Heterogeneity, contact patterns and modeling options

Jonathan Dushoff, McMaster University

`http://lalashan.mcmaster.ca/DushoffLab`

The resilience of infectious disease

1967: It's time to close the book on infectious diseases

Pathogen evolution

Human heterogeneity

1 Homogeneous disease models

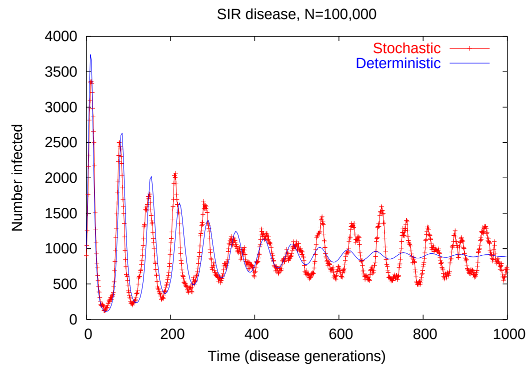
Expanding our models

- **Homogeneous** models assume everyone has the same:
  - disease characteristics (e.g. susceptibility, tendency to transmit)
  - mixing rate
  - probability of mixing with each person
- **Heterogeneous** models allow people to be different

The basic reproductive number

- $\mathcal{R}_0$ is the number of people who would be infected by an infectious individual *in a fully susceptible population.*
- $\mathcal{R}_0 = \beta/\gamma = \beta D = (cp)D$
  - c: Contact Rate
  - p: Probability of transmission (infectivity)
  - D: Average duration of infection
- A disease can invade a population if and only if $\mathcal{R}_0 > 1$.

Equilibrium



Equilibrium analysis

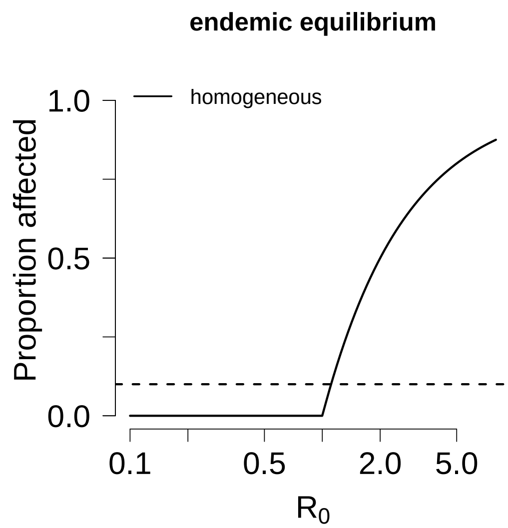
- $\mathcal{R}_{\text{eff}}$ is the number of people who would be infected by an infectious individual *in a general population.*

