

# Heterogeneity, contact patterns and modeling options

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## Goals

- Explain the importance of heterogeneity on patterns of disease spread
  - Focus on different types of human heterogeneity
- Discuss ways in which homogeneous models fail to match observed dynamics
- Use simple models to explore qualitative effects of heterogeneity on modeling conclusions
- Briefly introduce some methods that are used to incorporate heterogeneity in models

## The resilience of infectious disease

1967: It's time to close the book on infectious diseases

## Pathogen evolution

## Human heterogeneity

# 1 Homogeneous disease models

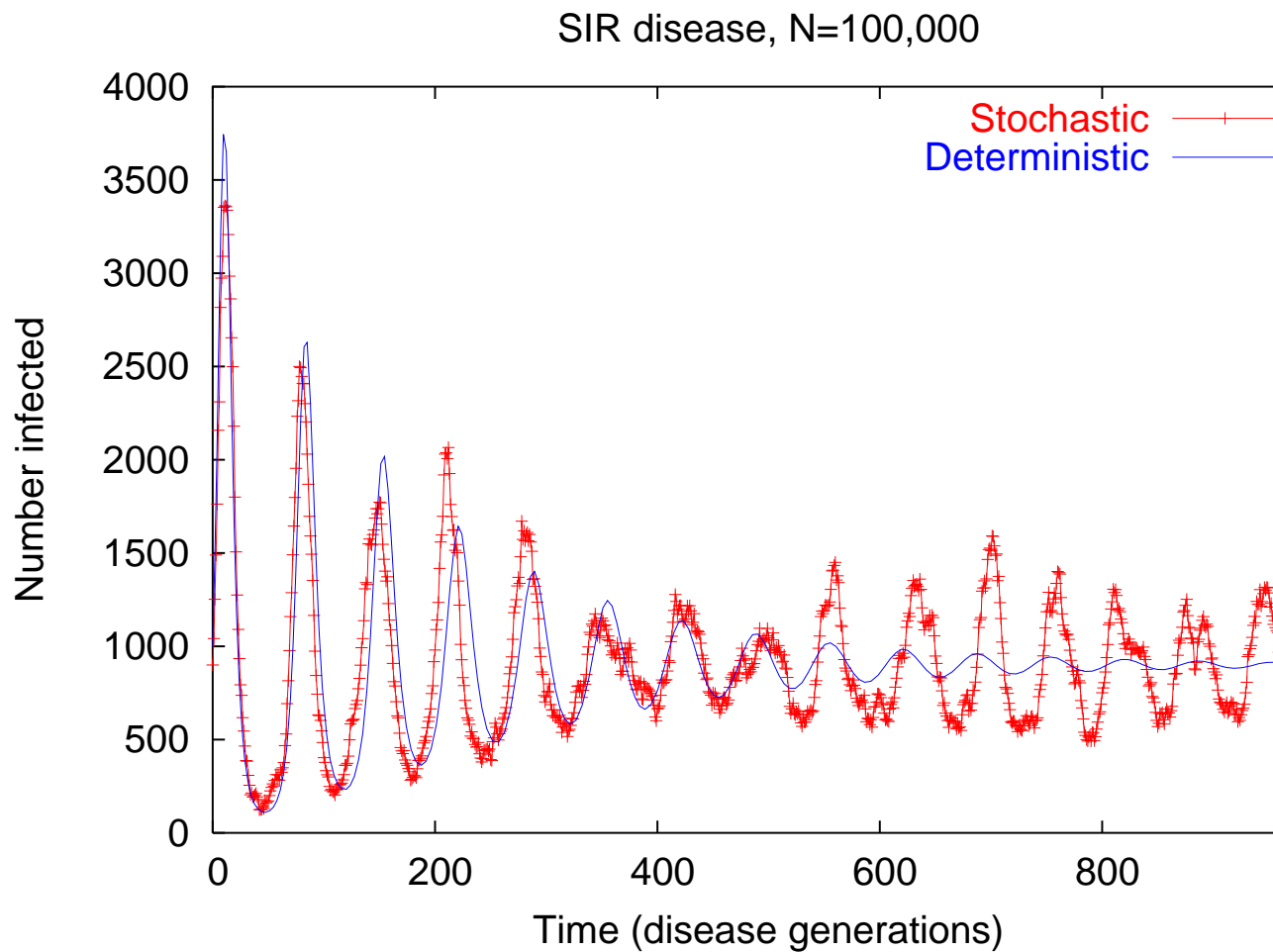
## Expanding our models

- **Homogeneous** models assume everyone has the same:
  - disease characteristics (e.g. susceptibility, tendency to transmit)
  - mixing rate
  - probability of mixing with each person
- **Heterogeneous** models allow people to be different

