Epidemic strength and speed: rethinking metrics for infectious disease spread and control.

SMB 2017

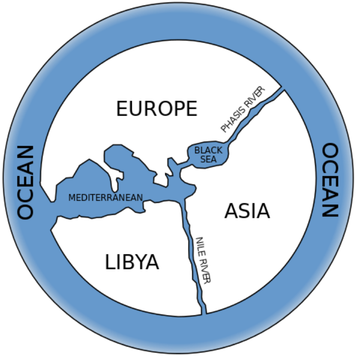
Jonathan Dushoff

http://lalashan.mcmaster.ca/DushoffLab
http://www.ici3d.org
https://github.com/dushoff/Generation_talks

Outline

Infectious diseases

Models



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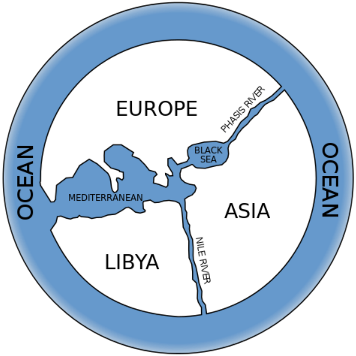

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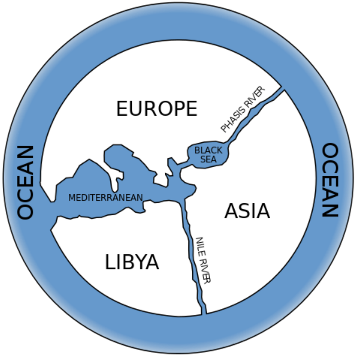

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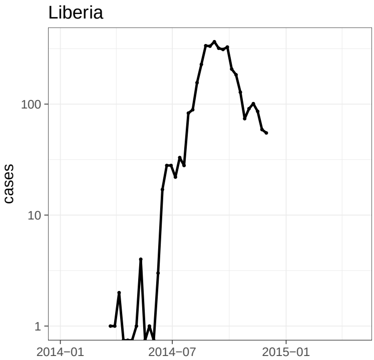
Ebola

Dynamic modeling connects scales



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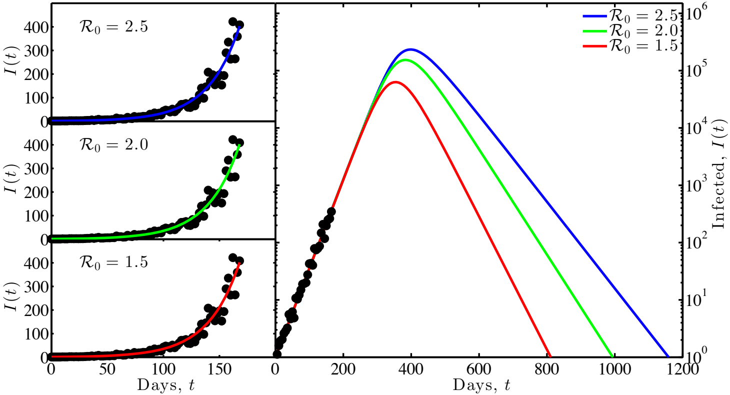

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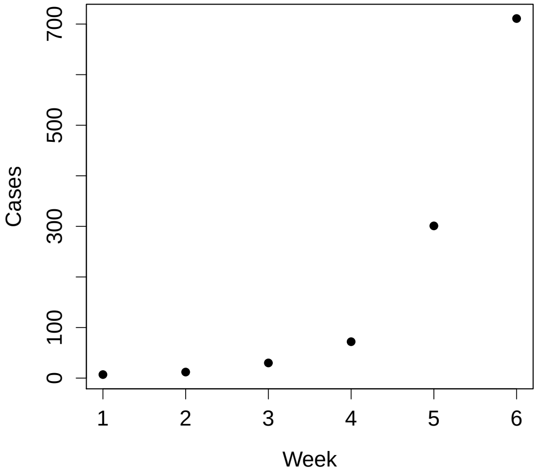

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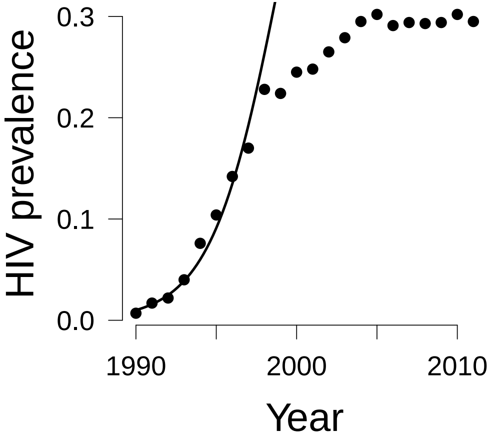

$C \approx 18\,\mathrm{month}$. Horrifyingly fast.

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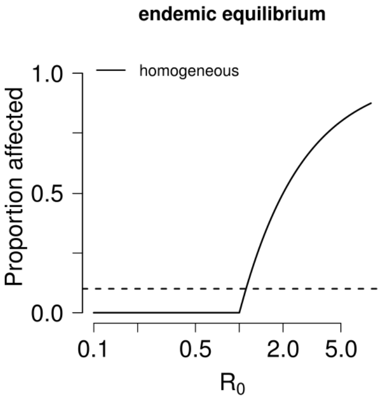

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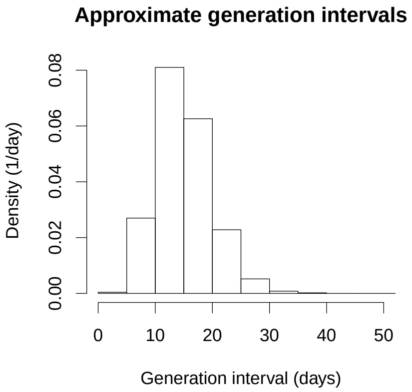

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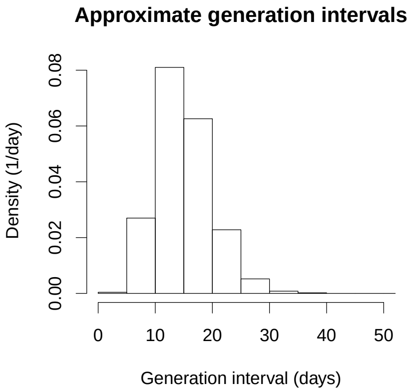

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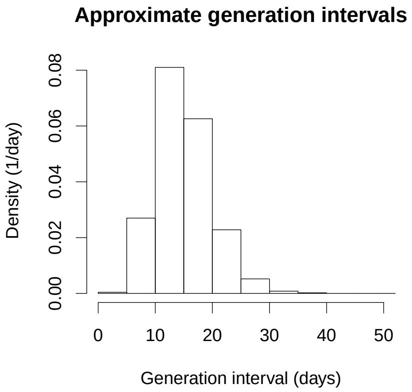

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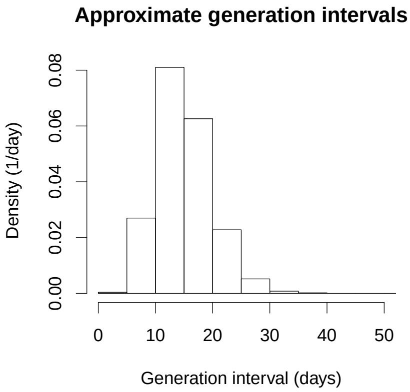

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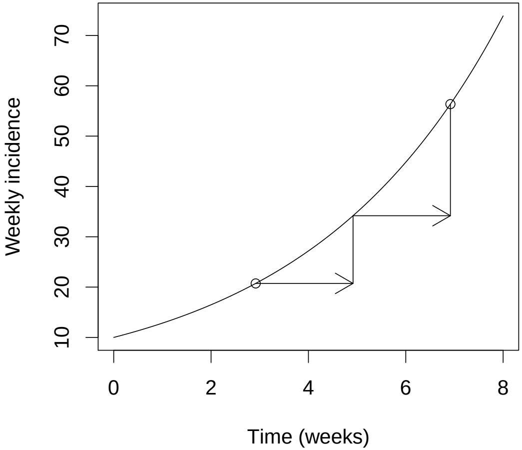

