

# Statistical philosophy

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# Outline

Statistical inference

P values and confidence intervals

Statistics and science

Paradigms for inference

Frequentist paradigm

Bayesian paradigm

Conclusion

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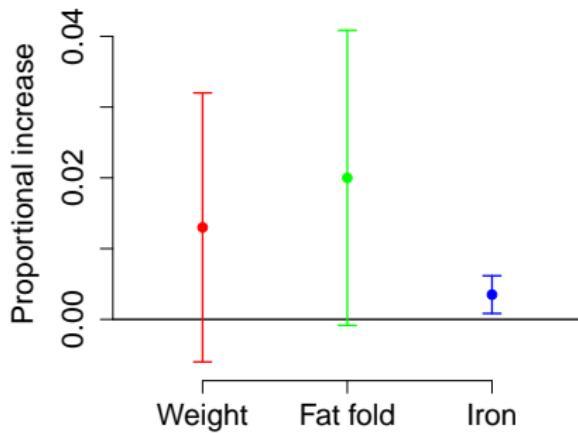
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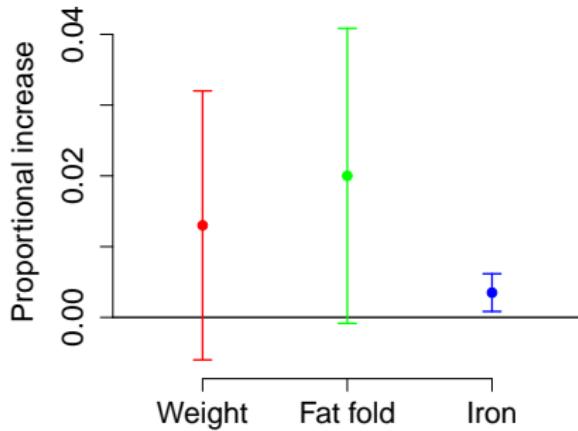
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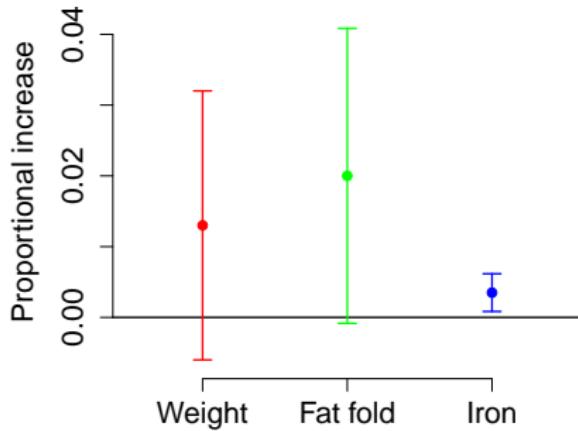
