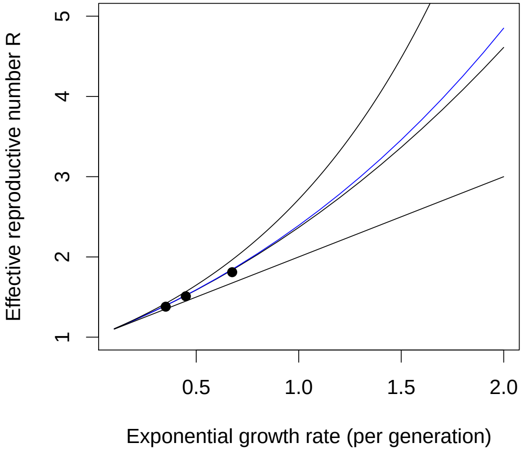

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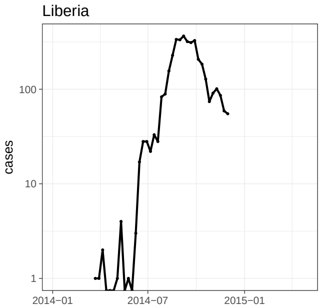

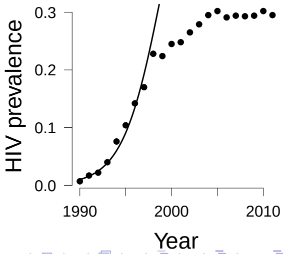

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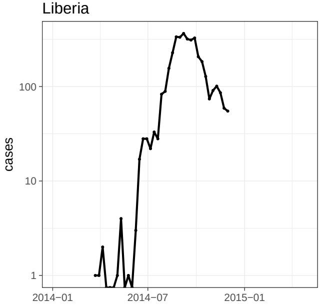

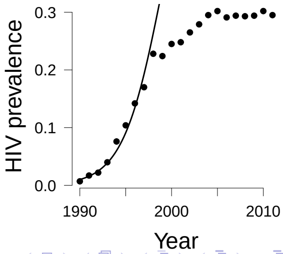

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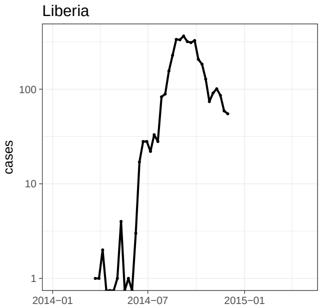

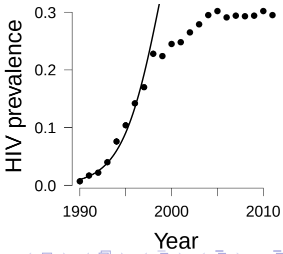

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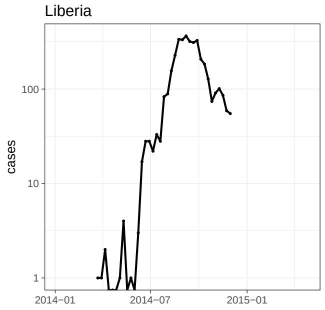

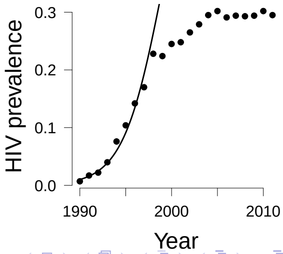

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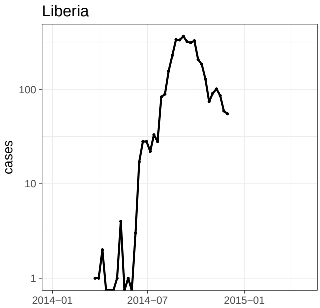

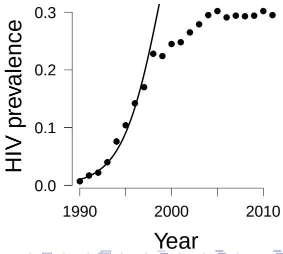

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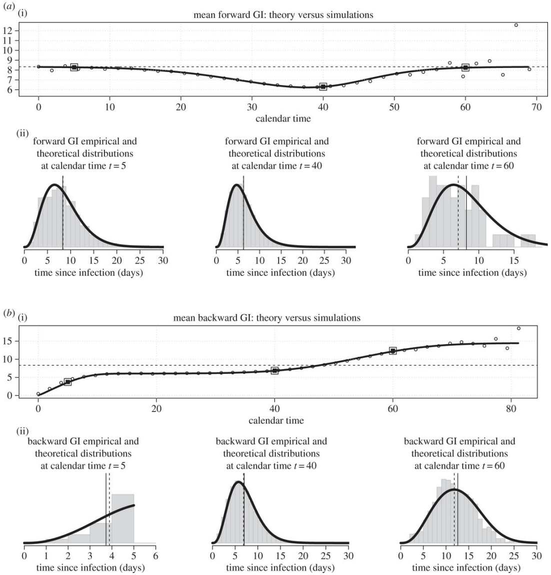
Theory and simulation



