

Heterogeneity, contact patterns and modeling options

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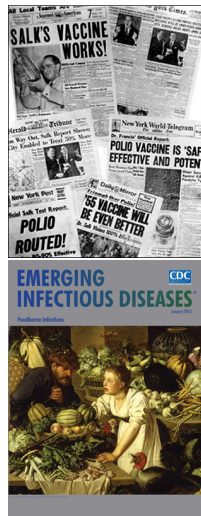
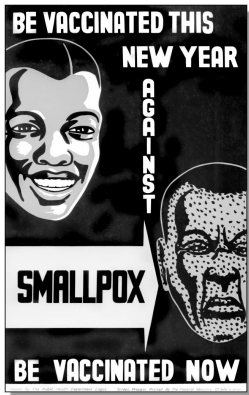
<http://lalashan.mcmaster.ca/DushoffLab>

2016 Summer Course on Mathematical Modeling and Analysis
of Infectious Diseases

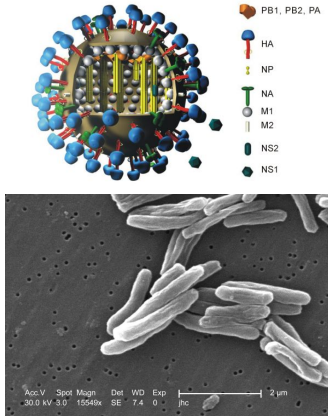
National Taiwan University

The resilience of infectious disease

1967: It's time to close the book on infectious diseases



Pathogen evolution



Human heterogeneity



Human heterogeneity



Human heterogeneity



Outline

Homogeneous disease models

The importance of heterogeneity

Effects of heterogeneity

Modeling approaches

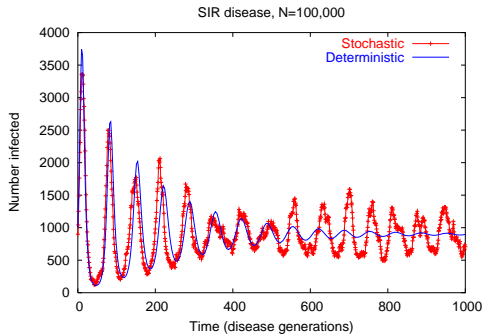
Expanding our models

- ▶ **Homogeneous** models assume everyone has the same:
 - ▶ disease characteristics (e.g. susceptibility, tendency to transmit)
 - ▶ mixing rate
 - ▶ probability of mixing with each person
- ▶ **Heterogeneous** models allow people to be different

The basic reproductive number

- ▶ \mathcal{R}_0 is the number of people who would be infected by an infectious individual *in a fully susceptible population*.
- ▶ $\mathcal{R}_0 = \beta/\gamma = \beta D = (cp)D$
 - ▶ c : Contact Rate
 - ▶ p : Probability of transmission (infectivity)
 - ▶ D : Average duration of infection
- ▶ A disease can invade a population if and only if $\mathcal{R}_0 > 1$.

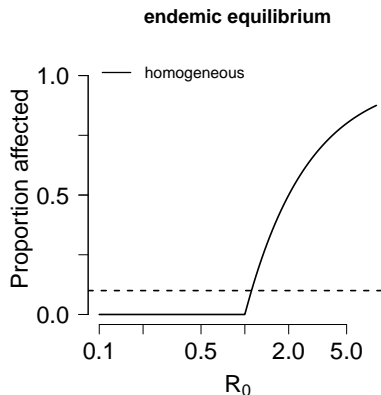
Equilibrium



Equilibrium analysis

- ▶ \mathcal{R}_{eff} is the number of people who would be infected by an infectious individual *in a general population*.
- ▶ $\mathcal{R}_{eff} = \mathcal{R}_0 \frac{S}{N} = pcD \frac{S}{N}$
- ▶ At equilibrium: $\mathcal{R}_{eff} = \mathcal{R}_0 \frac{S}{N} = 1$.
- ▶ Thus: $\frac{S}{N} = 1/R_0$.
- ▶ Proportion 'affected' is $V = 1 - S/N = 1 - 1/R_0$.

