## Heterogeneity, contact patterns and modeling options

#### Jonathan Dushoff, McMaster University

http://lalashan.mcmaster.ca/DushoffLab

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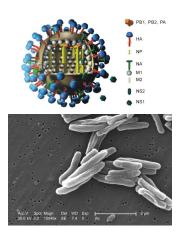
#### The resilience of infectious disease

1967: It's time to close the book on infectious diseases





# Pathogen evolution



# Human heterogeneity



# Human heterogeneity



Human heterogeneity





#### **Outline**

Homogeneous disease models

The importance of heterogeneity

Effects of heterogeneity

Modeling approaches

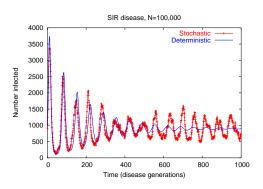
## Expanding our models

- Homogeneous models assume everyone has the same:
  - disease characteristics (e.g. susceptibility, tendency to transmit)
  - mixing rate
  - probability of mixing with each person
- ▶ Heterogeneous models allow people to be different

#### The basic reproductive number

- R<sub>0</sub> is the number of people who would be infected by an infectious individual in a fully susceptible population.
- $R_0 = \beta/\gamma = \beta D = (cp)D$ 
  - c: Contact Rate
  - p: Probability of transmission (infectivity)
  - D: Average duration of infection
- ▶ A disease can invade a population if and only if  $\mathcal{R}_0 > 1$ .

## Equilibrium



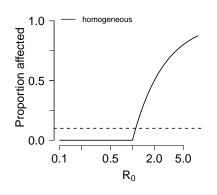
#### Equilibrium analysis

R<sub>eff</sub> is the number of people who would be infected by an infectious individual in a general population.

$$\blacktriangleright \ \mathcal{R}_{\textit{eff}} = \mathcal{R}_0 \frac{\mathcal{S}}{\textit{N}} = \textit{pcD} \frac{\mathcal{S}}{\textit{N}}$$

- ► At equilibrium:  $\mathcal{R}_{eff} = \mathcal{R}_0 \frac{\mathcal{S}}{N} = 1$ .
- ► Thus:  $\frac{S}{N} = 1/R_0$ .
- ▶ Proportion 'affected' is  $V = 1 S/N = 1 1/R_0$ .

## Homogeneous endemic curve

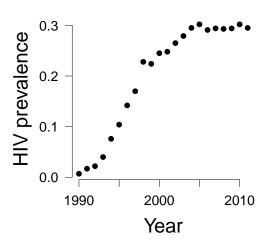


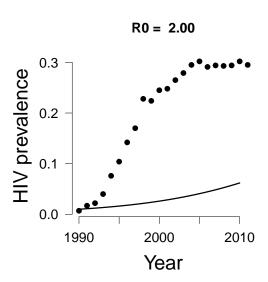
- ► Threshold value
- Sharp response to changes in factors underlying transmission
- Works sometimes

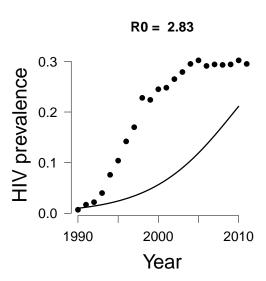
## Proportion affected and disease prevalence

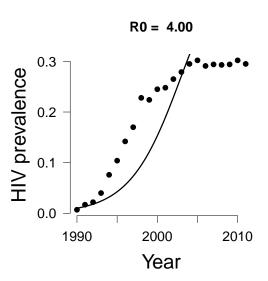
- ► For diseases with no recovery, *V* is the disease prevalence
- For other diseases, the equilibrium value of P = I/N will be equal to V times the ratio of time spent sick to the time spent immune.
- Example: measles before vaccination
  - V = 0.95
  - $\bar{P} = 0.95 \times (2wk/60yr)$ .

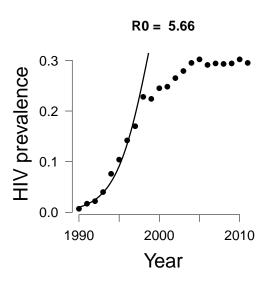
# Disease dynamics











#### **Outline**

Homogeneous disease models

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Effects of heterogeneity

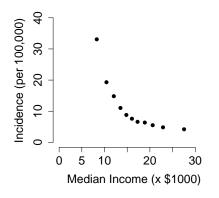
Modeling approaches

# Beyond homogeneity

- Flavors of heterogeneity
  - among hosts
  - spatial
  - demographic (discreteness of indviduals)
  - temporal
  - others

## Heterogeneity in TB

#### Tuberculosis Notifications in USA, 1980s

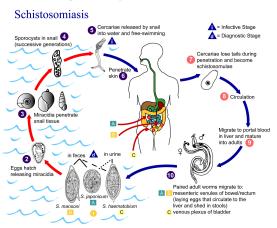


- Progression: Nutrition, stress
- Contact: Overcrowding, poor ventilation
- Cure: Access to medical care

## Heterogeneity in other diseases

- STDs: Sexual mixing patterns, access to medical care
- ▶ Influenza: Crowding, nutrition
- Malaria: Attractiveness to biting insects, geographical location, immune status
- Every disease!

#### Large-scale heterogeneity



- ▶ For schistosomiasis, the worldwide average  $\mathcal{R}_0 < 1$
- ▶ Disease persists because of specific populations with  $\mathcal{R}_0 > 1$ .
- This effect operates at many scales.



