THEMETITLE

PLAIN Statistical philosophy

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GOALS

- Discuss what statistics are used for, and why they are needed
- Explain what P values mean, and what they don't
  - Effect sizes and confidence intervals are usually better
- Explain the fundamentals of the two basic paradigms of statistical philosophy
- Discuss the role of statistics in science

OTHER NSLIDE Statistical philosophy

- Jonathan Dushoff, McMaster University
- Download pdf notes (should be this version): http://dushoff.github.io/notebook/materials/philosophy
- Download pdf slides (from some recent version): http://dushoff.github.io/notebook/materials/philoso

PIPE Long pipe

- Any piece of pipe longer than 30 feet shall be clearly labelled "long pipe" on each end
- Any piece of pipe longer than 100 feet shall also be labelled "long pipe" in the middle, so the plumber doesn't have to walk all the way to the end to find out whether it is long pipe or not.
- **WARNING: Long Lecture**

1 Statistical inference

- We use statistics to confirm effects, estimate parameters, and predict outcomes
- It usually rains when I'm in Cape Town, but mostly on Sunday
  - *Confirmation:* In Cape Town, it rains more on Sundays than other days
  - *Estimation:* In Cape Town, the *odds* of rain on Sunday are 1.6–2.2 times higher than on other days
  - *Prediction:* I am confident that it will rain at least one Sunday the next time I go

Raining in Cape Town

- How we interpret data like this necessarily depends on assumptions:
  - Is it likely our observations occured by chance?
  - Is it likely they *didn't*?

Vitamin A

- We compare health indicators of children treated or not treated with vitamin A supplements
  - *Estimate:* how much taller (or shorter) are the treated children on average?
  - *Confirmation:* are we sure that the supplements are helping (or hurting)?
  - *Range of estimates:* how much do we think the supplement is helping?

1.1 P values and confidence intervals

- We use *P values* to say how sure we are that we have seen a positive effect
- We use *confidence intervals* to say what we think is going on (with a certain level of confidence)
- P values are *over-rated*
- *Never* use a high P value as evidence for anything, e.g.:
  - that an effect is small
  - that two quantities are similar

Vitamin A example

- We want to know if vitamin A supplements improve the health of village children
  - Is height is a good measure of general health?
  - How will we know height differences are due to our treatment?
    * We want the two groups to start from the same point – independent randomization of each individual
    * We may measure *changes* in height
    * Or *control for* other factors

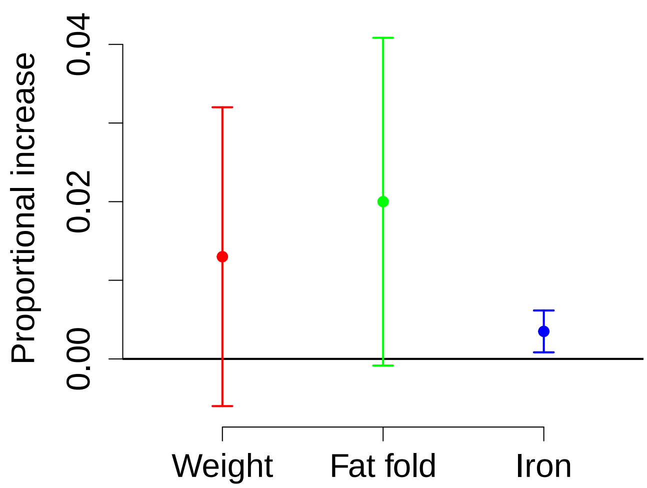
What do we hope to learn?

- Is vitamin A good for these children?
- How sure are we?
- How good do we think it is?
- How sure are we about that?

P values

- What does it mean if I find a ”significant P value” for some effect in this experiment?
- The difference is unlikely to be due to chance
  - So what! I already know vitamin A has strong effects on metabolism
- If I’m certain that the true answer isn’t exactly zero, why do I want the P value anyway?

Confidence intervals



- What do these results mean?
- Which are significant?

Confidence intervals and P values

- A high P value means we can’t see the sign of the effect clearly
- A low P value means we can

What do P values measure?