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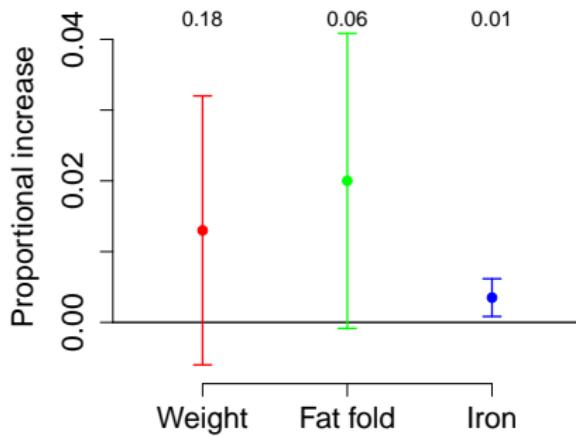
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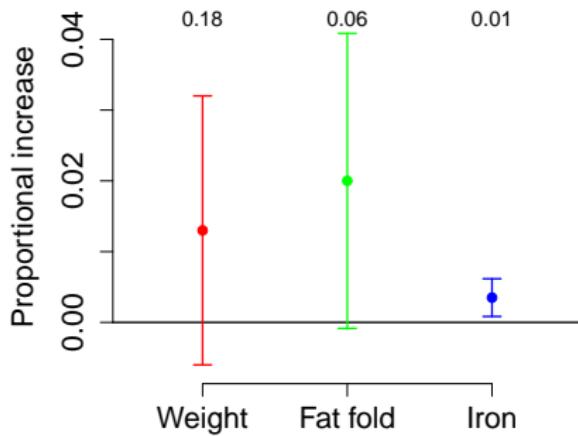
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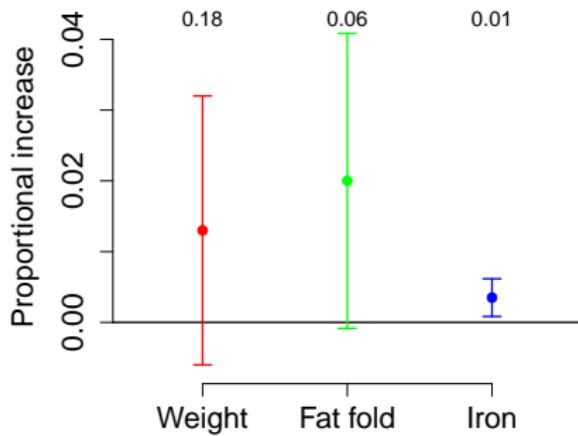
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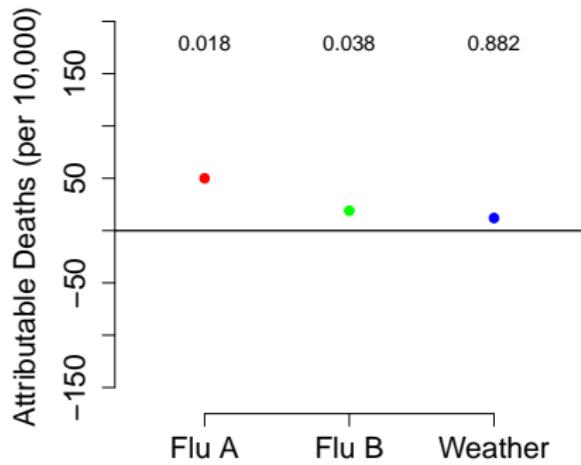
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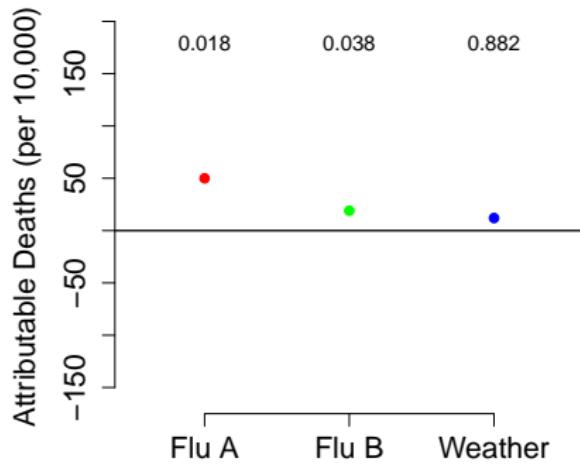
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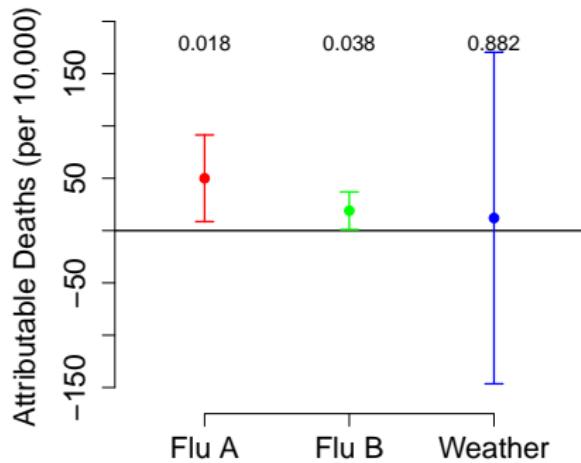
