Assignment

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Question: Consider the probability space $(\Omega, \mathcal{G}, P)$, where $\Omega = \{1, 2, 3, 4\}$, $\mathcal{G} = \{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$, $P(\{1\}) = \frac{1}{4}$. Let X be the random variable defined on the above probability space as $X(1) = 1$, $X(2) = X(3) = 2$, $X(4) = 3$. If $P(X \leq 2) = \frac{3}{4}$, then find $P(\{1, 4\})$ (rounded off to two decimal places).
(GATE ST 2023)
Solution:

TABLE 1: Probablity space

Probablity space	Value
Ω	$\{1, 2, 3, 4\}$
$\mathcal{G}$	$\{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$
$P(\{1\})$	$\frac{1}{4}$
$P(X \leq 2)$	$\frac{3}{4}$

TABLE 2: Random variable

$X(\Omega)$	Ω
$\{1\}$	1
$\{2, 3\}$	2
$\{4\}$	3

Pmf is defined as

$$p_x(k) = \begin{cases} P(\{1\}) & , k = 1 \\ P(\{2, 3\}) & , k = 2 \\ P(\{4\}) & , k = 3 \end{cases} \tag{1}$$

Values of P({2,3}), P({4}) are unknown, so let p, q be their respective values

$$p_x(k) = \begin{cases} \frac{1}{4} & , k = 1 \\ p & , k = 2 \\ q & , k = 3 \end{cases} \tag{2}$$

$$\Pr(\{1, 4\}) = p_X(1) + p_X(3) \tag{3}$$

We know

$$p_X(1) + p + q = 1 \tag{4}$$

We can express $\Pr(X \leq 2)$ as:

$$\Pr(X \leq 2) = p_X(1) + p \tag{5}$$

$$\tag{6}$$

We can expres above equations as:

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix} \tag{7}$$

$$p = \frac{1}{2}, q = \frac{1}{4} \tag{8}$$

Finally

$$\Pr(\{1,4\}) = P(\{1\}) + q \tag{9}$$

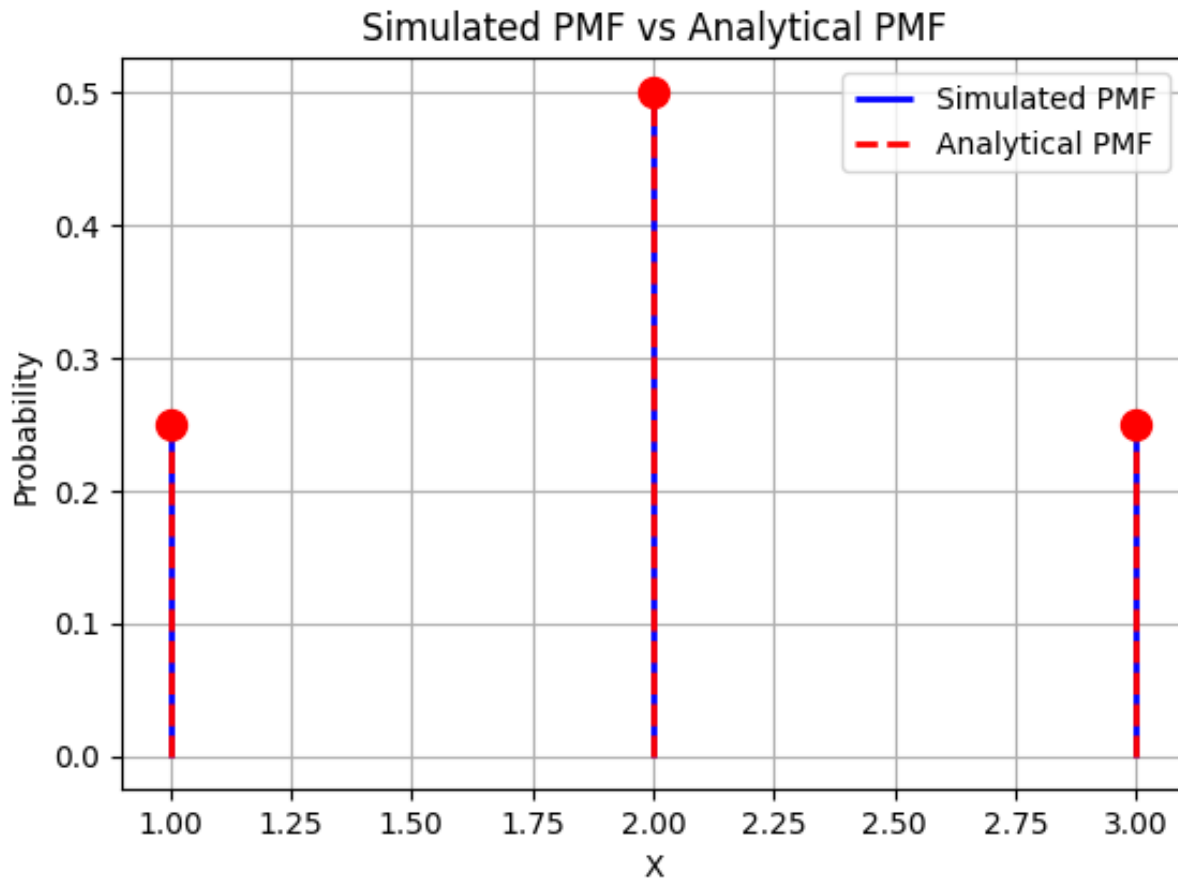$$\Pr(\{1,4\}) = \frac{1}{4} + \frac{1}{4} \tag{10}$$

$$\Pr(\{1,4\}) = 0.5 \tag{11}$$

Fig. 1: Analytical vs simulated