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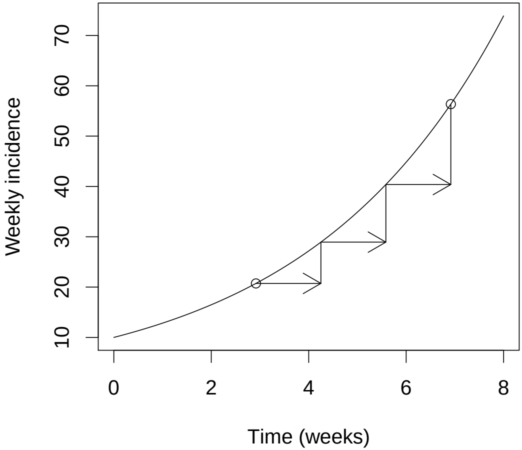

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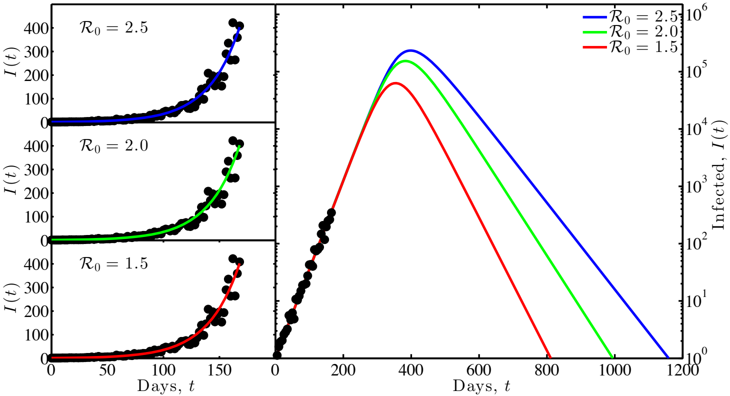

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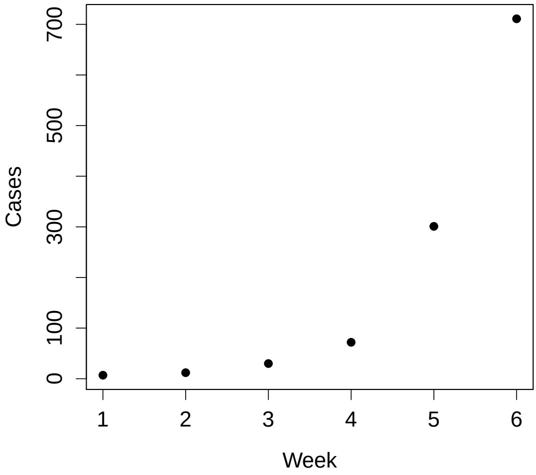

$C \approx 1\,\text{week},\ G \approx 3\,\text{day}$

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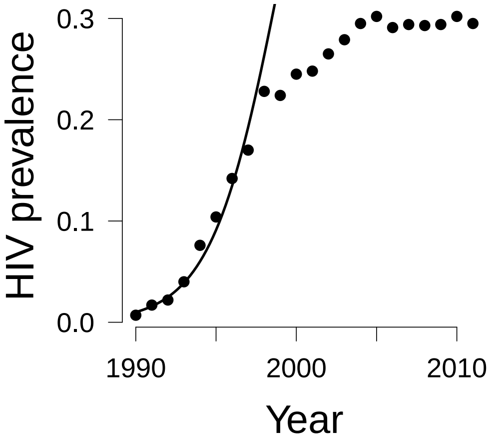

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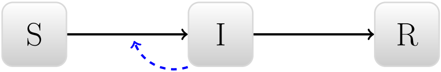

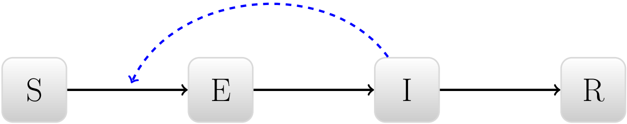

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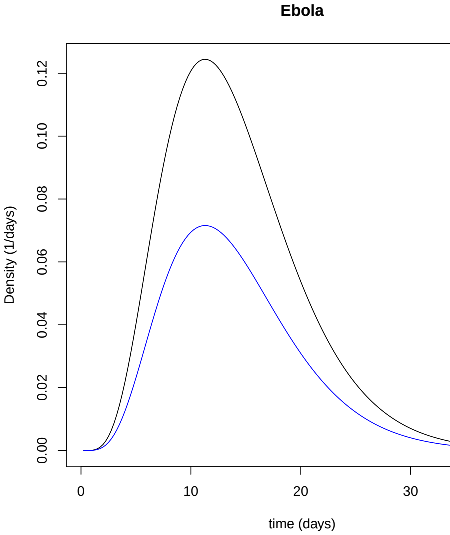

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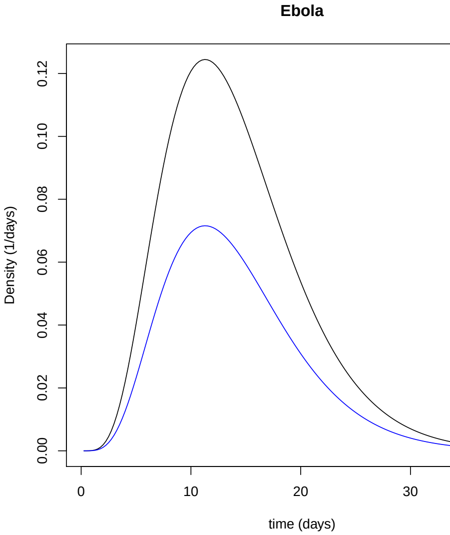

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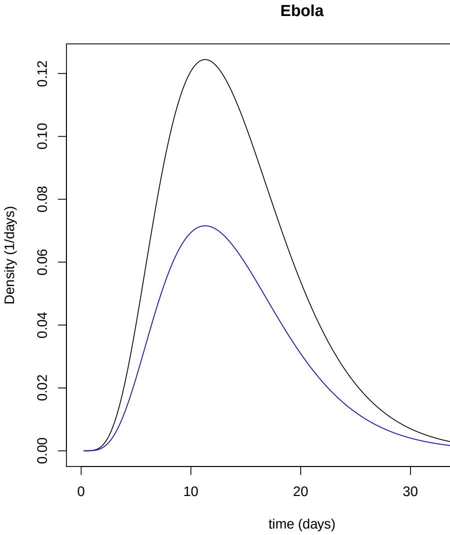

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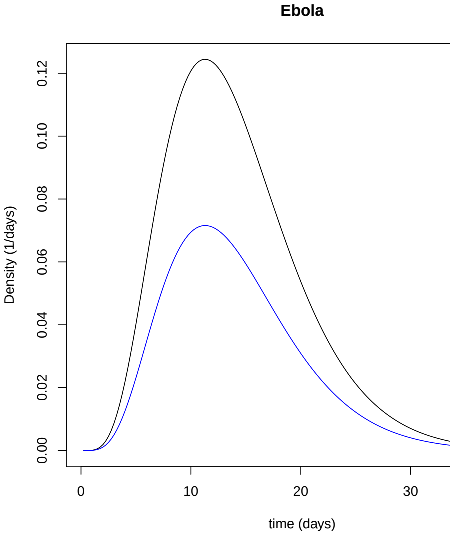

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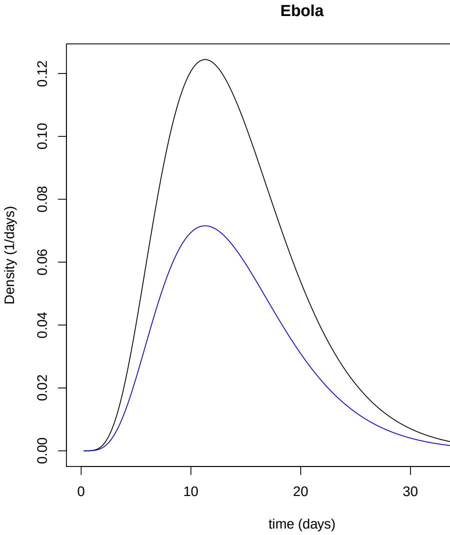

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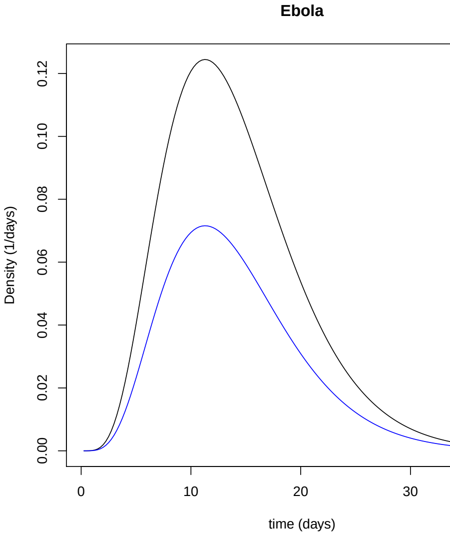

