Appendix

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1 Model structure

We developed a susceptible-infective-treated coinfection model with heterogeneous mixing. Model consists of 18 states which are classified by syphilis and HIV infection status, and risk group: SS_i, IS_i, TS_i, SI_i, II_i, TI_i, ST_i, IT_i, and TT_i. Given a state XY_i, X represents the HIV infection status; Y represents the syphilis infection status; and i represents the risk group. For simplicity, we do not consider progression through different stages of the disease, and assume that there are only two risk groups: $i = 1, 2$.

We assume that all individuals leave at-risk population at a rate μ, and individuals enter at-risk population as SS (susceptible to both diseas) at a constant rate that is proportional to the proportion of each risk group at a disease-free equilibrium: μN_0. Individuals infected syphilis receive treatment (acquiring partial immunity) at a rate γ and lose immunity (becoming susceptible) at a rate δ. Individuals infected with HIV

References

Champredon, D., S. Bellan, and J. Dushoff (2013). Hiv sexual transmission is predominantly driven by single individuals rather than discordant couples: a model-based approach. *PloS one 8*(12), e82906.

Cohen, M. S., Y. Q. Chen, M. McCauley, T. Gamble, M. C. Hosseinipour, N. Kumarasamy, J. G. Hakim, J. Kumwenda, B. Grinsztejn, J. H. Pilotto, et al. (2011). Prevention of hiv-1 infection with early antiretroviral therapy. *New England journal of medicine 365*(6), 493–505.

Collaboration, H.-C. et al. (2010). The effect of combined antiretroviral therapy on the overall mortality of hiv-infected individuals. *AIDS (London, England) 24*(1), 123.

Deschamps, M.-M., J. W. Pape, A. Hafner, and W. D. Johnson (1996). Heterosexual transmission of hiv in haiti. *Annals of internal medicine 125*(4), 324–330.

Table 1: Parameter values

Notation	Description	Value(s)	Source
c	Partnership change rate	40 and 5	Garnett et al. (1997)
N_0	Proportion of risk group	0.05 and 0.95	Assumption
ρ	Proportion of non-random contact	0.3	Assumption
μ	Rate of entry/exit from at risk population	0.05	Garnett et al. (1997)
β_{HIV}	HIV transmission probability per partnership	0.097	Grant et al. (1987)
ϵ_β	Relative HIV transmission ratio of people on ART	0.04	Cohen et al. (2011)
α	HIV induced mortality	0.125	Champredon et al. (2013)
ϵ_α	Relative mortality ratio of people on ART	0.5	Collaboration et al. (2010)
τ	ART treatment rate	1	Granich et al. (2009)
σ	ART failure/loss rate	0.015	Granich et al. (2009)
β_{syph}	Syphilis transmission probability per partnership	0.6	Garnett et al. (1997)
γ	Syphilis treatment rate	6	Grassly et al. (2005)
δ	Rate at which syphilis immunity is lost	0.05	Grassly et al. (2005)
ν_t	Relative HIV transmission ratio of people who are infected with syphilis	2	Deschamps et al. (1996)
ν_r	Relative HIV acquisition ratio of people who are infected with syphilis	3	Røttingen et al. (2001)
ν_{is}	Relative syphilis acquiring ratio due to ARV immunosuppression	3	Assumption

Garnett, G. P., S. O. Aral, D. V. Hoyle, W. Cates Jr, and R. M. Anderson (1997). The natural history of syphilis: implications for the transmission dynamics and control of infection. *Sexually transmitted diseases 24*(4), 185–200.

Granich, R. M., C. F. Gilks, C. Dye, K. M. De Cock, and B. G. Williams (2009). Universal voluntary hiv testing with immediate antiretroviral therapy as a strategy for elimination of hiv transmission: a mathematical model. *The Lancet 373*(9657), 48–57.

Grant, R. M., J. A. Wiley, and W. Winkelstein (1987). Infectivity of the human immunodeficiency virus: estimates from a prospective study of homosexual men. *The Journal of infectious diseases 156*(1), 189–193.

Grassly, N. C., C. Fraser, and G. P. Garnett (2005). Host immunity and syn-

chronized epidemics of syphilis across the united states. *Nature 433*(7024), 417–421.

Røttingen, J.-A., D. W. Cameron, and G. P. Garnett (2001). A systematic review of the epidemiologic interactions between classic sexually transmitted diseases and hiv: how much really is known? *Sexually transmitted diseases 28*(10), 579–597.