Little r: An under-rated epidemiological parameter

- Epidemics V
- Clearwater FL, December 2015

- Jonathan Dushoff
- McMaster University

Outline

How do we measure invading epidemics?

- Strength
- Speed
- Danger

How do we assess proposed control measures?

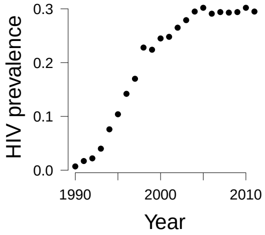
- Strength
- Speed
- Effectiveness

Strength: $\mathcal{R}$ – the reproductive number

- Expected number of new cases per cases
- $\mathcal{R} = \beta DS/N$
  - Disease increases iff $\mathcal{R} > 1$

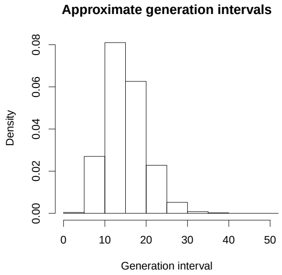
Speed: r – the growth rate

- $i(t) \approx i(0)\exp(rt)$
- $T_c = 1/r$
- $T_2 = \ln(2)/r$

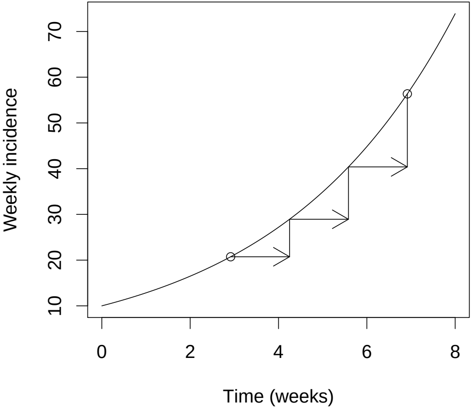


Generation intervals provide the link

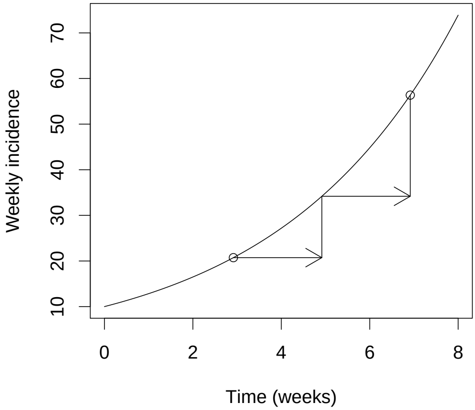
- Estimate the generation interval $G(\tau)$
- Calculate $\mathcal{R} = \exp(r\hat{G})$,
  - $\hat{G}$ is the effective generation interval



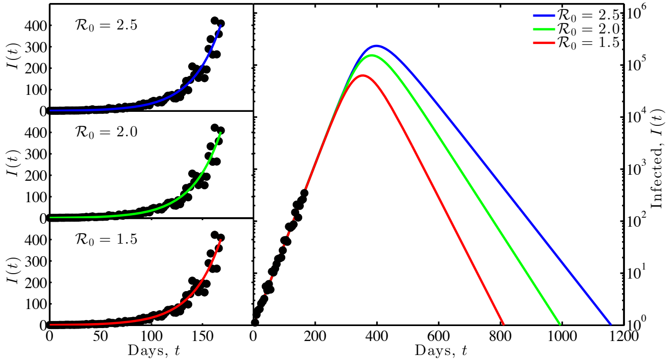
Faster generations mean lower $\mathcal{R}$



Faster generations mean lower $\mathcal{R}$



Ebola example



Weitz and Dushoff, Scientific reports

Standard approach

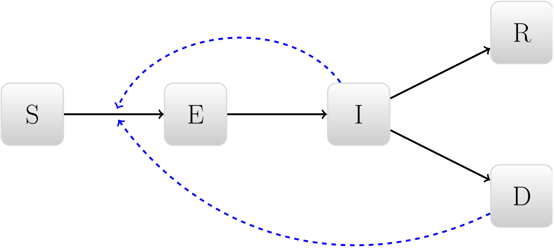
- Estimate $\mathcal{R}$
- Evaluate proposed control measures by comparing their "strength" to $\mathcal{R}$

Alternative approach

- Estimate r
- Evaluate proposed control measures by comparing their "speed" to r