Homogeneous endemic curve

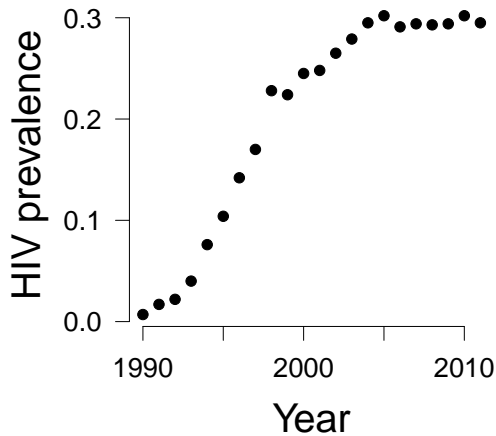


- ▶ Threshold value
- ▶ Sharp response to changes in factors underlying transmission
- ▶ Works – sometimes

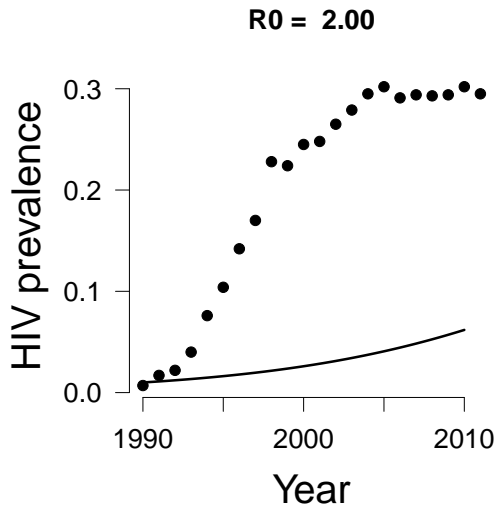
Proportion affected and disease prevalence

- ▶ For diseases with no recovery, V is the disease prevalence
- ▶ For other diseases, the equilibrium value of $P = I/N$ will be equal to V times the ratio of time spent sick to the time spent immune.
- ▶ Example: measles before vaccination
 - ▶ $V = 0.95$
 - ▶ $\bar{P} = 0.95 \times (2\text{wk}/60\text{yr})$.

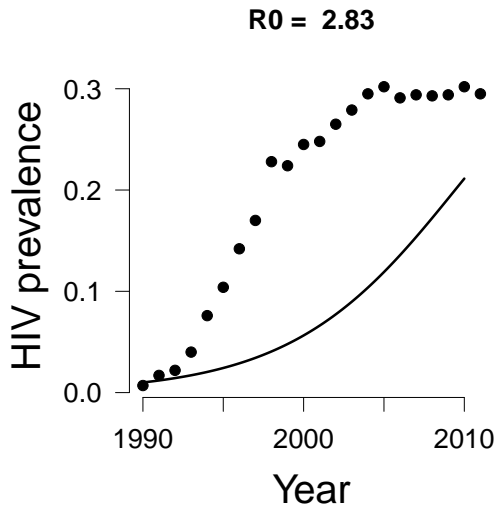
Disease dynamics



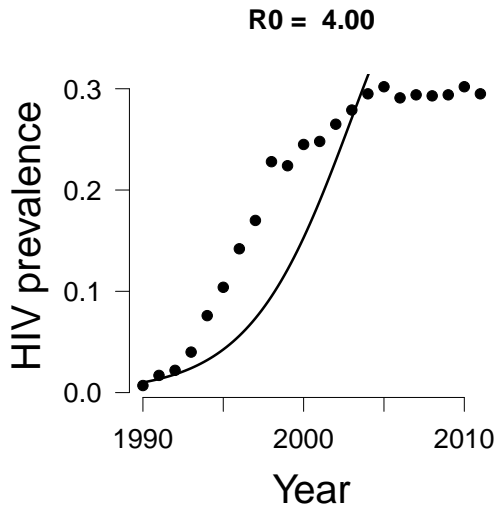
Homogeneous assumptions



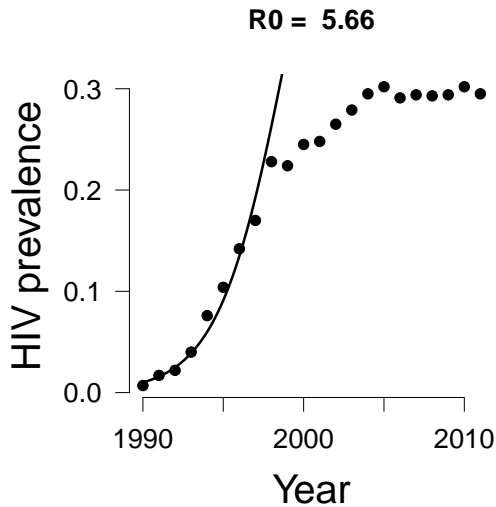
Homogeneous assumptions



Homogeneous assumptions



Homogeneous assumptions



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The importance of heterogeneity