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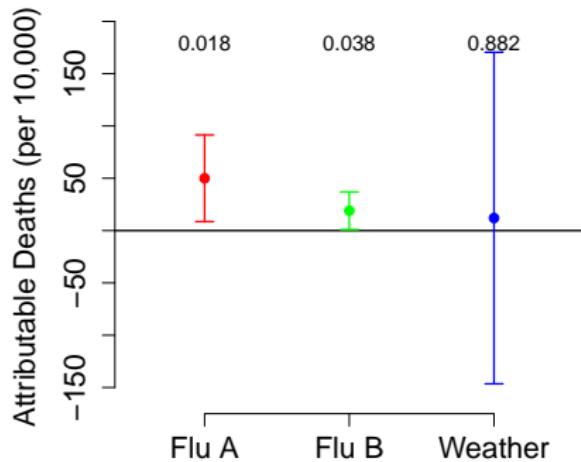
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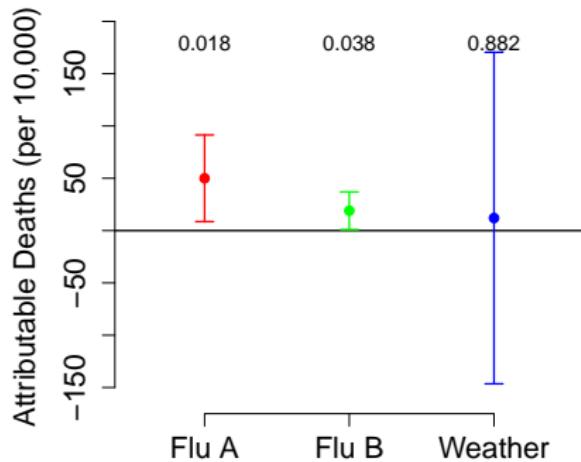
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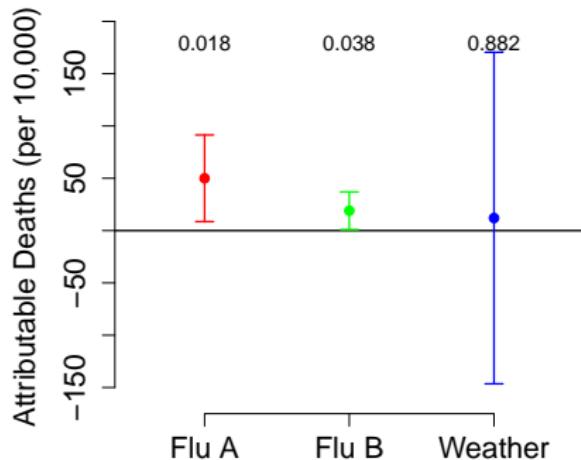
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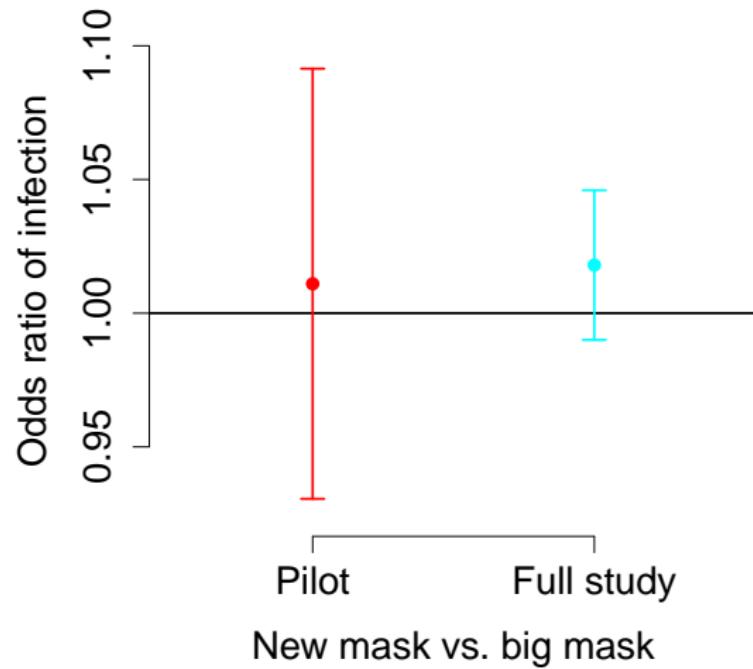
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  - ▶ They're not the same, so how close is close enough?