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## Equilibrium calculations

- Assume  $p = \sigma \tau$  has a susceptibility component and a transmission component:
  - $ightharpoonup \mathcal{R}_0 = \sigma \tau c D$
  - $\mathcal{R}_{eff} = \sigma \tau c D S / N$

## Equilibrium calculations with heterogeneity

- ▶  $\tau D$  applies to infectious individuals  $\rightarrow \tau_I D_I$
- $\sigma$  applies to susceptible individuals  $o \sigma_{\mathcal{S}}$
- c is complicated  $o c_{\mathcal{S}}c_{\mathcal{I}}/\bar{c}$

#### Example

- Imagine a disease spread by people who differ only in their effective mixing rates
- ▶ If the disease has just started spreading in the population, how do  $c_S$  and  $c_I$  compare to  $\bar{c}$ ?
  - $c_S \approx \bar{c}$ ;  $c_I > \bar{c}$ .
- If the disease is very widespread in the population?
  - $\qquad \qquad \boldsymbol{c_{\mathcal{S}}} < \boldsymbol{\bar{c}}; \, \boldsymbol{c_{\mathcal{I}}} \rightarrow \boldsymbol{\bar{c}}.$

## Simpson's paradox



- What happens when a peanut farmer is elected to the US Senate?
- The average IQ goes up in both places!

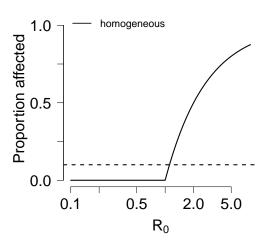
#### The basic reproductive number

- When the disease invades:
  - ▶ The susceptible population  $\approx$  the general population
  - The infectious population is likely to have higher values of c, D and/or τ
- R<sub>0</sub> is typically greater than you would expect from a homogeneous model

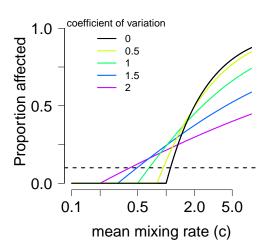
#### Equilibrium analysis

- As disease prevalence goes up:
  - Susceptible pool is the most resistant, or least exposed group
  - Infectious pool moves looks more like the general population.
- ▶  $\rightarrow$  lower proportion affected for a given value of  $\mathcal{R}_0$ .

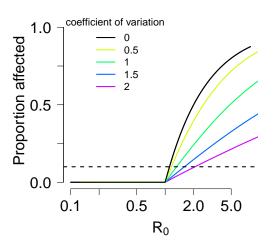
# Homogeneous endemic curve



## Heterogeneous endemic curves

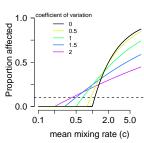


## Heterogeneous endemic curves



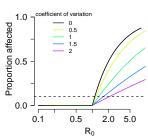
## Heterogeneity and disease

- Heterogeneity has a double-edged effect
  - Effects of disease are lower for a given value of R<sub>0</sub>.
  - But R<sub>0</sub> is higher for given mean values of factors underlying transmission



## Heterogeneous endemic curves

- Heterogeneity makes the endemic curve flatter
- Disease levels are more resistant to change



#### How diseases reach equilibrium

- Diseases that invade have high values of R<sub>0</sub>
- R<sub>eff</sub> must be 1 at equilibrium
  - Potentially infectious contacts are wasted
    - Many potential contacts are not susceptible (affected by disease)
    - Those not affected less susceptible than average
  - Infectious pool less infectious

#### Spatial and network models

- Individual-level, or spatial, heterogeneity also usually increases wasted contacts
- Infectious people meet:
  - people with similar social backgrounds
  - people with similar behaviours
  - people who are nearby geographically or in the contact network
- More wasted contacts further flatten the endemic curve

#### **Outline**

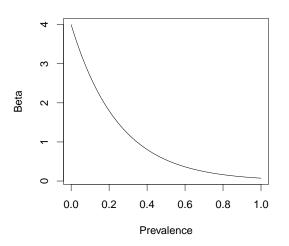
Homogeneous disease models

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# Phenomenological



- ▶ You can simply *make*  $\beta$  go down as prevalence goes up
  - Need to choose a functional form

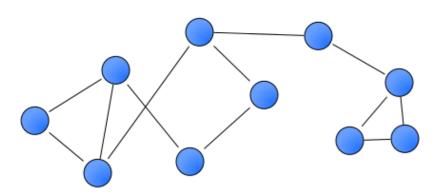


## Multi-group models

- Divide the population into groups.
  - cities and villages
  - rich and poor
  - high and low sexual activity
  - age, gender
  - **...**

#### Individual-based models





- Allow many possibilities:
  - vary individual characteristics
    - add a network of interactions
    - let the network change
- Individual-based approaches require stochastic models

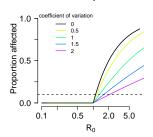


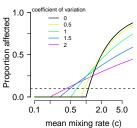
#### Summary



endemic equilibrium

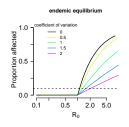
endemic equilibrium

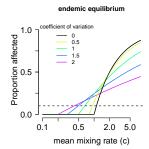




## Summary







People are heterogeneous in many ways

...and on many scales

Simple models give us important qualitative insights