Champredon and Dushoff, 2015. DOI:10.1098/rspb.2015.2026

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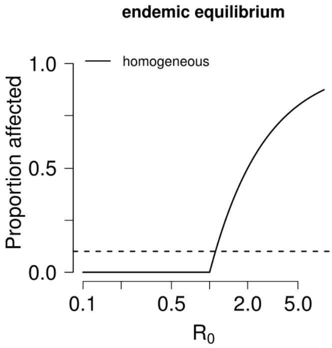

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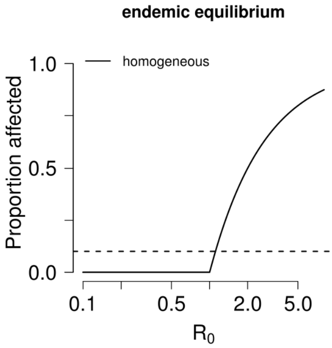

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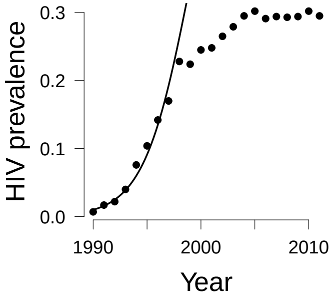

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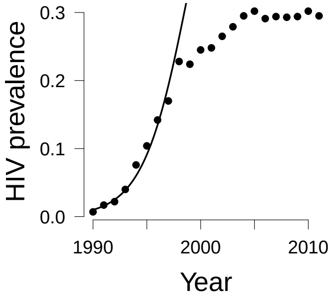

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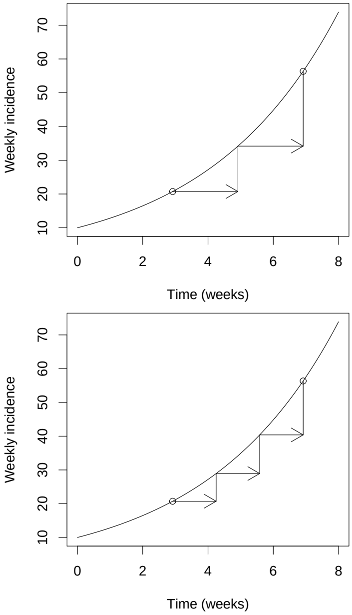

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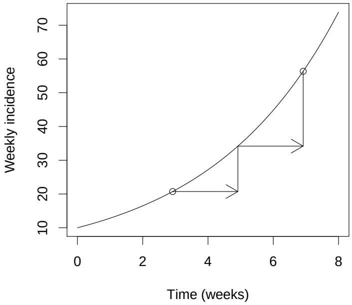

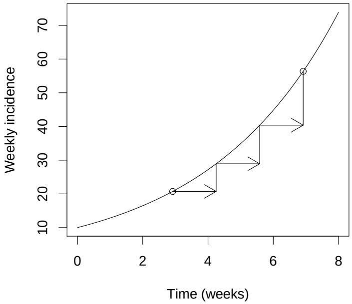

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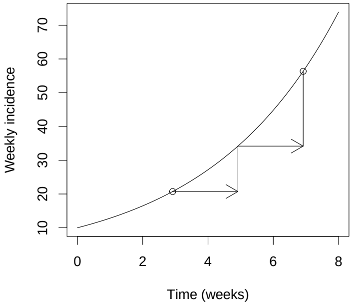

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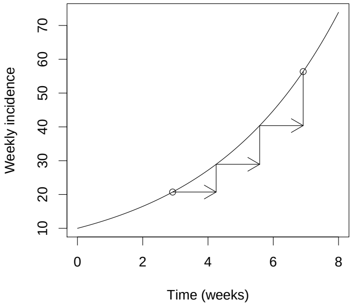

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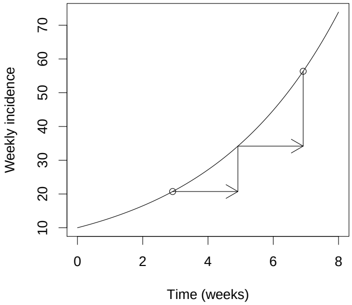

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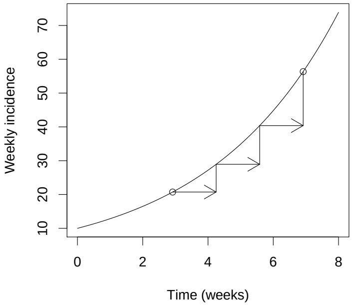

