Epidemic strength and speed: rethinking metrics for infectious disease spread and control.

SMB 2017

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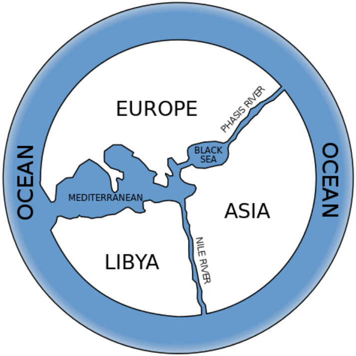
http://lalashan.mcmaster.ca/DushoffLab

Outline

Infectious diseases

Models



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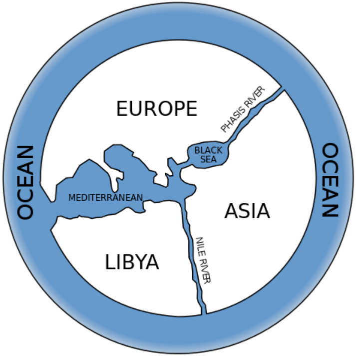

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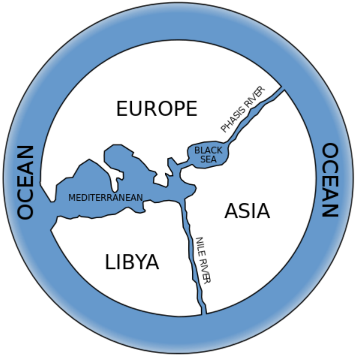

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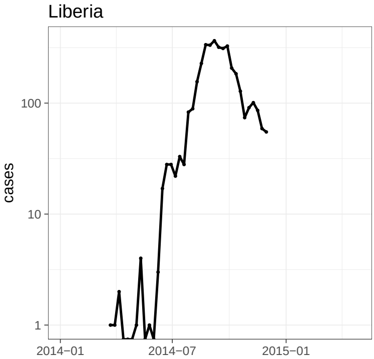
Ebola

Dynamic modeling connects scales



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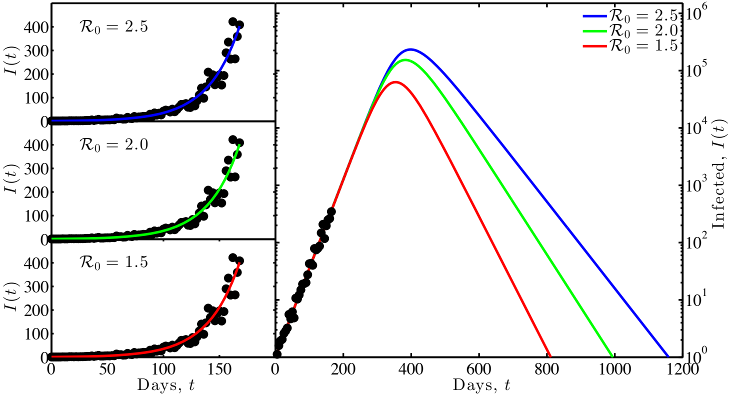

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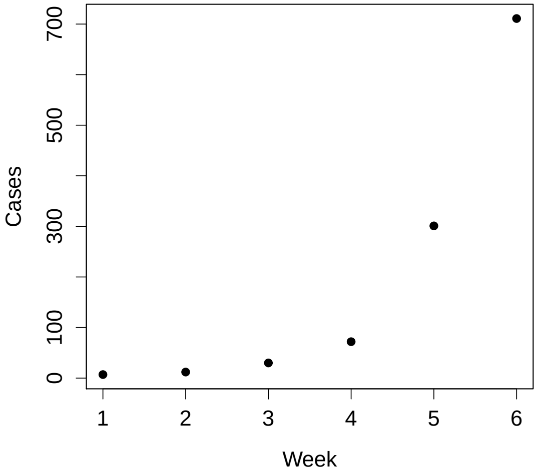

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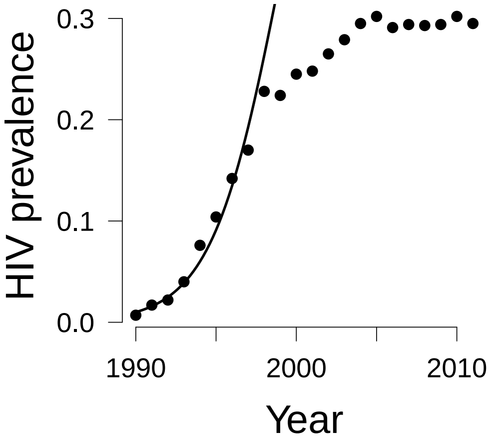

$C \approx 18\,\mathrm{month}$. Horrifyingly fast.

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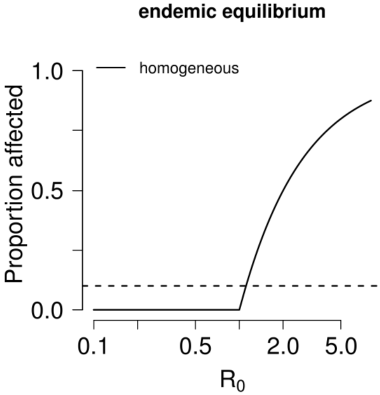

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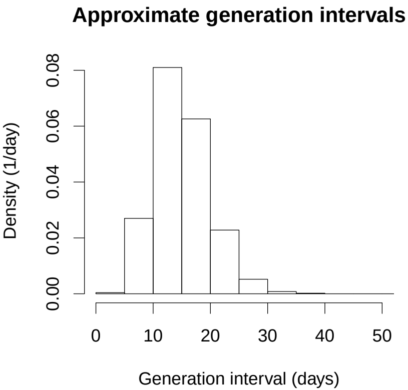

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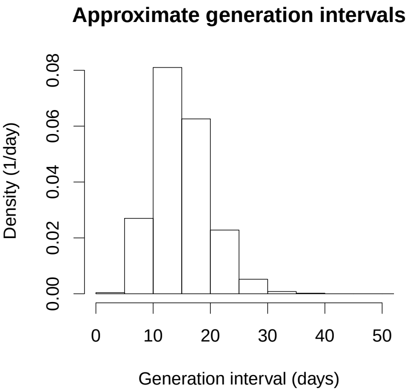

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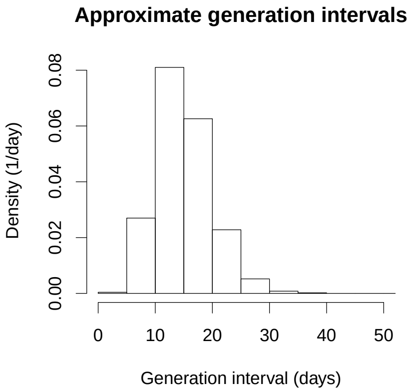

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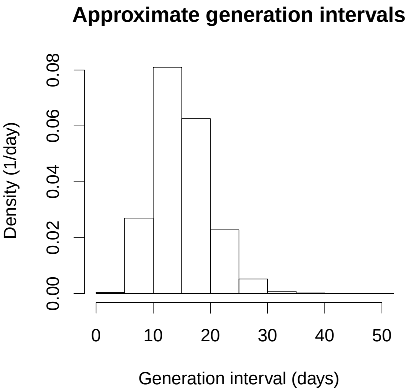

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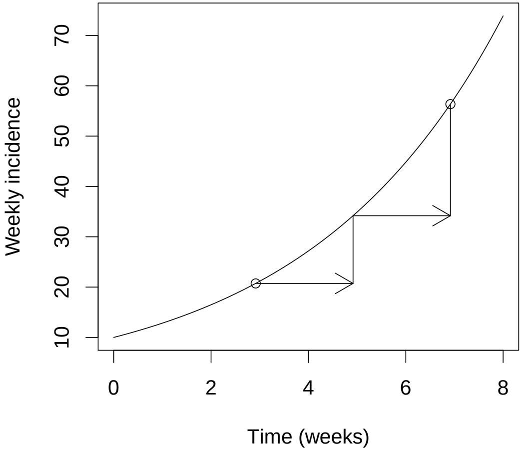