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-

Types of Error

- Type I (*False positive:*) concluding there is an effect when there isn't one
  - This doesn't happen in biology. There is always an effect.
- Type II (*False negative:*) concluding there is no effect when there really is
  - This *should* never happen, because we should never conclude there is no effect
- Type III Error is the error of using numerical codes for things that have perfectly good simple names
- Just say "false positive" or "false negative" when possible

Experimental design

- *False positive:* in the hypothetical case that the effect is exactly zero, what is the probability of falsely finding an effect
  - Should be less than or equal to my significance value
- *False negative:* what is the probability of failing to find an effect that is there?
  - Requires you specify a hypothetical effect *size*
  - This is a scientific judgment
- These are useful to analyze **power** and **validity** of a statistical design
  - You should do these analyses *before* you collect data

A new view of error

- *Sign error:* if I think an effect is positive, when it's really negative (or vice versa)
- *Magnitude error:* if I think an effect is small, when it's really large (or vice versa)
- Confidence intervals clarify all of this

Low P values

- If I have a low P value I can see something clearly
- But it's usually better to focus on what I see than the P value

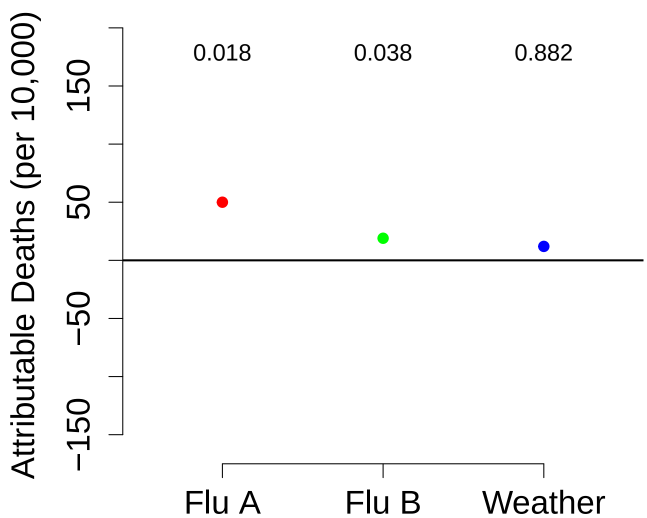
High P values

- If I have a high P value, there is something I *don't* see clearly
- It *may be* because this effect is small
- High P values should *not* be used to advance *any* conclusion

What causes high P values?

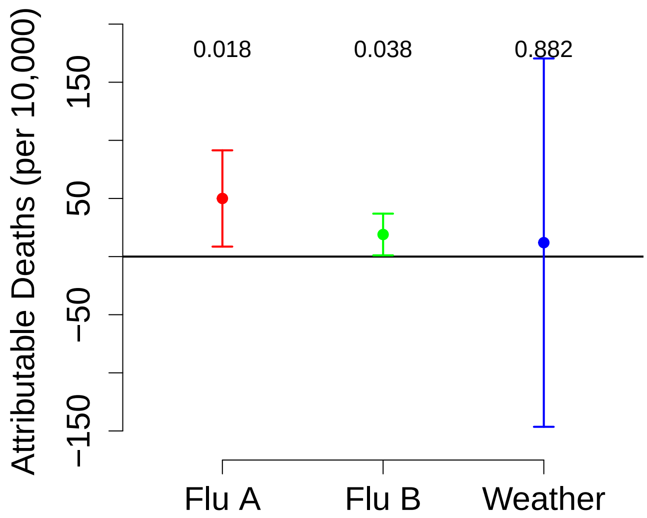
- Small differences
- Less data
- More noise
- An inappropriate model
- Less model resolution
- A lower P value means that your evidence for difference is better
- A higher P value means that your evidence for similarity is better – or worse!

Annualized flu deaths



- Why is weather not causing deaths at this time scale?