## The basic reproductive number

- $\mathcal{R}_0$  is the number of people who would be infected by an infectious individual *in a fully susceptible population*.
- $\mathcal{R}_0 = \beta/\gamma = \beta D = (cp)D$ 
  - $c$ : Contact Rate
  - $p$ : Probability of transmission (infectivity)
  - $D$ : Average duration of infection
- A disease can invade a population if and only if  $\mathcal{R}_0 > 1$ .

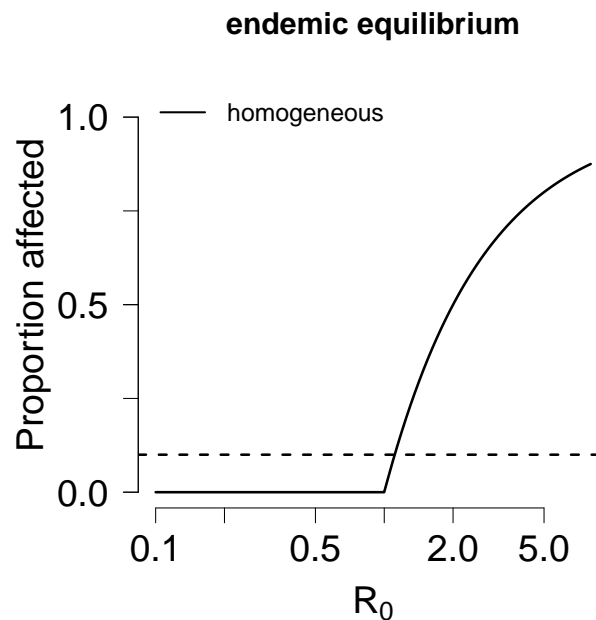
## Equilibrium



- Equilibrium is worth knowing even if the disease doesn't reach equilibrium
- System will move around the equilibrium

## Equilibrium analysis

- $\mathcal{R}_{\text{eff}}$  is the number of people who would be infected by an infectious individual *in a general population*.
- $\mathcal{R}_{\text{eff}} = \mathcal{R}_0 \frac{S}{N} = pcD \frac{S}{N}$
- At equilibrium:  $\mathcal{R}_{\text{eff}} = \mathcal{R}_0 \frac{S}{N} = 1$ .
- Thus:  $\frac{S}{N} = 1/R_0$ .
- Proportion 'affected' is  $V = 1 - S/N = 1 - 1/R_0$ .