- $\mathcal{R}_{\text{eff}} = \mathcal{R}_0 \dfrac{S}{N} = pcD\dfrac{S}{N}$

- At equilibrium: $\mathcal{R}_{\text{eff}} = \mathcal{R}_0 \dfrac{S}{N} = 1.$

- Thus: $\dfrac{S}{N} = 1/R_0.$

- Proportion 'affected' is $V = 1 - S/N = 1 - 1/R_0.$

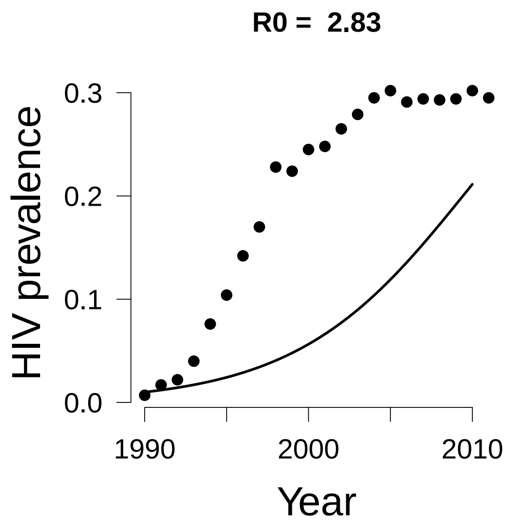


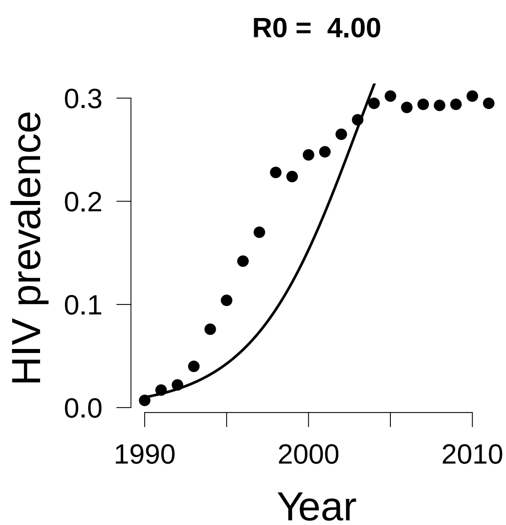
- Threshold value
- Sharp response to changes in factors underlying transmission
- Works – sometimes

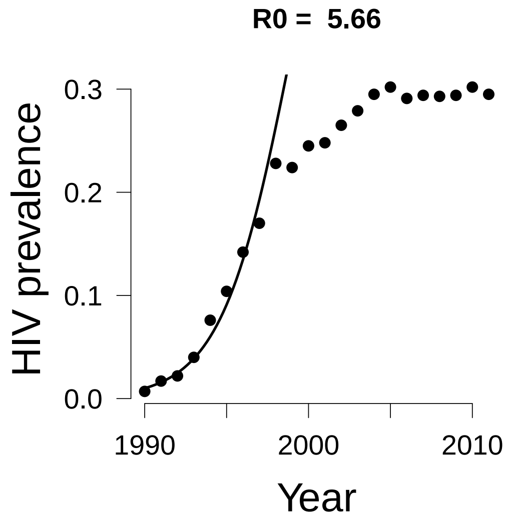
Proportion affected and disease prevalence

- For diseases with no recovery, V is the disease prevalence
- For other diseases, the equilibrium value of $P = I/N$ will be equal to V times the ratio of time spent sick to the time spent immune.
- Example: measles before vaccination
  - $V = 0.95$
  - $\bar{P} = 0.95 \times (2\text{wk}/60\text{yr})$.

Homogeneous assumptions





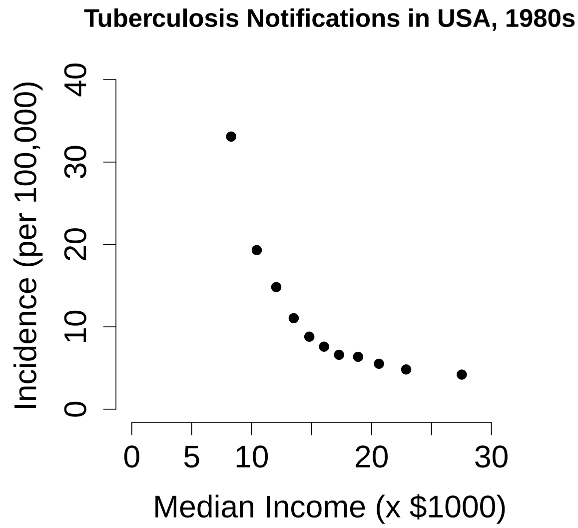


2 The importance of heterogeneity

Beyond homogeneity

- Flavors of heterogeneity
  - among hosts
  - spatial
  - demographic (discreteness of indviduals)
  - temporal
  - others

Heterogeneity in TB



- **Progression**: Nutrition, stress
- **Contact**: Overcrowding, poor ventilation
- **Cure**: Access to medical care

Heterogeneity in other diseases

- **STDs**: Sexual mixing patterns, access to medical care
- **Influenza**: Crowding, nutrition
- **Malaria**: Attractiveness to biting insects, geographical location, immune status
- **Every disease!**

Large-scale heterogeneity

- For schistosomiasis, the worldwide average $\mathcal{R}_0 < 1$
- Disease persists because of specific populations with $\mathcal{R}_0 > 1$.
- This effect operates at many scales.