... with confidence intervals



- **Never** say: A is significant and B isn't, so $A > B$
- **Instead:** Construct a statistic for the hypothesis $A > B$
  - May be difficult

Bad language

- Null effects of boot camps
  -
- Fat storage in vole populations is not correlated with elevation
  -
- As expected, the placebo group did not differ significantly from the control group
  -
- B and B showed that there is no statistically significant difference in sexual risk behaviour between men with and without clinic access in Zambia
  -
  -

Another way to talk

- Unclear effects of boot camps
- As expected, the sign of the difference between the placebo group and controls was unclear
- The direction of correlation between fat storage and elevation in vole populations is unclear in this study
- B and B showed that there is an unclear difference in sexual risk behaviour between men with and without clinic access in Zambia
  -

1.2 Statistics and science

Syllogisms

- All men are mortal
- Mohamed Salah is mortal
- Therefore, Mohamed Salah is a man

Syllogisms

- All men are mortal
- Fanny the elephant is mortal
- Therefore, Fanny the elephant is a man

Bad logic

- A lot of statistical practice works this way:
  - bad logic in service of conclusions that are (usually) correct
- This sort of statistical practice leads in the aggregate to bad science
- The logic can be fixed:
  - Estimate a difference, or an interaction

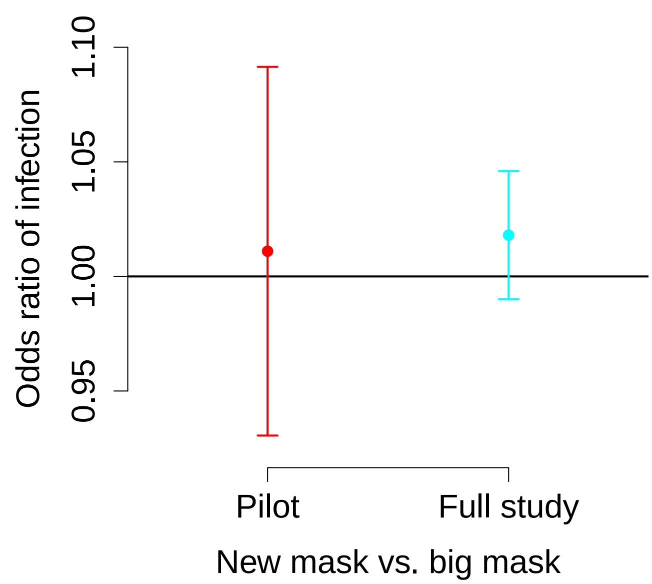
Small effects

- We can't build statistical confidence that something is small by failing to see it clearly
- We must instead see clearly that it is small
- This means we need a standard for what we mean by small

Flu mask example

- People who work in respiratory clinics sometimes have to wear bulky, uncomfortable, expensive masks
- They would like to switch to simpler masks, if those will do the job
- How can this be tested statistically? We don't want the masks to be "different".
  - We need to decide what we mean by different in this case!
  - They're not the same, so how close is close enough?

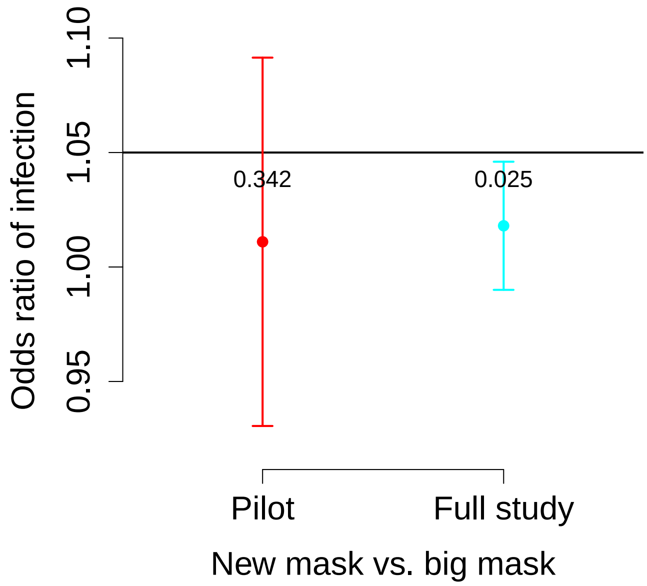
Tradiitonal approach



Non-inferiority approach

- Are we confident the new mask is "good enough"?
- There is no substitute for picking a standard

