Dynamical models of disease spread

Jonathan Dushoff, McMaster University

National Taiwan University College of Public Health

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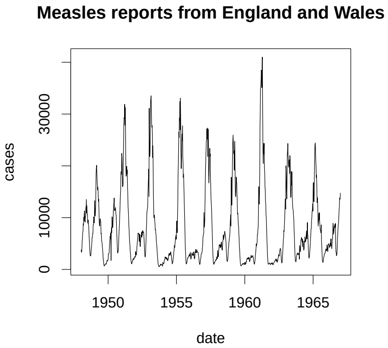
Outline

Dynamic modeling connects scales



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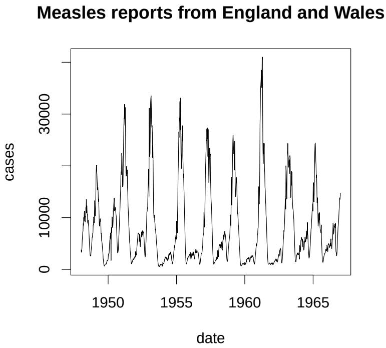

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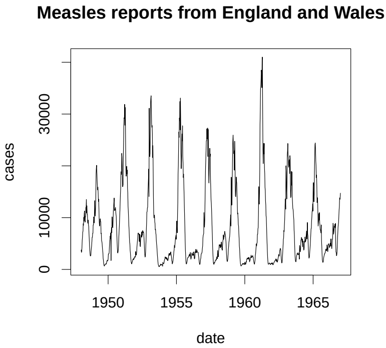

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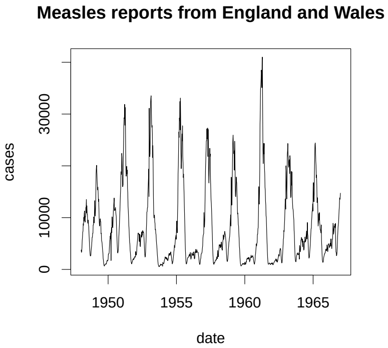

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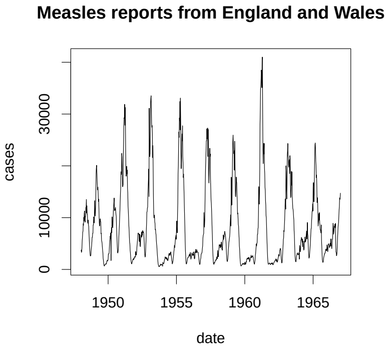

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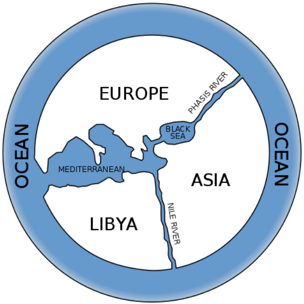

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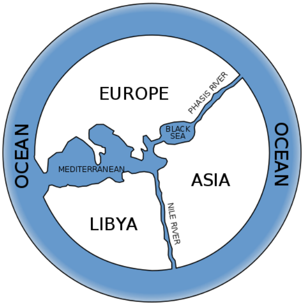

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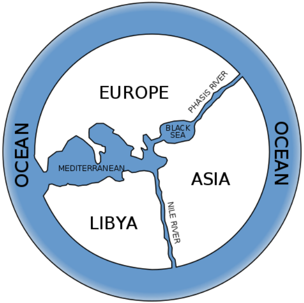

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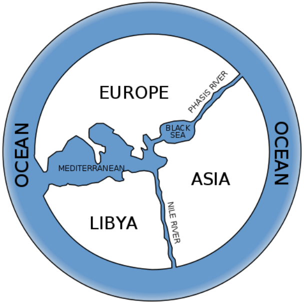

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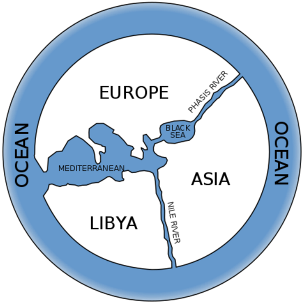

Simple models of disease spread

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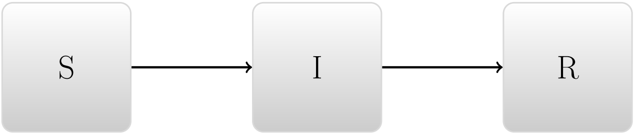

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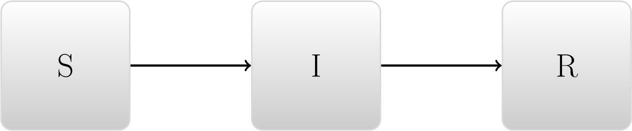

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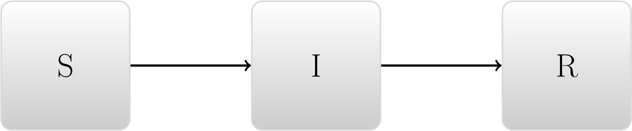

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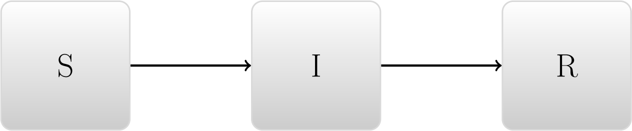

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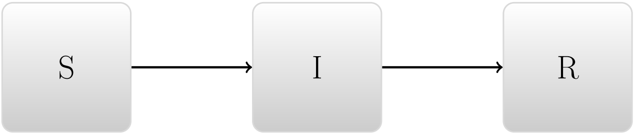

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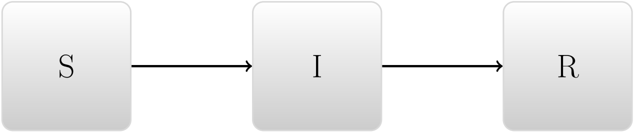
What determines transition rates? (preview)



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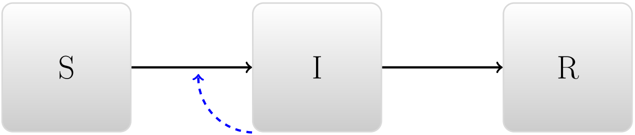

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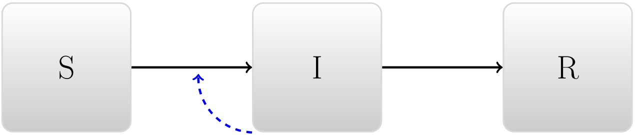

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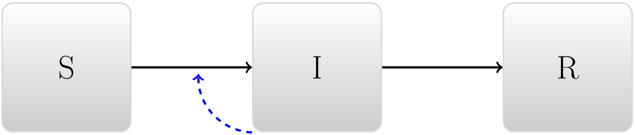

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What is dynamical modeling?

Modeling approaches
- Conceptual modeling
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- Deterministic models
- Stochastic models
- Statistical fitting

Limitations
- Heterogeneity
- Behavioural changes

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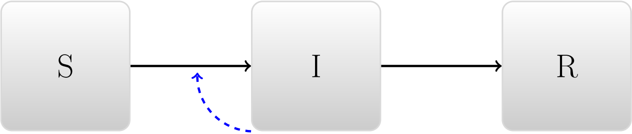
Conceptual modeling



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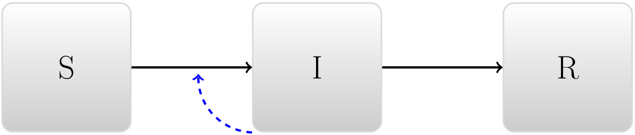

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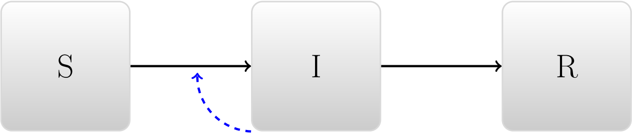

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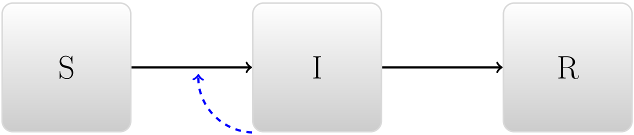

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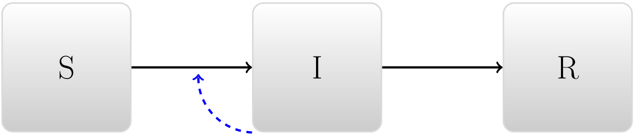