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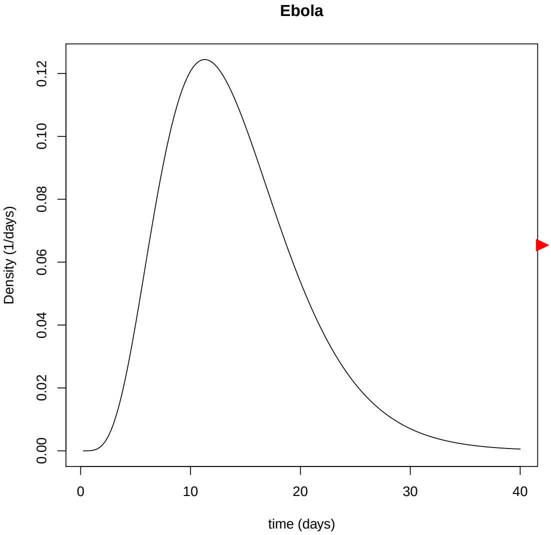

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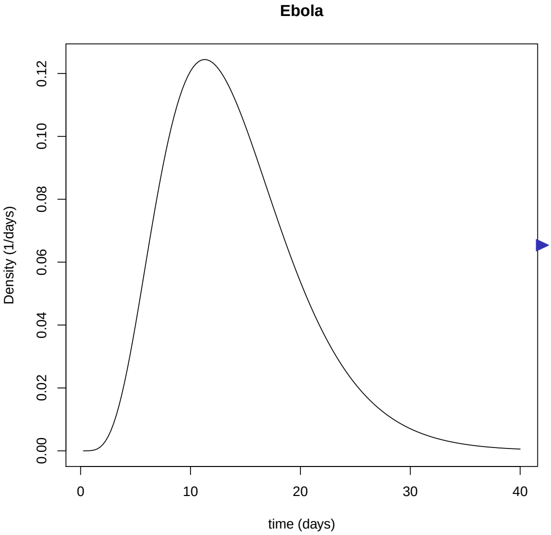

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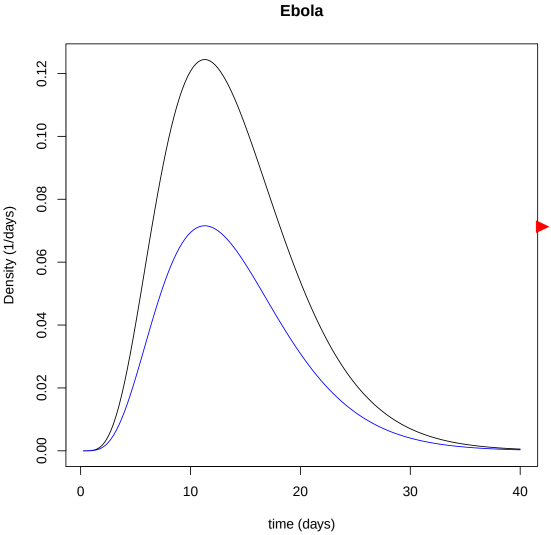

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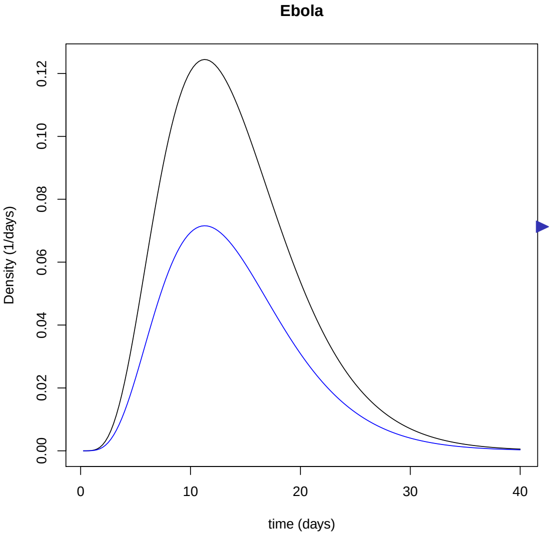

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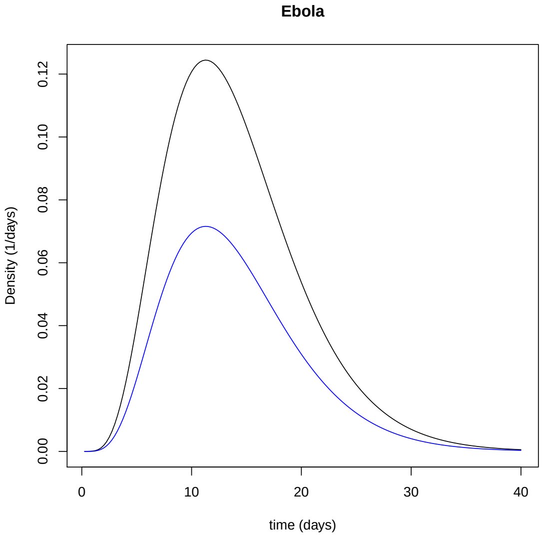