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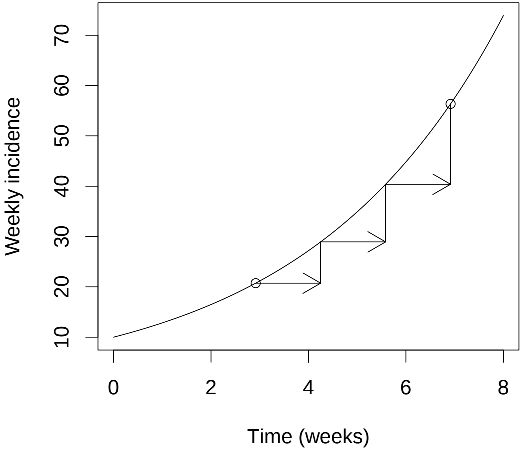

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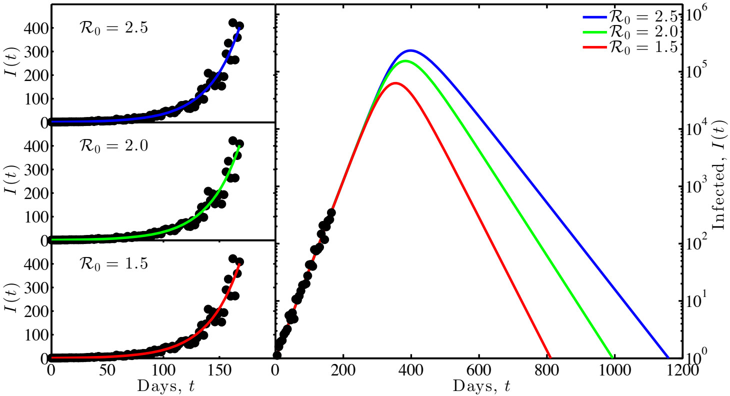

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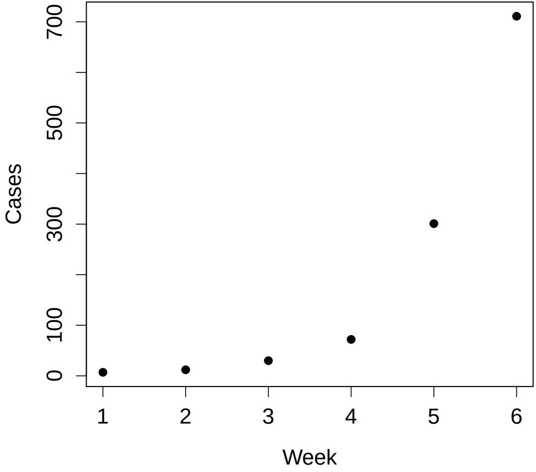

$C \approx 1\,\text{week}, \;\; G \approx 3\,\text{day}$

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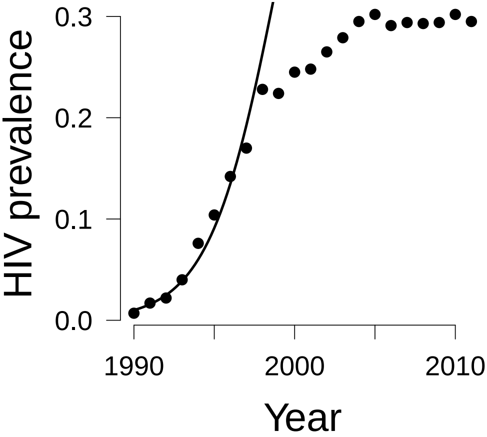

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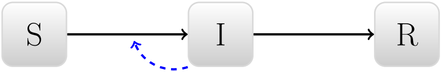

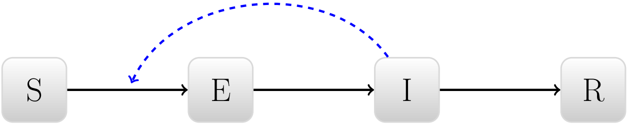

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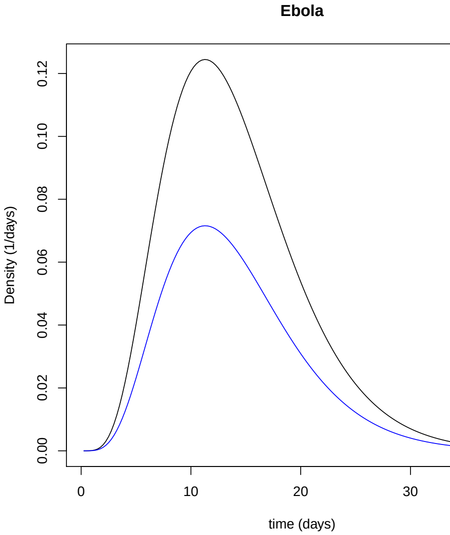

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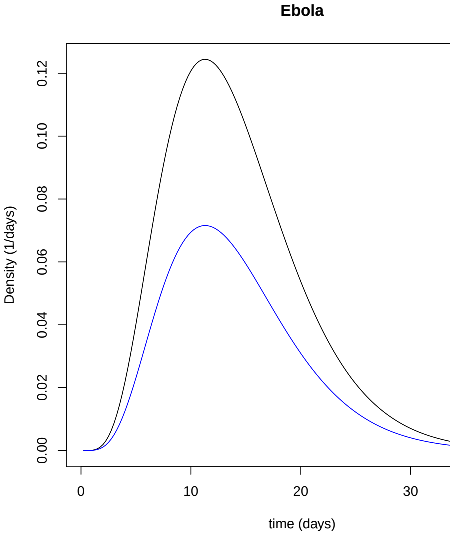

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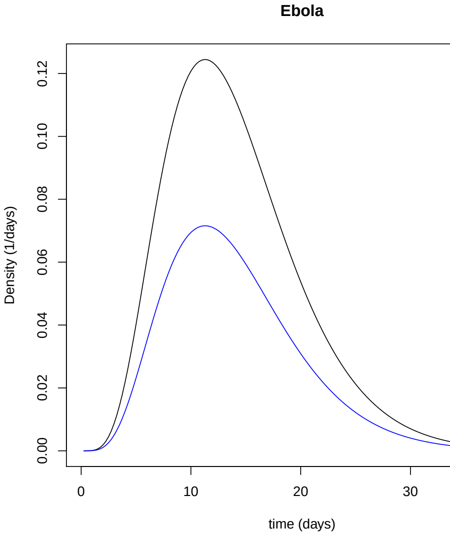

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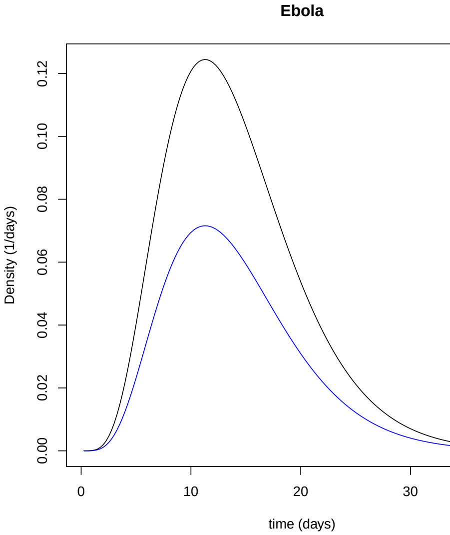

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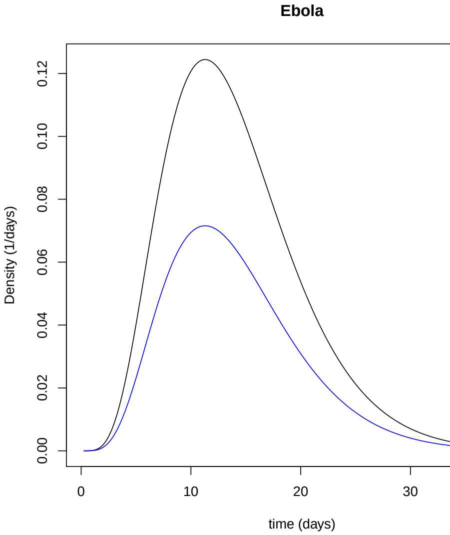

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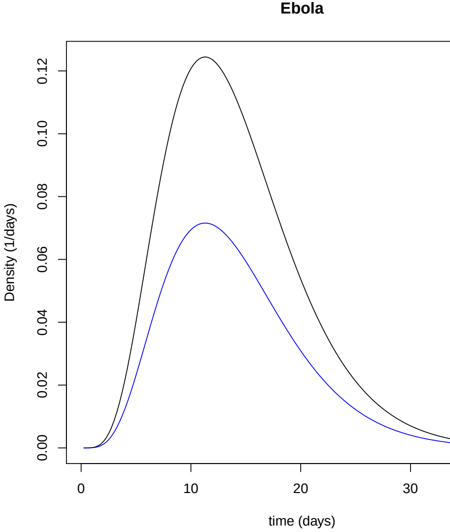

