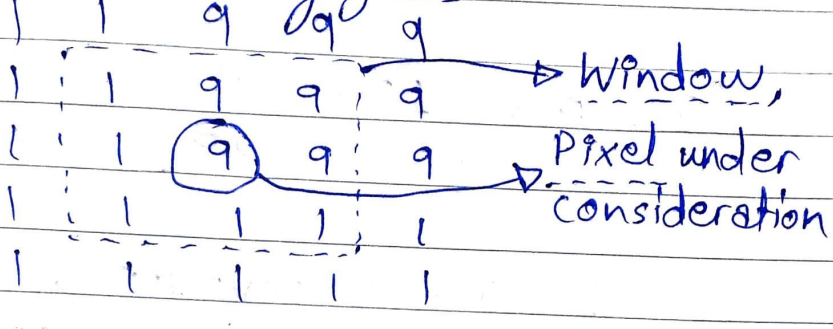
Given Matrix: $\begin{bmatrix} 1 & 9 & 9 \\ 1 & 9 & 9 \\ 1 & 1 & 1 \end{bmatrix} = I$

Symmetric Padding gives



(Q1) $I_x = I * \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} -8 & 0 & 0 \\ -8 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$I_y = I * \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 8 & 8 \\ 0 & 0 & 0 \end{bmatrix}$

$I_x^2 = \begin{bmatrix} 64 & 0 & 0 \\ 64 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $I_y^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 64 & 64 \\ 0 & 0 & 0 \end{bmatrix}$

$I_x \cdot I_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} 128 & 0 \\ 0 & 128 \end{bmatrix}$

$\left(\sum_{(x,y) \in W} \right)$

$$\det(M) = (128)(128) - (0)(0)$$
$$= \boxed{16384}$$

(Q2) $$M = \begin{bmatrix} 128 & 0 \\ 0 & 128 \end{bmatrix}$$

To find eigenvalues, find roots of the equation $\det(M - \lambda I) = 0$

$$\det\left(\begin{bmatrix} 128-\lambda & 0 \\ 0 & 128-\lambda \end{bmatrix} \right) = 0$$

$$\Rightarrow (128-\lambda)^2 = 0$$

$$\Rightarrow \boxed{\lambda_1 = \lambda_2 = 128}$$

$\Rightarrow$ Sum of the Eigen Values $= \lambda_1 + \lambda_2 = \boxed{256}$

(Q3) Harris Ranking Score

$$R = \det(M) - k \cdot tr(M)^2$$

$\det(M) = 16384$

$tr(M) = 128 + 128 = 256$

$tr(M)^2 = 65536$

$\Rightarrow R = 16384 - k(65536)$

$k \in [0.04, 0.06]$

$\therefore$ Let $k = 0.05$ (say)

$\Rightarrow R = 13107.2$

Q4) Shi-Tomasi

$R = \min(\lambda_1, \lambda_2) = \boxed{128}$