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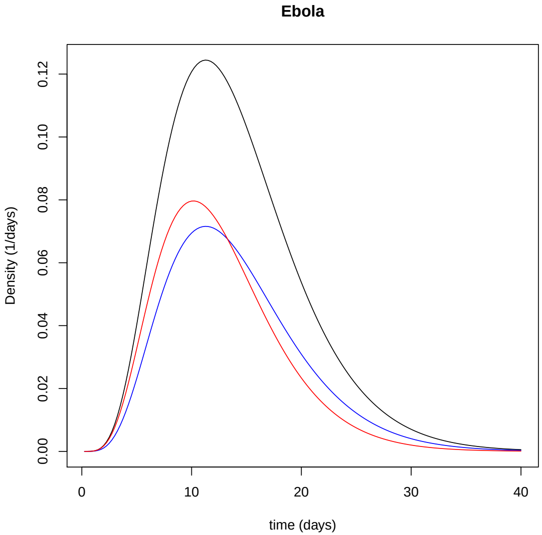

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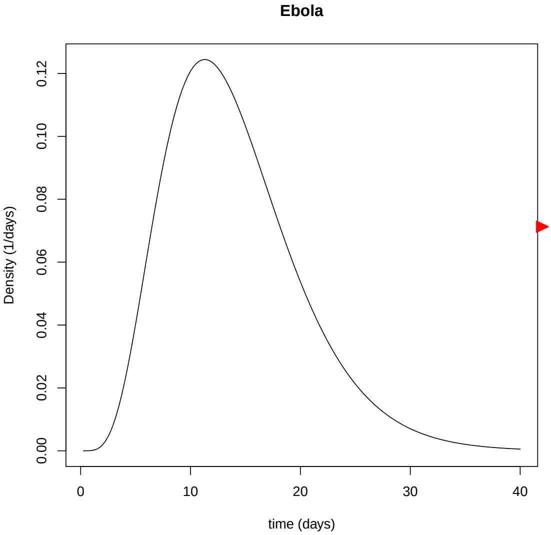

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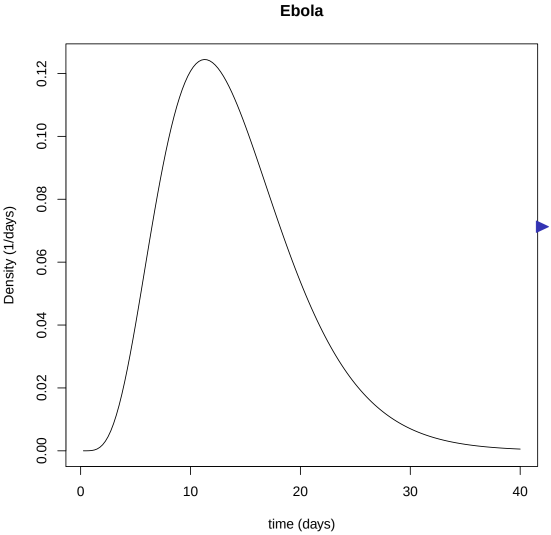

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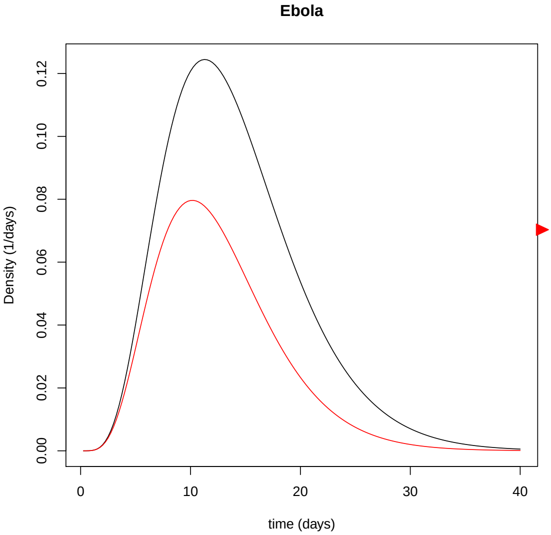

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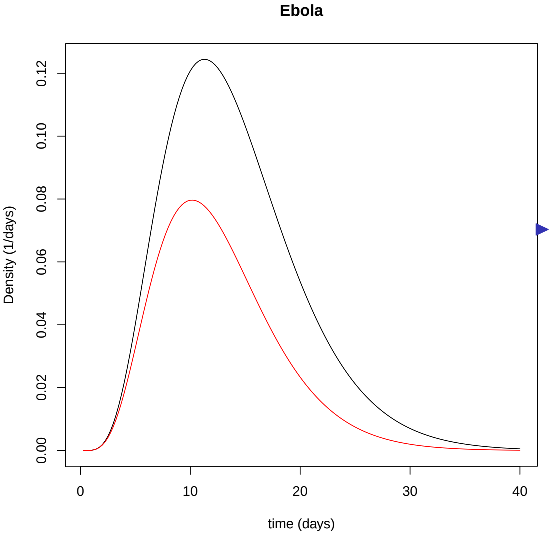

