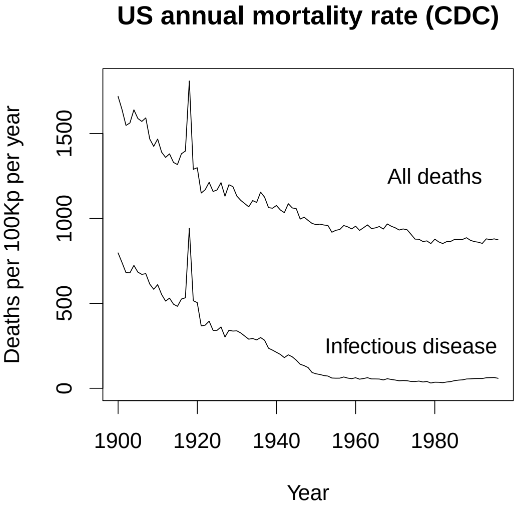
Epidemic strength and speed: rethinking metrics for infectious disease spread and control.

SMB 2017

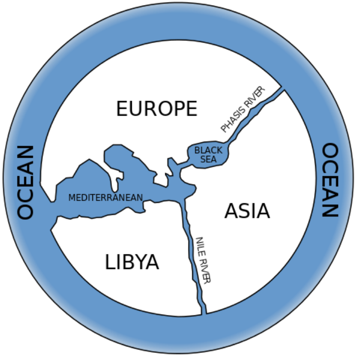
Jonathan Dushoff
http://lalashan.mcmaster.ca/DushoffLab

Outline

Infectious diseases


US annual mortality rate (CDC)
Deaths per 100Kp per year
0
500
1000
1500
All deaths
Infectious disease
1900
1920
1940
1960
1980
Year

Models



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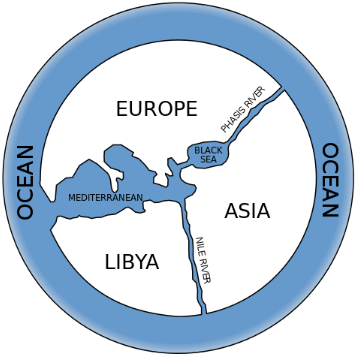

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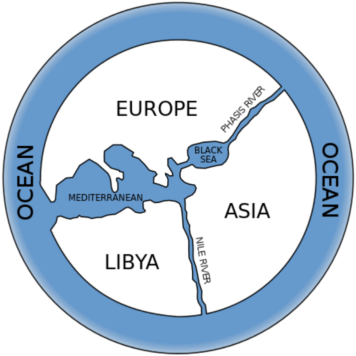

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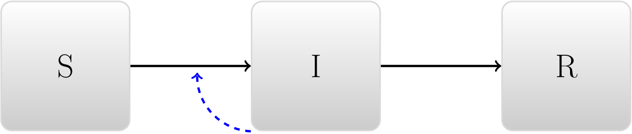
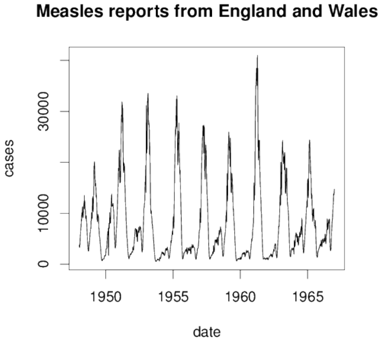


Small-scale events $\Leftrightarrow$ Large-scale patterns and outcomes

Measles

Dynamic modeling connects scales



Ebola

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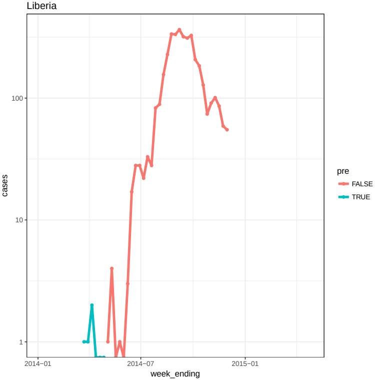

Yellow fever

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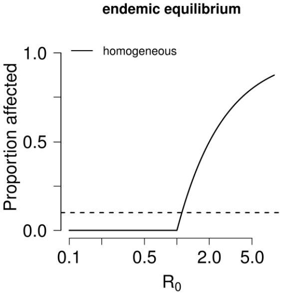

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Speed of epidemics

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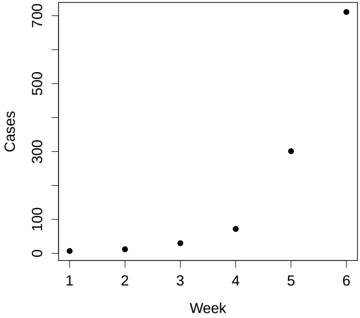

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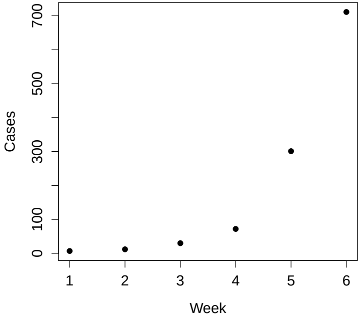

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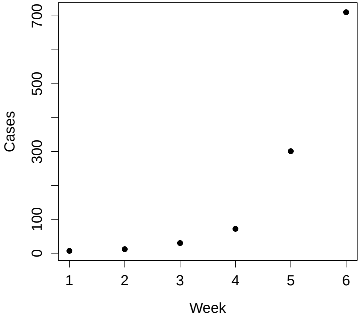

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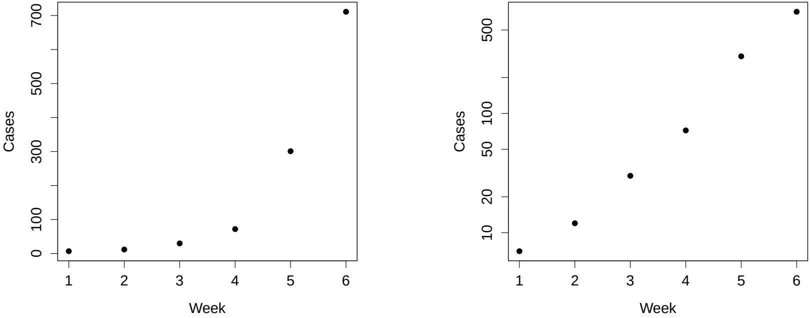

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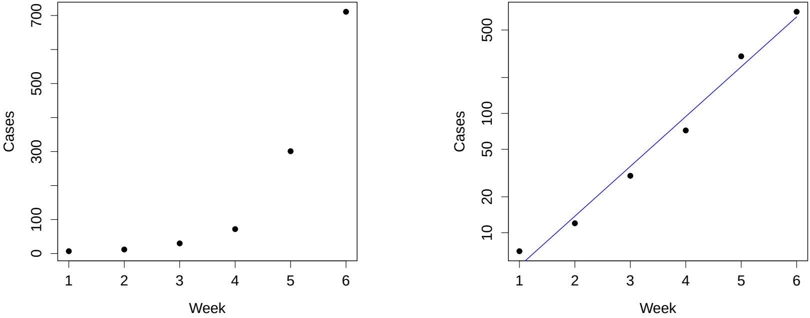

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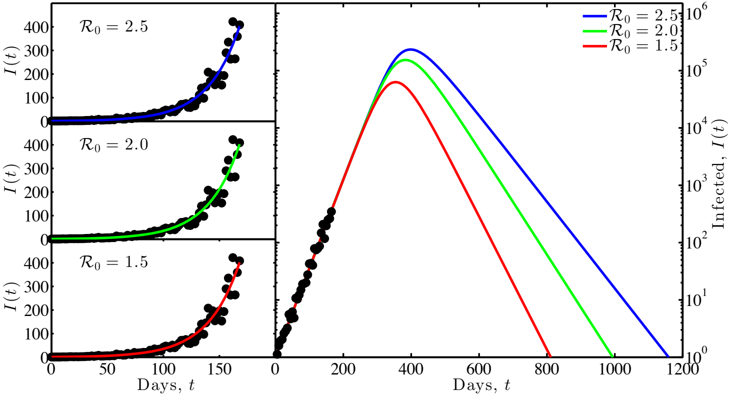
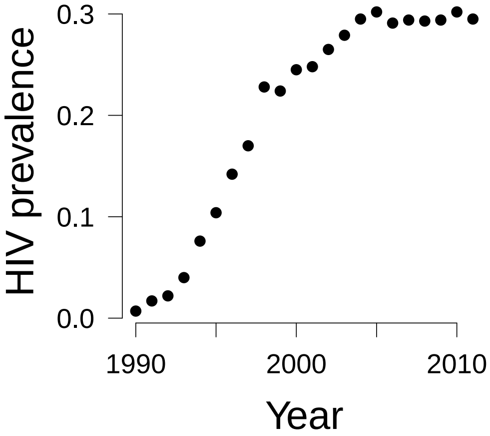


$C \approx 1\text{month}$. Fast.

