Heterogeneity, contact patterns and modeling options

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The resilience of infectious disease

1967: It's time to close the book on infectious diseases

Pathogen evolution

Human heterogeneity

# 1 Homogeneous disease models

Expanding our models

- **Homogeneous** models assume everyone has the same:

    - disease characteristics (e.g. susceptibility, tendency to transmit)

    - mixing rate

    - probability of mixing with each person

- **Heterogeneous** models allow people to be different

The basic reproductive number

- $\mathcal{R}_0$ is the number of people who would be infected by an infectious individual *in a fully susceptible population.*

- $\mathcal{R}_0 = \beta/\gamma = \beta D = (cp)D$

    - c: Contact Rate

    - p: Probability of transmission (infectivity)

    - D: Average duration of infection

- A disease can invade a population if and only if $\mathcal{R}_0 > 1$.

Equilibrium

SIR disease, N=100,000
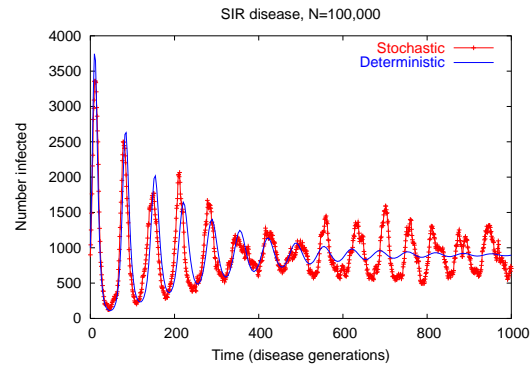
Equilibrium analysis

- $\mathcal{R}_{\text{eff}}$ is the number of people who would be infected by an infectious individual *in a general population.*

- $\mathcal{R}_{\text{eff}} = \mathcal{R}_0 \dfrac{S}{N} = pcD \dfrac{S}{N}$

- At equilibrium: $\mathcal{R}_{\text{eff}} = \mathcal{R}_0 \dfrac{S}{N} = 1$.

- Thus: $\dfrac{S}{N} = 1/R_0$.

- Proportion 'affected' is $V = 1 - S/N = 1 - 1/R_0$.

endemic equilibrium
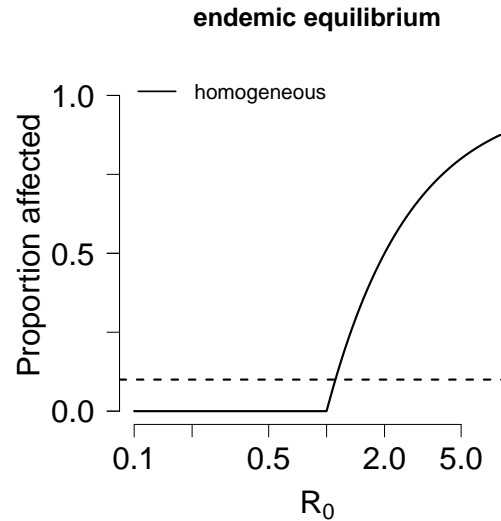
- Threshold value

- Sharp response to changes in factors underlying transmission

- Works – sometimes

Proportion affected and disease prevalence

- For diseases with no recovery, V is the disease prevalence

- For other diseases, the equilibrium value of $P = I/N$ will be equal to V times the ratio of time spent sick to the time spent immune.

- Example: measles before vaccination

  - $V = 0.95$
  - $\bar{P} = 0.95 \times (2\text{wk}/60\text{yr})$.