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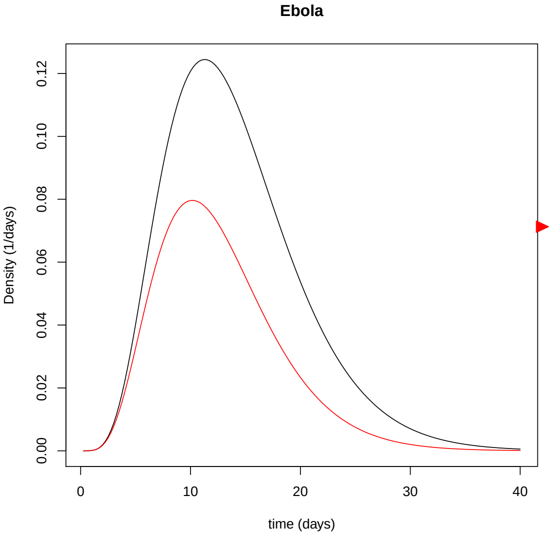

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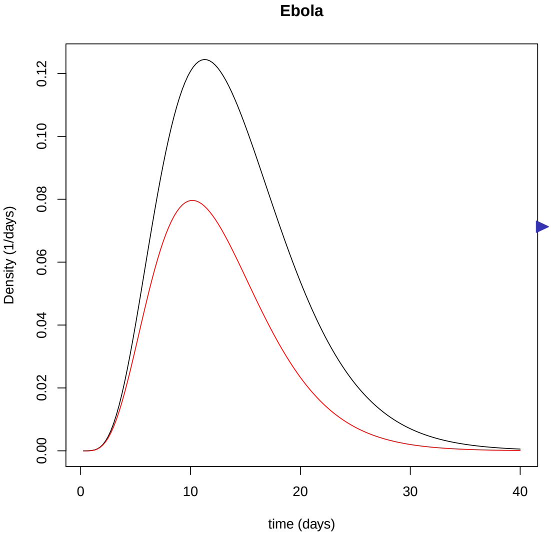

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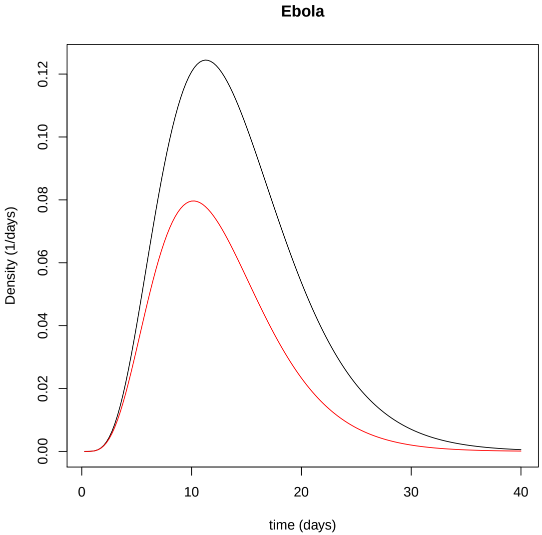

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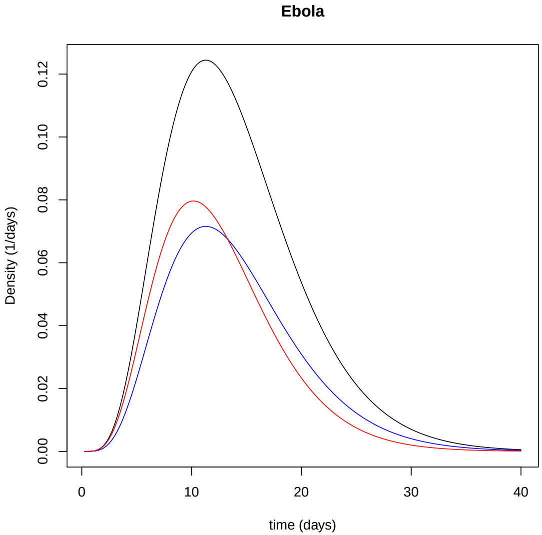

