1 Introduction

Goals

- Introduce the idea of individual-based models
- Discuss the need for stochasticity
- Work through some pedagogical examples
- Talk about how to investigate stochastic models

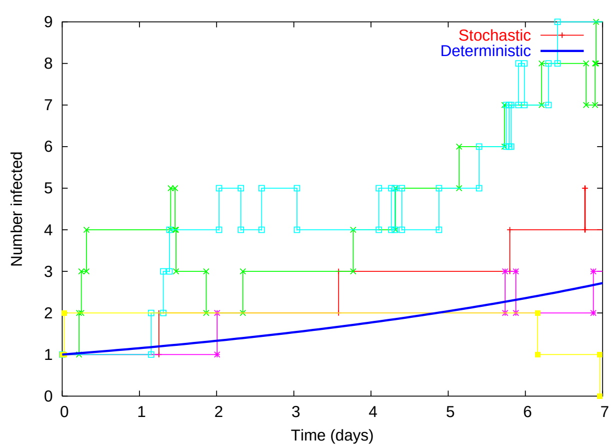
Modelling individual events

- Differential equations model continuous processes
- Disease spreads in the real world through discrete events
- Discrete events are fundamentally stochastic
  - Even in theory we don't know when the next event will occur, nor even what it will be

Types of stochasticity

- Demographic stochasticity is caused by the existence of individual people and discrete events
- Environmental stochasticity refers to events that affect more than one person at a time
  - Weather
  - Politics
  - Economics

Demographic spread

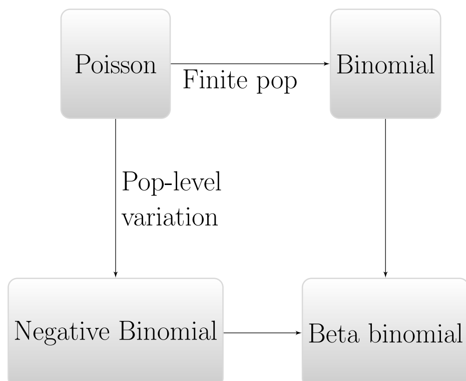


2 A discrete-time example

Model world

- Fixed $\mathcal{R}_0$
- Model spread by disease generation
  - Generations don't overlap *or* we're being sophisticated
- No recovery, birth or death
  - **Answer:** Maybe epidemic is fast enough that these don't matter

Practical simulation
Distribution diagram



Probability distributions

- Our practical example was equivalent to using which probability distribution?
  - **Answer:** Binomial
- What are other distributions we could have used, and what would that mean?
  - Poisson preserves the mean, but neglects the population size
    - Can be good when population size is unknown or large
    - **Answer:** compared to the mean
  - Poisson $\rightarrow$ negative binomial
    binomial $\rightarrow$ beta binomial
    allow for additional sources of variance

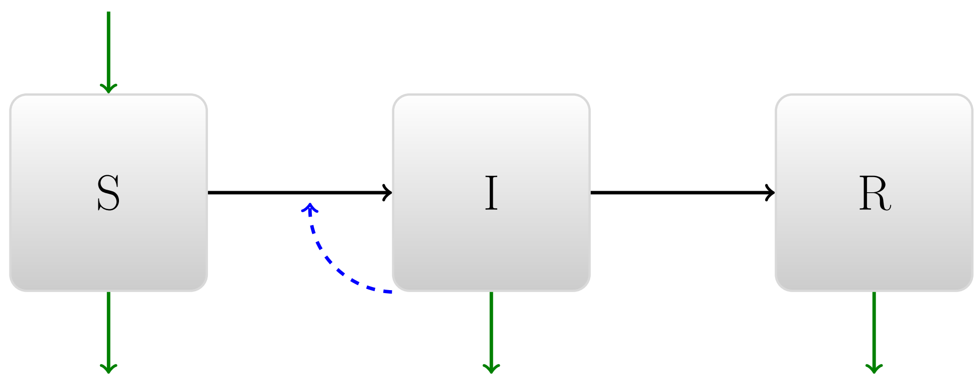
* **Answer:** Analogous to environmental stochasticity