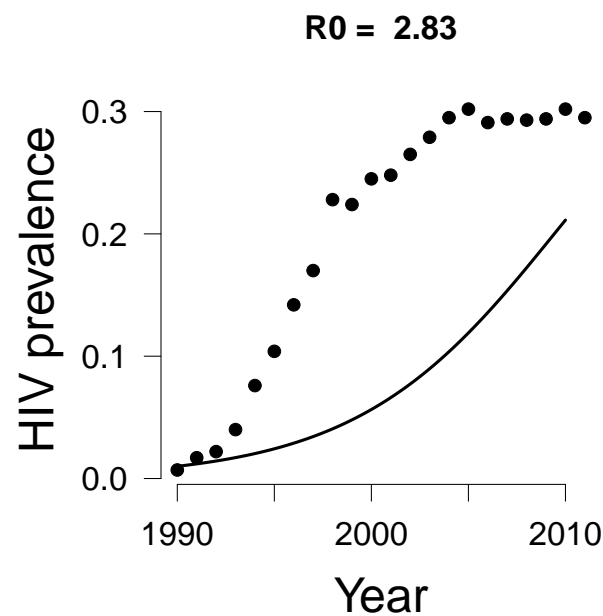


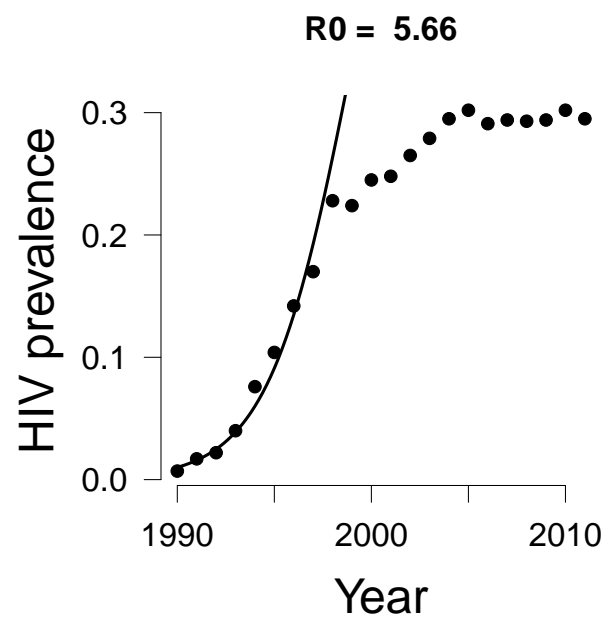
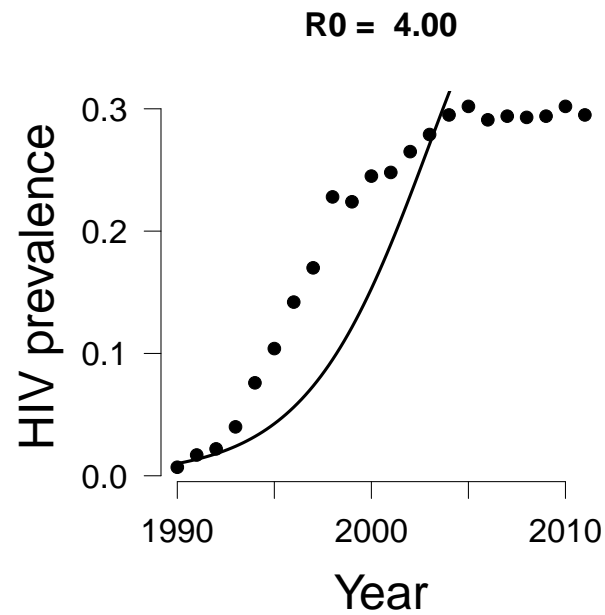
- Threshold value
- Sharp response to changes in factors underlying transmission
- Works – sometimes
- Sometimes predicts unrealistic sensitivity

## Proportion affected and disease prevalence

- For diseases with no recovery,  $V$  is the disease prevalence
- For other diseases, the equilibrium value of  $P = I/N$  will be equal to  $V$  times the ratio of time spent sick to the time spent immune.
- Example: measles before vaccination
  - $V = 0.95$
  - $\bar{P} = 0.95 \times (2\text{wk}/60\text{yr})$ .