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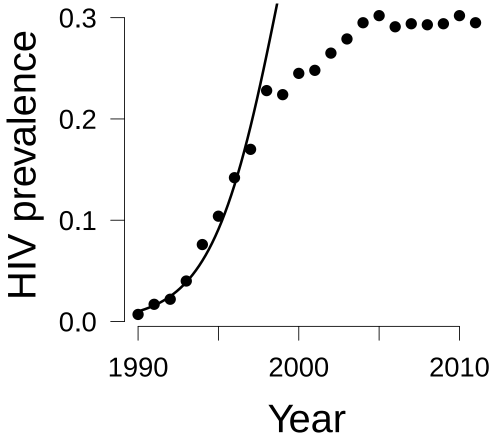
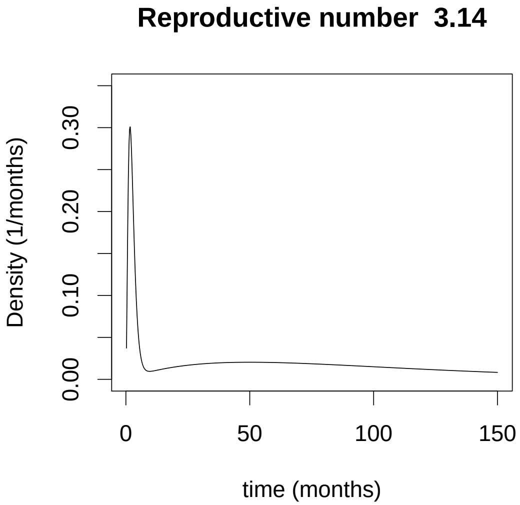
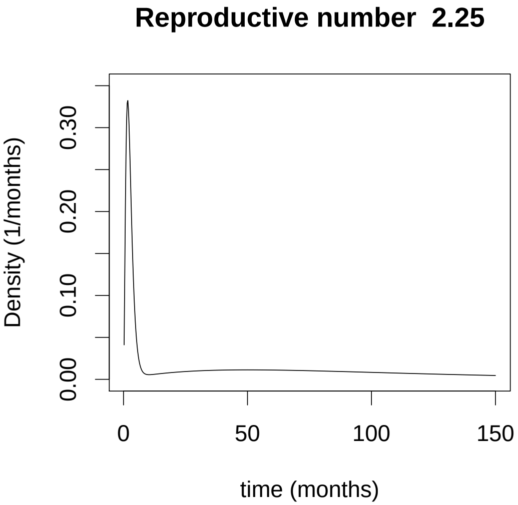