3 Effects of heterogeneity

Equilibrium calculations

- Assume $p = \sigma\tau$ has a susceptibility component and a transmission component:
  - $\mathcal{R}_0 = \sigma\tau c D$
  - $\mathcal{R}_{\text{eff}} = \sigma\tau c D S/N$

Equilibrium calculations with heterogeneity

- τD applies to infectious individuals $\rightarrow \tau_I D_I$
- σ applies to susceptible individuals $\rightarrow \sigma_S$
- c is complicated $\rightarrow c_S c_I/\bar{c}$

Example

- Imagine a disease spread by people who differ only in their effective mixing rates
- If the disease has just started spreading in the population, how do c_S and c_I compare to $\bar{c}$?
  - $c_S \approx \bar{c}$; $c_I > \bar{c}$.
- If the disease is very widespread in the population?
  - $c_S < \bar{c}$; $c_I \rightarrow \bar{c}$.

Simpson's paradox

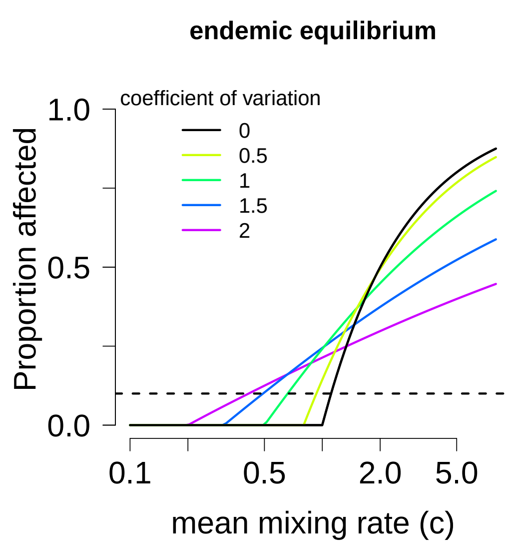
- What happens when a peanut farmer is elected to the US Senate?
- The average IQ goes up in both places!

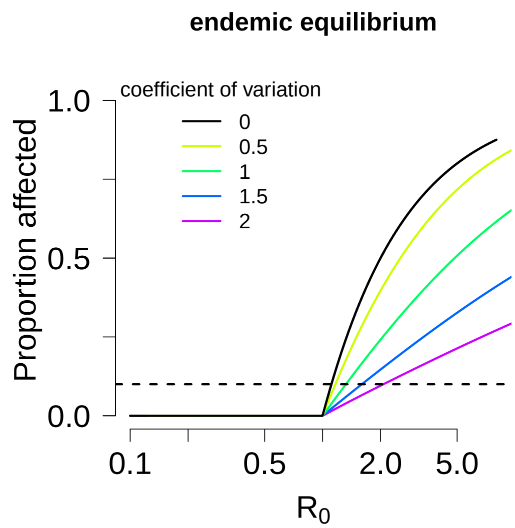
The basic reproductive number

- When the disease invades:
  - The susceptible population $\approx$ the general population
  - The infectious population is likely to have higher values of c, D and/or τ
- $\mathcal{R}_0$ is typically greater than you would expect from a homogeneous model

Equilibrium analysis

- As disease prevalence goes up:
  - Susceptible pool is the most resistant, or least exposed group
  - Infectious pool moves looks more like the general population.
- $\rightarrow$ lower proportion affected *for a given value of* $\mathcal{R}_0$.