Effects of heterogeneity

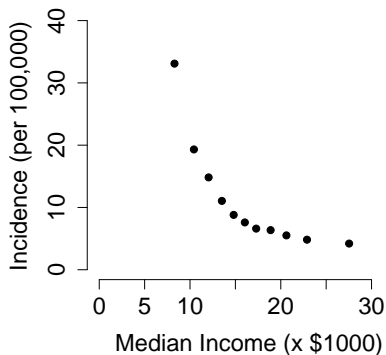
Modeling approaches

Beyond homogeneity

- ▶ Flavors of heterogeneity
 - ▶ among hosts
 - ▶ spatial
 - ▶ demographic (discreteness of individuals)
 - ▶ temporal
 - ▶ others

Heterogeneity in TB

Tuberculosis Notifications in USA, 1980s



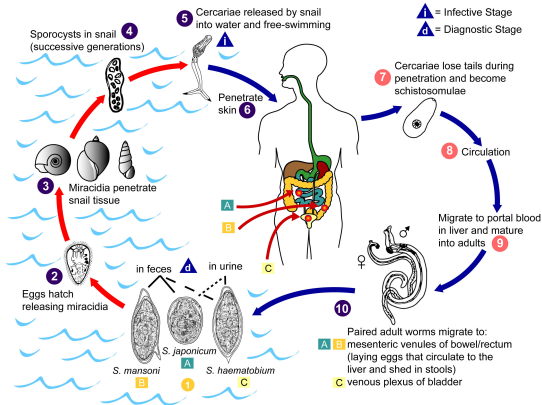
- ▶ **Progression:**
Nutrition, stress
- ▶ **Contact:**
Overcrowding, poor ventilation
- ▶ **Cure:** Access to medical care

Heterogeneity in other diseases

- ▶ **STDs:** Sexual mixing patterns, access to medical care
- ▶ **Influenza:** Crowding, nutrition
- ▶ **Malaria:** Attractiveness to biting insects, geographical location, immune status
- ▶ **Every disease!**

Large-scale heterogeneity

Schistosomiasis



- ▶ For schistosomiasis, the worldwide average $\mathcal{R}_0 < 1$
- ▶ Disease persists because of specific populations with $\mathcal{R}_0 > 1$.
- ▶ This effect operates at many scales.

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Equilibrium calculations

- ▶ Assume $p = \sigma\tau$ has a susceptibility component and a transmission component:
 - ▶ $\mathcal{R}_0 = \sigma\tau cD$
 - ▶ $\mathcal{R}_{eff} = \sigma\tau cDS/N$

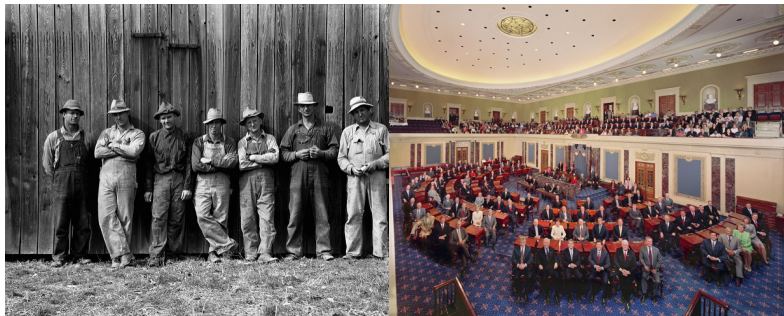
Equilibrium calculations with heterogeneity

- ▶ τD applies to infectious individuals $\rightarrow \tau_I D_I$
- ▶ σ applies to susceptible individuals $\rightarrow \sigma_S$
- ▶ c is complicated $\rightarrow c_S c_I / \bar{c}$

Example

- ▶ Imagine a disease spread by people who differ only in their effective mixing rates
- ▶ If the disease has just started spreading in the population, how do c_S and c_I compare to \bar{c} ?
 - ▶ $c_S \approx \bar{c}$; $c_I > \bar{c}$.
- ▶ If the disease is very widespread in the population?
 - ▶ $c_S < \bar{c}$; $c_I \rightarrow \bar{c}$.