HIV in sub-Saharan Africa



$C \approx 18\mathrm{month}$. Horrifyingly fast.

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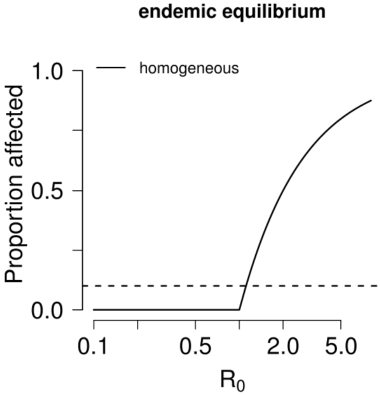

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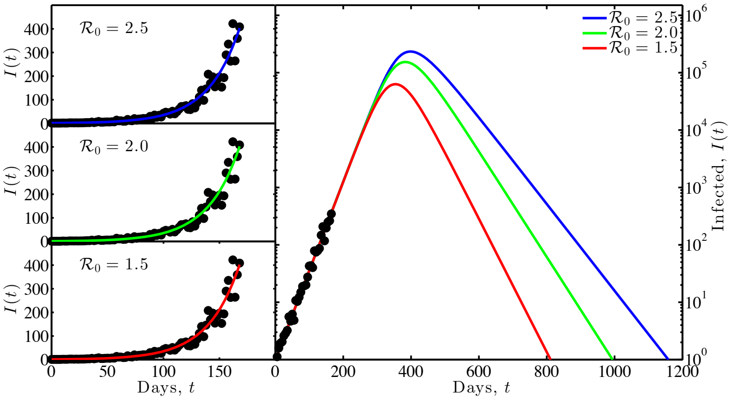

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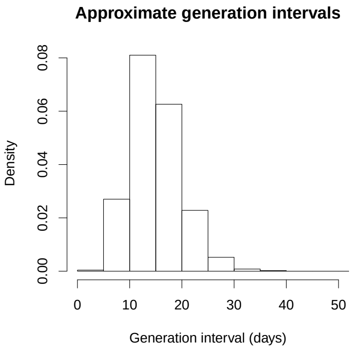

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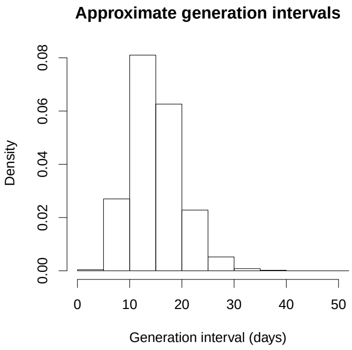

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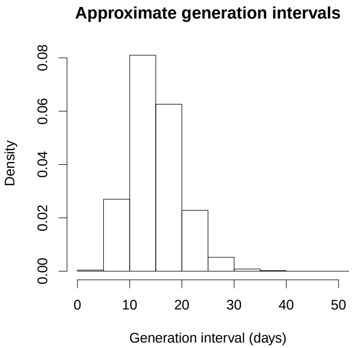

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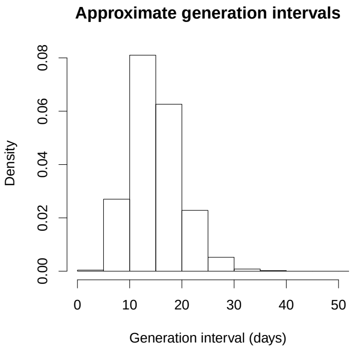

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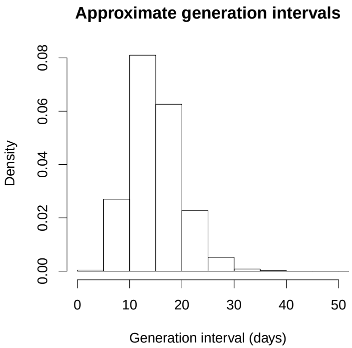

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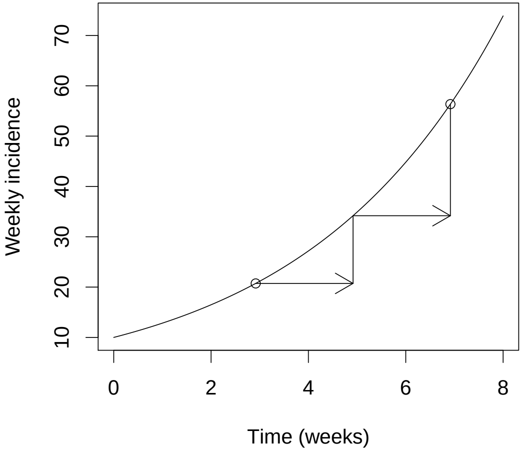

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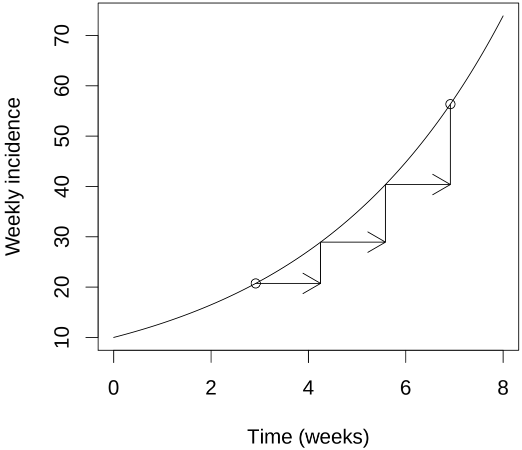

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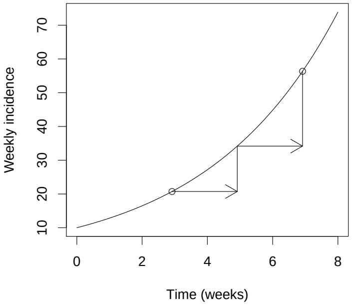

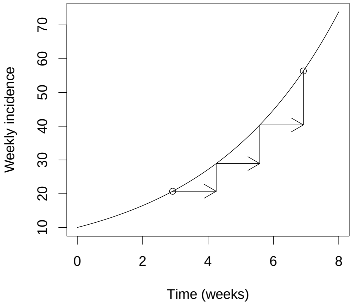

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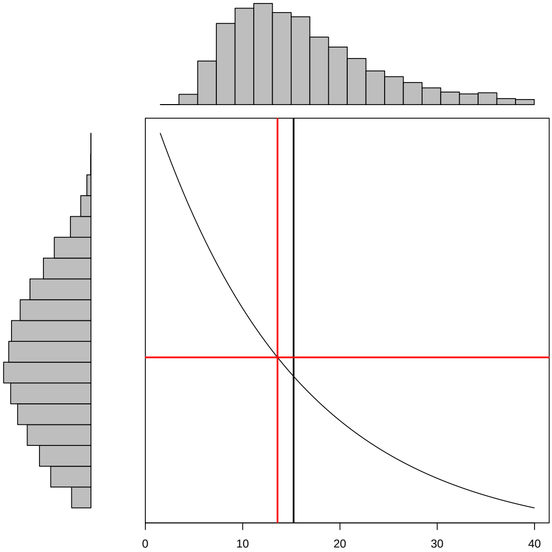
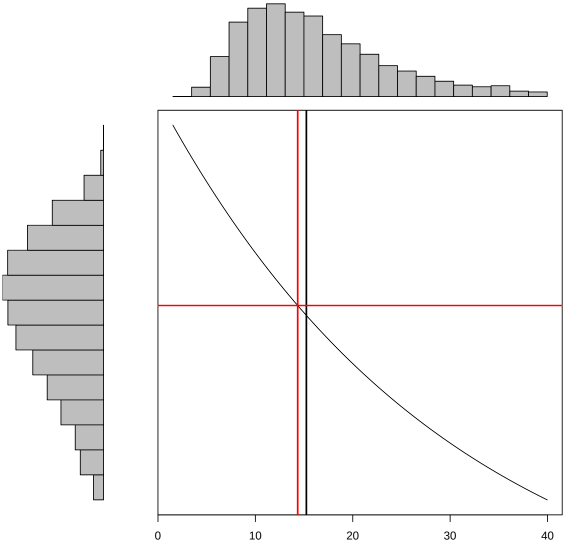
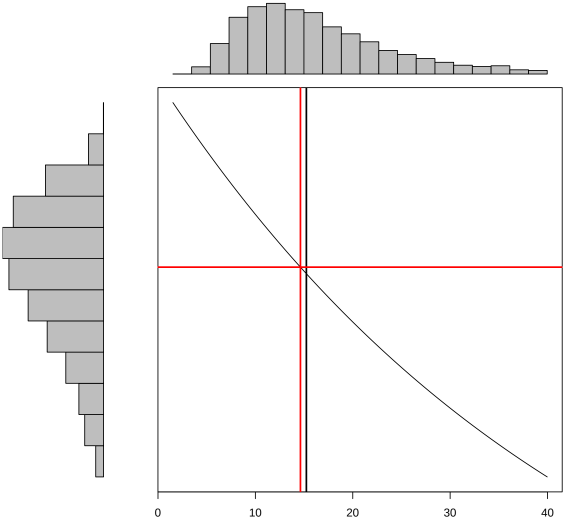