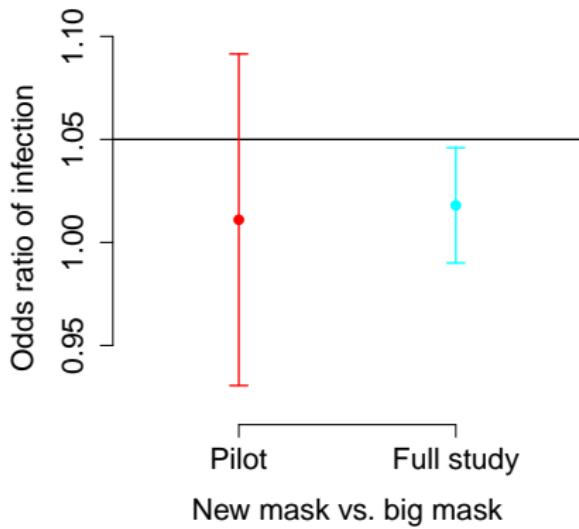
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- ▶ People who work in respiratory clinics sometimes have to wear bulky, uncomfortable, expensive masks
- ▶ They would like to switch to simpler masks, if those will do the job
- ▶ How can this be tested statistically? We don't want the masks to be "different".
  - ▶ We need to decide what we mean by different in this case!
  - ▶ They're not the same, so how close is close enough?

