

# Appendix

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## 1 Model

### 1.1 Model structure

We developed a susceptible-infective-treated coinfection model with heterogeneous mixing. Model consists of 9 states which are classified by syphilis and HIV infection status:  $SS_i$ ,  $IS_i$ ,  $TS_i$ ,  $SI_i$ ,  $II_i$ ,  $TI_i$ ,  $ST_i$ ,  $IT_i$ , and  $TT_i$ . Given a state  $XY_i$ ,  $X$  represents the HIV infection status;  $Y$  represents the syphilis infection status; and  $i$  represents the risk group. For simplicity, we do not consider progression through different stages of the disease, and assume that there are only two risk groups:  $i = 1, 2$ .

We assume that all individuals leave at-risk population at a rate  $\mu$ , and individuals enter at-risk population as  $SS$  (susceptible to both diseases) at a constant rate that is proportional to the proportion of each risk group at a disease-free equilibrium:  $\mu N_0$ . Individuals infected syphilis receive treatment (acquiring partial immunity) at a rate  $\gamma$  and lose immunity (becoming susceptible) at a rate  $\delta$ . Individuals infected with HIV receive ARV treatment at a rate  $\tau$ . They can also leave ARV treatment or treatment may fail (entering infective compartment) at a rate  $\sigma$ . Individuals infected with HIV die at a rate  $\alpha$  but if they're receiving ARV treatment, they die at a slower rate of  $\epsilon_\alpha \alpha$ , where  $\epsilon_\alpha < 1$ . Individuals infected with syphilis and HIV can infect a susceptible partner at a probability of  $\beta_{syph}$  and  $\beta_{HIV}$  per partnership, respectively. ARV treatment reduces HIV transmission rate by the factor of  $\epsilon_\beta$ , and syphilis infection increases acquisition and transmission rate of HIV by the factor of  $\nu_r$  and  $\nu_t$ , respectively (Deschamps et al., 1996; Røttingen et al., 2001). For simplicity, we do not consider explicit partnership dynamics but we assume assortative mixing.

### 1.2 Assortativity

Assortative mixing is modeled based on the work of Grassly et al. (2005). Proportion  $\rho$  of an individual's mixing is reserved for their own risk group, and the rest is used randomly in the whole population. Force of infection,  $\lambda_i$ , that susceptible individuals in a risk group,  $i$ , experience from a particular disease is given by the following equation:

$$\lambda_i = \rho\beta_i \frac{Y_i}{N_i} + (1 - \rho)\beta_i \frac{\sum_i c_i Y_i}{\sum_i c_i N_i}, \quad (1)$$

where  $i$  is the mixing group,  $c_i$  is the partnership rate,  $N_i$  is total number of people in the mixing group, and  $\beta_i = \beta * c_i$  ( $\beta$  is the transmission probability per partnership of the disease).

### 1.3 Transmission

In the equations introduced above,  $Y_i$  is the number of individuals in a risk group  $i$  that are infected with a particular disease. However, in order to account for varying transmissibility depending on the infection status, We define  $J_i^{HIV} = IS_i + \nu_t II_i + IT_i + \epsilon_\beta(TS_i + \nu_t TI_i + TT_i)$  and  $J_i^{syph} = SI_i + II_i + TI_i$ , which are used to calculate force of infection of two diseases:

$$\begin{aligned} \lambda_i^{HIV} &= \rho\beta_i^{HIV} \frac{J_i^{HIV}}{N_i} + (1 - \rho)\beta_i^{HIV} \frac{\sum_i c_i J_i^{HIV}}{\sum_i c_i N_i}, \\ \lambda_i^{syph} &= \rho\beta_i^{syph} \frac{J_i^{syph}}{N_i} + (1 - \rho)\beta_i^{syph} \frac{\sum_i c_i J_i^{syph}}{\sum_i c_i N_i}. \end{aligned} \quad (2)$$

Increased susceptibility to HIV due to syphilis is modeled by multiplying the term  $\nu_r$  to the infection term going from  $SI_i$  to  $II_i$ :  $SI'_i = -\nu_r \lambda_i^{HIV} SI_i$  and  $II'_i = -\nu_r \lambda_i^{HIV} SI_i$ .

