Foundations of dynamic modeling:
The SIR Model Family

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Goals

- This lecture will:
  - introduce the idea of dynamical modeling
  - explain why dynamical modeling is a key tool for understanding infectious disease
  - discuss and demonstrate simple dynamical models from the SIR model family
  - investigate some insights that can be gained from these models

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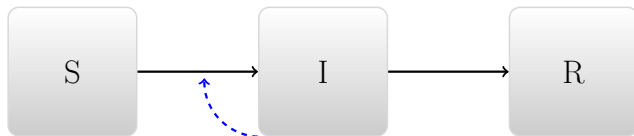
Dynamic modeling connects scales

Measles reports from England and Wales
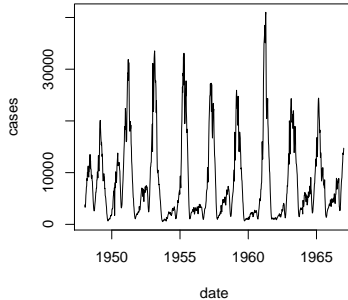
- ▶ Start with rules about how things change in short time steps
  - ▶ Usually based on *individuals*
- ▶ Calculate results over longer time periods
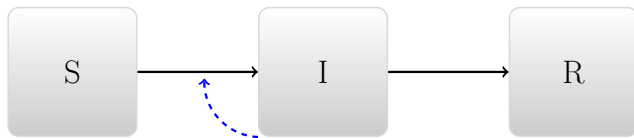 - ▶ Usually about *populations*
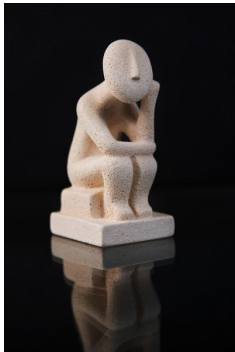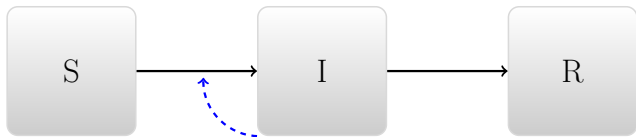
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Compartmental models

Divide people into categories:

- Susceptible → Infectious → Recovered
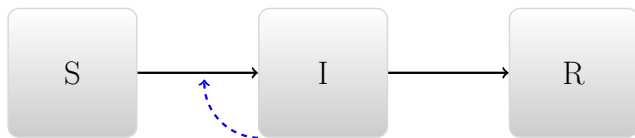
What determines transition rates?

- People get better independently

- People get infected by infectious people
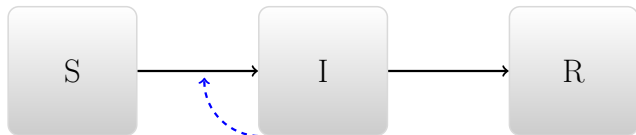
Conceptual modeling

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Conceptual modeling

- What is the final result?

- When does disease increase, decrease?

Dynamic implementation

- ▶ Requires assumptions about recovery and transmission

- ▶ The *conceptually simplest* implementation uses Ordinary Differential Equations (ODEs)
 - ▶ Other options may be more realistic
  - ▶ Or simpler in practice

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Recovery

- Infectious people recover at *per capita* rate γ
  - Total recovery rate is γI
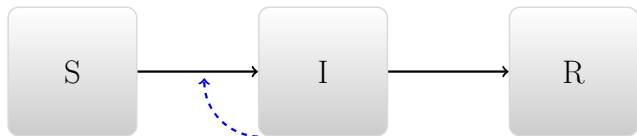 - Mean time infectious is $D = 1/\gamma$

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Transmission
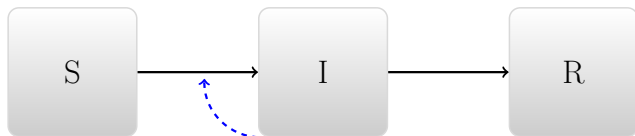
- Susceptible people get infected by:
  - Going around and contacting people (rate *c*)
  - Some of these people are infectious (proportion I/N)
  - Some of these contacts are effective (proportion *p*)
- Per capita rate of becoming infected is $cpI/N \equiv \beta I/N$
- Population-level transmission rate is $\mathcal{T} = \beta SI/N$

