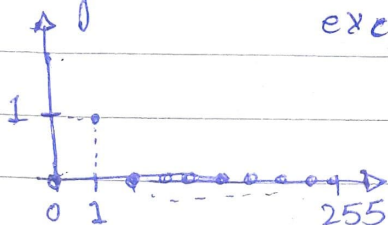


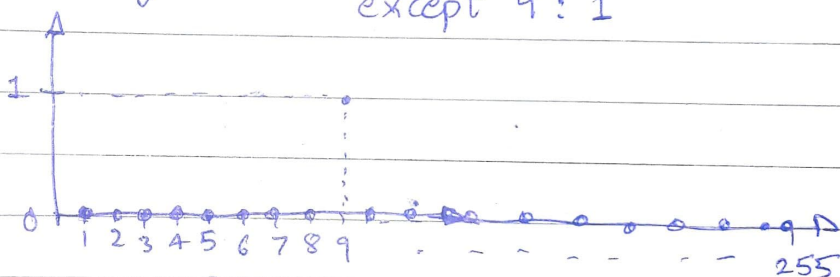
HW8

1 1 9 → FG  
 1 1 9 → BG  
 9 9 9

FG-histogram → 0-255 : 0  
except 1 : 1



BG-histogram → 0-255 : 0  
except 9 : 1



Source weights =  $\frac{\text{FG-histogram}(I_p)}{\max(\text{FG-histogram})}$   
for pixel  $p'$

∴ Q1)  $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  → Source to pixel weights

Sink weights =  $\frac{\text{BG-histogram}(I_p)}{\max(\text{BG-histogram})}$   
for pixel  $p'$

∴ Q1)  $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  → Sink weights

Q2) There are only 2 types of pixel values.

Neighboring edge weights only depend on the pixel value.

\* Central pixel:  $I_c = 1$

(a)  $I_c - I_1$  (b)  $I_c - I_9$

$$(a) \quad e^{-\underbrace{(1-1)^2}_{(I=1)}} \quad (b) \quad e^{-\underbrace{(1-9)^2}_{(I=9)}}$$

$$= 1 \quad \quad \quad = e^{-64}$$

∴ Edge weights will be as follows.

$$\begin{bmatrix} 1 & 1 & e^{-64} \\ 1 & - & e^{-64} \\ e^{-64} & e^{-64} & e^{-64} \end{bmatrix}$$

Q3)

R G B

$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$

1 0 1

0 1 1

9 9 8

$\begin{pmatrix} 9 & 9 & 9 \end{pmatrix}$

8 9 8

Let the 2 cluster centres be initialized to  $(1, 1, 1)$  and  $(9, 9, 9)$

Data	Dist. From (1,1,1)	Dist. from (9,9,9)
(1,0,1)	1 ✓	$\sqrt{81+64+64}$
(0,1,1)	1 ✓	$\sqrt{81+64+64}$
(9,9,8)	$\sqrt{64+64+49}$	1 ✓
(8,9,8)	$\sqrt{49+64+49}$	$\sqrt{2}$ ✓

Cluster 1: (1,1,1), (1,0,1), (0,1,1)

New center:  
(mean)  $\left(\frac{2}{3}, \frac{2}{3}, 1\right)$

Cluster 2: (9,9,8), (9,9,9), (8,9,8)

New center:  
(mean)  $\left(8\frac{2}{3}, 9, 8\frac{1}{3}\right)$

# It can be verified that points won't change the cluster further and the clusters won't change anymore.  
Omitted here because clear from figures.

Weights:

$$\text{GMM1} : 3/6 = 0.5$$

$$\text{GMM2} : 3/6 = 0.5$$

Covariance Matrix:

GMM 1

R G B

1 1 1  
1 0 1  
0 1 1

$$\Rightarrow \begin{bmatrix} 2/9 & -1/9 & 0 \\ -1/9 & 2/9 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

GMM2

$$\begin{array}{ccc}
 R & G & B \\
 9 & 9 & 8 \\
 9 & 9 & 9 \\
 8 & 9 & 8
 \end{array}
 \Rightarrow
 \begin{bmatrix}
 2/9 & 0 & 1/9 \\
 0 & 0 & 0 \\
 1/9 & 0 & 2/9
 \end{bmatrix}$$

Covariance Matrix

GMM1:  $(x - \bar{x}) = \begin{bmatrix} 1/3 & 1/3 & 0 \\ 1/3 & -2/3 & 0 \\ -2/3 & 1/3 & 0 \end{bmatrix}$

$$\text{cov} = \frac{1}{2} (x - \bar{x})^T (x - \bar{x}) = \begin{bmatrix} 1/3 & -1/6 & 0 \\ -1/6 & 1/3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

GMM2:  $(x - \bar{x}) = \begin{bmatrix} 1/3 & 0 & -1/3 \\ 1/3 & 0 & 2/3 \\ -2/3 & 0 & -1/3 \end{bmatrix}$

$$\text{cov} = \frac{1}{2} (x - \bar{x})^T (x - \bar{x}) = \begin{bmatrix} 1/3 & 0 & 1/6 \\ 0 & 0 & 0 \\ 1/6 & 0 & 1/3 \end{bmatrix}$$