Q U E S T I O N – B A N K
BCA SECOND SEMESTER - 2014
ADVANCE CALCULUS – 201

U N I T – I

Q. 1 : Prove by using definition of limit, that $\lim\limits_{(x, y) \to (1, 2)} 3x + 2y = 7$.

Q. 2 : Using definition $\lim\limits_{(x, y) \to (1, 2)} x^2 + 2y = 3$.

Q. 3 : Show that $\lim\limits_{(x, y) \to (u, u)} \dfrac{2x^3 + y^3 = 0}{x^2 + y^2}$

Q. 4 : Using definition (ε & δtechnique) prove that $\lim\limits_{x, y \to (2, 3)} xy =$ б.

Q. 5 : Show that $\lim\limits_{x, y \to (-1, 2)} \dfrac{x^3 + y^3 = 7/5}{x^2 + y^2}$

Q. 6 : Let $f(x, y) = \dfrac{xy}{x^2 + y^2}$ show that $\lim\limits_{x, y \to (u, u)} f(x, y)$ does not exist.

Q. 7 : Let $f(x, y) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$

Show that for any point $\lim\limits_{x, y \to (a, b)} s(x, y)$ does not exist.

Q. 8 : Examine whether the function $f(x,y) = \begin{cases} x^2+4y & \text{when } (x,y) \neq (1,2) \\ 0 & \text{when } (x, y) = (1, 2) \end{cases}$ in continuous at (1,2).

Q. 9 : Show that the function $f : R^2 \to R$ definition by $f(x, y) = \begin{cases} \dfrac{xy,}{\sqrt{x^2 + y^2}} & \text{when } (x, y) \neq (u, u) \\ & \text{when } (x, y) = (u, u). \end{cases}$ is continuous at the origin (u, u)

Q.10: Show that the function $f(x, y) = \begin{cases} \dfrac{\sin(x + y)}{x + y}, & (x, y) \neq (u, u). \\ 1 & (x, y) = (u, u) \end{cases}$

is continuous at the origin (u, u)

Q.11: Let $f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, u). \\ u & \text{if } (x, y) = (u, u) \end{cases}$ Show that f (x, y) is continuous but not differentiable at the origin.

Q.12: Show that the function f (x, y) define by $f(x, y) \begin{cases} \dfrac{xy^2,}{x^2 + y^2} & (x, y) \neq (u, u) \\ u & \text{otherwise} \end{cases}$

Show that f (x, y) is continuous but not differentiable.

Q.13: Show that $f(x, y) = e^{x+y}$ is differentiable at (1, 3).

Q.14: If $u = (1 - 2xy + y^2{}_1)^{-1/2}$ prove that $\partial/\partial x\, [(1 - x^2)\, \partial y/\partial x] + \partial/\partial y\, [y^2\, \partial x/\partial y] = 0$.

Q.15: If u = tan-1 (y/x) + sin-1 (x/y) prove that $x\, \partial 4/\partial x + y \partial 4/\partial y = 0$.

Q.16: If $u = u^2 - v^2$, $y = 2uv$ obtain $\partial 4/\partial x$, $\partial 4/\partial y$, $\partial v/\partial x$, $\partial v/\partial y$.

Q.17: If $u = (x^2 + y^2 + z^2)^{-1/2}$, $x^2 + y^2 + z^2 = 0$ then prove that.

(a) $x\partial 4/\partial x + y\partial 4/\partial y + z\partial 4/\partial z = -u$

(b) $\partial^2 4/\partial x^2 + \partial^2 4/\partial y^2 + \partial^2 4/\partial z^2 = u$

Q.18: If $u = e^{xyz}$, show that $\partial^3 4/\partial x\partial y\partial z = (1 + 3xyz + x^2y^2z^2)\, e^{xyz}$.

Q.19: If $u = \log (x^3 + y^3 + z^3 - 3xyz)$ prove that.

(a) $\partial 4/\partial z + \partial 4/\partial y + \partial 4/\partial z = 3/x+y+z$

(b) $(\partial/\partial x + \partial/\partial y + \partial/\partial z)^2\, u = -9/(x+y+z)^2$

(c) $\partial^2 4/\partial x^2 + \partial^2 4/\partial y^2 + \partial^2 4/\partial z^2 = -3/(x+y+z)^2$

Q.20: If $u = \tan\text{-}1 \dfrac{(x^3 + y^3)}{(x - y)}$ then show that $x\partial 4/\partial x + y\partial 4/\partial y = \sin 24$.

Q.21: Verify euler's theorem for function $u = \log \dfrac{x^2 + y^2}{xy}$

Q.22: If u = xy/x+y show that $x^2\partial^2 4/\partial x^2 + 2xy\, \partial^2 4/\partial x\partial y + y^2\partial^2 4/\partial y^2 = 0$.

Q.23: If $\sqrt{1-x^2} + \sqrt{1-y^2} = a\,(x - y)$ prove that $dy/dx = \sqrt{1-y^2}/\sqrt{1-x^2}$.

Q.24: If u = xlogxy where $x^3 + y^3 + 3xy = 1$ find d4/dx.