## Homogeneous assumptions





## Homogeneous dynamics

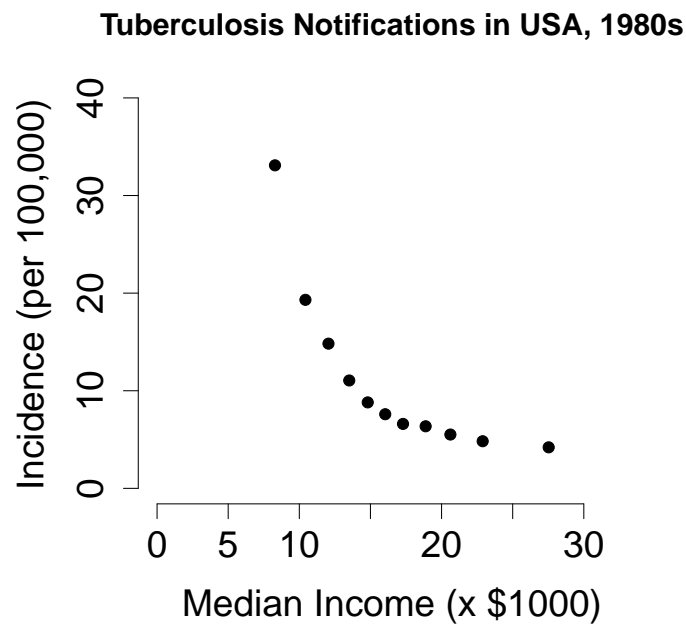
- For many diseases, homogeneous models tend to predict:
  - Too high of an equilibrium, when matching growth rate
  - Too low of a growth rate, when matching equilibrium

## 2 The importance of heterogeneity

### Beyond homogeneity

- Flavors of heterogeneity
  - among hosts
  - spatial
  - demographic (discreteness of individuals)
  - temporal
  - others

### Heterogeneity in TB



- **Progression:** Nutrition, stress
- **Contact:** Overcrowding, poor ventilation
- **Cure:** Access to medical care

### Heterogeneity in other diseases

- **STDs:** Sexual mixing patterns, access to medical care
- **Influenza:** Crowding, nutrition
- **Malaria:** Attractiveness to biting insects, geographical location, immune status
- **Every disease!**

## Large-scale heterogeneity

- For schistosomiasis, the worldwide average  $\mathcal{R}_0 < 1$
- Disease persists because of specific populations with  $\mathcal{R}_0 > 1$ .
- This effect operates at many scales.

## 3 Effects of heterogeneity

### Equilibrium calculations

- Assume  $p = \sigma\tau$  has a susceptibility component and a transmission component:
  - $\mathcal{R}_0 = \sigma\tau cD$
  - $\mathcal{R}_{\text{eff}} = \sigma\tau cDS/N$

### Equilibrium calculations with heterogeneity

- $\tau D$  applies to infectious individuals  $\rightarrow \tau_I D_I$
- $\sigma$  applies to susceptible individuals  $\rightarrow \sigma_S$
- $c$  is complicated  $\rightarrow c_S c_I / \bar{c}$

### Example

- Imagine a disease spread by people who differ only in their effective mixing rates
- If the disease has just started spreading in the population, how do  $c_S$  and  $c_I$  compare to  $\bar{c}$ ?
  - $c_S \approx \bar{c}$ ;  $c_I > \bar{c}$ .
- If the disease is very widespread in the population?
  - $c_S < \bar{c}$ ;  $c_I \rightarrow \bar{c}$ .

### Simpson's paradox

- What happens when a peanut farmer is elected to the US Senate?
- The average IQ goes up in both places!

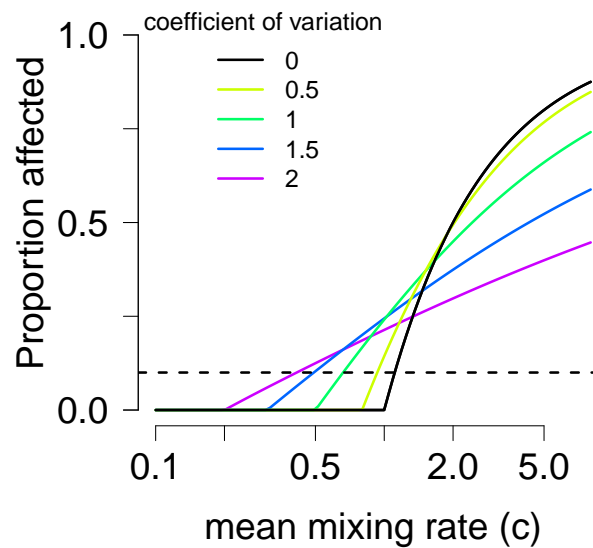
### The basic reproductive number

- When the disease invades:
  - The susceptible population  $\approx$  the general population
  - The infectious population is likely to have higher values of  $c$ ,  $D$  and/or  $\tau$
- $\mathcal{R}_0$  is typically greater than you would expect from a homogeneous model