Homogeneous assumptions
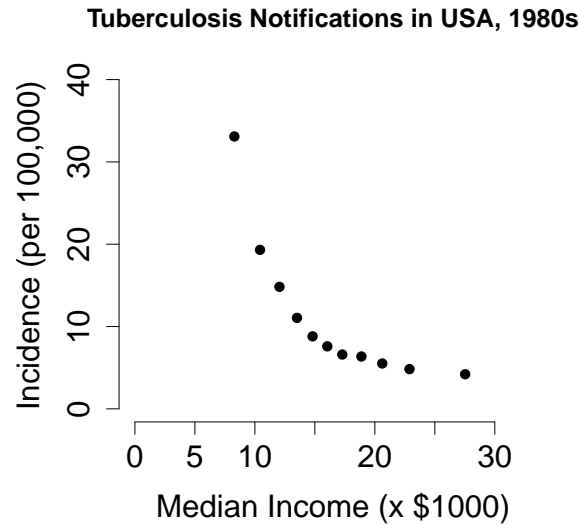
R0 = 2.83

R0 = 4.00

R0 = 5.66

HIV prevalence vs Year plot
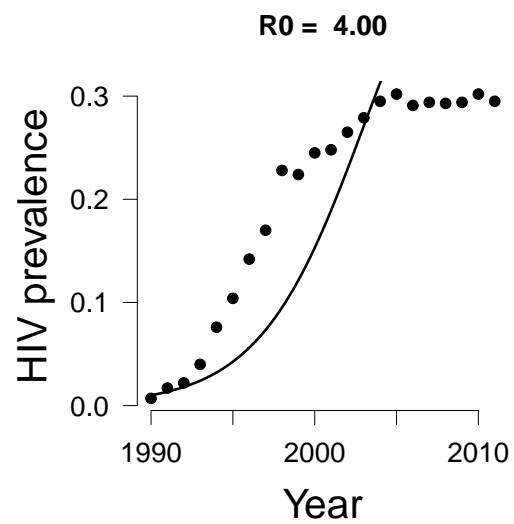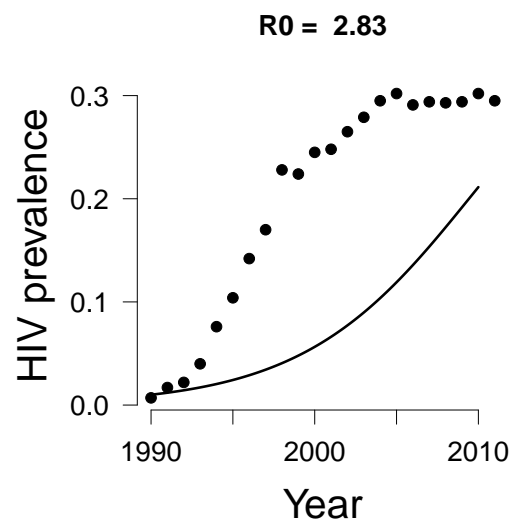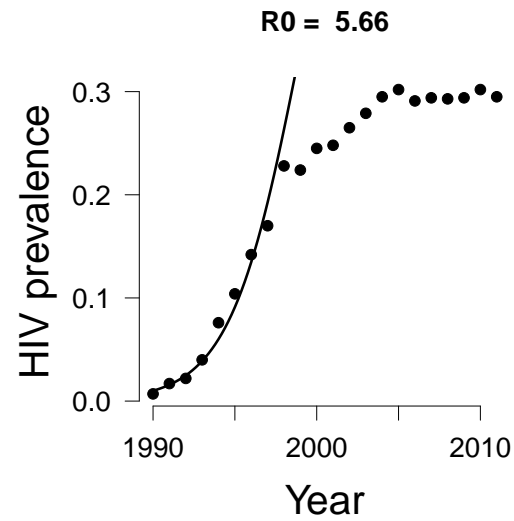
2   The importance of heterogeneity

Beyond homogeneity

- Flavors of heterogeneity

    - among hosts
    - spatial
    - demographic (discreteness of indviduals)
    - temporal
    - others

Heterogeneity in TB

Tuberculosis Notifications in USA, 1980s

Incidence (per 100,000) vs *Median Income (x $1000)*

- **Progression**: Nutrition, stress

- **Contact**: Overcrowding, poor ventilation

- **Cure**: Access to medical care

Heterogeneity in other diseases

- **STDs**: Sexual mixing patterns, access to medical care

- **Influenza**: Crowding, nutrition

- **Malaria**: Attractiveness to biting insects, geographical location, immune status

- **Every disease!**

Large-scale heterogeneity

- For schistosomiasis, the worldwide average $\mathcal{R}_0 < 1$

- Disease persists because of specific populations with $\mathcal{R}_0 > 1$.