Non-inferiority approach



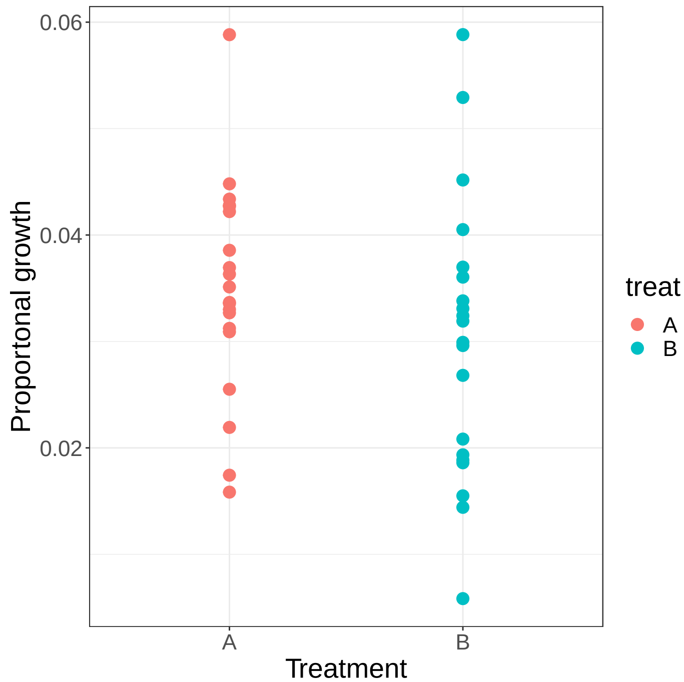
- We can even attach a P value by basing it on the "right" statistic.
- The right statistic is the thing whose sign we want to know:
  - The difference between the observed effect and the standard we chose

2 Paradigms for inference

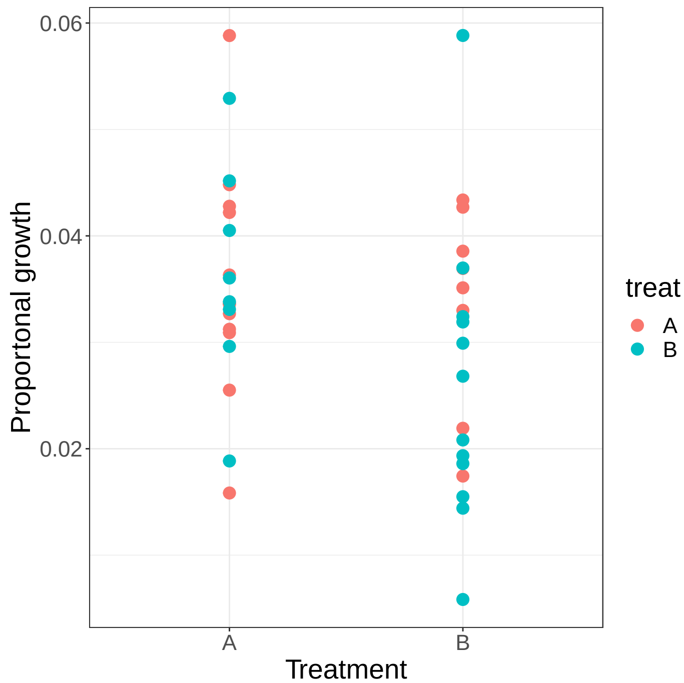
2.1 Frequentist paradigm

- Make a null model
- Test whether the effect you see could be due to chance
  - What is the probability of seeing a difference of exactly a 0.0048 in proportional growth?
- Test whether the effect you see *or a larger effect* could be due to chance
  - This probability is the P value