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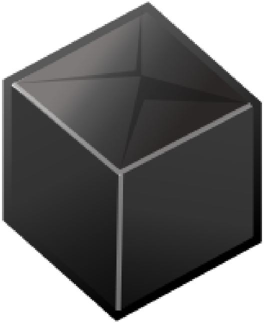

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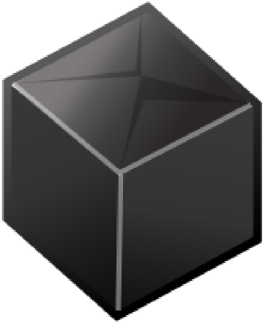

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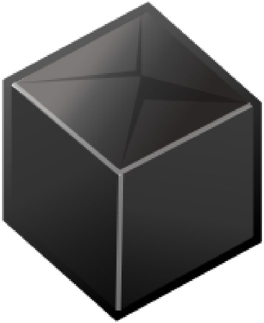

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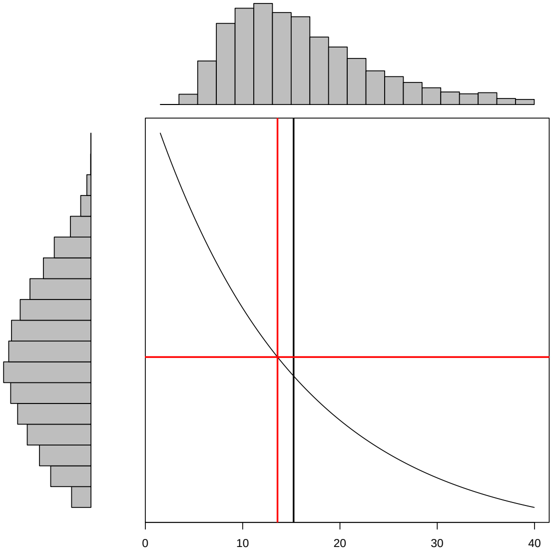
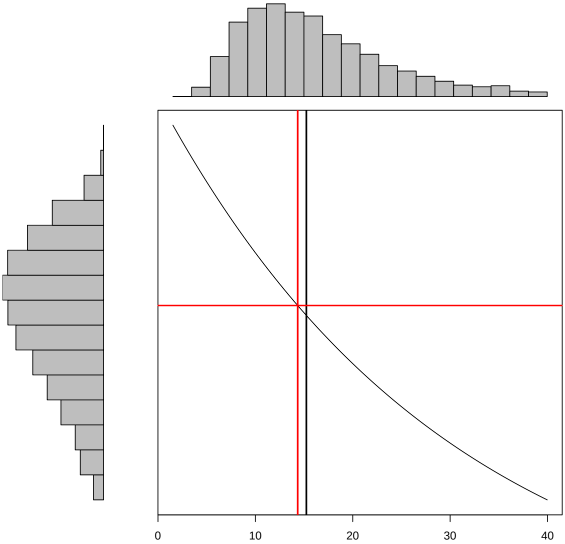
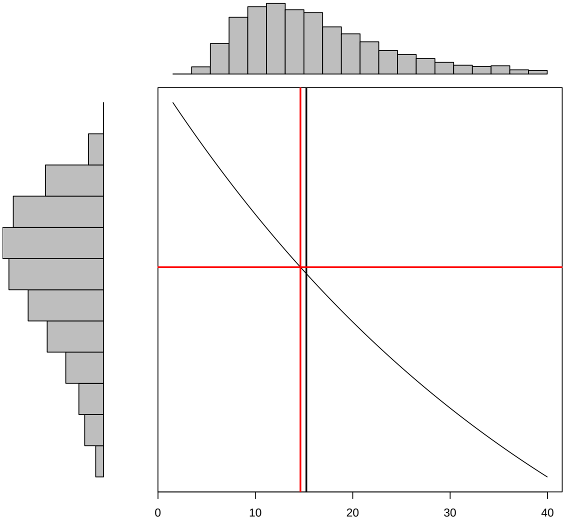


$C = 1/r = 30d$



$$C = 1/r = 45d$$



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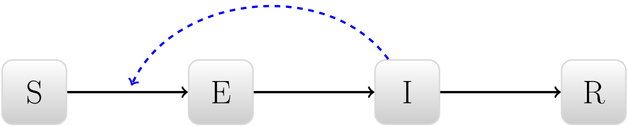

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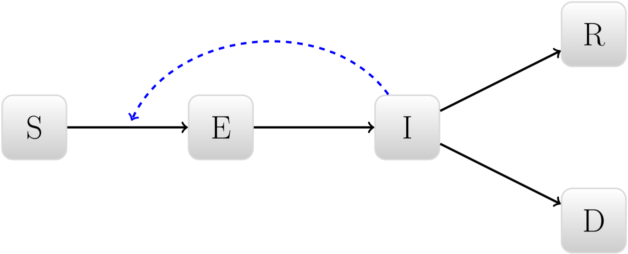

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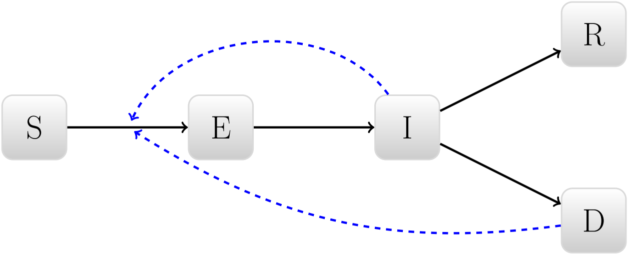
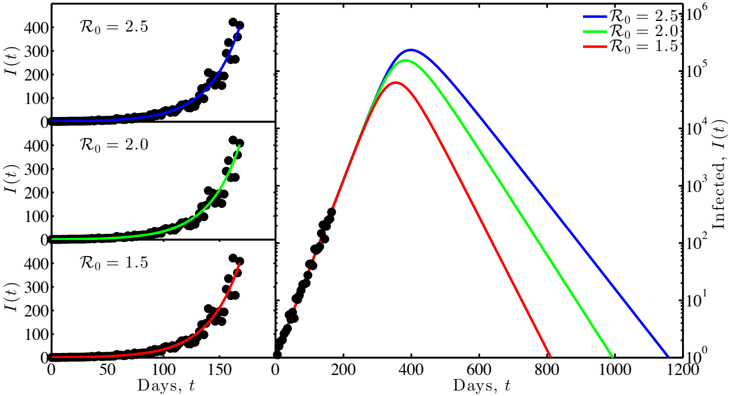


Scenarios



$\mathcal{R}_0 = 2.5$
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$I(t)$
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100
200
300
400
0
50
100
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Days, t
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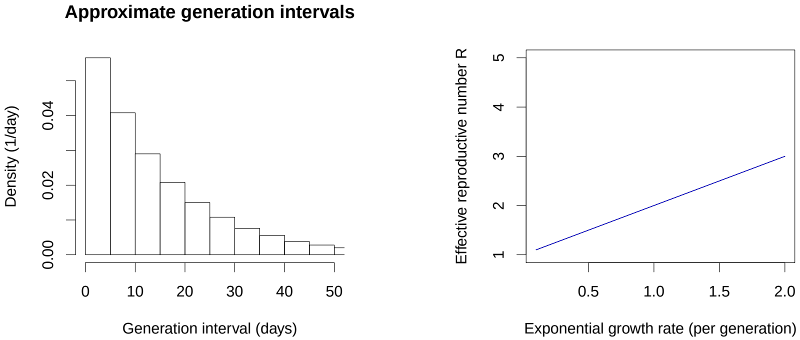

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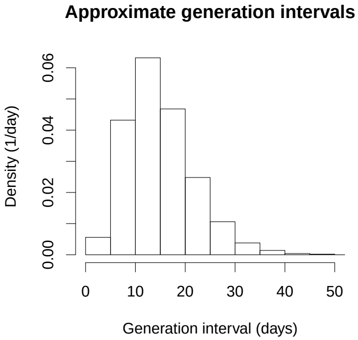