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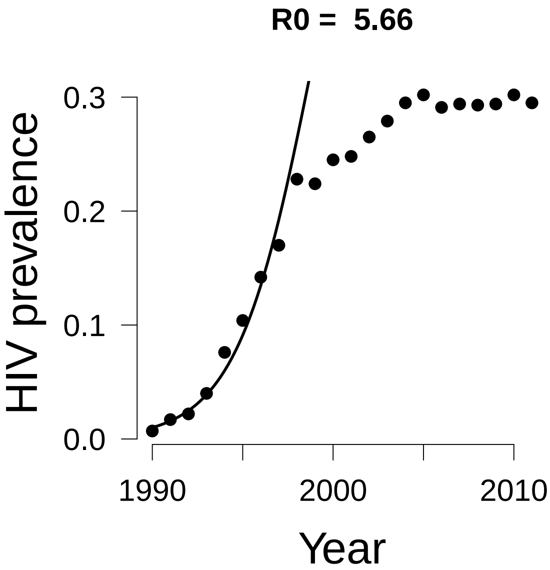

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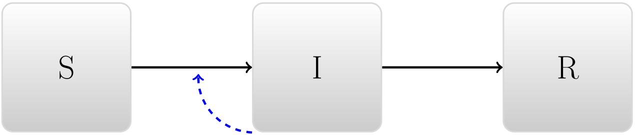

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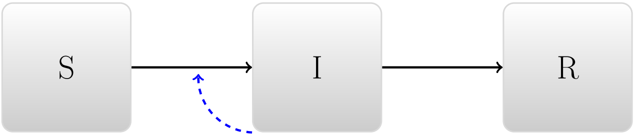

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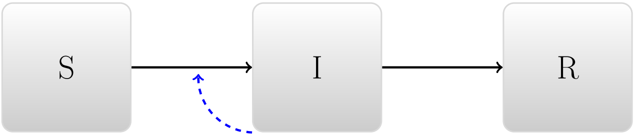

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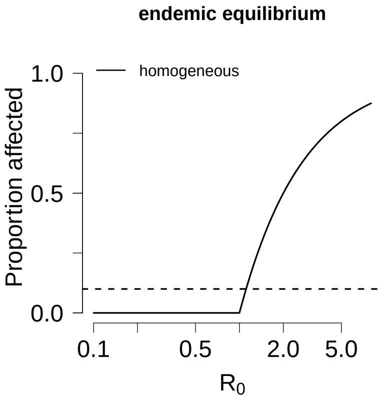
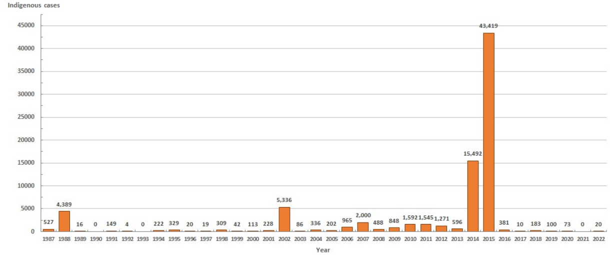
Yellow fever in Panama



Example: Dengue (Taiwan CDC)

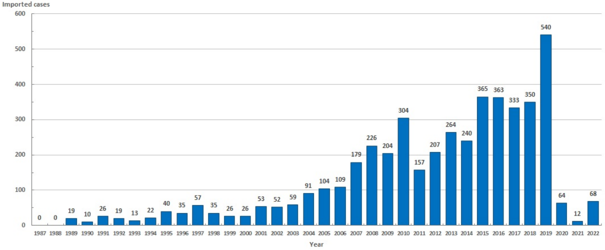
Indigenous cases

40,000



Imported cases

500



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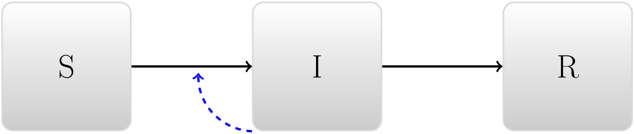

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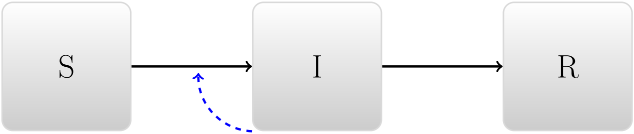

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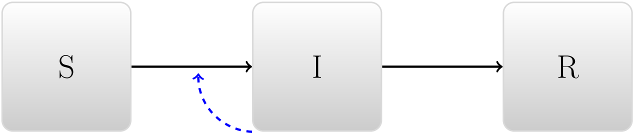

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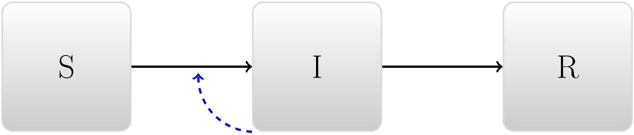

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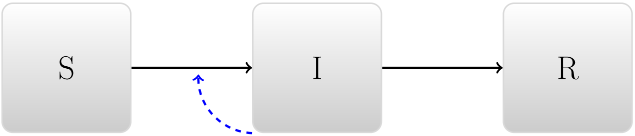

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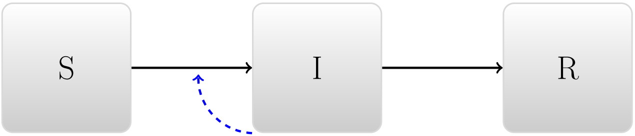
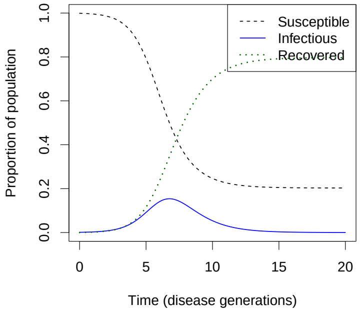
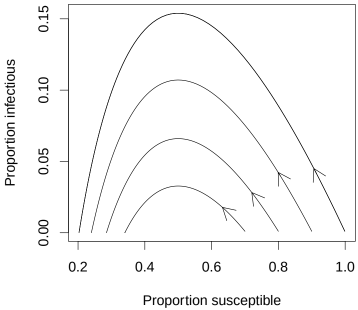
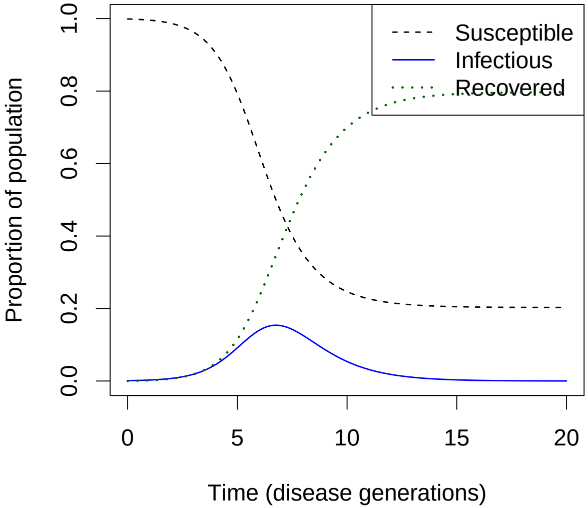






Simulations (repeat)



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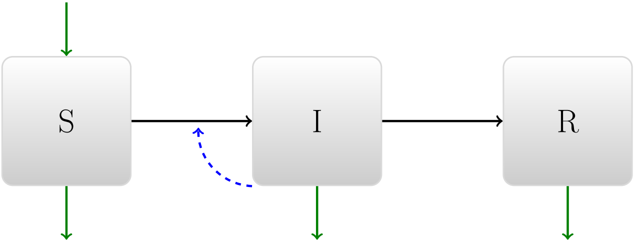

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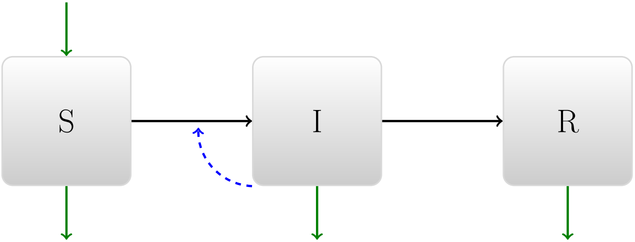

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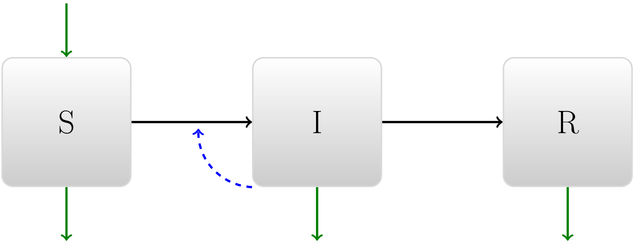