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Another perspective on transmission
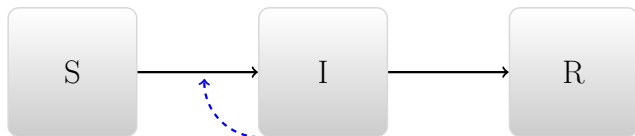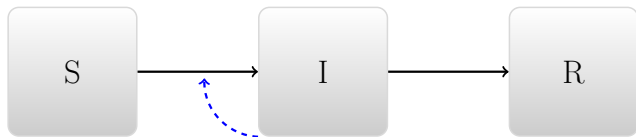

- Infectious people infect others by:
    - Going around and contacting people (rate *c*)
    - Some of these people are susceptible (proportion S/N)
    - Some of these contacts are effective (proportion *p*)

- Per capita rate of infecting others is $cpS/N \equiv \beta S/N$

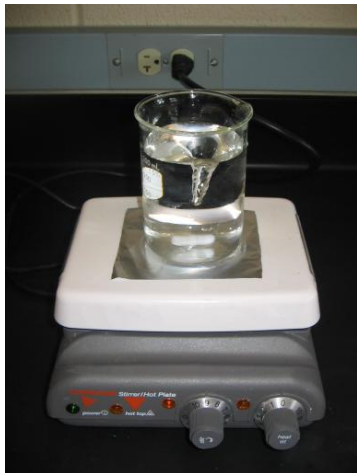- Population-level transmission rate is $\mathcal{T} = \beta SI/N$

ODE implementation

$$\frac{dS}{dt} = -\beta\frac{SI}{N}$$

$$\frac{dI}{dt} = \beta\frac{SI}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

Spreadsheet implementation

http://tinyurl.com/SIR-DAIDD-2017

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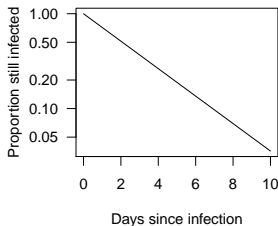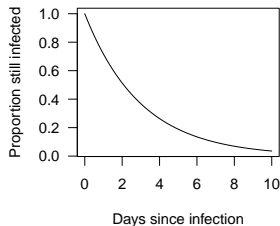# ODE assumptions

- Lots and lots of people

- Perfectly mixed

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ODE assumptions

- Waiting times are exponentially distributed

- Rarely realistic

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Scripts vs. spreadsheets

- ▶ Scripts are more transparent, less redundant

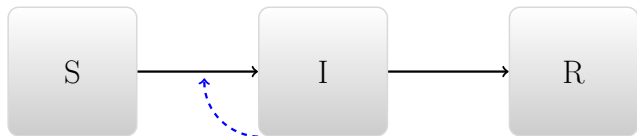- ▶ Spreadsheets are more intuitive for simple problems

More about transmission

- $\beta = pc$
  - What is a contact?
  - What is the probability of transmission?
- Sometimes this decomposition is clear
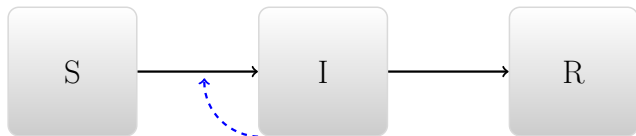- But usually it's not

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Population sizes

- How does β change with population size?

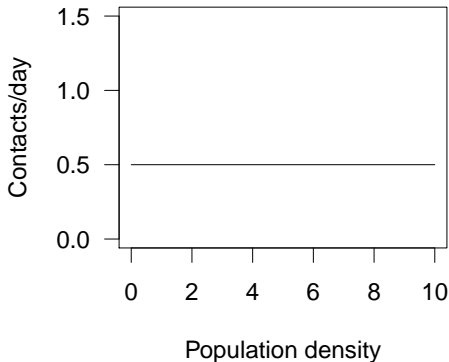- Recall that β is the *per capita* rate of contacts

Population sizes

$$\frac{dS}{dt} = -\beta \frac{SI}{N}$$
$$\frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