Renewal equation

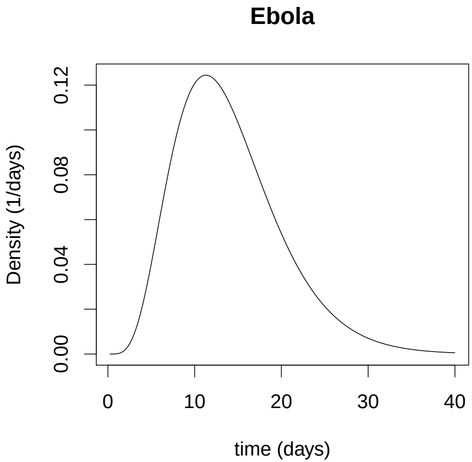
- Many disease models behave on average like:
  - $i(t) = \int k(\tau) i(t - \tau)\, d\tau$
- $i(t)$ is the incidence of infection at time t
- $k(\tau)$ describes the mean infectiousness of a person who has been infected for τ time units



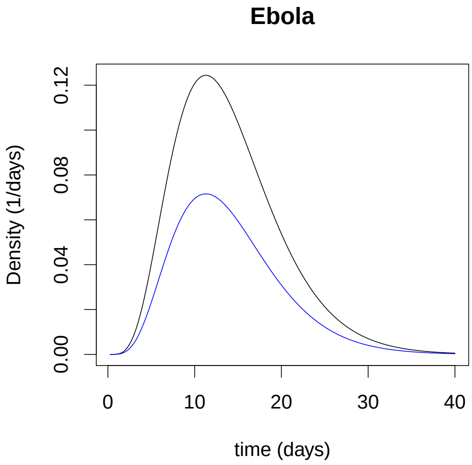
$\mathcal{R}$ and the generation interval

- $i(t) = \int k(\tau) i(t-\tau)\, d\tau$
- $\mathcal{R} = \int k(\tau) d\tau$
- Define the intrinsic generation interval distribution: $k(\tau) = \mathcal{R} g(\tau)$

$\mathcal{R}$ and the generation interval



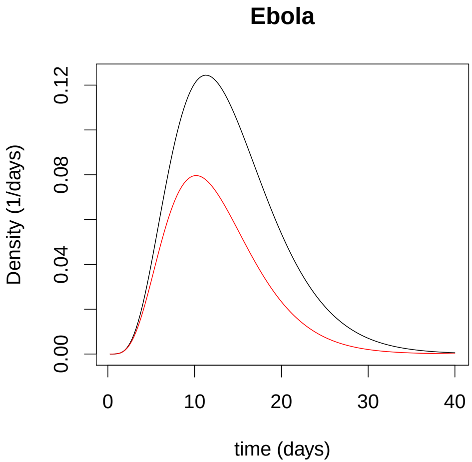
$\mathcal{R}$ and the generation interval

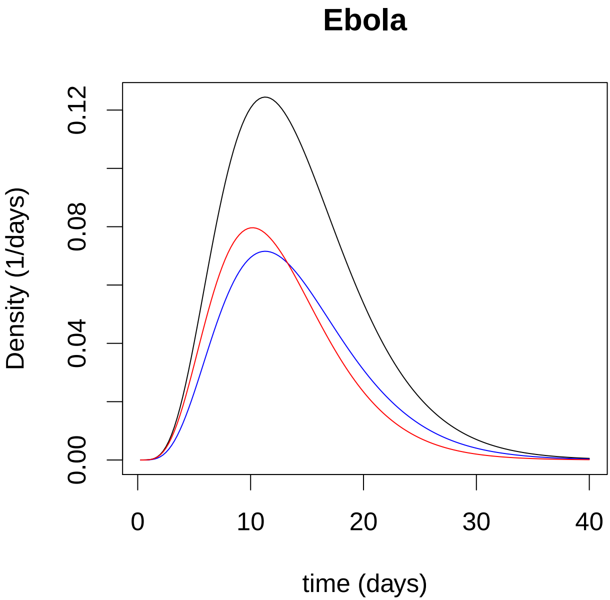


r and the (other) generation interval

- $i(t) = \int k(\tau) i(t-\tau)\, d\tau$
- if $i(t)$ grows like $\exp(rt)$, then
- $1 = \int k(\tau) \exp(-r\tau)\, d\tau$
- $b_0(\tau) = k(\tau)\exp(-r\tau)$ is the initial *backwards* generation interval