$$C = 1/r = 30d$$



$C = 1/r = 45d$



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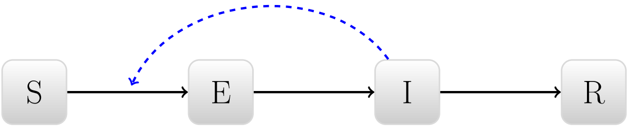

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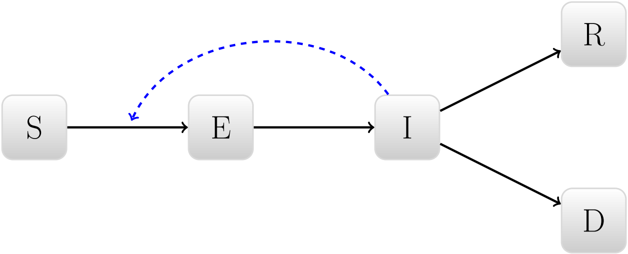

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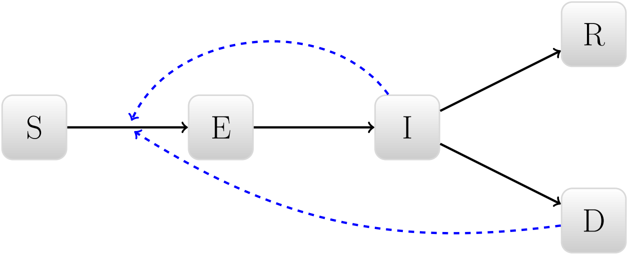
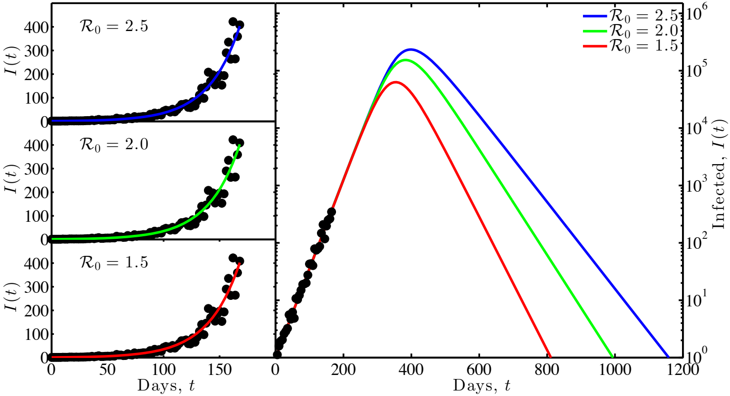


Scenarios



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$I(t)$
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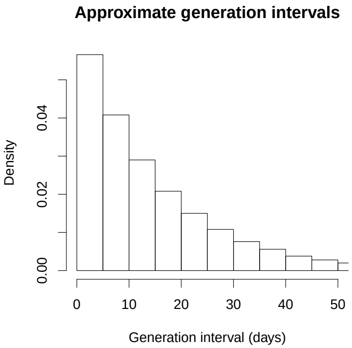

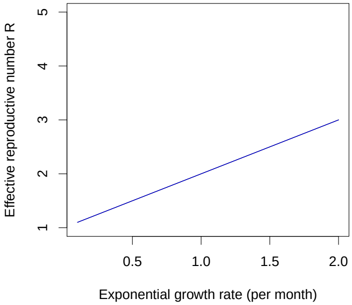

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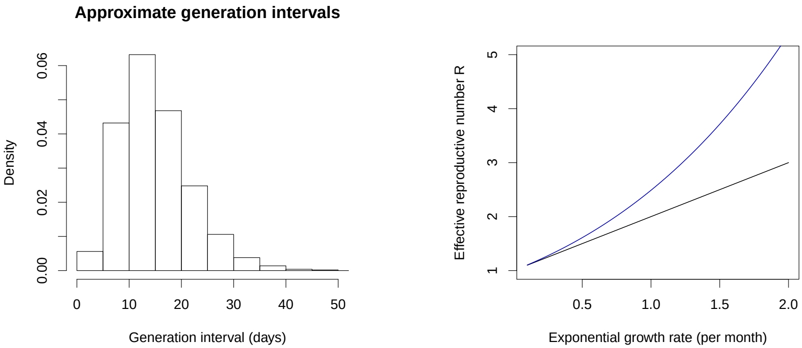

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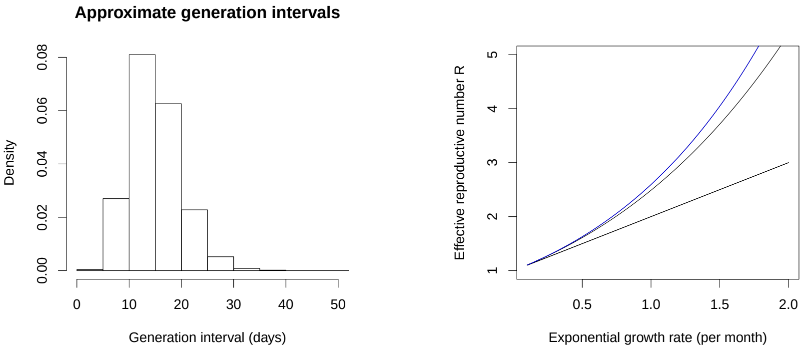

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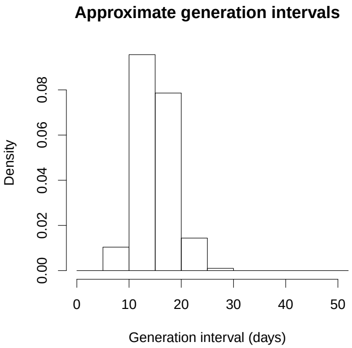

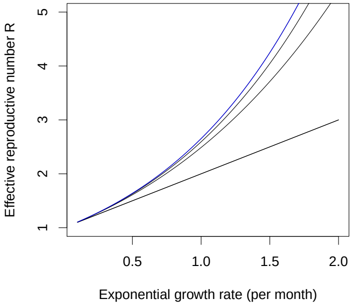

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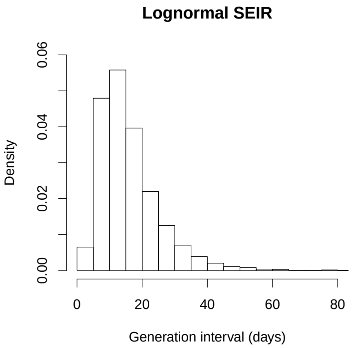

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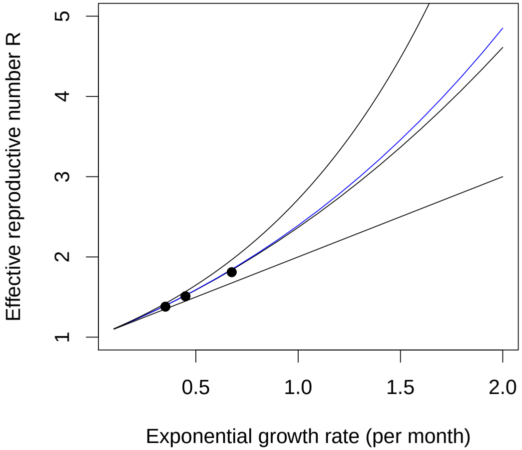