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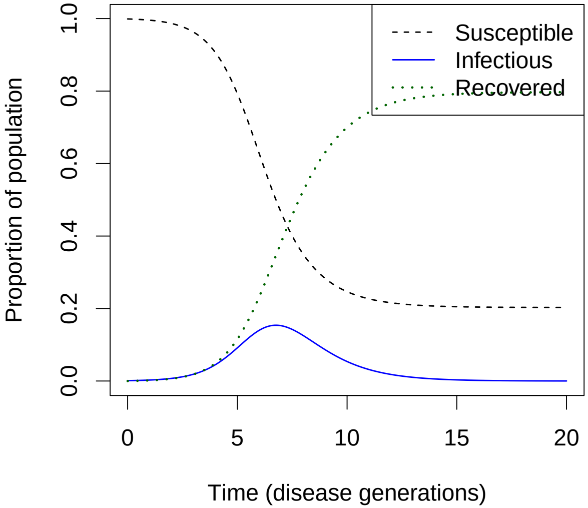

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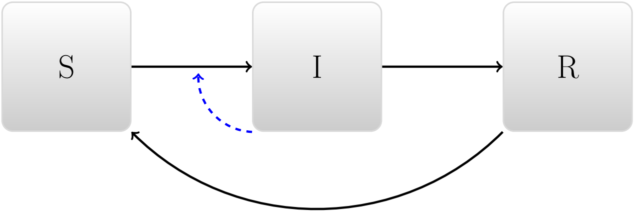

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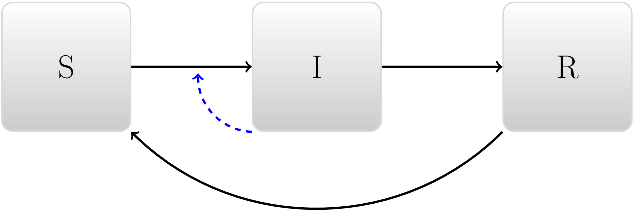

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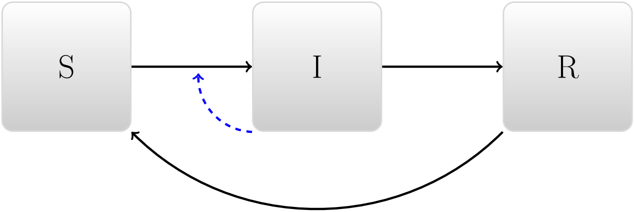

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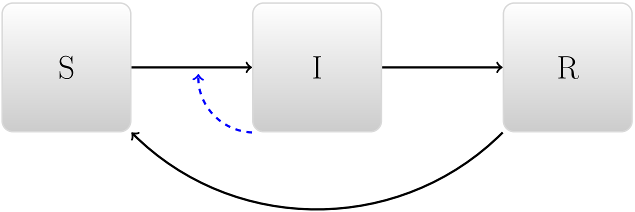
Processes and rates



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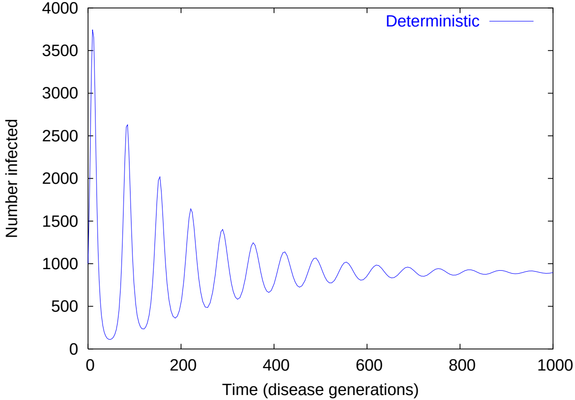
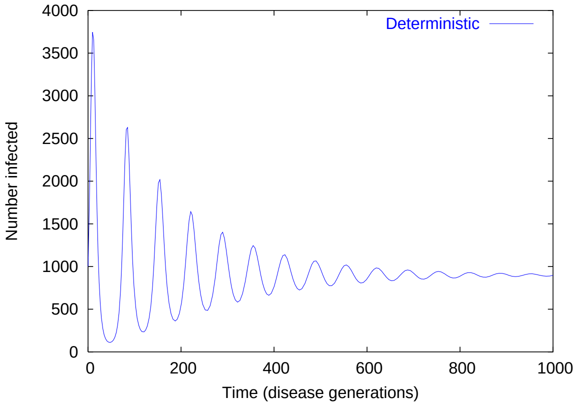
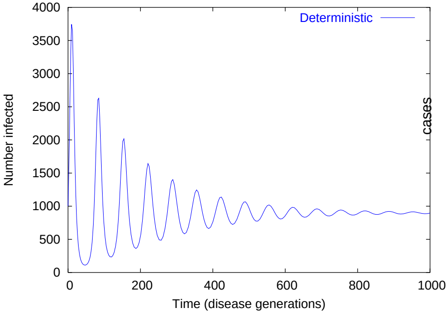
Result: Diseases tend to oscillate



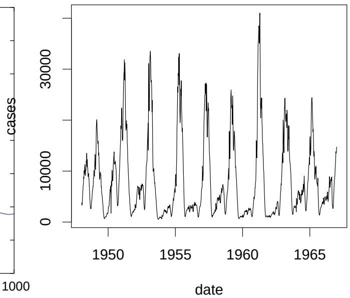
Result: Oscillations tend to be damped



What is missing from our model world? (repeat)



Measles reports from England and Wales



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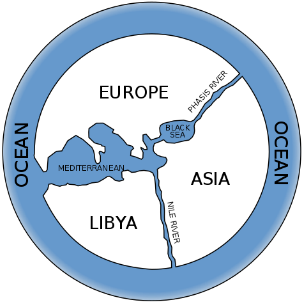

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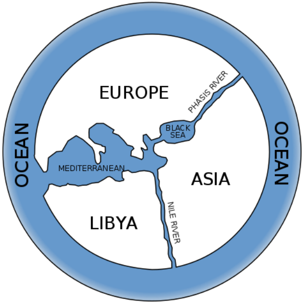

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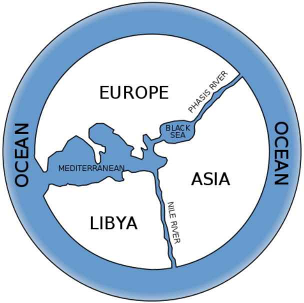

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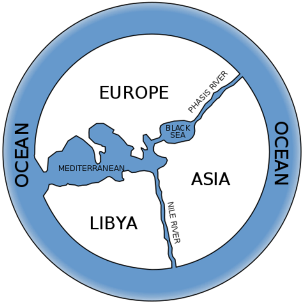

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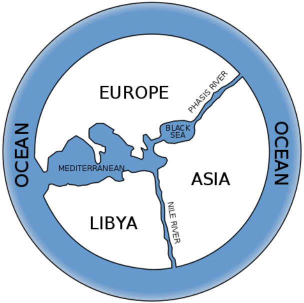

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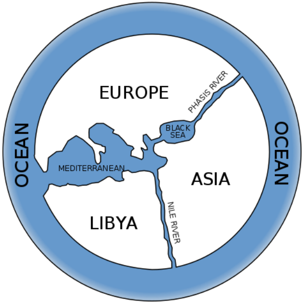

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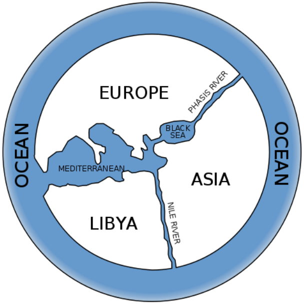

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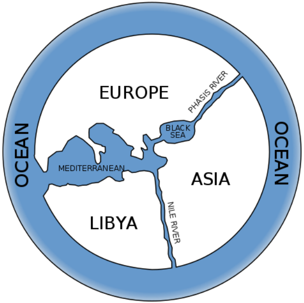

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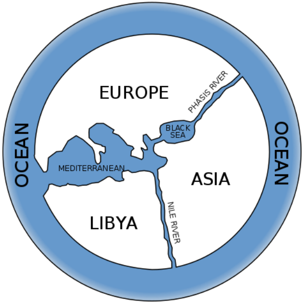

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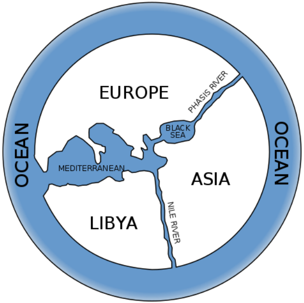

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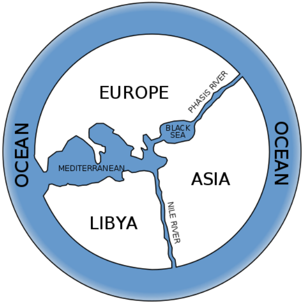