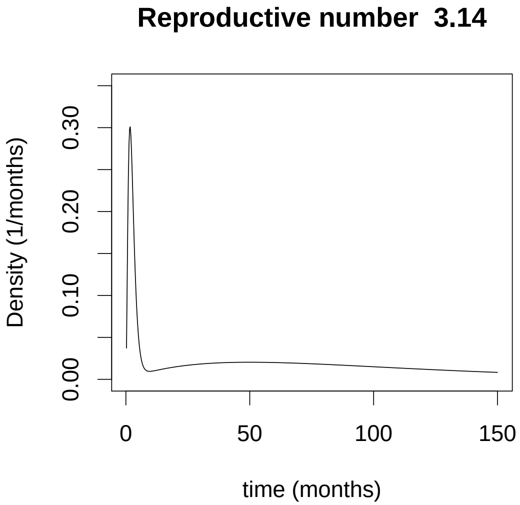
Baseline scenario



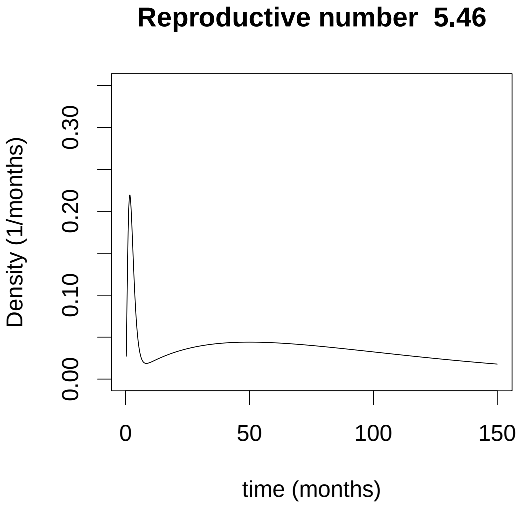
Fast scenario



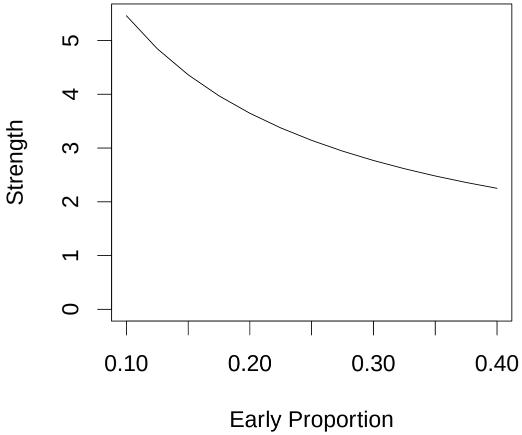
Baseline scenario



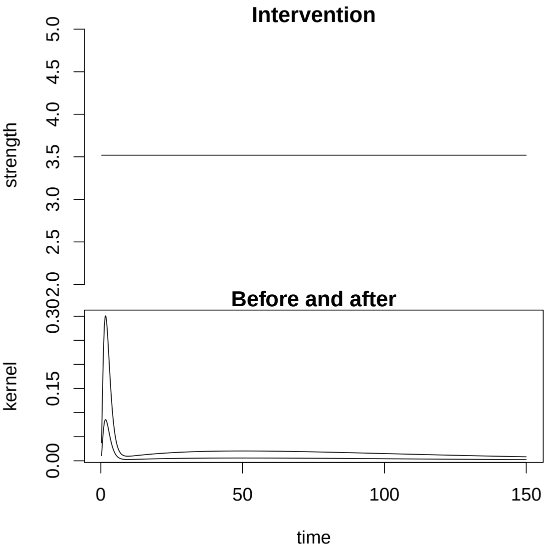
Slow scenario



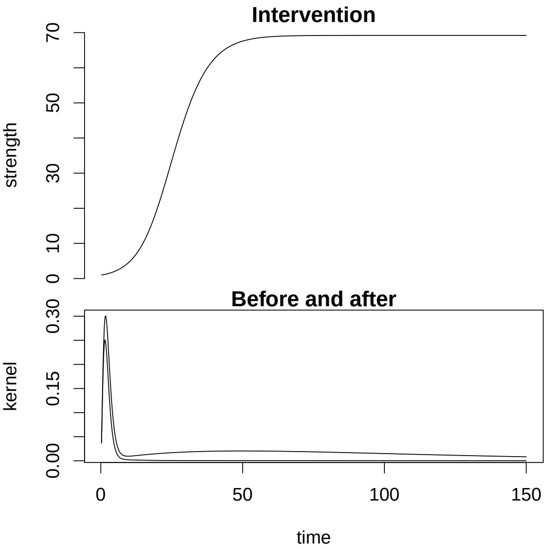
Range of estimates



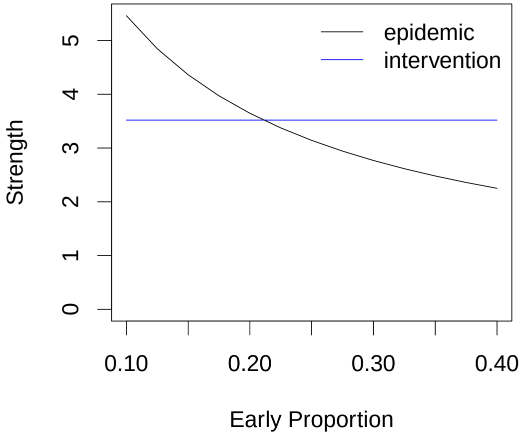
Condom intervention



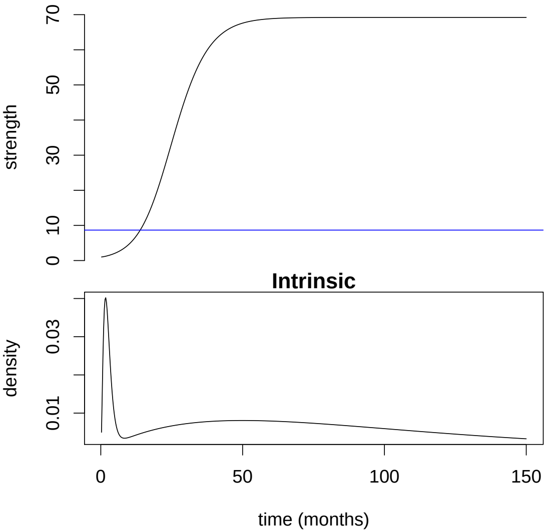
Test and treat



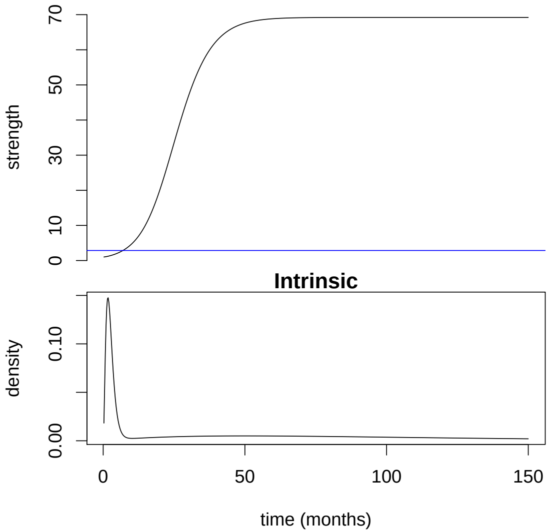
Condom intervention



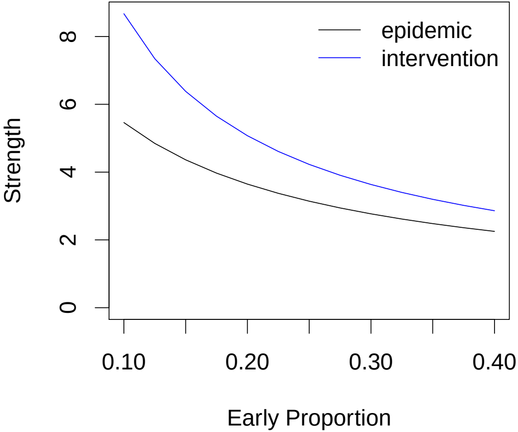
Test and treat (low early transmission)