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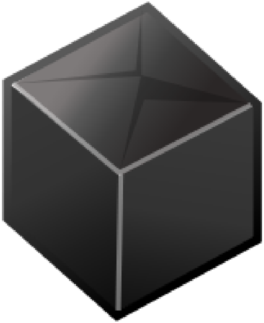

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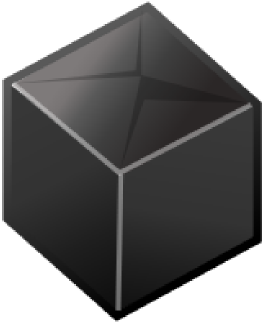

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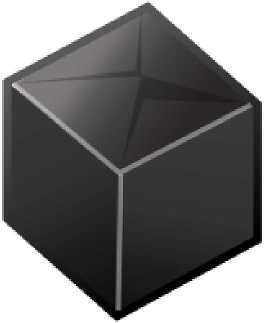

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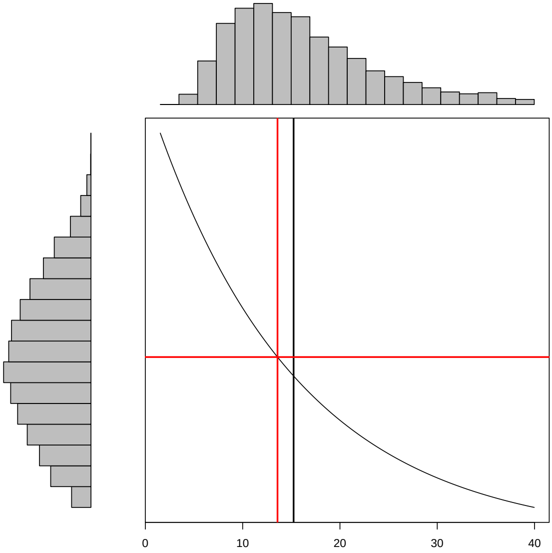
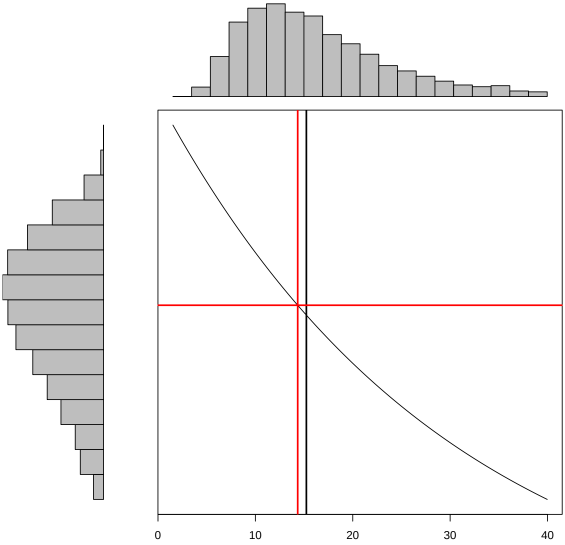
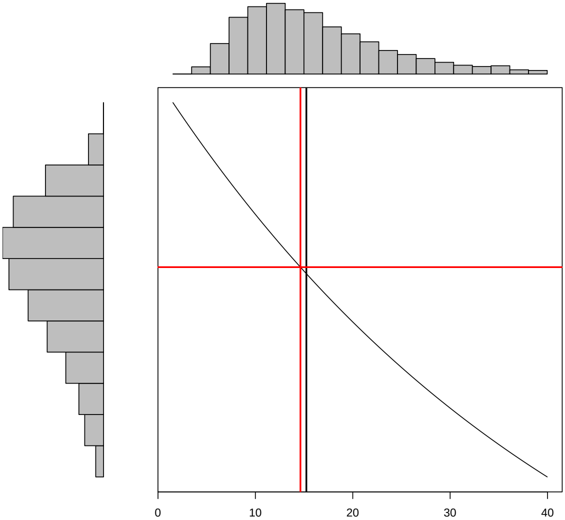


$C = 1/r = 30d$



$C = 1/r = 45d$



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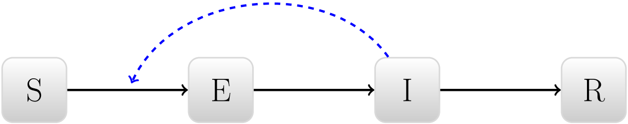

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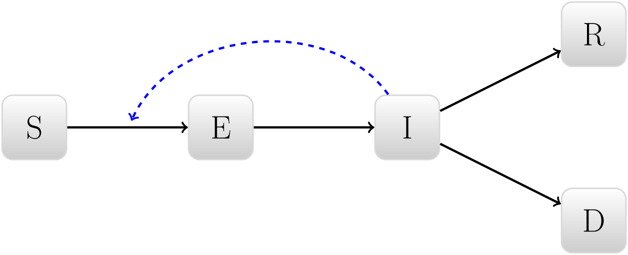

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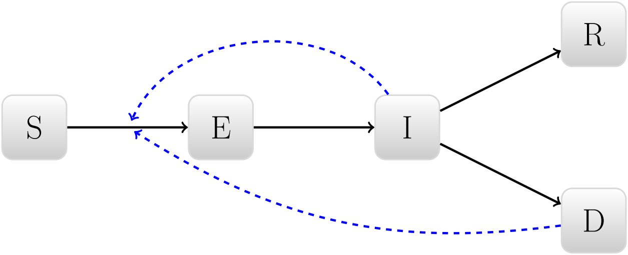
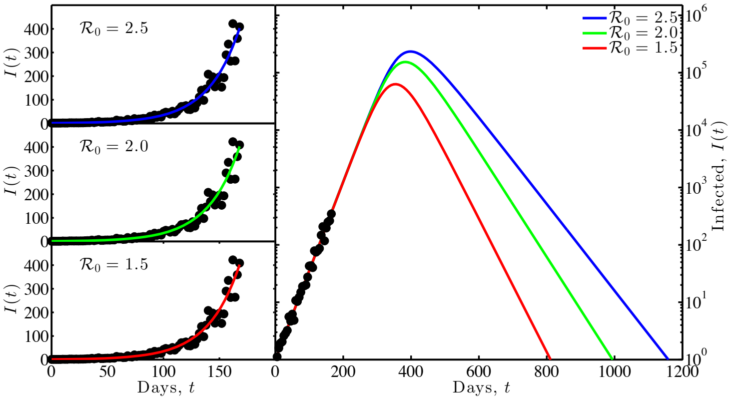


Scenarios



$\mathcal{R}_0 = 2.5$
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$I(t)$
0
100
200
300
400
0
50
100
150
Days, t
0
200
400
600
800
1000
1200
10^0
10^1
10^2
10^3
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10^5
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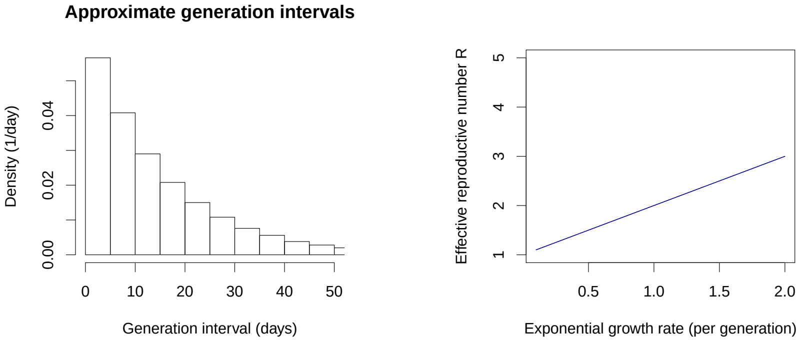

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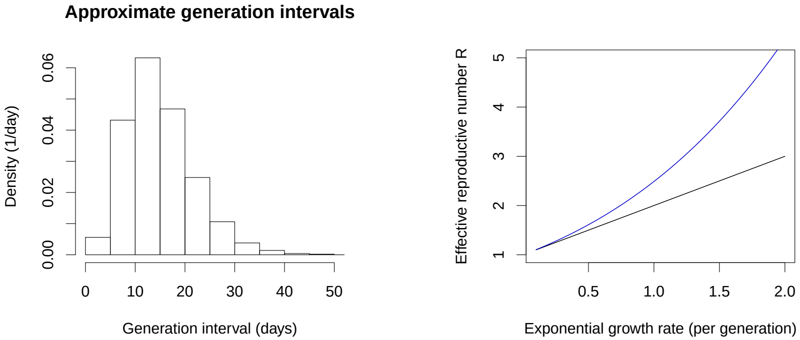

