Thoughts about modeling time since infection

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https://github.com/dushoff/Generation_talks

Why might we care about modeling TSI

- Questions
- Systems
- Mechanisms
- Implementation?

Questions

- Size, speed and strength
- Periodic dynamics
- Troughs and persistence
- Intervention and control
- Surveillance and elimination

Mechanisms

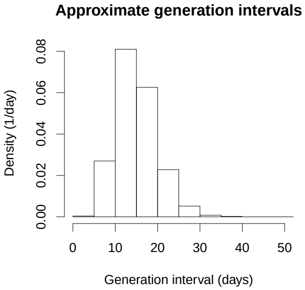
- Realistic distributions
- Incubation periods
- Courses of disease and routes of transmission
- Dormancy mechanisms

Implementation

- Boxes (Linear chain trick etc.)
- PDEs
- What specific distributions or mathematical mechanisms reflect the biology

$r\mathcal{R}$

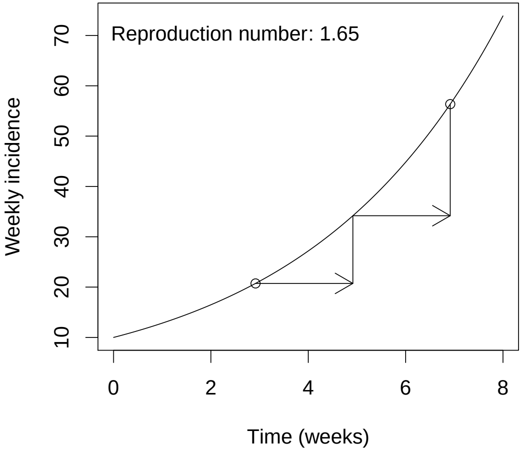
- We measure epidemic speed using little r:
- We measure epidemic "strength" using $\mathcal{R}$
- These are linked by the *distribution* of generation times



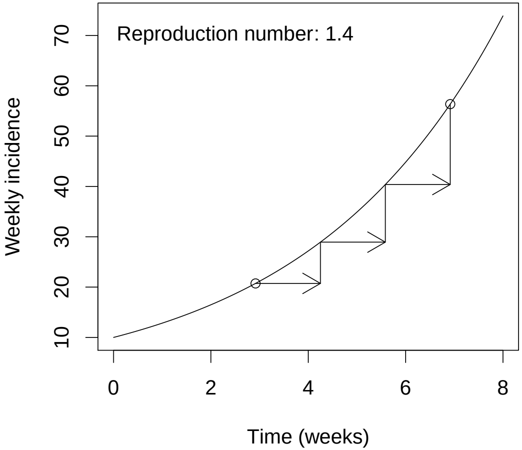
Conditional effect of generation time

- *Given* the reproductive number $\mathcal{R}$
  - faster generation time G means faster growth rate r
  - More danger
- *Given* the growth rate r
  - faster generation time G means *smaller* $\mathcal{R}$
  - Less danger

Generations and $\mathcal{R}$



Generations and $\mathcal{R}$



Linking framework

- Epidemic speed (r) is a *product*:
  - (something to do with) generation speed $\times$
  - (something to do with) epidemic strength
- How much does "generation speed" depend on the details of how infected people change through time?
- When can we just use the mean *generation interval?*
  - Not infectious period!

Euler-Lotka equation

- Provides the $r\mathcal{R}$ link during an exponential period
- $1 = \int k(\tau) \exp(-r\tau)\, d\tau$
  - i.e., the total of *discounted* contributions is 1
- $1/\mathcal{R} = \int g(\tau) \exp(-r\tau)\, d\tau$

Interpretation: "effective" generation times

- Define the effective generation time so that
  - $$\mathcal{R} = \exp(r\hat{G})$$
- Then:
  - $$1/\mathcal{R} = \int g(\tau)\exp(-r\tau)\,d\tau$$
  - $$\exp(-r\hat{G}) = \langle \exp(-r\tau) \rangle_g .$$
  - A filtered mean:
    - The discounted value of $\hat{G}$ is the expectation of the discounted values across the distribution

Linking framework

- Epidemic speed (r) is a *product*:
  - (something to do with) generation speed $\times$
  - (something to do with) epidemic strength
- In particular:
  - $r = (1/\hat{G}) \times \log(\mathcal{R})$
  - $\hat{G}$ is the effective mean generation time