Simpson's paradox



- ▶ What happens when a peanut farmer is elected to the US Senate?
- ▶ The average IQ goes up in both places!

The basic reproductive number

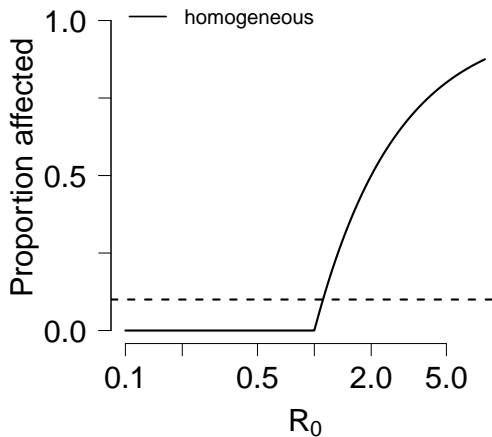
- ▶ When the disease invades:
 - ▶ The susceptible population \approx the general population
 - ▶ The infectious population is likely to have higher values of c , D and/or τ
- ▶ \mathcal{R}_0 is typically greater than you would expect from a homogeneous model

Equilibrium analysis

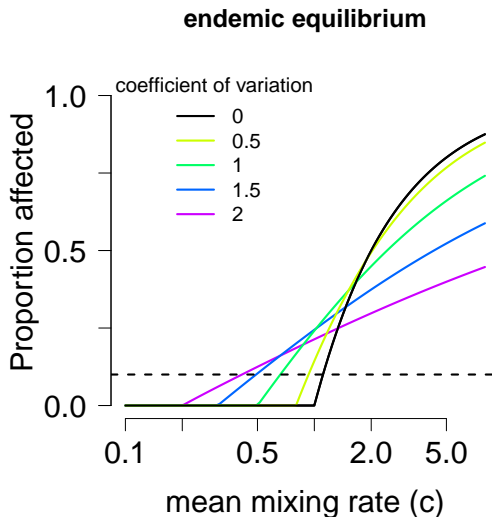
- ▶ As disease prevalence goes up:
 - ▶ Susceptible pool is the most resistant, or least exposed group
 - ▶ Infectious pool moves looks more like the general population.
- ▶ → lower proportion affected *for a given value of \mathcal{R}_0 .*

Homogeneous endemic curve

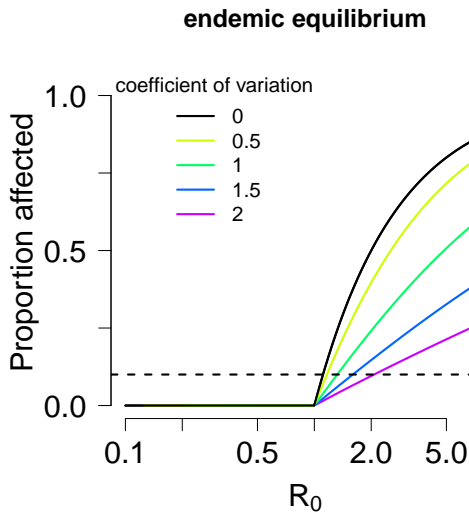
endemic equilibrium



Heterogeneous endemic curves

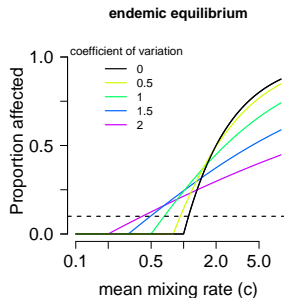


Heterogeneous endemic curves



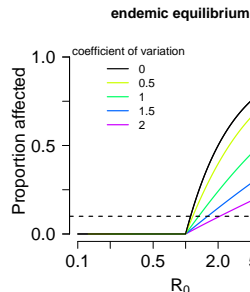
Heterogeneity and disease

- ▶ Heterogeneity has a double-edged effect
 - ▶ Effects of disease are *lower* for a given value of \mathcal{R}_0 .
 - ▶ But \mathcal{R}_0 is *higher* for given mean values of factors underlying transmission



Heterogeneous endemic curves

- ▶ Heterogeneity makes the endemic curve flatter
- ▶ Disease levels are more resistant to change