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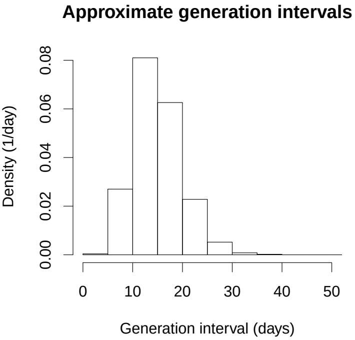

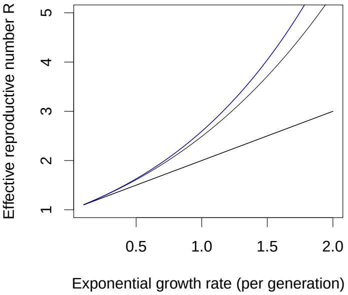

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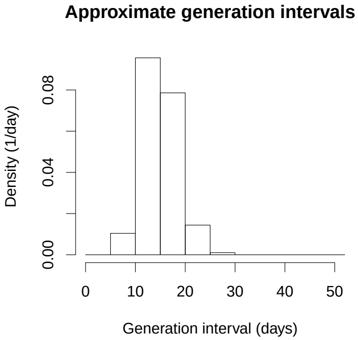

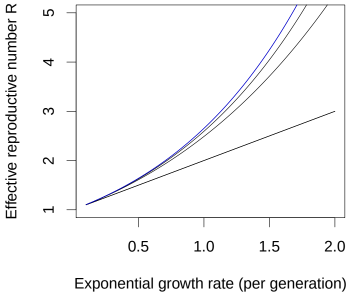

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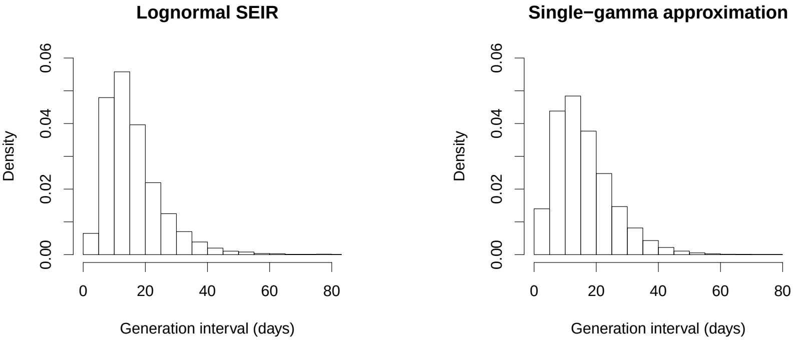

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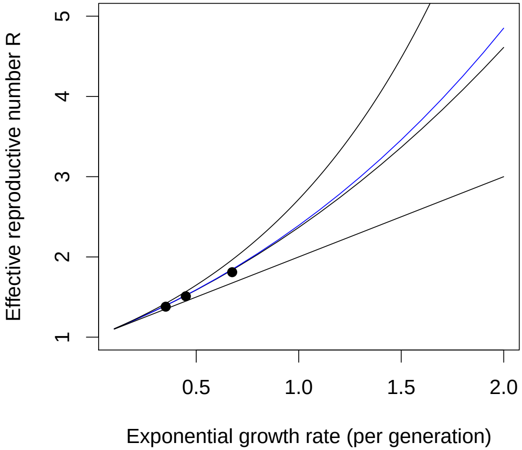

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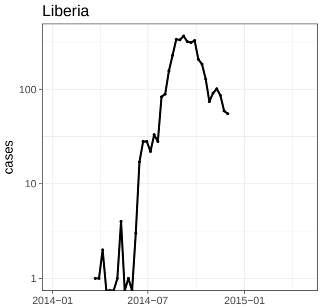

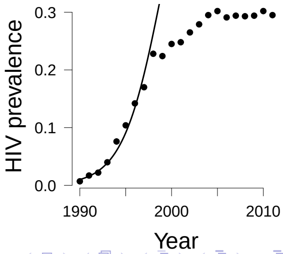

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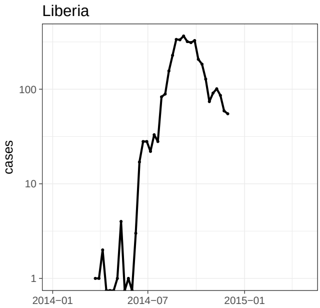

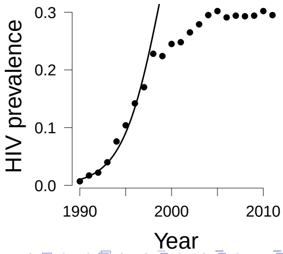

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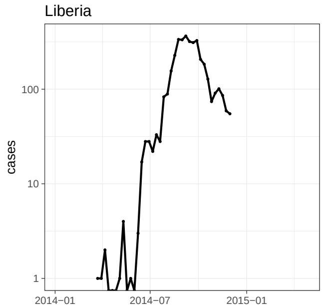

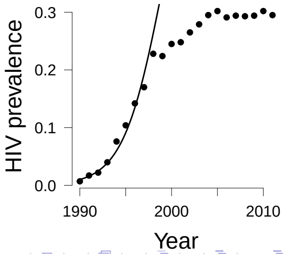

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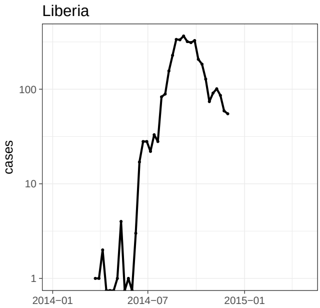

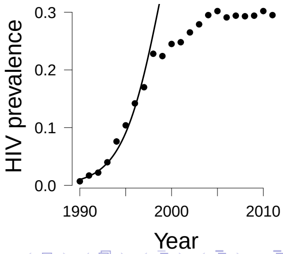

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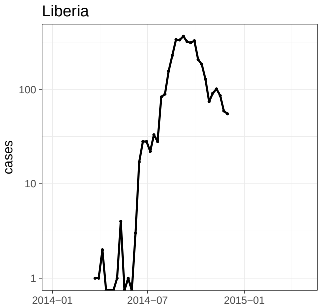

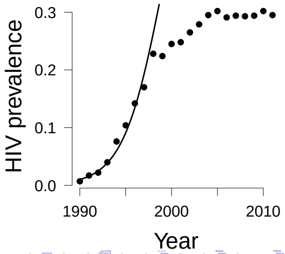

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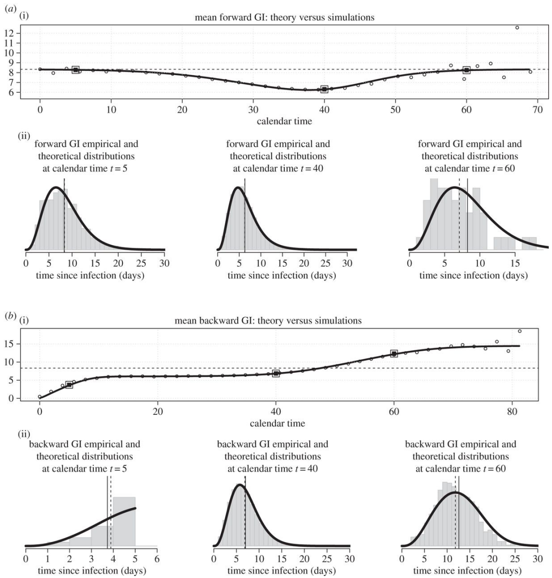

Champredon and Dushoff, 2015. DOI:10.1098/rspb.2015.2026

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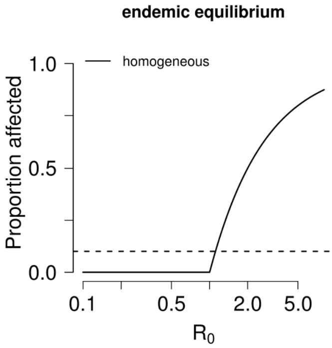
- Linking strength and speed
  - Generation intervals
  - "Effective" generation times
  - Moment approximations
- Generation intervals through time
- Strength and Speed of Epidemics
  - Intervention strength
  - Intervention speed
  - HIV example
  - Ways of looking