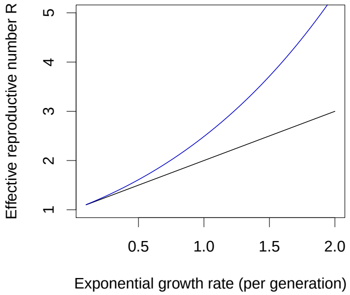

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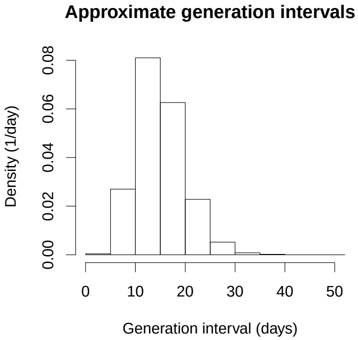

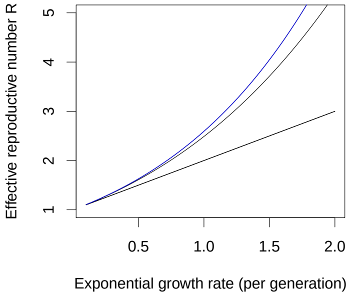

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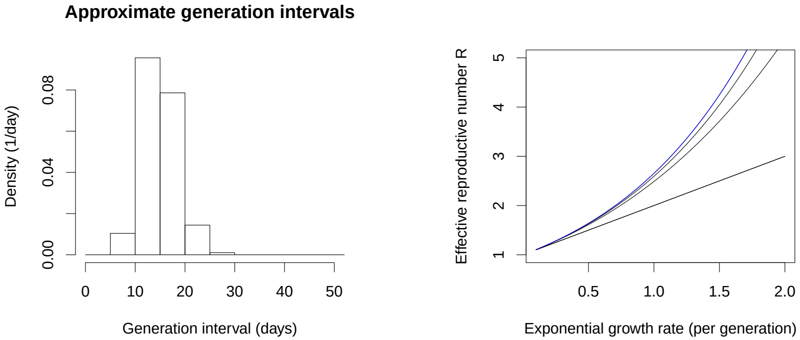

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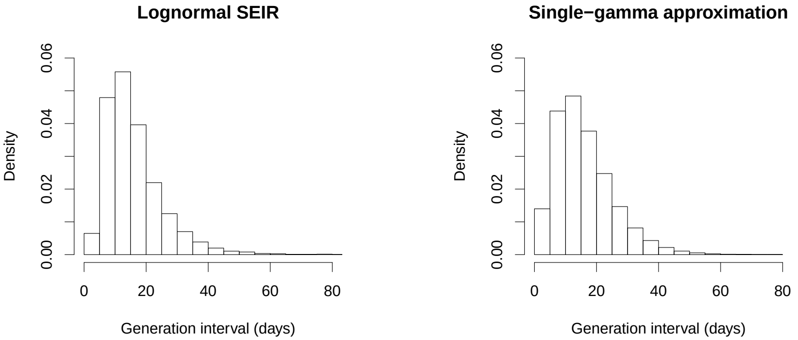

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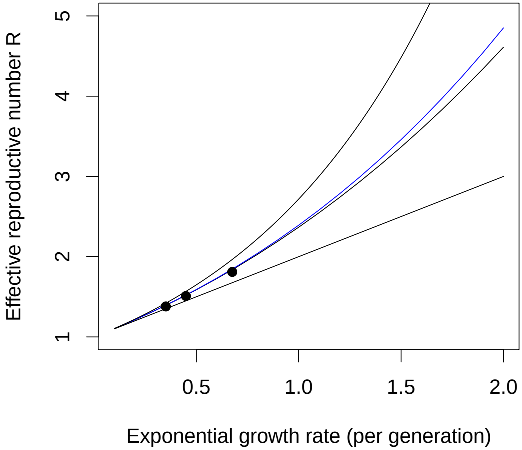

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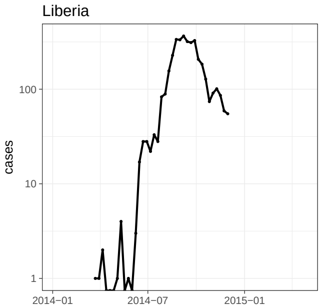

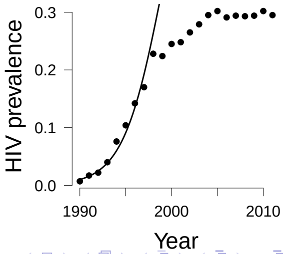

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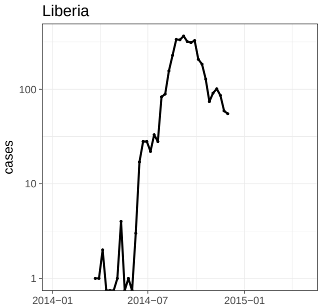

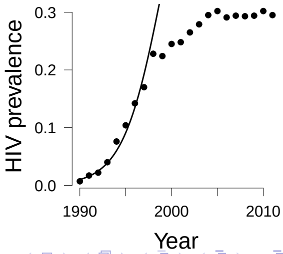

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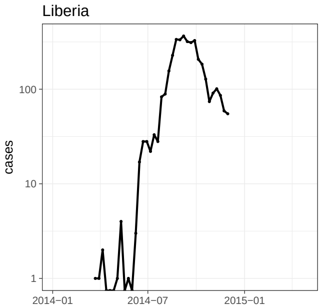

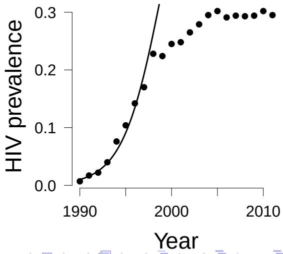

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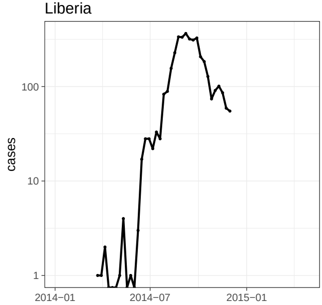

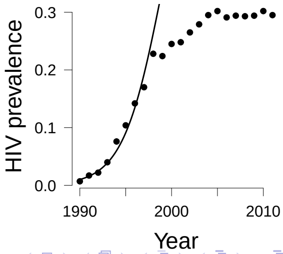

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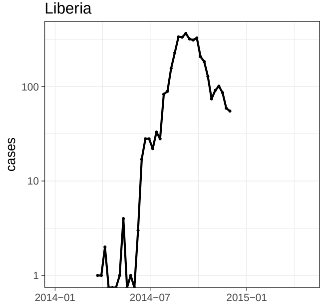

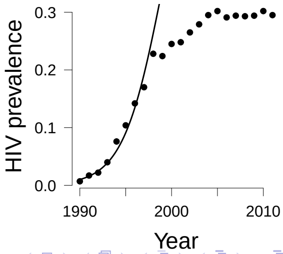

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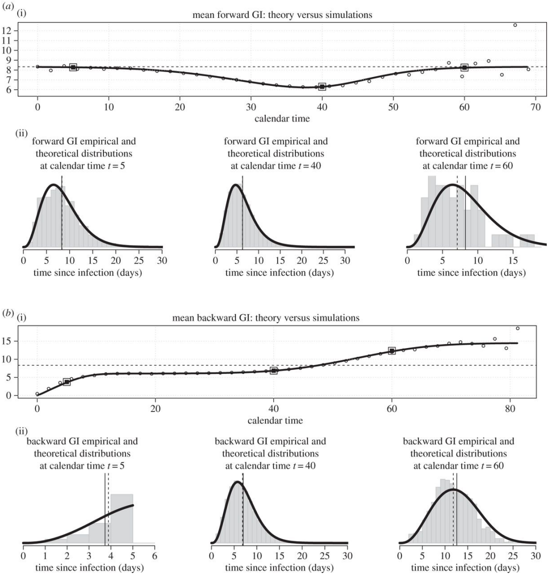

Champredon and Dushoff, 2015. DOI:10.1098/rspb.2015.2026

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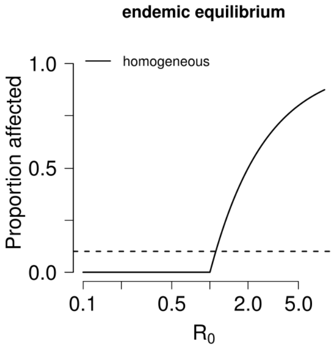

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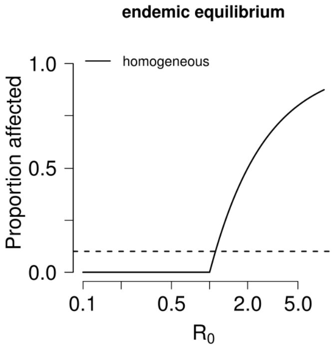