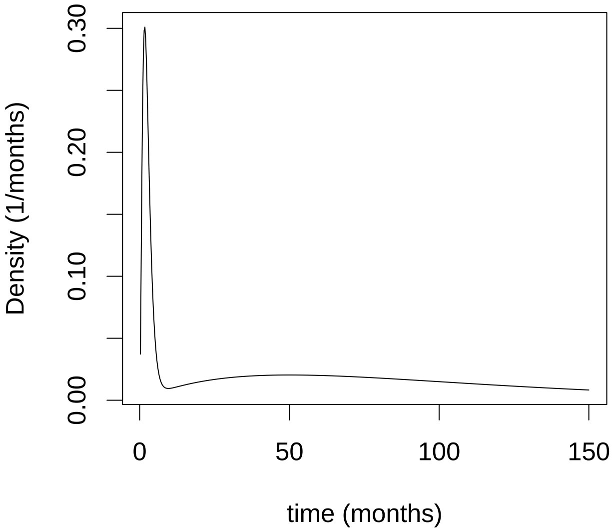
r and the (other) generation interval



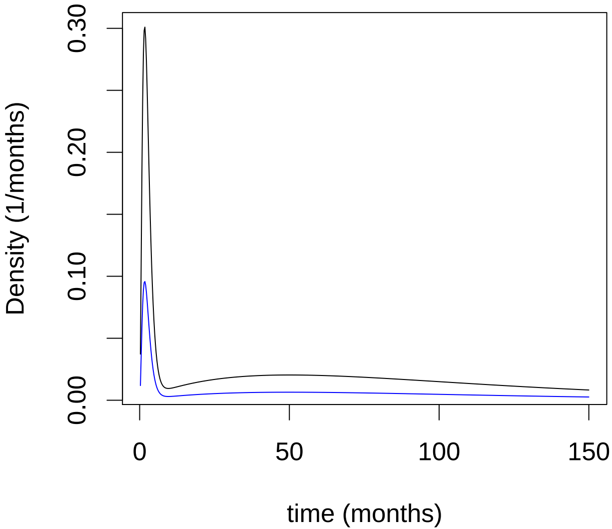


Ebola
Density (1/days)
0.00
0.04
0.08
0.12
0
10
20
30
40
time (days)