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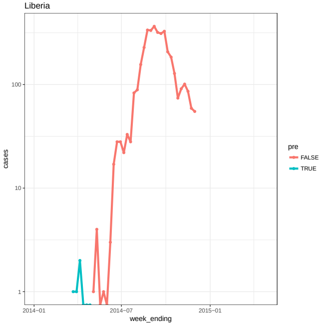

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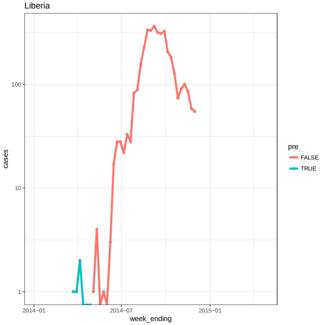

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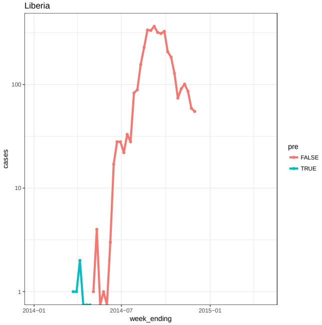

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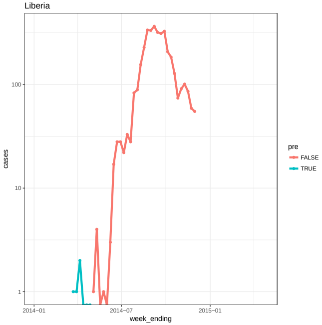

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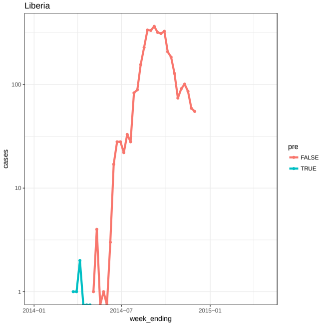
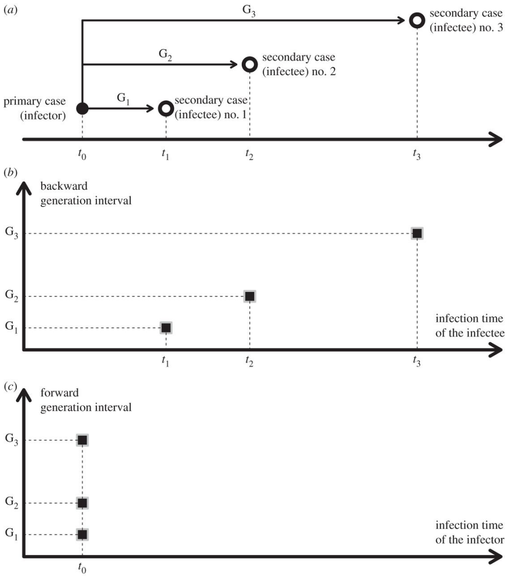


Forward and backward intervals



Champredon and Dushoff, 2015. DOI:10.1098/rspb.2015.2026

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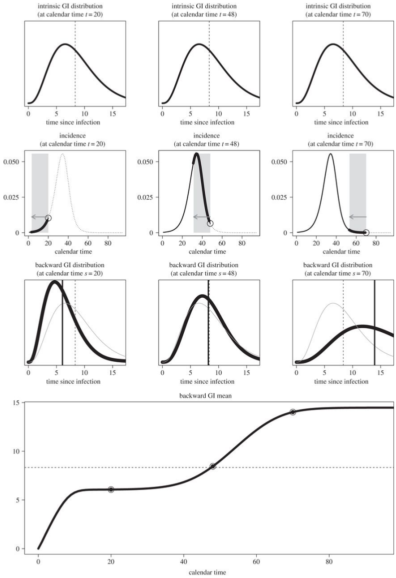

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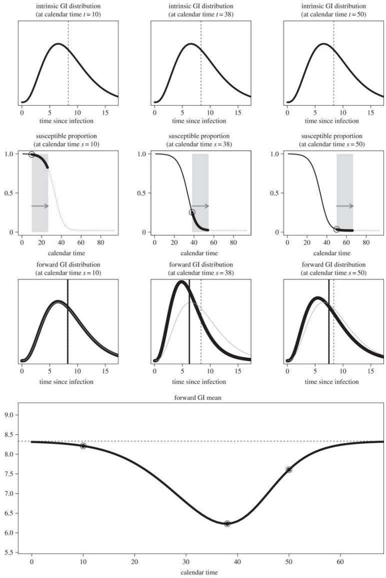
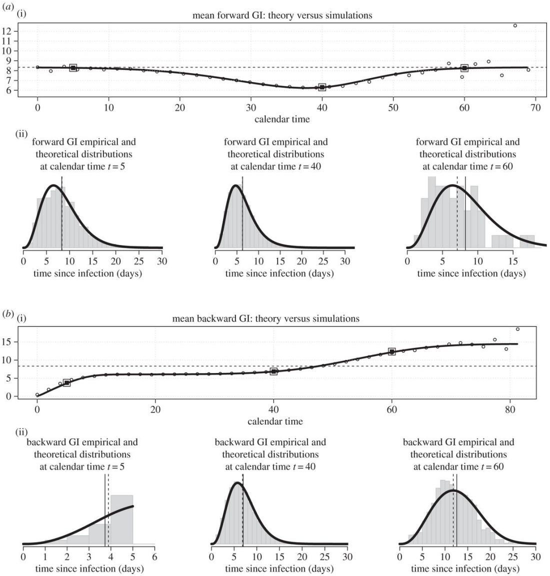


Comparison to simulations



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Outline

Strength: $\mathcal{R}$ – the reproductive number

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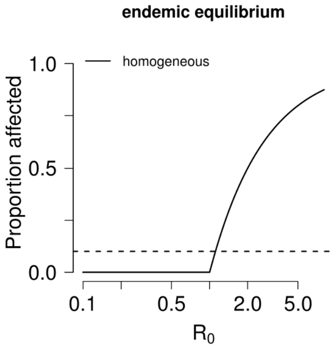

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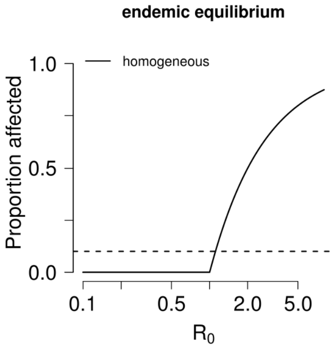

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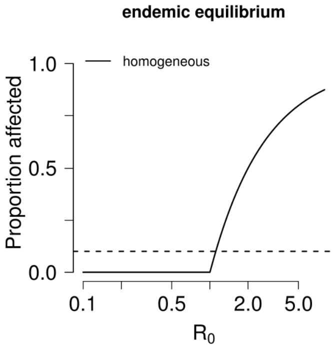

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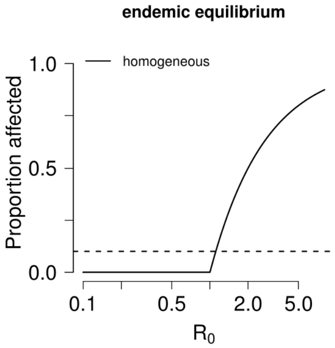

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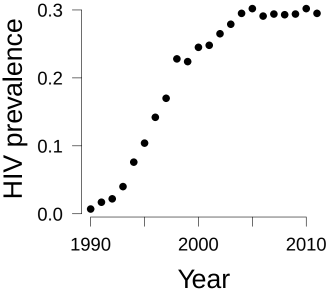

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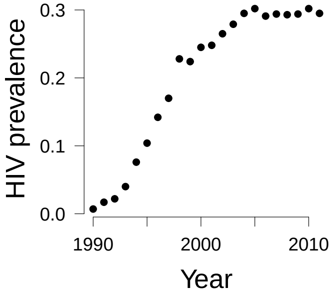

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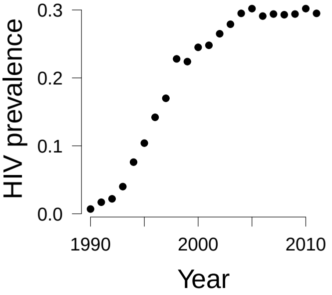

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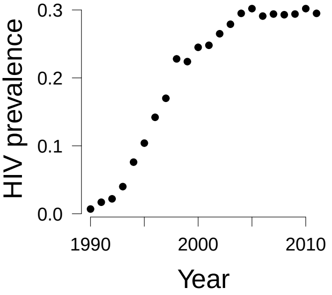

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Can treatment stop the HIV epidemic?