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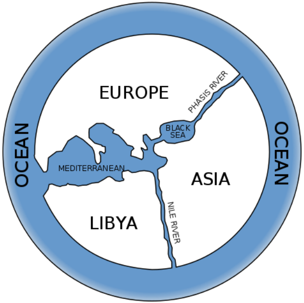

Example: Ebola transmission

- How much Ebola spread occurs before vs. after death

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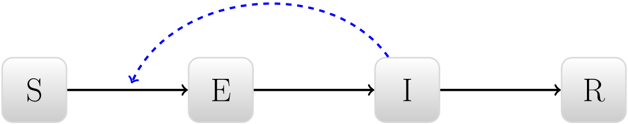
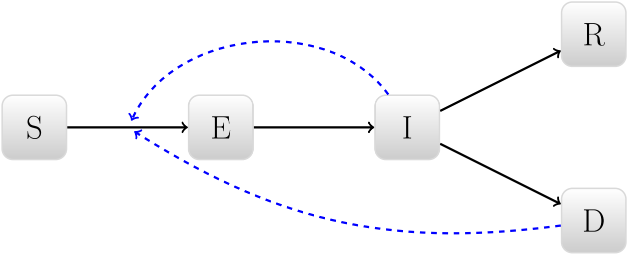
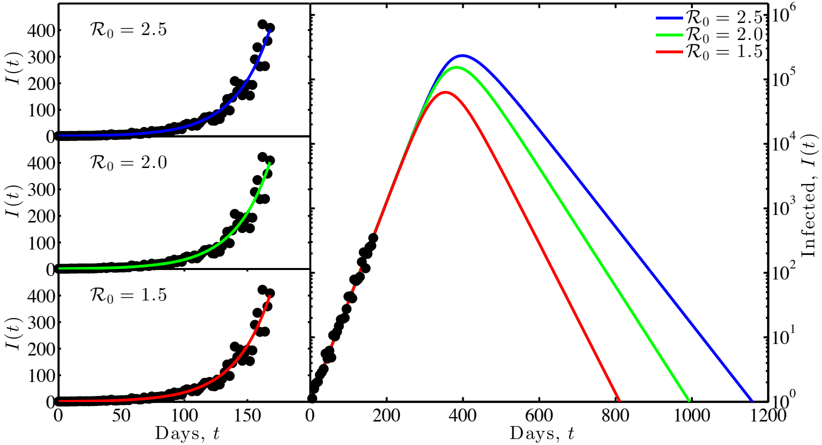
Model with latent period



Include post-death transmission



Result: generation interval links $r\mathcal{R}$(preview)



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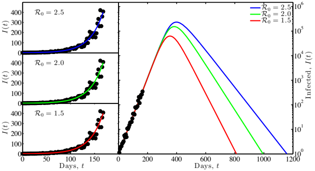

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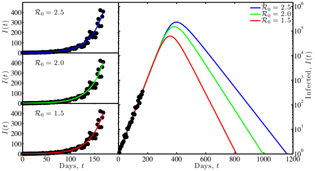

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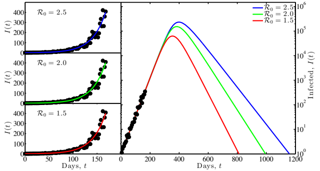
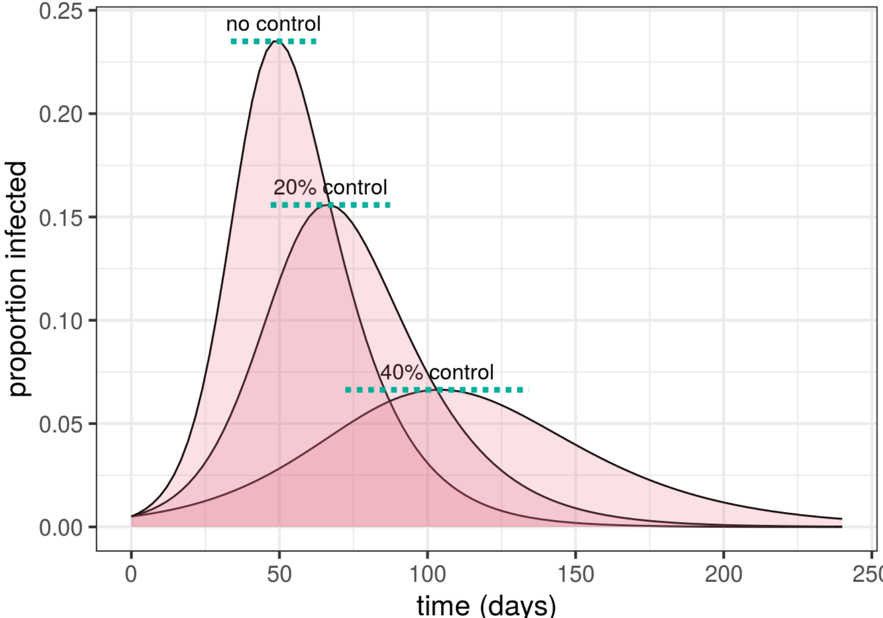


Example: COVID



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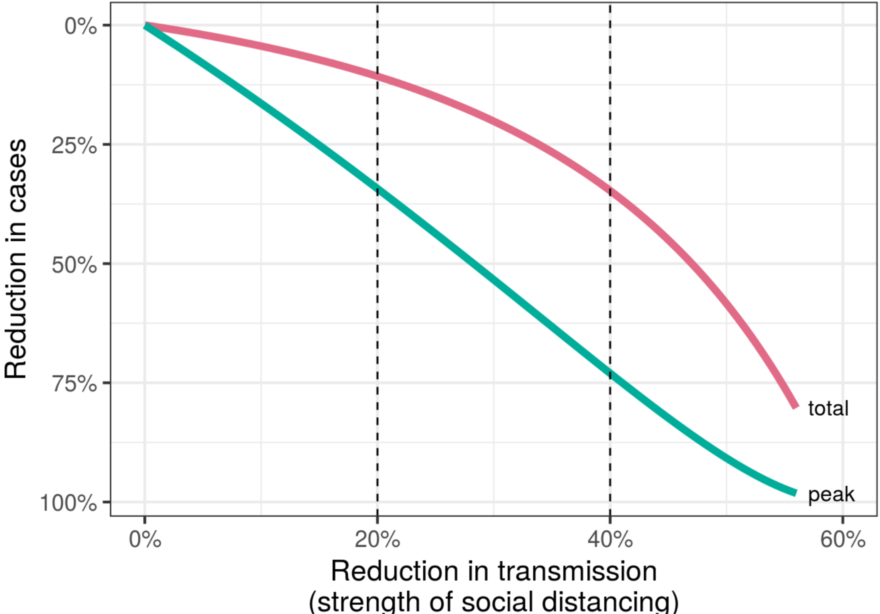
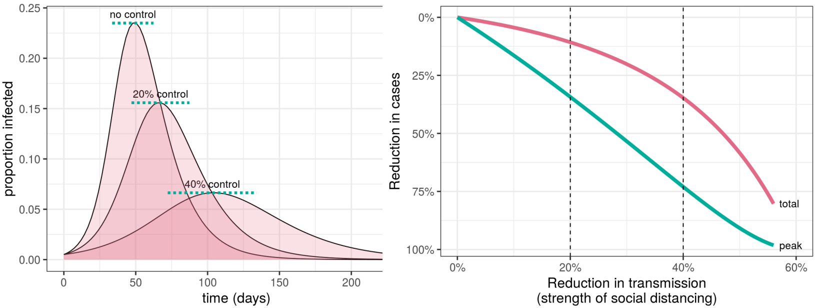
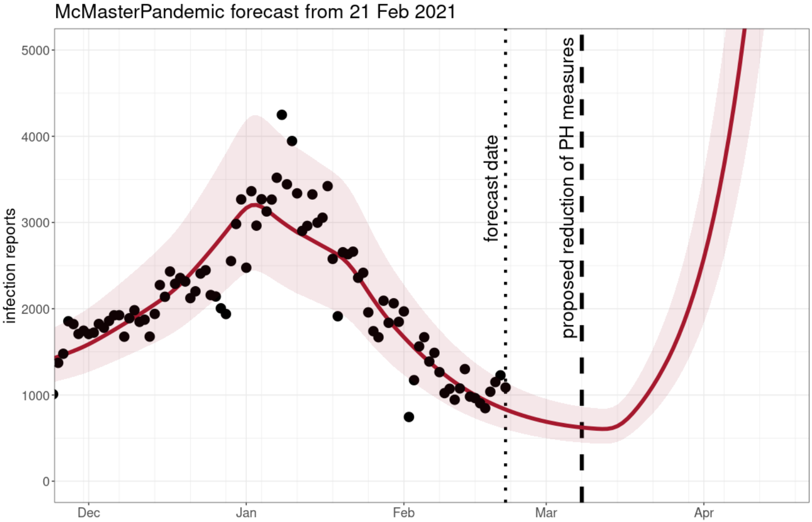