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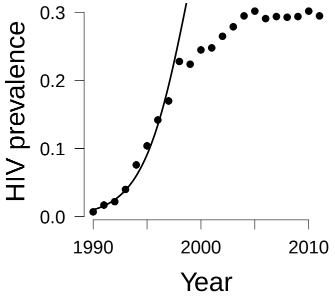

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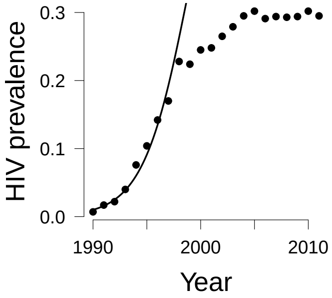

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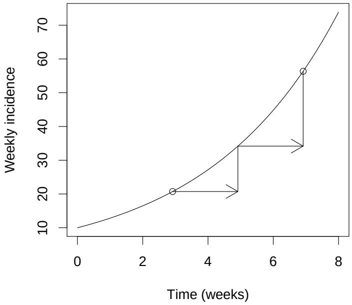

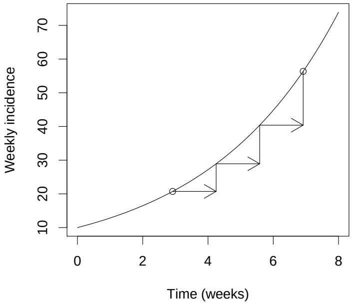

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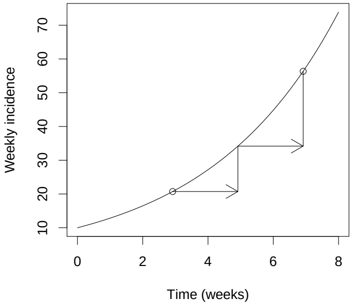

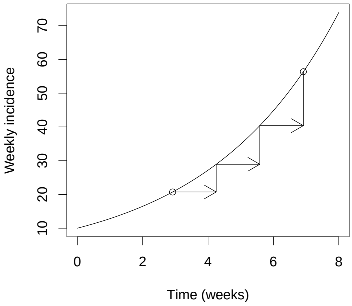

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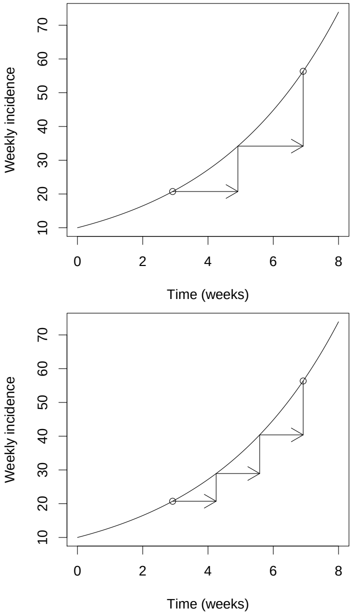

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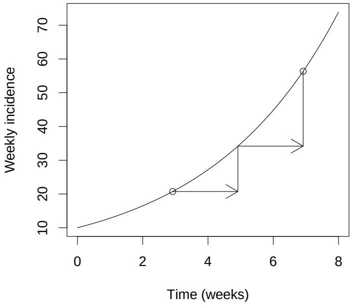

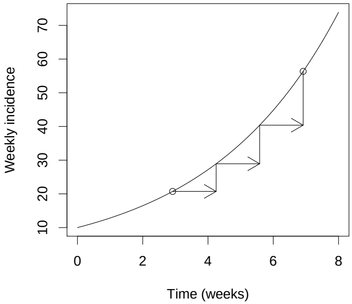

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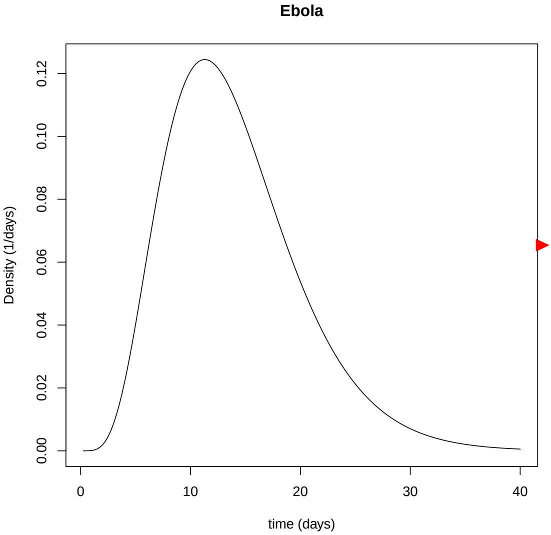

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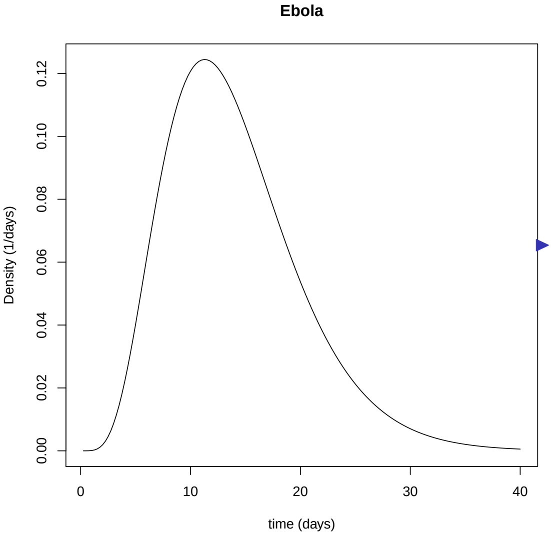

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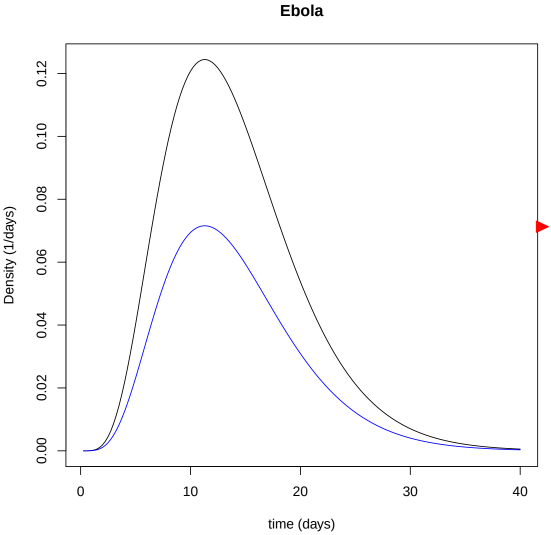

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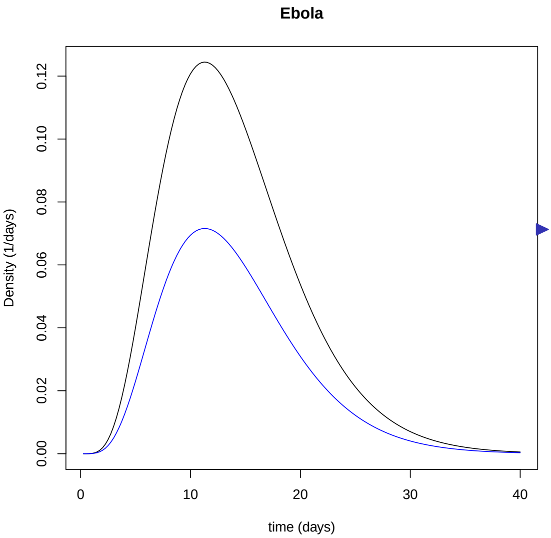

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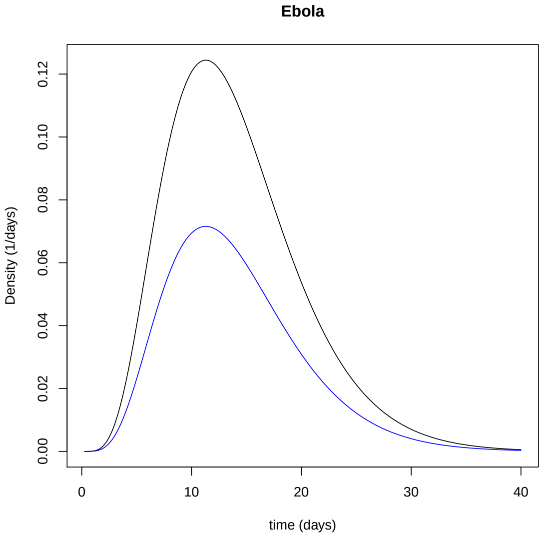

