Introduction to dynamical modeling

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Goals

- This lecture will:
  - introduce the idea of dynamical modeling
  - give simple examples of population modeling and disease modeling
  - discuss different types of model approaches

Dynamic modeling connects scales

Disease_data/ewmeas.Rout.pdf

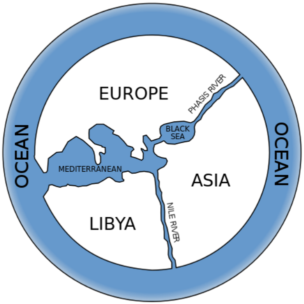
- Start with rules about how things change in short time steps
  - Usually based on *individuals*
- Calculate results over longer time periods
  - Usually about *populations*

Example: Dandelions

- Start with one dandelion; it produces 100 seeds, of which only 2% survive to reproduce.
- Seeds that survive to reproduce will produce 100 seeds after 1 year next year.
- How many dandelions after 3 years?

Model worlds

- A dynamic model is based on a model world
- The model world has *enough* assumptions to allow us to calculate dynamics
- The model world is *simpler* than the real world
- Essentially, all models are wrong, but some are useful. – Box and Draper (1987), *Empirical Model Building . . .*



Model result

- If each individual is reproducing independently at each time step, the population changes *exponentially*
  - it is *multiplied* by the same amount in each step.

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Scales

- The difference between 1 and 10 is the same as the difference between 10 and what?
  - *additive* difference:
    - * 19
  - *multiplicative* difference:
    - * 100

Scales

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Exponential_figures/ntu.Rout-

- Linear scale: 1 to 10 = 10 to 19
- Log scale: 1 to 10 = 10 to 100

Exponential change

- We can have exponential *growth* (population goes up)
- or exponential *decline* (population goes down)
- What if we spray the dandelions, so that each seed only has 0.5% chance of survival?

Exponential decline

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- Linear scale: 1 to 10 = 10 to 19
- Log scale: 1 to 10 = 10 to 100

Randomness

- Do our rules tell us exactly what is going to happen?
- If we have 1 dandelion this year, do we expect exactly two dandelions next year?
  - Do we expect exactly 1/2 of a dandelion?
- **Deterministic models**: rules describe exactly what will happen
- **Stochastic models**: rules describe a range of things that *might* happen

Stochastic model

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Time steps

- Dynamic models can use
  - *discrete time*: we model the population at specific time points
  - *continuous time*: we model time smoothly
- Which kind of model is the dandelion model?

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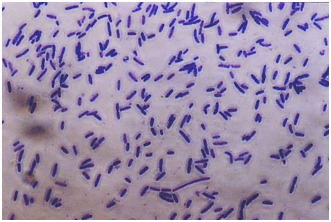
Bacteria

- Imagine we have some bacteria in a tank
- They are continuously dividing, and continuously dying



Bacteria

- Model world
  - The bacteria:
    - die at a constant *per capita* rate
    - divide at a constant *per capita* rate
- Model
  - $\frac{dN}{dt} = (b - d)N$

Model result

Exponential_figures/growth.Rout-2.pdf Exponential_figures/growth.Ro

- Population grows *exponentially*
- As long as $b > d$

Describing a model

- **Model structure:** what are the rules that our model is following?
  - Each individual is dividing and dying independently at a fixed rate
- **Parameters:** what quantities (with units) determine how the rules are working
  - Birth rate is 0.04/day
- **State variables:** what changing quantities are we modeling?
  - The number of bacteria

Simple models of disease spread

- Divide people into categories:

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What determines transition rates?

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Conceptual modeling

SIR_model_family/sir.three.pdf

Conceptual modeling

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Dynamic implementation

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Recovery

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Transmission

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Another perspective on transmission

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ODE implementation

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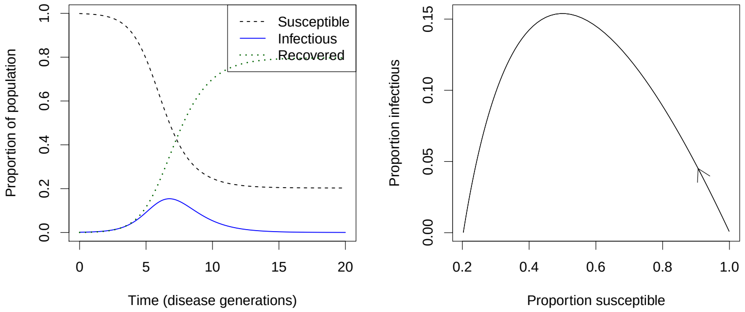
ODE implementation



- Not everyone will get infected
- Disease starts to decline when number of susceptibles is small

ODE assumptions

SIR_model_family/stirrer.jpg

- Lots and lots of people
- Perfectly mixed

Closing the circle

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Closing the circle

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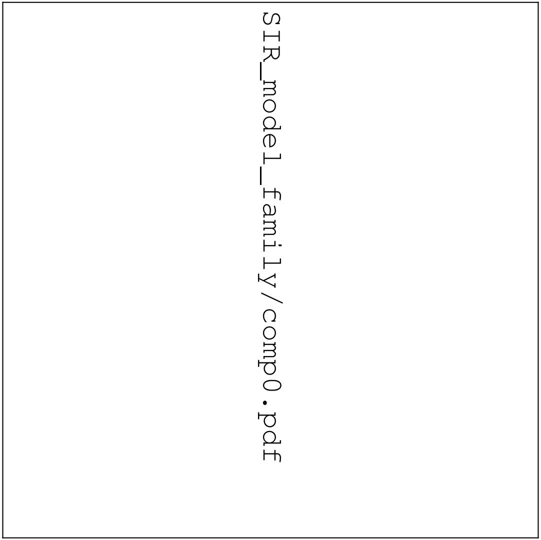
Tendency to oscillate



Modeling individuals as individuals usually requires a *stochastic* model

With individuality

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Even in the simplest form, this can cause large random oscillations even in large populations

Types

- **Discrete** vs. **Continuous** time steps
- **Deterministic** vs. **Stochastic** dynamics
    - Stochastic models may have **Discrete individuals**

Summary

- Dynamics are an essential tool to link scales
- Very simple models can provide useful insights
- More complex models can provide more detail, but also require more assumptions, and more choices