Result: It is easier to reduce the peak than the total cases



Example: COVID waves (preview)



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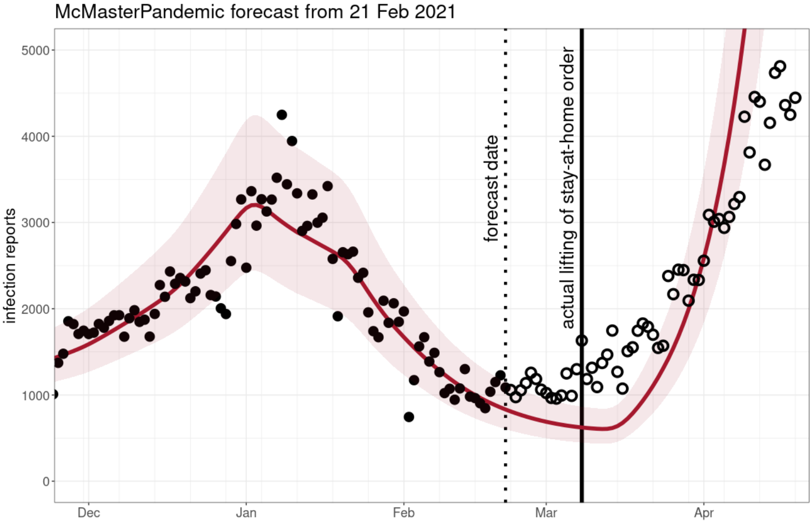

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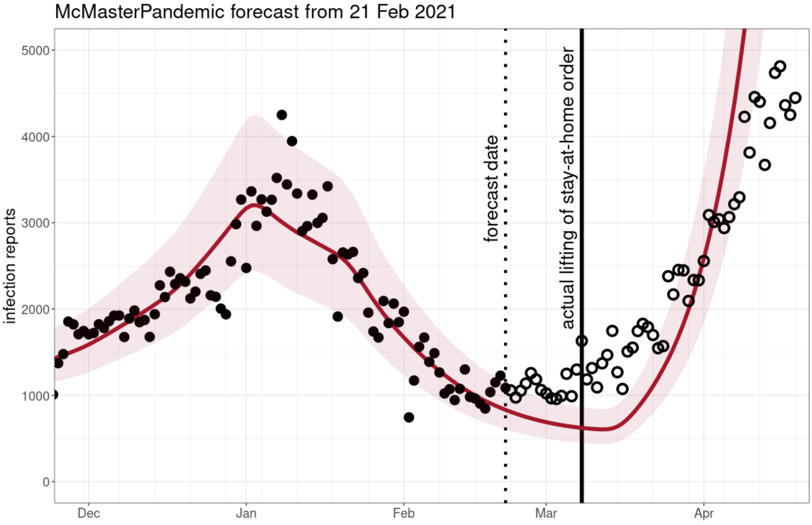

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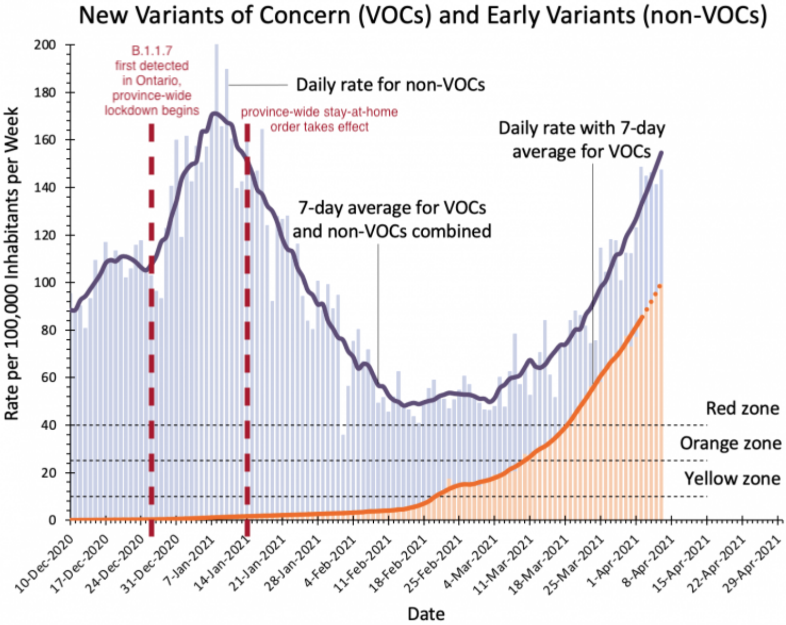

Example: COVID waves

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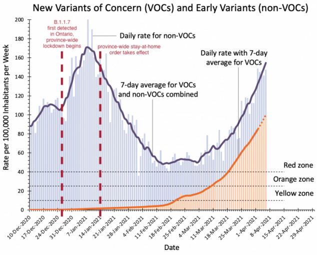

The daily rates of new variants of concern (VOCs) for the last 4 days are predicted.

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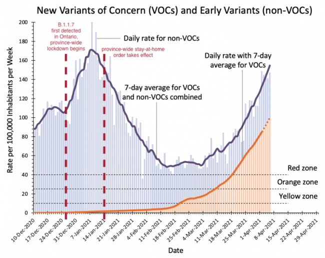

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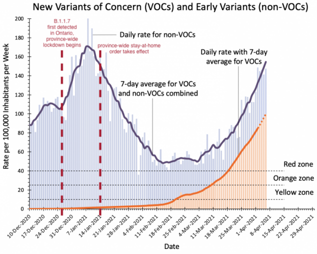

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Outline

Stochastic models

Event	transition	rate	Effect (S, I)
Infection	$S \to I$	$\beta SI/N$	$(-1, 1)$
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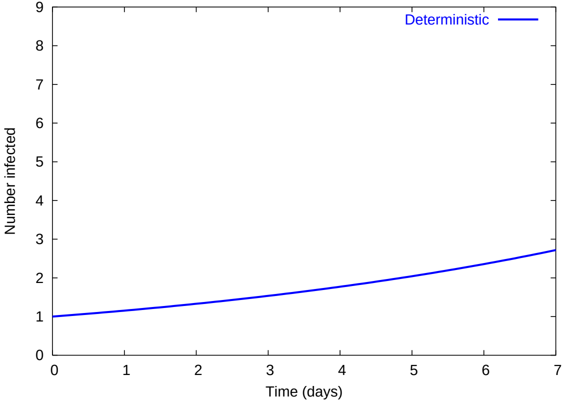
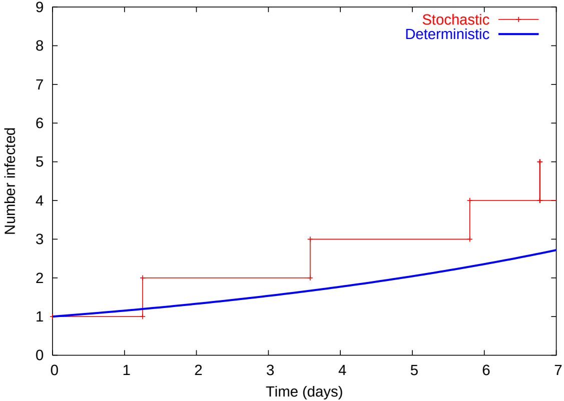


Demographic spread



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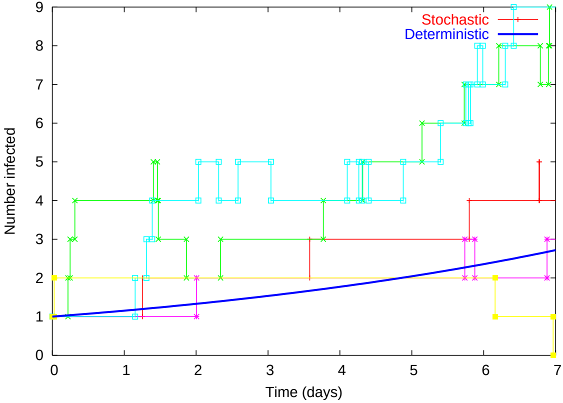

