Statistical philosophy

Jonathan Dushoff, McMaster University

GOALS

- Discuss what statistics are used for, and why they are needed
- Explain what P values mean, and what they don't
  - Effect sizes and confidence intervals are usually better
- Explain the fundamentals of the two basic paradigms of statistical philosophy
- Discuss the role of statistics in science

1 Statistical inference

- We use statistics to confirm effects, estimate parameters, and predict outcomes
- It usually rains when I'm in Cape Town, but mostly on Sunday
  - *Confirmation:* In Cape Town, it rains more on Sundays than other days
  - *Estimation:* In Cape Town, the *odds* of rain on Sunday are 1.6–2.2 times higher than on other days
  - *Prediction:* I am confident that it will rain at least one Sunday the next time I go

Raining in Cape Town

- How we interpret data like this necessarily depends on assumptions:
  - Is it likely our observations occured by chance?
  - Is it likely they *didn't*?

Vitamin A

- We compare health indicators of children treated or not treated with vitamin A supplements
  - *Estimate:* how much taller (or shorter) are the treated children on average?
  - *Confirmation:* are we sure that the supplements are helping (or hurting)?
  - *Range of estimates:* how much do we think the supplement is helping?

1.1 P values and confidence intervals

- We use *P values* to say how sure we are that we have seen a positive effect
- We use *confidence intervals* to say what we think is going on (with a certain level of confidence)
- P values are *over-rated*
- *Never* use a high P value as evidence for anything, e.g.:
  - that an effect is small
  - that two quantities are similar

Vitamin A example

- We want to know if vitamin A supplements improve the health of village children
  - Is height is a good measure of general health?
  - How will we know height differences are due to our treatment?
    * We want the two groups to start from the same point – independent randomization of each individual
    * We may measure *changes* in height
    * Or *control for* other factors

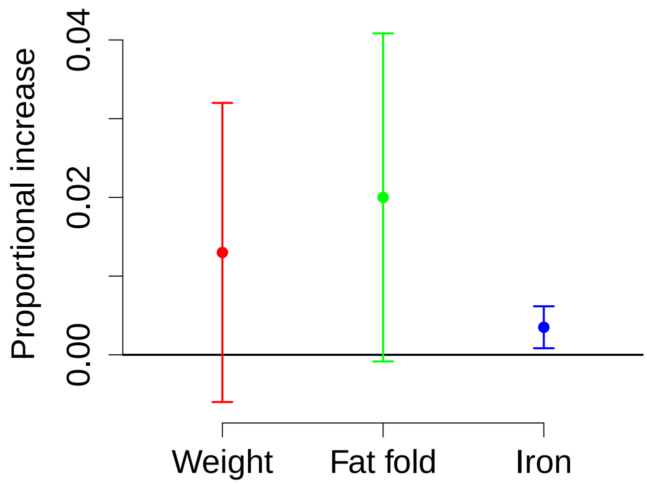
What do we hope to learn?

- Is vitamin A good for these children?
- How sure are we?
- How good do we think it is?
- How sure are we about that?

P values

- What does it mean if I find a "significant P value" for some effect in this experiment?
- The difference is unlikely to be due to chance
  - So what! I already know vitamin A has strong effects on metabolism
- If I'm certain that the true answer isn't exactly zero, why do I want the P value anyway?

Confidence intervals



- What do these results mean?
- Which are significant?