3 A continuous-time framework

States and rates

- We describe our system in terms of the *probability rates* of events happening
  - If the rate of event E is $r_E(t)$, the probability of the event occurring in the time interval $(t, t + dt)$ is $r_E(t)dt$
- If we assume that event rates depend on measurable states then waiting times are exponential
  - We'll talk about relaxing this assumption

States and rates (Demographic)



Event	**transition**	**rate**	**Effect** (S, I)
Infection	$S \to I$	$\beta SI/N$	$(-1, 1)$
Recovery	$I \to R$	γI	$(0, -1)$
Rebirth	$R \to S$	$\mu(N - S - I)$	$(1, 0)$
Rebirth	$I \to S$	μI	$(1, -1)$

Analogy

- The demographic model is an exact analogue of the deterministic one
  - Conceptually
  - In the limit as $N \to \infty$

4 A continuous time example

Model world

- Simple SIR
- No births, deaths or loss of immunity
- Homogeneous mixing

Practical simulation

Spreadsheet with event-based simulation

5 Analyzing stochastic systems

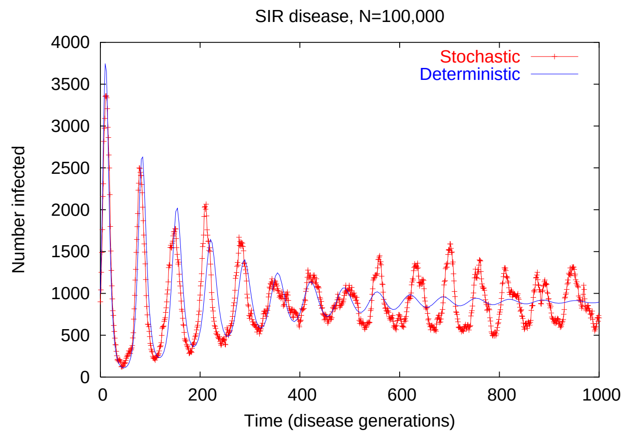
Realizations and ensembles

- How do we think about the behavior of a stochastic process?
  - A single example of how the process could go (e.g., from a stochastic simulation) is called a *realization*
  - The universe of possible realizations is called the *ensemble*.
  - The probability distribution that describes what state we expect the population to be in at time t is called the *ensemble distribution*

Some techniques

- Simulate one or many realizations
- Simulate the ensemble distribution
  - Requires one state variable for each possible state of the system
- Solve the ensemble distribution dynamics exactly!
  - Rarely possible
- Approximations to the ensemble distribution

Demographic model



Questions

- What kind of questions do we want to ask with a stochastic model?
  - How does stochasticity affect disease dynamics?
    - Spatial distribution
    - Establishment
    - Persistence
  - How much variance do we expect stochasticity to cause?
  - Under what circumstances can we eliminate or eradicate a disease?

The fate of infectious disease

- Fizzle
  - Disease fails to "establish"
- Burn-out
  - Disease goes extinct after first epidemic
- Fade-out
  - Disease goes extinct after system approaches quasi-equilibrium

Mathematics

- Simple models can approximate:
  - The probability of fizzle, and burn out
  - The average time until fade-out if the system survives fizzle and burn-out

- What about permanence
  - Mathematically impossible! Why?
  - **Answer:** In practice, corresponds to a very large average time until fade-out

6 Conclusions

- Stochasticity is real
  - People are individuals
  - The world has weather, and history
- Even minimal stochasticity can have dramatic effects on models of disease transmission
  - Amplified by acquired immunity
- Stochastic models are hard, and we usually combine techniques to understand them
- Even though it's real, you may not need it for your research question