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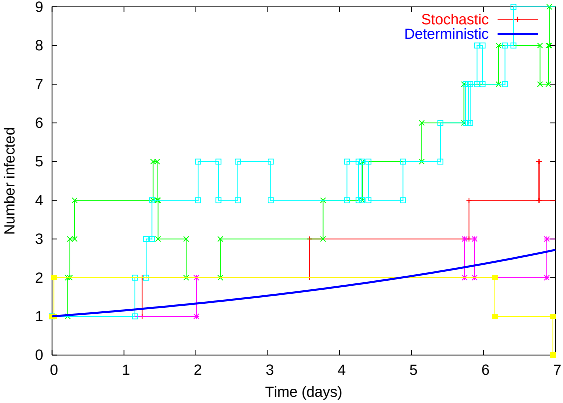

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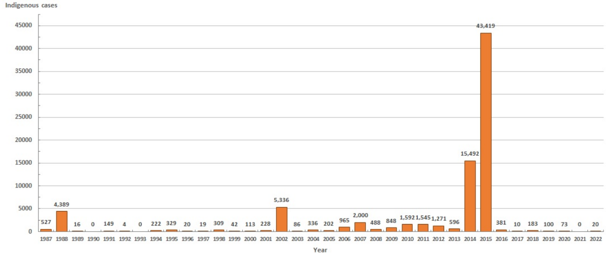
Result: Pattern of outbreak sizes is related to $\mathcal{R}$(repeat)

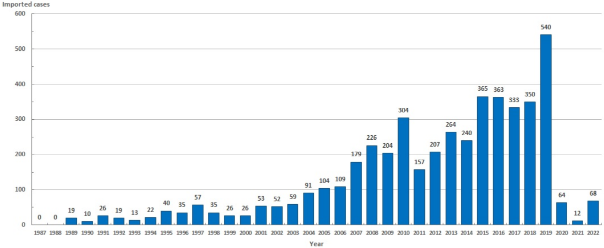
Indigenous cases

40,000

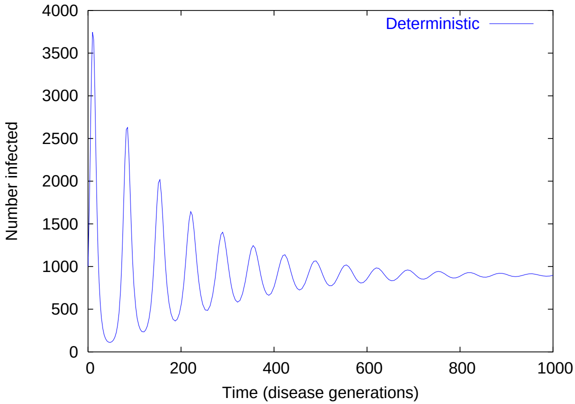


Imported cases

500



Result: stochasticity interacts with oscillations (preview)



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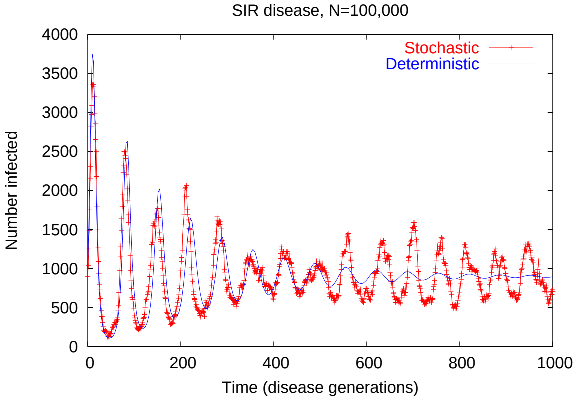


Outline

What is dynamical modeling?

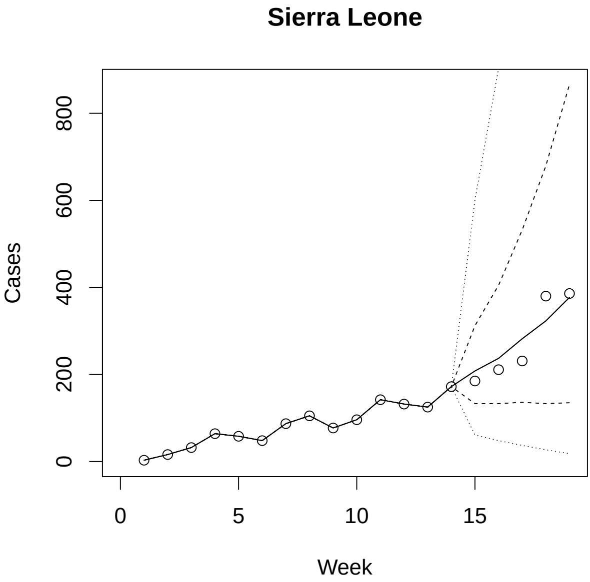
Modeling approaches

- Conceptual modeling
- Conceptual modeling
- Deterministic models
- Stochastic models
- Statistical fitting

Limitations

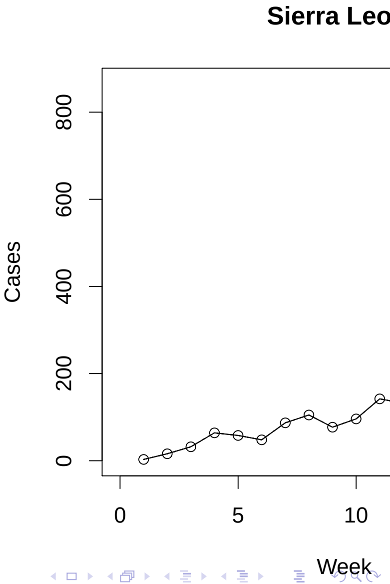
- Heterogeneity
- Behavioural changes

Statistical fitting



Statistical fitting

- How certain or uncertain are our projections?



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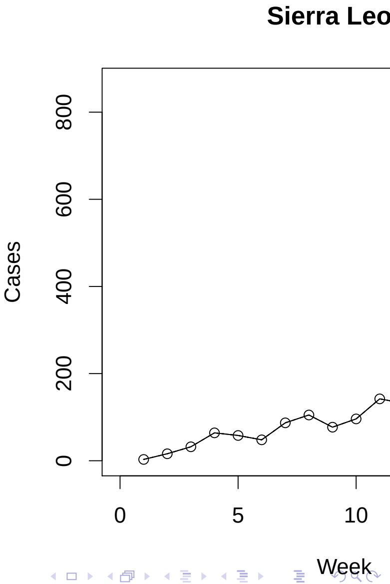

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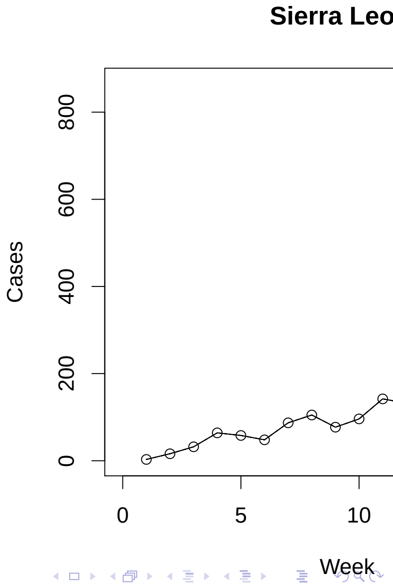

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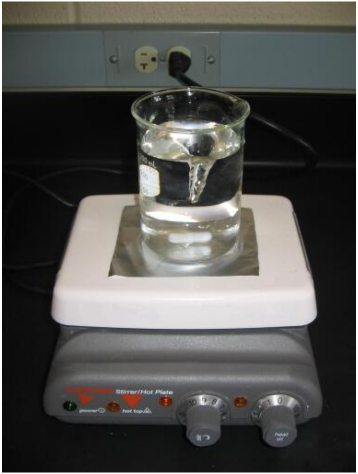

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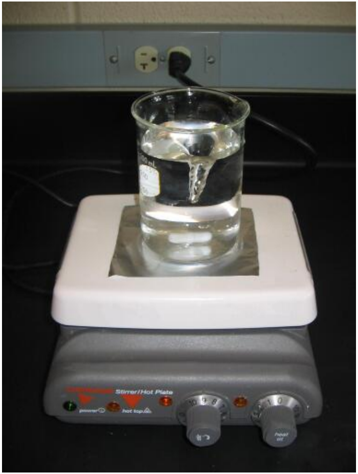

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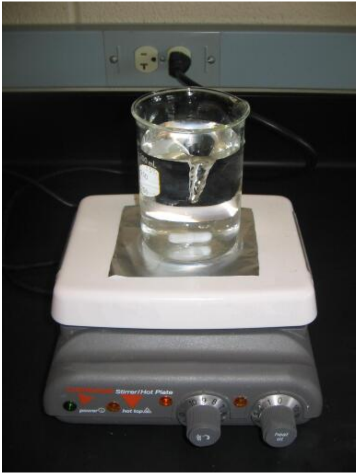

