ASSIGNMENT-1

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Question 1.3.3

D_1 is a point on $\mathbf{BC}$ such that $\mathbf{AD_1} \perp \mathbf{BC}$ and $\mathbf{AD_1}$ is defined to be the altitude. Find the equations of the altitude $\mathbf{BE_1}$ and $\mathbf{CF_1}$ to the sides $\mathbf{AC}$ and $\mathbf{AB}$ respectively.

Solution: Given:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

Direction vector

$$\mathbf{m}_{AB} = \mathbf{B} - \mathbf{A} \tag{2}$$
$$= \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{3}$$
$$= \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{4}$$
$$\mathbf{m}_{AC} = \mathbf{C} - \mathbf{A} \tag{5}$$
$$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{6}$$
$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{7}$$

Normal vector

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{8}$$
$$\mathbf{n}_{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{9}$$
$$= \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{10}$$
$$\mathbf{n}_{AC} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{11}$$
$$= \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{12}$$

Equation of line is represented by:

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{p}) = 0 \tag{13}$$

1) The equation of line $\mathbf{CF_1}$

$$\mathbf{n}_{AB}^\top (\mathbf{x} - \mathbf{C}) = 0 \tag{14}$$
$$\begin{pmatrix} 7 \\ 5 \end{pmatrix}^\top \left(\mathbf{x} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right) = 0 \tag{15}$$
$$\begin{pmatrix} 7 & 5 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right) = 0 \tag{16}$$
$$\begin{pmatrix} 7 \\ 5 \end{pmatrix}^\top \mathbf{x} = -46 \tag{17}$$

2) The equation of line $\mathbf{BE_1}$

$$\mathbf{n}_{AC}^\top (\mathbf{x} - \mathbf{B}) = 0 \tag{18}$$
$$\begin{pmatrix} -4 \\ 4 \end{pmatrix}^\top \left(\mathbf{x} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \right) = 0 \tag{19}$$
$$\begin{pmatrix} -4 & 4 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \right) = 0 \tag{20}$$
$$\begin{pmatrix} -4 \\ 4 \end{pmatrix}^\top \mathbf{x} = 40 \tag{21}$$

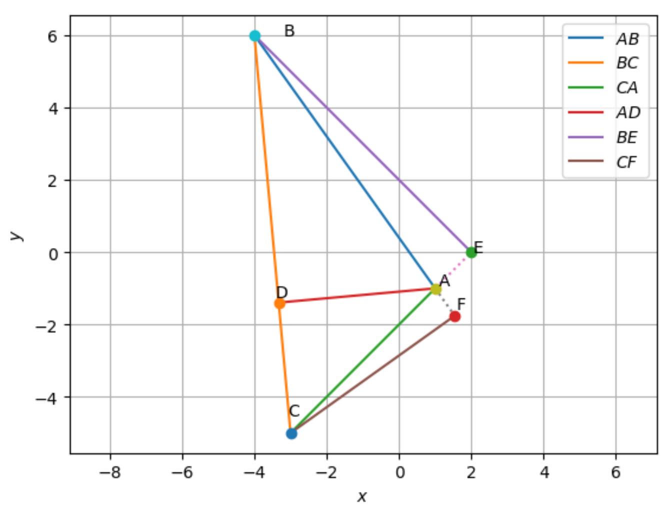

Fig. 2. Lines $\mathbf{BE}_1$ and $\mathbf{CF}_1$