

Assignment

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Question: Consider the probability space (Ω, \mathcal{G}, P) , where $\Omega = \{1, 2, 3, 4\}$, $\mathcal{G} = \{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$, $P(\{1\}) = \frac{1}{4}$. Let X be the random variable defined on the above probability space as $X(1) = 1$, $X(2) = X(3) = 2$, $X(4) = 3$. If $P(X \leq 2) = \frac{3}{4}$, then find $P(\{1, 4\})$ (rounded off to two decimal places).
(GATE ST 2023)

Solution:

TABLE 1: Probability space

| Probability space | Value |
|-------------------|---|
| Ω | $\{1, 2, 3, 4\}$ |
| \mathcal{G} | $\{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$ |
| $P(\{1\})$ | $\frac{1}{4}$ |
| $P(X \leq 2)$ | $\frac{3}{4}$ |

TABLE 2: Random variable

| $X(\Omega)$ | Ω |
|-------------|----------|
| $\{1\}$ | 1 |
| $\{2, 3\}$ | 2 |
| $\{4\}$ | 3 |

Pmf is defined as

$$p_x(k) = \begin{cases} P(\{1\}) & , k = 1 \\ P(\{2, 3\}) & , k = 2 \\ P(\{4\}) & , k = 3 \end{cases} \quad (1)$$

Values of $P(\{2, 3\})$, $P(\{4\})$ are unknown, so let p , q be their respective values

$$p_x(k) = \begin{cases} \frac{1}{4} & , k = 1 \\ p & , k = 2 \\ q & , k = 3 \end{cases} \quad (2)$$

$$\Pr(\{1, 4\}) = \Pr(X = 1 \text{ or } X = 3) \quad (3)$$

We know

$$\Pr(\{1\}) + \Pr(\{2, 3\}) + \Pr(\{4\}) = 1 \quad (4)$$

We can express $\Pr(X \leq 2)$ as:

$$\Pr(X \leq 2) = \Pr(\{1\}) + p \quad (5)$$

$$\frac{3}{4} = p + \frac{1}{4} \quad (6)$$

$$p = \frac{1}{2} \quad (7)$$

Using

$$\Pr(\{1\}) + p + q = 1 \quad (8)$$

$$\frac{1}{4} + \frac{1}{2} + q = 1 \quad (9)$$

$$q = \frac{1}{4} \quad (10)$$

Finally

$$\Pr(\{1, 4\}) = P(\{1\}) + q \quad (11)$$

$$\Pr(\{1, 4\}) = \frac{1}{4} + \frac{1}{4} \quad (12)$$

$$\Pr(\{1, 4\}) = 0.5 \quad (13)$$

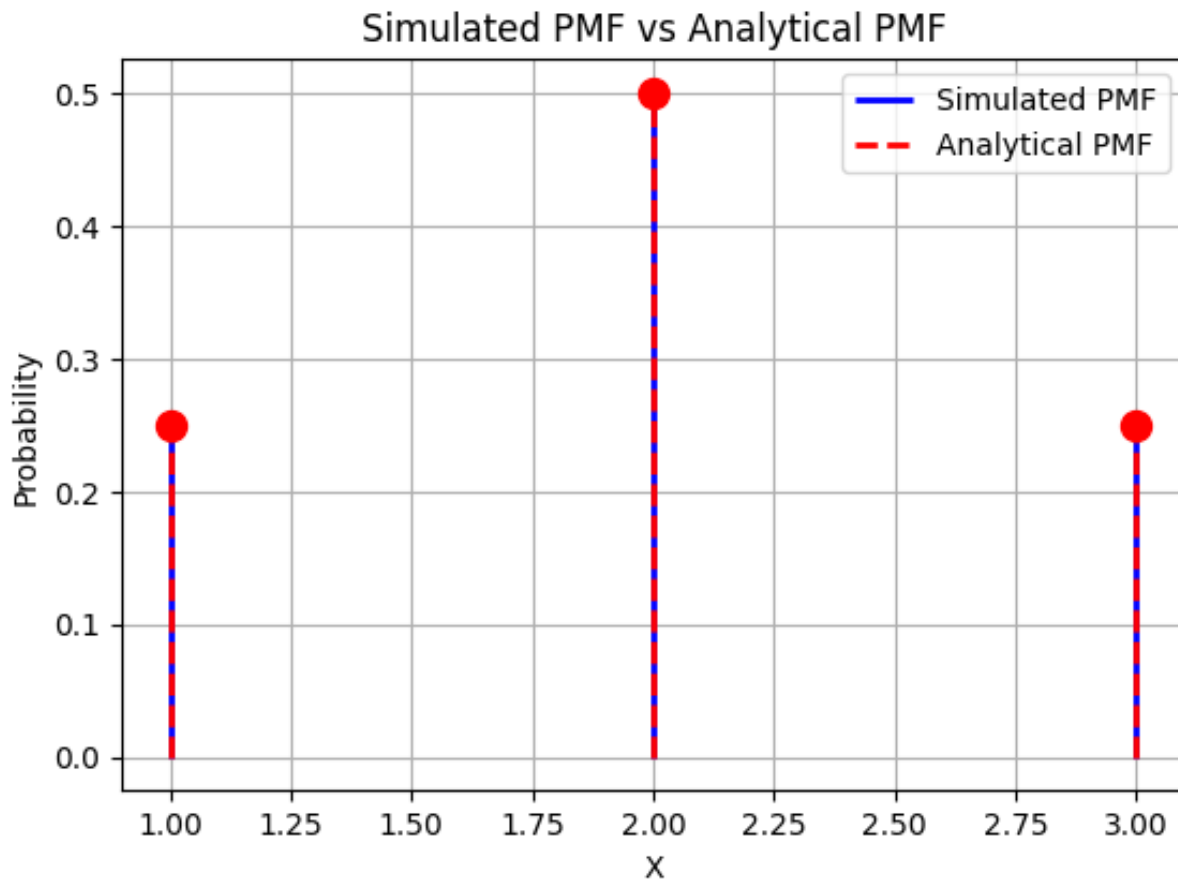


Fig. 1: Analytical vs simulated