P – 6146

B. C. A. (Third Semester) EXAMINATION, Nov./Dec., 2014

Paper – 301

DISCRETE MATHEMATICS

Time : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 32

Note–Attempt *all* questions. All questions carry equal marks.

Unit – I

1. (a) Prove that : 8

$$(p \Rightarrow q) \vee r \equiv (p \vee r) \Rightarrow (q \vee r)$$

Or

Prove that :

$p \wedge q \wedge \sim q \Rightarrow p$ is a tautology or contradiction.

P.T.O.

(b) Let A and B are two subsets of a universal set U then prove that : 8

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$.

Or

If A, B, C are non-empty sets then prove that–

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B - C) = A \times B - A \times C$.

Unit – II

2. (a) If R^{-1} and S^{-1} are inverse relation of R and S respectively then show that $(SOR)^{-1} = R^{-1} O S^{-1}$. 8

Or

If I be a set of non-zero

integers and a relation R defined on I such that $xRy \Rightarrow x^y = y^x, \forall x, y \in I$ then show that relation R is an equivalence relation on I.

(b) Define following with an example– 8

(i) Constant function

(ii) Identify function

(iii) One-one mapping

(iv) Onto mapping.

Or

If $f: x \to y$ is a one-one and onto mapping then prove that $f o f^{-1} = I_y$ and $f^{-1} o f = I_x$ where I_x and I_y are identify functions on x and y respectively.

Unit – III

3. (a) In a Boolean algebra $(B, +, \cdot, ')$ prove that : 8

(i) $a + a.b = a$

(ii) $a + (b + c) = (a + b) + c$ where $a, b, c \in B$.

Or

Design a circuit of a following boolean function and convert it to simple one :

$$F(x, y, z) = x.z + [y.(y' + z).(x' + x.z')]$$

(b) Prove that the number of vertices of odd degree in a graph is always even. 8

Or

Show that a tree with n-vertices has $(n - 1)$ edges.

4.

Unit – IV

4. (a) Prove that – 8

$$(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos\left(\frac{n\pi}{4}\right)$$

Or

If $x_r = \cos\left(\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}\right)$ then prove that $x_1\, x_2\, x_3$........infinite $= -1$ where $r = 1, 2, 3$..........∞.

(b) Find the rank and nullity of following matrix– 8

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \\ 2 & 4 & 6 & 4 \end{bmatrix}$$

Or

Find Normal form of a matrix–

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 5 \\ -2 & 3 & 2 & 2 \end{bmatrix}$$

Unit – V

5. (a) Solve the following system of equations by matrix method– 8

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

Or

Solve :

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0$$

(b) Find eigen values and eigen vectors of the matrix– 8

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Or

Show that matrix :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

satisfies Cayley - Hamilton theorem.

$x + \cancel{y} + z = 6$

$x - \cancel{y} + z = 2$

$x + z = 8$

$2 + \cancel{x} = 8$

$x = 8 - 2$