Strength: $\mathcal{R}$ – the reproductive number

- Expected number of new cases per cases



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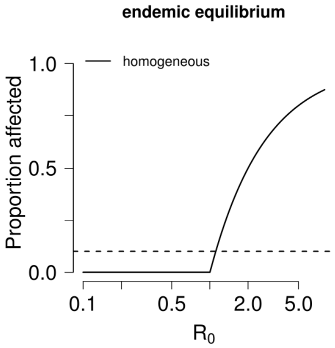

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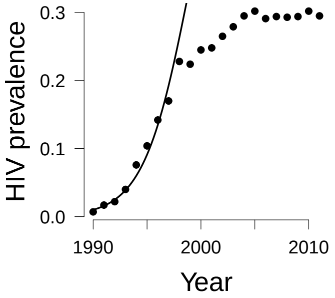

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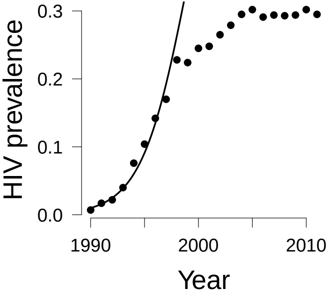

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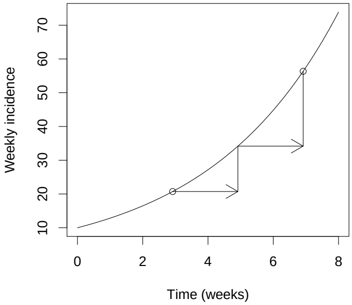

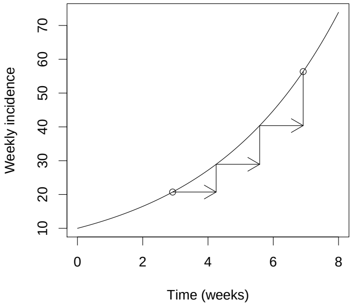

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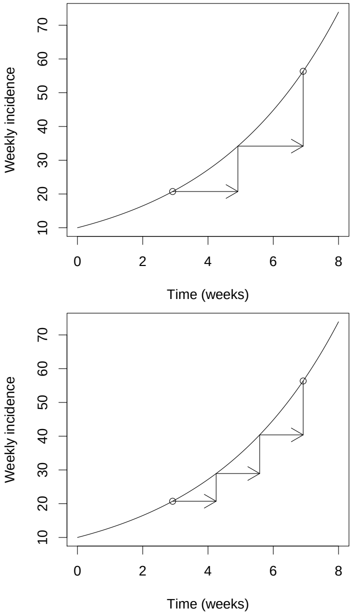

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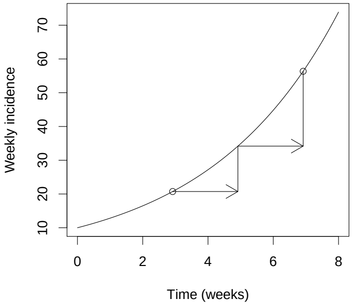

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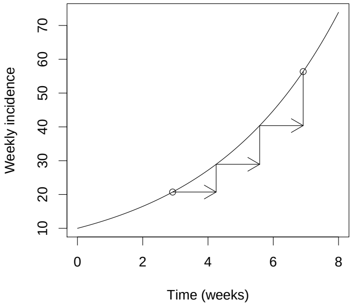

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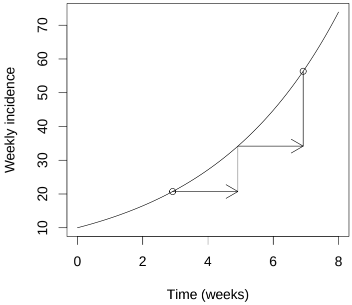

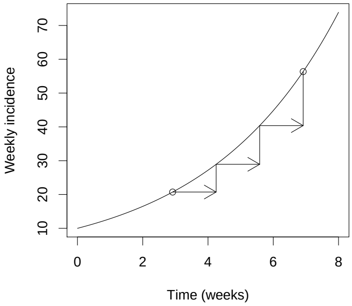

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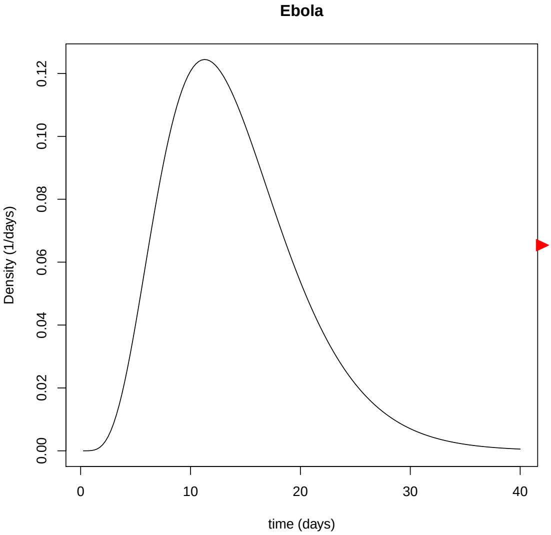

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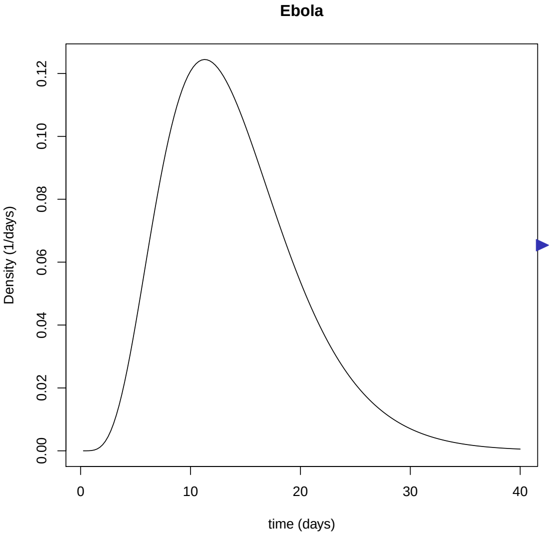

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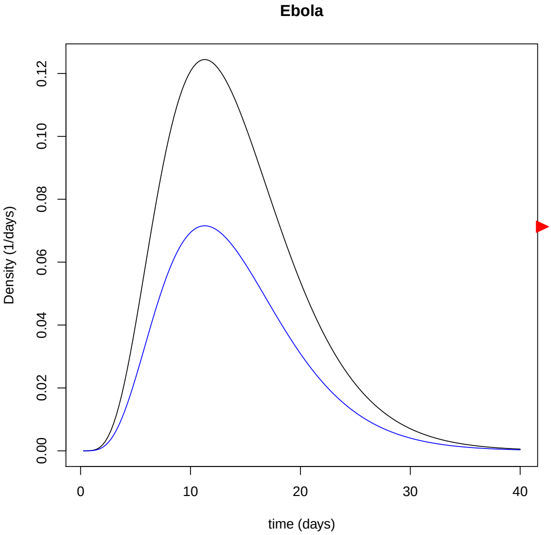

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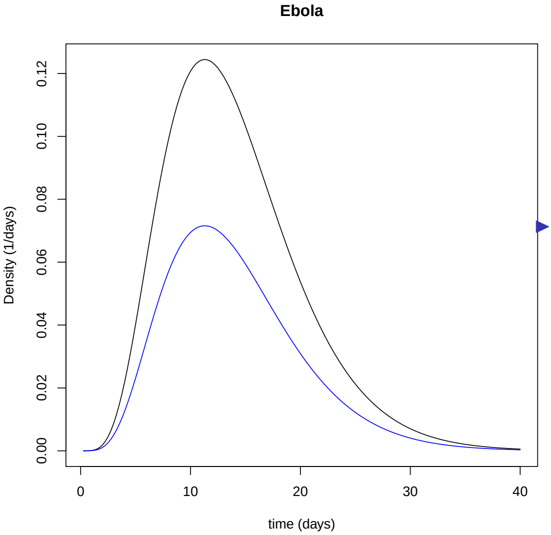

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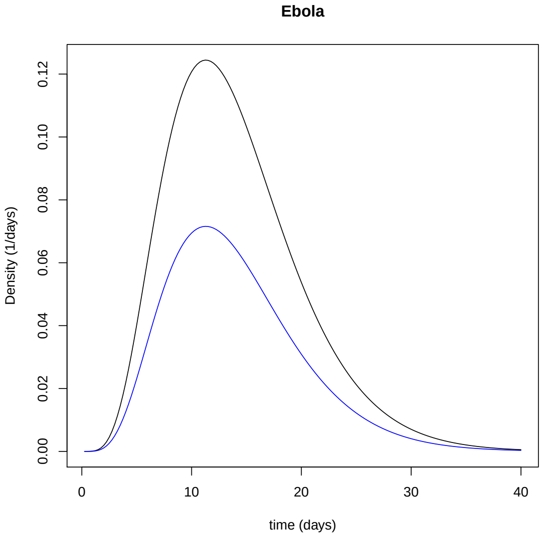

