The SIR Model Family

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DAIDD 2015

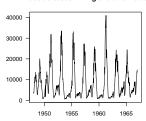
http://www.ici3d.org/daidd/

Dynamic modeling

Connects scales

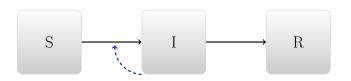


Measles cases in England and Wales



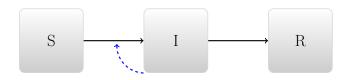
Compartmental models

Divide people into categories:



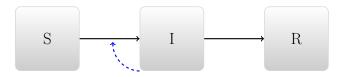
 $\blacktriangleright \ \, \text{Susceptible} \to \text{Infectious} \to \text{Recovered}$

What determines transition rates?



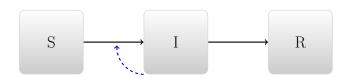
- People get better independently
- People get infected by infectious people

Conceptual modeling



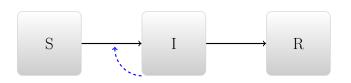


Conceptual modeling



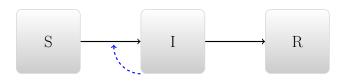
- What is the final result?
- When does disease increase, decrease?

Dynamic implementation



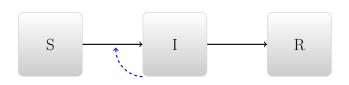
- Requires assumptions about recovery and transmission
- The conceptually simplest implementation uses Ordinary Differential Equations (ODEs)
 - Other options may be more realistic
 - Or simpler in practice

Recovery



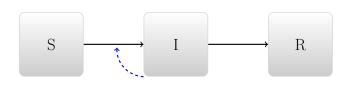
- ▶ Infectious people recover at *per capita* rate γ
 - ▶ Total recovery rate is γI
 - Mean time infectious is $D = 1/\gamma$

Transmission



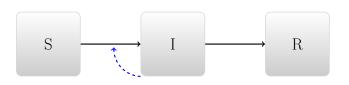
- Susceptible people get infected by:
 - Going around and contacting people (rate c)
 - ▶ Some of these people are infectious (proportion I/N)
 - Some of these contacts are effective (proportion p)
- ▶ Per capita rate of becoming infected is $cpI/N \equiv \beta I/N$
- ▶ Population-level transmission rate is $T = \beta SI/N$

Another perspective on transmission



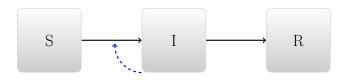
- Infectious people infect others by:
 - Going around and contacting people (rate c)
 - ▶ Some of these people are susceptible (proportion S/N)
 - Some of these contacts are effective (proportion p)
- ▶ Per capita rate of infecting others is $cpS/N \equiv \beta S/N$
- ▶ Population-level transmission rate is $T = \beta SI/N$

ODE implementation



$$\begin{array}{rcl} \frac{dS}{dt} & = & -\beta \frac{SI}{N} \\ \frac{dI}{dt} & = & \beta \frac{SI}{N} - \gamma I \\ \frac{dR}{dt} & = & \gamma I \end{array}$$

Spreadsheet implementation



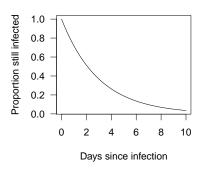
http://tinyurl.com/SIR-DAIDD-2015

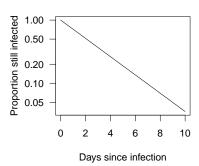
ODE assumptions



- Lots and lots of people
- Perfectly mixed

ODE assumptions





- Waiting times are exponentially distributed
- Rarely realistic

Scripts vs. spreadsheets

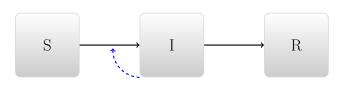
```
Susceptibles | Infectious | Remover | Total | People | Pe
```

More about transmission



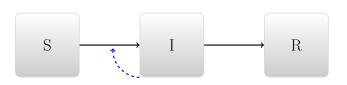
- $\beta = pc$
- Sometimes this decomposition is clear
- ▶ But usually it's not

Population sizes



$$\begin{array}{rcl} \frac{dS}{dt} & = & -\beta \frac{SI}{N} \\ \frac{dI}{dt} & = & \beta \frac{SI}{N} - \gamma I \\ \frac{dR}{dt} & = & \gamma I \end{array}$$

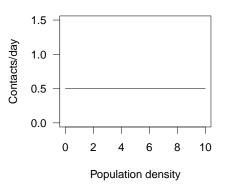
Population sizes



$$\begin{array}{rcl} \frac{dS}{dt} & = & -\beta(N)\frac{SI}{N} \\ \frac{dI}{dt} & = & \beta(N)\frac{SI}{N} - \gamma I \\ \frac{dR}{dt} & = & \gamma I \end{array}$$

Standard incidence

Standard incidence

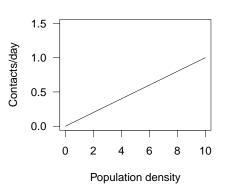


- $\beta(N) = \beta_0$ $T = \frac{\beta_0 SI}{N}$
- Also known as frequency-dependent transmission

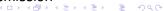


Mass action

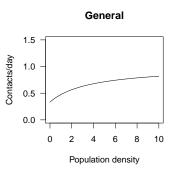
Mass action



- $\beta(N) = \beta_1 N$
- \triangleright $\mathcal{T} = \beta_1 SI$
- Also known as density-dependent transmission



Other



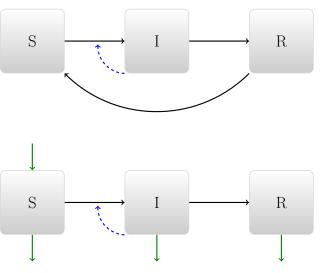
- May not go to zero when N does
- ► May not go to ∞ when N does



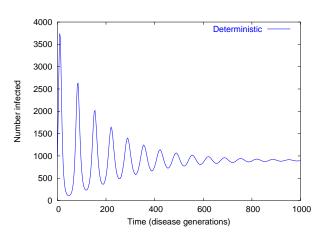
Digression – units

- $\mathcal{T} = \beta SI/N$: [ppl/time]
- $\triangleright \beta : [1/time]$
 - $\beta/\gamma = \beta D : [1]$
 - Standard incidence, β_0 : [1/time]
 - ▶ Mass-action incidence, β_1 : [1/(people · time)]

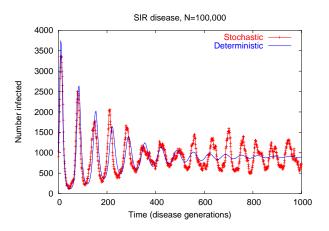
Closing the circle



Tendency to oscillate



With individuality



Summary

- Dynamics are an essential tool to link scales
- Very simple models can provide useful insights
- More complex models can provide more detail, but also require more assumptions, and more choices

Conclusions from simple models

- Threshold behaviour
- Tendency to oscillate