Assignment

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Question: Consider the probability space $(\Omega, \mathcal{G}, P)$, where $\Omega = \{1, 2, 3, 4\}$, $\mathcal{G} = \{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$, $P(\{1\}) = \frac{1}{4}$. Let X be the random variable defined on the above probability space as $X(1) = 1$, $X(2) = X(3) = 2$, $X(4) = 3$. If $P(X \leq 2) = \frac{3}{4}$, then find $P(\{1, 4\})$ (rounded off to two decimal places).
(GATE ST 2023)
Solution:

TABLE 1: Probablity space

Probablity space	Value
Ω	$\{1, 2, 3, 4\}$
$\mathcal{G}$	$\{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$
$P(\{1\})$	$\frac{1}{4}$
$P(X \leq 2)$	$\frac{3}{4}$

TABLE 2: Random variable

$X(\Omega)$	Ω
$\{1\}$	1
$\{2, 3\}$	2
$\{4\}$	3

Pmf is defined as

$$P_X(k) = \begin{cases} P(\{1\}) & , k = 1 \\ P(\{2, 3\}) & , k = 2 \\ P(\{4\}) & , k = 3 \end{cases} \tag{1}$$

$$Pr(\{1, 4\}) = Pr(X = 1 \text{ or } X = 3) \tag{2}$$

We know

$$Pr(X = 1) + Pr(X = 2) + Pr(X = 3) = 1 \tag{3}$$

We can express $Pr(X \leq 2)$as:

$$Pr(X \leq 2) = Pr(X = 1) + Pr(X = 2) \tag{4}$$

$$\frac{3}{4} = Pr(X = 2) + \frac{1}{4} \tag{5}$$

$$Pr(X = 2) = \frac{1}{2} \tag{6}$$

Using

$$Pr(X = 1) + Pr(X = 2) + Pr(X = 3) = 1 \tag{7}$$

$$\frac{1}{4} + \frac{1}{2} + Pr(X = 3) = 1 \tag{8}$$

$$Pr(X = 3) = \frac{1}{4} \tag{9}$$

Finally

$$Pr(\{1, 4\}) = Pr(X = 1) + Pr(X = 3) \tag{10}$$

$$Pr(\{1, 4\}) = \frac{1}{4} + \frac{1}{4} \tag{11}$$

$$Pr(\{1, 4\}) = 0.5 \tag{12}$$