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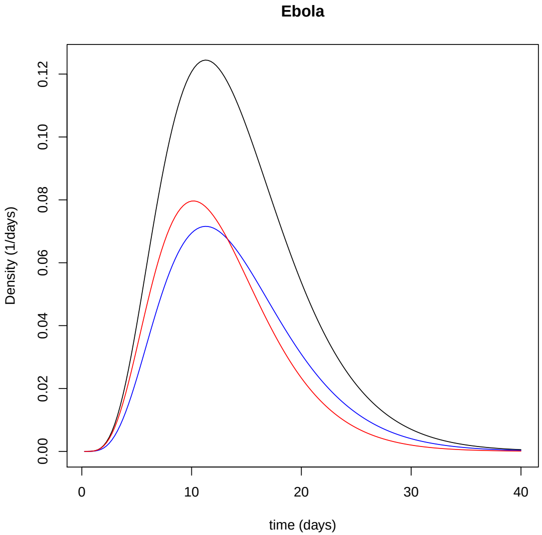

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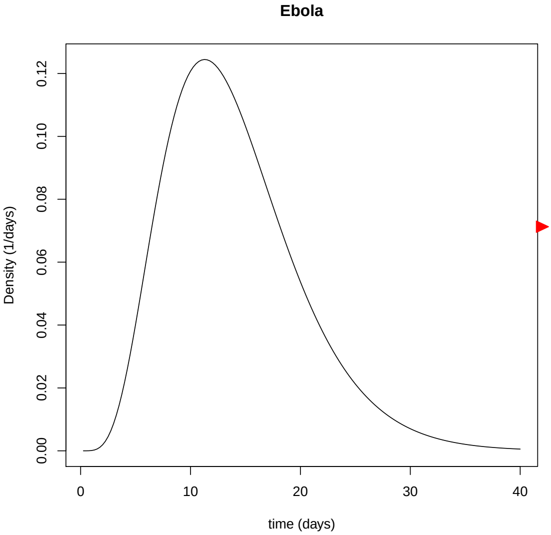

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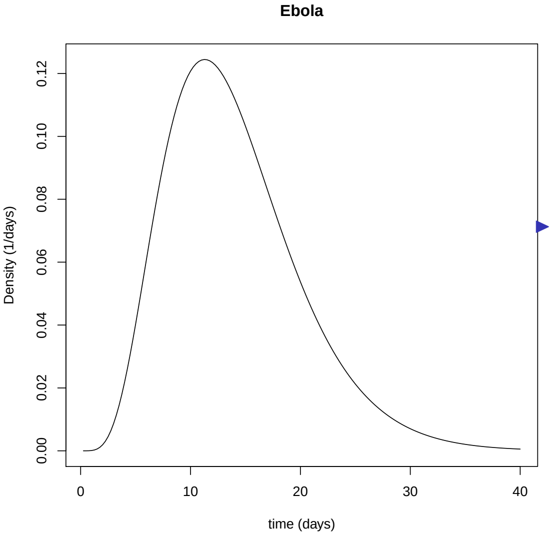

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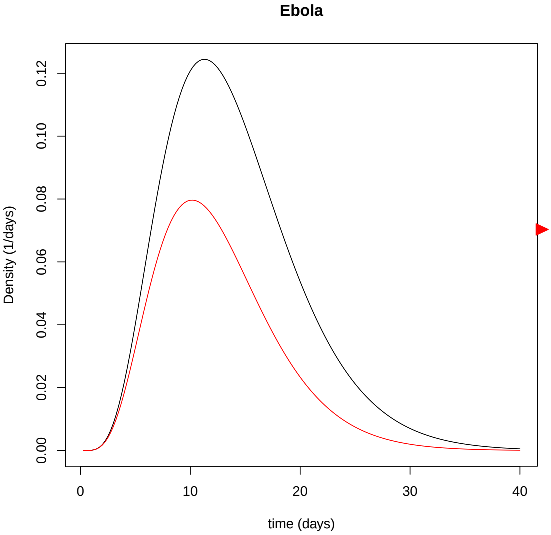

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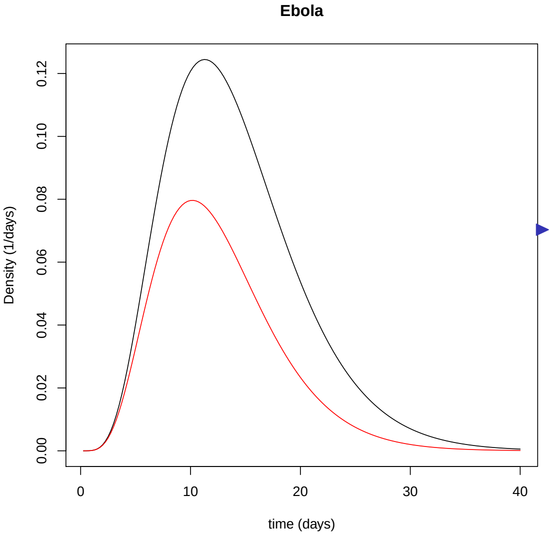

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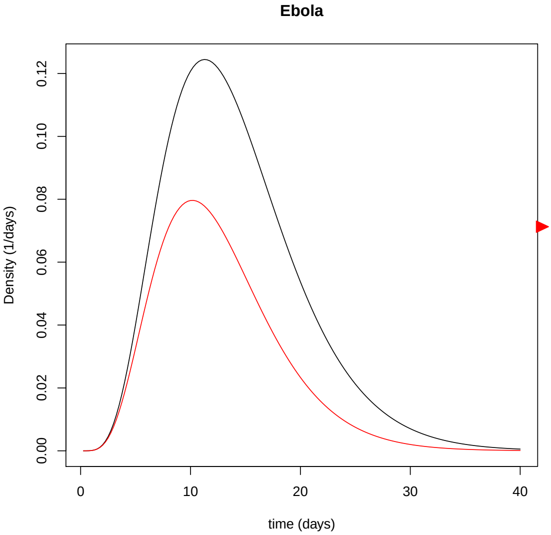

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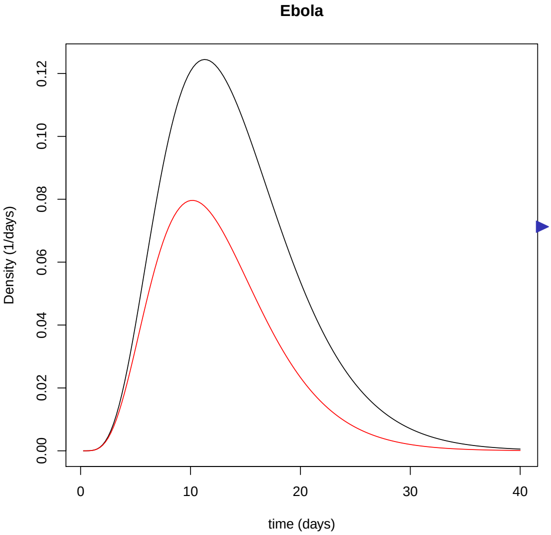

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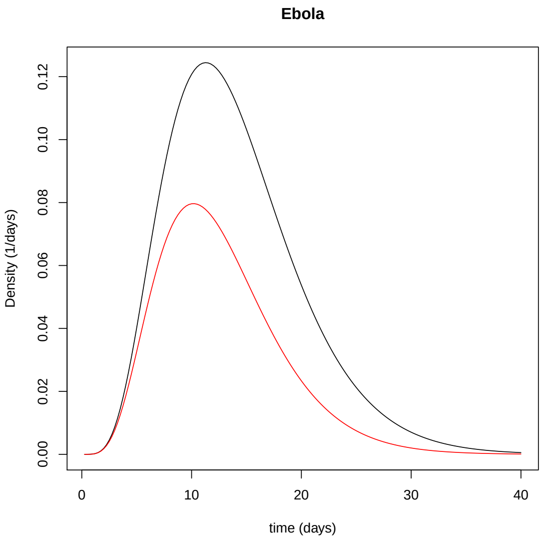