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Population sizes

$$\frac{dS}{dt} = -\beta(N)\frac{SI}{N}$$

$$\frac{dI}{dt} = \beta(N)\frac{SI}{N} - \gamma I$$

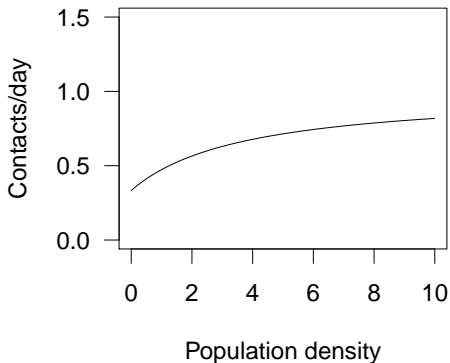$$\frac{dR}{dt} = \gamma I$$

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Standard incidence

- $\beta(N) = \beta_0$
- $\mathcal{T} = \frac{\beta_0 SI}{N}$
- Also known as *frequency-dependent* transmission

Mass action
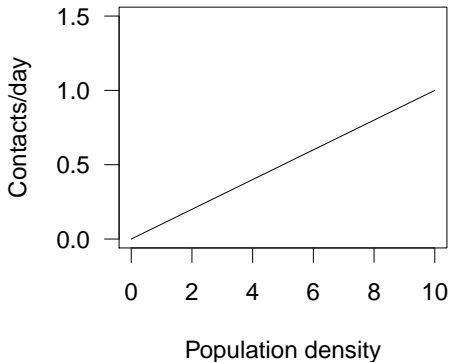
- $\beta(N) = \beta_1 N$
- $\mathcal{T} = \beta_1 SI$
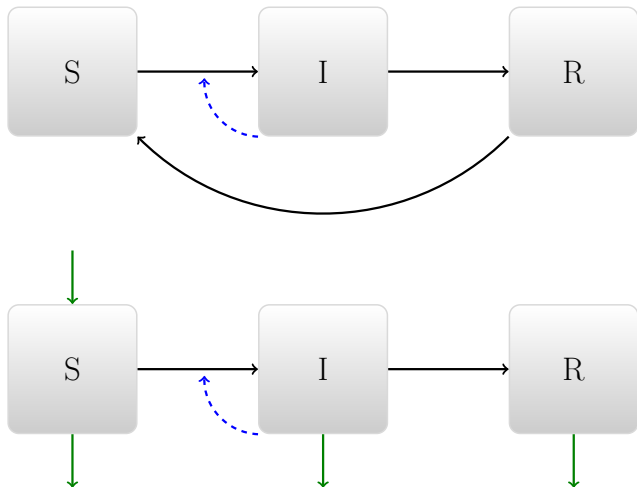- Also known as *density-dependent* transmission

General

- Per-capita rate:
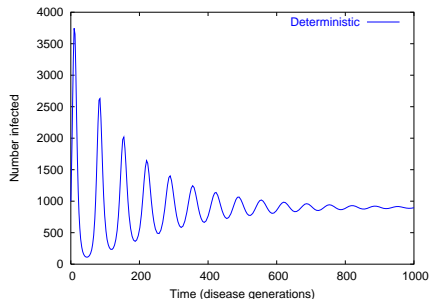 - May not go to zero when *N* does
  - May not go to ∞ when *N* does

Digression – units

- $\mathcal{T} = \beta SI / N : [\text{ppl/time}]$

- $\beta : [1/\text{time}]$
  - $\beta / \gamma = \beta D : [1]$
  - Standard incidence, $\beta_0 : [1/\text{time}]$
  - Mass-action incidence, $\beta_1 : [1/(\text{people} \cdot \text{time})]$
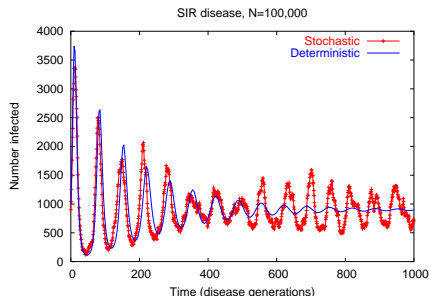
Closing the circle

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Tendency to oscillate

- Many susceptibles → many infections → few susceptibles → few infections → . . .

- Oscillations in simple models tend to be "damped"

With individuality

SIR disease, N=100,000

- ▶ Treating individuals as individuals can produce substantial oscillations even in large populations

- ▶ Interaction between random effects and the different time scales (of infection and recovery)

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Summary

- Dynamic models are an essential tool because they allow us to link between scales

- There are many ways to construct and implement dynamic models

- Very simple models can provide useful insights
  - Reproductive numbers and thresholds
  - Tendency for oscillation (and tendency for damping)

- More complex models can provide more detail, but also require more assumptions, and more choices

- Understanding simple models can help guide our understanding of more complicated models

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