Q.25: If u = f (y – z, z – x, x - y) prove that $\partial 4/\partial x + \partial 4/\partial y + \partial 4/\partial z = 0$.

Q.26: Find Taylor's expansion of the function f (x, y) = e^{xy} about that the point (1, 1) upto third degree terms.

Q.27: Expand the function f (x, y) = $x^2 + xy - y^2$ by taylor's theorem in powers uf (x - 1) and (y + 2).

U N I T – II

Q. 1 : Find the envelop of straight line x/a Cos Θ + y/b sin Θ = 1 where Θ is a parameter, Interpret the result geometrically.

Q. 2 : Show that the envelop of the pulsars of points on ellipse $x^2/h^2 + y^2/k^2 = 1$ with respect to the ellipse $x^2/a^2 + y^2/b^2 = 1$ in $h^2x^2/a^4 + k^2y^2/b^4 = 1$.

Q. 3 : Obtain the envelope of the family of curve given by $x^2/\alpha^2 + y^2/k^2\text{-}\alpha^2 = 1$ where α is the parameter.

Q. 4 : Find the envelope of straight line x/a + y/b = 1 where a^2 + b2 = c^2 and c is a constant.

Q. 5 : Fine the envelope of the circle described on the radis vector of the curve $r^n = a^n$ CosnΘ as diameter.

Q. 6 : Find the evaluate of the parabola $y^2 = 4ax$.

Q. 7 : Show that the evaluate ufa cycloid is another cyloid.

Q. 8 : Discuss the maximum or minimum value of the function f (x, y) = $x^3 - 4xy + 2y^2$.

Q. 9 : Discuss the maxima and minima of function $ax^3y^2 - x^4y^2 - x^3y^3$.

Q.10: If f (x, y) = $2x^4 + y^4 - 2x^2 - 2y^2$ then show that has a maximum at (u, u) and a minimum (1/√2, 1).

Q.11: Discuss the maxima and minima ufu = sinx + siny + sin (x + y).

Q.12: Find the point whish in a triangle such that the sum of the square of its distance from the three vertices is a minimum.

Q.13: Find the minimum value of $x^2 + y^2 + z^2$ when ax + by + cz = p.

Q.14: Find the maxima and minima ufu = $x^2 + y^2$ + z2 where $ax^2 + by^2 + cz^2 = 1$.

Q.15: Find the maxima and minima ufx2 + y2 +z2 subject to the following condition
ax + by + cz = 1, a'x + b'y + c'z = 1.

Q.16: Find the maxima and minima value of u = $a^2x^2 + b^2y^2 + c^2z^2$ where $x^2 + y^2 + z^2 = 1$ and lxtmytnz = 0.

UNIT – III

Q. 1 : To prove that
(a) Γ1 = 1 (b) $\sqrt{n + 1}$= n n (c) Γ n+1 = ∟n,n= 1,2,3.....

Q. 2 : Prove that $\Gamma n/z^n = \int_0^\infty e^{-zx} x^{n-1}$ dx.

Q. 3 : Prove that Γn Γ1-n = π/sinn π where u<n<1.

Q. 4 : Express the following integral in terms of beta function.

(a) $\int_u^1 x^3/1-x^7$ dx (b) $\int_u^1 x^4 (1-x)^3$ dx

Q. 5 : Show that $\int_u^1 x (8 -x^3)^{1/3}$ dx = 16π/9√3.

Q. 6 : Show that $\int_u^1 dx//\sqrt{c1-x^n}$ = √ π Γ1/n/[Γ1/n + 1/2].

Q. 7 : Express $\int_u^1 x^m(1-x^n)^p$ dx in terms of beta sanction and hence evaluate $\int_u^1 x^5 (1-x^3)^{10}$ dx.

Q. 8 : Show that $\int_u^{Ty2}$ ten^nx dx = π /2 sec nπ/2 where -1<n<1.

Q. 9 : Show that $\int_u^1 x^2/(1-x^4)^{1/2}$ dx-x $\int_u^1 1/(1+x^4)^{1/2}$ dx = π/4√2.

Q.10: Γ(-1/2) Γ(-3/2) evaluate.

Q.11: Evaluate $\int_u^\infty x^5e^{-x}$dx.

Q.12: Show that Γp Γ1-p/2 = π Γp/2/2^{1-p} cos (1/2pπ) where u<p<1.

Q.13: Show that $B(m, m) = 2^{1-2m} B(m,1/2)$.

Q.14: Prove that $B(m, m) = B(m+1, n) + B(m, n+1)$ where $m, n > 0$.

Q.15: Prove that $\int_u^1 \log \Gamma(x)\, dx = ½ \log (2\pi)$.

Q.16: Prove that $\int_u^1 x^m (\log)^n\, dx = (-1)^n \llcorner n/(m+1)^{n+1}$ where n is a positive integer and m>-1.