- This effect operates at many scales.

3 Effects of heterogeneity

Equilibrium calculations

- Assume $p = \sigma\tau$ has a susceptibility component and a transmission component:

  - $\mathcal{R}_0 = \sigma\tau cD$
  - $\mathcal{R}_{\text{eff}} = \sigma\tau cDS/N$

Equilibrium calculations with heterogeneity

- τD applies to infectious individuals $\rightarrow \tau_I D_I$

- σ applies to susceptible individuals $\rightarrow \sigma_S$

- c is complicated $\rightarrow c_S c_I / \bar{c}$

Example

- Imagine a disease spread by people who differ only in their effective mixing rates

- If the disease has just started spreading in the population, how do c_S and c_I compare to $\bar{c}$?

  - $c_S \approx \bar{c}$; $c_I > \bar{c}$.

- If the disease is very widespread in the population?

 - $c_S < \bar{c}$; $c_I \rightarrow \bar{c}$.

Simpson's paradox

- What happens when a peanut farmer is elected to the US Senate?

- The average IQ goes up in both places!

The basic reproductive number

- When the disease invades:

  - The susceptible population $\approx$ the general population
  - The infectious population is likely to have higher values of c, D and/or τ

- $\mathcal{R}_0$ is typically greater than you would expect from a homogeneous model

Equilibrium analysis

- As disease prevalence goes up:

  - Susceptible pool is the most resistant, or least exposed group
  - Infectious pool moves looks more like the general population.

- $\rightarrow$ lower proportion affected *for a given value of* $\mathcal{R}_0$.

endemic equilibrium

endemic equilibrium
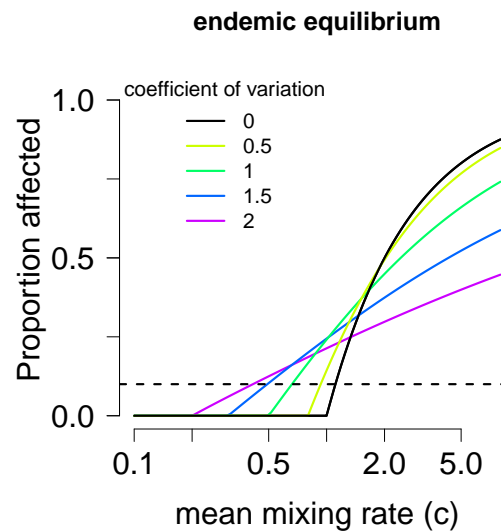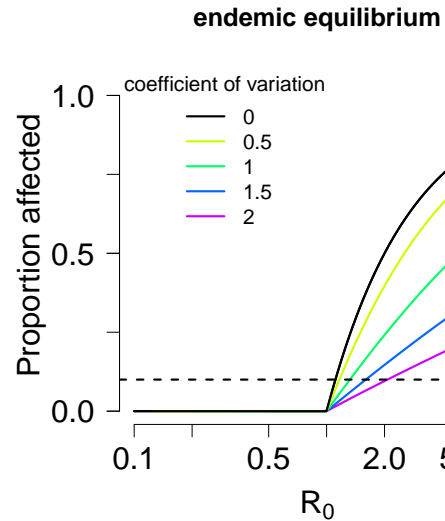
Heterogeneity and disease

- Heterogeneity has a double-edged effect

    - Effects of disease are *lower* for a given value of $\mathcal{R}_0$.
    - But $\mathcal{R}_0$ is *higher* for given mean values of factors underlying transmission