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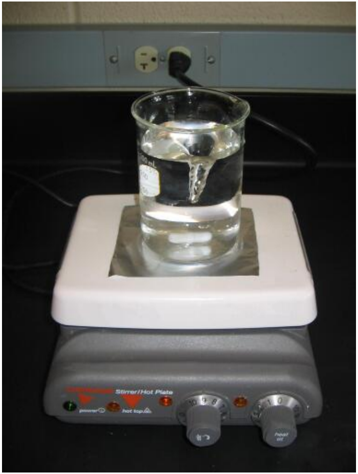

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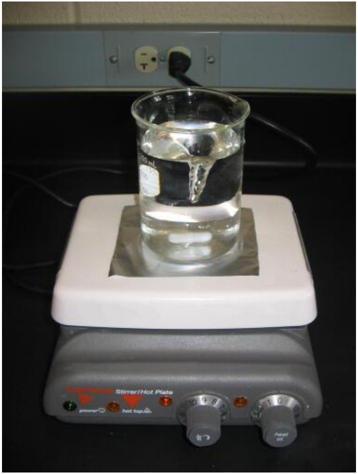

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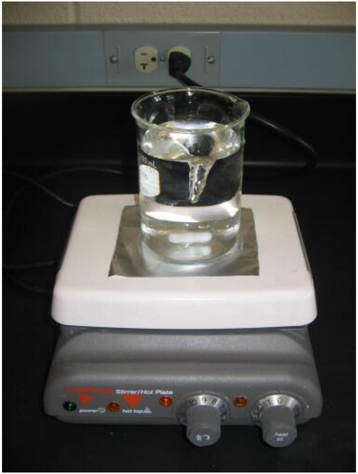

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Human heterogeneity

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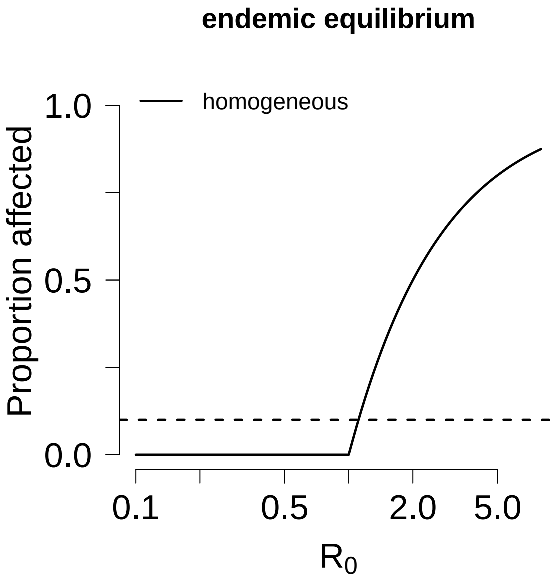
Example: Gonorrhea



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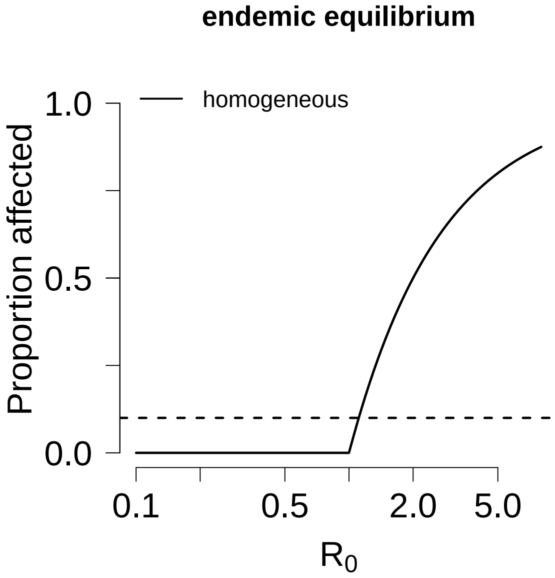

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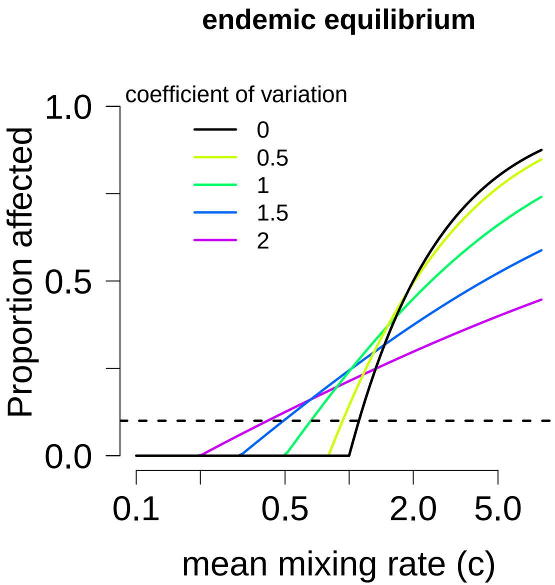
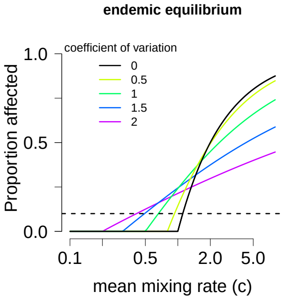


Result: heterogeneity makes incidence robust

- Disease levels are more resistant to change



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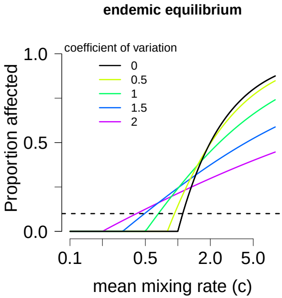

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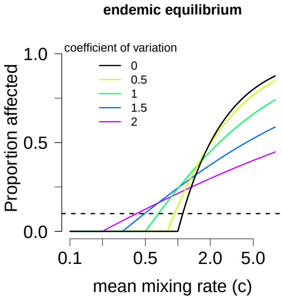

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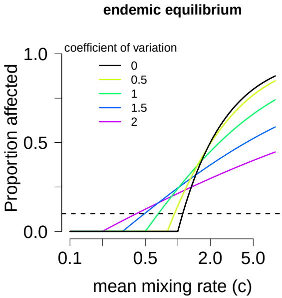
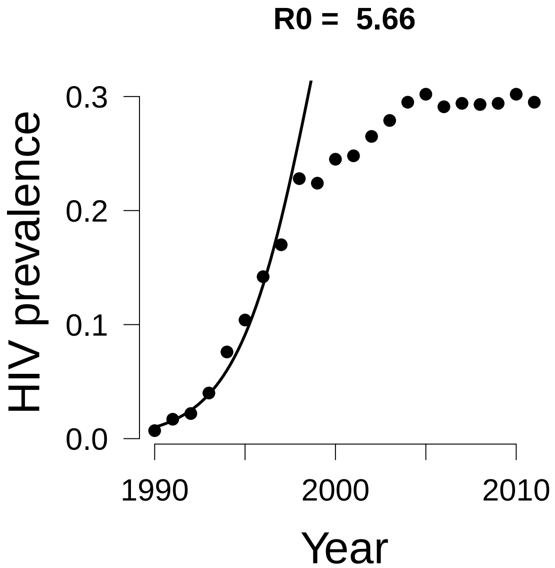


Example: HIV



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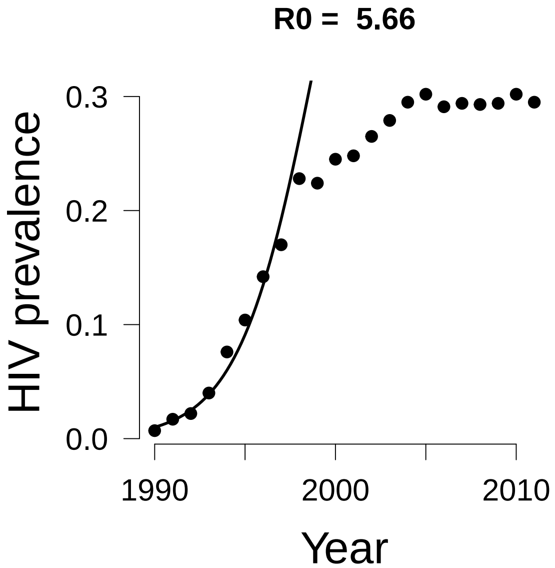
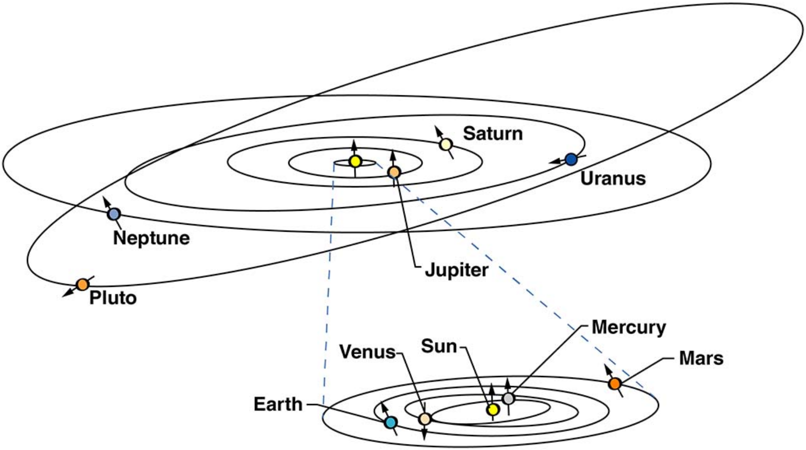