Confidence intervals and P values

- A high P value means we can't see the sign of the effect clearly
- A low P value means we can

The meaning of P values

- More broadly, a P value measures whether we are seeing *something* clearly
  - It's usually the sign ($\pm$) of some quantity, but doesn't need to be

Types of Error

- Type I (*False positive:*) concluding there is an effect when there isn't one
  - This doesn't happen in biology. There is always an effect.
- Type II (*False negative:*) concluding there is no effect when there really is
  - This *should* never happen, because we should never conclude there is no effect
- Type III Error is the error of using numerical codes for things that have perfectly good simple names
- Just say "false positive" or "false negative" when possible

Experimental design

- *False positive:* in the hypothetical case that the effect is exactly zero, what is the probability of falsely finding an effect
  - Should be less than or equal to my significance value
- *False negative:* what is the probability of failing to find an effect that is there?
  - Requires you specify a hypothetical effect *size*
  - This is a scientific judgment
- These are useful to analyze **power** and **validity** of a statistical design
  - You should do these analyses *before* you collect data

A new view of error

- *Sign error:* if I think an effect is positive, when it's really negative (or vice versa)
- *Magnitude error:* if I think an effect is small, when it's really large (or vice versa)
- Confidence intervals clarify all of this

Low P values

- If I have a low P value I can see something clearly
- But it's usually better to focus on what I see than the P value

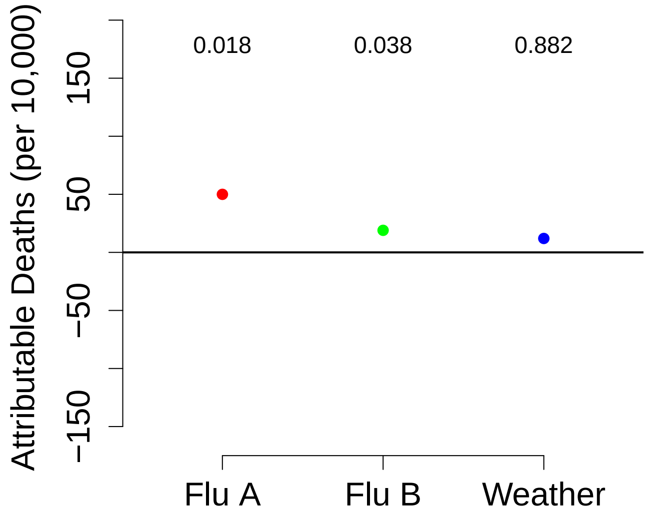
High P values

- If I have a high P value, there is something I *don't* see clearly
- It *may be* because this effect is small
- High P values should *not* be used to advance your conclusion

What causes high P values?

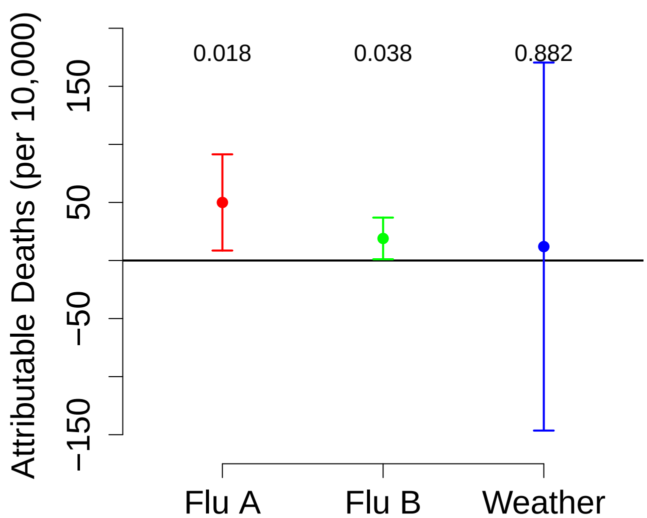
- Small differences
- Less data
- More noise
- An inappropriate model
- Less model resolution
- A lower P value means that your evidence for difference is better
- A higher P value means that your evidence for similarity is better – or worse!

Annualized flu deaths



- Why is weather not causing deaths at this time scale?