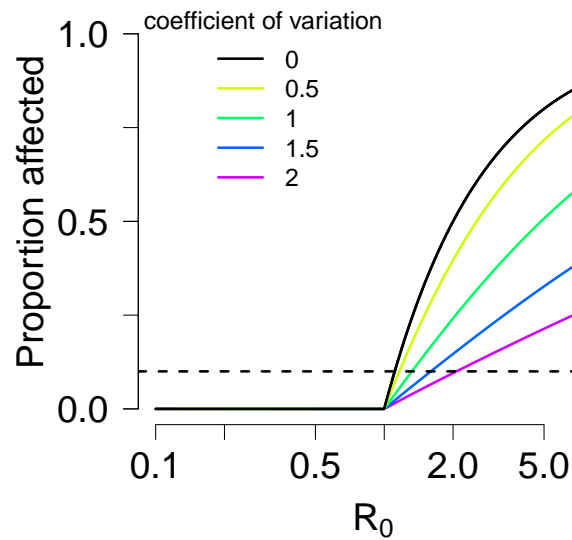
## Equilibrium analysis

- As disease prevalence goes up:
  - Susceptible pool is the most resistant, or least exposed group
  - Infectious pool moves looks more like the general population.
- → lower proportion affected *for a given value of  $\mathcal{R}_0$ .*

### endemic equilibrium



### endemic equilibrium





## Heterogeneity and disease

- Heterogeneity has a double-edged effect
  - Effects of disease are *lower* for a given value of  $\mathcal{R}_0$ .
  - But  $\mathcal{R}_0$  is *higher* for given mean values of factors underlying transmission

## Heterogeneous endemic curves

- Heterogeneity makes the endemic curve flatter
- Disease levels are more resistant to change

## How diseases reach equilibrium

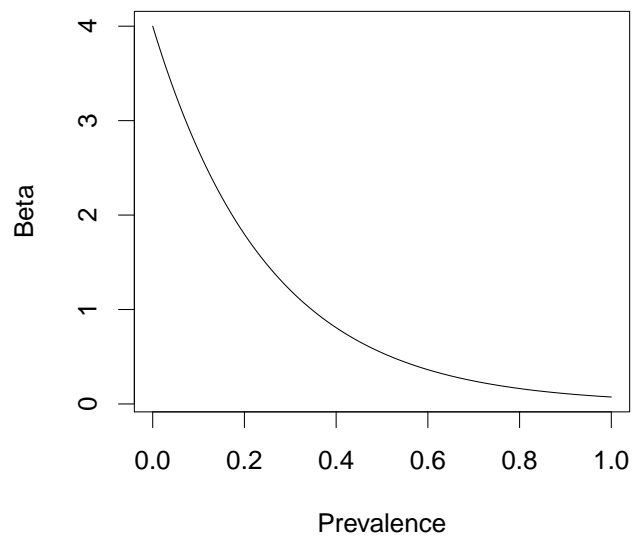
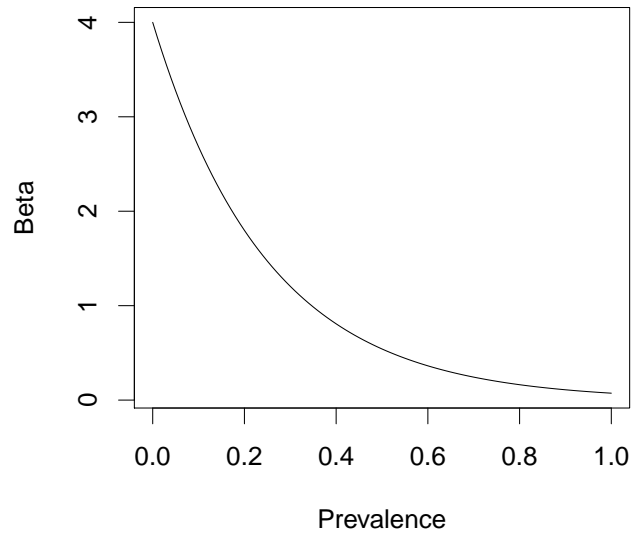
- Diseases that invade have high values of  $\mathcal{R}_0$
- $\mathcal{R}_{\text{eff}}$  must be 1 at equilibrium
  - Potentially infectious contacts are wasted
    - \* Many potential contacts are not susceptible (affected by disease)
    - \* Those not affected less susceptible than average
  - Infectious pool less infectious