Heterogeneous endemic curves

- Heterogeneity makes the endemic curve flatter

- Disease levels are more resistant to change

How diseases reach equilibrium

- Diseases that invade have high values of $\mathcal{R}_0$

- $\mathcal{R}_{\mathit{eff}}$ must be 1 at equilibrium

    - Potentially infectious contacts are wasted
        * Many potential contacts are not susceptible (affected by disease)
        * Those not affected less susceptible than average
    - Infectious pool less infectious

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Spatial and network models

- Individual-level, or spatial, heterogeneity also usually increases wasted contacts

- Infectious people meet:

  - people with similar social backgrounds
  - people with similar behaviours
  - people who are nearby geographically or in the contact network

- More wasted contacts further flatten the endemic curve

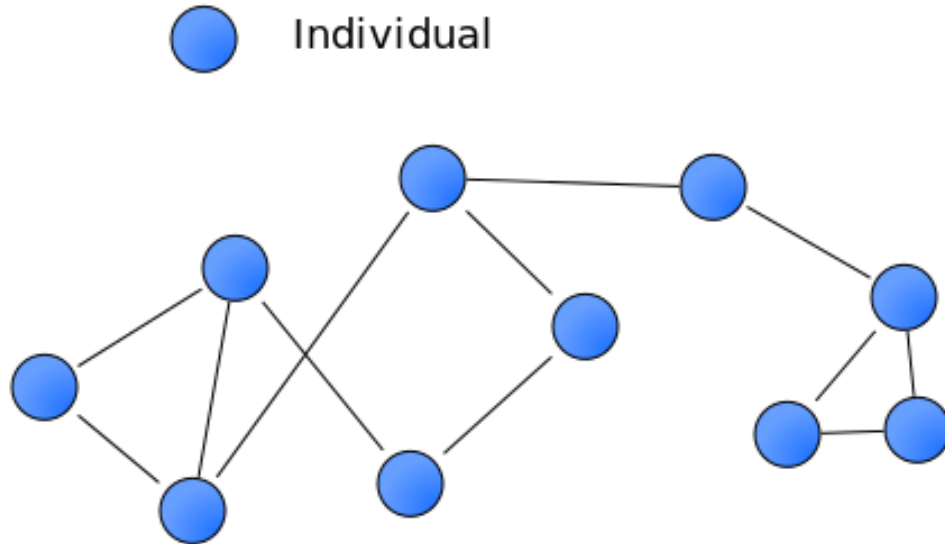# 4 Modeling approaches

Phenomenological
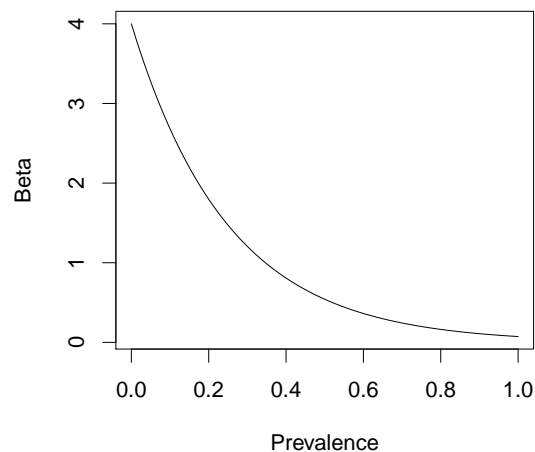
- You can simply *make β* go down as prevalence goes up

  - Need to choose a functional form

Multi-group models

- Divide the population into groups.

    - cities and villages
    - rich and poor
    - high and low sexual activity
    - age, gender
    - ...

Individual-based models

- Allow many possibilities:

    - vary individual characteristics
    - add a network of interactions
    - let the network change

- Individual-based approaches require stochastic models

Summary

People are heterogeneous in many ways

- ...and on many scales

Simple models give us important qualitative insights