… with confidence intervals



- **Never** say: A is significant and B isn't, so $A > B$
- **Instead:** Construct a statistic for the hypothesis $A > B$
  - May be difficult

1.2 Statistics and science

Syllogisms

- All men are mortal
- Mohamed Salah is mortal
- Therefore, Mohamed Salah is a man

Syllogisms

- All men are mortal
- Fanny the elephant is mortal
- Therefore, Fanny is a man

Bad logic

- A lot of statistical practice works this way:
  - bad logic in service of conclusions that are (usually) correct
- This sort of statistical practice leads in the aggregate to bad science
- The logic can be fixed:
  - Estimate a difference, or an interaction

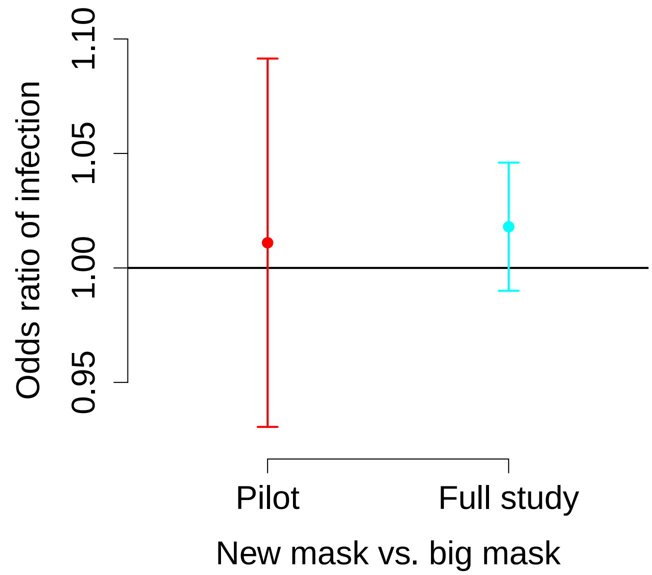
Small effects

- We can't build statistical confidence that something is small by failing to see it clearly
- We must instead see clearly that it is small
- This means we need a standard for what we mean by small

Flu mask example

- People who work in respiratory clinics sometimes have to wear bulky, uncomfortable, expensive masks
- They would like to switch to simpler masks, if those will do the job
- How can this be tested statistically? We don't want the masks to be "different".
  - We need to decide what we mean by different in this case!
  - They're not the same, so how close is close enough?

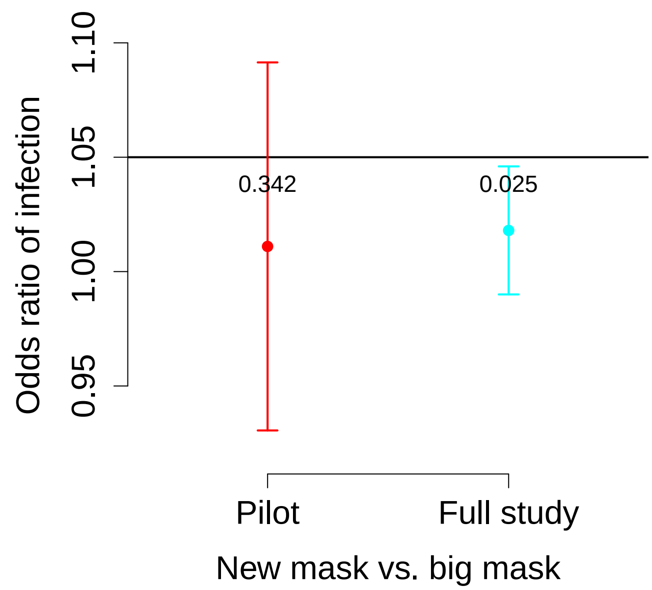
Tradiitonal approach



Non-inferiority approach

- Are we confident the new mask is "good enough"?
- There is no substitute for picking a standard

Non-inferiority approach



- We can even attach a P value by basing it on the "right" statistic.