## Spatial and network models

- Individual-level, or spatial, heterogeneity also usually increases wasted contacts
- Infectious people meet:
  - people with similar social backgrounds
  - people with similar behaviours
  - people who are nearby geographically or in the contact network
- More wasted contacts further flatten the endemic curve

## 4 Modeling approaches

## Phenomenological

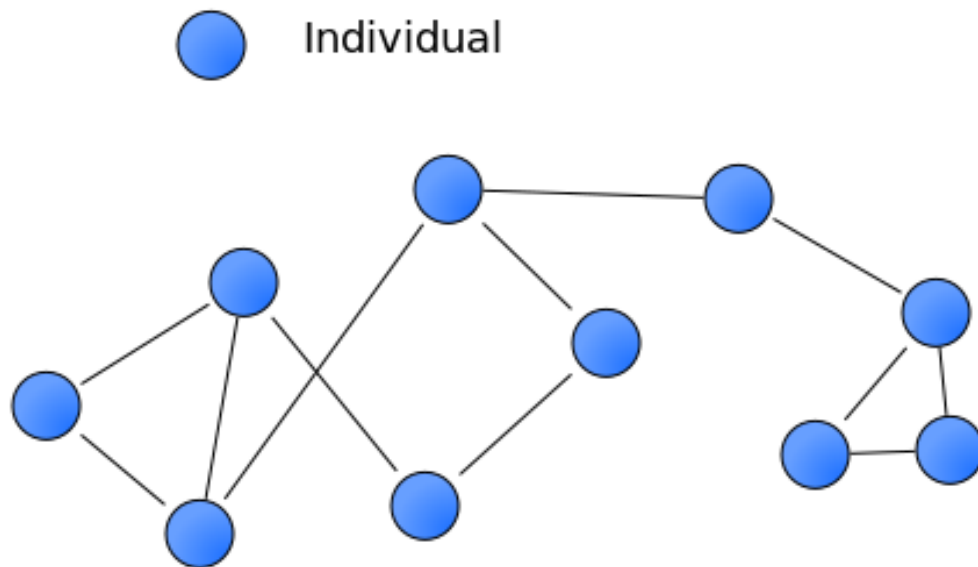


- Simply *make*  $\beta$  go down with prevalence,  $\beta = B \times$ :
  - $e^{-\alpha P}$
  - $(1 - P)^s$
  - $(1 - P/s)^{\alpha s}$

## Multi-group models

- Divide the population into groups.
  - cities and villages
  - rich and poor
  - high and low sexual activity
  - age, gender
  - ...
- Even if details are not correct, heterogeneity will emerge and move model in the right direction

## Individual-based models



- Allow many possibilities:
  - vary individual characteristics
  - add a network of interactions
  - let the network change
- Individual-based approaches require stochastic models

## Summary

- People are heterogeneous in many ways
  - ...and on many scales
- Simple models give us important qualitative insights

- Diseases in heterogeneous populations are likely to be more robust to change than expected from homogeneous models
- More complicated models can help address relevant detail