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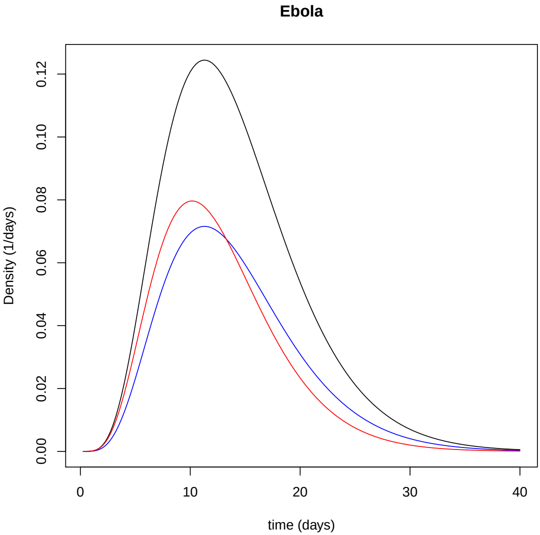

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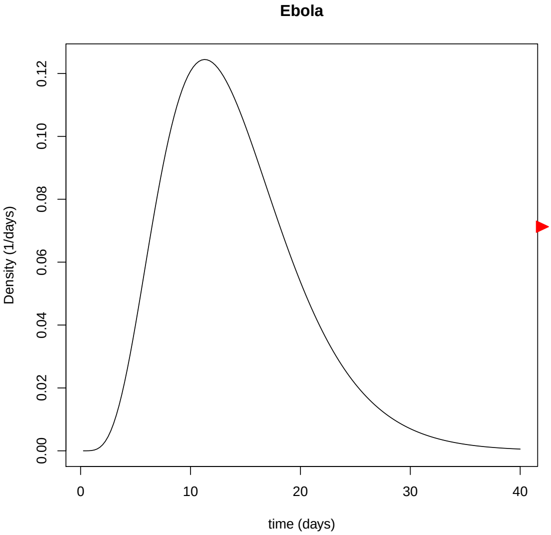

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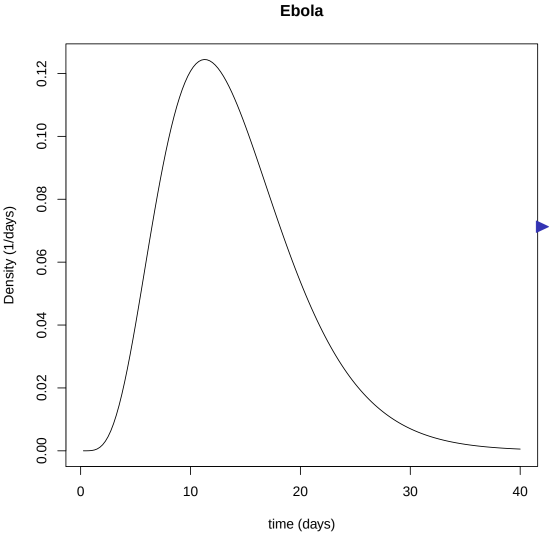

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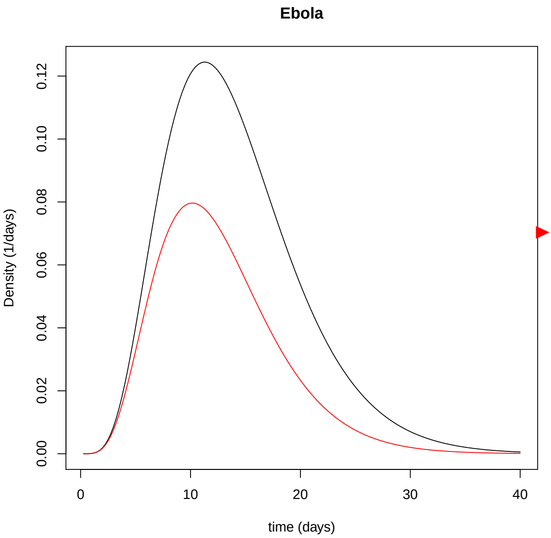

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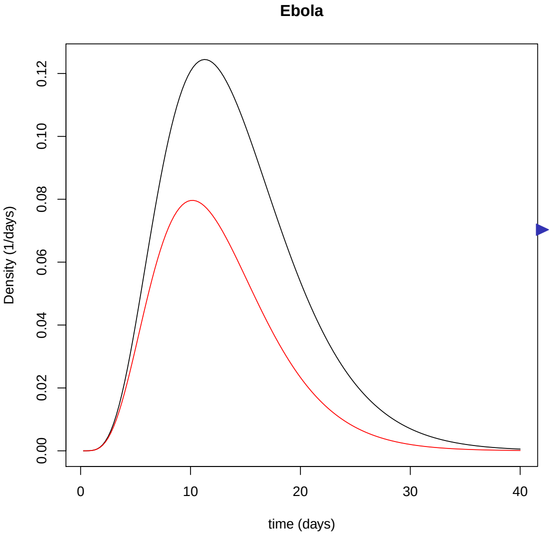

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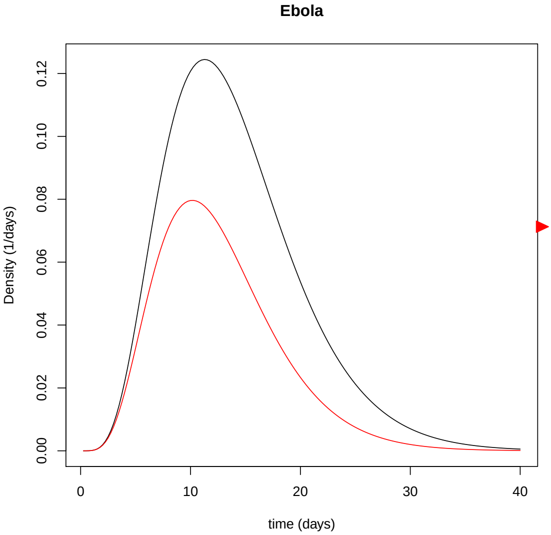

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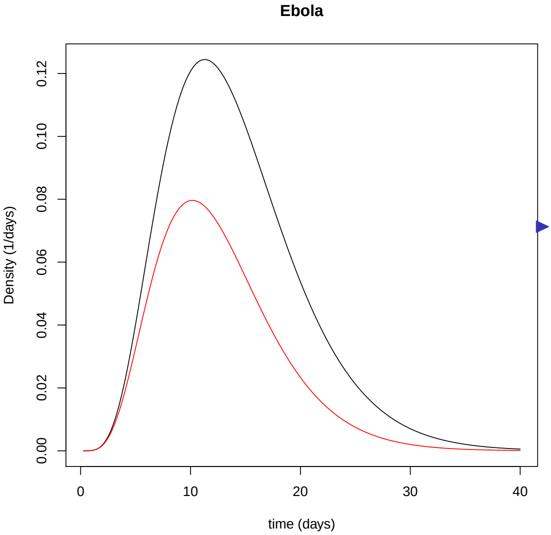

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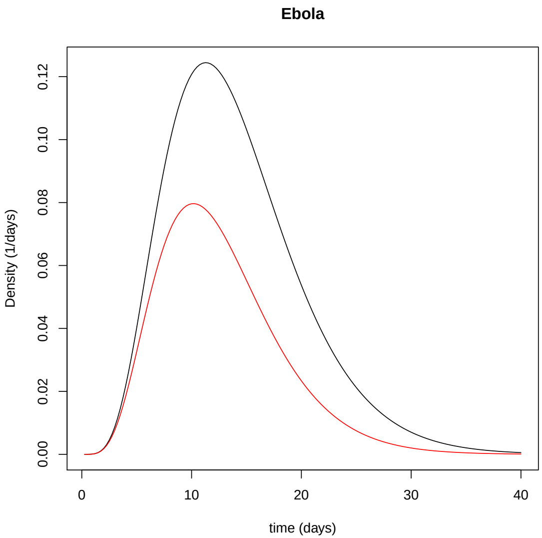