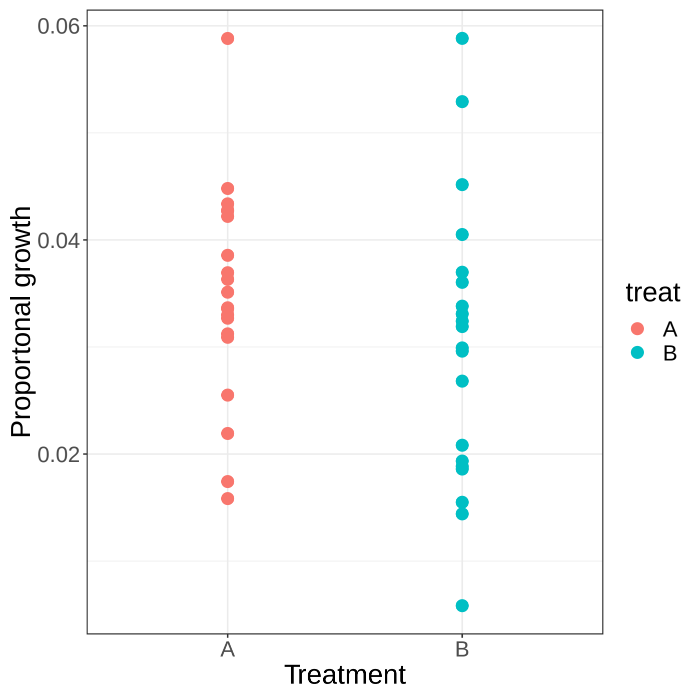
- The right statistic is the thing whose sign we want to know:
  - The difference between the observed effect and the standard we chose

2 Paradigms for inference

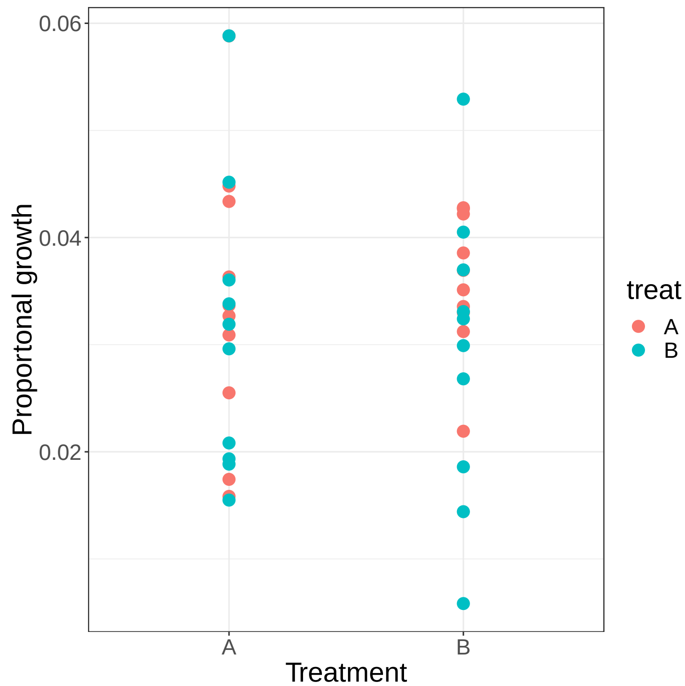
2.1 Frequentist paradigm

- Make a null model
- Test whether the effect you see could be due to chance
  - What is the probability of seeing a difference of exactly a 0.0048 in proportional growth?
- Test whether the effect you see *or a larger effect* could be due to chance
  - This probability is the P value