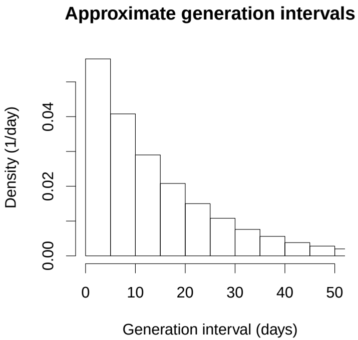
An approximation

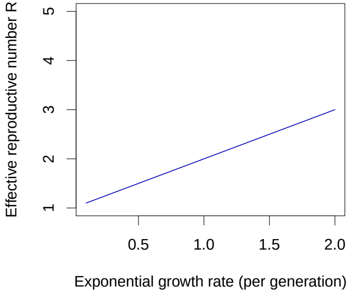
- We connect these quantities with a moment approximation
- Define $\kappa = \sigma_G^2/\mu_G^2$ – the squared coefficient of variation of the generation distribution
- $\mathcal{R} \approx (1 + r\kappa\bar{G})^{1/\kappa}$
  - Equal when $g(\tau)$ has a gamma distribution
  - Simple and straightforward
  - When is it a useful approximation?

Compound-interest interpretation

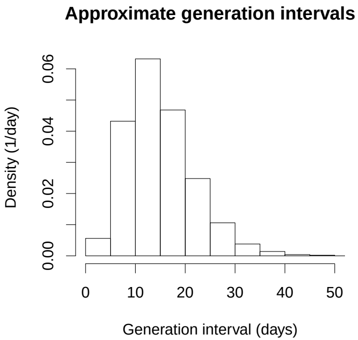
- Define $\mathcal{R} \approx (1 + r\kappa\bar{G})^{1/\kappa} \equiv X(r\bar{G}; 1/\kappa)$
- X is the compound-interest approximation to the exponential
  - Linear when $\kappa = 1$ (i.e., when g is exponential)
  - Approaches exponential as $\kappa \to 0$

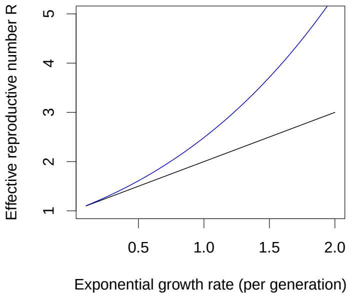
Moment approximation



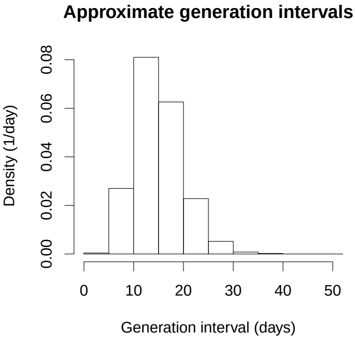


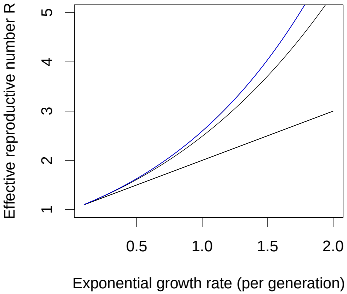
Moment approximation



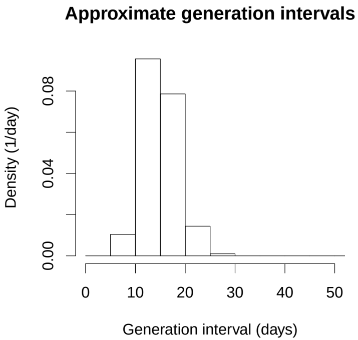


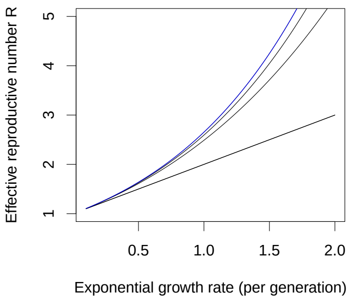
Moment approximation





Moment approximation





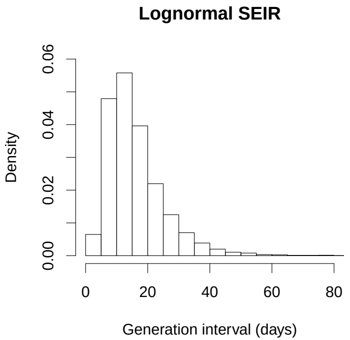
Qualitative response

- For a given value of $\bar{G}$, smaller values of κ mean:
  - less variation in generation interval
  - less compounding of growth
  - greater $\mathcal{R}$ required for a given r

Fitting to Ebola

- Simulate generation intervals based on data and approach from WHO report
- Use both lognormals and gammas
  - WHO used gammas
  - Lognormals should be more challenging

Approximating the distribution



Approximating the curve