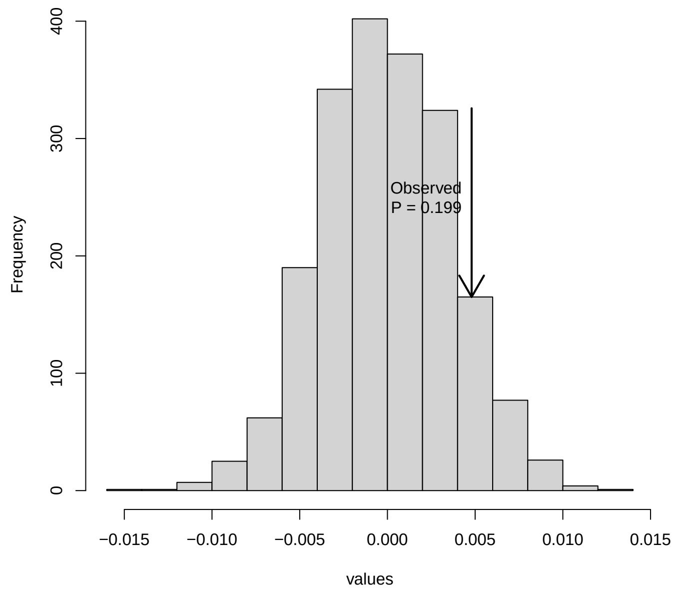
Height measurements



Scrambled measurements



The null distribution



2.2 Bayesian paradigm

- Make a complete model world
- Use conditional probability to calculate the probability you want

A powerful framework

- More assumptions $\implies$ more power
- With great power comes great responsibility

Bayesian inference

- We want to go from a *statistical model* of how our data are generated, to a probability model of parameter values
  - Requires *prior* distributions:
    * the assumed likelihood of parameters before these observations are made
  - Use Bayes theorem to calculate *posterior* distributions:
    * the inferred likelihood of parameters after taking the data into account
- Provides a strong framework for combining information from different sources and for propagating uncertainty

Vitamin A study

- A frequentist can do a clear analysis right away
- A Bayesian needs a ton of assumptions – will often try to make "uninformative" assumptions

Cape Town weather

- Frequentist: how unlikely is the observation, from a random perspective?
- Bayesian: what's my model world? What is my prior belief about weather-weekday interactions?

BAYESCALC Example: MMEV

- MMEV is a viral infection that can cause a serious disease (called MMED)
- LIT MMED patients are unable to control their urge to fit models to data
- The rapid MMEV test gives a positive result:
  - 100% of the time for people with the virus
  - 5% of the time for people without the virus

MMED MMEV questions

- The rapid MMEV test gives a positive result:
  - 100% of the time for people with the virus
  - 5% of the time for people without the virus
- The population prevalence of MMEV is 1%
- You pick a person from this population at random, and test them, and the test is positive.
  - What is the probability that they have MMEV?
- This calculation is the core of Bayes theorem

MMED MMEV questions

- You learn that your friend has had a positive rapid test for MMEV
  - What do you tell them?
- This is what Bayesian philosophy is about: combining information from different sources

3 Conclusion

Your philosophy

- Statistics are not a magic machine that gives you the right answer
- If you are to be a serious scientist in a noisy world, you should have your own philosophy of statistics
  - Be pragmatic: your goal is to do science, not get caught by theoretical considerations
  - Be honest: it's harder than it sounds.

Honesty

- You can always keep analyzing until you find a "significant" result
  - If you do this you will make a lot of mistakes
- You may also keep analyzing until you find a result that you already "know" is true.
  - This is confirmation bias; you're probably right, but your project is not advancing science
- Good practice
  - Keep a data-analysis journal
  - Start *before* you look at the data

PIPE PSLIDE Summary

Summary

- P values are over-rated
- High P values should not be used as evidence for anything ever.
  - They can provide indirect evidence. Wonderful. Find the direct evidence and use that instead.
- Use effect sizes and confidence intervals when you can
- Otherwise, find ways to make significant P values do the work
  - Non-inferiority tests, interactions

- Frequentist statistics makes weak assumptions, and finds logically weak formal conclusions:
  - These parameters are unlikely to produce a statistic this extreme by chance
- Bayesian statistics makes strong assumptions:
  - prior distributions must be fully specified
- ... and finds logically strong formal conclusions:
  - The probability that the effect value is in this range is X
  - These strong conclusions can be used directly for prediction with uncertainty

- Statistics are a key component of data-based science
  - You should think about statistical analysis from the beginning of your project
- You need a basic understanding of statistical principles
- You need your own statistical philosophy
  - If you're a theoretician, it should be ideological and honest
  - If you're a scientist, it should be pragmatic and honest