

Assignment

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Question: Consider the probability space (Ω, \mathcal{G}, P) , where $\Omega = \{1, 2, 3, 4\}$, $\mathcal{G} = \{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$, $P(\{1\}) = \frac{1}{4}$. Let X be the random variable defined on the above probability space as $X(1) = 1$, $X(2) = X(3) = 2$, $X(4) = 3$. If $P(X \leq 2) = \frac{3}{4}$, then find $P(\{1, 4\})$ (rounded off to two decimal places).
(GATE ST 2023)

Solution:

TABLE 1: Probability space

Probability space	Value
Ω	$\{1, 2, 3, 4\}$
\mathcal{G}	$\{\emptyset, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$
$P(\{1\})$	$\frac{1}{4}$
$P(X \leq 2)$	$\frac{3}{4}$

TABLE 2: Random variable

$X(\Omega)$	Ω
$\{1\}$	1
$\{2, 3\}$	2
$\{4\}$	3

$$Pr(\{1, 4\}) = Pr(X = 1 \text{ or } X = 3) \quad (1)$$

We know

$$Pr(X = 1) + Pr(X = 2) + Pr(X = 3) = 1 \quad (2)$$

We can express $Pr(X \leq 2)$ as:

$$Pr(X \leq 2) = Pr(X = 1) + Pr(X = 2) \quad (3)$$

$$\frac{3}{4} = Pr(X = 2) + \frac{1}{4} \quad (4)$$

$$Pr(X = 2) = \frac{1}{2} \quad (5)$$

Using

$$Pr(X = 1) + Pr(X = 2) + Pr(X = 3) = 1 \quad (6)$$

$$\frac{1}{4} + \frac{1}{2} + Pr(X = 3) = 1 \quad (7)$$

$$Pr(X = 3) = \frac{1}{4} \quad (8)$$

Finally

$$Pr(\{1, 4\}) = Pr(X = 1) + Pr(X = 3) \quad (9)$$

$$Pr(\{1, 4\}) = \frac{1}{4} + \frac{1}{4} \quad (10)$$

$$Pr(\{1, 4\}) = 0.5 \quad (11)$$