How diseases reach equilibrium

- ▶ Diseases that invade have high values of \mathcal{R}_0
- ▶ \mathcal{R}_{eff} must be 1 at equilibrium
 - ▶ Potentially infectious contacts are wasted
 - ▶ Many potential contacts are not susceptible (affected by disease)
 - ▶ Those not affected less susceptible than average
 - ▶ Infectious pool less infectious

Spatial and network models

- ▶ Individual-level, or spatial, heterogeneity also usually increases wasted contacts
- ▶ Infectious people meet:
 - ▶ people with similar social backgrounds
 - ▶ people with similar behaviours
 - ▶ people who are nearby geographically or in the contact network
- ▶ More wasted contacts further flatten the endemic curve

Outline

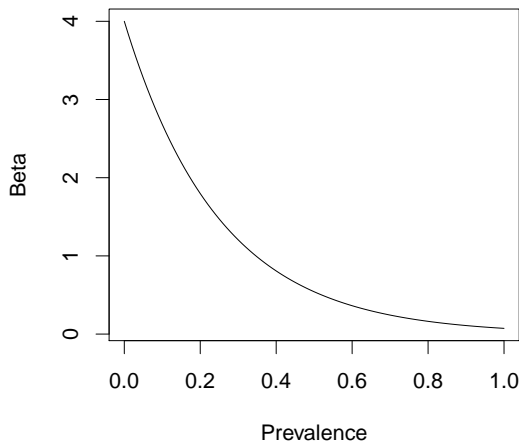
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Modeling approaches

Phenomenological



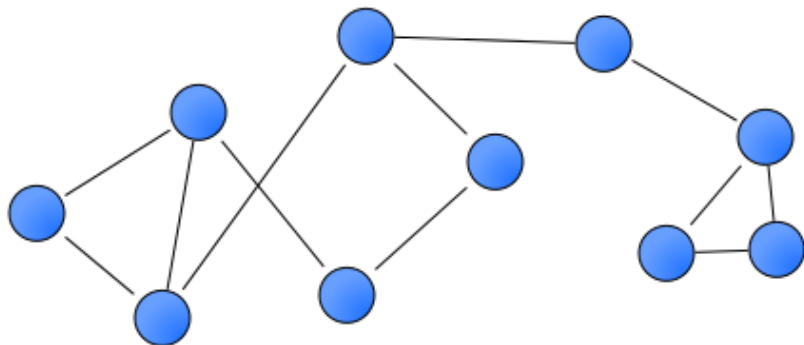
- ▶ You can simply *make* β go down as prevalence goes up
 - ▶ Need to choose a functional form

Multi-group models

- ▶ Divide the population into groups.
 - ▶ cities and villages
 - ▶ rich and poor
 - ▶ high and low sexual activity
 - ▶ age, gender
 - ▶ ...

Individual-based models

 Individual

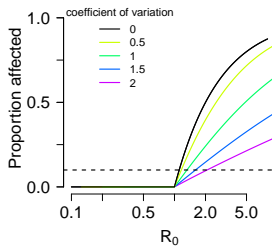


- ▶ Allow many possibilities:
 - ▶ vary individual characteristics
 - ▶ add a network of interactions
 - ▶ let the network change
- ▶ Individual-based approaches require stochastic models

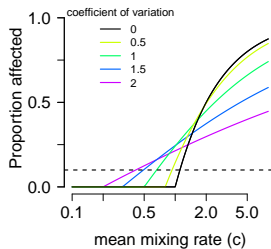
Summary



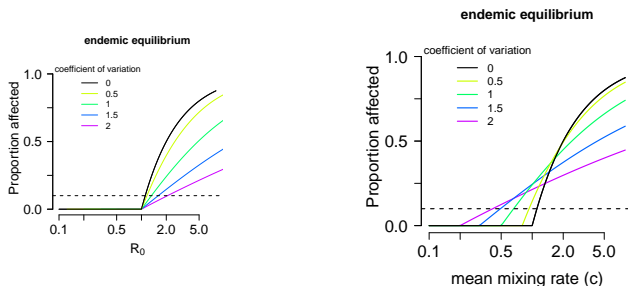
endemic equilibrium



endemic equilibrium



Summary



People are heterogeneous in many ways

- ...and on many scales

Simple models give us important qualitative insights