SEMESTER

Assignment :-

Q1 Find rank ?

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$
$R_4 \rightarrow R_4 - 6R_1$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \longleftrightarrow R_3$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \Rightarrow -\frac{R_2}{4}$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_1 = R_1 - 2R_2$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 = R_4 + 4R_2$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 = -\frac{R_3}{3}$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_1 = R_1 + R_3$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_2 = R_2 - 2R_3$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_4 = R_4 + 3R_3$$

$$A \Rightarrow \left[\begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right]$$

Rank Of Matrix is "3"

Q2) Let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be the Linear Transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)+(b-c)x+(c-a)x^2$

Find rank & nullity of T.

Since the ~~dimension~~ maximum degree of polynomial $T \Rightarrow 2$.
So $\dim(P_2) \Rightarrow 3$

Kernel

So a subset of Kernel T is $T(A)=0$

$$(a-b)+(b-c)x+(c-a)x^2 \Rightarrow 0$$

$$\hookrightarrow \boxed{a=b=c=t \text{ (let)}}$$

Kew matrix $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

dimension of Kernel is 1, because there's only one independent parameter as 't'.

Acc. to rank nullity Theorem $\Rightarrow$
$$\text{rank}(T) + \text{nullity}(T) \Rightarrow \dim(W)$$
$$\text{rank}(T) + 1 = 4$$

So, rank of T is 3, & nullity is 1.

Q3) Let $A \Rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find eigen-values and eigen vectors of A^{-1} & $A + 4I$.

$$A - \lambda I \Rightarrow 0$$

$$\det \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$(2-\lambda) \Rightarrow \pm 1$$

$$\lambda \Rightarrow 1, 3$$

for $\lambda = 1$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x - y = 0$$

$$x \Rightarrow y \quad , \text{ let } x = t, \; y \Rightarrow t$$

Eigen vector V_1, $t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Ans for $\lambda = 3$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x - y = 0$

$x = -y$, let $x = t$

$y = -t$

So, eigen value $V_2 \Rightarrow t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Now, find for A^{-1}

→ eigen values of A^{-1} will be $\frac{1}{\lambda_1}$ & $\frac{1}{\lambda_2}$ $\Rightarrow 1, \frac{1}{3}$

→ and eigen vectors are same as of A.

$V_1 \Rightarrow t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $V_2 = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Now, for $A + 4I$

→ eigen values for $A + 4I$ will be $\lambda_1 + 4$, $\lambda_2 + 4 \Rightarrow 5, 7$

→ and eigen vectors are same as as of A

$$V_1 = t\begin{bmatrix}1\\1\end{bmatrix} \qquad V_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$$

Q4) Solve by Gauss-Siedel Method (Take three Iterations)

$$3x - 0.1y - 0.2z \Rightarrow 7.85$$
$$0.1x - 7y - 0.3z \Rightarrow -19.3$$
$$0.3x - 0.2y + 10z \Rightarrow 71.4$$

with initial values $x(0) \Rightarrow 0$, $y(0)=0$, $z(0)=0$.

eqn $\Rightarrow$ $x^{k+1} \Rightarrow \dfrac{7.85 + 0.1y^{k} + 0.2z^{k}}{3}$

$$y^{k+1} \Rightarrow \frac{-19.3 - 0.1x^{k+1} - 0.3z^{k}}{-7}$$

$$z^{k+1} \Rightarrow \frac{71.4 - 0.3x^{k+1} + 0.2y^{k+1}}{10}$$

We know $x(0)=0$, $y(0)=0$, $z(0)=0$

teratⁿ-1 :→

$x(1) \Rightarrow \frac{7.85 + 0.1(0) + 0.2(0)}{3} \Rightarrow 2.6167$

$y(1) \Rightarrow \frac{-19.3 - 0.1(2.6167) - 0.3(0)}{-7} \Rightarrow 2.7956$

$z(1) \Rightarrow \frac{71.4 - 0.3(2.6167) - 0.2(2.9756)}{10} \Rightarrow 7.1373$

teratⁿ-2 :→

$x(2) \Rightarrow \frac{7.85 + 0.1(2.7956) + 0.2(7.1373)}{3} = 3$

$y(2) \Rightarrow \frac{-19.3 - 0.1(3) - 0.3(7.1373)}{-7} = 3$

$z(2) \Rightarrow \frac{71.4 - 0.3(3) - 0.2(3)}{10} = 3$

Iteratⁿ-3 :→

$x(3) = (7.85 + 0.1(3) + 0.2(3)) / 3 \rightarrow 3$
$y(3) \rightarrow (-19.3 - 0.1(3) - 0.3(3)) / -7 \Rightarrow 3$
$z(3) \Rightarrow (71.4 - 0.3(3) + 0.2(3)) / 10 \rightarrow 3$

After, three iteratⁿ $x, y, z \simeq 3$
So value of $x = 3$, $y = 3$ & $z = 3$

Q5. Define consistent or inconsistent system of equations.