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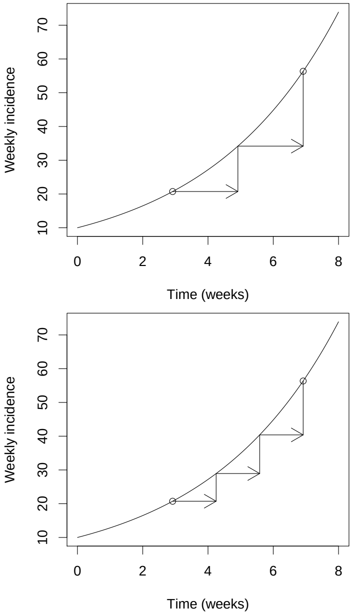

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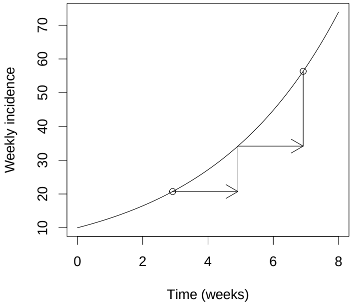

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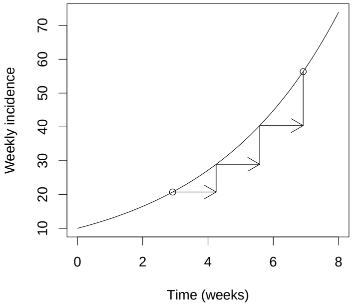

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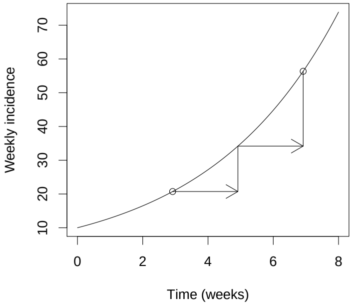

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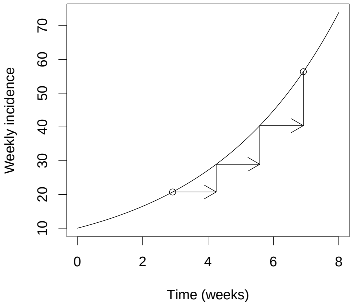

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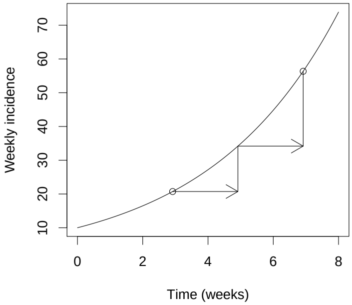

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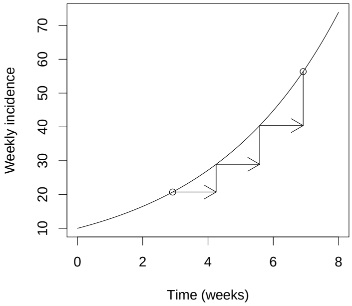

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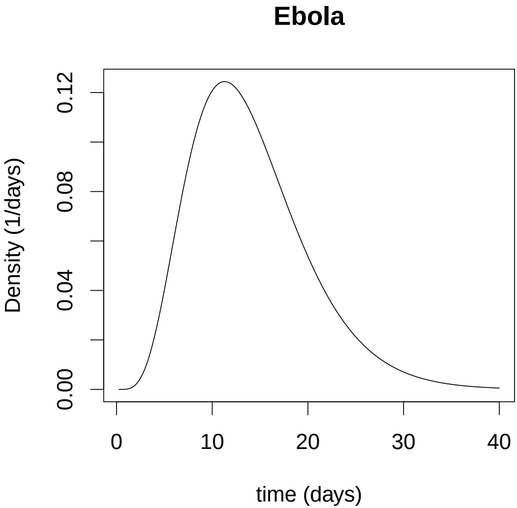

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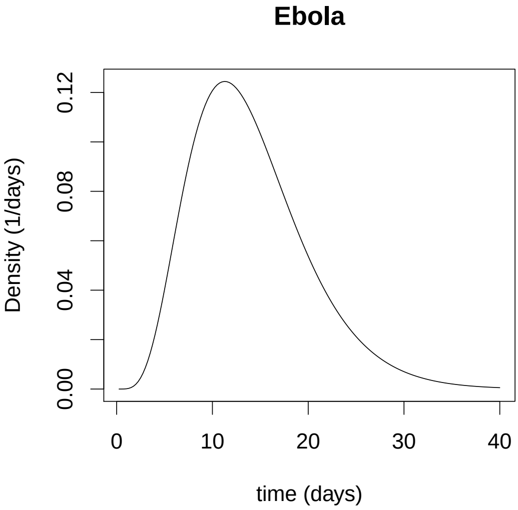

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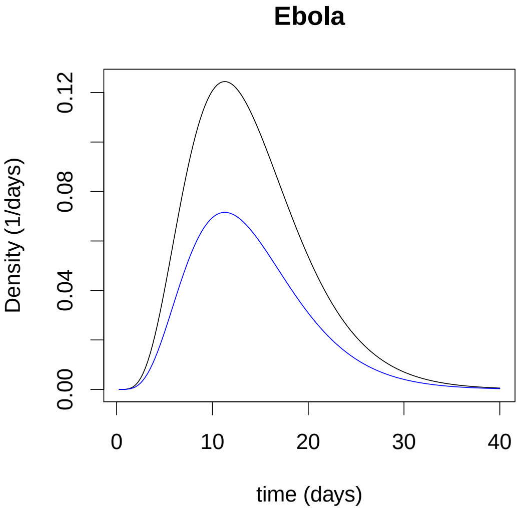

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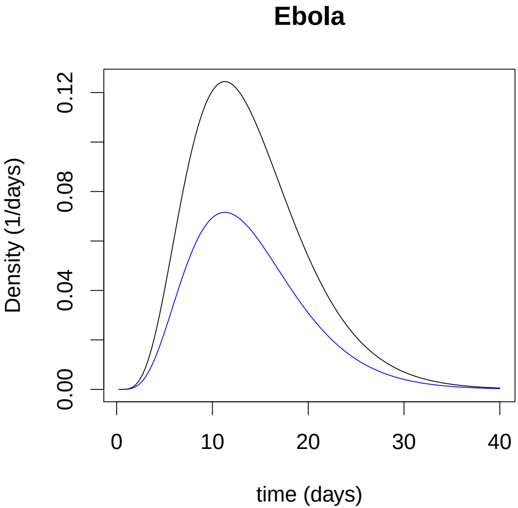

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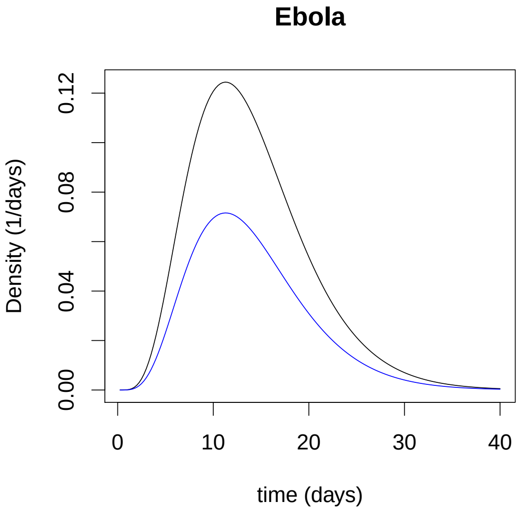

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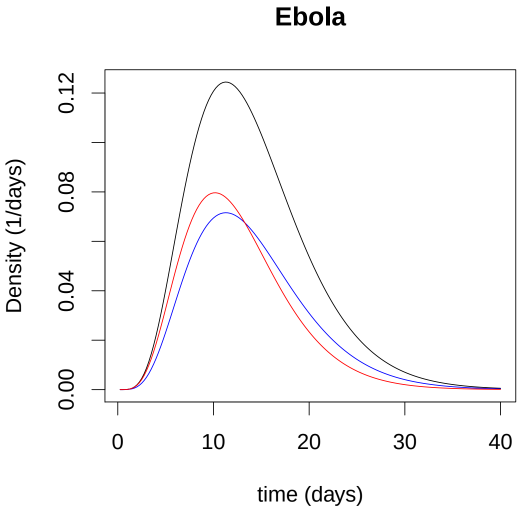