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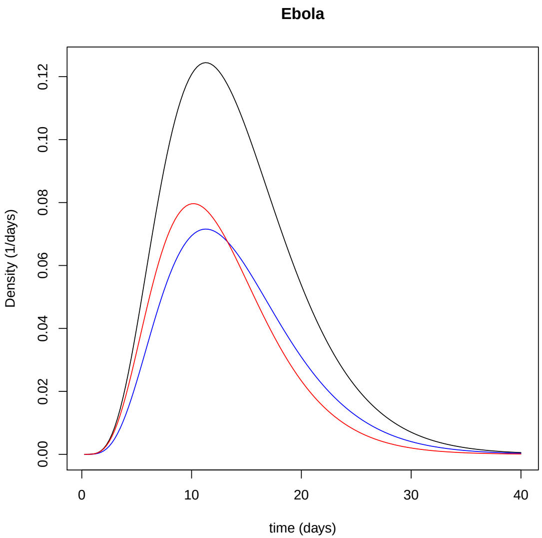

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- In general, ϕ is a (weird) mean of h

The speed paradigm

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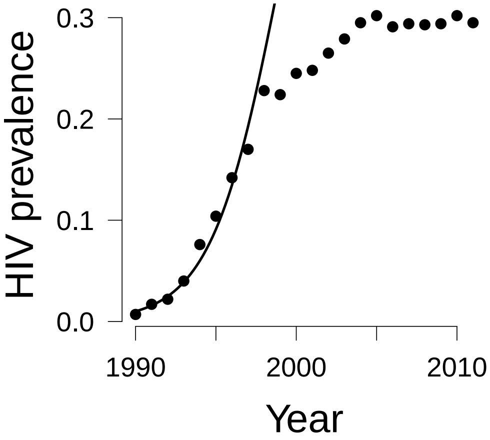
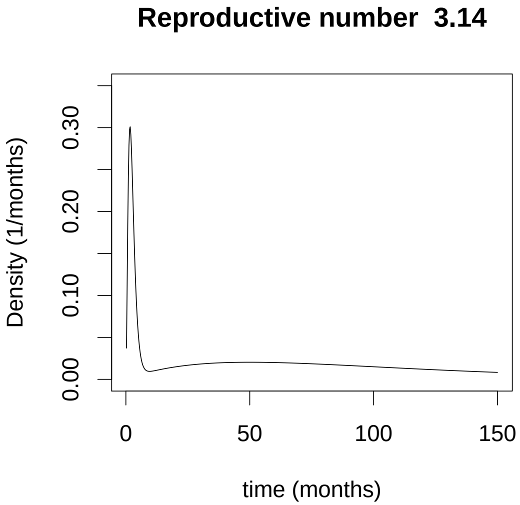
Outline

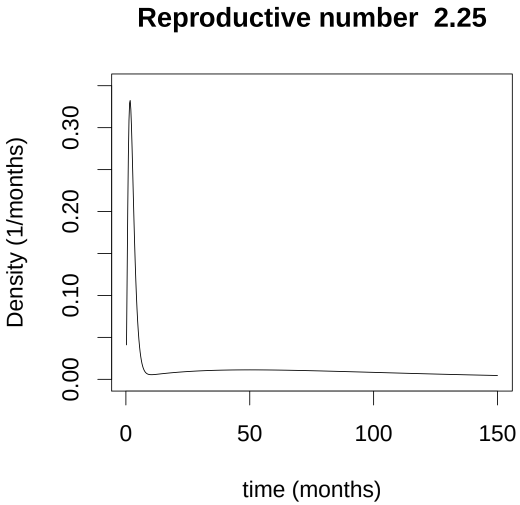
Epidemic speed



Baseline scenario



High early transmission



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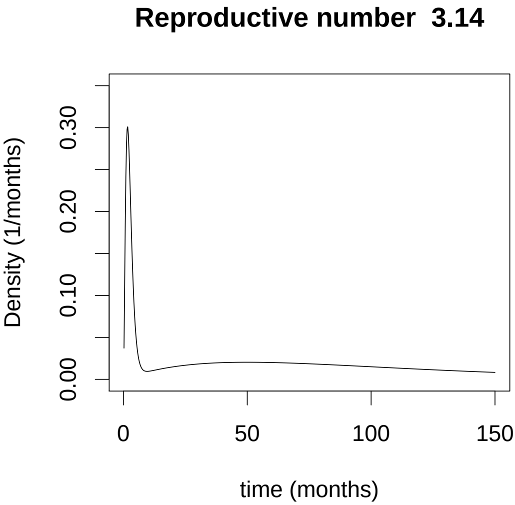
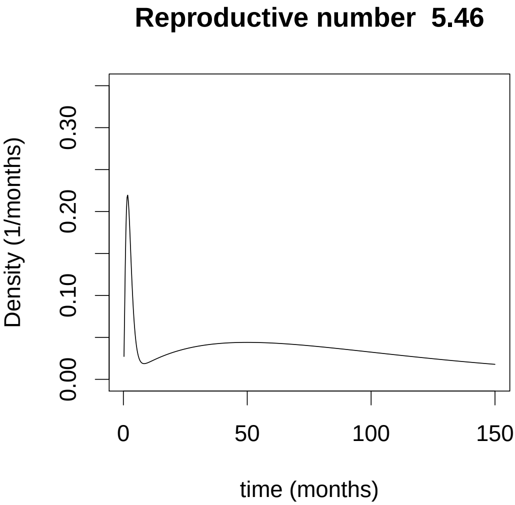
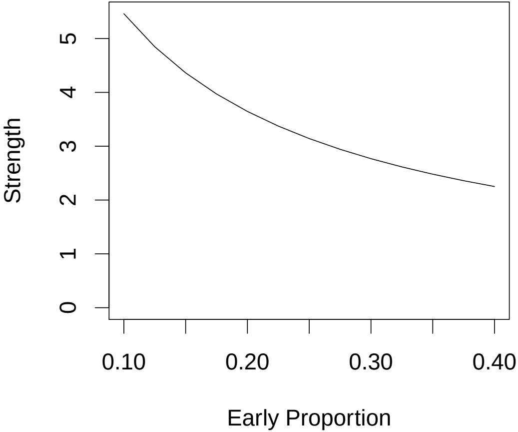


