



$$b_1 = b_2 = 0.35 \quad b_3 = b_4 = 0.6$$

$$w_1 = 0.15 \quad w_2 = 0.2 \quad w_3 = 0.25 \quad w_4 = 0.3$$

$$w_5 = 0.4 \quad w_6 = 0.45 \quad w_7 = 0.5 \quad w_8 = 0.55$$

Let $\sigma \equiv \text{sigmoid } f^n$

FORWARD PASS:

$$h_1: z_1 = w_1 i_1 + w_2 i_2 + b_1 \quad a_1 = \sigma(z_1)$$

$$h_2: z_2 = w_3 i_1 + w_4 i_2 + b_2 \quad a_2 = \sigma(z_2)$$

$$o_1: z_3 = w_5 a_1 + w_6 a_2 + b_3 \quad a_3 = \sigma(z_3)$$

$$o_2: z_4 = w_7 a_1 + w_8 a_2 + b_4 \quad a_4 = \sigma(z_4)$$

$$\text{Loss: } L = \frac{1}{2} (a_3 - o_1)^2 + \frac{1}{2} (a_4 - o_2)^2$$

BACKWARD PASS:

$$\frac{\partial L}{\partial w_6}$$

$$\begin{aligned} \frac{\partial L}{\partial w_6} &= \frac{\partial z_3}{\partial w_6} \cdot \frac{\partial L}{\partial z_3} = \frac{\partial z_3}{\partial w_6} \cdot \frac{\partial a_3}{\partial z_3} \cdot \frac{\partial L}{\partial a_3} \\ &= [a_2] [\sigma'(z_3)] [a_3 - o_1] \end{aligned}$$

$$\sigma'(z) = \sigma(z) (1 - \sigma(z))$$

$$\Rightarrow \sigma'(z_3) = \sigma(z_3) (1 - \sigma(z_3)) = a_3 (1 - a_3)$$

$$\therefore \left[\frac{\partial L}{\partial w_6} = a_2 \cdot a_3 \cdot (1 - a_3) \cdot (a_3 - 0.01) \right]$$

~~Now~~ ~~$z_1 = 0.3775$~~

$$h_1: z_1 = 0.3775 \quad a_1 = \sigma(z_1) = 0.5933$$

$$h_2: z_2 = 0.3925 \quad a_2 = \sigma(z_2) = 0.5969$$

$$o_1: z_3 = 1.1059 \quad a_3 = \sigma(z_3) = 0.7514$$

$$o_2: z_4 = 1.2249 \quad a_4 = \sigma(z_4) = 0.7729$$

$$\begin{aligned} \frac{\partial L}{\partial w_6} &= (0.5969)(0.7514)(1 - 0.7514)(0.7514 - 0.01) \\ &= 0.08266 \end{aligned}$$

UPDATE: Let $\eta = 0.5$

$$w_6 := w_6 - \eta \frac{\partial L}{\partial w_6}$$

$$\begin{aligned} w_6 &:= 0.45 - (0.5)(0.08266) \\ &= \cancel{0.26734} \quad 0.45 - 0.04133 \\ &= 0.40867 \end{aligned}$$

$$\boxed{\cancel{\text{UPDATED-} w_6 = 0.3673}}$$

$$\therefore \boxed{\text{UPDATED } w_6 = 0.40867}$$