Cohort scribbles

Cohort consortium

in progress

Continuous-time SIR

We have an initial cohort of infectees I_0, a recovery rate V (analogous to SIR γ), and a transmission factor F (analogous to SIR β). These are all scalars. I_0 is conventionally set to 1 and ignored. We are keeping it here because it might be a vector later.

The cohort of remaining infectees at age of infection τ is given by

$$I(\tau) = I_0 \exp(-V\tau).$$

The generated force of infection is

$$\Lambda(\tau) = F I_0 \exp(-V\tau).$$

and the next cohort is given by the integral:

$$I' = F I_0 / V$$

The reproductive number $\mathcal{R}_0$ is

$$I'/I_0 = F/V$$

and the renewal kernel is

$$k(\tau) = \Lambda(\tau)/(\mathcal{R}_0 I_0) = V \exp(-V\tau),$$

with moments

$$k_n = \Gamma(n+1)/V^n.$$

In particular, the mean $mu_k = k_1 = 1/V$, and the squared CV $\kappa_k = k_2/k_1^2 - 1 = 1$.

Continuous-time with compartments

The initial cohort of infectees I_0 is now an $i \times 1$ column vector, where i is the number of initial-infection boxes. To model flow through the c infected compartments (containing individuals who can infect without infecting again), we multiply by a $c \times i$ "expansion matrix" X, with 1s on the main diagonal and 0s elsewhere.

NB: We don't really need X if we're willing to just put a lot of extra zeroes into F, which I guess is the more standard way to do it.

V is now a $c \times c$ open flow matrix, with outflows representing recovery.

The cohort of infectees at age of infection τ is now

$$I(\tau) = \exp(-V\tau)XI_0,$$

basically the same as above, but with the correct linear-algebra book-keeping.

The generated force of infection is based on a $i \times c$ transmission matrix (F), describing how each of the infected compartments moves individuals into the initial-infection compartments, so

$$\Lambda(\tau) = F\exp(-V\tau)XI_0.$$

and the next-generation integral is:

$$I' = FV^{-1}XI_0 = GI_0,$$

where $G = FV^{-1}X$ is an next-generation operator, and $\mathcal{R}_0$ is the dominant eigenvalue, with associated eigenvector I^*.

Taking stock

None of the above is new.

Does it give a good way to think about or calculate things? We can straightforwardly extend our formal calculation to the kernel when $i = 1$, even if $c > 1$, but I guess there's a sensible way to do it with right eigenvectors when $c > 1$. Note that this would involve *defining* the mean kernel in a non-obvious multi-compartment case. We could also explore the circumstances under which you *don't* need to do that (e.g., if there are different tracks ($i > 1$), but they are synchronized (the only difference is how the groups interact with each other, not the timing).

Future

I expect whatever we do in continuous time to be translatable to discrete time. I wsa thinking of staying on this side until the concepts are ironed out, since it seems likely that the algebra is simpler. On the other hand, I could imagine switching to the discrete side if things get confusing in a way that makes us want to test ourselves with simple simulations (or with MacPan itself).