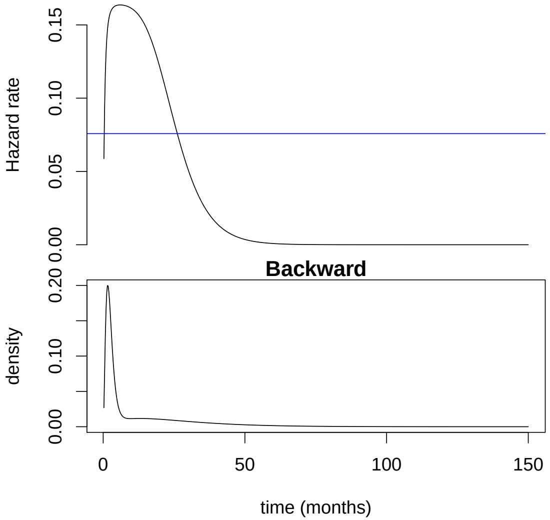
Test and treat (high early transmission)



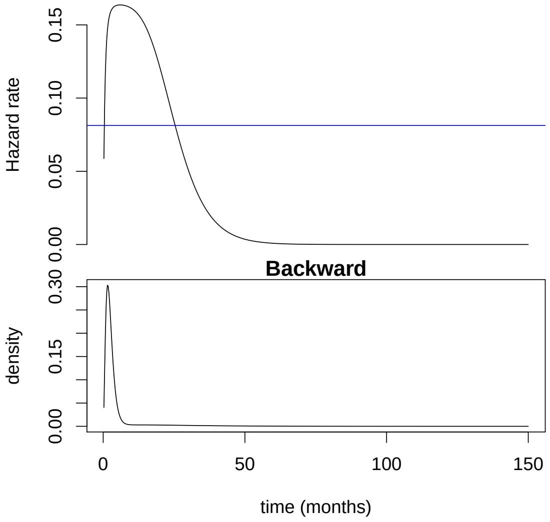
Range of estimates



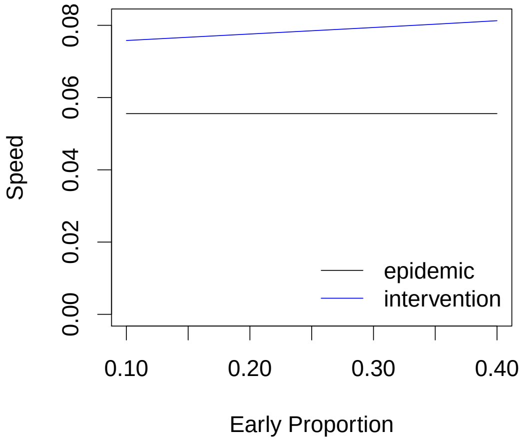
Low early transmission



High early transmission



Range of estimates



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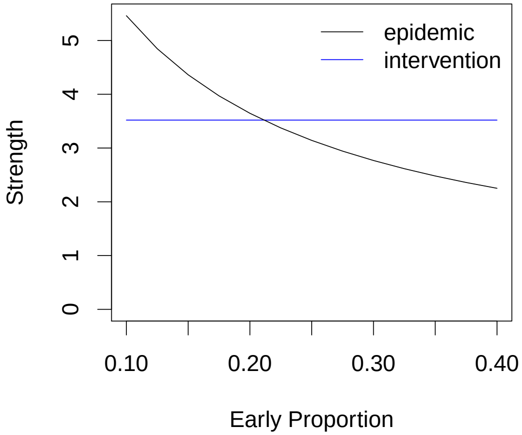

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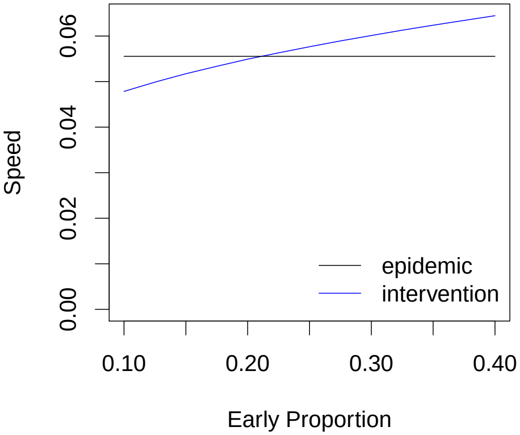
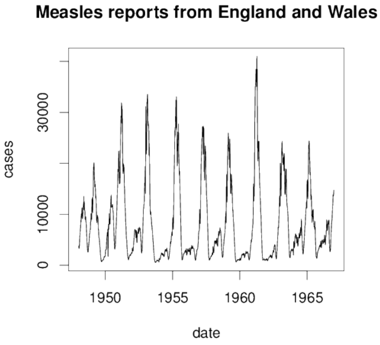
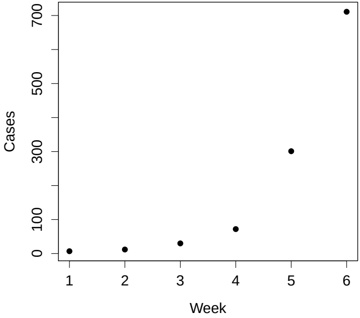


Outline

Ways of looking

Measuring the epidemic





Measuring the intervention

Examples

- Measles

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  - Information (long-term) is strength-like

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  - Information (long-term) is strength-like
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Thanks

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- Audience

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