### 1.4 Mathematical model

$$\begin{aligned} SS'_i &= \mu N(0)_i - (\lambda_i^{HIV} + \lambda_i^{syph})SS_i + \delta ST_i - \mu SS_i \\ IS'_i &= -\lambda_i^{syph} IS_i + \lambda_i^{HIV} SS_i - \tau IS_i + \sigma TS_i + \delta IT_i - \alpha IS_i - \mu IS_i \\ TS'_i &= -\lambda_i^{syph} TS_i + \tau IS_i - \sigma TS_i + \delta TT_i - \epsilon_\alpha \alpha TS_i - \mu TS_i \\ SI'_i &= -\nu_r \lambda_i^{HIV} SI_i + \lambda_i^{syph} SS_i - \gamma SI_i - \mu SI_i \\ II'_i &= \nu_r \lambda_i^{HIV} SI_i + \lambda_i^{syph} IS_i - \tau II_i + \sigma TI_i - \gamma II_i - \alpha II_i - \mu II_i \\ TI'_i &= \lambda_i^{syph} TS_i + \tau II_i - \sigma TI_i - \gamma TI_i - \epsilon_\alpha \alpha TI_i - \mu TI_i \\ ST'_i &= -\lambda_i^{HIV} ST_i + \gamma SI_i - \delta ST_i - \mu ST_i \\ IT'_i &= \lambda_i^{HIV} ST_i - \tau IT_i + \sigma TT_i + \gamma II_i - \delta IT_i - \alpha IT_i - \mu IT_i \\ TT'_i &= \tau IT_i - \sigma TT_i + \gamma TI_i - \delta TT_i - \epsilon_\alpha \alpha TT_i - \mu TT_i \end{aligned} \quad (3)$$

### 1.5 Delayed introduction of ARV

Instead of using the model as it is above, we introduce ARV treatment 20 years after the beginning of the simulation. It is simply done by multiplying  $T_{start}$  to  $\tau$  ( $\tau_{adj} = T_{start}\tau$ ) and setting  $T_{start} = 0$  when  $t < 20$  and 1 otherwise. With the introduction of ARV, we introduce two more variables to the model:  $\nu_{IS}$

and  $c_{inc}$ .  $\nu_{IS}$  is the ratio of increased susceptibility to syphilis due to ARV immunosuppression effect and  $c_{inc}$  is the effect of ARV on behaviour (also given as a ratio).  $\nu_{IS}$  is simply multiplied to syphilis infection term going from  $TS_i$  to  $TI_i$ .  $TS'_i = -\nu_{IS}\lambda_i^{syph}TS_i$  and  $TI'_i = \nu_{IS}\lambda_i^{syph}TS_i$ .

With the introduction of ARV, behaviour change is introduced to people who are receiving ARV. The behaviour change is modeled by multiplying  $c_{inc}$  to the partnership change rate of those who are receiving ARV. It is done by modifying  $J$ ,  $N$ , as well as the infection term going from  $TS_i$  to  $TI_i$ :

$$\begin{aligned} J_i^{HIV} &= IS_i + \nu_t II_i + IT_i + c_{inc}\epsilon_\beta(TS_i + \nu_t TI_i + TT_i), \\ J_i^{syph} &= SI_i + II_i + c_{inc}TI_i, \\ N_i &= SS_i + IS_i + SI_i + II_i + ST_i + TT_i + c_{inc}(TS_i + TI_i + TT_i) \quad (4) \\ TS'_i &= -c_{inc}\nu_{IS}\lambda_i^{syph}TS_i \\ TI'_i &= c_{inc}\nu_{IS}\lambda_i^{syph}TS_i. \end{aligned}$$

Increase in behaviour change is modeled by using the exponential function:  $c_{inc} = c_f + (c_0 - c_f)\exp((T_{start} - t)/T_c)$ . These equations replace the equations in the above model.

## References

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Table 1: Parameter values

Notation	Description	Value(s)	Source
$c$	Partnership change rate	40 and 5	<a href="#">Garnett et al. (1997)</a>
$N_0$	Proportion of risk group	0.05 and 0.95	Assumption
$\rho$	Proportion of non-random contact	0.3	Assumption
$\mu$	Rate of entry/exit from at risk population	0.05	<a href="#">Garnett et al. (1997)</a>
$\beta_{HIV}$	HIV transmission probability per partnership	0.097	<a href="#">Grant et al. (1987)</a>
$\epsilon_\beta$	Relative HIV transmission ratio of people on ART	0.04	<a href="#">Cohen et al. (2011)</a>
$\alpha$	HIV induced mortality	0.125	<a href="#">Champredon et al. (2013)</a>
$\epsilon_\alpha$	Relative mortality ratio of people on ART	0.5	<a href="#">Collaboration et al. (2010)</a>
$\tau$	ART treatment rate	1	<a href="#">Granich et al. (2009)</a>
$\sigma$	ART failure/loss rate	0.015	<a href="#">Granich et al. (2009)</a>
$\beta_{syph}$	Syphilis transmission probability per partnership	0.6	<a href="#">Garnett et al. (1997)</a>
$\gamma$	Syphilis treatment rate	6	<a href="#">Grassly et al. (2005)</a>
$\delta$	Rate at which syphilis immunity is lost	0.05	<a href="#">Grassly et al. (2005)</a>
$\nu_t$	Relative HIV transmission ratio of people who are infected with syphilis	2	<a href="#">Deschamps et al. (1996)</a>
$\nu_r$	Relative HIV acquisition ratio of people who are infected with syphilis	3	<a href="#">Røttingen et al. (2001)</a>
$\nu_{is}$	Relative syphilis acquiring ratio due to ARV immunosuppression	3	Assumption