## Traditional approach

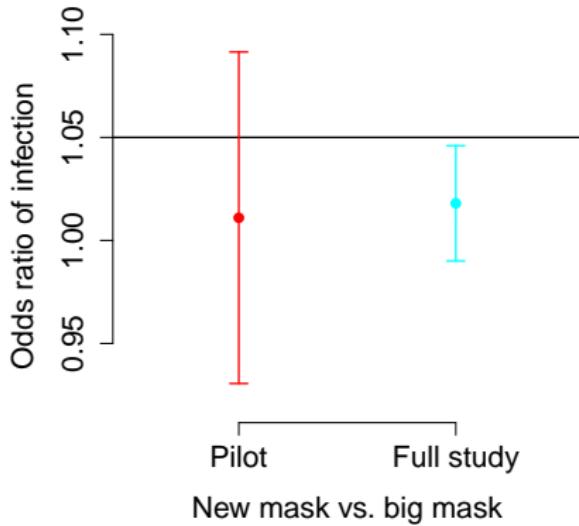


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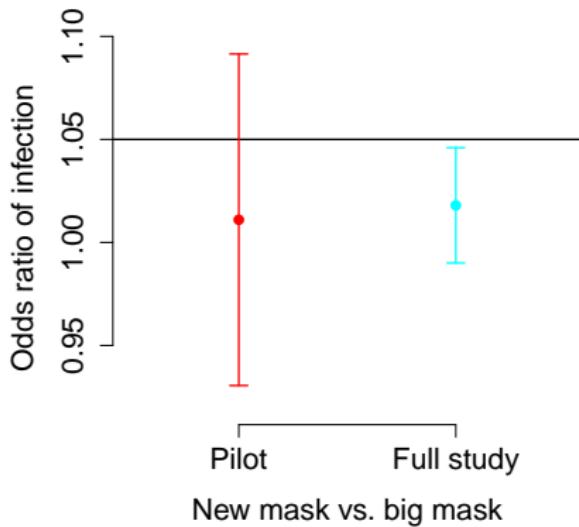
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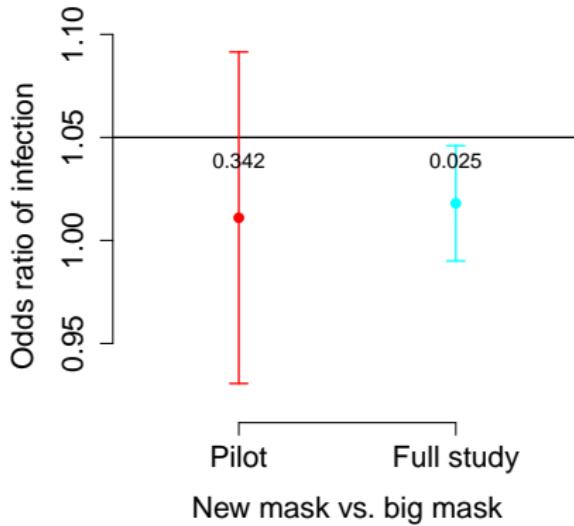
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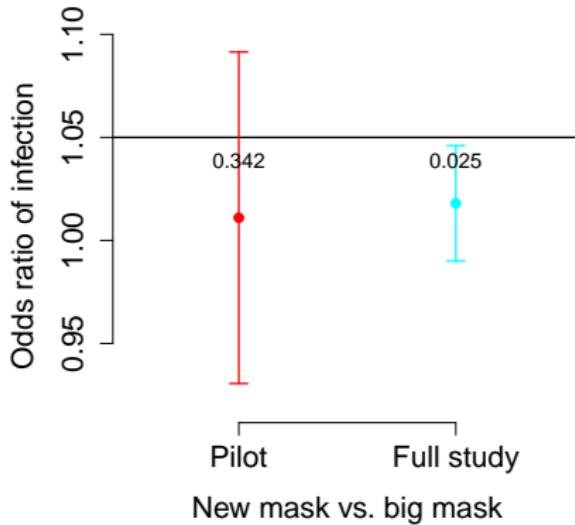