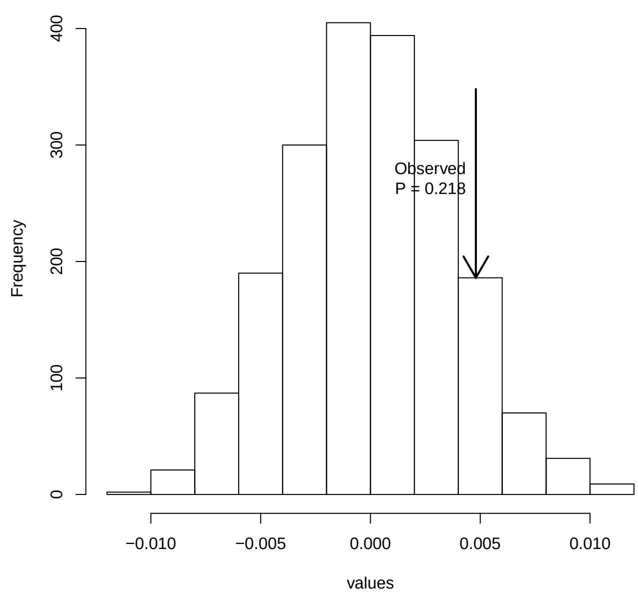
Height measurements



Scrambled measurements



The null distribution



2.2 Bayesian paradigm

- Make a complete model world
- Use conditional probability to calculate the probability you want

A powerful framework

- More assumptions $\implies$ more power
- With great power comes great responsibility

Bayesian inference

- We want to go from a *statistical model* of how our data are generated, to a probability model of parameter values
  - Requires *prior* distributions:
    * the assumed likelihood of parameters before these observations are made
  - Use Bayes theorem to calculate *posterior* distributions:
    * the inferred likelihood of parameters after taking the data into account
- Provides a strong framework for combining information from different sources and for propagating uncertainty

Vitamin A study

- A frequentist can do a clear analysis right away
- A Bayesian needs a ton of assumptions – will try to make "uninformative" assumptions

Cape Town weather

- Frequentist: how unlikely is the observation, from a random perspective?
- Bayesian: what's my model world? What is my prior belief about weather-weekday interactions?

Example: MMEV

- MMEV is a viral infection that can cause a serious disease (called MMED)
- MMED patients are unable to control their urge to fit models to data
- The rapid MMEV test gives a positive result:
  - 100% of the time for people with the virus
  - 5% of the time for people without the virus

MMED MMEV questions

- The rapid MMEV test gives a positive result:
  - 100% of the time for people with the virus
  - 5% of the time for people without the virus
- The population prevalence of MMEV is 1%
- You pick a person from this population at random, and test them, and the test is positive.
  - What is the probability that they have MMEV?
- This calculation is the core of Bayes theorem

MMED MMEV questions

- You learn that your friend has had a positive rapid test for MMEV
  - What do you tell them?
- This is what Bayesian philosophy is about: combining information from different sources

3 Conclusion

Your philosophy

- Statistics are not a magic machine that gives you the right answer
- If you are to be a serious scientist in a noisy world, you should have your own philosophy of statistics
  - Be pragmatic: your goal is to do science, not get caught by theoretical considerations
  - Be honest: it's harder than it sounds.

Honesty

- You can always keep analyzing until you find a "significant" result
  - If you do this you will make a lot of mistakes
- You may also keep analyzing until you find a result that you already "know" is true.
  - This is confirmation bias; you're probably right, but your project is not advancing science
- Good practice
  - Keep a data-analysis journal
  - Start *before* you look at the data

Summary

- P values are over-rated
- High P values should not be used as evidence for anything ever.
  - They can provide indirect evidence. Wonderful. Find the direct evidence and use that instead.
- Use effect sizes and confidence intervals when you can
- Otherwise, find ways to make significant P values do the work
  - Non-inferiority tests, interactions

- Frequentist statistics makes weak assumptions, and finds logically weak formal conclusions:
  - These parameters are unlikely to produce a statistic this extreme by chance
- Bayesian statistics makes strong assumptions:
  - prior distributions must be fully specified
- . . . and finds logically strong formal conclusions:
  - The probability that the effect value is in this range is X
  - These strong conclusions can be used directly for prediction with uncertainty

- Statistics are a key component of data-based science
  - You should think about statistical analysis from the beginning of your project
- You need a basic understanding of statistical principles
- You need your own statistical philosophy
  - If you're a theoretician, it should be ideological and honest
  - If you're a scientist, it should be pragmatic and honest