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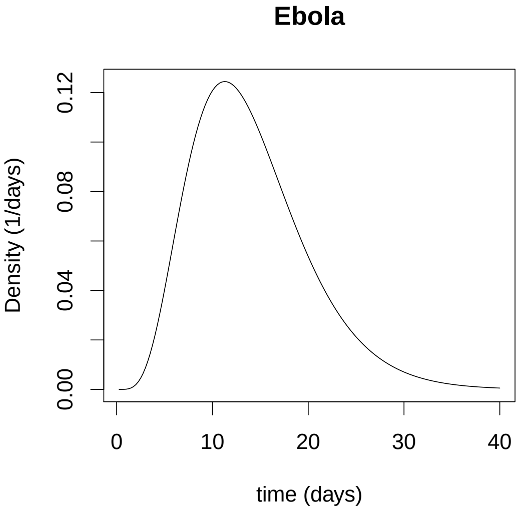

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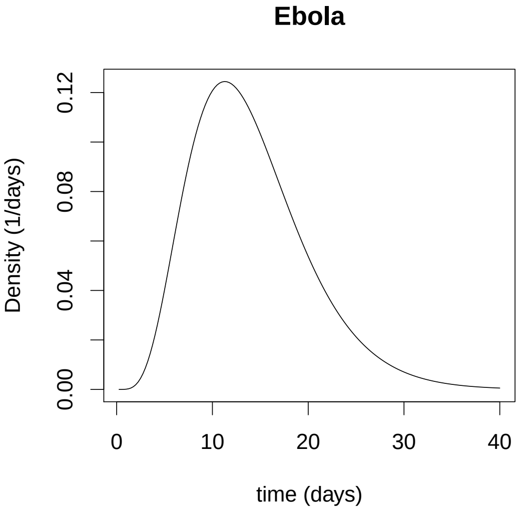

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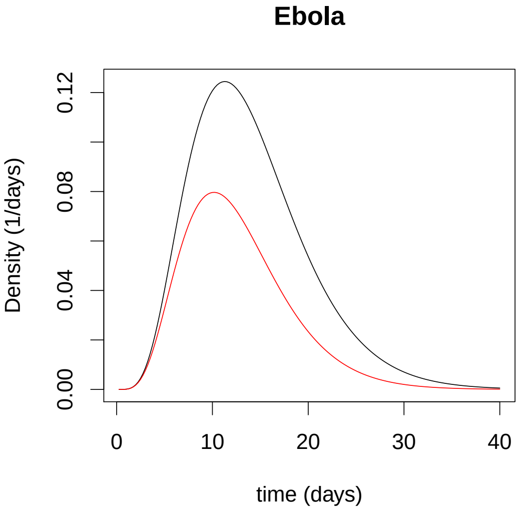

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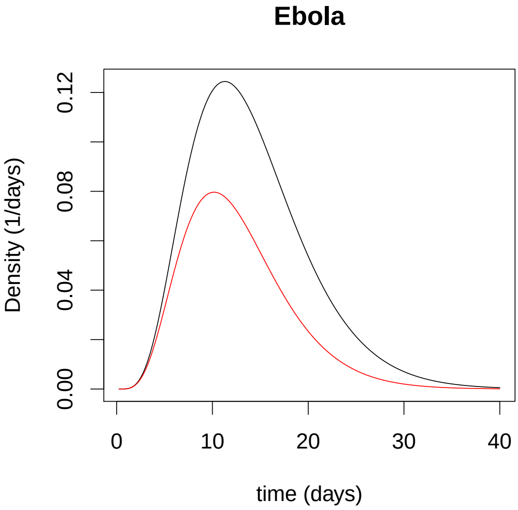

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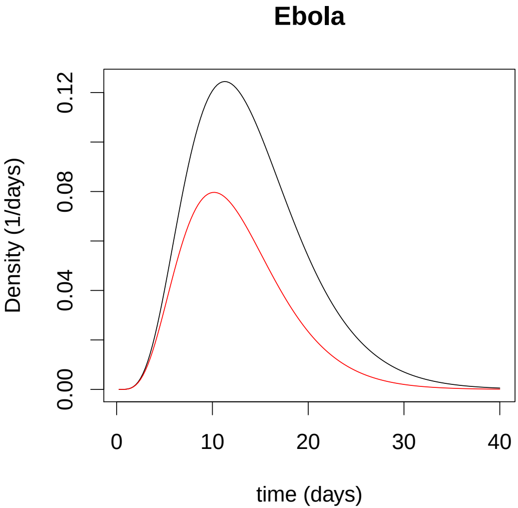

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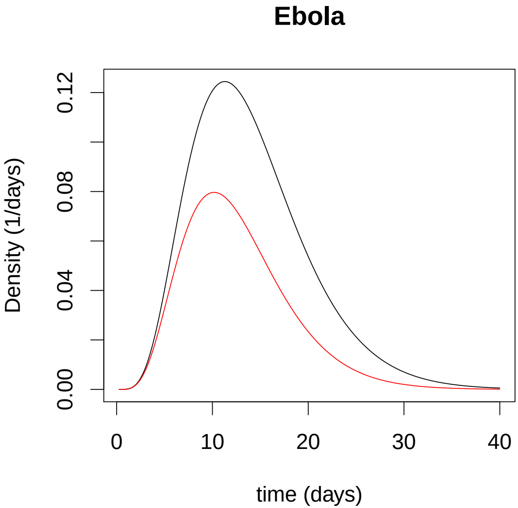

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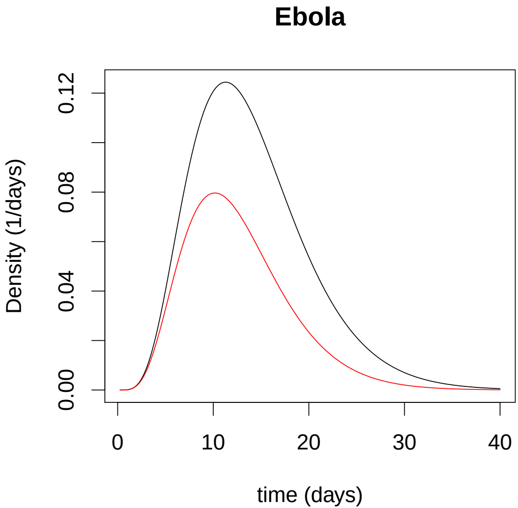

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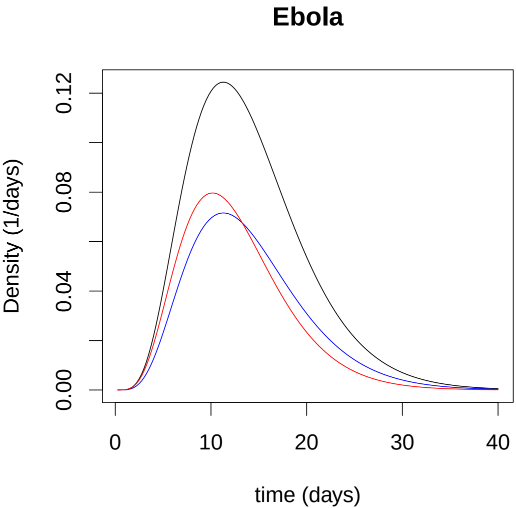

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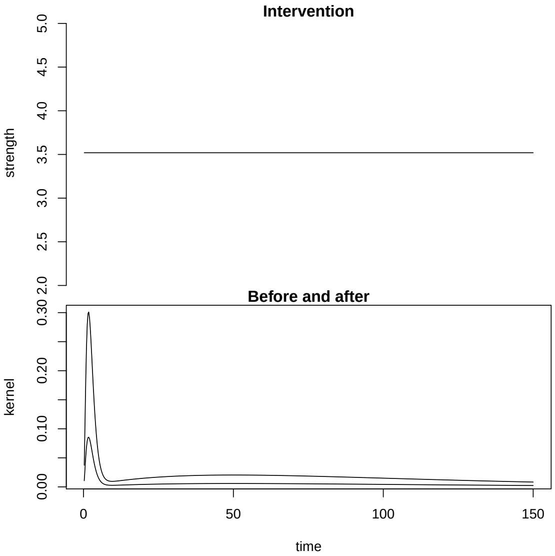
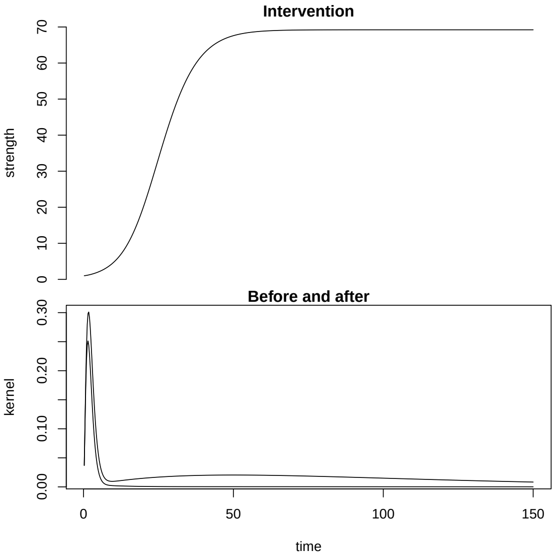
Outline