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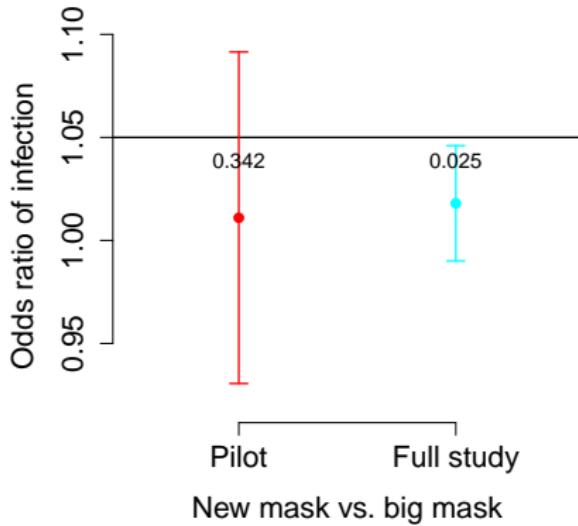
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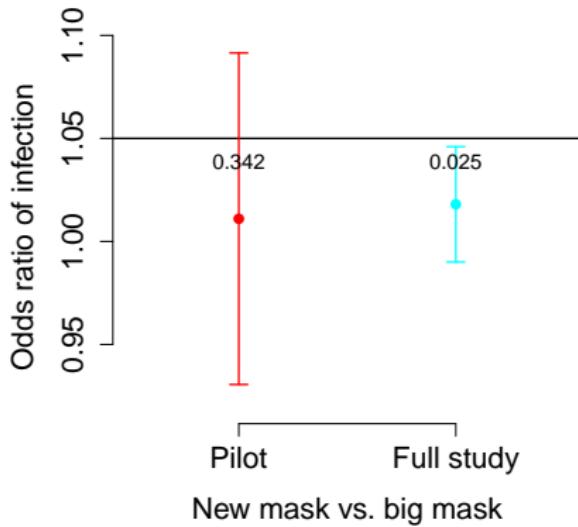
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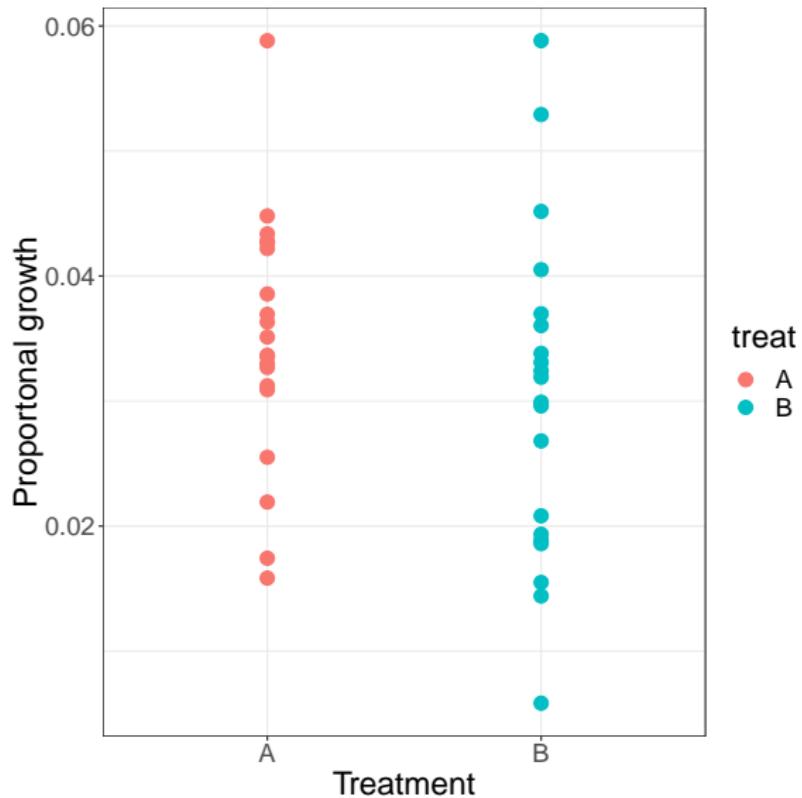
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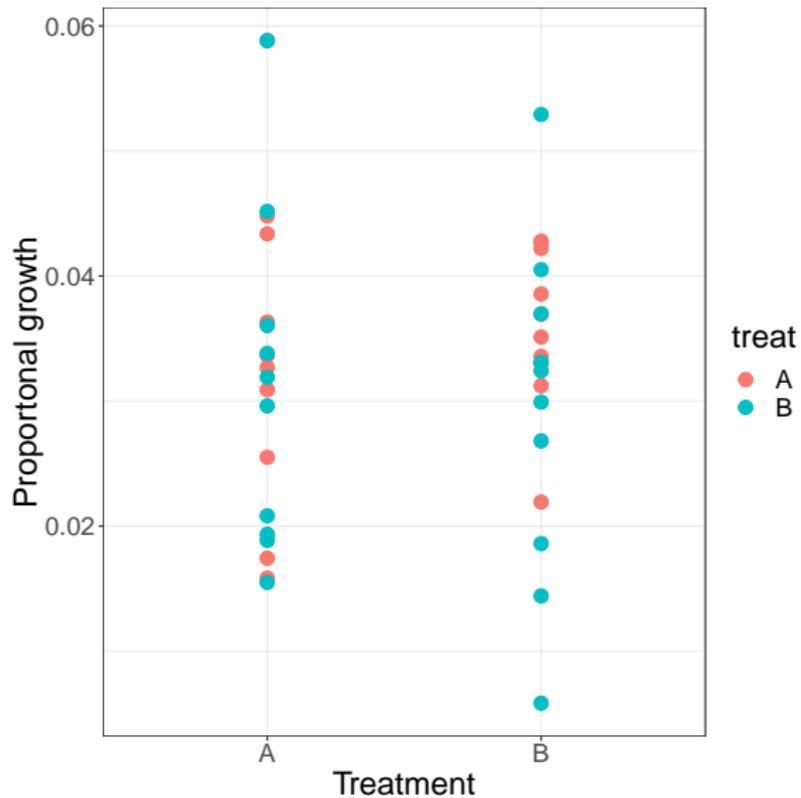