UNIT – IV

Q. 1 : Evaluate $\int_u^2 \int_u^1 (x^2+y^2)\, dxdy$.

Q. 2 : Evaluate $\int_u^2 \int_u^{\sqrt{1+x^2}} dxdy/1+x^2+y^2$.

Q. 3 : Evaluate $\int\int \frac{x}{(a^2 - y^2)}$ dxdy over the positive quadrant of the circle $x^2+y^2 = a^2$.

Q. 4 : When the region of integration R is the tringle bounded by y = 0, y = x and x = 1 show that $\iint_R 4x^2-y^2\, dxdy = 1/3 (\pi/3 +\sqrt{3}/2)$.

Q. 5 : Evaluate $\iint_R \sqrt{xy-y^2}\, dydx$ where R is a tringle with vertices (u, u), (20, 2) and (2, 2).

Q. 6 : Evaluate $\int_u^{\pi} \int_u^{a(1-GQ)} r^2 \sin\Theta dr d\Theta$.

Q. 7 : Evaluate $\int_u^3 \int_u^1 \int_u^1 (x+y+z)\, dxdydz$.

Q. 8 : Evaluate $\iiint_v (x^2+y^2+z^2)\, dzdydx$ where v is the volume of the cube bounded by the coordinator plans and plans x = y = z = a.

Q. 9 : Change the order of integration in $\int_u^a \int_{\sqrt{a^2-x^2}}^{x+2a} f(x, y)\, dxdy$.

Q.10: By changing the order of integration of $\int_u^{\infty} \int_u^{\infty} e^{-xy} \sin px$.

Q.11: Prove that $\int_u^{\pi/2} \int_u^{\pi/2} \sin x \sin^{-1} (\sin x \sin y)\, dxdy = \pi/4 – \pi/2$.

Q.12: Transform the integral $\int_u^a \int_u^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\, dxdy$ by changing to polar coordinate and hence evaluate it.

Q.13: Evaluate $\int_u^a \int_u^{\sqrt{a^2-y^2}} (x^2+y^2)dxdy$ by changing into pulars.

Q.14: Evaluate $\int\int\int z (x^2+y^2+z^2)\, dxdydz$ through the valume of the cylinder $x^2+y^2 = a^2$ intercepted by planes z = 0 and z = h.

Q.15: By using the transformation x + y = u, y = uv show that $\int_u^1 \int_u^{1-x} e\, y/x+y\, dydx = ½ (e-1)$.

Q.16: Prove that $\int\int_D x^{l-1} y^{m-1}\, dxdy = \Gamma l\, \Gamma m/\sqrt{l+m+1}\, h^{l+m}$ where D is the domain $x \geq 0$ and $x+y \leq h$.

Q.17: Prove that when $x+y < h \iint f^1 (x+y).x^{l-1} y^{-l}\, dxdy = \pi/\sin l\pi [f (h) – f (u)]$.

Q.18: Prove that $\int\int\int dxdydz/\sqrt{(1-x^2-y^2-z^2)} = \pi^2/8$ the integral being extended to all positive values of the variable for which the expression in real.

Q.19: Find the area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

Q.20: Find the ehere area of the ellips $x^2/a^2 + y^2/b^2 = 1$.

Q.21: Find the area of the curve $r = a\ (1+\cos\Theta)$.

Q.22: Find the area common to the circle $r = a\sqrt{2}$ and $r = 2a\cos\Theta$.

Q.23: Find the volume of paraboloid generated by the revolution of the parabola $y^2 = 4ax$ about the x axis from $x = 0$ to $x = h$.

Q.24: Find the volume of the solid bounded by the surface $x = 0$, $y = 0$, $x+y+z = 1$ and $z = 0$.

Q.25: Prove that the volume enclosed by the cylinders $x^2+y^2=zax$, $z^2=zax$ is $128a^3/15$.

U N I T – V

Q. 1 : Show that the integral $\int_a^\infty dx/\sqrt{1+x^2}$, $a > u$ converge.

Q. 2 : Test the convergence of $\int_2^\infty dx/x^{2-1}$.

Q. 3 : The test convergence of $\int_1^\infty dx/x^{1/3}\,(1+x^{1/2})$.

Q. 4 : Examine the convergence of $\int_a^\infty \sin x/\sqrt{x}\ dx$ where $a > o$.

Q. 5 : Show that $\int_1^\infty \sin x/x^4\ dx$ converges absolutely.

Q. 6 : Show that $\int_1^\infty \sin mx/a^2+x^2\ dx$ converges absolutely.

Q. 7 : Show that $\int_1^1 \sec x/x\ dx$ diverges.

Q. 8 : Test convergence of $\int_u^1 dx/x^{1/2}(1-x)^{1/3}$.

Q. 9 : Prove that the integral $\int_u^b dx/(x-a)\sqrt{b-x}$.

Q.10: Show that $\int_u^{\pi/2} \log \sin x\ dx$ converges.

Q.11: Discuss the convergence of the gammu function $\int_u^\infty x^{n-1}\ e^{-x}\ dx$.

Q.12: Show that $\int_u^{\pi/2} \mathrm{Cos}2nx \log \sin n\ dx$ converges.

Q.13: Show that $\int_u^1 \sec x/x\ dx$ diverges.