

# Assignment

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**Question:** Consider the probability space  $(\Omega, \mathcal{G}, P)$ , where  $\Omega = \{1, 2, 3, 4\}$ ,  $\mathcal{G} = \{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$ ,  $P(\{1\}) = \frac{1}{4}$ . Let  $X$  be the random variable defined on the above probability space as  $X(1) = 1$ ,  $X(2) = X(3) = 2$ ,  $X(4) = 3$ . If  $P(X \leq 2) = \frac{3}{4}$ , then find  $P(\{1, 4\})$  (rounded off to two decimal places).  
(GATE ST 2023)

**Solution:**

TABLE 1: Probability space

Probability space	Value
$\Omega$	$\{1, 2, 3, 4\}$
$\mathcal{G}$	$\{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$
$P(\{1\})$	$\frac{1}{4}$
$P(X \leq 2)$	$\frac{3}{4}$

TABLE 2: Random variable

$X(\Omega)$	$\Omega$
$\{1\}$	1
$\{2, 3\}$	2
$\{4\}$	3

Pmf is defined as

$$P_X(k) = \begin{cases} P(\{1\}) & , k = 1 \\ P(\{2, 3\}) & , k = 2 \\ P(\{4\}) & , k = 3 \end{cases} \quad (1)$$

Values of  $P(\{2, 3\})$ ,  $P(\{4\})$  are unknown, so let  $p$ ,  $q$  be their respective values

$$P_X(k) = \begin{cases} \frac{1}{4} & , k = 1 \\ p & , k = 2 \\ q & , k = 3 \end{cases} \quad (2)$$

$$\Pr(\{1, 4\}) = \Pr(X = 1 \text{ or } X = 3) \quad (3)$$

We know

$$\Pr(X = 1) + \Pr(X = 2) + \Pr(X = 3) = 1 \quad (4)$$

We can express  $\Pr(X \leq 2)$  as:

$$\Pr(X \leq 2) = \Pr(X = 1) + p \quad (5)$$

$$\frac{3}{4} = p + \frac{1}{4} \quad (6)$$

$$p = \frac{1}{2} \quad (7)$$

Using

$$\Pr(X = 1) + p + q = 1 \quad (8)$$

$$\frac{1}{4} + \frac{1}{2} + q = 1 \quad (9)$$

$$q = \frac{1}{4} \quad (10)$$

Finally

$$\Pr(\{1, 4\}) = \Pr(X = 1) + q \quad (11)$$

$$\Pr(\{1, 4\}) = \frac{1}{4} + \frac{1}{4} \quad (12)$$

$$\Pr(\{1, 4\}) = 0.5 \quad (13)$$