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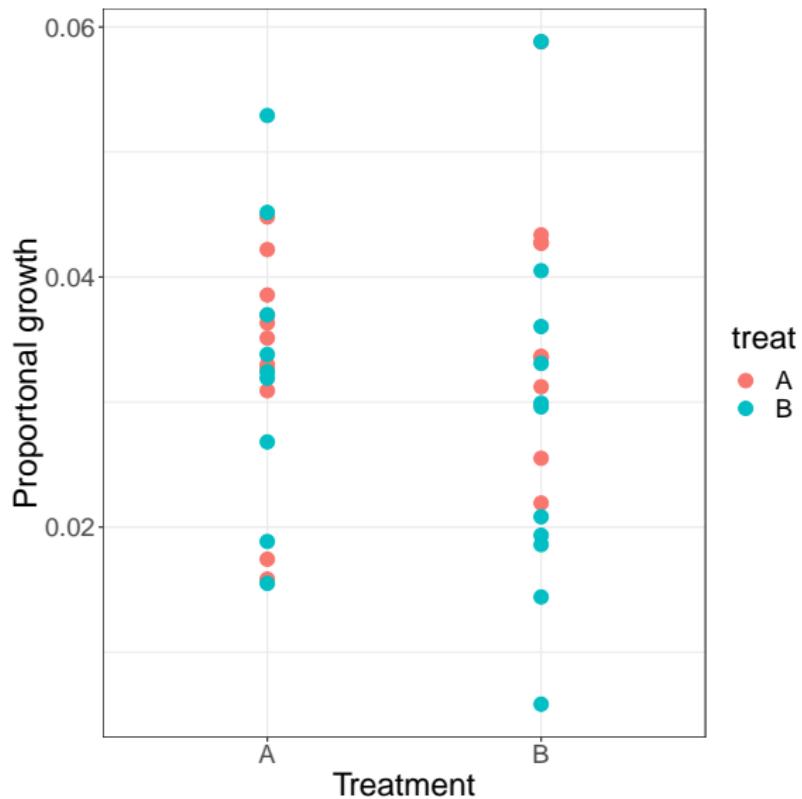
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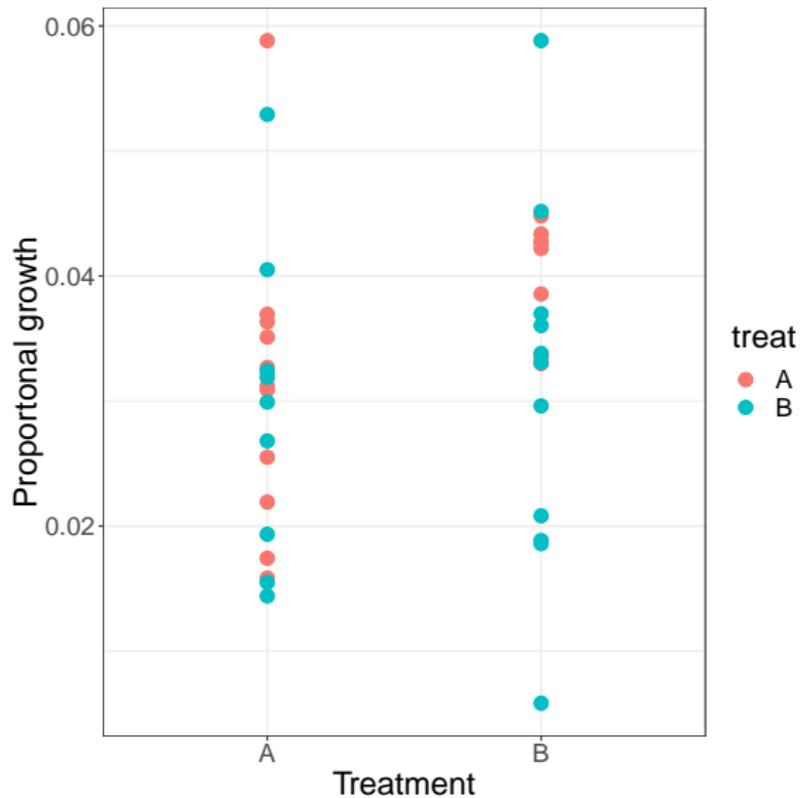
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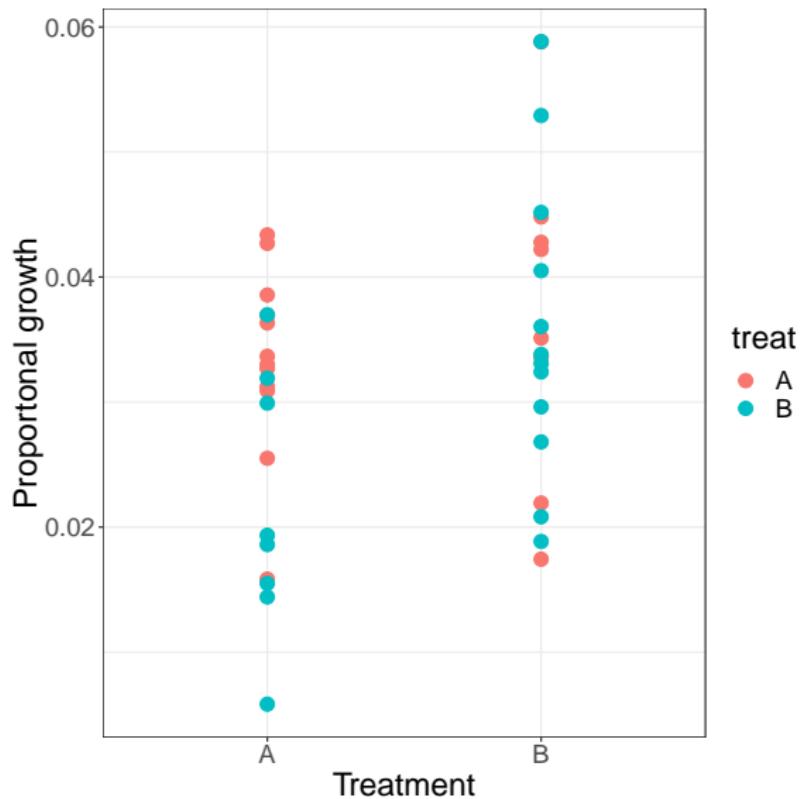
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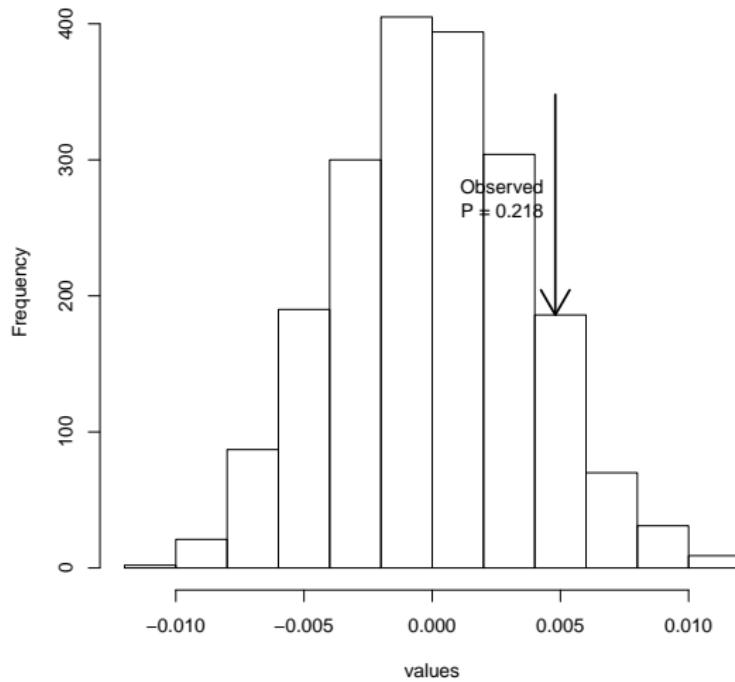