Consistent
(atleast one solution)

↓ Dependent (infinite solⁿ)

↓ Independent (unique solⁿ)

Inconsistent
(No solution)

$$A \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$, $R_4 \rightarrow R_4 - R_1$

$$\Rightarrow \begin{array}{cccc} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}$$

$R_3 \rightarrow R_3 - 2R_2$, $R_4 \rightarrow R_4 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) \Rightarrow 2$
$\rho(A:B) \Rightarrow 2$
$n \Rightarrow 3$

$\rho(A) = \rho(A:B) \neq n$

Consistent, but infinite solⁿ.

<u>Q6)</u> Determine whether fxn $T: P_2 \to P_2$ is linear transformatⁿ or not.

$$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$$

1.) <u>Addictive</u>

$$T(u+v) \Rightarrow T(u) + T(v)$$
$$u = a_1 + b_1 x + c_1 x^2$$
$$v = a_2 + b_2 x + c_2 x^2$$

$$T(u+v) \Rightarrow T\left((a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^2\right)$$

$$\Rightarrow (a_1+a_2+1) + (b_1+b_2+1)x + (c_1+c_2+1)x^2$$

$$\Rightarrow (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$$

$$\Rightarrow T(u) + T(v)$$

Hence Proved

2.) Homogenity

$T(kU) \Rightarrow kT(u)$

$\Rightarrow kT$

$T(k(a+bx+cx^2))$

$T(ka+kbx+kcx^2)$

$\Rightarrow (ka+kb+kc+1)+(ka+kb+kc+1)x$
$+(ka+kb+kc+1)x^2$

$\Rightarrow k(a+1)+k(b+1)x+k(c+1)x^2$

$\Rightarrow kT(u)$

Hence Proved

Hence, It's a Linear Transformatⁿ

Q7) Determine whether set $S \Rightarrow \{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis, determine the dimⁿ & basis of subspace spanned by S.

$a(1,2,3)+b(3,1,0)+c(-2,1,3) = (0,0,0)$

$a+3b-2c=0$
$2a+b+c=0$
$3a+3c=0$

$c \Rightarrow -a \quad , b \Rightarrow -a$

only one sol^n is possible is
a = b = c = 0. So linearly
Independent.

Since dim^n of $V_3(R)$ is 3 and
S also contains 3 vector.
And S → LI, then it spans $V_3(R)$
making it a basis for $V_3(R)$.

Q8) Using Jacobi's method (perform
3 iterat's)

$$3x - 6y + 2z \Rightarrow 23$$
$$-4x + y - z \Rightarrow -15$$
$$x - 3y + 7z \Rightarrow 16$$

$x_0 \Rightarrow 1$, $y_0 \Rightarrow 1$, $z_0 \Rightarrow 1$

⇒ first eq^n. $x = \frac{1}{3}(23 + 6y - 2z)$

second eq^n $y \Rightarrow (-15 + 4x + z)$

thrid eq^n. $z \Rightarrow \frac{1}{7}(16 - x + 3y)$

$x(0) = 1$, $y(0) = 1$, $z(0) = 1$

Iterat^n-1 :→

$x(1) \rightarrow (23 + 6 - 2)/3 \Rightarrow 9$
$y(1) \rightarrow (-15 + 4 + 1) \Rightarrow -10$
$z(1) \Rightarrow (16 - 1 + 3)/7 \Rightarrow 18/7$

Q9b

Affine Transformat^n^ Rotat^n^

Suppose we have a 2-D image represented as grid or pixels. we can use AT matrix to rotate around centre.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\longrightarrow$ rotat^n^ of image by θ

to rotate it around centre.

1.) Translat^n^ to origin

translate the image, so that it centre alligns with origin.

2.) Rotat^n^

Apply rotat^n^ matrix.

3.) Translat^n^ Back

translate it back with the original pos^n^ by adding coordinates of centre.

10) Brief Description of Linear Transformat^n for computer vision for rotating 2-D image.

⇒ Linear Transformation for rotat^g 2D images involves applying a rotat^n matrix to each pixel coordinate. This matrix rotates points counterclockwise by an angle θ around the origin. It preserves geometric properties like parallelism and distances. Rotat^n is essential in tasks like image allignment and object detection in computer vision.

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