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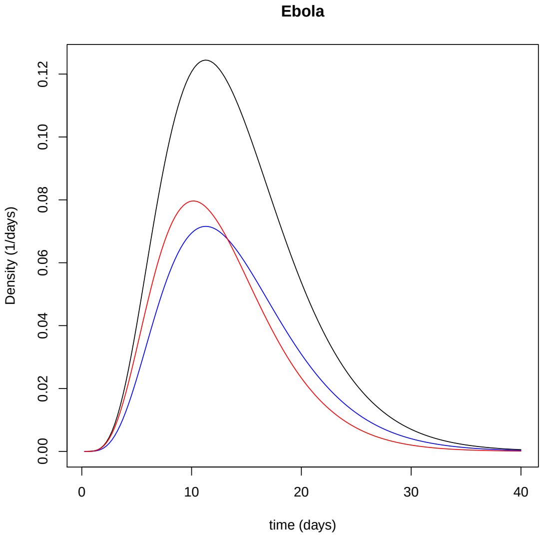

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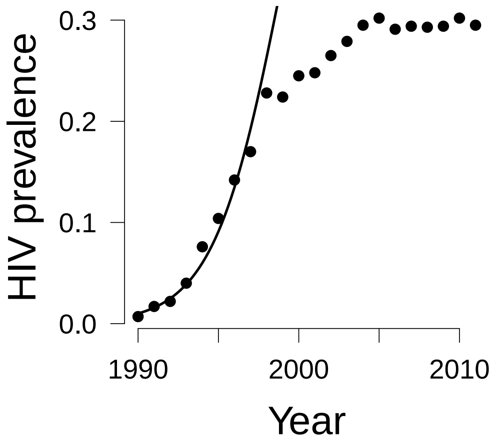
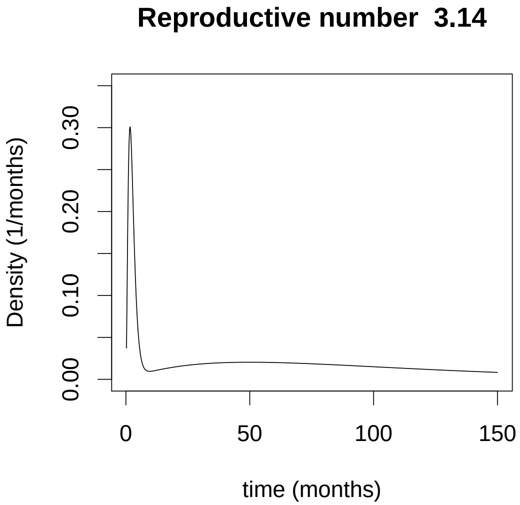
Outline