Heterogeneity and disease

- Heterogeneity has a double-edged effect
  - Effects of disease are *lower* for a given value of $\mathcal{R}_0$.
  - But $\mathcal{R}_0$ is *higher* for given mean values of factors underlying transmission

Heterogeneous endemic curves

- Heterogeneity makes the endemic curve flatter
- Disease levels are more resistant to change

How diseases reach equilibrium

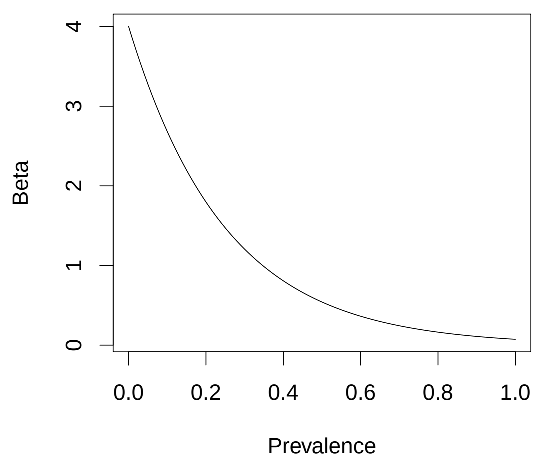
- Diseases that invade have high values of $\mathcal{R}_0$
- $\mathcal{R}_{\mathit{eff}}$ must be 1 at equilibrium
  - Potentially infectious contacts are wasted
    - Many potential contacts are not susceptible (affected by disease)
    - Those not affected less susceptible than average
  - Infectious pool less infectious

Spatial and network models

- Individual-level, or spatial, heterogeneity also usually increases wasted contacts
- Infectious people meet:
  - people with similar social backgrounds
  - people with similar behaviours
  - people who are nearby geographically or in the contact network
- More wasted contacts further flatten the endemic curve

4 Modeling approaches

Phenomenological

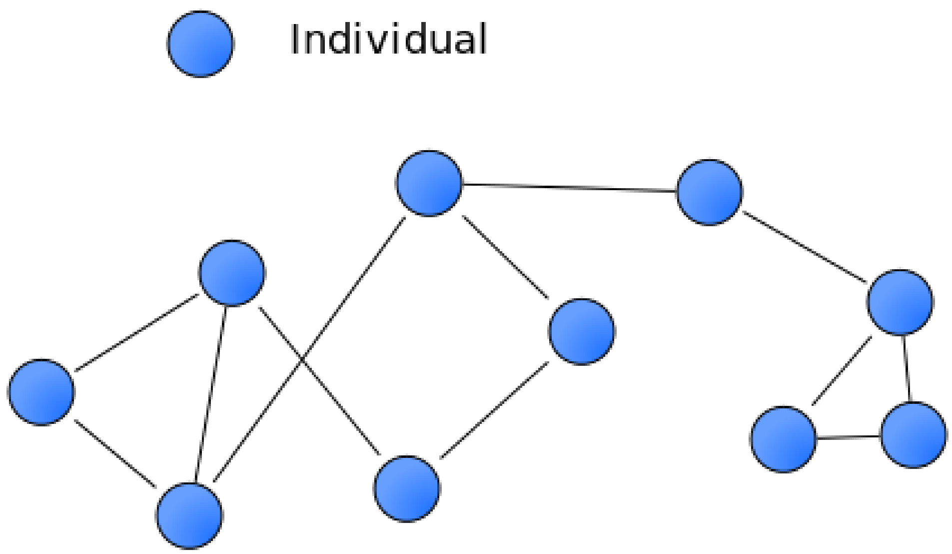


- You can simply *make* β go down as prevalence goes up
  - Need to choose a functional form

Multi-group models

- Divide the population into groups.
  - cities and villages
  - rich and poor
  - high and low sexual activity
  - age, gender
  - ...

Individual-based models



- Allow many possibilities:
  - vary individual characteristics
  - add a network of interactions
  - let the network change
- Individual-based approaches require stochastic models

Summary

People are heterogeneous in many ways

- ...and on many scales

Simple models give us important qualitative insights