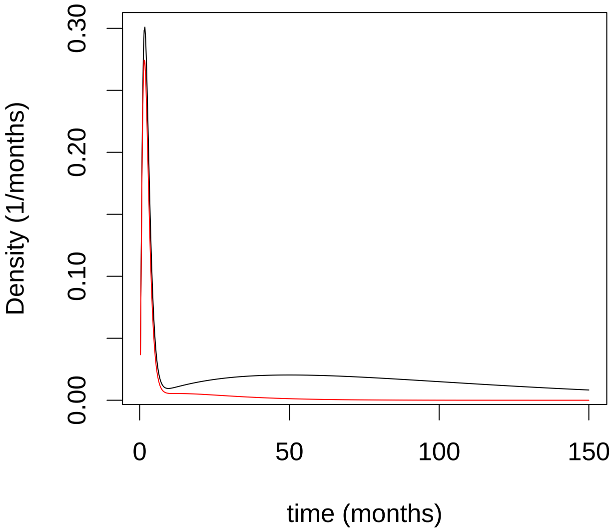
HIV



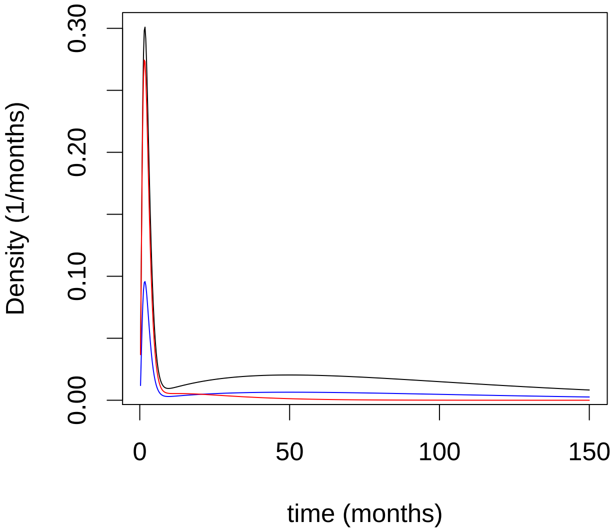
HIV



HIV



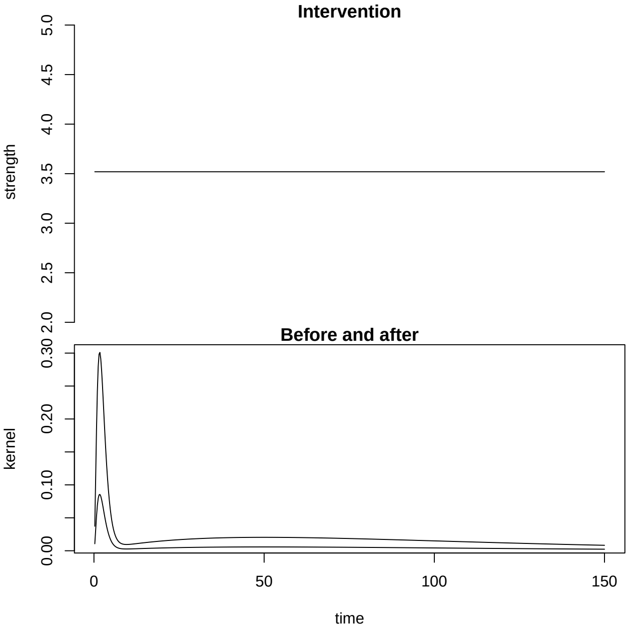
HIV



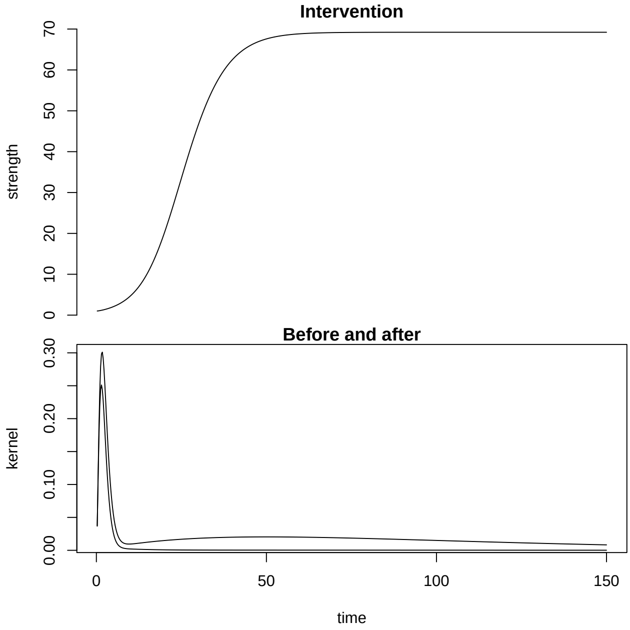
Outline

Strength of intervention

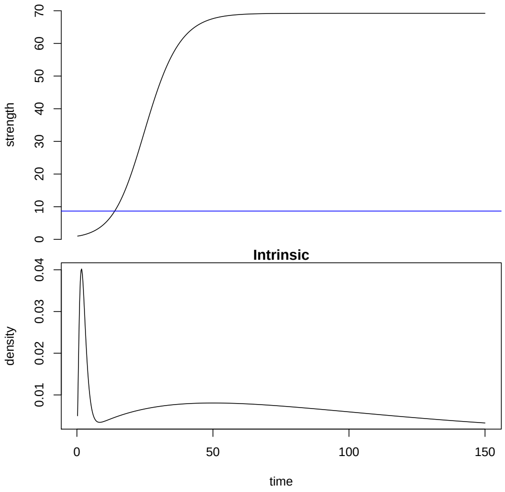
- Imagine we have an intervention that reduces transmission
  - $k(\tau) \to k(\tau)/L(\tau)$
  - Define *strength* $\theta = \mathcal{R}/\hat{\mathcal{R}}$ – the proportional amount by which the intervention reduces transmission.
- We then have:
  - $\theta = 1/\left\langle 1/L(\tau)\right\rangle_{g(\tau)}$
  - θ is the harmonic mean of L, weighted by the generation distribution g.
- Outbreak can be controlled if $\theta > \mathcal{R}$