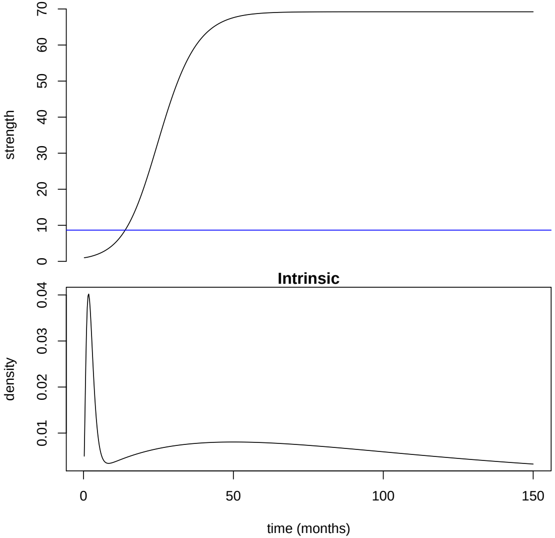
Condom intervention



Find and treat



Low early transmission



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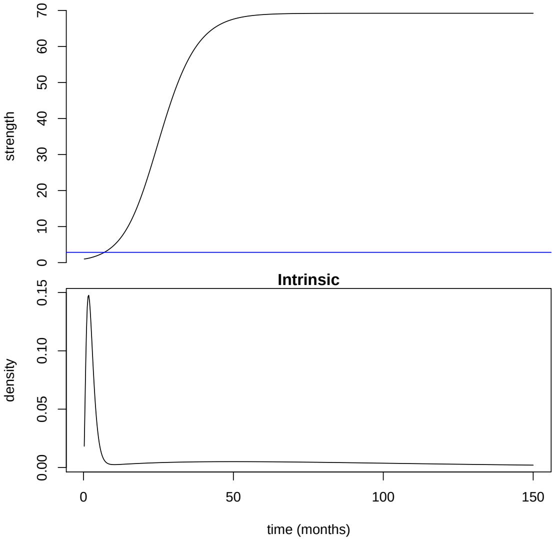
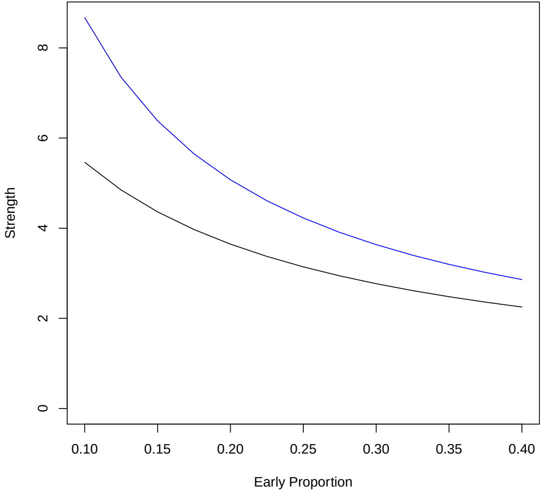


Range of estimates



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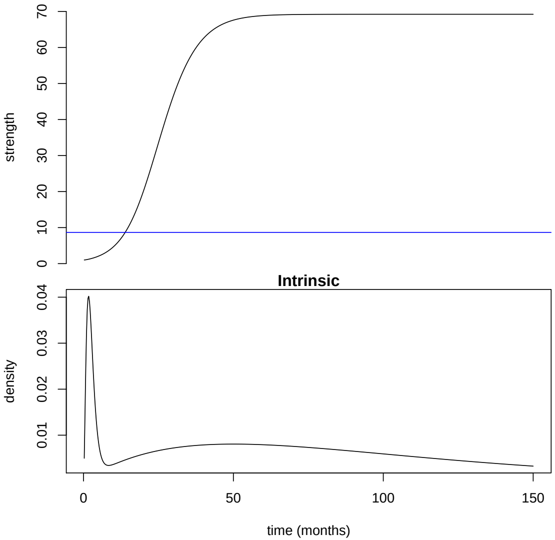

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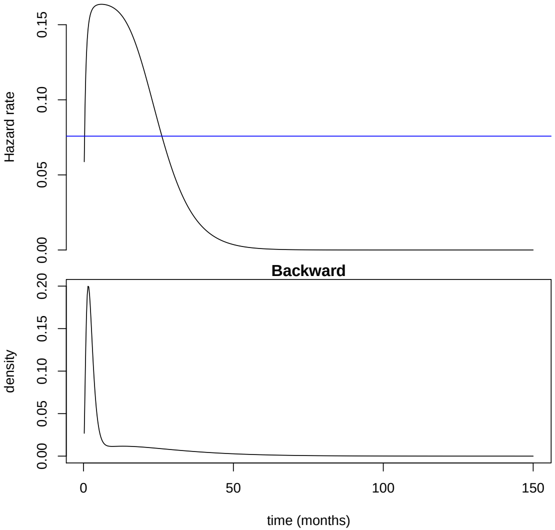

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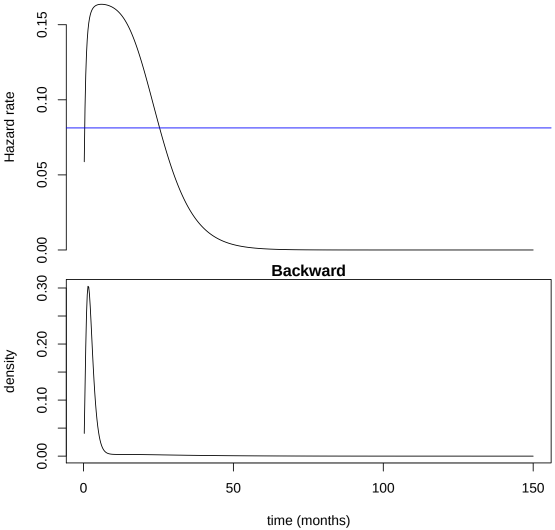

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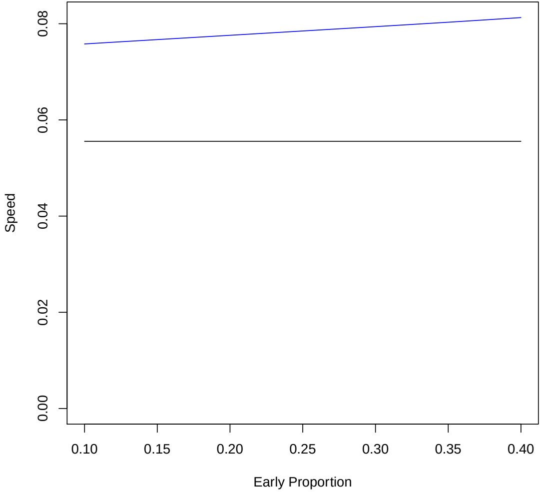
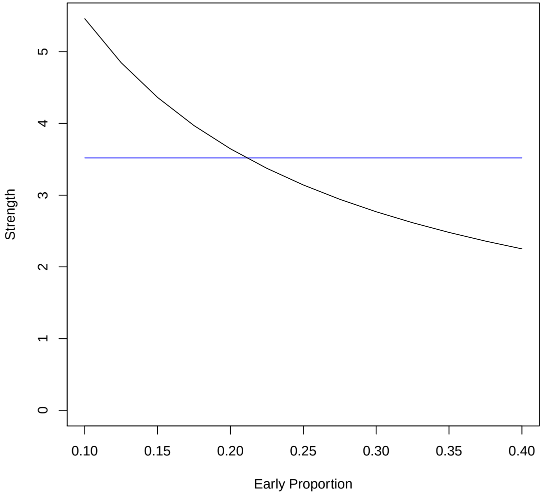