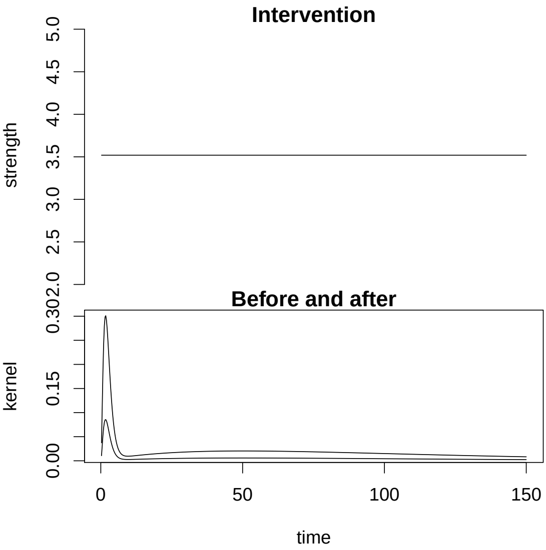
Low early transmission



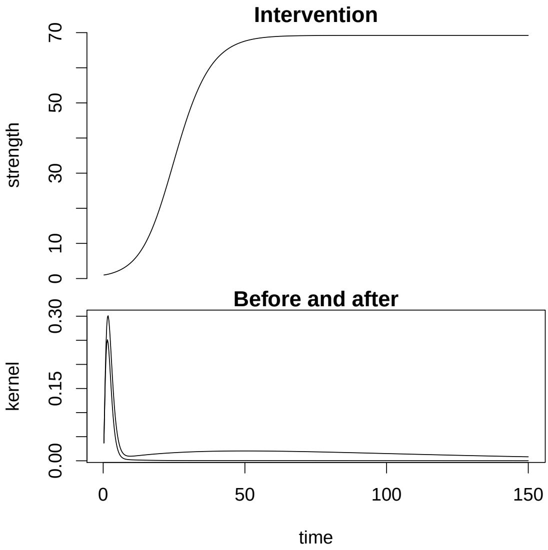
Range of estimates



Condom intervention



Test and treat



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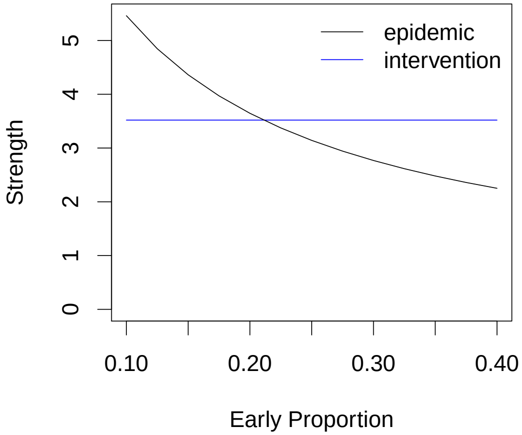
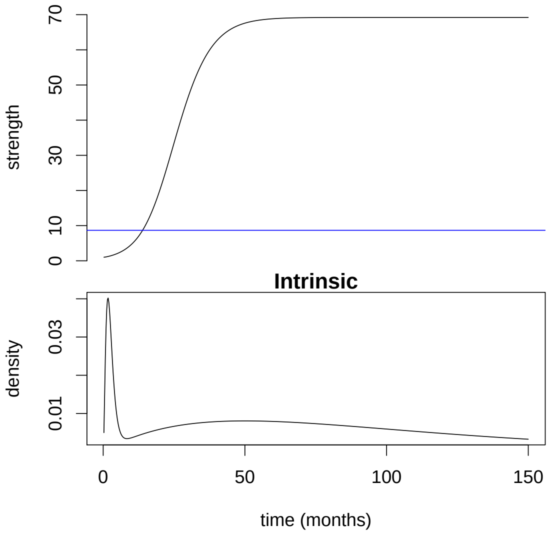
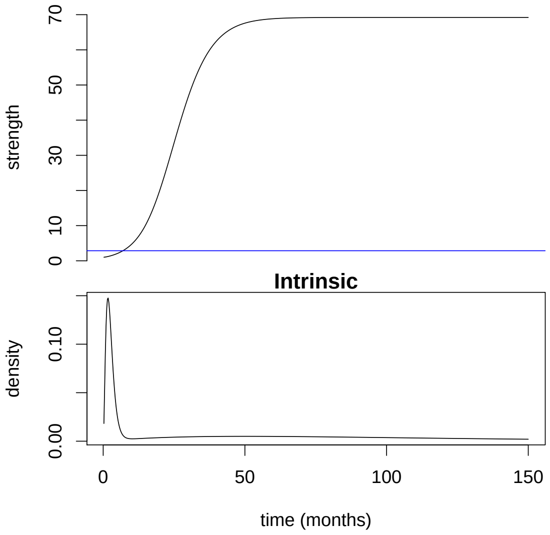


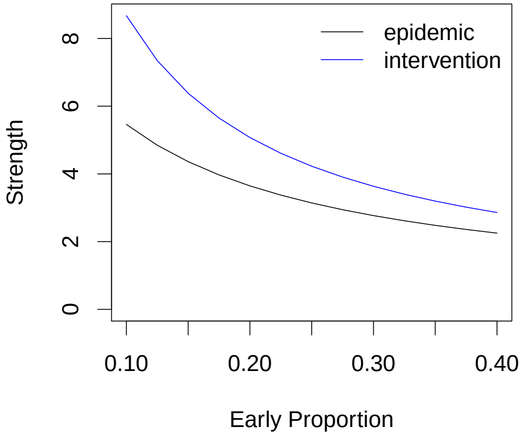
Test and treat (low early transmission)



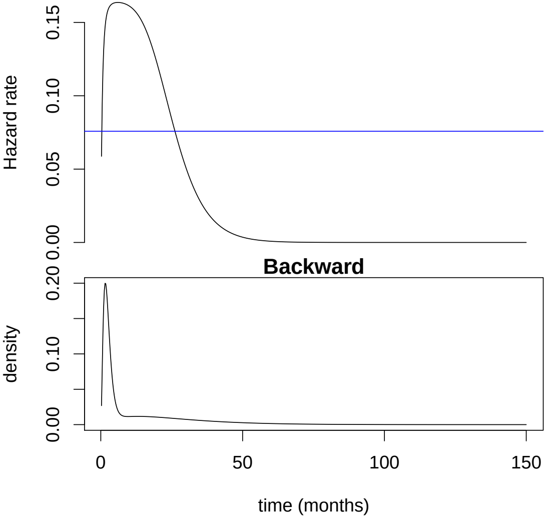
Test and treat (high early transmission)



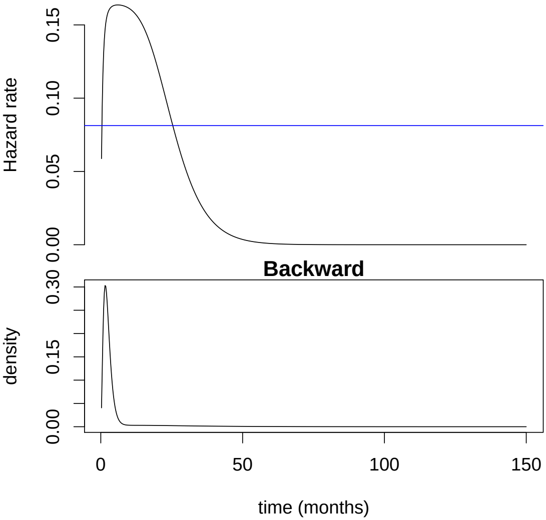
Range of estimates



Low early transmission



High early transmission



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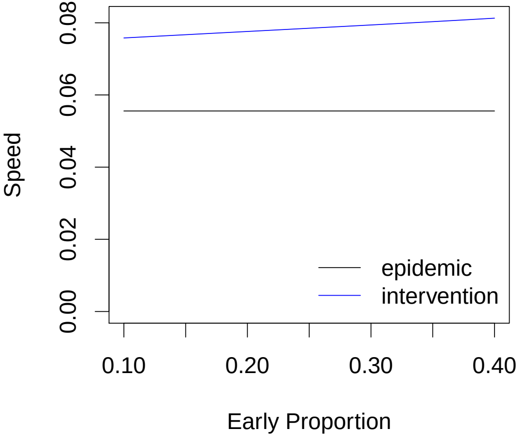

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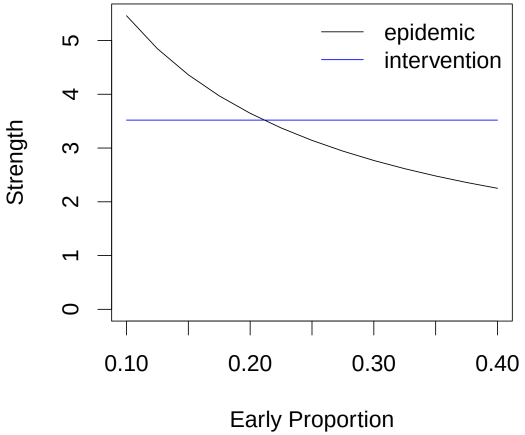

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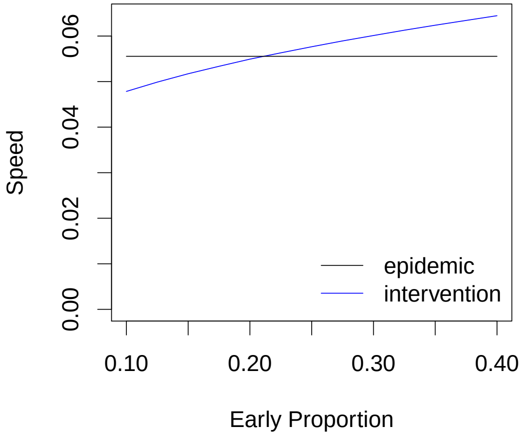
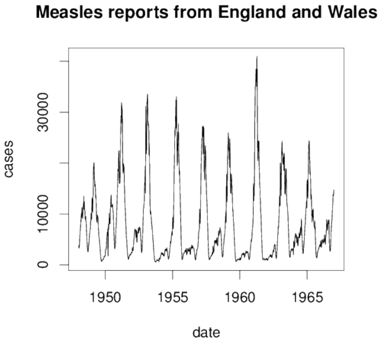
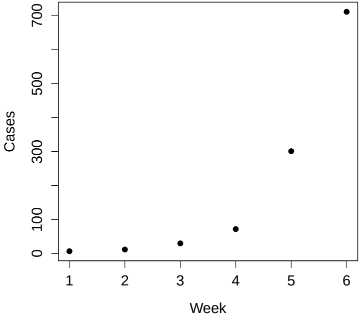


Outline

Ways of looking

Measuring the epidemic





Measuring the intervention

Examples

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  - Information (long-term) is strength-like

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Thanks

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