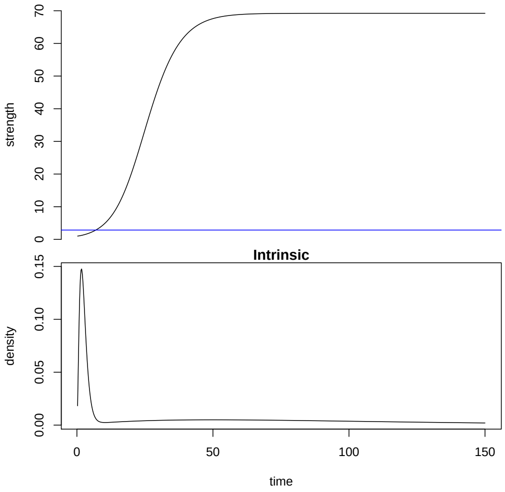
Intervention
strength
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Before and after
kernel
0.30
0.20
0.10
0.00
0
50
100
150
time

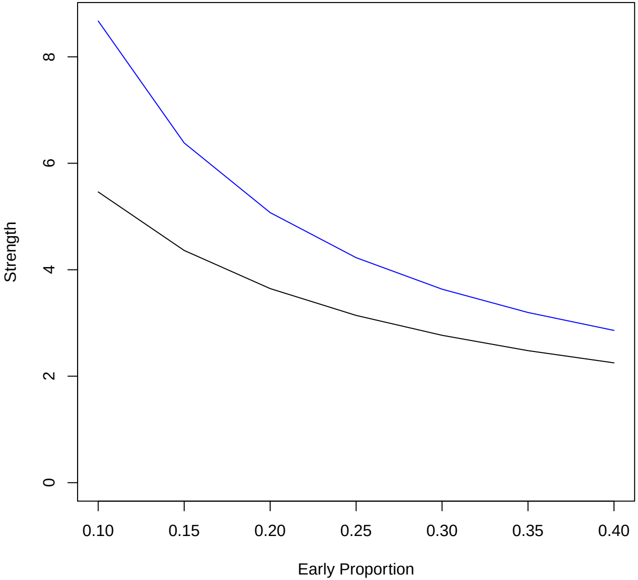

Intervention
strength
0
10
20
30
40
50
60
70
Before and after
kernel
0.00
0.10
0.20
0.30
0
50
100
150
time

Low early transmission



High early transmission





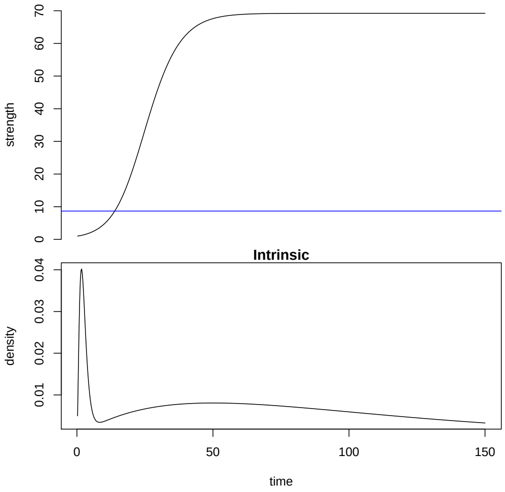
Strength
0
2
4
6
8
0.10
0.15
0.20
0.25
0.30
0.35
0.40
Early Proportion

Outline

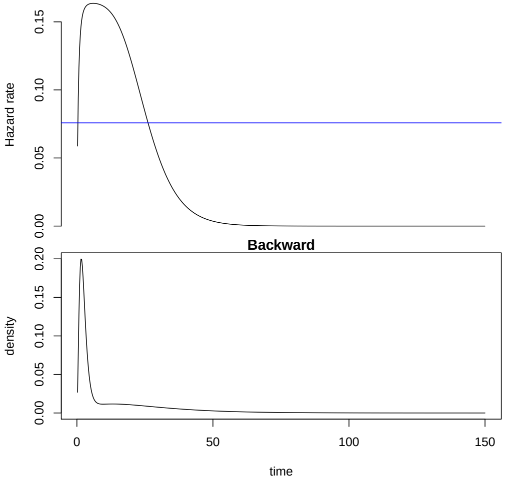
Speed of intervention

- Define the *speed* of an intervention be $\phi = r - \hat{r}$ – the amount by which the intervention slows down spread.
- $1 = \left\langle \frac{\exp(\phi\tau)}{L(\tau)} \right\rangle_{b(\tau)}$
- ϕ is sort of a mean of the *hazard* associated with L
  - Averaged over the initial *backwards* generation interval
- Outbreak can be controlled if $\phi > r$.