Outline

Behavioural changes

- I can calculate the motion of heavenly bodies, but not the madness of people. – Isaac Newton



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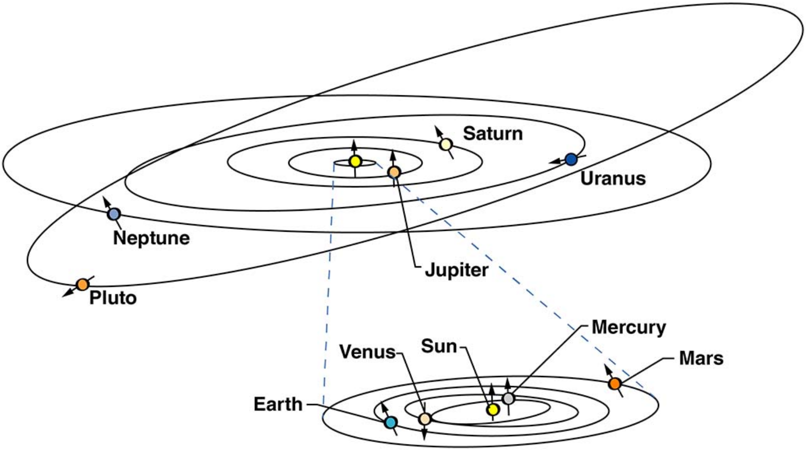
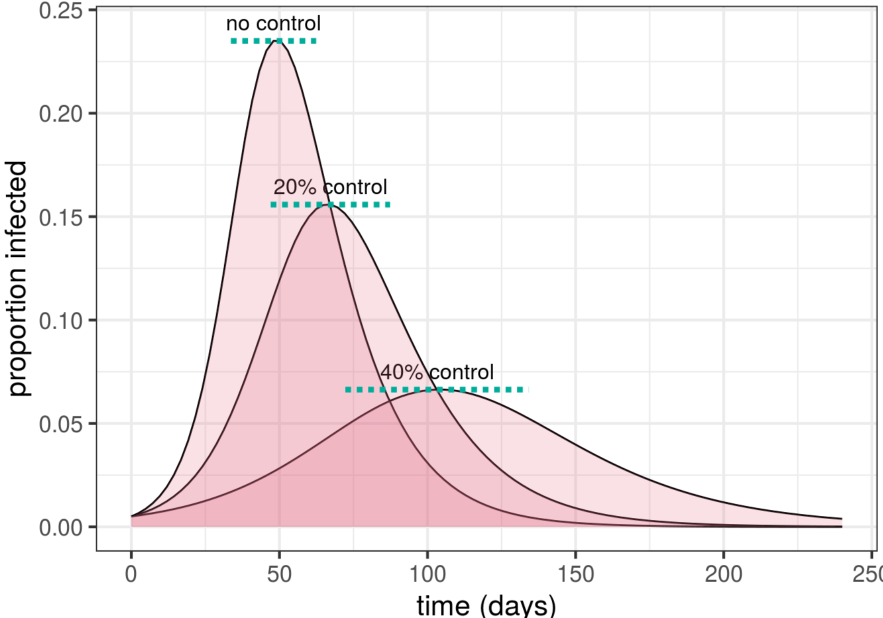


Example: COVID



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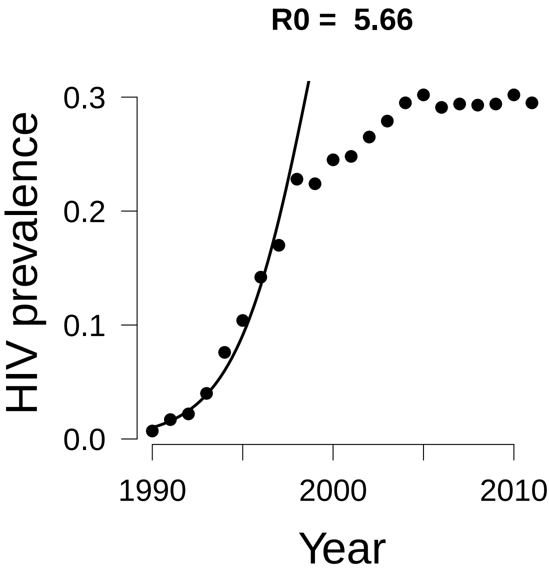
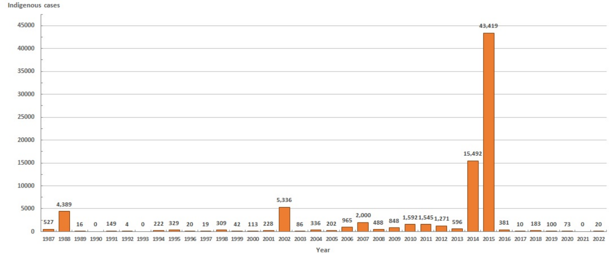


Example: Dengue (Taiwan CDC)

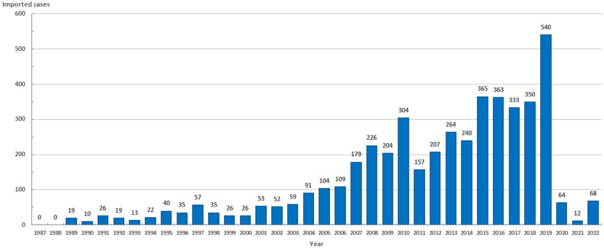
Indigenous cases

40,000

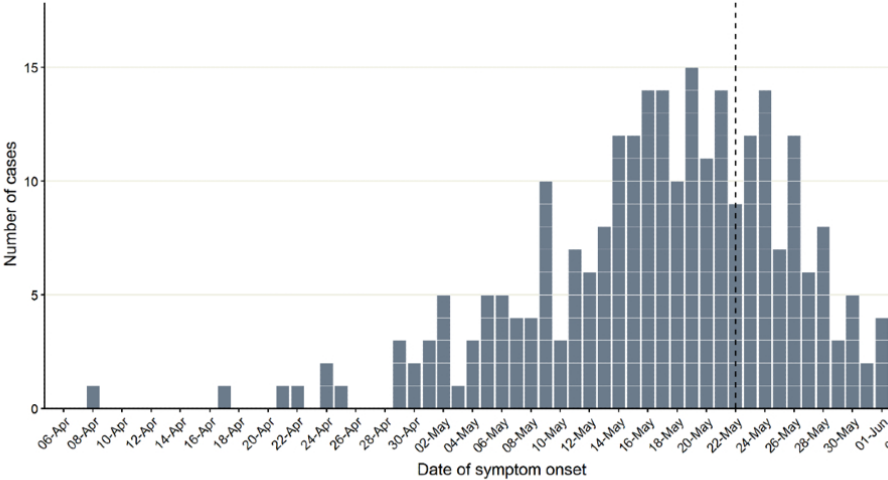


Imported cases

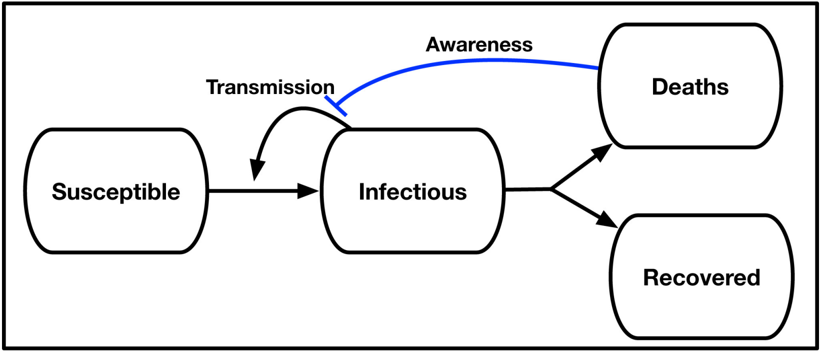
500



Example: monkeypox



Example: COVID awareness



Weitz et al.

https://www.pnas.org/doi/10.1073/pnas.2009911117

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