Assignment

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Question: Consider the probability space $(\Omega, \mathcal{G}, P)$, where $\Omega = \{1,2,3,4\}$, $\mathcal{G} = \{\emptyset, \Omega, \{1\}, \{4\}, \{2,3\}, \{1,4\}, \{1,2,3\}, \{2,3,4\}\}$, $P(\{1\}) = \frac{1}{4}$. Let X be the random variable defined on the above probability space as $X(1) = 1$, $X(2) = X(3) = 2$, $X(4) = 3$. If $P(X \leq 2) = \frac{3}{4}$, then find $P(\{1,4\})$ (rounded off to two decimal places).
(GATE ST 2023)
Solution:

TABLE 1: Probablity space

Probablity space	Value
Ω	$\{1,2,3,4\}$
$\mathcal{G}$	$\{\emptyset, \Omega, \{1\}, \{4\}, \{2,3\}, \{1,4\}, \{1,2,3\}, \{2,3,4\}\}$
$P(\{1\})$	$\frac{1}{4}$
$P(X \leq 2)$	$\frac{3}{4}$

TABLE 2: Random variable

$X(\Omega)$	Ω
$\{1\}$	1
$\{2,3\}$	2
$\{4\}$	3

Pmf is defined as

$$p_x(k) = \begin{cases} P(\{1\}) & ,k = 1 \\ P(\{2,3\}) & ,k = 2 \\ P(\{4\}) & ,k = 3 \end{cases} \tag{1}$$

Values of P({2,3}), P({4}) are unknown, so let p, q be their respective values

$$p_x(k) = \begin{cases} \frac{1}{4} & ,k = 1 \\ p & ,k = 2 \\ q & ,k = 3 \end{cases} \tag{2}$$

$$\Pr(\{1,4\}) = p_X(1) + p_X(3) \tag{3}$$

We know

$$p_X(1) + p_X(2) + p_X(3) = 1 \tag{4}$$

We can express $\Pr(X \leq 2)$as:

$$\Pr(X \leq 2) = p_X(1) + p_X(2) \tag{5}$$

$$\frac{3}{4} = p + \frac{1}{4} \tag{6}$$

$$p = \frac{1}{2} \tag{7}$$

Using

$$\Pr(\{1\}) + p + q = 1 \tag{8}$$

$$\frac{1}{4} + \frac{1}{2} + q = 1 \tag{9}$$

$$q = \frac{1}{4} \tag{10}$$

Finally

$$\Pr(\{1,4\}) = P(\{1\}) + q \tag{11}$$

$$\Pr(\{1,4\}) = \frac{1}{4} + \frac{1}{4} \tag{12}$$

$$\Pr(\{1,4\}) = 0.5 \tag{13}$$

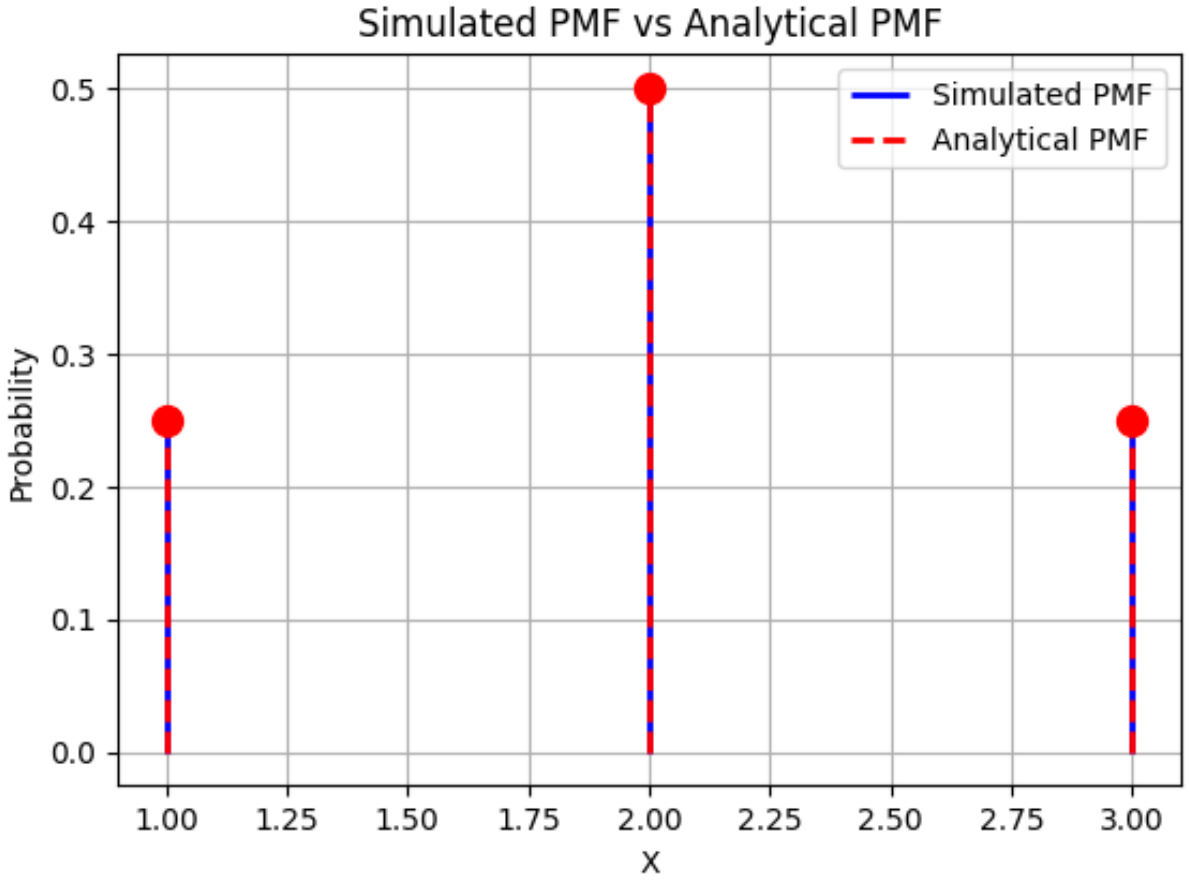

Fig. 1: Analytical vs simulated