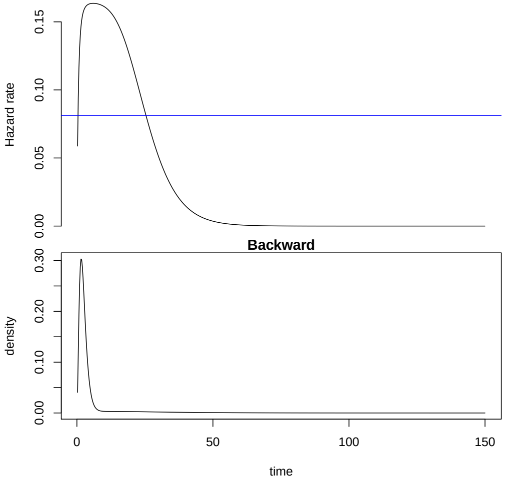
Low early transmission

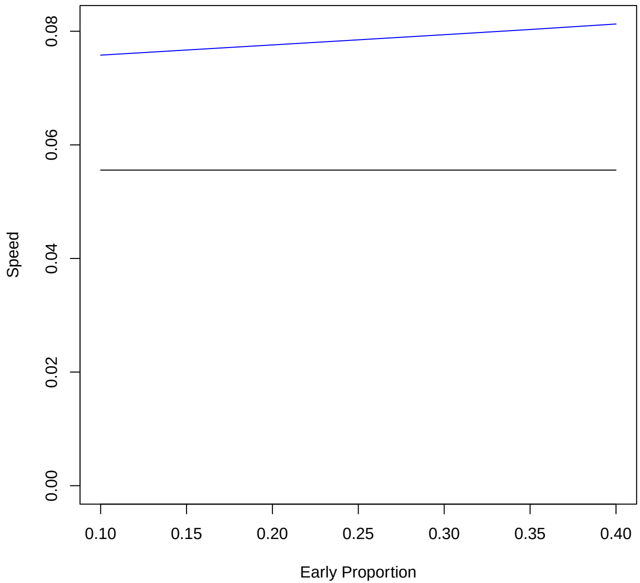


Low early transmission



High early transmission

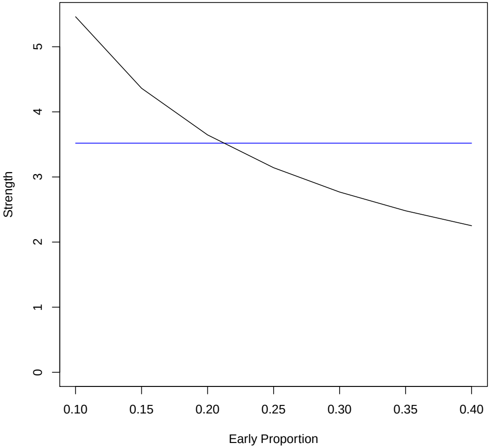




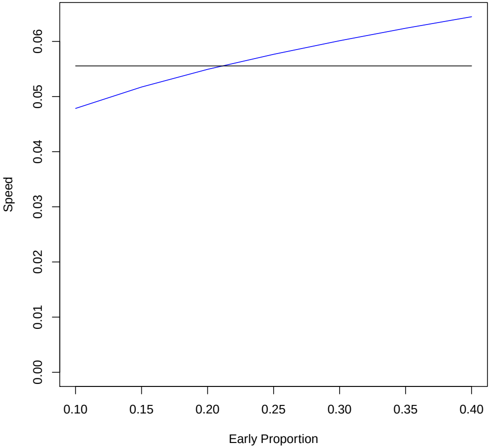
Speed
0.00
0.02
0.04
0.06
0.08
0.10
0.15
0.20
0.25
0.30
0.35
0.40
Early Proportion

Outline

Proportional intervention



Proportional intervention



Similarities

- "Factor" $k(\tau)$ into a risk and a generation interval
- Measure the effect of an intervention
- Compare effect to a threshold
- Two ways of looking at the same picture
  - Why does it matter?

Differences

- r vs. $\mathcal{R}$
- hazard-like vs. proportional-like interventions
- intrinsic vs. initial backwards generation intervals
- Long-term implications

Conclusion

Thanks

- Organizers
- Audience
- Collaborators
- Funders: NSERC, CIHR

Filtered mean slides