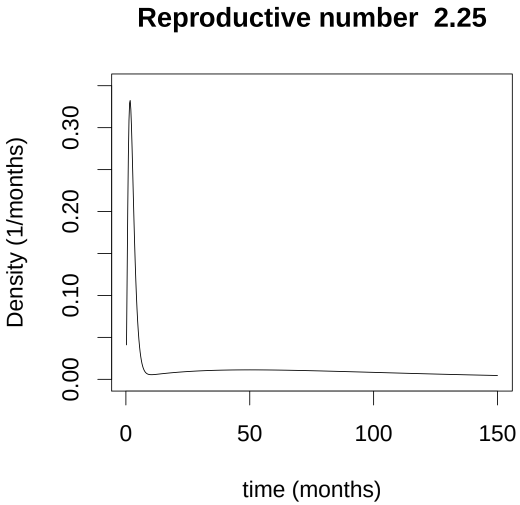
Epidemic speed



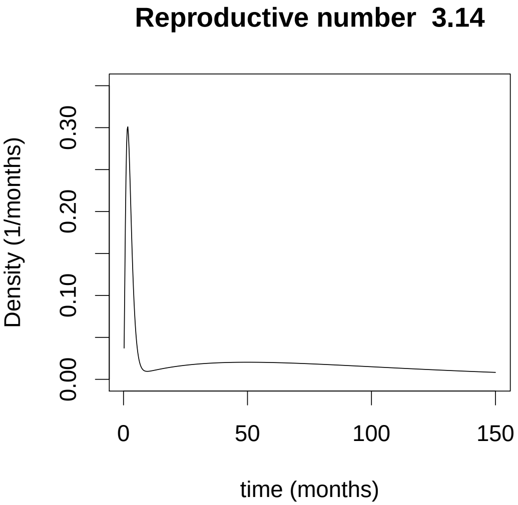
Baseline scenario



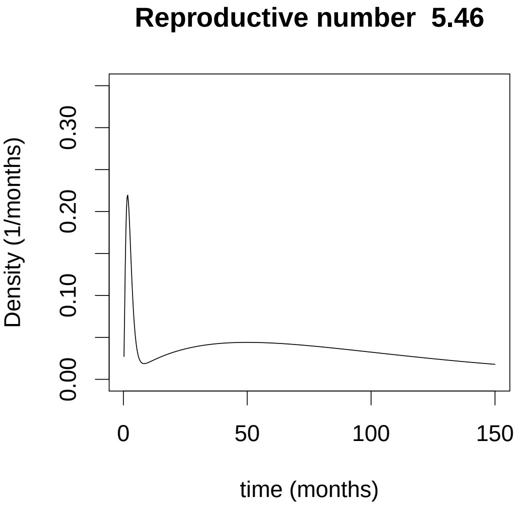
High early transmission



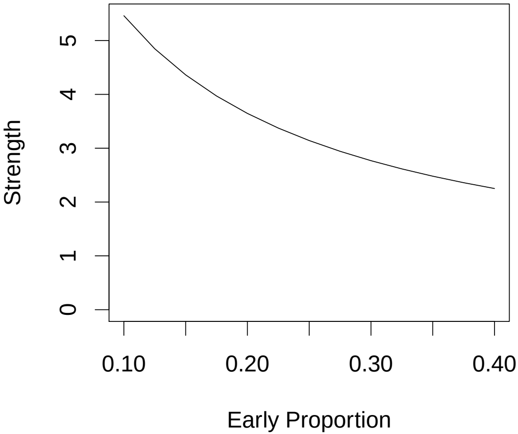
Baseline scenario



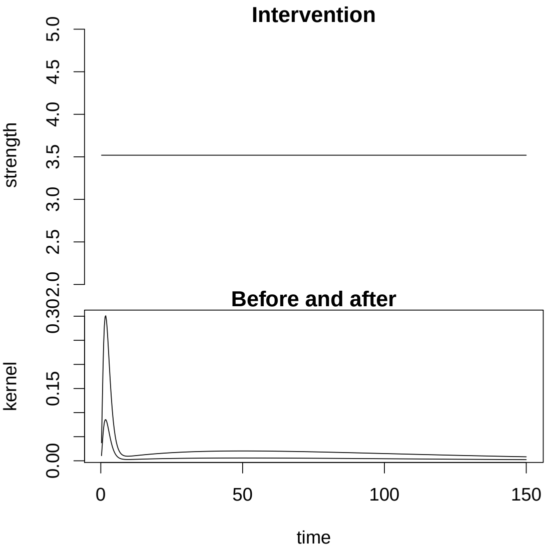
Low early transmission



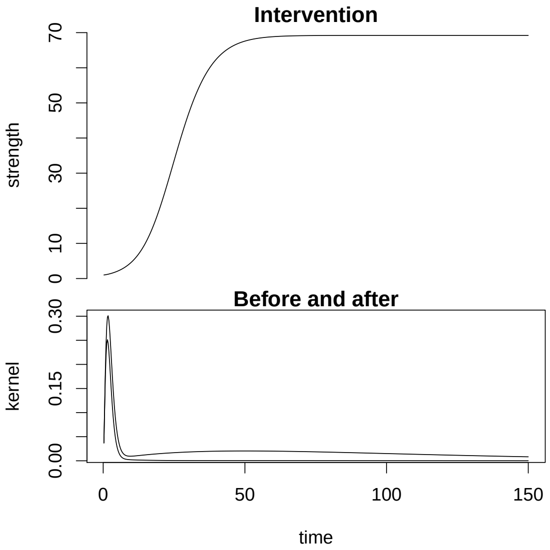
Range of estimates



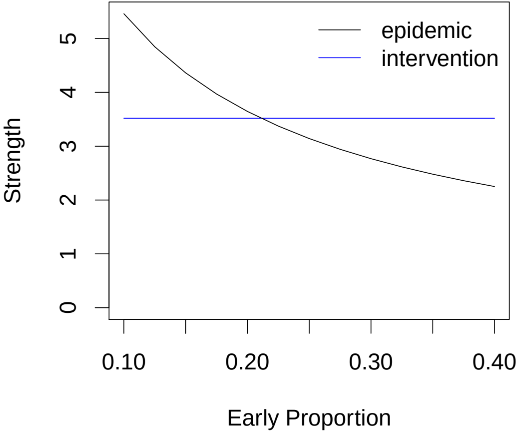
Condom intervention



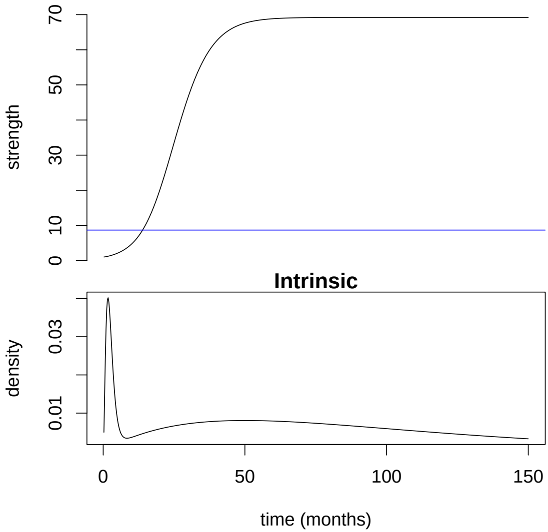
Test and treat



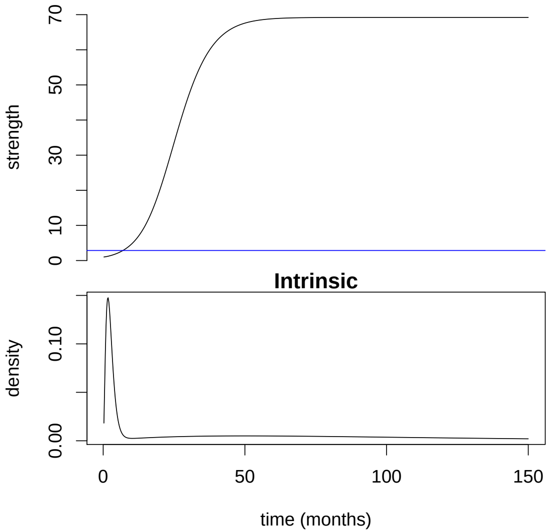
Condom intervention



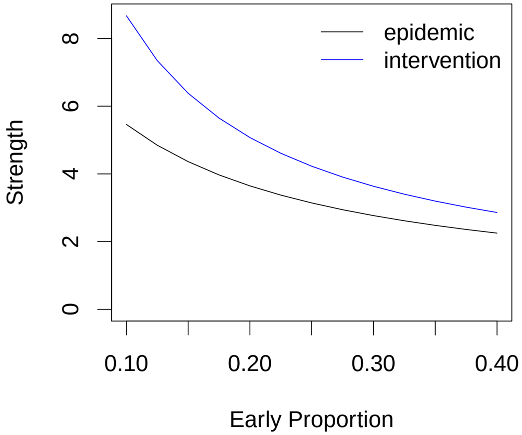
Test and treat (low early transmission)



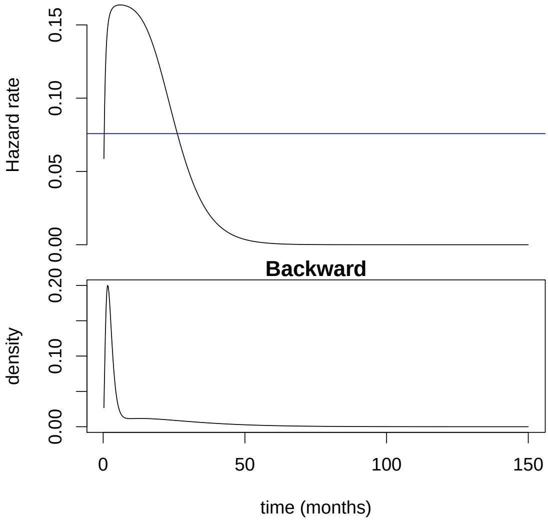
Test and treat (high early transmission)



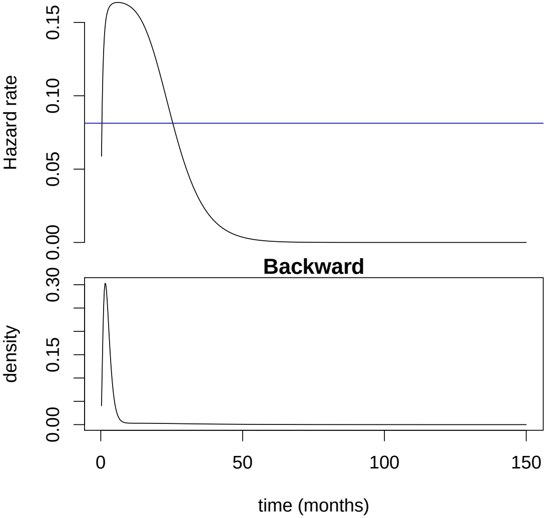
Range of estimates



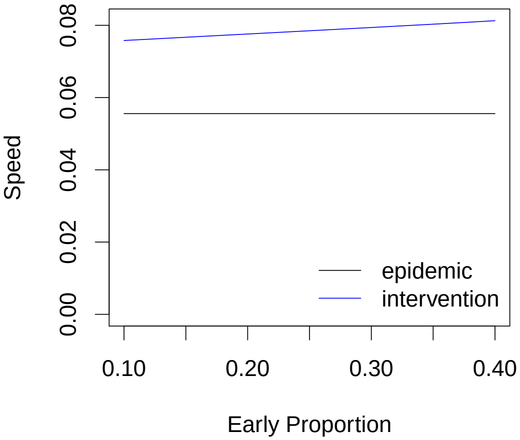
Low early transmission



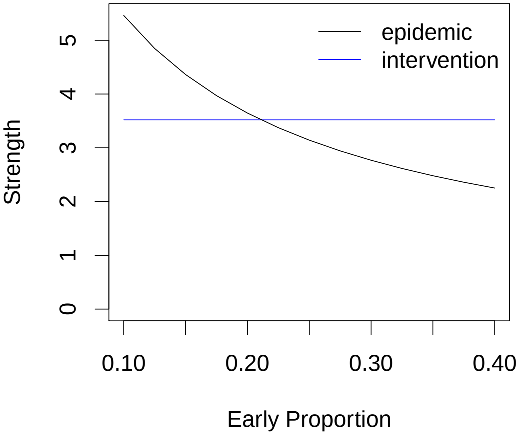
High early transmission



Range of estimates



Condom intervention



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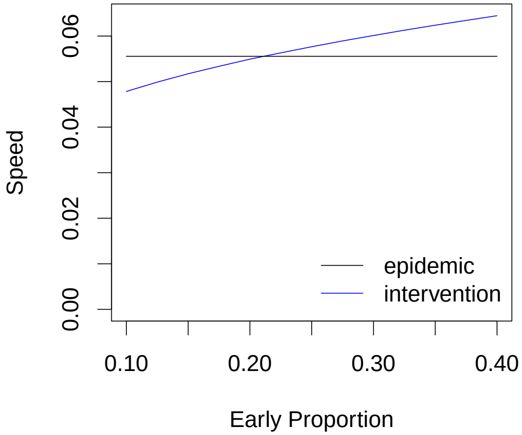
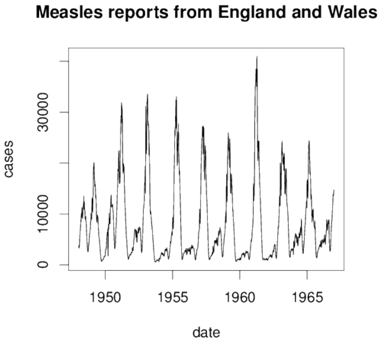
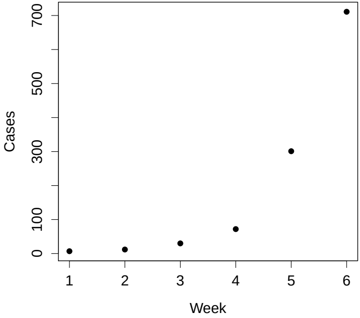


Outline

Ways of looking

Measuring the epidemic





Measuring the intervention

Examples

- Measles

Examples

- Measles
  - Information (long-term) is strength-like

Examples

- Measles
  - Information (long-term) is strength-like
  - Intervention (vaccine) also strength-like

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  - Information is speed-like

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- Ebola vaccination
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  - Interventions: vaccination (strength?); isolation and control (speed?)

Examples

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  - Intervention (vaccine) also strength-like
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Thanks

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- Audience

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