Condom intervention



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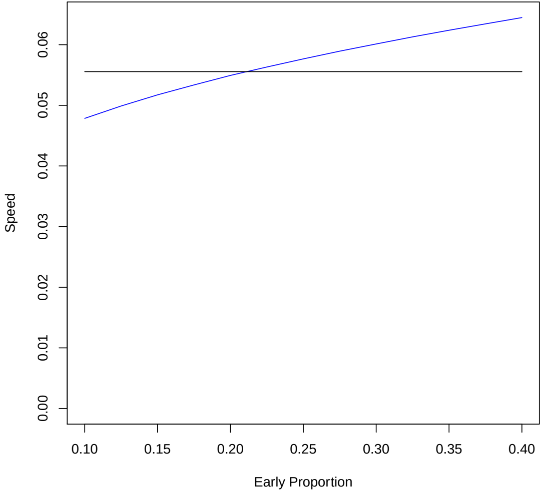
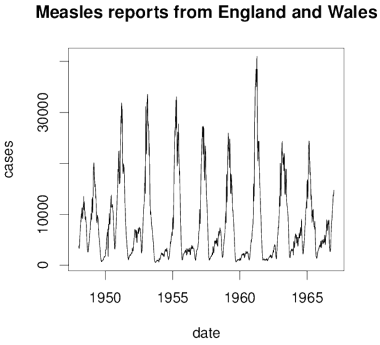
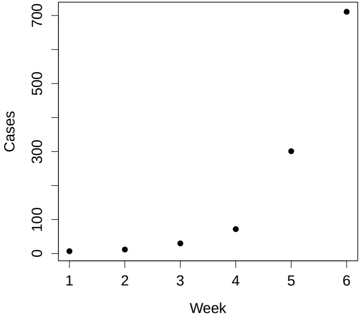


Outline

Ways of looking

Measuring the epidemic





Measuring the intervention

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Thanks

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- Audience

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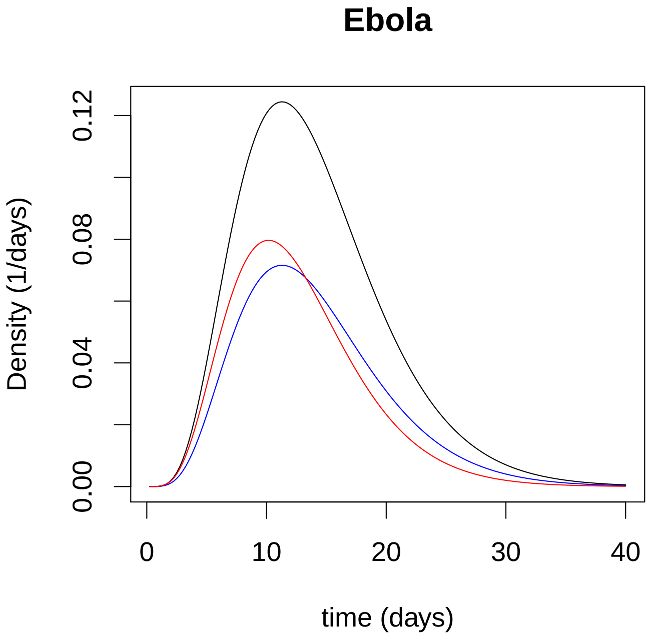
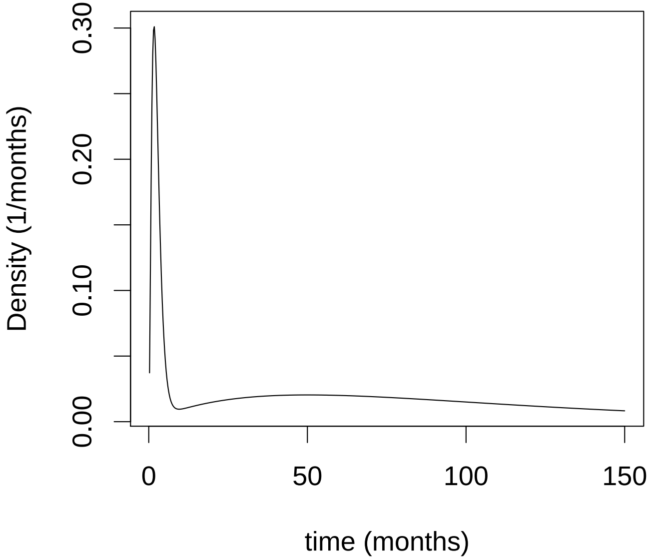
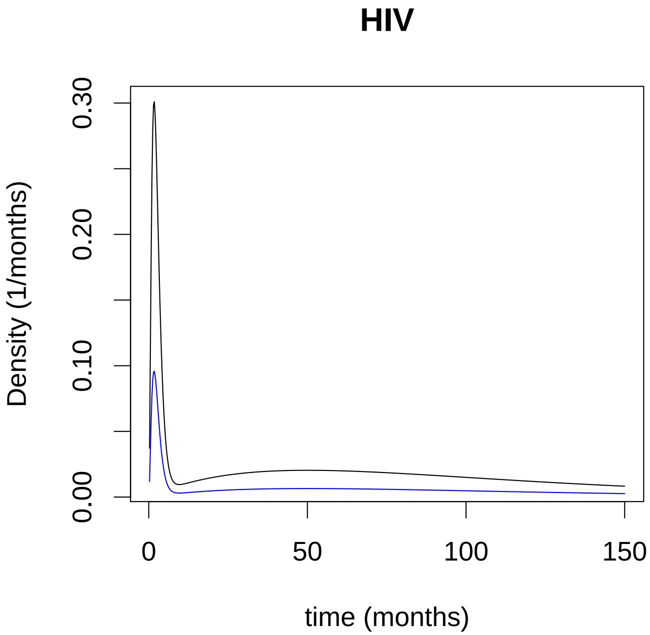
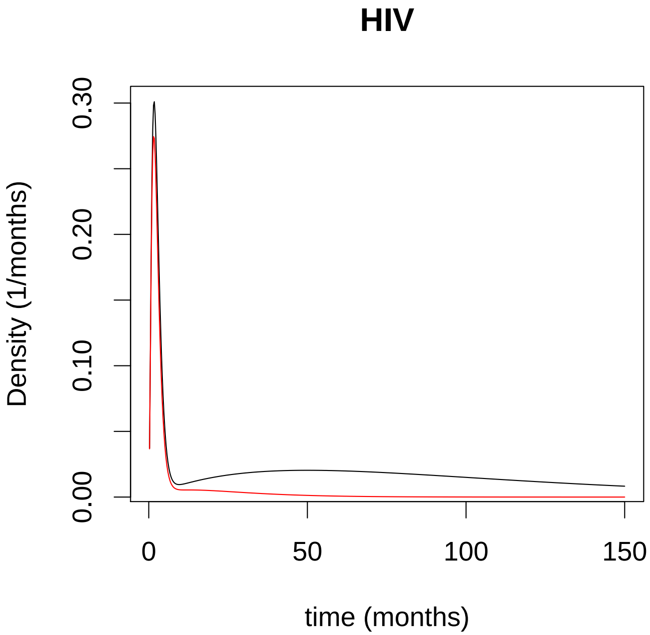

Ebola
Density (1/days)
0.00
0.04
0.08
0.12
0
10
20
30
40
time (days)

HIV




HIV
Density (1/months)
0.00
0.10
0.20
0.30
0
50
100
150
time (months)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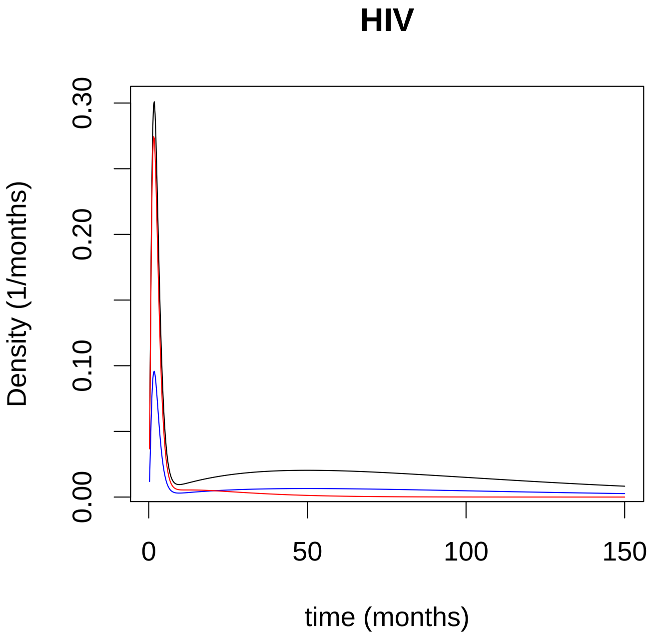
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