Assignment

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Question: Consider the probability space $(\Omega, \mathcal{G}, P)$, where $\Omega = \{1,2,3,4\}$, $\mathcal{G} = \{\emptyset, \Omega, \{1\}, \{4\}, \{2,3\}, \{1,4\}, \{1,2,3\}, \{2,3,4\}\}$, $P(\{1\}) = \frac{1}{4}$. Let X be the random variable defined on the above probability space as $X(1) = 1$, $X(2) = X(3) = 2$, $X(4) = 3$. If $P(X \leq 2) = \frac{3}{4}$, then find $P(\{1,4\})$ (rounded off to two decimal places).
(GATE ST 2023)
Solution:

TABLE 1: Probablity space

Probablity space	Value
Ω	$\{1,2,3,4\}$
$\mathcal{G}$	$\{\emptyset, \Omega, \{1\}, \{4\}, \{2,3\}, \{1,4\}, \{1,2,3\}, \{2,3,4\}\}$
$P(\{1\})$	$\frac{1}{4}$
$P(X \leq 2)$	$\frac{3}{4}$

TABLE 2: Random variable

$X(\Omega)$	Ω
$\{1\}$	1
$\{2,3\}$	2
$\{4\}$	3

Pmf is defined as

$$p_x(k) = \begin{cases} P(\{1\}) & ,k = 1 \\ P(\{2,3\}) & ,k = 2 \\ P(\{4\}) & ,k = 3 \end{cases} \tag{1}$$

Values of P({2,3}), P({4}) are unknown, so let p, q be their respective values

$$p_X(k) = \begin{cases} \frac{1}{4} & ,k = 1 \\ p & ,k = 2 \\ q & ,k = 3 \end{cases} \tag{2}$$

$$\Pr(\{1,4\}) = p_X(1) + p_X(3) \tag{3}$$

We know

$$p_X(1) + p + q = 1 \tag{4}$$

We can express $\Pr(X \leq 2)$as:

$$\Pr(X \leq 2) = p_X(1) + p \tag{5}$$

(6)

We can expres above equations as:

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix} \tag{7}$$

$$p = \frac{1}{2}, q = \frac{1}{4} \tag{8}$$

Finally

$$\Pr(\{1,4\}) = P(\{1\}) + q \tag{9}$$

$$\Pr(\{1,4\}) = \frac{1}{4} + \frac{1}{4} \tag{10}$$

$$\Pr(\{1,4\}) = 0.5 \tag{11}$$

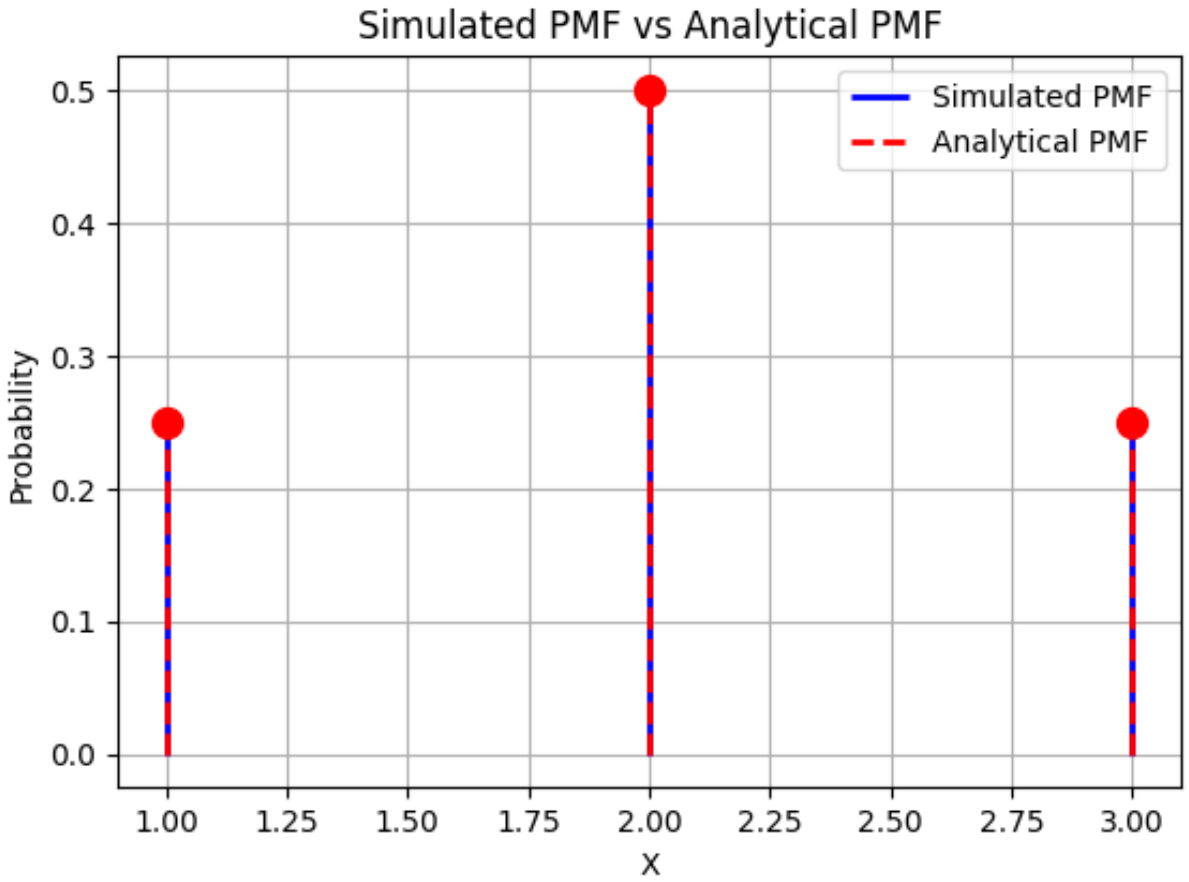

Fig. 1: Analytical vs simulated

Steps for simulating random variable.

1) Define the simulation size for datast (samples).
2) Assign calculated probablity for each probablity space p1, p2, p3, p4.
3) Define Random to generate a random number between 0 and 1.
4) Define the loop such that it generated number 1, 2, 3 for defined probablity space.
5) Store the simulated data in a .dat file.

6) Using matplotlib lib of python generate a V-line graph from the data in .dat file by counting the number of 1, 2, 3 .