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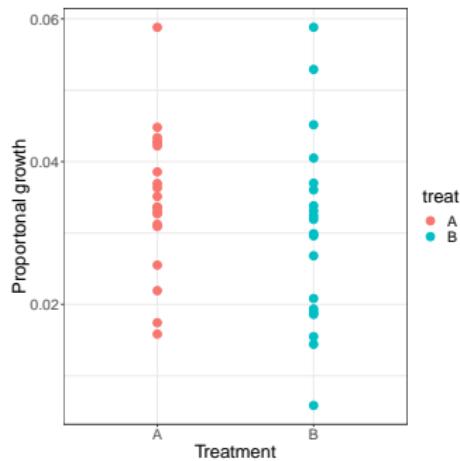
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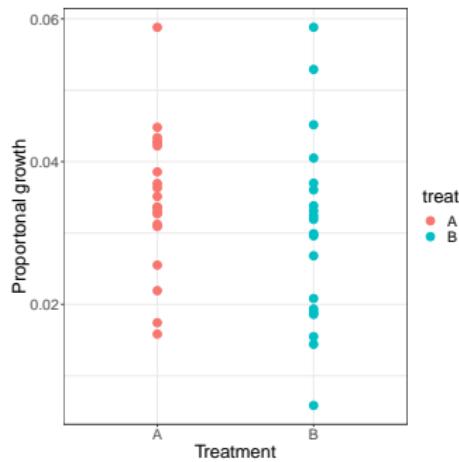
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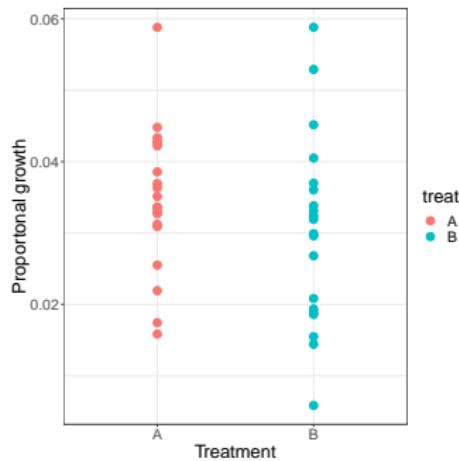
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*Tessa Wessels, Faces on a Train*

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